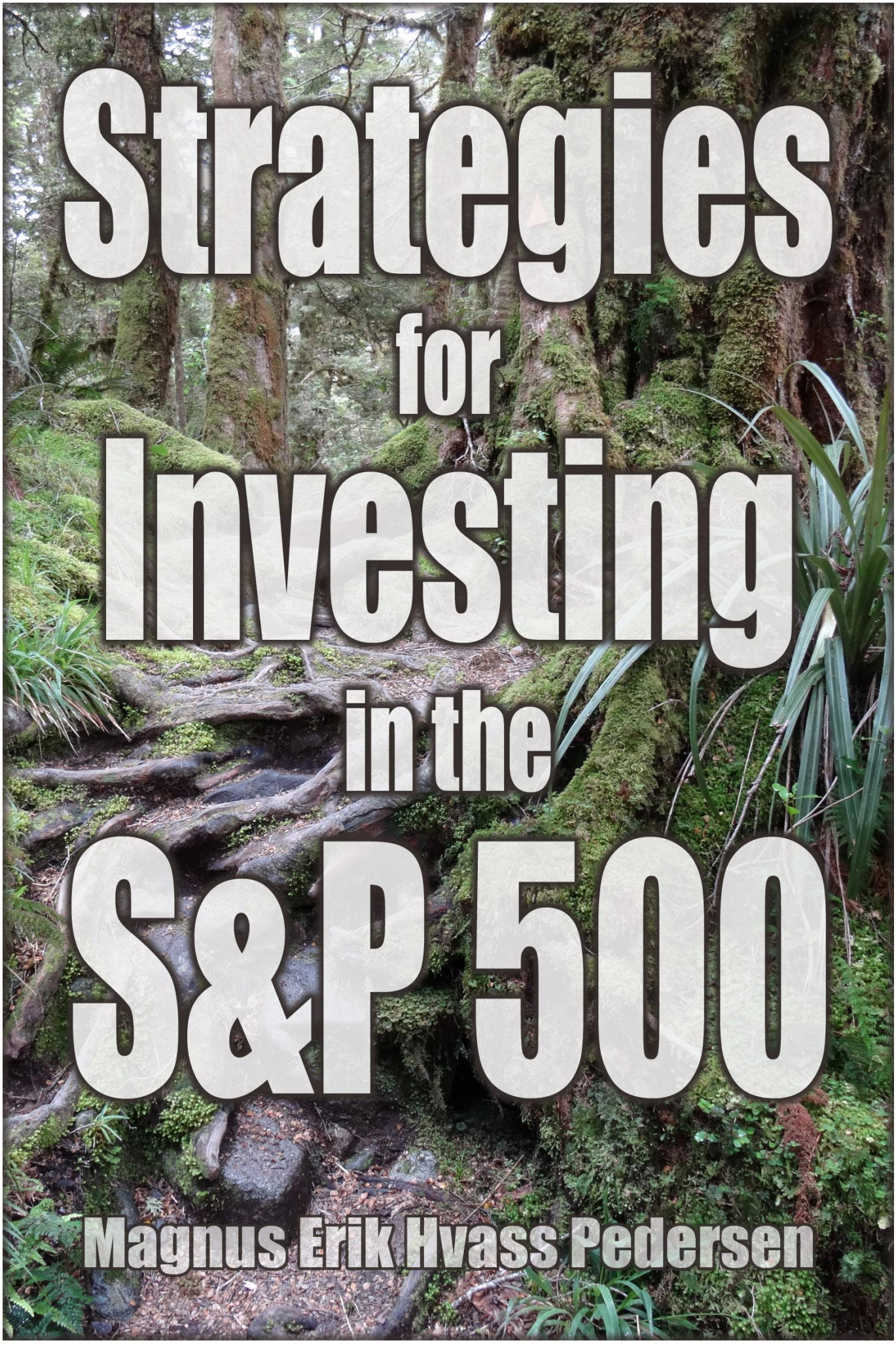
Strategies
for
Investing
in the
S&P 500
Magnus Erik Hvass Pedersen

Strategies for Investing in the S&P 500

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Summary

This book presents different investment strategies for the S&P 500 stock-market index. Some of the strategies use fixed rebalancing in which a predetermined part of the portfolio is invested in the S&P 500 while the remainder is invested in government bonds. Other strategies adapt the rebalancing using the P/Book of the S&P 500 and the yield on government bonds. Stop-loss strategies are also presented. The strategies are tested for all starting days and investment periods up to ten years during 1978-2013, which gives a total of more than 75,000 test periods. Extensive performance statistics and comparisons show the advantages and disadvantages of the different investment strategies.

Warning

The warnings in section 4 should be read before using the investment strategies.
The author is not responsible for your investment results.

About the Author

The author has a BSc degree in Computer Science and a PhD degree in Engineering Science. The author's previous work in finance includes a comprehensive theory on share buyback valuation and new models for financial Monte Carlo simulation. The work is available on the internet:

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1. Introduction

Investing in individual companies is challenging because it requires insight about the future of the company's products, management, competitors, finances, etc. Some industries change so rapidly and dramatically that they are impossible to predict even for insiders. Investors can guard against such risks by diversifying their investments in many different companies and industries. But it is laborious to maintain portfolios of many different stocks and it is expensive in terms of trading commissions and fees. A simple solution is to invest in a low-cost, broadly diversified index fund.

The so-called Standard & Poor's 500 stock-market index (S&P 500) consists of 500 large companies in USA that operate in a wide variety of industries including energy and utility, financial services, health care, information technology, heavy industry, manufacturers of consumer products, etc. The S&P 500 index may be used as a proxy for the entire US stock market as it covers about 80% of that market [**1**]. It is not possible to invest directly in the S&P 500 index but there are several funds that closely track the index at a low cost to the investor. Rather than having to assess the future of individual companies the investor now only needs to assess the general future of companies in USA.

There are still risks when investing in a broadly diversified index such as the S&P 500 because the returns have historically been volatile and yearly losses of nearly (50%) have occasionally been experienced. It may therefore be desirable to combine an investment in the volatile S&P 500 index with a guaranteed return from US government bonds. The question is how much of one's portfolio to invest in each of these.

1.1. Aim

The aim of this book is to present simple strategies for allocating a portfolio between the S&P 500 index and US government bonds. Some of the strategies use a fixed portfolio allocation that is rebalanced annually, while other strategies adapt the allocation by using financial data such as the P/Book ratio of the S&P 500 index and the yield on government bonds. Historically, the adaptive strategies had much lower probability and magnitude of loss, and significantly lower probability of under-performing bond-only and stock-only investments, when compared to the fixed rebalancing strategies. Furthermore, the adaptive strategies sometimes experienced greater gains than the fixed rebalancing strategies. Numerous statistics and examples throughout the book show the mutual advantages and disadvantages of the strategies. An overview is given in section 5.

1.2. Requirements

The writing style is academic for the sake of conciseness and clarity. Knowledge of basic statistics is required, such as quartiles, box-plots and simple probabilities. Readers who are unfamiliar with these concepts may learn about them from Wikipedia, Khan Academy or Coursera.

1.3. Negative Numbers

Negative percentages are written as (x%) rather than –x%. For example, (43.2%) means –43.2%.

2. Literature Survey

This section briefly surveys the literature on rebalancing a portfolio between stocks and bonds. There is no consensus amongst the leading authorities and there is a lack of both theoretical and empirical evidence.

2.1. Warren Buffett

One of the wealthiest people in the world is Warren Buffett who has accumulated his wealth through investing. Buffett believes laymen should invest in a broadly diversified stock-market index such as the S&P 500 and his own heirs should do the same, as he explains [**2**]:

> *"In aggregate, American business has done wonderfully over time and will continue to do so (though, most assuredly, in unpredictable fits and starts). In the 20th Century, the Dow Jones Industrials index advanced from 66 to 11,497, paying a rising stream of dividends to boot. The 21st Century will witness further gains, almost certain to be substantial. The goal of the non-professional should not be to pick winners – neither he nor his "helpers" can do that – but should rather be to own a cross-section of businesses that in aggregate are bound to do well. A low-cost S&P 500 index fund will achieve this goal.*
>
> *(...) The main danger is that the timid or beginning investor will enter the market at a time of extreme exuberance and then become disillusioned when paper losses occur. (...) The antidote to that kind of mistiming is for an investor to accumulate shares over a long period and never to sell when the news is bad and stocks are well off their highs. Following those rules, the "know-nothing" investor who both diversifies and keeps his costs minimal is virtually certain to get satisfactory results.*
>
> *(...) What I advise here is essentially identical to certain instructions I've laid out in my will. One bequest provides that cash will be delivered to a trustee for my wife's benefit. (...) My advice to the trustee could not be more simple: Put 10% of the cash in short-term government bonds and 90% in a very low-cost S&P 500 index fund. (...) I believe the trust's long-term results from this policy will be superior to those attained by most investors – whether pension funds, institutions or individuals – who employ high-fee managers."*

So Buffett recommends increasing one's investment in the S&P 500 index over time to avoid guessing when the price is high or low. Buffett also recommends a portfolio consisting of 90% S&P 500 and 10% short-term government bonds. Unfortunately, it is unclear if the portfolio should be rebalanced and how often.

2.2. Ben Graham

Warren Buffett's teacher was Ben Graham who is known as the founder of so-called "value investing". After decades of investing, Graham also spoke in favour of investing in a broadly diversified index [**3**]:

> *"(...) I have a feeling that the way in which institutional funds should be managed, at least a number of them, would be to start with the index concept – the equivalent of index results, say 100 or 150 stocks out of the Standard & Poor's 500. Then turn over to managers the privilege of making a variation, provided they would accept personal responsibility for the success of the variation that they introduced. (...) I think any experience of the last 20 years, let's say, would indicate that one could have done as well with Standard & Poor's than with a great deal of work, intelligence, and talk."*

This was from an interview in 1976, around the time a financial company named Vanguard was founded for providing index-based investing to the general public, see below. It seems plausible that Graham would have suggested both institutions and laymen should invest in a broadly diversified index if it had been widely available at the time. But without a fund such as Vanguard, the trading commissions would have been excessive for the individual small investor to invest in all or part of the S&P 500 index.

Around the same time, Graham gave his opinion on allocation between stocks and bonds [**4**]:

> *"(...) the investor should always have a minimum percentage of his total portfolio in common stocks and a minimum percentage in bond equivalents. I recommend at least 25% of the total at all times in each category. A good case can be made for a consistent 50-50 division here, with adjustments for changes in the market level. This means the investor would switch some of his stocks into bonds on significant rises of the market level, and vice-versa when the market declines. I would suggest, in general, an average seven- or eight-year maturity for his bond holdings."*

Note how Graham's allocation between stocks and bonds differs from Buffett's above.

2.3. Ed Thorp

Edward Thorp was a professor of mathematics who discovered ways of improving the chances of winning in some gambling games e.g. by counting cards in Blackjack [**5**]. Thorp and Kassouf also devised a system for increasing the returns from certain financial instruments without increasing the risk [**6**]. Thorp managed a hedge-fund for decades using these and other proprietary techniques.

In a recent interview [**7**] Thorp recommends people without special knowledge should invest in a broadly diversified stock-market index:

> *"Basically, people should be putting money into index funds when they can't demonstrate that an investment with similar characteristics is better.*
>
> *(...) Warren Buffett has knowledge about the fundamentals of a lot of companies, among many other things. So in that area, the markets are inefficient from his point of view, but there are a hundred million people out there who don't have that knowledge, and they should behave as if the markets are efficient because, from their point of view, they don't have any edge at all."*

In the same interview, Thorp also discusses portfolio allocation between different asset classes:

> *"(...) the board [of a university endowment] that I sit on has investment guidelines that allocate in a certain range to various categories, such as real estate, private equity, bonds, domestic equity, international equity, and so forth. They move these guidelines around. (...) I'm not sure that the time and energy spent get us very much, though. They'll debate whether to have 20% or 25% in domestic equity, and the finance committee will spend a lot of time offering opinions about this. Maybe they'll decide to move the guideline from 20% to 25%, but no matter what happens, it will only have an incremental effect on returns that is so small that it's hardly noticeable and appears to me to be almost random."*

In his own investing, Thorp used the so-called Kelly Criterion to allocate his portfolio according to the probability distributions for returns of different assets. In one of Thorp's papers [**8**] (Example 7.2) he gives

an example of applying the Kelly criterion to the S&P 500 stock-market index, where it is assumed the mean return is 11% with standard deviation 15% and the yield on US government bonds is 6%. The Kelly-optimal size of the S&P 500 investment is 222% of one's portfolio, that is, borrow an additional 122% of the available cash and make a leveraged investment in the S&P 500. But if such an investment was made in October 2007 when the S&P 500 was around USD 1550 and the investment was held until March 2009 when the S&P 500 had decreased to around USD 680, see Figure 1, then the investor would be bankrupt because of the leverage.

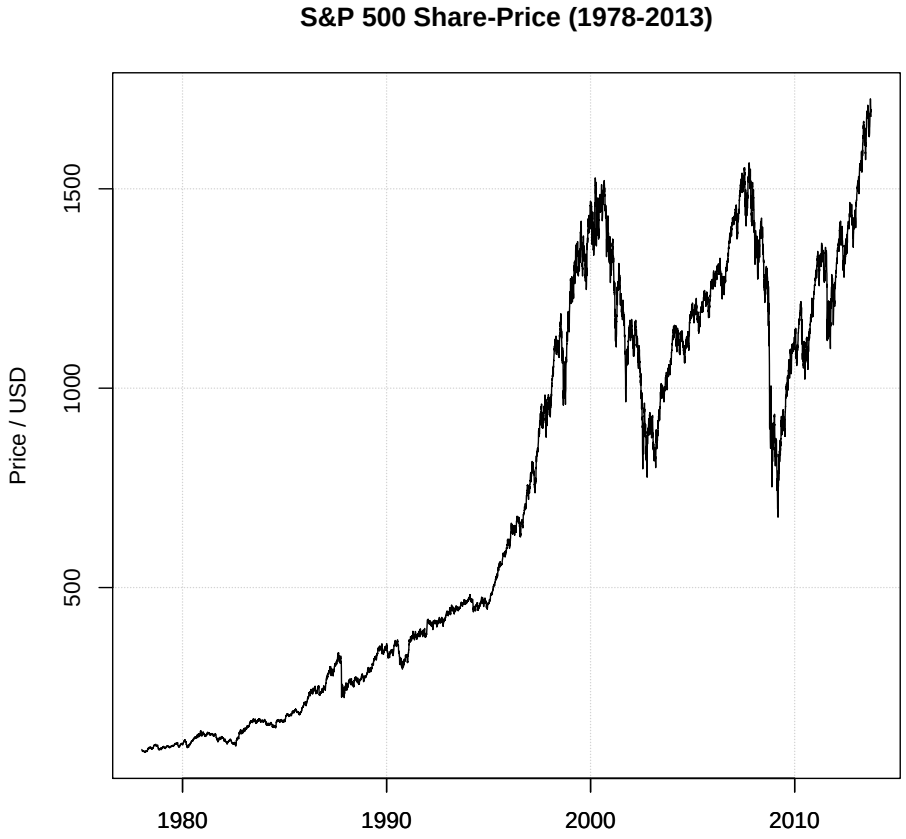

Figure 1: S&P 500 index price during the period 1978-2013. Data source [9].

Thorp and Mizusawa [**10**] studied portfolios consisting of a stock-market index, options on that index, and government bonds, under certain assumptions for the return distributions and option pricing models, etc. An advantage of adding options to the portfolio seems to be protection from large losses. However, such portfolios are not considered in this book.

2.4. John Bogle & Vanguard

John Bogle founded The Vanguard Group in 1975 to provide index-based mutual funds for individual investors. The motivation is that investors on average are incapable of outperforming the stock-market because they themselves are the market. An investor should therefore diversify broadly with low costs so as to get a return that closely matches that of the overall stock-market. A book by Bogle [**11**] argues this from several angles, although it does not provide detailed analysis of different rebalancing strategies between stocks and bonds which is useful for lowering the volatility of the portfolio.

