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ABSTRACT

In this paper, an approach to obtain an accurate yet simple model for vehicle handling is proposed. The approach involves linearizing a full-car MBD model and dividing the large-order system states into two groups depending on their effect on ride quality and handling performance. Then, the singular perturbation method is used to reduce the model size and to compensate for the steady state relationship between the two state groups. Based on simulation using ADAMS, the results show an accurate matching between the reduced model and the original MBD model. The benefits of the approach are illustrated by comparing step responses with the conventional nominal model. Also, the reduced-order model obtained by the proposed approach resulted in improved performances under sliding mode control even with smaller switching gains than what are required for the conventional bicycle model.

NOTATION

m sprung mass, kg
I_z yaw moment of inertia of vehicle, kg-m^2
a, b distance from vehicle c.g. to front and rear wheel a, b respectively m
C_f, C_r tire cornering stiffness N-m/rad
x_s, y_s longitudinal and lateral displacement of vehicle c.g. [m]
v_x, v_y longitudinal and lateral velocity of vehicle c.g. [m/s]
ϕ_s, p_s roll angle, roll rate of vehicle c.g.
θ_s, q_s pitch angle, pitch rate of vehicle c.g.
ψ_s, r_s yaw angle, yaw rate of vehicle c.g.
z_s vertical displacement of vehicle c.g.
z_{ui} vertical displacements of four wheels ($i = 1,2,3,4$)
τ_1, τ_2 front and rear steering torque
δ_1, δ_2 front and rear steer angle
$\boldsymbol{x}$ state vector of original model
x_H, x_R handling and ride related state vectors
A, B, C system matrices of the large-order system
A_m, B_m system matrices of the reference model

INTRODUCTION

In passive suspension systems, there always exists a “trade-off” between ride quality and handling performance. Various control systems have been developed to address this problem including active suspension, 4WS(Four Wheel Steering), ABS(Anti-lock Brake System), TCS(Traction Control System), ARCS(Active Roll Control System)…etc [2]. As a result, active control of suspension systems has been generally accepted as a means of improving both ride quality and handling performance.

Some active suspension works have dealt with the handling performance based on quarter car suspension systems [3],[4]. In these, tire deflection is considered as a key variable in evaluating the road holding performance. However, tire deflection is not enough for evaluating the whole handling performance because of other important factors in handling such as quickness of steer response, roll angle, yaw rate, and lateral slip angle.

In order to analyze and simulate these factors, various kinds of models have been developed so far. As early as the 1950s, some aerospace engineers began to develop a very simple handling model [1], verifying it with experiment data. It is called a “bicycle model” since it includes a rigid car body with front and rear tires. Recent works including [18] on automated steering have used this model for designing their control systems. This model provides a quick and moderately accurate analysis of vehicle directional motions, but it is too simple to account for important handling characteristics such as roll axis, roll steer, and tire load transfer effect.

More recently, a 3-DOF(degree of freedom) model has been developed to include roll-related characteristics. The basic model structure is the same as the bicycle model, but, with this, suspension geometric characteristics such as roll steer, roll camber, the reduction of the tire lateral force due to the load transfer can be accounted for. One of the key concepts of the model is the roll axis that can be obtained from the roll centers of front and rear suspensions. A different roll axis may change the roll stiffness that results in different handling performances. Using the model, it is possible to simulate the lateral motion with moderate accuracy even when the lateral acceleration is high (e.g., greater than 0.4g).

For both the lateral and the longitudinal dynamic behaviors of a vehicle, a spinning motion of each wheel needs to be modeled. The so-called “17-DOF model” has been developed to accommodate the need. The total degrees of freedom are composed of six sprung mass (car body) motions, four unsprung mass motions, four spinning motions of the wheels, two road wheel steer angles, and the steering wheel angle. Geometric and compliance characteristics (e.g., camber, caster, toe, lateral/longitudinal compliance, torsional stiffness of steering wheel…etc.) are included. This model makes it possible to simulate a four wheel drive system and braking during a turning motion. Although the model provides a more accurate vehicle behavior than the bicycle model, it is difficult and costly to obtain all set of data.

With the advent of computer-aided kinematics/dynamics software (e.g., ADAMS, DADS, …etc.) and improved hardware techniques, it became easier and faster to analyze and synthesis a complex mechanical system for full vehicle modeling. In the multi-body dynamic model, every suspension linkage and compliance element can be included. Once a new suspension system is drawn into the model, it is possible to predict the behavior of a full vehicle as well as the load history of every connecting element. However, such a model is rarely used in designing an active system due to its complexity and computation overhead.

