SELECTIVE - REDUCED INTEGRATION

Why Selective - reduce integration?

- The previous 2 sections were focused on the case where we use a single quadrature point in the element, to resolve issues associated with locking. We saw that the use of uniform reduced integration involve the need of hourglass control. An interesting question that could arise at this point is: Why not try to use reduced integration only for the part ~~that~~ of the stiffness that locks? This is the basis premise behind selective-reduced integration SRI procedures.

- Our discussion will focus on the context of volumetric locking, where SRI uses a reduced integration rule only for the volumetric part of the stiffness matrix - the part that is associated with volumetric strain

Formulation:

The constitutive law for the three-dimensional linear elasticity and isotropic material can be written ~~in~~:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

We can seperate the material stiffness matrix in 2 parts:

$$\sigma = D\varepsilon = (\bar{D} + \bar{\bar{D}})\varepsilon$$

where:

$$\bar{D} = \begin{bmatrix} 2\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} ; \quad \bar{\bar{D}} = \begin{bmatrix} \lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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- Liked the bulk modulus K_v, Lame's constant λ also tends to infinity for incompressible materials (when $\nu \to 0.5$) while the constant $\mu = G$ remain finite (and retain a "reasonable" value) even for incompressible materials

Stiffness matrix (per unit thickness):

$$K^e = \iint_{\Omega^e} (B^{(e)})^T D^{(e)} B^e \, dV = \iint_{\Omega^e} (B^e)^T (\bar{\bar{D}}^e + \bar{D}^e) B^e \, dV = K^e_\lambda + K^e_\mu$$

where:

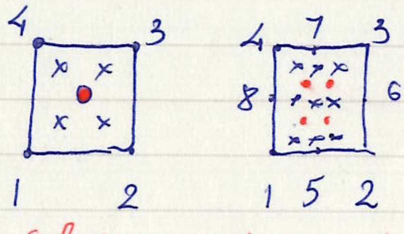
$$K^e_\lambda = \iint_{\Omega_e} (B^e)^T \bar{\bar{D}}^e B^e \, dV \rightsquigarrow \text{volumetric strain} \sim \text{"lock" (overstiff)}$$

$$K^e_\mu = \iint_{\Omega^e} (B^e)^T \bar{D}^e B^e \, dV \rightsquigarrow \text{deviatoric strain}$$

- In SRI procedure, the K^e_λ part of the stiffness matrix is generated using reduced integration (for Q4 elements, using one-point quadrature), and K^e_μ part is intergrated using full integration (Q4 → 4 gauss points)

⇒ { Avoid locking problem
  Retain the correct rank for the element stiffness matrix

Q4 element   Q8 Serendipity Element



{ x : Quadrature point for $K^{(e)}_\lambda$
  o : Quadrature point for K_μ

Selective-reduced integration for Quadrilateral element

Q. Is Cam Clay model

Drawback of ~~Cam Clay~~ SRI

The only problem with the method is that it cannot be easily extended to analyzes involving materials for which the constitutive law cannot be decomposed into a volumetric and a deviatoric part