Vanguard researchers Jaconetti et al. [**12**] use broadly diversified stock and bond index data for USA during the period 1926-2009. They study various rebalancing strategies for a target allocation of 60% stocks and 40% bonds. The conclusion is that there is no significant advantage to monthly or quarterly rebalancing and that annual or semi-annual rebalancing with a 5% threshold should be sufficient, especially considering taxes and trading fees. However, the study is limited as it only considers one target allocation (60/40) and the full period is apparently used so the investment and rebalancing begins in 1926 and ends in 2009. The results would likely be very different for other investment periods, as demonstrated below.

2.5. Sector Rebalancing

Bunn and Shiller [**13**] compiled data for three sectors of the US stock-market during the period 1872-2013: Industrials, utilities and railroads. A modified P/E (Price-To-Earnings) ratio is used for taking into account business cycles and inflation, which is calculated essentially just by averaging the earnings for several years. This valuation ratio is found to be useful in predicting future stock returns because lower valuation ratios tend to precede higher stock returns in the following years, and vice versa.

The authors then propose a scheme for scoring the three sectors according to their mutual valuation ratios and their stock price momentum. The score is used to allocate a portfolio consisting of the three sectors which is rebalanced quarterly. In the period 1903-2013 this rebalancing scheme had an annual excess return of about 1% compared to investing in the stock-market without rebalancing. However, it is unclear how the rebalancing scheme performed for shorter investment periods and different starting dates.

2.6. Mean-Variance Portfolio Optimization

Mean-variance portfolio optimization was proposed by Markowitz [**14**] [**15**] for combining assets so as to maximize the portfolio's mean return while minimizing the variance. Such portfolios are called mean-variance efficient or optimal. The investor can choose from the set of such efficient portfolios depending on the investor's personal preference for return versus volatility. See Luenberger [**16**] for a good explanation.

If a risk-free asset such as a government bond is available, then there exists a single fund of assets so that all mean-variance efficient portfolios are combinations of the government bond and the single fund. The Capital Asset Pricing Model (CAPM) extends on this by making a number of assumptions, such as all investors having complete and identical interpretation of information so the single fund is the total market. An investor can then get a mean-variance optimal portfolio simply by combining government bonds and a fund containing all stocks in the market. The market fund could be approximated by the S&P 500 index.

However, there are several problems with this hypothesis. Firstly, variance is not a useful measure of investment risk as demonstrated below. Secondly, the historical average return of the stock-market is not a good estimate of the future return. Lastly, the assumption of CAPM that all investors have complete and identical interpretation of information is unrealistic.

2.6.1. Variance is Not Risk

Mean-variance portfolios are commonly believed to minimize risk for a given level of expected return, because variance is believed to measure risk. But this is an incorrect notion as proven with a short example reprinted from Pedersen [**17**].

Let asset A be an asset with possible negative returns (4%), (5%) or (6%) and let asset B be an asset with possible positive returns 5%, 10% or 15%. The returns have equal probability of occurring and are dependent in the order they are listed so that if asset A has return (4%) then asset B has return 5%, etc.

The long-only, minimum-variance portfolio lies on the efficient frontier and is when the weight for asset A is 5/6 and the weight for asset B is 1/6. This gives a portfolio with mean (2.5%) and zero variance, that is, all possible returns of the portfolio are losses of exactly (2.5%). But an investor could instead have chosen a portfolio consisting entirely of asset B which would always give a positive return of either 5%, 10% or 15%. An investment entirely in asset B is clearly superior to an investment in the minimum-variance portfolio.

The above example had an asset with negative returns which could be avoided by adding the constraint that returns must be positive; but the problem also exists for assets with partly negative returns or all positive returns. To see this, change asset A's possible returns to 3%, 2% or 1%. Then the minimum variance portfolio still has asset A weight 5/6 and asset B weight 1/6 which gives a portfolio return of about 3.3% with zero variance. But asset B alone would give a higher return of either 5%, 10% or 15%. So although the minimum-variance portfolio has no return spread, it has a lower return with certainty.

The problem also exists for assets that have overlapping return distributions. Let asset A's possible returns be 6%, 5% or 4%. Then the minimum variance portfolio still has asset A weight 5/6 and asset B weight 1/6 which always gives a portfolio return of about 5.8% with zero variance. But asset B alone would give a higher return of either 10% or 15% with probability 2/3 (or about 67%) and a slightly lower return of 5% with probability 1/3 (or about 33%).

The reason asset A is included in the efficient frontier and minimum-variance portfolio is that its return has a low (sample) standard deviation of 1% while asset B has a higher standard deviation of 5%. The two assets have negative correlation so combining them in a portfolio lowers the combined standard deviation. The mean-variance efficient frontier is optimized for low variance (and hence a low standard deviation) which gives a low spread of the possible returns from the portfolio. But the spread of possible returns is not a useful measure of risk because it does not consider the probability of loss, and the probability of other assets having a higher return. So the mean-variance portfolio is not optimized for risk in the traditional sense of the word defined in a dictionary as "*the chance of injury or loss*".

3. Investment Periods

It is common in the literature to study the performance of investment strategies over long periods of time, e.g. 20, 50 or even 100 years. But it is demonstrated repeatedly below that the performance of an investment strategy depends on the starting date and duration. An investment strategy may perform well during one period and perform poorly during another period. For example, consider a full investment in the S&P 500 index. In the nine years ending on March 24, 2000, the S&P 500 increased from around USD 367 to 1527, a gain of about 316%. But in the following nine years ending on March 24, 2009, the S&P 500 decreased from about USD 1527 to 806, a loss of about (47%). This demonstrates the necessity of considering the performance of an investment strategy for all possible starting dates and durations.

3.1. Multiple Years

Another common problem with studies in the literature is that they only consider investment returns for one-year periods and assume that the same return distributions are possible for consecutive years. But the returns are sequentially dependent and must therefore be considered for multiple years consecutively.

For example, during the period 1978-2013 the largest annual return of the S&P 500 occurred in the year following March 9, 2009. The price on this date was around USD 677 and a year later the price was about USD 1140, a price gain of about 68% and adding the dividend yield of 3% gives a total return of about 71%. But during the period 1978-2013 there were no annual returns of 71% for the S&P 500 that were followed by another year with a return of 71%. In fact, the following year from March 9, 2010 to 2011 only had a price return of about 16% and the dividend yield was another 2% for a total return of about 18%.

It is therefore necessary to consider the performance of investment strategies on multiple consecutive years rather than single years individually.

3.2. Duration

This book uses financial data for the period 1978-2013 because certain financial data for the S&P 500 index is difficult to obtain from before 1978. The period covers about 35 years, for which data is available on 8,925 trading days. The maximum investment period considered is 10 years because it gives starting dates in the period 1978-2003. If longer investment periods were considered then the starting dates would be limited to a shorter period which might have had particular economic circumstances that would distort the performance statistics for the investment strategies.

For example, if 30 year investment periods were considered, then the starting dates would only be in the period 1978-1983 during which the P/Book for the S&P 500 was historically very low, see Figure 2, and the yield on US government bonds was historically very high, see Figure 3. So the performance statistics would be biased for this historically unusual period and would not be representative of other economic conditions.

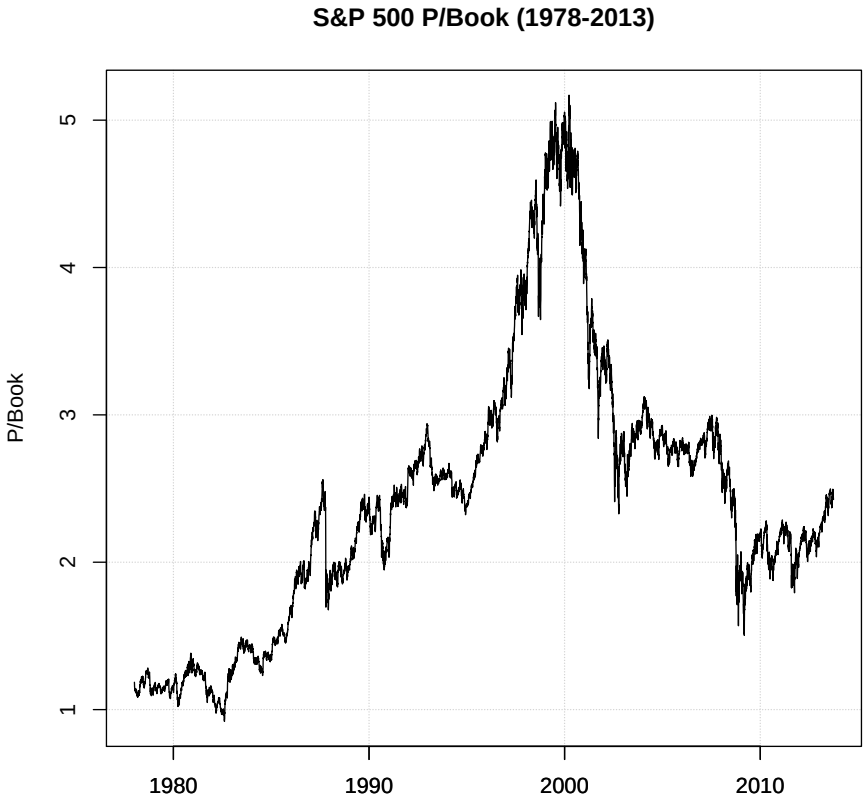

Figure 2: S&P 500 P/Book during the period 1978-2013. Data source [18**].**

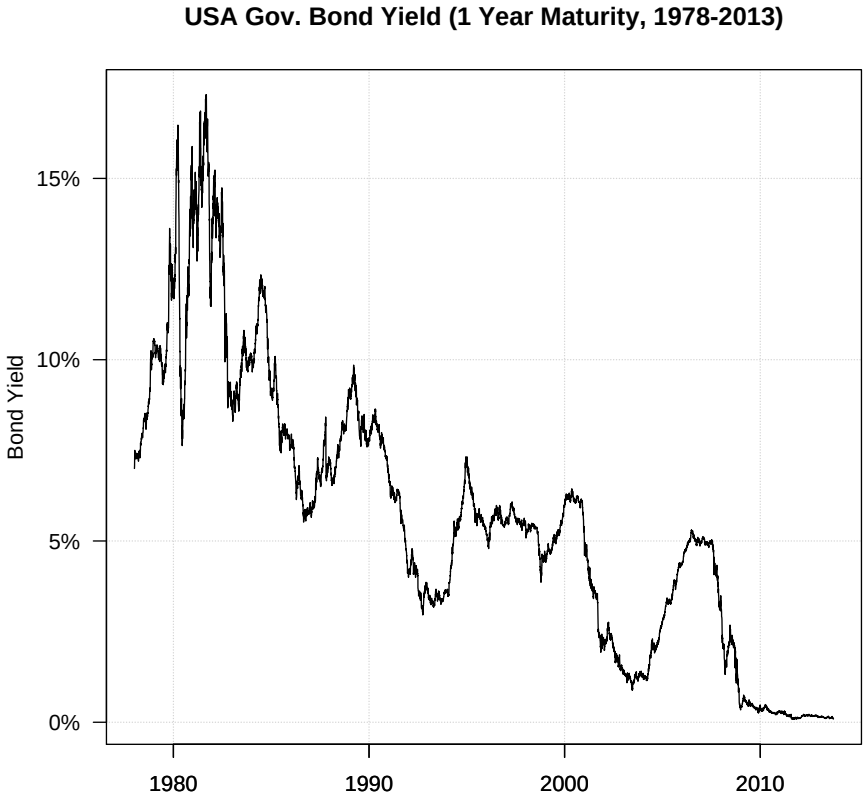

Figure 3: US government bond yields with 1 year maturity for the period 1978-2013. Data source [19**].**

3.3. Number of Combinations

This book tests the investment strategies for all starting dates and investment periods up to 10 years during 1978-2013. The first date in the data-set is January 3, 1978 so investments starting on this date and lasting one, two, three, and up to ten years are considered, and similarly for all other dates in the period 1978-2013. There are a total of 8,672 combinations with one year duration, and 8,421 combinations with two year duration, and 8,170 combinations with three year duration, etc. The number of combinations decreases with longer investment durations because the data-set is limited towards the end, e.g. investments starting in 2012 can only last one year because data is only available until year 2013.

In total there are 75,446 combinations of starting dates and durations for which the performance of an investment strategy is being considered.

3.4. Statistics

The performance of investment strategies on all these combinations of starting dates and durations are summarized using statistics such as quartiles, box-plots, and the probabilities of loss and under-performing bond-only and stock-only investments. Although the main conclusions are also described in the text, the reader is encouraged to study and compare the statistical tables and figures to gain a better understanding of the advantages and disadvantages of different investment strategies.

3.4.1. Example

Further below, Table 5 shows an example of the statistics that are reported throughout the book. This particular table shows statistics for the annualized return from annual rebalancing with 50% / 50% allocation between the S&P 500 and US government bonds. The strategy is tested for all starting dates between 1978 and 2013 and for all investment periods between one and ten years. For each investment period the table shows statistics for the annualized return including the minimum, maximum, quartiles, median, mean and standard deviation. These give an overview of the distribution of returns. For example, for investment periods of five years using the 50/50 rebalancing strategy, the minimum annualized return was (1.7%), the median was 10.2%, the mean was 8.9%, the maximum was 21.0%, and the standard deviation was 5.1%.

Table 5 also shows the probability of loss, that is, the probability that the annualized return was negative. For investment durations of five years the probability of loss was 0.004 (or 0.4%), which means that investing using the 50/50 rebalancing strategy for any five year period during 1978-2013 would have resulted in a loss after five years with probability 0.004.

Table 5 also shows the probability of under-performing bond-only investments was 0.20 (or 20%) for five-year investment periods, which means that one out of every five investments using the 50/50 rebalancing strategy experienced returns that were worse than a bond-only investment after five years. Table 5 also shows the probability of under-performing stock-only investments was 0.73 (or 73%) for five-year investment periods, so almost 3 out of 4 investments using the 50/50 rebalancing strategy under-performed a full investment in the S&P 500 index.

Whether loss or under-performance relative to bond-only or stock-only investments were actually experienced depends on the starting date of the investment. Detailed examples of this will be given later.

4. Warning

The performance of each investment strategy is tested for all starting dates and investment periods up to ten years during 1978-2013. Various statistics are used to summarize the investment performance. There is no strategy that is universally best. The strategies are sometimes better and sometimes worse than a full investment in either the S&P 500 or government bonds. Sometimes the performance of an investment strategy is radically different when the starting dates are only months apart.

It is important to stress that the statistics are for the historical performance during 1978-2013. Although the period covers 35 years which had many different economic conditions, crises of varying kinds, wars, etc. the future may still be very different. The investment strategies are devised so as to limit the historical magnitude of losses while maximizing the average return. Some of the strategies use the P/Book of the S&P 500 and the government bond yield in deciding how to allocate the portfolio. Historically the strategies had limited losses of e.g. (5%) or (15%), but the future may experience greater losses because the economic conditions may be different.

Large losses of the S&P 500 may occur for other reasons than overvaluation as indicated by an abnormally high P/Book (this ratio is explained in more detail in section 13.1). An example of such a great loss is the financial crisis of 2008-2009 where the S&P 500 lost (46.8%) consisting of a price-loss of (48.8%) offset slightly by a positive dividend yield of 2%. This great loss occurred in the year starting March 5, 2008 where the P/Book of the S&P 500 was 2.51 which was only slightly higher than the historical average of 2.4.

If losses are to be strictly limited then stop-loss orders must be used. Investment strategies are devised specifically for this, but the strategies may be difficult to execute in practice because the stop-price may be crossed numerous times during a day and the shares have to be sold and repurchased each time this happens, with each trade possibly incurring a small loss that may accumulate to a large loss over a year.

All liability is waived and the reader is responsible for any outcome from using the strategies.

5. Overview of Strategies

This section gives an overview of the investment strategies and their performance.

5.1. Bond-Only & Stock-Only Investments

Full investments in either US government bonds with one-year maturity (aka. bond-only) or the S&P 500 index (aka. stock-only) serve as benchmarks throughout the book for comparing the performance of investment strategies to either a guaranteed return from government bonds or a full investment in the S&P 500. Table 1 shows statistics for the annualized returns of bond-only investments and Table 2 shows it for stock-only investments. The annualized return is defined and explained in section 9.2.