The motivation of this study comes from an effort "bridge" a gap between the simple bicycle model and a multi-body dynamics model. There will always exist a "trade-off" between the simplicity and the accuracy of a model. But, the work shown in this paper proposes a way to obtain both simple and accurate model by taking an advantage of the accuracy of ADAMS model in determining an accurate set of parameter data for the bicycle model. To achieve this, a linearization technique and a model reduction technique are used to obtain the goal. Finally, a four wheel steer system is used to demonstrate the benefits of the process.

The rest of this paper is organized as follows. In section 2, obtaining a linearized model for a full vehicle is described. In section 3, a simple handling model is obtained using a model reduction technique. In section 4, the merits of the resulting model are illustrated in design of a four wheel steering (4WS) system. Finally, the conclusion section reviews and recommends further topics.

A LINEARIZED FULL-CAR MODEL

FULL VEHICLE MODELS FOR HANDLING SIMULATION - Figure 1 shows a simple bicycle model which has been used frequently in the field of vehicle directional control studies. The simple model is composed of the car body and two tires that are to be steered. The model is simple yet captures important characteristics of a vehicle including two major behaviors (yaw motion and lateral slip). When the roll effects are included, it becomes a 3-DOF model. The key to modeling the bicycle model is determining the vehicle parameters such as mass, moment of inertia, and tire cornering stiffness. In most literatures, these parameters are assumed to be known. Mostly, they are obtained from component data provided by part vendors. A complex MBD model used to analyze a full car handling performance is not shown. But, in the model, each suspension link is modeled as a rigid body, and every connecting bushing is modeled as a kinematic joint (e.g., revolutional joint, universal joint, spherical joint, …etc.). For the simulation of 4WS, the rear steering system is modeled the same as the front system.

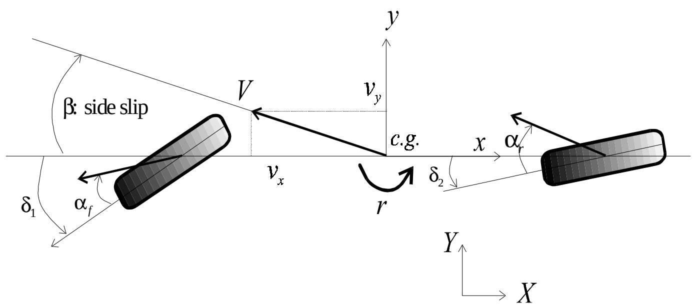


Figure 1 Simple bicycle model for a vehicle handling simulation

In this study, the limitation of the "*nominal simple model*" (a model that is based on component data) is first investigated. It is obvious that the vehicle parameters should include the characteristics of a suspension system. The effect of suspension structure for a quarter car model is shown in the previous study[5]. Here, the effect of suspension structure for a full car handling model is investigated. Obtaining an accurate simple handling model is also proposed.

LINEARIZATION OF A COMPLEX ADAMS MODEL - A complex ADAMS model in Figure 1a can be described by a set of nonlinear equations of motion:

$$\dot{x} = f(x, \tau_1, \tau_2) \tag{1a}$$

$$y = g(x) \tag{1b}$$

where, x, τ_1, and τ_2 are system state vector and front/rear steering torque inputs respectively. Using ADAMS, it is not possible to produce the linearized equation directly with a displacement(e.g., steering wheel angle) as a control input. The steering wheel torque is chosen as a control variable instead. After some modifications of the equation of motion, it is possible to change the control variable to the steering wheel angle.

Linearizing the equations around an equilibrium position (when it is running straight with constant speed), a linearized equation of motion can be obtained as follows.

$$\dot{x} = Ax + B_1\tau_1 + B_2\tau_2 \tag{2a}$$

$$y = Cx \tag{2b}$$

The system output y is defined as a 2×1 vector of the lateral velocity(v_y), and the yaw rate(r). For the system in Figure 1a, the d.o.f. (degree of freedom) of the system is 12. The number of states of the linearized system is 24 which include displacements of generalized coordinates and their velocity terms.

The linearized equation of motion can be expressed as follows:

$$\begin{bmatrix} \dot{x}_H \\ \dot{x}_R \end{bmatrix} = A \begin{bmatrix} x_H \\ x_R \end{bmatrix} + B_1 \delta_1 + B_2 \delta_2 , \tag{3}$$

