Metric Learning in Optimal Transport for Domain Adaptation

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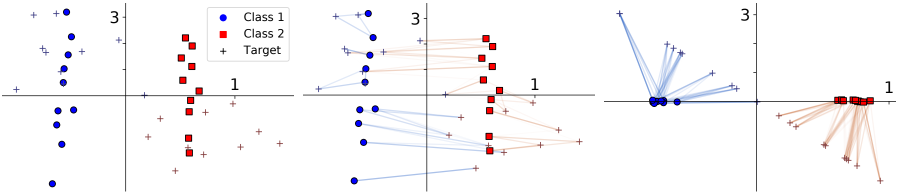
23 juin 2020



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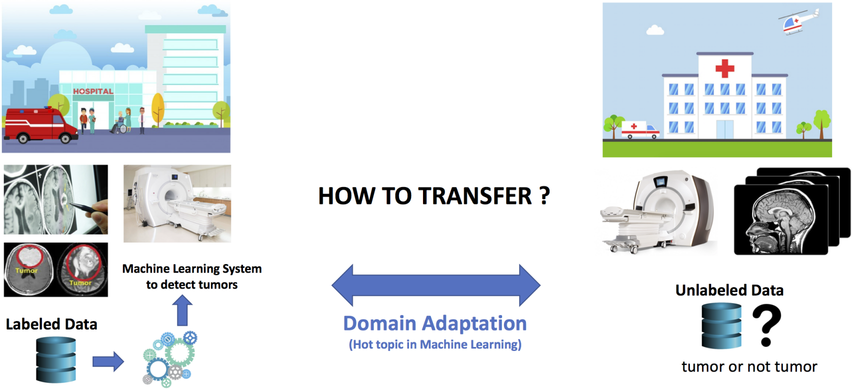
Domain Adaptation (DA) : Intuition



Domain Adaptation (DA) : Notations

x_i^s : Image

x_j^t : Image

- In DA, both source $(x_i^s)_i$ and target $(x_i^t)_j$ samples are available

Domain Adaptation (DA) : Notations

x_i^s : Image
y_i^s : Chair

x_j^t : Image
y_j^t : Unknown

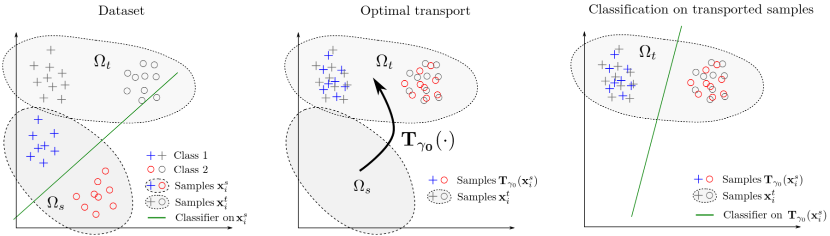
- In DA, both source $(x_i^s)_i$ and target $(x_i^t)_j$ samples are available
- Only source labels $(y_i^s)_i$ are given

Discrete Optimal Transport (OT)

Discrete Optimal Transport

$$\mathcal{W}^2(\hat{\mu}_s, \hat{\mu}_t) = \min_{\gamma \in \hat{\Pi}(\hat{\mu}_s, \hat{\mu}_t)} \sum_{i=1}^{m_s} \sum_{j=1}^{m_t} \|x_i^s - x_j^t\|_2^2 \gamma_{ij} = \min_{\gamma \in \hat{\Pi}(\hat{\mu}_s, \hat{\mu}_t)} \langle \gamma, C^2 \rangle.$$

With $\hat{\Pi}(\hat{\mu}_s, \hat{\mu}_t) = \{\gamma \in \mathbb{R}_+^{m_s \times m_t} | \gamma \mathbf{1}_{m_t} = \hat{\mu}_s, \gamma^T \mathbf{1}_{m_s} = \hat{\mu}_t\}$



Optimal Transport for Domain Adaptation (OTDA) [Courty et al., 2017]

Previous methods : entropy and classes regularization

Entropy regularization [Cuturi, 2013]

$$\min_{\gamma \in \hat{\Pi}(\hat{\mu}_s, \hat{\mu}_t)} \langle \gamma, C^2 \rangle - \lambda_e \left(- \sum_{i=1}^{m_s} \sum_{j=1}^{m_t} \gamma_{ij} \log(\gamma_{ij}) \right) \tag{1}$$