During 1978-2013 the bond-only investments had median annualized return between 5.2-5.5% depending on the investment duration. For example, the median annualized return was 5.5% for one-year investment periods while it was 5.3% for ten-year investment periods. The lowest one-year return was 0.08% while the highest was 17.3%. For ten-year investment periods, the lowest annualized return was 1.80% while the highest was 10.4%.

For stock-only investments the median annualized return was between 12.5-14.0% depending on the investment duration. For example, the median annualized return was 14.0% for one-year investment periods while it was 13.8% for ten-year investment periods. The greatest one-year loss was (46.8%) while the greatest one-year gain was 71.8%. For ten-year investment periods the lowest annualized return was (4.0%) while the highest was 19.8%.

The government bonds did not experience any losses during 1978-2013 so the historical probability (i.e. frequency) of loss was zero for bond-only investments. Stock-only investments experienced losses for all investment periods up to ten years, although the probability of loss decreased from 0.19 (or 19%) for one-year investment periods down to a probability of 0.07 (or 7%) for ten-year investment periods.

Figure 20 shows the stock-only investment performed much better than the bond-only investment over the full period covering 1978-2013, but Figure 21 and Figure 22 give examples of the stock-only investment performing both better and worse than the bond-only investment over ten-year periods. The mutual performance depends on the starting date and investment duration. Table 2 shows the stock-only investments frequently under-performed bond-only investments, with the probability going from 0.25 (or 25%) for one-year investment periods down to a probability of 0.12 (or 12%) for ten-year investment periods.

The full analyses of bond-only and stock-only investments are given in sections 9 and 10.

5.2. Annual Rebalancing

The following investment strategies are based on annual rebalancing. At the beginning of each year the portfolio is allocated between the S&P 500 index and US government bonds with one-year maturity. After a year has passed, the portfolio has grown from the yield on the government bonds and the dividend yield on the S&P 500, while the price change of the S&P 500 may cause either a loss or a gain.

5.3. Random Rebalancing

Another useful benchmark is random rebalancing. At the beginning of each year the allocation between the S&P 500 index and US government bonds is done randomly. This lowers the probability of extreme gains and losses but does not eliminate the possibility of them occurring. The mean annualized return is about the average of the S&P 500 and government bonds. This is analysed in more detail in section 11.

5.4. Fixed Rebalancing

Fixed rebalancing uses a predetermined weight between the S&P 500 index and US government bonds. The portfolio is rebalanced annually using the same weight.

The statistics for the fixed rebalancing strategy are not simply weighted from the statistics of the bond-only and stock-only investments because of the annual rebalancing. Section 12 analyses fixed rebalancing with different weights: 25% stock / 75% bond, 50/50 and 75/25. It is found that the mean annualized return increases with the stock-weight, as does the probability and magnitude of loss. But the probabilities of under-performing bond-only and stock-only investments are remarkably similar for these stock-weights, and it is the magnitude of the under-performance that differs.

The fixed rebalancing with 50% stock / 50% bond has approximately the same mean annualized return as random rebalancing, but the 50/50 rebalancing strategy has the advantage that losses are limited while random rebalancing is potentially fully exposed to the losses of the S&P 500.

5.5. Adaptive Rebalancing using P/Book

The P/Book ratio is defined as the price of the S&P 500 index divided by its book-value or equity-value, which can be used to assess whether the S&P 500 is over- or under-priced relative to its historical average P/Book. This is discussed in more detail in section 13.1.

Investment strategies are presented in section 13 that use the P/Book of the S&P 500 index to allocate the portfolio between the S&P 500 and US government bonds. This is done using simple linear formulas taking as input the P/Book and outputting the stock-weight. The portfolio is rebalanced annually using these stock-weights.

Strategies are given for low-, medium- and high-risk which had increasing magnitudes and probabilities of loss during the period 1978-2013. The greatest loss was (4.5%) for the low-risk strategy, it was (13.9%) for the medium-risk strategy, and (46.8%) for the high-risk strategy. Conversely, the greatest one-year gain was 50.9% for the low-risk strategy, while it was 65.6% for the medium-risk strategy, and 71.8% for the high-risk strategy.

The mean annualized return for the low-risk strategy was 7.9% for one-year investment periods and it decreased gradually to 7.3% for ten-year investment periods. For the medium-risk strategy the mean annualized return was 9.3% for one-year investment periods and it decreased to 8.3% for ten-year investment periods. For the high-risk strategy the mean annualized return was 12.9% for one-year investment periods and it decreased to 11.9% for ten-year investment periods.

All three strategies experienced occasional under-performance compared to bond-only investments. All the strategies experienced frequent under-performance compared to stock-only investments, except the high-risk strategy which only experienced occasional under-performance compared to stock-only investments.

The adaptive strategies had several advantages over the fixed rebalancing strategies. For similar levels of mean annualized returns, the adaptive strategies had much lower probability and magnitude of loss, and significantly lower probability of under-performing bond-only and stock-only investments, compared to the fixed rebalancing strategies. Furthermore, the adaptive strategies sometimes experienced greater gains than were possible with the fixed rebalancing strategies, because the adaptive strategies were sometimes fully invested in the S&P 500 if the P/Book was sufficiently low. However, the median annualized return was significantly lower for the adaptive strategies compared to the fixed rebalancing strategies, with the difference typically being 2 percentage points or more.

5.6. Adaptive Rebalancing using P/Book & Bond Yield

The adaptive strategies are extended in section 14 to use both the P/Book of the S&P 500 index and the yield on US government bonds to allocate the portfolio between the S&P 500 and government bonds. The strategies again use simple linear formulas for calculating the stock-weights. The portfolio is rebalanced annually using these stock-weights.

Although there were some advantages to using the government bond yield in the adaptive strategies, the advantages were not consistent, they were not always significant, and sometimes there were also disadvantages compared to the strategies that only use the P/Book in the portfolio allocation. See section 14 for details.

5.7. Composite Strategies

The adaptive rebalancing strategies can be combined to better match an investor's personal risk-tolerance. This can be done by dividing the portfolio into four parts. The first part is invested in government bonds because the investor cannot tolerate any losses on this part of the portfolio. The second part is invested according to the low-risk strategy because small losses can be tolerated for a year or two. The third part is invested according to the medium-risk strategy because medium losses can be tolerated for a few years. The fourth part is invested according to the high-risk strategy because large losses can be tolerated for several years. The allocation between these parts depends on the investor's personal preferences. In practice, the allocation is done by calculating the stock-weights for all three adaptive strategies and weighting them according to the investor's preferences. Examples are given in sections 13.6 and 14.4.

5.8. Stop-Loss

Although the adaptive strategies had limited losses during the period 1978-2013, there is no guarantee that the future losses will also be limited. If losses must be strictly limited then stop-loss orders must be used.

Section 15 presents different stop-loss strategies which use annual rebalancing according to stock-weights calculated much like the adaptive strategies described above. But during a year the investment in the S&P 500 is sold whenever the price becomes lower than a so-called stop-price. If the price again increases above the stop-price then the shares are repurchased. If this selling and repurchasing can be done exactly at the stop-price and the stop-price is set to equal the purchase price at the beginning of each year, then the mean annualized return is about 15% and the median is about 14-16% depending on investment duration.

However, it may not be possible to sell and repurchase exactly at the stop-price and this may cause small losses that compound into a great loss during a year. Assuming a frictional cost of (0.5%) per trade and max one trade per day, then annual losses of (14%) were sometimes experienced and the mean annualized

return decreases to around 11% and the median is about 10-12% depending on investment duration. But the assumption that only one trade is executed per day may not be realistic because the S&P 500 may cross the stop-price multiple times during a day, so the losses may be greater and hence the mean and median annualized returns may be lower.

The probability and number of stop-price crossings decreases when the stop-price is further below the purchase price. The low-risk strategy has a stop-price (5%) below the purchase price and combined with its formula for the stock-weight, the strategy has historically had annual losses lower than (5%), while the mean annualized return was around 9% and the median was around 7-8% depending on investment duration.

The medium-risk strategy has a stop-price (15%) below the purchase price and combined with its formula for the stock-weight, the strategy has historically had losses lower than about (15%), while the mean annualized return was around 10-12% and the median was around 10-11% depending on investment duration.

The performance of the stop-loss strategies depend greatly on how close to the stop-price the investor can execute the selling and repurchasing of the S&P 500 shares. The investor may do much better or much worse. It is recommended that the investor experiments with these strategies using a small part of the portfolio to understand the difficulties of executing the strategies. Automated computer trading may also be helpful.

6. Practical Issues

This section briefly describes some practical issues on how to obtain data and invest in the S&P 500 index.

6.1. Website

The stock-weight formulas for the adaptive strategies are automatically updated on this internet website:

www.value-yield.com/strategies

6.2. Input Data

Manually calculating the stock-weights for the adaptive rebalancing strategies requires the P/Book of the S&P 500 index which can be obtained from this internet website:

www.spdrs.com/product/fund.seam?ticker=spy

Some of the adaptive strategies also require the yield on government bonds with one-year maturity. For US government bonds the yield can be obtained from this internet website:

www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield

6.3. Spreadsheet

The stock-weight formulas for the adaptive strategies are available in a spreadsheet for Microsoft Excel:

www.hvass-labs.org/books/files/pedersen2015strategies-sp500.xlsx

6.4. Video Talks

The main aspects of this book are explained in several video talks that can be viewed on the internet:

www.youtube.com/user/hvasslabs/playlists

6.5. Investing in the S&P 500

Investing in the S&P 500 can be done through mutual funds e.g. from the Vanguard Group as previously mentioned. It is also possible to invest in the S&P 500 through Exchange Traded Funds (ETF) which are traded like ordinary shares of stock. For example, Vanguard provides an ETF for the S&P 500 which trades under the ticker symbol VOO, while SPDR provides a similar ETF trading under the ticker symbol SPY.

In general, the investor should invest in whichever S&P 500 fund that offers the lowest expense ratio, unless there are specific reasons to invest in another fund.

6.5.1. Foreigners

Foreigners who want to invest in the S&P 500 may be able to buy the ETFs described above which are traded on US stock exchanges and sometimes also available for purchase through foreign stock-brokers.

In recent years, ETFs for the S&P 500 have also become available on non-US stock exchanges. For example, Vanguard in Canada provides an ETF trading under the ticker symbol VFV and for the UK the ticker symbol is VUSA, while SPDR provides ETFs trading under the ticker symbol SPY5 in several European countries. Note that these ETFs are un-hedged and therefore fully exposed to changes in the currency exchange rate.

Currency Hedging

Several ETFs for the S&P 500 have recently become available that also hedge the currency exposure for non-US investors. For example, Vanguard in Canada provides an ETF with the ticker symbol VSP which hedges for the exchange rate between Canadian and US Dollars. The company BlackRock provides ETFs hedged for several different currencies: XSP is their ETF hedged for Canadian Dollars, IGUS is hedged for British Pounds, IUSE is hedged for the Euro, and IHVV is hedged for Australian Dollars. Note that some of these ETFs may accumulate and reinvest dividends rather than pay dividends to investors.

Currency hedging does not eliminate currency fluctuations but merely dampens them. The effectiveness of the hedge depends on the magnitude and direction of currency fluctuations as well as changes to the price of the S&P 500. Furthermore, there may be hedging costs depending on the difference between the interest rate in USA and the foreign country. The mathematics of currency hedging is explained in section 8.2 and considerations for when to use currency hedging are summarized in section 8.2.5.

6.6. Investing in Government Bonds

Government bonds can be bought through banks, brokers, or directly from a government agency.

People from outside USA should instead consider buying bonds from their own government because there might be tax advantages and it also avoids the currency risks described in section 8.

7. Tax

Taxes are different amongst investors and may depend on the investor's income-level, the size and type of the investment, and whether the investment is short- or long-term. Some investors may have previous losses to credit against future income. Tax-laws also change over time and are differ amongst countries.

It is possible to devise investment strategies that take specific taxes into account, but due to the wide range of possible taxes this book ignores taxes and instead gives some general advice in this section.

As demonstrated below, it is generally best to make long-term investments in a deferred-tax account such as a retirement savings account. This is because of the compounding nature of returns in which annual taxation causes substantial under-performance compared to deferred taxation.

An investor who cannot use a deferred-tax account should limit trading activity as much as possible so as to defer the taxation that way. The investor should keep in mind that the rebalancing strategies in this book are inherently imprecise and small adjustments of the stock / bond allocation may have almost random impacts on the pre-tax returns, while the taxes on small portfolio adjustments may compound into a substantial penalty over many years. It is therefore suggested that investors only sell stocks when the target allocation between stocks and bonds differs significantly from the current allocation if profits are taxable. Instead of selling stocks to meet a small change in target allocation, the investor may instead add bonds when making new contributions to the portfolio so as to approach the target allocation that way.

7.1. Deferred Tax

When investments compound for several years it is better to have deferred taxation rather than annual taxation. This can be demonstrated with a few examples.

Let *n* be the number of investment years. Let *Annual Return* denote the rate of return for each year and let *Tax Rate* denote the tax-rate which is either deferred to the end of the investment period or levied annually.

The compounded return after *n* years with deferred tax is:

$$\textit{Return with Deferred Tax} = ((1 + \textit{Annual Return})^n - 1) \times (1 - \textit{Tax Rate})$$

Eq. 7-1

The compounded return after *n* years with annual taxation is:

$$\textit{Return with Annual Tax} = (1 + \textit{Annual Return} \times (1 - \textit{Tax Rate}))^n - 1$$

Eq. 7-2

For example, if *Annual Return = 10%, n = 10* years and *Tax Rate = 25%* then the compounded return with deferred tax is calculated using Eq. 7-1:

$$\textit{Return with Deferred Tax} = ((1 + \textit{Annual Return})^n - 1) \times (1 - \textit{Tax Rate}) = ((1 + 10\%)^{10} - 1) \times (1 - 25\%) \approx 1.195$$

The return with annual taxation is calculated using Eq. 7-2:

$$\textit{Return with Annual Tax} = (1 + \textit{Annual Return} \times (1 - \textit{Tax Rate}))^n - 1 = (1 + 10\% \times (1 - 25\%)^{10} - 1 \approx 1.061$$

So the compounded return after ten years is 1.195 (or 119.5%) with deferred taxation and it is only 1.061 (or 106.1%) with annual taxation.

The advantage of deferred tax increases for longer investment periods. For $n = 30$ years the return with deferred tax is 12.34 (or 1,234%) while it is only 7.75 (or 775%) with annual taxation. For $n = 60$ years the return with deferred tax is 227.61 (or 22,761%) while it is only 75.65 (or 7,565%) with annual taxation.

7.2. Rebalancing & Tax

This book presents investment strategies that use annual rebalancing. If the investment is made in a tax-deferred account, such as a retirement savings account where the tax is paid when the funds are withdrawn after retirement, then the rebalancing normally does not incur taxation. But if the rebalancing is done in a taxable account then it is important to sell as little of the portfolio as possible when rebalancing, because the annual taxation incurs a large penalty when compounded over many years, as shown above.

7.2.1. Example

Suppose we have invested our entire portfolio of USD 100 in stocks and it has been a very good year with a return of 100% so the total portfolio is now worth USD 200 before tax. If we sell all the stocks then we have to pay tax on the profits, e.g. if the tax-rate is 25% then the tax would be USD 25. But what if we only want to rebalance the portfolio by changing the stock-weight from 1.0 to 0.9? How much of the portfolio should we then sell and how much tax should we pay?

The portfolio consists of USD 200 so our first estimate is to invest USD 180 in stocks corresponding to a stock-weight of 0.9. This means we have to sell USD 20 worth of stocks which has a profit of USD 10 that incurs a tax of 25% or USD 2.50. But now our total portfolio value is only USD 197.50 because USD 2.50 was paid in tax, so a stock-weight of 0.9 means that only USD 177.75 should be invested in stocks instead of USD 180. This in turn means that USD 22.25 worth of stocks must be sold from our portfolio of USD 200. The profit on this is about USD 11.13 and the tax is then about USD 2.78. After paying this tax our total portfolio value has decreased to about USD 197.22. So we must recalculate the amount of stocks to sell and the tax. This can be done a number of times and eventually the numbers converge, so the amount of stocks that must be sold is about USD 22.54 which incurs a tax of about USD 2.82 so the remaining portfolio value is about USD 197.18 and hence USD 177.46 should be invested in stocks for a stock-weight of 0.9.

