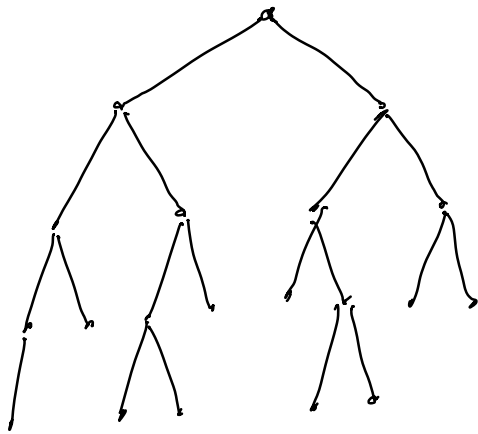
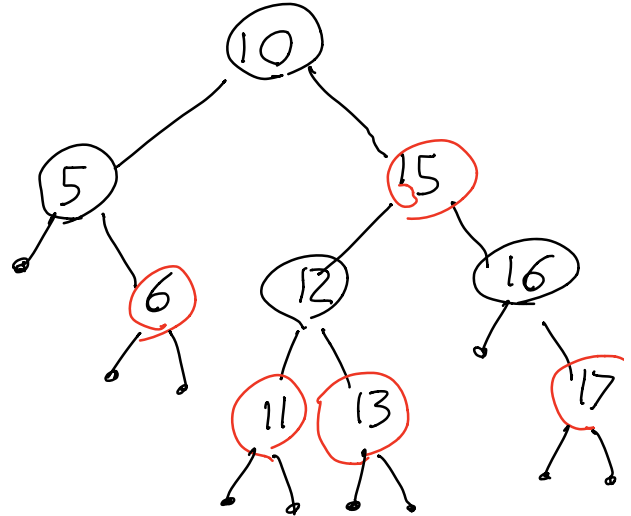


Recitation 15: Balanced Trees



n nodes
 $d \leq \log_2(n) + 1$



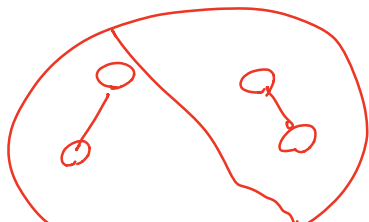
Red-Black trees

Global Invariant:

Every root-leaf path has
same # of black nodes.

Local Invariant:

No two red nodes can have
parent-child relationship.

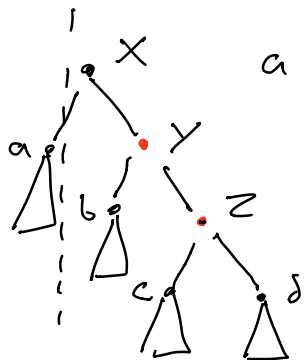


4 black nodes
3 red nodes

n nodes

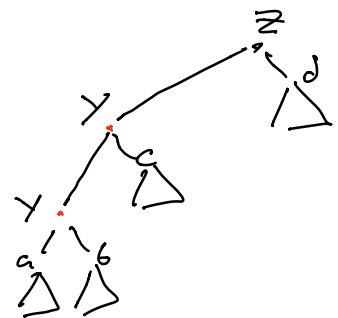
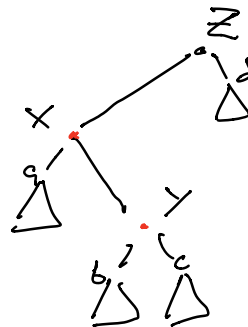
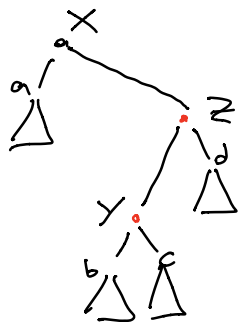
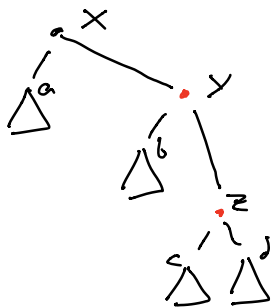
$$h \leq 2 \log_2(n)$$

Balance Operation



$$a < b < c < d$$

4 cases



b black nodes
 $b-1$ red nodes

$$2b-1$$

$$x < y < z$$

$$x < y < z$$