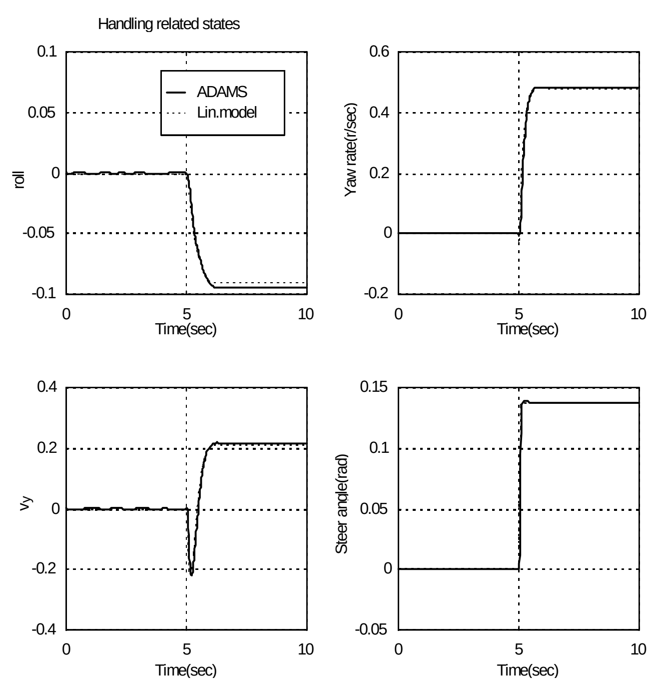
where, $A \in \Re^{24\times24}$, $B_1, B_2 \in \Re^{24\times1}$, $x = [x_H^T \quad x_R^T]^T$,

$$x_H = \begin{bmatrix} x_s & y_s & \psi_s & v_x & v_y & r_s \end{bmatrix}^T ,$$

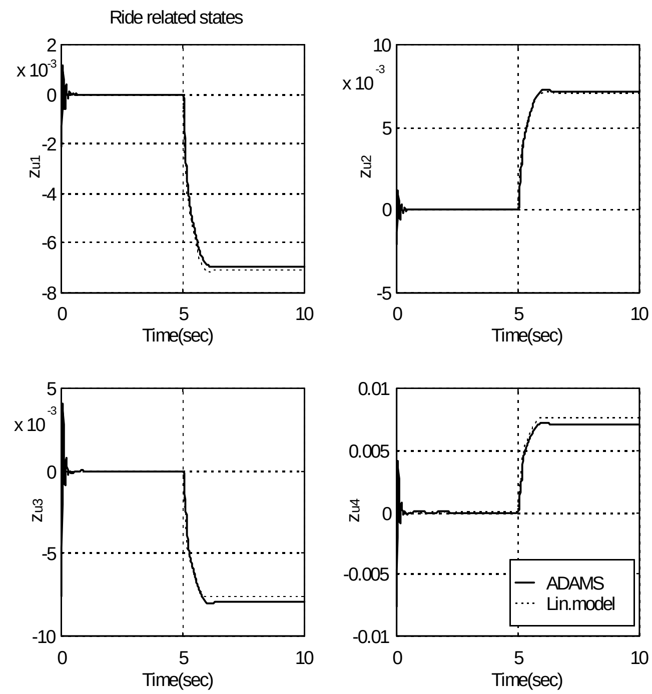
$$x_R = [\phi_s \quad \theta_s \quad z_s \quad z_{u1} \quad z_{u2} \quad z_{u3} \quad z_{u4} \quad p_s \quad q_s \quad \dot{z}_s \quad \dot{z}_{u1} \quad \dot{z}_{u2} \quad \dot{z}_{u3} \quad \dot{z}_{u4}]^T$$

Subscripts "*s*" and "*u*" stand for the sprung mass and the unsprung mass respectively. Numbers are used to denote four sides of a vehicle (i.e., 1: front right, 2: front left, 3: rear right, and 4: rear left). States are divided into two groups: one is "handling related" states and the other is "ride related" states. In general, these two groups are believed to be weakly coupled in a physical sense. Therefore this could be a good example to use a model reduction technique.

In the following section, the linearized equation will be reduced to produce a small-sized equation to be used in designing a control system. Here, the comparison between the original model and the large-order linear model is shown in Figure 2. A step steer input is given to systems and responses of several important state variables are plotted. The results shows that the large-order linear system is almost same as the original ADAMS model.



(a) Handling related states(yaw rate, lateral velocity, roll, and steer angle)



(b) Ride quality related states(vertical displacements of unsprung masses)

Figure 2. The comparison between the original ADAMS model and the linearized model

MODEL REDUCTION OF A FULL CAR MODEL

From a perspective of controller design, the large-order linearized model of Equation (2) has little practical use unless it is reduced to a manageable size without sacrificing accuracy.

Model order reduction has been a major topic in systems theory for decades. Various kinds of techniques have been developed since it first appeared in late 1960s[6-8],[10]. Pade approximation, modal approximation, and continued fraction expansions were the typical methodologies in the beginning. Singular perturbation was also applied to model reduction [9]. Singular perturbation is very similar to the dominant model technique in the sense of separating the modes based on their "fastness" [9].

