Math 335 Portfolio

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1 Induction Proofs

1.1 Ordinary Induction

Exercise 1. Prove, for all natural numbers n, that

$$\sum_{k=0}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \tag{1}$$

Proof. We prove this by induction on $n \in \mathbb{N}$. In the base case, $n = 0$, and (1) becomes

$$\sum_{k=0}^{n} k = \sum_{k=0}^{0} k = 0 = \frac{0(1)}{2} = \frac{n(n+1)}{2}$$

Now, let $n > 0$ be arbitrary, and assume (1). We show $\sum_{k=0}^{n+1} k = \frac{(n+1)(n+2)}{2}$. To that end note

$$\begin{aligned}
\sum_{k=0}^{n+1} k &= \left(\sum_{k=0}^{n} k\right) + (n+1) && \text{(sum definition)}\\
&= \frac{n(n+1)}{2} + (n+1) && \text{(induction hypothesis)}\\
&= \frac{n(n+1)}{2} + \frac{2n+2}{2} && \text{(common denominator)}\\
&= \frac{n^2+n}{2} + \frac{2n+2}{2} && \text{(distribute)}\\
&= \frac{n^2+3n+2}{2} && \text{(combine like terms)}\\
&= \frac{(n+1)(n+2)}{2} && \text{(factor the numerator)}
\end{aligned}$$

In all cases, (1) is true, so $\forall n \in \mathbb{N}$, $\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$ ∎