



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/24

Paper 2 Further Pure Mathematics 2

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 (a)** Find the values of k for which the system of equations

$$\begin{aligned}x + 2y + 3z &= 1, \\kx + 5y + 6z &= 2, \\7x + 2ky + 9z &= 3,\end{aligned}$$

does not have a unique solution.

[3]

- (b) Given that $k = 1$, show that the system of equations in part (a) is consistent. Interpret this situation geometrically. [3]





- 2 Find the exact value of $\int_1^{\frac{5}{2}} \frac{1}{\sqrt{x^2 - 2x + 5}} dx$, giving your answer in logarithmic form. [6]





- 3** Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 13e^{3x}$$

given that $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$.

[10]





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- ## 4 A curve has parametric equations

$$x = t^3 - t^2 + t - 1 \quad \text{and} \quad y = te^t.$$

- (a) Show that 1 is the only real value of t for which $x = 0$.

[1]

- (b) Show that $\frac{dy}{dx} = \frac{(t+1)e^t}{3t^2 - 2t + 1}$.

[3]





- (c) Find the Maclaurin's series for y up to and including the term in x^2 .

[6]





- 5 (a)** Use de Moivre's theorem to show that

$$\sin 7\theta = -64 \sin^7 \theta + 112 \sin^5 \theta - 56 \sin^3 \theta + 7 \sin \theta.$$

[5]





(b) Hence find all roots of the equation

$$64x^6 - 112x^4 + 56x^2 - 7 = 0$$

in the form $\sin q\pi$, where q is a rational number.

[3]





- 6** Find the solution of the differential equation

$$x \frac{dy}{dx} - y = 2x^2 \tan^{-1} x$$

for which $y = \frac{1}{2}\pi$ when $x = 1$. Give your answer in the form $y = f(x)$.

[9]





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7 The matrix A is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 7 & 11 \\ 0 & 2 & 5 \\ 0 & 0 & -3 \end{pmatrix}.$$

- (a) Find a matrix P and a diagonal matrix D such that $A^6 = PDP^{-1}$.

[7]





(b) Use the characteristic equation of \mathbf{A} to show that

$$\mathbf{A}^6 = a\mathbf{A}^2 + b\mathbf{A} + c\mathbf{I},$$

where a , b and c are integers to be determined.

[4]





- 8** The curve C has equation $y = \tanh x$ for $x \geq 0$.

- (a) Sketch C and state the equation of the asymptote.

[2]

- (b) By considering a suitable set of N rectangles of unit width, use your sketch to show that

$$\sum_{r=1}^N \tanh r > \ln(\cosh N). \quad [3]$$





(c) The arc of C joining the point where $x = 0$ to the point where $x = \frac{1}{2}\ln 3$ is rotated through one complete revolution about the x -axis. The area of the surface generated is denoted by S .

(i) Use the substitution $u = \sqrt{1 + \operatorname{sech}^4 x}$ to show that

$$S = \pi \int_{\frac{5}{4}}^{\sqrt{2}} \frac{u^2}{u^2 - 1} du. \quad [7]$$





- (ii) Find the exact value of $\pi \int_{\frac{5}{4}}^{\sqrt{2}} \frac{u^2}{u^2 - 1} du$. You need not simplify your answer. [3]





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