Metric Learning in Optimal Transport for Domain Adaptation

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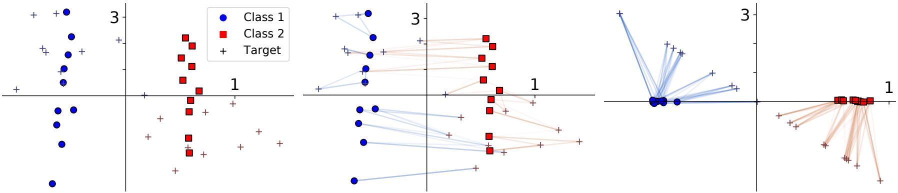
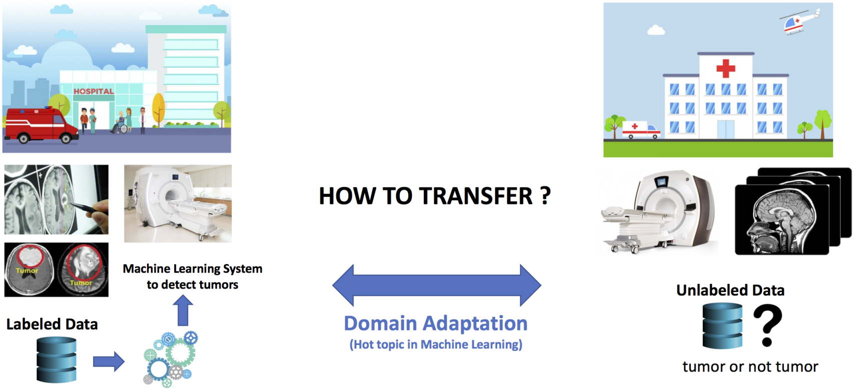


Domain Adaptation (DA) : Intuition

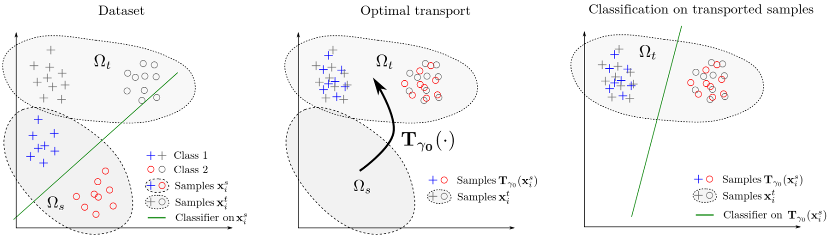


Discrete Optimal Transport (OT)

Discrete Optimal Transport

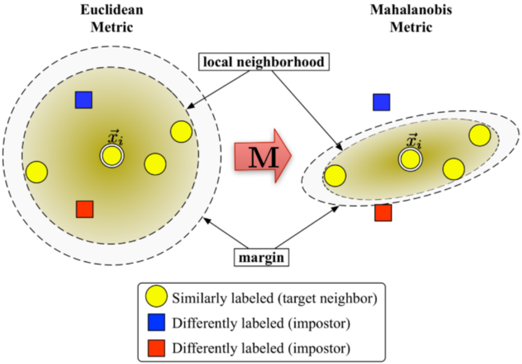
$$\mathcal{W}^2(\hat{\mu}_s, \hat{\mu}_t) = \min_{T \in \hat{\Pi}(\hat{\mu}_s, \hat{\mu}_t)} \sum_{i=1}^{m_s} \sum_{j=1}^{m_t} \|x_i^s - x_j^t\|_2^2 T_{ij} = \min_{T \in \hat{\Pi}(\hat{\mu}_s, \hat{\mu}_t)} \langle T, C^2 \rangle.$$

With $\hat{\Pi}(\hat{\mu}_s, \hat{\mu}_t) = \{T \in \mathbb{R}_+^{m_s \times m_t} | T\mathbf{1}_{m_t} = \hat{\mu}_s, T^T\mathbf{1}_{m_s} = \hat{\mu}_t\}$



Optimal Transport for DA [Courty et al., 2017]

Metric learning



LMNN [Weinberger et al., 2006]

Mahalanobis metric

- $C_M^2(x, x') = (x - x')M(x - x')^T = \|Lx - Lx'\|_2^2$ with $L^T L = M$

Table of Contents

Bound on the Target Risk

Classical form of Target Risk Bound [Ben-David et al., 2007]

$$\begin{aligned} \text{target risk} \leq\ & \text{source risk} \\ & + \textit{distance}\,(\mu_s, \mu_t) \\ & + \lambda\ (\text{error of the best classifier}) \end{aligned} \tag{1}$$

Bound on the Target Risk

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Our Target Risk Bound

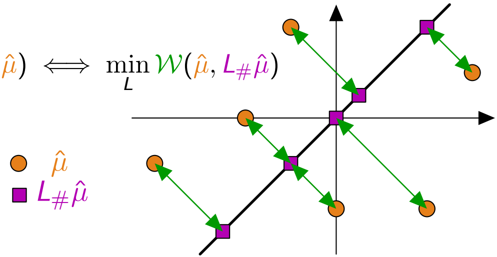
$$\begin{aligned} \text{target risk} \leq \ & \text{source risk} + \mathcal{W}(L_{s\#}\hat{\mu}_s, L_{t\#}\hat{\mu}_t) \\ & + \mathcal{W}(\hat{\mu}_s, L_{s\#}\hat{\mu}_s) + \mathcal{W}(L_{t\#}\hat{\mu}_t, \hat{\mu}_t) \\ & + \mathcal{W}(\mu_s, \hat{\mu}_s) + \mathcal{W}(\hat{\mu}_t, \mu_t) + \lambda \end{aligned} \tag{2}$$

