(T1)

$\xi \sim R[0, \Theta], \quad \Theta > 0$

вер. модель $\Theta \in (0, +\infty)$

$\tilde{\Theta}_1 = 2\overline{X} = 2 \cdot \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$

$\tilde{\Theta}_2 = X_{min}$

$\tilde{\Theta}_3 = X_{max}$

$\tilde{\Theta}_4 = x_1 + \frac{1}{(n-1)} \sum_{i=2}^{n} x_i$

$M\xi = \int_{-\infty}^{a} x\, df(x, \Theta) = \int_{0}^{\Theta} x \cdot \frac{1}{\Theta} dx = \frac{\Theta}{2}$

$M\xi^2 = \int_{0}^{\Theta} x^2 \frac{1}{\Theta} dx = \frac{\Theta^2}{3}$

$D\xi = \frac{\Theta^2}{3} - \left(\frac{\Theta}{2}\right)^2 = \frac{\Theta^2}{12}$

- несмещённость: $\forall \Theta \in \Theta \quad M\tilde{\Theta} = \Theta$
- состоятельность: $\forall \Theta \in \Theta \quad \forall \varepsilon > 0 \quad P(|\tilde{\Theta} - \Theta| \geq \varepsilon) \xrightarrow[n \to \infty]{} 0$

$\tilde{\Theta}_1: \forall \Theta > 0 \quad M\tilde{\Theta}_1 = M\left(2 \cdot \frac{1}{n} \sum_{i=1}^{n} x_i\right) =$

$= \frac{2}{n} \sum_{i=1}^{n} M x_i = \frac{2}{n} \cdot n M\xi = \Theta$ несмещена

$D\tilde{\Theta}_1 = D\left(2 \cdot \frac{1}{n} \cdot \sum_{i=1}^{n} x_i\right) = \frac{4}{n^2} \sum_{i=1}^{n} D x_i = \frac{4}{n^2} \cdot n D\xi =$

$= \frac{\Theta^2}{3n} \xrightarrow[n \to \infty]{} 0 \quad \forall \Theta > 0 \Rightarrow$ состоятельна по дост. усл.

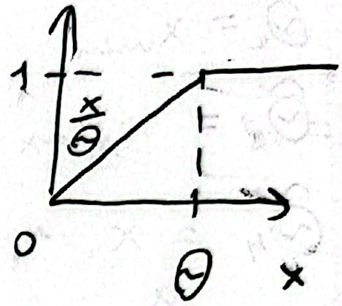
$\tilde{\Theta}_2 = X_{min}$

$\forall \Theta > 0 \quad M\tilde{\Theta}_2 = M x_{min}$

$\xi_{min} \sim 1 - (1 - F(x))^n, \quad \varphi(x) = (1 - F(x))^n$

$f(x) = \varphi'(x) = n(1 - F(x))^{n-1} F'(x) = f(x)$

$= n\left(1 - \frac{x}{\Theta}\right)^{n-1} \cdot \frac{1}{\Theta} \{(0, \Theta)\}$



$M x_{min} = \int_0^{\Theta} x n \left(1 - \frac{x}{\Theta}\right)^{n-1} \cdot \frac{1}{\Theta} \, dx =$

$= \left\{ \begin{array}{l} t = 1 - \frac{x}{\Theta} \\ x = \Theta(1 - t) \end{array} \right\} = -\int_1^0 n(1-t) t^{n-1} \Theta \, dt =$

$= n\Theta \int_0^1 (t^{n-1} - t^n) \, dt = n\Theta\left(\frac{1}{n} - \frac{1}{n+1}\right) =$

$= \frac{\Theta}{n+1}$ – смещена

$\tilde{\Theta}_2' = x_{min}(n+1); \quad M\tilde{\Theta}_2' = (n+1) M x_{min}$ – несмещ.

$D\tilde{\Theta}_2' = D(x_{min}(n+1)) = D x_{min} \cdot (n+1)^2$

$M x_{min}^2 = \int_0^{\Theta} x^2 n \left(1 - \frac{x}{\Theta}\right)^{n-1} \cdot \frac{1}{\Theta} \, dx =$

$= \int_0^1 \Theta^2 (1-t)^2 n t^{n-1} \, dt = n\Theta^2 \int_0^1 (t^{n-1} - 2t^n + t^{n+1}) \, dt$

$= n\Theta^2\left(\frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}\right) = \frac{2\Theta^2}{(n+1)(n+2)} \Rightarrow$

$D x_{min} = \frac{2\Theta^2}{(n+1)(n+2)} - \frac{\Theta^2}{(n+1)^2} = \frac{n\Theta^2}{(n+1)^2(n+2)}$

$\Rightarrow D\tilde{\Theta}_2' = \frac{n\Theta^2}{(n+2)} \underset{n\to\infty}{\not\to} 0$ не дост.

$\forall \Theta > 0 \ \forall \varepsilon > 0 \quad P(|\tilde{\Theta}_2' - \Theta| \geq \varepsilon) \underset{n\to\infty}{\longrightarrow} 0$

$P(|\tilde{\Theta}_2' - \Theta| \geq \varepsilon) \geq P(\tilde{\Theta}_2' \geq \Theta + \varepsilon) =$

$= P((n+1)\, x_{min} \geq \Theta + \varepsilon) = P\left(x_{min} \geq \frac{\Theta+\varepsilon}{n+1}\right) =$

$= 1 - \varphi\left(\frac{\Theta+\varepsilon}{n+1}\right) = 1 - \left(1 - \left(1 - \frac{\Theta+\varepsilon}{\Theta(n+1)}\right)^n\right) =$

$= \left(1 - \frac{\Theta+\varepsilon}{\Theta(n+1)}\right)^n \underset{n\to\infty}{\longrightarrow} e^{-\frac{\Theta+\varepsilon}{\Theta}} > 0$ -

не состоятельна

$P(|x_{min} - \Theta| \geq \varepsilon) = P(x_{min} \leq \Theta - \varepsilon) =$

$= \varphi(\Theta - \varepsilon) = 1 - (1 - f(\Theta - \varepsilon))^n =$

$= 1 - \left(1 - \frac{\Theta - \varepsilon}{\Theta}\right)^n = 1 - \left(\frac{\varepsilon}{\Theta}\right)^n \underset{n\to\infty}{\longrightarrow} 1$

$\frac{\varepsilon}{\Theta} < 1$;

$\exists \varepsilon > 0, \exists \Theta > 0$ - отрицательно опр. $\Rightarrow$

x_{min} - не состоятельна

$\tilde{\Theta}_3 = x_{max}$; $M\tilde{\Theta}_3 = x_{max}$

$x_{max} \sim (f(x))^n = (\varphi(x))^n$