The optimal tax was found to be about USD 2.82 while it would have been USD 25 if the entire portfolio had first been sold and then rebalanced. Although the remaining tax will eventually have to be paid (unless future losses erase the profits), the remaining tax is deferred which may give greater compounded returns for the investor over time, as demonstrated above.

7.2.2. Formulas

The above example showed an iterative calculation of the optimal tax when rebalancing a portfolio. Although the calculation can be performed easily in a computer spreadsheet as provided in section 6.3, it is also possible to calculate the optimal tax directly using the following formulas. The derivation of the formulas has been omitted.

Let *Start Value* denote the starting value of the stock investment and let *Return* denote the return on the stock investment. The portfolio value is now the sum of the start value and the return.

We want to rebalance the portfolio according to a given stock-weight. The value of the portfolio after it has been rebalanced and the appropriate tax has been paid is called the portfolio's target value. In order to calculate this we first need to calculate an adjustment factor:

$$Adjustment = 1 - Return \times Tax\ Rate \times Stock\ Weight / (Start\ Value + Return)$$

Eq. 7-3

The target value for the portfolio is then:

$$Target\ Value = (Start\ Value + Return \times (1 - Tax\ Rate)) / Adjustment$$

Eq. 7-4

The part of the portfolio's target value invested in stocks is:

$$Target\ Stock\ Investment = Target\ Value \times Stock\ Weight$$

Eq. 7-5

The amount of stock that has to be sold to meet the portfolio's target allocation is:

$$Sell\ Value = Start\ Value + Return - Target\ Stock\ Investment$$

Eq. 7-6

The tax to be paid is:

$$Tax = Start\ Value + Return - Target\ Value$$

Eq. 7-7

7.2.3. Example

Consider again the example from section 7.2.1 where the starting value for the portfolio was USD 100 which was entirely invested in stocks and the return was USD 100 for a total portfolio value of USD 200 before 25% tax on the profit. Now we wish to rebalance the portfolio so 90% is invested in stocks. The formulas above are used to calculate the amount of stocks to sell and how much tax to pay.

In order to calculate the portfolio's target value we first calculate the adjustment factor using Eq. 7-3:

$$Adjustment = 1 - Return \times Tax\ Rate \times Stock\ Weight / (Start\ Value + Return)$$

$$= 1 - USD\ 100 \times 25\% \times 90\% / (USD\ 100 + USD\ 100) = 0.8875$$

The portfolio's target value is then calculated using Eq. 7-4:

$$Target\ Value = (Start\ Value + Return \times (1 - Tax\ Rate)) / Adjustment$$

$$= (USD\ 100 + USD\ 100 \times (1 - 25\%)) / 0.8875 \approx USD\ 197.18$$

From this we calculate the target amount invested in stocks using Eq. 7-5:

$$\textit{Target Stock Investment} = \textit{Target Value} \times \textit{Stock Weight} \approx \textit{USD } 197.18 \times 0.9 \approx \textit{USD } 177.46$$

The amount of stocks we have to sell is calculated using Eq. 7-6:

$$\textit{Sell Value} = \textit{Start Value} + \textit{Return} - \textit{Target Stock Investment} \approx \textit{USD } 100 + \textit{USD } 100 - \textit{USD } 177.46 \approx \textit{USD } 22.54$$

The amount of tax we have to pay is calculated using Eq. 7-7:

$$\textit{Tax} = \textit{Start Value} + \textit{Return} - \textit{Target Value} \approx \textit{USD } 100 + \textit{USD } 100 - \textit{USD } 197.18 \approx \textit{USD } 2.82$$

So we have to sell stocks in the amount of USD 22.54, pay USD 2.82 in tax, and keep USD 177.46 invested in stocks. The portfolio is then rebalanced to 90% stocks while the tax payment has been minimized.

8. Foreign Currencies

This section is intended for foreigners who invest in the S&P 500 index and are therefore also exposed to fluctuations in the exchange rate between the foreign currency and the US Dollar (USD), which should be taken into account when assessing the annualized returns from a given investment strategy. The section first explains currency hedging which is intended to limit the effect of currency fluctuations. This is followed by statistical studies of currency fluctuations between the US Dollar and several foreign currencies.

8.1. Government Bonds

A foreigner investing in US government bonds would also be exposed to changes in the exchange rate between USD and the foreign currency. This may be desirable if the investor believes the USD will appreciate in value over the foreign currency. But unless the probability of this is deemed high, it may be advisable to instead invest in government bonds of the foreign currency, to avoid being exposed to currency exchange rates which can turn modest bond yields into losses.

Note, however, that foreign government bonds may have higher risks of default than US government bonds which are generally considered completely safe especially in the short-term.

8.2. Currency Hedging

In recent years, investors in some countries have been able to invest in the S&P 500 through Exchange Traded Funds (ETFs) that are also hedged for the currency exchange rate between the foreign currency and US Dollar. See section 6.5.1 for a few examples of such ETFs. Ideally such ETFs would allow foreigners to invest in the S&P 500 and obtain the same returns as American investors without the effect of currency fluctuations between the US Dollar and the foreign currency, which can be substantial as shown below. But in order for the currency hedging to work perfectly, the future price of the S&P 500 must be known in advance which is of course impossible. This section studies the possible imprecision of currency hedging.

8.2.1. Effective Exchange Rate

Assume we are European and invest in the S&P 500. We wish to avoid the currency risk by purchasing a so-called forward contract where we agree to exchange a predetermined amount of USD into EUR after one year. The cost of the forward contract is ignored for now and studied further below.

We do not know in advance whether the return on the S&P 500 is positive or negative so we purchase a forward currency contract that covers the value of the investment at the beginning of the year. Any gain on the investment is therefore un-hedged and will be exchanged at the future exchange rate. Conversely, any loss on the investment means that we do not have enough money to perform the contracted currency exchange and must therefore borrow the remaining amount in USD, then do the currency exchange to EUR for the contracted amount, and finally exchange an amount back into USD so as to repay the loan.

The effective exchange rate takes both the hedged and un-hedged rates into account. It is the amount we ultimately get in EUR after a year, divided by the investment value in USD after the year. The actual amount factors out in the mathematical formula which only uses the exchange rates and return on investment.

Let $Exchange\ Rate_1$ denote today's rate for exchanging one USD to EUR and let $Exchange\ Rate_2$ denote the exchange rate a year from now. Let *Return on Investment* denote the return on the investment for the year

without the effect of currency fluctuations. In our case, this would be the price return on the S&P 500 plus the dividend yield. The effective exchange rate is calculated as:

$$\textit{Effective Exchange Rate}$$

$$= (\textit{Exchange Rate}_1 + (\textit{Return on Investment} - 1) \times \textit{Exchange Rate}_2) / \textit{Return on Investment}$$

Eq. 8-1

Example: Currency Loss

For example, assume today's exchange rate is $\textit{Exchange Rate}_1 = 0.8$ so that 1 USD buys 0.8 EUR and after one year the exchange rate has decreased (20%) to $\textit{Exchange Rate}_2 = 0.64$ so that 1 USD only buys 0.64 EUR. Further assume that the investment in the S&P 500 has gained 30% during the year. Because the 30% gain is un-hedged, only the original part of the portfolio is exchanged into EUR at the higher exchange rate of 0.8 through the forward contract, and the remaining part of the portfolio is exchanged at the lower exchange rate of 0.64. The effective exchange rate is 0.76 as calculated using Eq. 8-1:

$$\textit{Effective Exchange Rate}$$

$$= (\textit{Exchange Rate}_1 + (\textit{Return on Investment} - 1) \times \textit{Exchange Rate}_2) / \textit{Return on Investment}$$

$$= (0.8 + (1.3 - 1) \times 0.64) / 1.3 \approx 0.76$$

The effective exchange rate of 0.76 is slightly worse than the desired exchange rate of 0.8 but much better than the actual exchange rate of 0.64.

If instead there had been a (30%) loss on the investment, then the effective exchange rate would have been about 0.87, which is significantly higher than the desired exchange rate of USD 0.8 and effectively means the European investor would make up some of the loss on the investment in the S&P 500 through the currency hedging. This is because the forward contract for the currency demands that we exchange the full amount prior to the year's loss, so after a year we have to borrow the money in USD to make up for the loss and then do the currency exchange from USD to EUR at the contracted rate of 0.8. Then we exchange a smaller amount in EUR back to USD at today's lower currency exchange rate of 0.64 to repay the loan.

Example: Currency Gain

Now consider the case where there is a 20% gain on the currency so that $\textit{Exchange Rate}_1 = 0.8$ and $\textit{Exchange Rate}_2 = 0.96$. If there was a 30% gain on the investment then the effective exchange rate would be about 0.84 as calculated using Eq. 8-1, which would be better for the European investor than the starting exchange rate of 0.8. Conversely, if there was a (30%) loss on the investment then the effective exchange rate would be 0.73 which would be significantly worse than the desired exchange rate of 0.8.

8.2.2. Malfunctioning Currency Hedge

The hedging malfunctions when the effective exchange rate is lower than the desired exchange rate from the beginning of the year, so the foreign investor loses money on the hedged currency exchange:

$$\textit{Effective Exchange Rate} < \textit{Exchange Rate}_1$$

Eq. 8-2

It follows from Eq. 8-1 that this occurs when there is a gain on the investment and a loss on the currency:

$$\textit{Return on Investment} > 1 \text{ and } \textit{Exchange Rate}_1 > \textit{Exchange Rate}_2$$

Eq. 8-3

The hedging also malfunctions when there is a loss on the investment and a gain on the currency:

$$\textit{Return on Investment} < 1 \text{ and } \textit{Exchange Rate}_1 < \textit{Exchange Rate}_2$$

Eq. 8-4

Note that in this case the effective exchange rate is lower than both the exchange rate at the beginning and at the end of the year, so an un-hedged investment would have performed better.

8.2.3. Monthly Hedging

In practice, the hedging is typically reset monthly so the fluctuations in both the currency and investment value are smaller and hence the hedging is more accurate. The hedging contract is merely 'rolled over' to the next month and actual cash transfers are only needed so as to reset the hedging.

Example

For example, assume we have EUR 8,000 which we exchange to USD 10,000 and invest in the S&P 500. The exchange rate from USD to EUR is 0.8 and to lower the effect of currency fluctuations we also purchase a forward currency contract at this exchange rate.

Recall from Eq. 8-4 that the hedging malfunctions the most when there is a loss on the investment and a gain on the currency. So let us consider such an example. Assume the S&P 500 decreases (10%) so the investment is only worth USD 9,000 and the currency gains 5% thus making the exchange rate 0.84 after the first month. These are large but realistic monthly changes that do occur in the real world.

The effective exchange rate is then about 0.796 as calculated using Eq. 8-1, which means the investment's value of USD 9,000 is only worth EUR 7.164 instead of EUR 7,200 if the hedging had worked perfectly at the desired rate of 0.8. The difference is due to the hedging contract being for the amount USD 10,000 but our investment value had decreased to USD 9,000, so we need to borrow an additional USD 1,000, do the contracted currency exchange at the forward rate of 0.8 to obtain EUR 8,000, then subtract EUR 840 and exchange them back to USD 1,000 and repay the loan. The result is that we have EUR 7,160 left. Note the small difference from the EUR 7,164 calculated above which is due to rounding errors as the effective exchange rate was actually 0.7955555... which was rounded to 0.796.

In practice, the monthly cash transfers and currency exchanges can be avoided by 'rolling over' the forward currency contracts. In this example, the financial institution would merely need to be paid EUR 40 or about USD 47.62 at the end of the month – in addition to the cost of the hedge which is ignored here and described further below.

Rate of Return

Now consider the rate of return. After the first month our investment is worth EUR 7,160 which is 89.5% of the original amount of EUR 8,000 so there was a (10.5%) loss. This means we lost (10%) from the investment in the S&P 500 and (0.5%) from the currency fluctuations. The currency actually gained 5% which would have offset some of the loss on the S&P 500 if we had not used a currency hedge, but we

could not have predicted this and the goal was to lower the effect of currency fluctuations, which was achieved reasonably well as the 5% increase in the exchange rate only had a hedged effect of (0.5%).

Resetting the Hedge

After the first month our portfolio is worth EUR 7,160 corresponding to about USD 8,523.81 at the exchange rate of 0.84. We now make another currency hedge to try and lock in this rate of 0.84 for the next month.

Recall from Eq. 8-3 that another case where the currency hedge malfunctions is when there is a gain on the investment and a loss on the currency, which causes the effective exchange rate to be lower than the desired exchange rate of 0.84. Assume the S&P 500 increases 8% so the value of our portfolio has grown from about USD 8,523.81 to about USD 9,205.71 after the second month. Also assume the currency exchange rate decreases 6% to 0.7896. The effective exchange rate is then about 0.836 as calculated using Eq. 8-1, which means the investment's value of USD 9,205.71 is only worth about EUR 7,695.97 instead of EUR 7,732.80 if the hedging had worked perfectly at the desired rate of 0.84. The difference is EUR 36.83.

Now consider the rate of return. After the second month our hedged investment is worth EUR 7,695.97 which is about 107.5% of the amount EUR 7,160 at the beginning of the month so there was a 7.5% gain. This means we gained 8% from the investment in the S&P 500 and lost about (0.5%) from the currency hedge while the actual currency lost (6%). So the hedging performed reasonably well.

For the two months combined, the S&P 500 had a loss of (2.8%) and the currency had a loss of (1.3%). Without the currency hedge the initial EUR 8,000 would have decreased to EUR 7,674.91 while the perfectly hedged value would only have decreased to EUR 7,776 corresponding to the (2.8%) loss of the S&P 500. The currency hedged result was about EUR 7,695.97 corresponding to a hedged currency loss of about (1%) as opposed to the actual currency loss of (1.3%).

This example was deliberately chosen so as to demonstrate conditions under which the currency hedging malfunctions. Even so, the end result was slightly better than an un-hedged investment in the S&P 500. However, the hedging costs were ignored and must also be taken into account, which is done next.

8.2.4. Cost of Hedging

In the above, it was assumed that a forward currency contract was available for free. But how is the hedging actually implemented and how much does it cost?

How to Hedge

A currency hedge from USD to EUR can be implemented by first borrowing the hedged amount in USD, then immediately exchanging the money to EUR at today's exchange rate, and then buying a risk-free bond in EUR to earn some interest for the duration of the hedging period. After the hedging period is over the loan in USD must be repaid and the amount in EUR is delivered. This locks in today's exchange rate minus the cost of the hedge which depends on the interest rate on the USD loan and the yield on the EUR bonds.

Example

For example, assume you want to hedge USD 10,000 for one year. You first borrow USD 10,000 and immediately exchange it to EUR at today's exchange rate. If the exchange rate is 0.8 then you get EUR 8,000. You then purchase a risk-free European bond for EUR 8,000 whose maturity period matches the

hedging period. When the period is over you receive EUR 8,000 plus the bond yield. If the bond yields 4% then you receive a total amount of EUR 8,320. You also have to repay the original loan of USD 10,000 plus interest, e.g. if the interest is 5% then you have to repay USD 10,500. So we have hedged USD 10,500 to EUR 8,320 for an effective exchange rate of about 0.792 as opposed to the desired exchange rate of 0.8. If instead the US interest rate had been 3% we would only have to repay USD 10,300 and the effective exchange rate would be almost 0.808, that is, slightly better than the desired exchange rate of 0.8.

