B-bar METHOD

Why B-bar?

- The selective-reduced integration procedure can help us avoid volumetric locking for incompressible material, without the effect of "hour-glassing"
- While the method is very efficient for isotropic elastic materials, it may not be possible ~~case~~ to extend the SRI to the general case where, for example, the volumetric and deviatoric parts of the constitutive model are coupled
- For this reason, another method, called the B-bar method (Hughes, 1980), is commonly employed ~~to~~ in lieu of SRI
- The B-bar method is equivalent to SRI for isotropic elastic materials, but it can also be used for general linearly elastic or inelastic material
- Like selective reduce integration, the B-bar method works by treating the volumetric and deviatoric parts of the stiffness matrix seperately. ∗
- Instead of seperating the volume~~tric~~ integral into two parts, however, the B-bar method modifies the definition of the strain in the element.
- In this method, shear strain is calculated with full integration as normal FEM; however, volumetric strain is calculated with one order lower than the standard of FEM.

Formulation:

- According to small strain theory, total strain is the summation of shear strain and volumetric strain:

$$\varepsilon_{ij} = e_{ij} + \frac{\varepsilon_{KK}}{\cancel{2}\,3}\delta_{ij} \xrightarrow{} 3D$$

in which e_{ij} is shear strain; $\varepsilon_{KK}\cancel{\delta_{ij}}$: volumetric strain
- We replace this volumetric strain by:

$$w = \frac{1}{V_{el}}\int_{V_{el}} \varepsilon_{KK}\, dV$$

B- bar METHOD

Formulation (continued)

- Total strain is rewritten:

$$\bar{\varepsilon}_{ij} = e_{ij} + \frac{\omega}{2}\delta_{ij}$$

Plain strain → 2 ; 3: 3D

- To implement this to FEM framework, a new $\bar{B}$ matrix is defined so that: $\bar{\varepsilon} = \bar{B}\, u^{el}$

$$\frac{\partial \bar{N}^a}{\partial X_j} = \frac{1}{V^{el}} \int_{V^{el}} \frac{\partial N^a}{\partial X_j}\, dV$$

Why Q4, but $\bar{B}$ N_1, N_2, N_3 only?

$$\bar{B} = \begin{bmatrix} \frac{\partial N_1}{\partial x_1} & 0 & \frac{\partial N_2}{\partial x_1} & 0 & \frac{\partial N_3}{\partial x_1} & 0 & \cdots \\ 0 & \frac{\partial N_1}{\partial x_2} & 0 & \frac{\partial N_2}{\partial x_2} & 0 & \frac{\partial N_3}{\partial x_2} & \cdots \\ \frac{\partial N_1}{\partial x_2} & \frac{\partial N_1}{\partial x_1} & \frac{\partial N_2}{\partial x_2} & \frac{\partial N_2}{\partial x_1} & \frac{\partial N_3}{\partial x_2} & \frac{\partial N_3}{\partial x_2} \end{bmatrix}$$

Plain strain 2D → 2 ; 3D : 3

$$-\frac{1}{2}\begin{bmatrix} \frac{\partial N_1}{\partial x_1} & \frac{\partial N_1}{\partial x_2} & \frac{\partial N_2}{\partial x_1} & \frac{\partial N_2}{\partial x_2} & \frac{\partial N_3}{\partial x_1} & \frac{\partial N_3}{\partial x_2} & \cdots \\ \frac{\partial N_1}{\partial x_1} & \frac{\partial N_1}{\partial x_2} & \frac{\partial N_2}{\partial x_1} & \frac{\partial N_2}{\partial x_2} & \frac{\partial N_3}{\partial x_1} & \frac{\partial N_3}{\partial x_2} & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \frac{\partial \bar{N}_1}{\partial x_1} & \frac{\partial \bar{N}_1}{\partial x_2} & \frac{\partial \bar{N}_2}{\partial x_1} & \frac{\partial \bar{N}_2}{\partial x_2} & \frac{\partial \bar{N}_3}{\partial x_1} & \frac{\partial \bar{N}_3}{\partial x_2} & \cdots \\ \frac{\partial \bar{N}_1}{\partial x_1} & \frac{\partial \bar{N}_1}{\partial x_2} & \frac{\partial \bar{N}_2}{\partial x_1} & \frac{\partial \bar{N}_2}{\partial x_2} & \frac{\partial \bar{N}_3}{\partial x_1} & \frac{\partial \bar{N}_3}{\partial x_2} & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- Finally, stiffness matrix is calculated as follow

$$K^{el} = \int_{V^{el}} \bar{B}^T D\, B\, dV$$

- Notice that ε_v in the element is everywhere equal to its mean value in B-bar method.

- In this study, we used 4-nodes isoparametric element in which ε_v is integrated with only 1 Gauss point, and shear strain is fully integrated with 4-Gauss points.
- In $\bar{B}$ method, the value of $\bar{B}_v$ at any location (ξ, η) in the interior of the element, is set equal to the value of the specific array at the origin of the element $\xi = 0, \eta = 0$