In the previous study[19], model reduction techniques have been applied to obtain an accurate set of equivalent parameters of a two mass model for the quarter car suspension system. Based on the result, reduced-order models obtained using the model reduction techniques can represent the system more accurately compared to the nominal model. In this section, similar process is applied to a full vehicle system to obtain an accurate simple model for the handling simulation.

Among the mentioned techniques, singular perturbation is used in this study. Since the choice of remaining states are not based on the size of eigenvalues, the dominant mode technique is not used. Two different reduced-order models are obtained with/without the compensation of the steady state relationship between two sets of states(remaining states and eliminated states).

The ADAMS model used in this study does not include the bushing compliance effects for convenience. Therefore removing a high frequency states is not necessary using dominant mode techniques. In case of including all the bushing compliances, it is recommended to apply those techniques before applying the process in this section.

The large-order linear equation can be rewritten by separating two states groups as follows.

$$\begin{bmatrix} \dot{x}_H \\ \dot{x}_R \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_H \\ x_R \end{bmatrix} + \begin{bmatrix} B_{H1} \\ B_{R1} \end{bmatrix} \delta_1 + \begin{bmatrix} B_{H2} \\ B_{R2} \end{bmatrix} \delta_2 \tag{4}$$

or

$$\dot{x}_H = A_{11} x_H + A_{12} x_R + B_{H1}\delta_1 + B_{H2}\delta_2 \tag{5a}$$

$$\dot{x}_R = A_{21} x_H + A_{22} x_R + B_{R1}\delta_1 + B_{R2}\delta_2 \,. \tag{5b}$$

Since the goal is obtaining a reduced-order handling model, the first set of differential equations is to remain. The second set of equations could be used to compensate the static relation between two state vectors, x_H and x_R.

REDUCED-ORDER MODEL WITHOUT COMPENSATING THE STEADY STATE RELATIONSHIP ('DEL-MODEL') - Without compensating the steady state relationship, a reduced-order model can be obtained as follows. The following reduced-order model is obtained by deleting the second set of equations.

$$\dot{x}_H = A_{11} x_H + B_{H1}\delta_1 + B_{H2}\delta_2 \tag{6}$$

or

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{r} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 7.6798\text{e-}05 & 1 & -7.6001\text{e-}06 & 0 \\ 0 & 0 & -1.2000\text{e+}01 & 7.6723\text{e-}06 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 3.1569\text{e-}04 & 5.1906\text{e-}05 & 3.1459\text{e-}04 & 2.6331\text{e-}03 \\ 0 & 0 & -1.5660\text{e-}05 & -2.2794\text{e-}05 & -4.7534\text{e+}00 & 1.0971\text{e+}01 \\ 0 & 0 & 4.1321\text{e-}06 & -3.1525\text{e-}05 & -5.4860\text{e-}01 & -5.0047\text{e+}00 \end{bmatrix} \begin{Bmatrix} x \\ y \\ \psi \\ v_x \\ v_y \\ r \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 6.3606\text{e-}08 \\ 0 & -2.6348\text{e-}08 \\ -8.8933\text{e-}03 & 1.3781\text{e-}02 \\ -2.9095\text{e+}01 & -2.7294\text{e+}01 \\ 1.8351\text{e+}01 & -2.4851\text{e+}01 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \tag{7}$$

Notice that the relationships between the horizontal coordinates(x, y) and yaw angle(ψ) comes from the trigonometric relationships based on the linear transformation. The following equations should be used to find the vehicle trajectory (x and y) by integrating velocity components .

$$\dot{x} = v_x \cos(\psi) - v_y \sin(\psi) \tag{8a}$$

$$\dot{y} = v_x \sin(\psi) + v_y \cos(\psi) \,. \tag{8b}$$

Selecting only two states from the above equations, the reduced order model for the vehicle handling simulation can be obtained. That is,

$$\begin{Bmatrix} \dot{v}_y \\ \dot{r} \end{Bmatrix} = \begin{bmatrix} -4.7534e+00 & 1.0971e+01 \\ -5.4860e-01 & -5.0047e+00 \end{bmatrix} \begin{Bmatrix} v_y \\ r \end{Bmatrix} + \begin{bmatrix} -2.9095e+01 \\ 1.8351e+01 \end{bmatrix} \delta_1 + \begin{bmatrix} -2.7294e+01 \\ -2.4851e+01 \end{bmatrix} \delta_2 \tag{9}$$

