

Recitation 19: Type Checking

Type System

Decides whether

e / ill typed
 / well typed
 \ what is the type?

$\text{HasType}(e, t, \text{ctx})$

$\text{ctx} \vdash e : t$

Defn: An expression e is well-typed in context ctx if there is type t with $\text{ctx} \vdash e : t$

Lexer tokens

Parser exprs

Semantic Analysis { Type Checking

Evaluator values

$\text{true} + 3 \rightarrow$
 stack

$\text{env} : \text{Var} \rightarrow \text{Val}$

$\text{ctx} : \text{Var} \rightarrow \text{Types}$

$\langle \text{env}, \text{let } x = 5 \text{ in } x+x \rangle \Rightarrow^{10} v$

if

$\langle \text{env}[x \rightarrow 5], x+x \rangle \rightarrow^{10} v$

$\text{let } f x = x+1$

$f : \text{int} \rightarrow \text{int}$

$\text{ctx} \{ f : \text{int} \rightarrow \text{int} \}$

$f y : \text{int}$

Type System for SimPL

$e ::= x \mid i \mid b \mid e_1 \text{ bop } e_2$
 $\mid \text{if } e_1 \text{ then } e_2 \text{ else } e_3$
 $\mid \text{let } x = e_1 \text{ in } e_2$

$\text{bop} ::= + \mid * \mid \leq$

$\text{ctx} : e \vdash t$

$t ::= \text{int} \mid \text{bool}$

Base Cases

$$\begin{aligned} \text{ctx} &\vdash i : \text{int} \\ \text{ctx} &\vdash b : \text{bool} \\ \{x:t, \dots\} &\vdash x : t \end{aligned}$$

Inductive cases

Binop

$$\begin{aligned} &\text{if } \text{ctx} \vdash e_1 : \text{int} \\ &\text{and } \text{ctx} \vdash e_2 : \text{int} \\ &\text{ctx} \vdash e_1 + e_2 : \text{int} \\ &\text{ctx} \vdash e_1 * e_2 : \text{int} \\ &\text{ctx} \vdash e_1 \leq e_2 : \text{bool} \end{aligned}$$

Let

$$\begin{aligned} &\text{if } \text{ctx} \vdash e_1 : t_1 \\ &\text{and } \text{ctx}[e_1 \rightarrow t_1] \vdash e_2 : t \\ &\text{ctx} \vdash \text{let } x = e_1 \text{ in } e_2 : t \end{aligned}$$

Conditionals

$$\begin{aligned} &\text{if } \text{ctx} \vdash e_1 : \text{bool} \\ &\text{and } \text{ctx} \vdash e_2 : t \\ &\text{and } \text{ctx} \vdash e_3 : t \\ &\text{ctx} \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : t \end{aligned}$$