MA325: Project# 2, Due: 2022/05/23

The nonlinear Allen–Cahn equation is given as

$$u_t = \epsilon(u_{xx} + u_{yy}) + \frac{1}{\epsilon} f(u), \quad (x, y) \in (0, 2\pi)^2,$$

subject to periodic boundary conditions and the random initial value, where $f(u) = u - u^3$. Taking the semi-implicit Euler method with a stabilized term given in red in time with time step Δt and the central finite difference scheme with grid width h, and denoting u_{ij}^n as the numerical approximation of the exact solution $u(ih, jh, n\Delta t)$ results the discrete scheme

$$\frac{u_{ij}^{n+1} - u_{ij}^n}{\Delta t} = \epsilon \frac{u_{i,j-1}^{n+1} + u_{i-1,j}^{n+1} - 4u_{ij}^{n+1} + u_{i+1,j}^{n+1} + u_{i,j+1}^{n+1}}{h^2} + \frac{1}{\epsilon} f(u_{ij}^n) - \beta \left(u_{ij}^{n+1} - u_{ij}^n\right). \tag{1}$$

a. Set $N = 255,\ h = 2\pi/(N+1),\ \Delta t = 0.01,\ \epsilon = 0.1,\ \beta = \frac{2}{\epsilon}$, respectively.

b. Use the following code in Matlab to generate the initial condition, which has a zero mean

```
uin=0.05*(2*rand(N,N)-1); aver=sum(sum(uin))/N^2; uin=uin-aver;
```

c. Use whatever methods you prefer to solve the system (1) (fast solver is recommended).

d. In each time loop, compute the discrete form of following energy

$$E(u) = \int_0^1 \int_0^1 \left(\frac{1}{4\epsilon}(u^2 - 1)^2 - \frac{\epsilon}{2} u(u_{xx} + u_{yy}) \right) dxdy.$$

e. Use 'subplot' in Matlab plot the numerical solutions at $T = 0$ (initial data), $T = 0.4$, $T = 1$, $T = 2$, and $T = 6$. Below are lines of code for your reference.

```
if kk*dt==4 %% kk is the loop number, and dt is the time step, un is the solution.
  subplot(2,3,2); imagesc(un); axis off; title(['T=0.4']);
end
```

f. Plot the energy (computed in step d) evolution against time as the last subfigure.

```
subplot(2,3,6); plot...
```

Requirement: submit the **runnable Matlab** codes together with **the** 2×3 **figure**.