✓ Smooth solution, the Transport Plan γ is **not sparse** anymore

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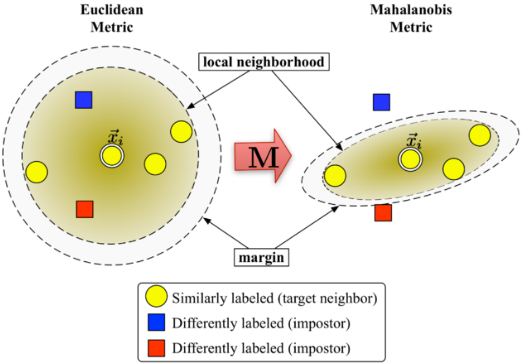
Optimal Transport for Domain Adaptation [Courty et al., 2017]

$$\min_{\gamma \in \hat{\Pi}(\hat{\mu}_s, \hat{\mu}_t)} \langle \gamma, C^2 \rangle - \lambda_e \Omega_e(\gamma) + \lambda_c \left(\sum_{j=1}^{m_t} \sum_{cl=1}^{c} \| \gamma(\mathcal{I}_{cl}, j) \|_2 \right) \tag{2}$$

✓ Points of different classes should **not** be send to same location

Metric learning



LMNN [Weinberger et al., 2006]

Mahalanobis metric

- $C_M^2(x,x') = (x-x')M(x-x')^T = \|Lx - Lx'\|_2^2$ with $L^T L = M$

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Bound on the Target Risk

Classical form of Target Risk Bound [Ben-David et al., 2007]

$$\begin{aligned} \text{target risk} \leq \ & \text{source risk} \\ & + \mathit{distance}\left(\mu_s, \mu_t\right) \\ & + \lambda \text{ (error of the best classifier)} \end{aligned} \tag{3}$$

Bound on the Target Risk

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Our Target Risk Bound

$$\begin{aligned} \text{target risk} \leq\ & \text{source risk} + \mathcal{W}(L_{s\#}\hat{\mu}_s, L_{t\#}\hat{\mu}_t) \\ & + \mathcal{W}(\hat{\mu}_s, L_{s\#}\hat{\mu}_s) + \mathcal{W}(L_{t\#}\hat{\mu}_t, \hat{\mu}_t) \\ & + \mathcal{W}(\mu_s, \hat{\mu}_s) + \mathcal{W}(\hat{\mu}_t, \mu_t) + \lambda \end{aligned} \tag{4}$$

Our Target Risk Bound

$$
\begin{aligned}
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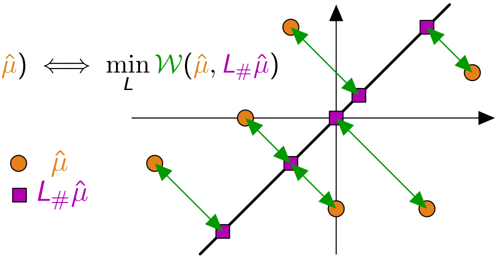
\end{aligned} \tag{5}
$$

$$
PCA(\hat{\mu}) \iff \min_L \mathcal{W}(\hat{\mu}, L_{\#}\hat{\mu})
$$



Metric Learning for Optimal Transport (MLOT)

Input : η (gradient step) X_s X_t Y_s

$V_s = PCA(X_s)$, $V_t = PCA(X_t)$
$L_s = V_s^T V_s$, $L_t = V_t^T V_t$
for $i = 1$ **to** N **do**
  $\gamma =$ Solve $OT(C(L_s, L_t))$
  $L_s = L_s - \eta \nabla_{L_s}(\langle \gamma, C^2(L_s, L_t)\rangle + \lambda_l \Omega_l(L_s))$
end for
$\tilde{X}_s = \gamma L_t X_t$
return classifier$((\tilde{X}_s, Y_s), X_t)$

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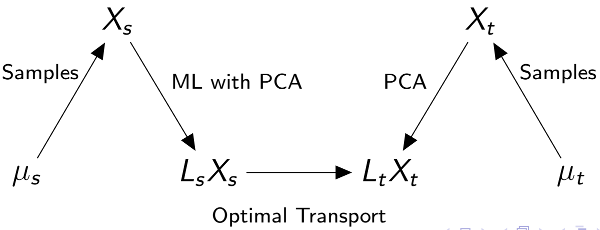
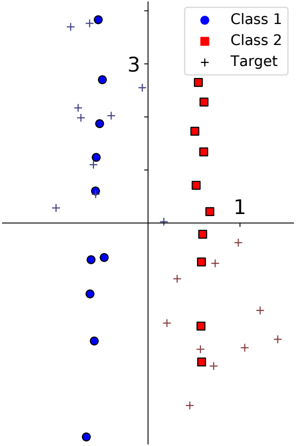
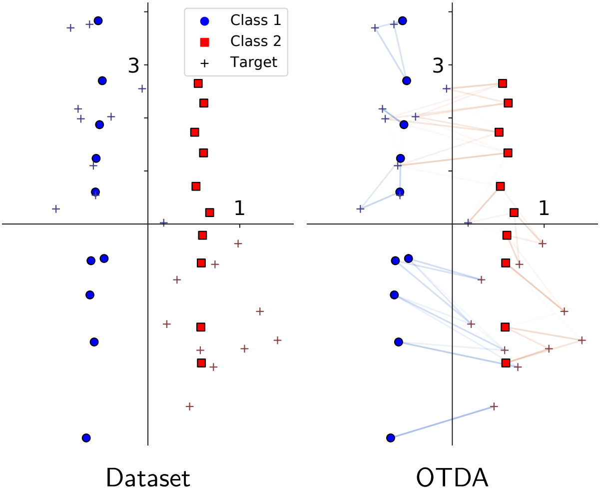