Formulas

In the example above we wanted to hedge USD 10,000 but we had actually hedged USD 10,300 because of the interest on the USD loan that also had to be paid. To get a hedge for an exact amount the interest on the USD loan must be taken into account. The relation between the hedged and the borrowed amount is:

$$\textit{Hedged Amount in USD} = \textit{Borrowed Amount in USD} \times (1 + \textit{Interest Rate for USD})$$

Eq. 8-5

This can be rewritten as:

$$\textit{Borrowed Amount in USD} = \textit{Hedged Amount in USD} / (1 + \textit{Interest Rate for USD})$$

Eq. 8-6

To calculate the hedged amount that we will ultimately receive in EUR, we first need to calculate the ratio between the bond yield we will get in Europe and the interest rate we have to pay in USA:

$$\textit{Interest Ratio} = (1 + \textit{Bond Yield for EUR}) / (1 + \textit{Interest Rate for USD})$$

Eq. 8-7

Today's exchange rate is denoted $\textit{Exchange Rate}_1$ so the hedged amount we ultimately receive in EUR is:

$$\textit{Hedged Amount in EUR} = \textit{Hedged Amount in USD} \times \textit{Exchange Rate}_1 \times \textit{Interest Ratio}$$

Eq. 8-8

The hedged exchange rate is the amount we ultimately get in EUR divided by the hedged amount in USD:

$$\textit{Hedged Exchange Rate} = \textit{Hedged Amount in EUR} / \textit{Hedged Amount in USD}$$

Eq. 8-9

The cost of the hedge is the difference between the desired amount in EUR and the actual amount we get:

$$\textit{Hedging Cost in EUR} = \textit{Hedged Amount in USD} \times \textit{Exchange Rate}_1 - \textit{Hedged Amount in EUR}$$

Eq. 8-10

There is a hedging cost when the interest rate from borrowing in USA is greater than the bond yield we will get in Europe:

$$\textit{Hedging Cost} > 0 \Leftrightarrow \textit{Interest Rate for USD} > \textit{Bond Yield for EUR}$$

Eq. 8-11

If the hedging cost is negative it means there is a profit from the currency hedging because the interest rate we have to pay in USA is lower than the bond yield we will get in Europe.

The hedging cost can also be expressed as a ratio which is useful for calculating the cost as a percentage:

$$Hedging\ Cost\ Ratio = (1 + Interest\ Rate\ for\ USD) / (1 + Bond\ Yield\ for\ EUR) - 1$$

Eq. 8-12

Example

Continuing the example from above, assume we want to hedge USD 10,000 and the interest rate in USA is 5%. The amount that must be borrowed in USD is calculated using Eq. 8-6:

$$Borrowed\ Amount\ in\ USD = Hedged\ Amount\ in\ USD / (1 + Interest\ Rate\ for\ USD)$$

$$= USD\ 10{,}000 / (1 + 5\%) \approx USD\ 9{,}523.81$$

So we borrow USD 9,523.81 and exchange it immediately to EUR and then invest the money in risk-free European bonds. If the bond yield in Europe is 4% then the interest ratio is calculated using Eq. 8-7:

$$Interest\ Ratio = (1 + Bond\ Yield\ for\ EUR) / (1 + Interest\ Rate\ for\ USD) = (1 + 4\%) / (1 + 5\%) \approx 0.99$$

If today's exchange rate is 0.8 then the hedged amount in EUR is calculated using Eq. 8-8:

$$Hedged\ Amount\ in\ EUR = Hedged\ Amount\ in\ USD \times Exchange\ Rate_1 \times Interest\ Ratio$$

$$\approx USD\ 10{,}000 \times EUR/USD\ 0.8 \times 0.99 \approx EUR\ 7{,}920$$

The hedged exchange rate is calculated using Eq. 8-9:

$$Hedged\ Exchange\ Rate = Hedged\ Amount\ in\ EUR / Hedged\ Amount\ in\ USD \approx EUR\ 7{,}920 / USD\ 10{,}000$$

$$\approx 0.792$$

So the exchange rate that we get by hedging is about 0.792 which is slightly lower than today's exchange rate of 0.8 that we desired. The difference is the cost of the hedge which is calculated using Eq. 8-10:

$$Hedging\ Cost\ in\ EUR = Hedged\ Amount\ in\ USD \times Exchange\ Rate_1 - Hedged\ Amount\ in\ EUR$$

$$\approx USD\ 10{,}000 \times EUR/USD\ 0.8 - EUR\ 7{,}920 \approx EUR\ 80$$

If instead the interest rate in USA had been 3% then the hedging cost would have been negative EUR (78), that is, a profit of about EUR 78 because we earn more on the EUR bond yields than we have to pay interest on borrowing money in USA.

The hedging cost can also be expressed as a percentage which is calculated using Eq. 8-12:

$$Hedging\ Cost\ Ratio = (1 + Interest\ Rate\ for\ USD) / (1 + Bond\ Yield\ for\ EUR) - 1 = (1 + 5\%) / (1 + 4\%) - 1 \approx 1\%$$

So the hedging cost is about 1%. If instead the interest rate in USA had been 3% then the hedging cost would have been negative at about (1%), that is, a profit of about 1%.

The hedging cost increases with the difference between the interest rate in USA and the bond yield in Europe. For example, if the interest rate in USA is 5% and the bond yield in Europe is 0.5% then the hedging cost is about 4.5% as calculated using Eq. 8-12. If instead the interest rate in USA is 3% and the bond yield in Europe is 8% then the hedging cost is about (4.6%), that is, a profit of about 4.6%.

Note that the hedging cost is approximately equal to the difference (i.e. subtraction) between the interest rate we have to pay in USA and the bond yield we get in Europe, but the correct calculation uses division while subtraction gives a small error. In the previous example where the interest rate in USA was 3% and the bond yield in Europe was 8% the actual hedging cost was (4.6%) as correctly calculated using Eq. 8-12 while merely subtracting the rates gives (5%) which has a small error.

8.2.5. Summary

A currency hedge should ideally remove the effect of currency fluctuations on investment returns, but the reality is more complicated as demonstrated above.

The hedging cost is approximately the difference between the interest rate in the foreign country and USA. If the interest rate in USA is lower than that of the foreign country, then the currency hedge actually results in a profit. If the investor intends to hold the investment for many years or decades, then the cost of the currency hedge should be taken into account because a small annual cost may compound into substantial under-performance over many years.

Furthermore, the effectiveness of the currency hedge depends on the magnitude and direction of currency fluctuations as well as changes in the value of the investment that is being hedged. This may cause the currency hedge to perform either better or worse than intended.

Altogether, currency hedging should not be expected to completely eliminate the effect of currency fluctuations but rather dampen the effect.

Currency hedging is useful for investors who are concerned about short-term currency fluctuations, or who believe the USD will soon decrease in value against the foreign currency. Because future currency fluctuations are unpredictable and the hedging is imperfect and potentially costly to long-term investors, it may be useful to hedge only a part of the portfolio. For example, half of the investment in the S&P 500 can be made in a currency hedged ETF and the remaining half can be made in an un-hedged ETF. The ratio between hedged and un-hedged investments depends on the investor's personal beliefs and tolerance for future currency fluctuations.

8.3. Forecasting Exchange Rates

The currency exchange rate between two countries depends on many aspects of the economies of the two individual countries as well as their mutual relationship and is therefore complicated to forecast with any accuracy. But a simple way of estimating a possible range of future exchange rates is to assume the historical exchange rate will repeat itself in random order. This is a reasonable forecasting method if the economies of the two countries can be assumed to develop somewhat similarly to their past.

The exchange rate within the next few years is likely biased by the exchange rate of today because changes usually occur gradually over time. So the full historical distribution of exchange rates must be limited and

biased somehow if used to forecast the exchange rate within a few years. For ten years into the future, it seems reasonable that the full historical distribution can be used without any bias or limitation.

Note that this simple method of using the historical exchange rate for estimating the range of future exchange rates will likely not work for countries whose economies are undergoing radical change.

8.4. Annualized Return

The annualized return that takes the currency exchange rate into account is calculated as follows. Let *n* denoted the number of years for the investment. At the beginning of the period the exchange rate from USD to the foreign currency is denoted $Exchange\ Rate_1$. The investor makes the opposite exchange from the foreign currency to USD and hence divides the amount in foreign currency by this exchange rate. During the investment period of *n* years, the investment accumulates in USD. After the period of *n* years has ended, the investor exchanges the total return on the investment from USD back into the foreign currency at the rate denoted $Exchange\ Rate_n$.

The investor's return on the currency exchange is:

$$Currency\ Return_n = Exchange\ Rate_n / Exchange\ Rate_1$$

Eq. 8-13

The investor's return consists of both the return on the currency and the investment itself. The annualized return over *n* years is the *n*'th root of the combined return. The formula is:

$$Annualized\ Return = (Total\ Return\ on\ Investment_n \times Currency\ Return_n)^{1/n} - 1$$

$$= (Total\ Return\ on\ Investment_n)^{1/n} \times (Currency\ Return_n)^{1/n} - 1$$

Eq. 8-14

For example, assume the investment period is $n = 10$ years and the total return at the end of this period is 2.25 thus giving a 125% gain on the investment itself without taking the exchange rate into account. The annualized return on the investment is $2.25^{1/10} - 1 \approx 8.4\%$. The exchange rate at the beginning is $Exchange\ Rate_1 = 1.1$ which means that 1 USD equals 1.1 of the foreign currency, or equivalently 1 of the foreign currency equals about 0.9091 of USD. After ten years the exchange rate is $Exchange\ Rate_{10} = 0.88$ which means that 1 USD equals 0.88 of the foreign currency. This gives a (20%) loss on the currency over ten years. The annualized return on the currency is $(0.88/1.1)^{1/10} - 1 \approx (2.2\%)$. The combination is an annualized return of 6.05% which is calculated using Eq. 8-14 as follows:

$$Annualized\ Return = (Total\ Return\ on\ Investment_n)^{1/n} \times (Currency\ Return_n)^{1/n} - 1$$

$$= 2.25^{1/10} \times (0.88/1.1)^{1/10} - 1 \approx 6.05\%$$

Note that the returns must be multiplied like this rather than added, although the results are approximately equal for modest returns and exchange rate differences. That is, 8.4% minus 2.2% equals 6.2% but this is an incorrect calculation and the correct result is about 6.05% as calculated using the formula above.

8.5. Exchange Rate for USD-CAD

Figure 4 shows the historical currency exchange rate for converting one US Dollar (USD) to Canadian Dollar (CAD). The data period is from January 1971 to January 2015. On January 6, 2015 the exchange rate was 1.1770 which means that 1 USD would buy 1.1770 CAD.

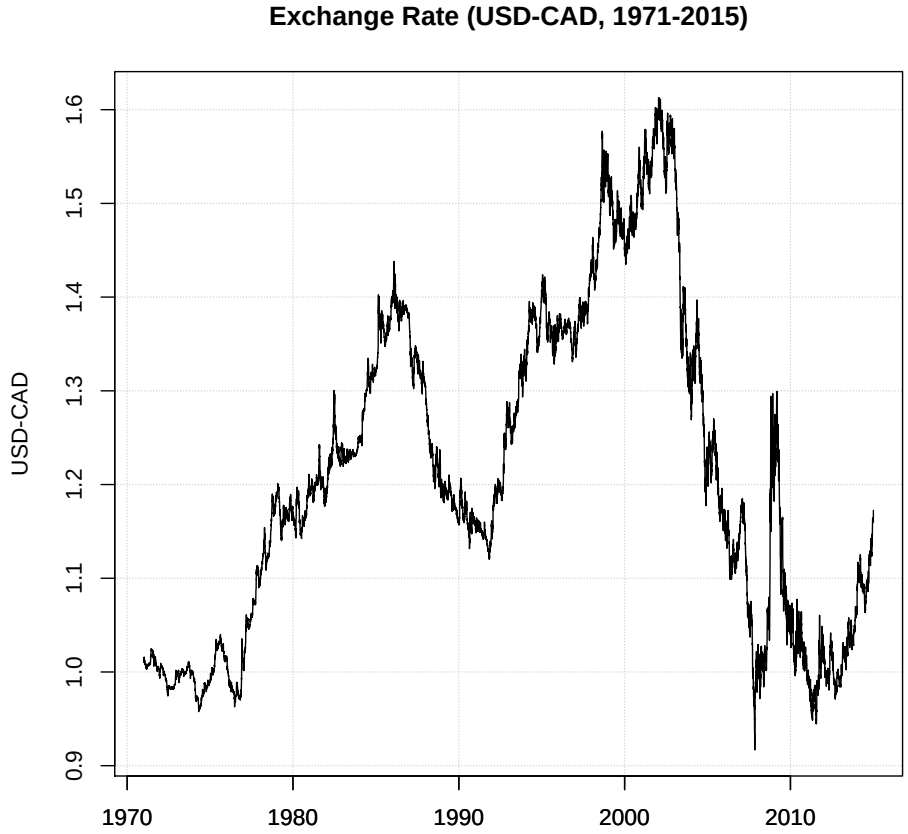

Figure 4: USD-CAD currency exchange rate for converting one US Dollar (USD) to Canadian Dollar (CAD) during 1971-2015. Data source [20**].**

8.5.1. Cumulative Distribution Function

Figure 5 shows the Cumulative Distribution Function (CDF) which gives the historical probability of the exchange rate being lower than some number. For example, the probability was about 0.1 (or 10%) that the exchange rate was lower than 1.0, which means that during the period 1971-2015 the exchange rate was lower than 1.0 about 10% of the time. The exchange rate is 1.1770 at the time of this writing in early January 2015 and according to the CDF the historical exchange rate was lower than 1.1770 about 0.45 (or 45%) of the time during the period 1971-2015.

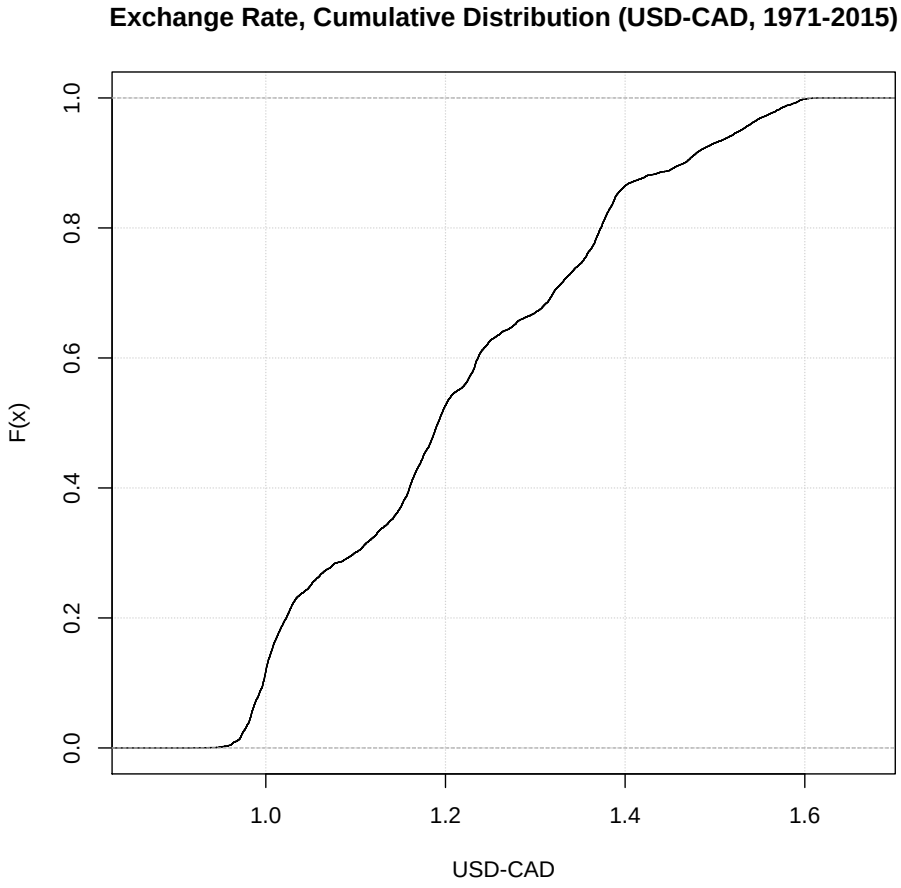

Figure 5: USD-CAD Cumulative Distribution Function (CDF) for the currency exchange rate during 1971-2015.