PCA and Wasserstein Distance

Our Target Risk Bound

$$\begin{aligned} \text{target risk} \leq \text{source risk} &+ \mathcal{W}(L_{s\#}\hat{\mu}_s, L_{t\#}\hat{\mu}_t) \\ &+ \mathcal{W}(\hat{\mu}_s, L_{s\#}\hat{\mu}_s) + \mathcal{W}(\hat{\mu}_t, L_{t\#}\hat{\mu}_t) \\ &+ \mathcal{W}(\mu_s, \hat{\mu}_s) + \mathcal{W}(\hat{\mu}_t, \mu_t) + \lambda \end{aligned} \tag{3}$$

$$PCA(\hat{\mu}) \iff \min_L \mathcal{W}(\hat{\mu}, L_{\#}\hat{\mu})$$



Metric Learning for Optimal Transport (MLOT)

Input : η (gradient step) X_s X_t Y_s

$V_s = PCA(X_s)$, $V_t = PCA(X_t)$
$L_s = V_s^T V_s$, $L_t = V_t^T V_t$
for $i = 1$ **to** N **do**
    $\gamma =$ Solve $OT(C(L_s, L_t))$
    $L_s = L_s - \eta \nabla_{L_s} (\langle \gamma, C^2(L_s, L_t) \rangle + \lambda_l \Omega_l(L_s))$
end for
$\tilde{X}_s = \gamma L_t X_t$
return classifier$((\tilde{X}_s, Y_s), X_t)$

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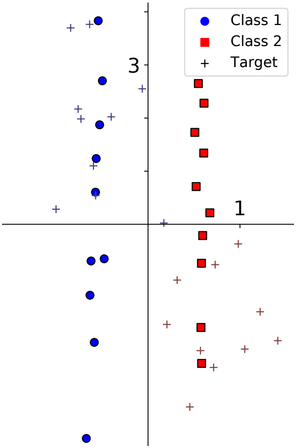
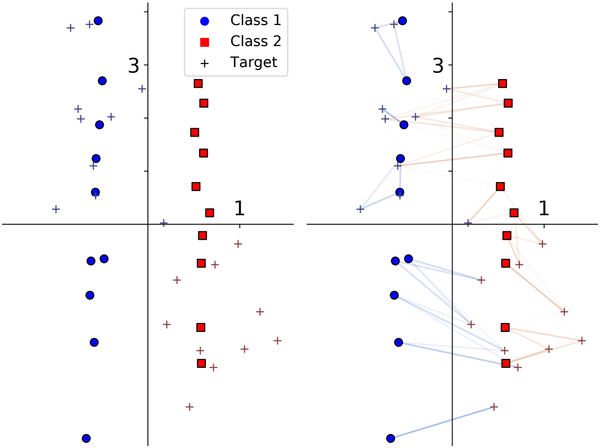
$$\begin{tikzcd}
& X_s \arrow[dr, "\text{ML with PCA}"] & & & X_t \arrow[dl, "\text{PCA}"'] & \\
\mu_s \arrow[ur, "\text{Samples}"] & & L_s X_s \arrow[r, "\text{Optimal Transport}"'] & L_t X_t & & \mu_t \arrow[ul, "\text{Samples}"']
\end{tikzcd}$$

Illustration of the usefulness of Metric Learning



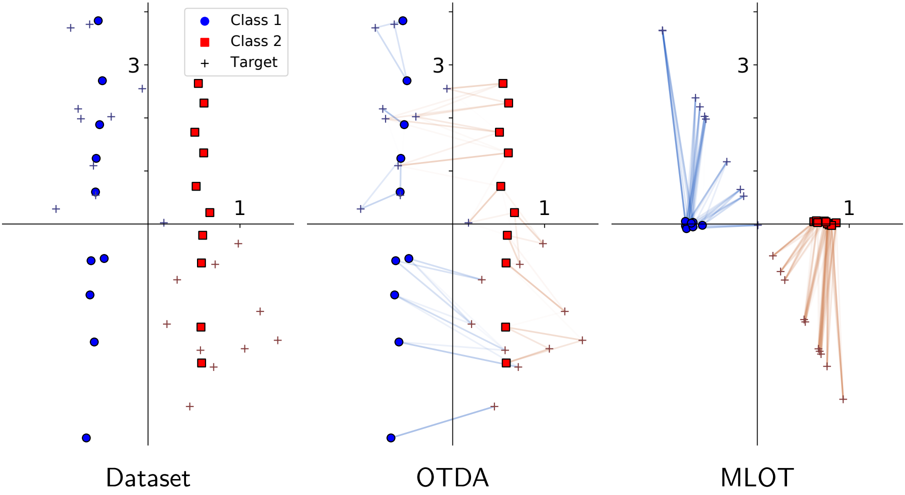
Dataset

Illustration of the usefulness of Metric Learning



Dataset OTDA

Illustration of the usefulness of Metric Learning



References

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Optimal Transition between slide

- To use the Optimal Transition of the presented work see my github : `https://github.com/Hv0nnus/TransitionPDF`