REDUCED-ORDER MODEL WITH COMPENSATING THE STATIC RELATIONSHIP('MDC-MODEL') - Using the singular perturbation technique, a reduced-order differential equation can be obtained as follows:

$$\dot{x}_H = A_H x_H + B_1\delta_1 + B_2\delta_2 \,, \tag{10}$$

where,

$$A_H = A_{11} - A_{12}A_{22}^{-1}A_{21},\ B_1 = A_{12}A_{22}^{-1}B_{R1} + B_{H1},\ B_2 = A_{12}A_{22}^{-1}B_{R2} + B_{H2}$$

or

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{v}_x \\ \dot{v}_y \\ \dot{r} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 6.2600\text{e-}02 & 1 & -1.0597\text{e-}02 & 1.0063\text{e-}06 \\ 0 & 0 & -1.2000\text{e+}01 & 1.0599\text{e-}02 & 1 & -9.3568\text{e-}05 \\ 0 & 0 & 0 & 0 & 2.3644\text{e-}08 & 1 \\ 0 & 0 & 2.2967\text{e-}02 & 2.7389\text{e-}06 & 3.0287\text{e-}04 & 6.7259\text{e-}02 \\ 0 & 0 & -3.7620\text{e-}05 & -2.7287\text{e-}02 & -4.9880\text{e+}00 & 1.0916\text{e+}01 \\ 0 & 0 & -1.5325\text{e-}04 & -5.1963\text{e-}03 & -5.7443\text{e-}01 & -5.0121\text{e+}00 \end{bmatrix} \begin{Bmatrix} x \\ y \\ \psi \\ v_x \\ v_y \\ r \end{Bmatrix} + \begin{bmatrix} 2.4097\text{e-}05 & 2.5419\text{e-}05 \\ -2.2984\text{e-}03 & -2.3449\text{e-}03 \\ 1.4269\text{e-}07 & 1.4524\text{e-}07 \\ -9.5733\text{e-}03 & 1.3211\text{e-}02 \\ -3.0785\text{e+}01 & -2.9073\text{e+}01 \\ 1.8175\text{e+}01 & -2.5074\text{e+}01 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \tag{11}$$

Therefore, the desired reduced order system for vehicle handling simualtion is as follows:

$$\begin{Bmatrix} \dot{v}_y \\ \dot{r} \end{Bmatrix} = \begin{bmatrix} -4.9880e+00 & 1.0916e+01 \\ -5.7443e-01 & -5.0121e+00 \end{bmatrix} \begin{Bmatrix} v_y \\ r \end{Bmatrix} + \begin{bmatrix} -3.0785e+01 \\ 1.8175e+01 \end{bmatrix} \delta_1 + \begin{bmatrix} -2.9073e+01 \\ -2.5074e+01 \end{bmatrix} \delta_2 \tag{12}$$

NOMINAL MODEL BASED ON COMPONENT DATA(NOMINAL MODEL) - In general, a nominal model is obtained using a set of component data. From the bicycle model shown the Figure 1b, the following equation of motion can be derived. Table 1 shows the component data used in this study.

$$\begin{Bmatrix} \dot{v}_y \\ \dot{r} \end{Bmatrix} = \frac{1}{v_x} \begin{bmatrix} \dfrac{C_f + C_r}{m} & -v_x^2 + \dfrac{-aC_f + bC_r}{m} \\ \dfrac{-aC_f + bC_r}{I_z} & \dfrac{a^2C_f + b^2C_r}{I_z} \end{bmatrix} \begin{Bmatrix} v_y \\ r \end{Bmatrix} + \begin{bmatrix} -\dfrac{C_f}{m} & -\dfrac{C_r}{m} \\ \dfrac{aC_f}{I_z} & -\dfrac{bC_r}{I_z} \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} , \tag{13}$$

or

$$\begin{Bmatrix} \dot{v}_y \\ \dot{r} \end{Bmatrix} = \begin{bmatrix} -5.8685e+00 & 1.0498e+01 \\ -8.5333\text{e-}01 & -6.2935\text{e+}00 \end{bmatrix} \begin{Bmatrix} v_y \\ r \end{Bmatrix} + \begin{bmatrix} -3.5211e+01 & -3.5211e+01 \\ 2.1880e+01 & -3.2120e+01 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \tag{14}$$

Table 1 Vehicle parameter set used to obtain a nominal model

C_f, C_r	20,000 [Nm/rad]	a	1.094 m
m	568 kg	b	1.606 m
I_z	1,000 $kg \cdot m^2$	v_x	-12 m/sec

COMPARISON OF STEP RESPONSES (NOMINAL MODEL AND REDUCED-ORDER MODELS) - Figure 3 shows the comparison of step responses for three kinds of simple models(two reduced-order models and nominal model) with the original ADAMS model. A step steer is given to the front steering (where, $\delta_1 = 0.1333$ [rad]).

