

MA325: Project# 1, Due: 2022/05/10

The following elliptic PDE

$$u_{xx} + u_{yy} - 100u = (12x^2 - 12x + 2)y^2(1-y)^2 + x^2(1-x)^2(12y^2 - 12y + 2) - 100x^2y^2(1-x)^2(1-y)^2,$$

in the domain $(0, 1) \times (0, 1)$ with the zero Dirichlet boundary condition has the exact solution $u(x, y) = x^2y^2(1-x)^2(1-y)^2$. Following below steps to solve above PDE numerically.

- a. Set $h_x = h_y = 2^{-4}, 2^{-5}, 2^{-6}, 2^{-7}, 2^{-8}$ respectively
- b. Use the five-point central finite difference method to discretize above PDE.
- c. Solve the linear system by Discrete Sine Transform. (Hint: there is no 'dst2' function in Matlab (at least on my laptop), but there is 'dct2' function. You can follow this way to get 'dst2': (1) in matlab command window 'edit dct2', (2) copy it to another function file, (3) change all 'dct' to 'dst', (4) save it as 'dst2.m'. Make sure your main function and this 'dst2.m' in the same file folder)
- d. Choose a suitable initial guess and solve above system by SOR iteration method with the optimal relaxation parameter $\omega = \frac{2}{1 + \sin \frac{\pi}{N}}$
- e. Compute the error in L^2 -norm and max-norm.
- f. **Plot** the error against meshsize using 'loglog' in matlab to show the second order convergence. (learn "subplot" to present your results in an elegant way)

Requirement: submit the runnable **matlab codes** together with **error figures in (g)**.

Plus: pay attention to the number of iterations in the SOR method.