Illustration of the usefulness of Metric Learning



Dataset

Illustration of the usefulness of Metric Learning



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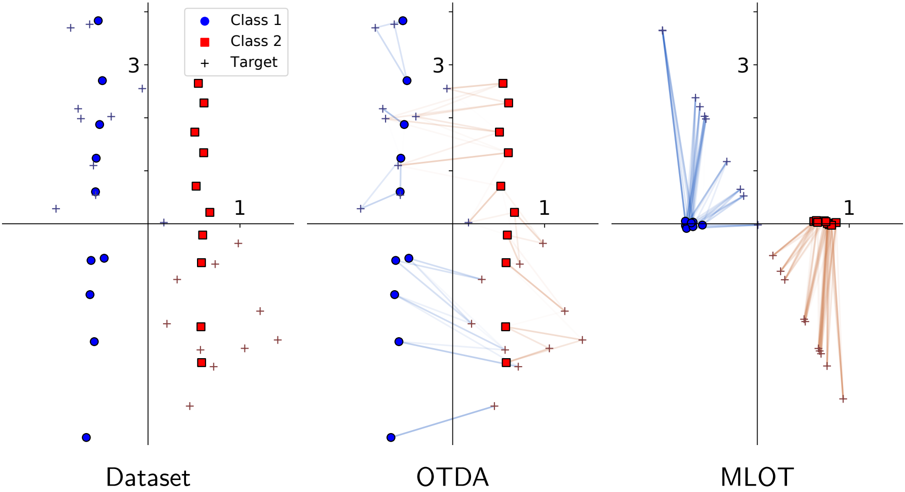

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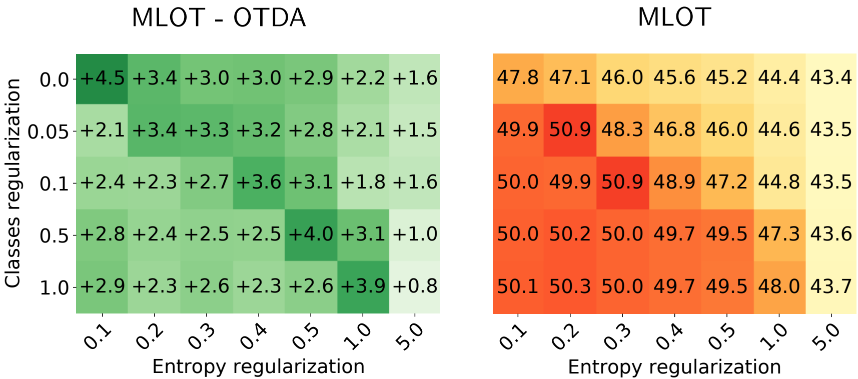
Cross validation of hyperparameters

- In **Unsupervised** Domain Adaptation the target label are **not available**
- We use the Reverse Validation of [Zhong et al., 2010]

Method	OT	TCA	LMNN	SA	JDOT	OTDA	OTDA_p	MLOT
Classical Validation	45.3	45.3	47.9	47.4	48.5	52.8	54	**55.1**
Reverse Validation	42.2	44.5	44.8	45.8	47.0	48.2	48.8	**49.7**

Table – Accuracy on Office-Caltech dataset [Gong et al., 2012] with SURF features.

OTDA vs MLOT



Accuracy of OTDA and MLOT on Office-Caltech dataset [Gong et al., 2012] with SURF features.

References

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Zhong, E., Fan, W., Yang, Q., Verscheure, O., and Ren, J. (2010).
Cross validation framework to choose amongst models and datasets for transfer

Optimal Transition between slide

- To use the Optimal Transition of the presented work see my github : `https://github.com/Hv0nnus/TransitionPDF`