It can be seen from figures that the reduced-order model using the singular perturbation('mdc'-model) shows the most accurate response. The differences of two reduced-order models ('mdc'-model and 'del'-model) reflect the effect of steady state compensation. On the other hand, a huge discrepancy exists in the steady state of the lateral velocity (v_y) between the nominal model and the ADAMS model. The steady state values of the lateral velocity is directly related to the "side slip angle" or "heading angle" of the car body (shown in Fiugre 1b). It plays an important role in the subjective evaluation of vehicle handling performance.

The differences are little in the yaw rate responses. This reveals that the tire cornering stiffness is the most important factor in anticipating the yaw rate response. It can be inferred from the fact that the same tire data are used in both the complex model and the nominal model. However, there would be some discrepancies in yaw rate when the original ADAMS includes compliance(e.g, bushing) effects in the lateral direction.

The result clearly shows the benefits of model reduction and the limitation of the nominal model in the process of obtaining an accurate simple handling model.

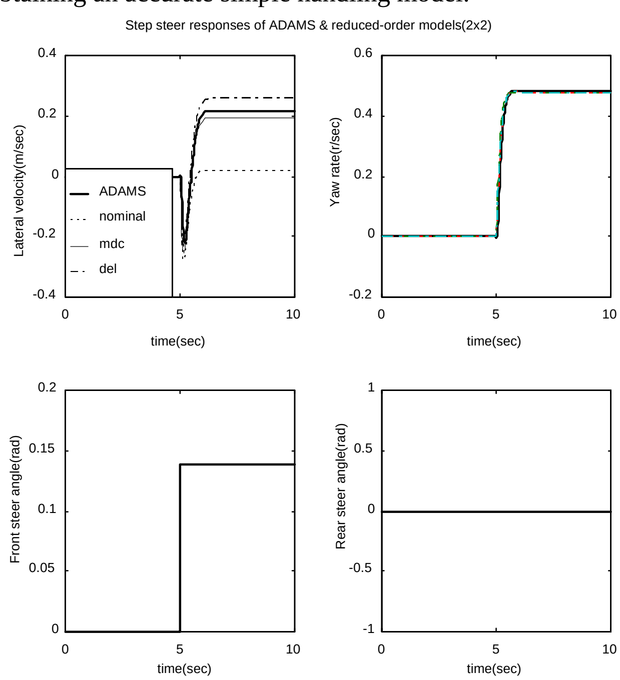


Figure 3 The comparison of step responses for three different models(nominal, mdc-reduced, del-reduced) with ADAMS model.

DESIGN OF 4WS SYSTEM USING A REDUCED VEHICLE MODEL

In this section, advantages of using the reduced-order model are examined in the process of designing a 4WS(Four Wheel Steering) system.

In a conventional front wheel steering system, the rear wheel generates cornering force from the side slip angle due to the vehicle dynamic attitude. Only the front wheel is actively involved in controlling the lateral motion of the vehicle. The basic idea of the 4WS system is to improve the lateral response of the vehicle by steering both the front and real wheels. The rear wheel steering control mechanism is a secondary controller in the 4WS vehicle system while the human driver acts as the main controller.

Various kinds of 4WS systems have been proposed and studied in recent years. Some of them have appeared in the market already. One of the first types of the 4WS vehicle was suggested by Sano et. al.[11], where the rear-to-front steering ratio is pre-determined with respect to the vehicle speed. In the low speed range, rear wheels are steered in the direction opposite to the front wheels. This is to make the turning radius of the vehicle smaller. In the high speed range, the ratio is determined by minimizing the steady-state side slip angle of the vehicle in the high speed range. A mechanical link is used to determine the rear wheel steering angle only by the front steering angle. It utilizes the fact that the driver corrects the steering angle in smaller magnitude as the forward speed increase.

Adaptive techniques(e.g., self-tuning control, model reference control) have been applied to 4WS systems. In those systems, the vehicle is controlled to follow a desirable reference model to the driver's steering wheel input[12-14]. Nonlinear control schemes(e.g., sliding mode control, neural network control) also have been used to solve the robustness issues[15],[16]. Since the main focus of this study is to show the benefits of the reduced-order model, one of the mentioned control schemes(sliding mode control) is used in designing a 4WS system.

The design of sliding mode controller can be classified to two steps. The first step of sliding mode control is to choose a switching surface. Then a control law is determined to guarantee that all the states outside the surface be attracted toward it. Once the states falls into the surface ("sliding mode"), the system dynamics is governed by the equations that define the surface. This is so called "invariance property", which provides the control scheme with the robustness in the presence of system uncertainties. In 4WS systems, side wind disturbances, inaccurate suspension parameters are examples of system uncertainties.