8.5.2. One-Year Returns on Currency

Figure 6 shows a histogram of the one-year returns in the USD-CAD exchange rate, most of which were between ±10% but some were below (20%) and some above 30%. So there can be substantial gains or losses within a single year on the currency, which must be taken into account when a foreign resident invests in the S&P 500 index.

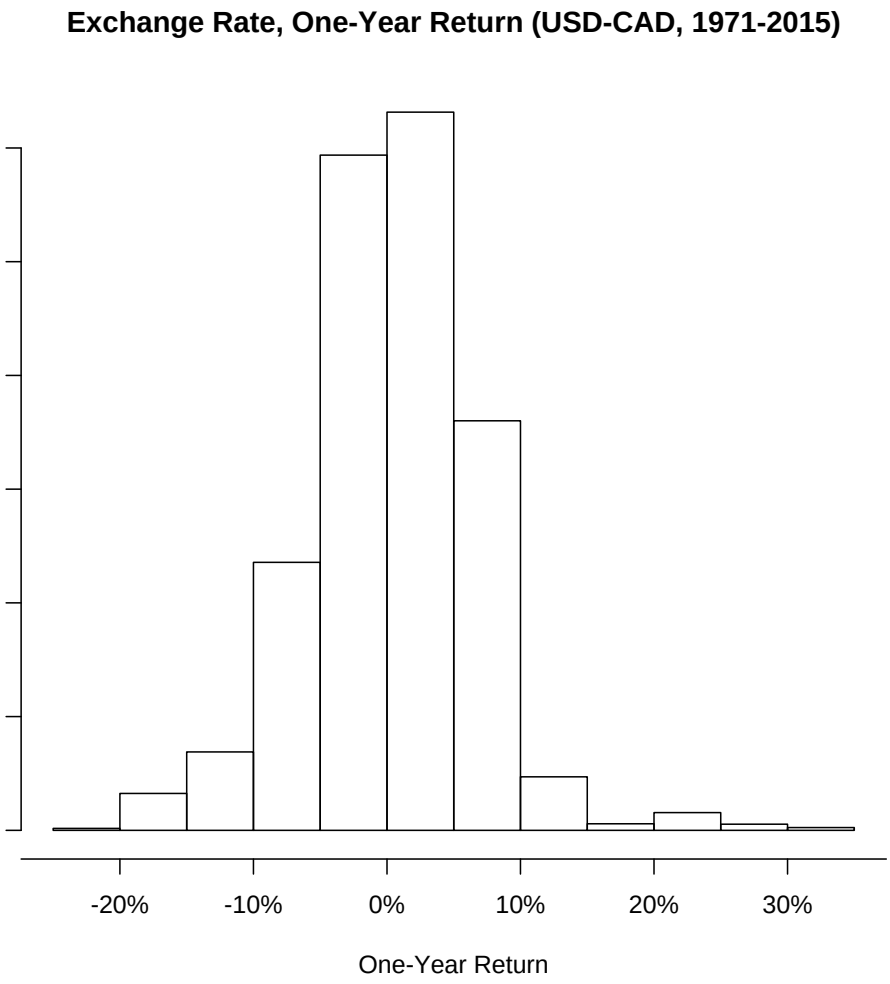

Figure 6: USD-CAD histogram of one-year returns on the currency exchange rate between US Dollar (USD) and Canadian Dollar (CAD) during 1971-2015.

8.5.3. Annualized Return on Currency

Figure 7 shows the Cumulative Distribution Function for the annualized return on the currency, which is calculated using the currency exchange part of Eq. 8-14. The period is assumed to be n = 10 years and the exchange rate at the beginning of the period is assumed to be 1.1770 as it was at the time of this writing. Assuming the future exchange rate is random from its historical distribution, the CDF in Figure 7 gives the probability of the annualized return being lower than some number.

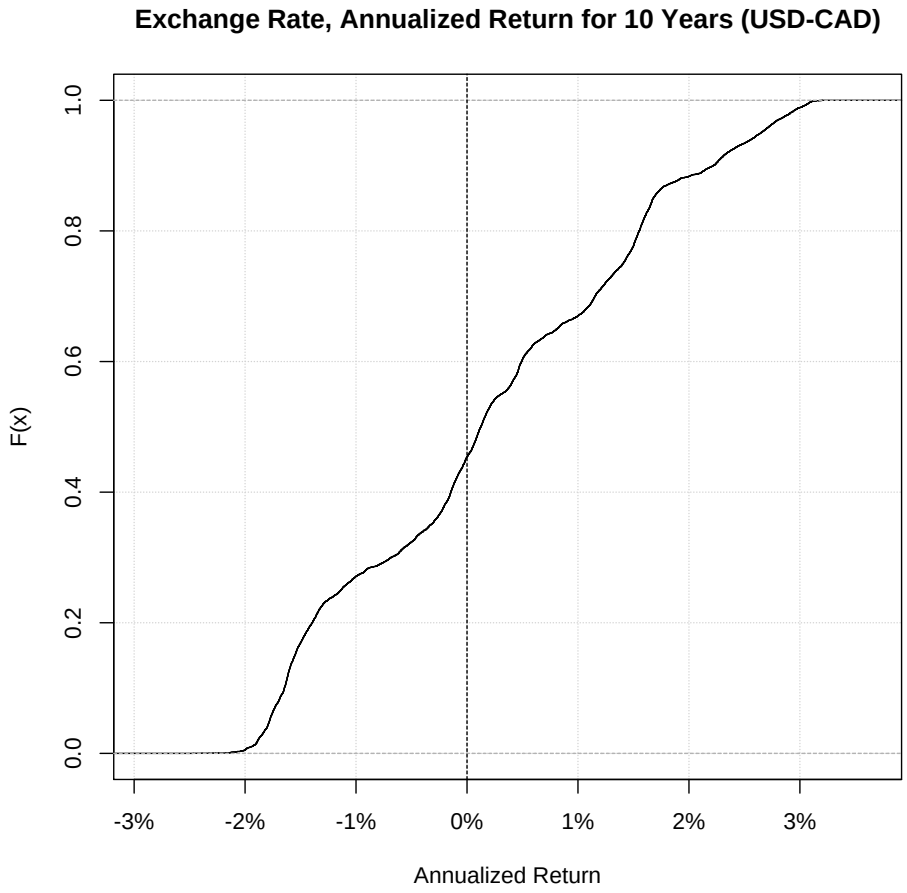

Figure 7: USD-CAD Cumulative Distribution Function (CDF) for the annualized returns when buying USD at the exchange rate 1 USD = 1.1770 CAD and then after 10 years buying CAD at an exchange rate from the historical distribution in Figure 4.

For example, the probability is about 0.45 (or 45%) that the annualized return is negative which means there is a loss on the currency. The probability is about 0.28 (or 28%) that the annualized return on the currency is less than (1%), and the probability is almost zero that the annualized return is less than (2%).

Conversely, the probability is about 0.33 (or 33%) that the annualized return on the currency is a gain greater than 1%, while the probability is about 0.12 (or 12%) that the annualized return is greater than 2%, and the probability is almost zero that the annualized return on the currency is greater than 3%. Note that this plot depends on both the exchange rate at the beginning of the period (here 1.1770) and the duration of the period (here 10 years), so the plot of the CDF must be re-done for other numbers.

8.5.4. Currency-Adjusted Returns for the S&P 500

The correct way of calculating the currency-adjusted return on the S&P 500 is to use Eq. 8-14. However, an approximation that is simpler to calculate is to merely add the annualized returns from the currency to the annualized returns from the investment in the S&P 500. The small numerical imprecision may be deemed acceptable because the true probability distribution is not known anyway.

For example, if the exchange rate for USD-CAD is 1.1770 at the beginning of the investment, then Figure 7 shows that there is a probability of about 0.45 (or 45%) that there will be a loss on the currency after ten years, while there is almost zero probability that the annualized return on the currency is less than (2%). Conversely, there is almost zero probability that the annualized return on the currency is greater than 3%.

If the mean annualized return for the S&P 500 is estimated at 8% over the next ten years, then the currency-adjusted return is likely between 6% and 11%, calculated simply by adding the estimated return for the S&P 500 and the lower and upper bounds for the currency returns.

Similar calculations can be done for other return estimates of the S&P 500. For example, if the lowest annualized return for the S&P 500 is estimated at (10%), then the currency-adjusted return is likely between (7%) and (12%). Conversely, if the greatest annualized return for the S&P 500 is estimated at 20% then the currency-adjusted return is likely between 18% and 23%.

8.6. Exchange Rate for USD-GBP

Figure 8 shows the historical currency exchange rate for converting one US Dollar (USD) to British Pound Sterling (GBP). The data period is from January 1971 to January 2015. On January 6, 2015 the exchange rate was 0.6569 which means that 1 USD would buy 0.6569 GBP.

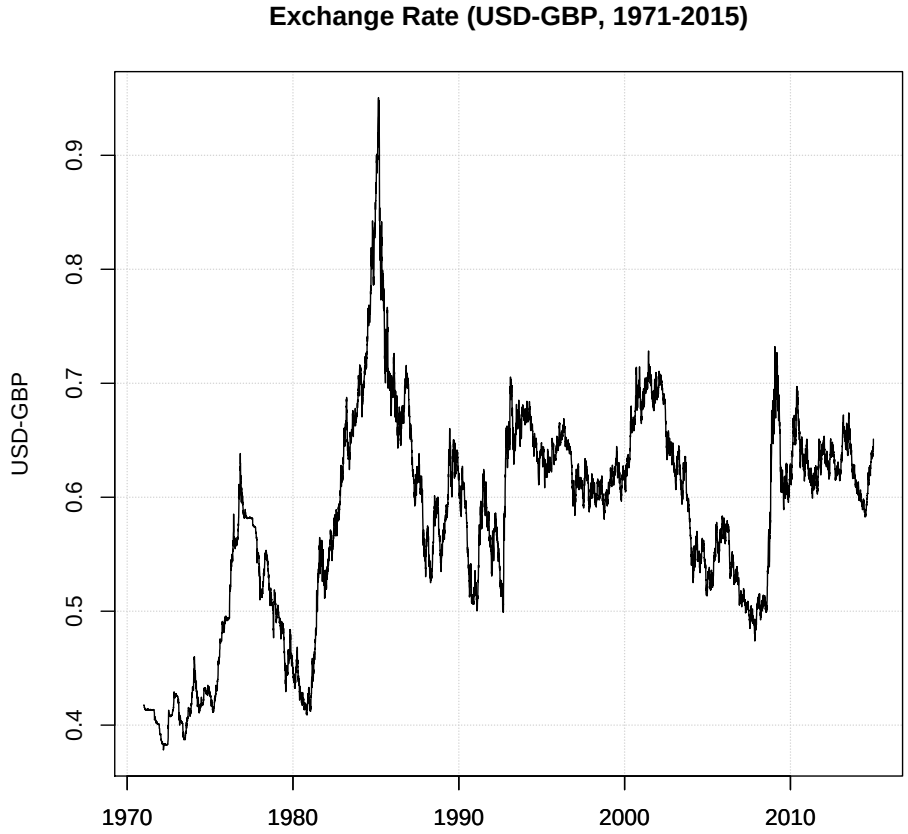

Figure 8: USD-GBP currency exchange rate for converting one US Dollar (USD) to British Pound Sterling (GBP) during 1971-2015. Data source [21**].**

8.6.1. Cumulative Distribution Function

Figure 9 shows the Cumulative Distribution Function (CDF) which gives the historical probability of the exchange rate being lower than some number. For example, the exchange rate is 0.6569 at the time of this writing in early January 2015 and according to the CDF the historical exchange rate was lower than 0.6569 about 0.82 (or 82%) of the time during the period 1971-2015.

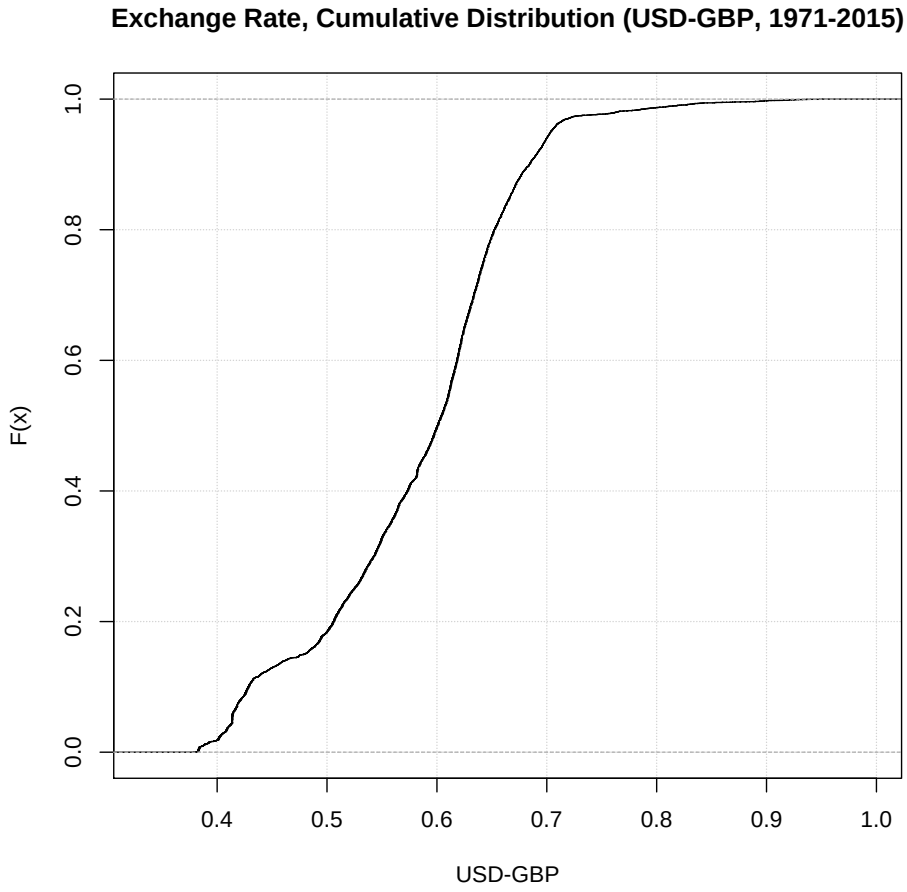

Figure 9: USD-GBP Cumulative Distribution Function (CDF) for the currency exchange rate during 1971-2015.

8.6.2. One-Year Returns on Currency

Figure 10 shows a histogram of the one-year returns in the USD-GBP exchange rate, most of which were between ±15% but some were below (20%) and some above 40%. So there can be substantial gains or losses within a single year on the currency, which must be taken into account when a foreign resident invests in the S&P 500 index.

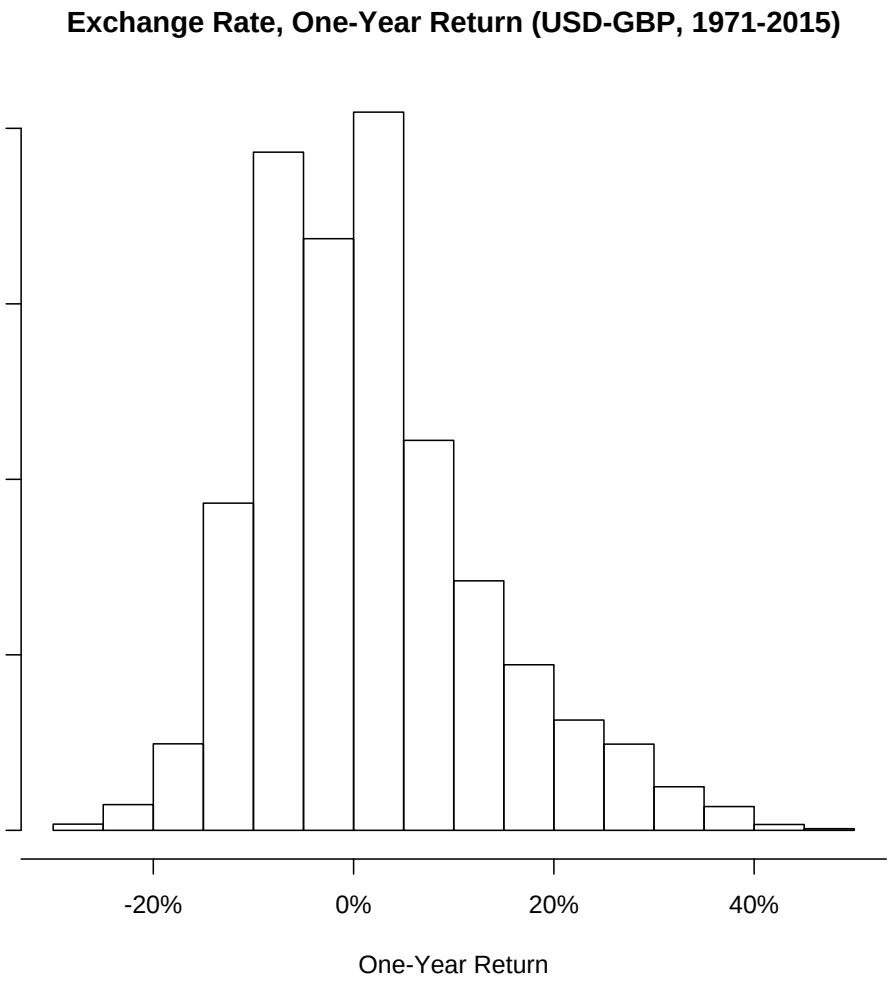

Figure 10: USD-GBP histogram of one-year returns on the exchange rate between US Dollar (USD) and British Pound Sterling (GBP) during 1971-2015.

8.6.3. Annualized Return on Currency

Figure 11 shows the Cumulative Distribution Function for the annualized return on the currency, which is calculated using the currency exchange part of Eq. 8-14. The period is assumed to be $n = 10$ years and the exchange rate at the beginning of the period is assumed to be 0.6569 as it was at the time of this writing. Assuming the future exchange rate is random from its historical distribution, the CDF in Figure 11 gives the probability of the annualized return being lower than some number.

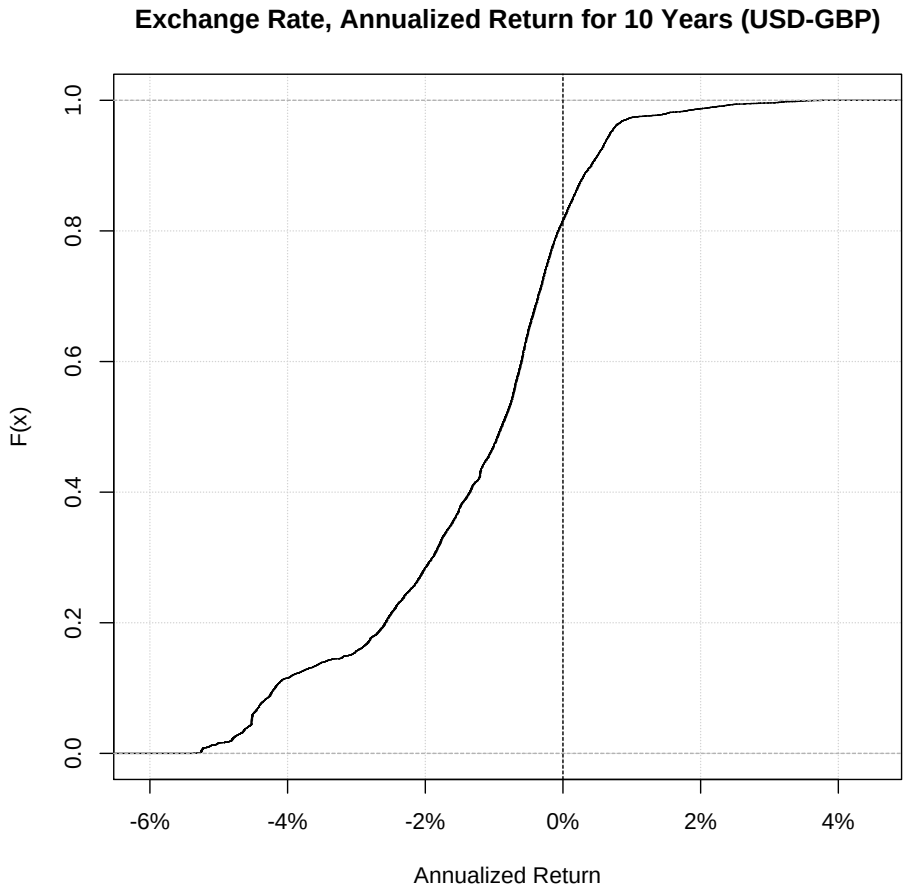

Figure 11: USD-GBP Cumulative Distribution Function (CDF) for the annualized returns when buying USD at the exchange rate 1 USD = 0.6569 GBP and then after 10 years buying GBP at an exchange rate from the historical distribution in Figure 8.

For example, the probability is about 0.82 (or 82%) that the annualized return is negative which means there is a loss on the currency. The probability is about 0.28 (or 28%) that the annualized return on the currency is less than (2%), and the probability is almost zero that the annualized return is less than (5%).

Conversely, the probability is about 0.03 (or 3%) that the annualized return is greater than 1% and the probability is almost zero that the annualized return on the currency is greater than 3%. Note that this plot depends on both the exchange rate at the beginning of the period (here 0.6569) and the duration of the period (here 10 years), so the plot of the CDF must be re-done for other numbers.

8.6.4. Currency-Adjusted Returns for the S&P 500

The correct way of calculating the currency-adjusted return on the S&P 500 is to use Eq. 8-14. However, an approximation that is simpler to calculate is to merely add the annualized returns from the currency to the annualized returns from the investment in the S&P 500. The small numerical imprecision may be deemed acceptable because the true probability distribution is not known anyway.

For example, if the exchange rate for USD-GBP is 0.6569 at the beginning of the investment, then Figure 11 shows that there is a probability of about 0.82 (or 82%) that there will be a loss on the currency after ten years, while there is almost zero probability that the annualized return on the currency is less than (5%). Conversely, there is almost zero probability that the annualized return on the currency is greater than 3%.

If the mean annualized return for the S&P 500 is estimated at 8% over the next ten years, then the currency-adjusted return is likely between 3% and 11%, calculated simply by adding the estimated return for the S&P 500 and the lower and upper bounds for the currency returns.

Similar calculations can be done for other return estimates of the S&P 500. For example, if the lowest annualized return for the S&P 500 is estimated at (10%), then the currency-adjusted return is likely between (7%) and (15%). Conversely, if the greatest annualized return for the S&P 500 is estimated at 20% then the currency-adjusted return is likely between 15% and 23%.

8.7. Exchange Rate for USD-EUR

Figure 12 shows the historical currency exchange rate for converting one US Dollar (USD) to Euro (EUR). The data period is from January 1999 to January 2015. On January 6, 2015 the exchange rate was 0.8380 which means that 1 USD would buy 0.8380 EUR.

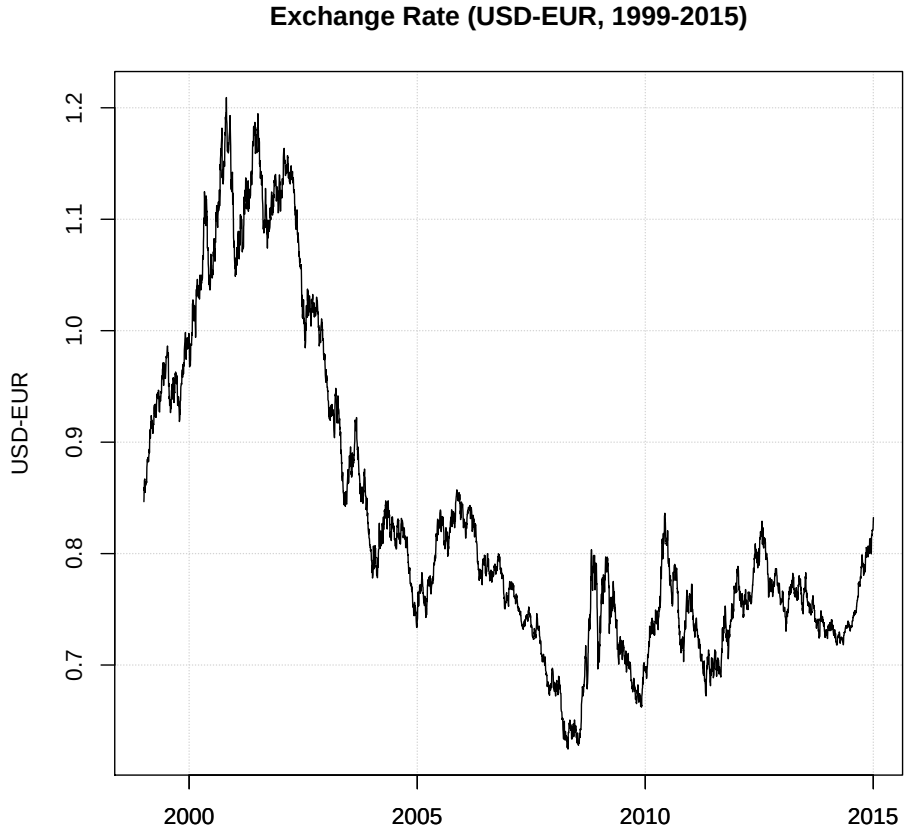

Figure 12: USD-EUR currency exchange rate for converting one US Dollar (USD) to Euro (EUR) during 1999-2015. Data source [22].

8.7.1. Cumulative Distribution Function

Figure 13 shows the Cumulative Distribution Function (CDF) which gives the historical probability of the exchange rate being lower than some number. For example, the exchange rate is 0.8380 at the time of this writing in early January 2015 and according to the CDF the historical exchange rate was lower than 0.8380 about 0.68 (or 68%) of the time during the period 1999-2015.

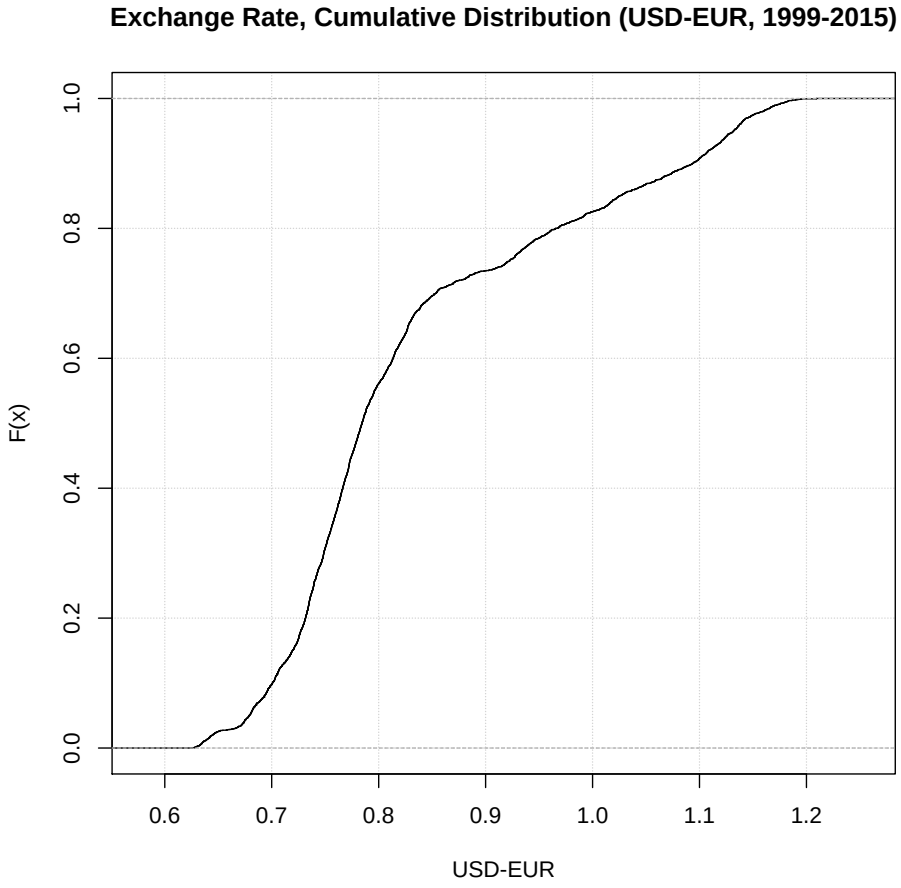

Figure 13: USD-EUR Cumulative Distribution Function (CDF) for the currency exchange rate during 1999-2015.

8.7.2. One-Year Returns on Currency

Figure 14 shows a histogram of the one-year returns in the USD-EUR exchange rate, which range between about (25%) and 30%. So there can be substantial gains or losses within a single year on the currency, which must be taken into account when a foreign resident invests in the S&P 500 index.

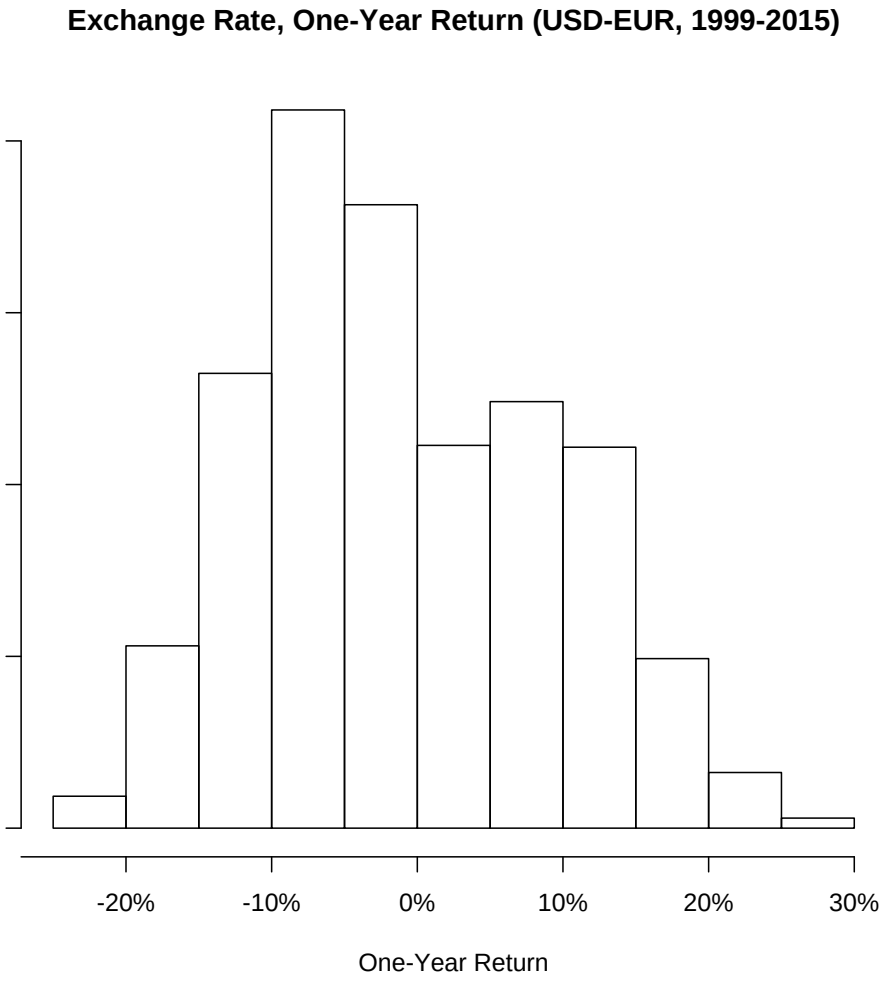

Figure 14: USD-EUR histogram of one-year returns on the exchange rate between US Dollar (USD) and Euro (EUR) during 1999-2015.

8.7.3. Annualized Return on Currency

Figure 15 shows the Cumulative Distribution Function for the annualized return on the currency, which is calculated using the currency exchange part of Eq. 8-14. The period is assumed to n = 10 years and the exchange rate at the beginning of the period is assumed to be 0.8380 as it was at the time of this writing. Assuming the future exchange rate is random from its historical distribution, the CDF in Figure 15 gives the probability of the annualized return being lower than some number.