In this study, a switching function is chosen as an error function of responses between the system and a reference model. A reference model should have a good handling performance with a shorter rise time and a shorter settling time. In order to realize the reference model, a “sports car like” model (with high tire cornering stiffness and small yaw moment of inertia) is chosen.

SYNTHESIS OF SLIDING MODE 4WS SYSTEM - The equations of motion for the system and the reference model are as follows:

$$\dot{x} = Ax + B_1\delta_1 + B_2\delta_2 \tag{15}$$

$$\dot{x}_m = A_m x_m + B_m\delta_1 \tag{16}$$

where, δ_1 is steer angle given by the driver and δ_2 is the

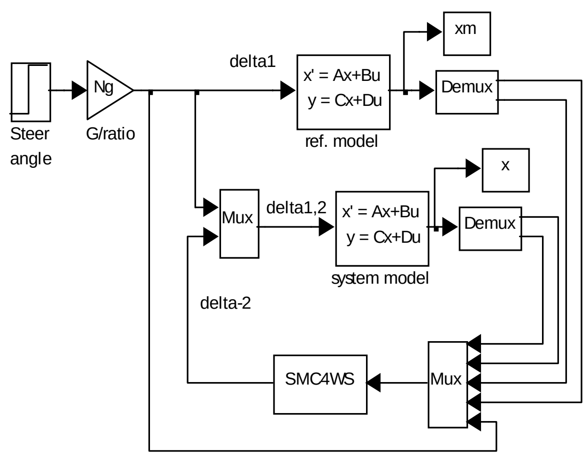


Figure 4 Schematic diagram of sliding mode 4WS system

Define the switching surface in terms of state error vectors, $S \equiv Ge$, where, $e = x_m - x \in \Re^{2\times 1}$, and $G = [\lambda_1 \quad \lambda_2]$. Using the equivalent control method, the following control law can be obtained. That is,

$$\hat{\delta}_2 = (GB_2)^{-1} G[A_m x_m - Ax + (B_m - B_1)\delta_1]. \tag{17}$$

The equivalent control law works fine for a system with no uncertainty. In real situation, however, there always exists uncertainties (parameter uncertainty or structural uncertainty). In order to compensate the following form of control law is usually used with the equivalent control [17].

$$\delta_2 = \hat{\delta}_2 - K\,\mathrm{sgn}(S) \tag{18}$$

where, sgn is the sign function and K is a switching gain.

In general, the choice of switching gain depends on the boundary of uncertainties. If the gain increases, the effect of uncertainties decreases. However, the gain can not be increased with certain amount since there is a hardware limitation in real situation. Therefore, an accurate model is crucial in order to design a sliding mode controller with small switching gain. In this section, this fact is shown by comparing two different models(nominal model and accurate reduced-order model).

Using the scheme, two different sliding mode control laws can be obtained as follows depending on the estimation of the plant(i.e., nominal model and reduced-order model).

(1) nominal model based sliding mode 4WS system

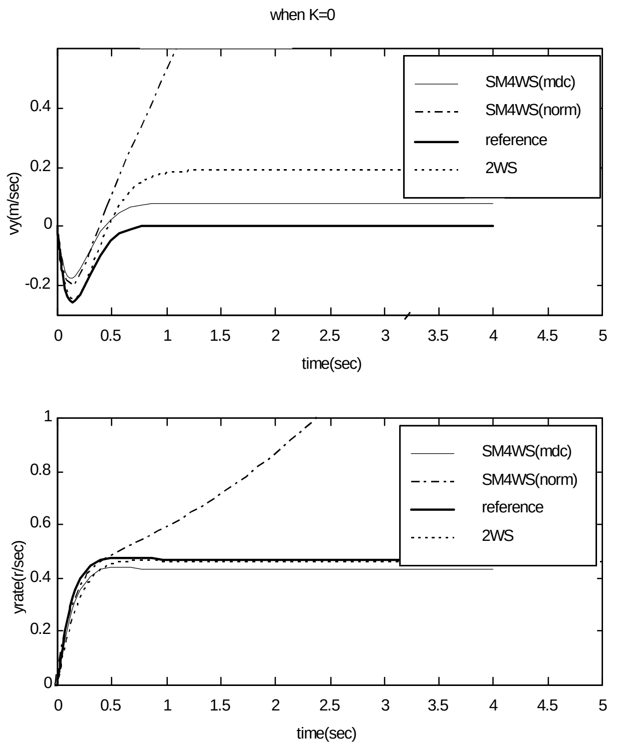
$$\delta_2 = 0.0819v_{ym} + 0.0483r_m - 0.0762v_y - 0.0210r - 0.0876\delta_1 - K\,\mathrm{sgn}(S) \tag{20}$$

(2) reduced-order model based sliding mode 4WS system

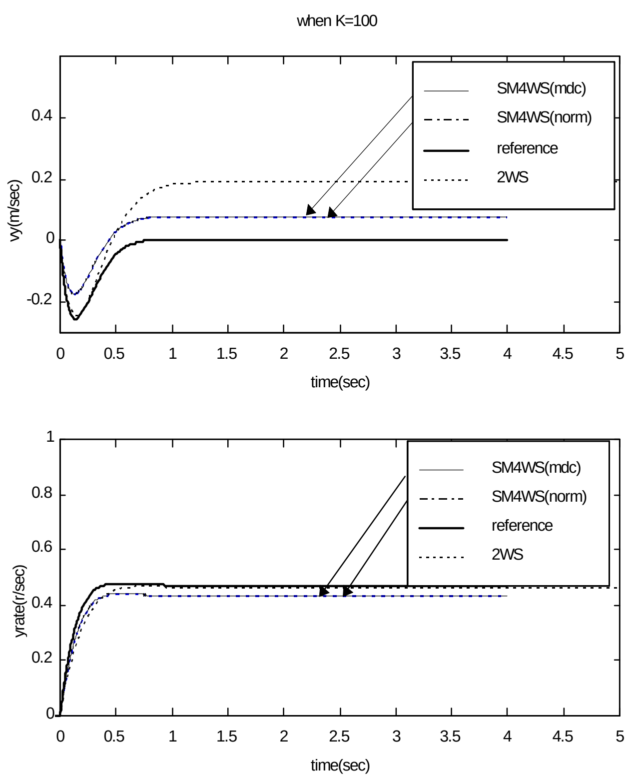
$$\delta_2 = 0.1028v_{ym} + 0.0606r_m - 0.0775v_y + 0.0113r - 0.1477\delta_1 - K\,\mathrm{sgn}(S) \tag{21}$$

SIMULATION RESULTS FOR TWO SLIDING MODE 4WS SYSTEMS - Figure 5 shows the J-turn simulation result in which two different switching gains(K=0, 100) are used to show the effect of the uncertainty. Notice that the system becomes unstable for the nominal model based SM4WS system when the switching gain is small. This is the situation that the switching gain is too small to overcome the system uncertainty. However, both SM4WS systems become close to the reference model when K is increased. The result clearly shows the benefits of using accurate model over the nominal model. With a small switching gain it is possible to obtain the desired handling performance using the reduced-order model. On the other hand, the nominal model requires a bigger switching gain. In real situations, the switching gain can not be increased infinitely due to hard limitations.

It should be mentioned here that the proposed control law is a scalar function. This means that two different variables (v_y and r) are controlled with one control input(δ_2). Therefore, there is certain limitation of the performance although sliding mode 4WS is better than 2WS system. In order to obtain a better performance, both steer angles are suggested to be controlled, which becomes so called “all wheel steer” system.



(5a) J-turn simulation for different 4WS systems (when K is small)



(5b) J-turn simulation for different 4WS systems (when K is big)

Figure 5 J-turn simulation for different 4WS systems with reference model and 2WS system,

where, SM4WS-mdc; reduced-order model based SM, SM4WS-norm; nominal model based SM.

CONCLUSIONS

In this paper, an accurate simple model for vehicle handling simulation is obtained. Linearization and model reduction techniques are used to find the simple bicycle model from a complex full vehicle system. First, a large-order model for a full vehicle system is obtained using the linearization from a ADAMS model. Second, the model is divided into two groups: ride motions and handling motions. Among the states, the handling related states are chosen using a model reduction technique (singular perturbation method). Then a simple bicycle model is obtained for the handling simulation and synthesis.

The reduced-order handling model results in very accurate responses compared to the nominal model (based on component data). On the other hand, the nominal model shows discrepancy with the original ADAMS model, specifically in the lateral velocity response. The steady state of the lateral velocity is one of important factors which give effects to the driver (e.g., heading angle). Using the process in the study, it becomes possible to obtain a both accurate and simple model for handing analysis/synthesis.

Finally, the model has been used in designing a four wheel steering system in order to demonstrate the benefits of the accurate model. In general, an accurate model reduces the tuning process when the control system is applied back to the original system. In this study, it is shown that the accurate simple model results in a small switching gain of sliding mode four wheel steering system. The gain can not be made arbitrarily large when it is implemented. Therefore, it is clear that obtaining such an accurate handling model becomes crucial for designing vehicle control systems.

The ADAMS full vehicle model in this study does not include compliance effects(e.g. bushing). Therefore, it is recommended that all kinematic joints be replaced with bushing. Then it may possible to obtain a simple model with increased accuracy. Also, a more complex tire force function should be included (camber thrust, load transfer, …etc.) in order to compensate the effect of roll and other nonlinear effects.

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