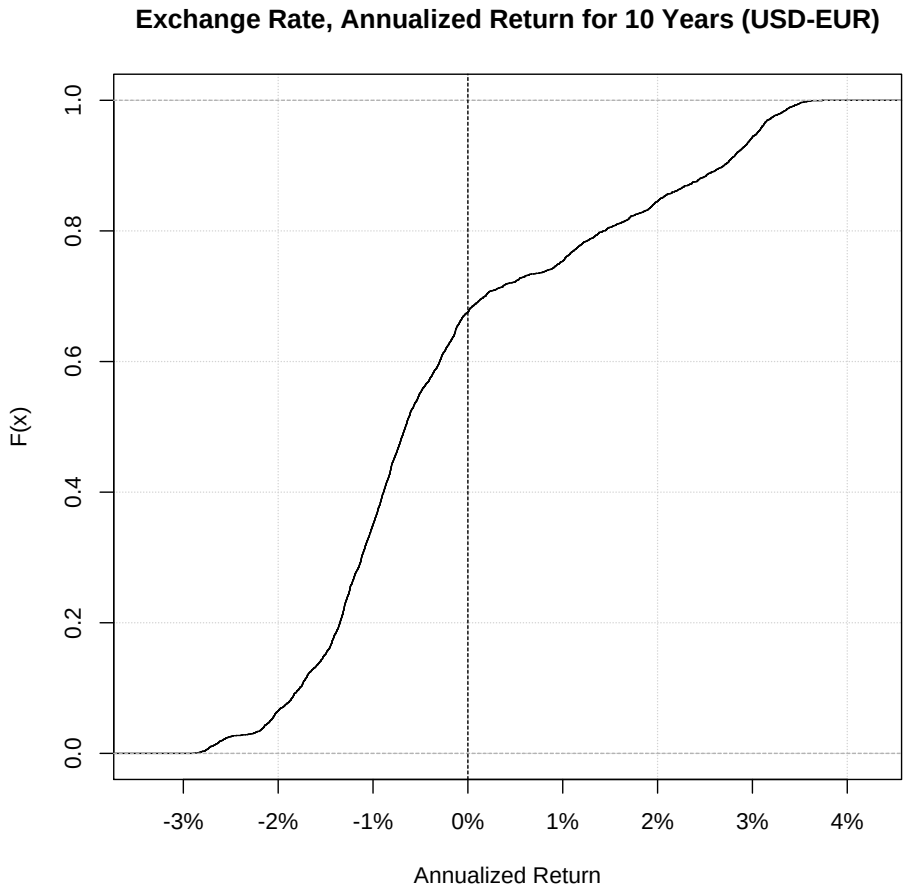

Figure 15: USD-EUR Cumulative Distribution Function (CDF) for the annualized returns when buying USD at the exchange rate 1 USD = 0.8380 EUR and then after 10 years buying EUR at an exchange rate from the historical distribution in Figure 12.

For example, the probability is about 0.68 (or 68%) that the annualized return is negative which means there is a loss on the currency. The probability is about 0.32 (or 32%) that the annualized return on the currency is less than (1%), and the probability is zero that the annualized return is less than (3%).

Conversely, the probability is about 0.26 (or 26%) that the annualized return on the currency is a gain greater than 1%, while the probability is zero that the annualized return is greater than 4%. Note that this plot depends on both the exchange rate at the beginning of the period (here 0.8380) and the duration of the period (here 10 years), so the plot of the CDF must be re-done for other numbers.

8.7.4. Currency-Adjusted Returns for the S&P 500

The correct way of calculating the currency-adjusted return on the S&P 500 is to use Eq. 8-14. However, an approximation that is simpler to calculate is to merely add the annualized returns from the currency to the annualized returns from the investment in the S&P 500. The small numerical imprecision may be deemed acceptable because the true probability distribution is not known anyway.

For example, if the exchange rate for USD-EUR is 0.8380 at the beginning of the investment, then Figure 15 shows that there is a probability of about 0.68 (or 68%) that there will be a loss on the currency after ten years, while there is zero probability that the annualized return on the currency is less than (3%). Conversely, there is zero probability that the annualized return on the currency is greater than 4%.

If the mean annualized return for the S&P 500 is estimated at 8% over the next ten years, then the currency-adjusted return is likely between 5% and 12%, calculated simply by adding the estimated return for the S&P 500 and the lower and upper bounds for the currency returns.

Similar calculations can be done for other return estimates of the S&P 500. For example, if the lowest annualized return for the S&P 500 is estimated at (10%), then the currency-adjusted return is likely between (6%) and (13%). Conversely, if the greatest annualized return for the S&P 500 is estimated at 20% then the currency-adjusted return is likely between 17% and 24%.

9. Bond-Only Investment

The investment available in USA with the lowest risk of loss is government bonds, which are considered risk-free because it is deemed impossible for the government of USA to default on its debt obligations.

US government bonds that mature in one year or less are commonly known as Treasury Bills and do not pay any interest during the year but are instead sold at a discount of the face (or par) value which is repaid in full at the time of maturity so as to create a positive yield.

We consider government bonds with one year maturity because the investment strategies are designed for rebalancing intervals of one year. That is, an investment allocation is made between US government bonds and the S&P 500 index. After a year the bond matures and is repaid in full by the government. In addition there is a dividend yield from the S&P 500 and either a gain or loss on the price of the S&P 500. A new investment allocation is then made between the S&P 500 index and government bonds. This process continues for a number of years.

The investment portfolio that has the least risk of loss is a full investment in US government bonds and nothing invested in the S&P 500 index. This investment strategy is called 'bond-only' in the following and will serve as a benchmark of the risk-free return that could have been earned during some period, which can then be compared to the return of other investment strategies so as to assess their performance relative to a risk-free investment.

There are also government bonds with much longer maturity than one year, e.g. 10 or 30 years, which typically have much higher yield to maturity than the bonds with one year maturity. But the prices of bonds change inversely with their yield, and the price change becomes larger for bonds with longer maturity. So if a bond with long maturity is sold before it matures it may actually result in a large loss. This problem is avoided by restricting ourselves to government bonds of one year maturity and investment strategies that are rebalanced annually.

9.1. Historical Data

Figure 3 shows the yield for US government bonds with 1 year maturity for the period 1978-2013. In this period the average bond yield was 5.6% and the standard deviation was 3.8%. But these two summary statistics do not reveal much about the distribution of the bond yields. For example, in the early 1980's the bond yield was above 15% and in 2013 the bond yield was almost 0%. Figure 16 shows the histogram which can be used to assess the shape of the distribution. Figure 17 shows the Cumulative Distribution Function (CDF) which can be used to answer questions such as: How often were the bond yields below 2% (the answer is about 20% of the time), or how often were the bond yields above 10% (the answer is about 12% of the time).

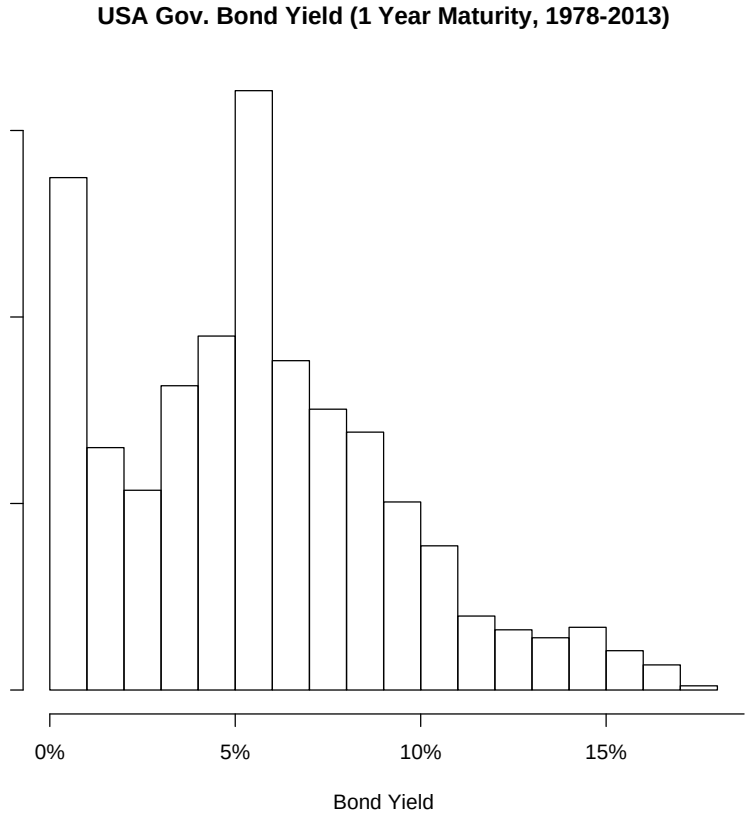

Figure 16: US government bond yields with 1 year maturity. Histogram for the data in Figure 3 covering the period 1978-2013.

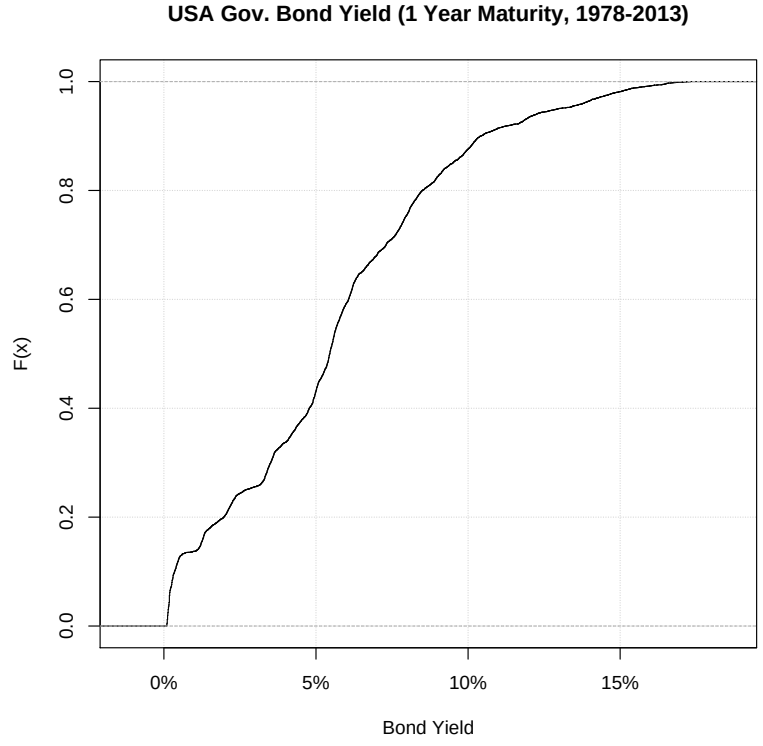

Figure 17: US government bond yields with 1 year maturity. Cumulative Distribution Function (CDF) for the data in Figure 3 covering the period 1978-2013.

9.2. Annualized Return

We are interested in knowing the return an investor would have earned by investing in government bonds during this period. If the investment period was one year then we know from above that the return could have been as low as nearly 0% at some time and above 15% at another time. On average the return was 5.6% with standard deviation 3.8%. But this does not extend to multi-year returns because the bond yield was not completely random as shown in Figure 3. Periods of high bond yields are usually followed by lower yields and vice versa. The starting date and duration of an investment therefore affects the return.

9.2.1. Formulas

Before we can study the returns for multiple years we first need a few mathematical formulas to express different kinds of returns. Taxes are ignored in these formulas and discussed in section 7.

The return for year *t* is denoted $Return_t$ and in this case equals the bond yield. The compounded return is the multiplication of the returns for several years so as to obtain the cumulative return for the entire investment period. This corresponds to getting $Return_1$ on the investment the first year, then reinvesting all the proceeds and getting $Return_2$ the second year, and so on. For *n* years the compounded return is:

$$Compounded\ Return_n = (1 + Return_1) \times (1 + Return_2) \times \ldots \times (1 + Return_n) - 1$$

Eq. 9-1

The annualized return is defined from the compounded return as follows:

$$Annualized\ Return_n = (1 + Compounded\ Return_n)^{1/n} - 1$$

Eq. 9-2

The annualized return should be understood as the rate of return that compounds over *n* years to become the original compounded return. This can be seen by rewriting Eq. 9-2:

$$Compounded\ Return_n = (1 + Annualized\ Return_n)^n - 1$$

Eq. 9-3

The total return is simply the compounded return plus one:

$$Total\ Return_n = Compounded\ Return_n + 1$$

Eq. 9-4

9.2.2. Example

Consider an example. On September 3, 1981 the bond yield was 17.31%. If an investor bought bonds on this date then a year later on September 3, 1982 the investor would get the purchase price plus an additional 17.31% in return. On September 3, 1982 the bond yield had decreased to 10.88%, so if on this date the investor again bought bonds for the entire portfolio including the return from the first year, then on September 3, 1983 the compounded return would be:

$$Compounded\ Return_2 = (1 + 17.31\%) \times (1 + 10.88\%) - 1 \approx 30.07\%$$

Eq. 9-5

That is, the investor's portfolio would have grown about 30.07% over two years from investing the portfolio entirely in these government bonds.

There is no bond data available for September 3, 1983 possibly because the markets were closed. The next available date is September 6, 1983 on which the bond yield was 10.46%. So if the investor again invested the entire portfolio in these bonds then on September 6, 1984 the portfolio's value would have grown by:

$$Compounded\ Return_3 = (1 + 17.31\%) \times (1 + 10.88\%) \times (1 + 10.46\%) - 1 \approx 43.68\%$$

It took three years of investing to obtain this gain of 43.68%. In Eq. 9-5 the gain was 30.07% for two years of investing. These returns can be compared independently of the investment period by calculating their annualized returns. For the two-year investment period the annualized return is calculated using Eq. 9-2:

$$Annualized\ Return_2 = (1 + Compounded\ Return_2)^{1/2} - 1 = (130.07\%)^{1/2} - 1 \approx 14.05\%$$

That is, the gain of 30.07% over two years corresponds to a single-year gain of 14.05% which is compounded for two years. We can check that this is correct using Eq. 9-3:

$$(1 + Annualized\ Return_2)^2 - 1 = (1 + 14.05\%) \times (1 + 14.05\%) - 1 \approx 30.07\%$$

For the three years of investing the gain was 43.68% so the annualized return was:

$$Annualized\ Return_3 = (1 + Compounded\ Return_3)^{1/3} - 1 = (143.68\%)^{1/3} - 1 \approx 12.84\%$$

That is, the gain of 43.68% over three years corresponds to a single-year gain of 12.84% which is compounded for three years. We can check that this is correct by compounding the return three times:

$$(1 + Annualized\ Return_3)^3 - 1 = (1 + 12.84\%) \times (1 + 12.84\%) \times (1 + 12.84\%) - 1 \approx 43.68\%$$

So a bond investment started on September 3, 1981 would have returned 17.31% after one year, an annualized return of 14.05% after investing one year and reinvesting the proceeds a second year, and an annualized return of 12.84% after investing and reinvesting the proceeds for all three years. Note the gradual decrease in annualized return which is because the reinvestments were made at lower bond yields.

9.2.3. Example

Consider another example. On January 14, 1993 the bond yield was 3.48%. A year later in 1994 the bond yield was 3.55% and another year later in 1995 the bond yield was 6.99%. The compounded return of reinvesting for the first two years was 7.15% and it was 14.64% for reinvesting all three years. The annualized return for investing the first two years was about 3.51% and it was 4.66% for the three years. In this case the annualized returns increased as the investment period got longer because the reinvestments were made at higher bond yields.

9.2.4. Statistics

The above examples demonstrate that the return from investing in government bonds depends on the starting date and the number of years of reinvesting. It is therefore necessary to consider all possible starting dates and investment periods when assessing the historical returns that could have been earned from investing in government bonds.

Figure 18 shows a box-plot of the annualized return of investing in US government bonds for all possible starting dates between 1978 and 2013 and investment periods up to 10 years. (Note that the whiskers are located 1.5 IQR (inter-quartile range) from the quartiles and the scale of the y-axis is identical to the other box-plots below for easy comparison.) Table 1 shows the same statistical information in tabular form along with some additional statistics.

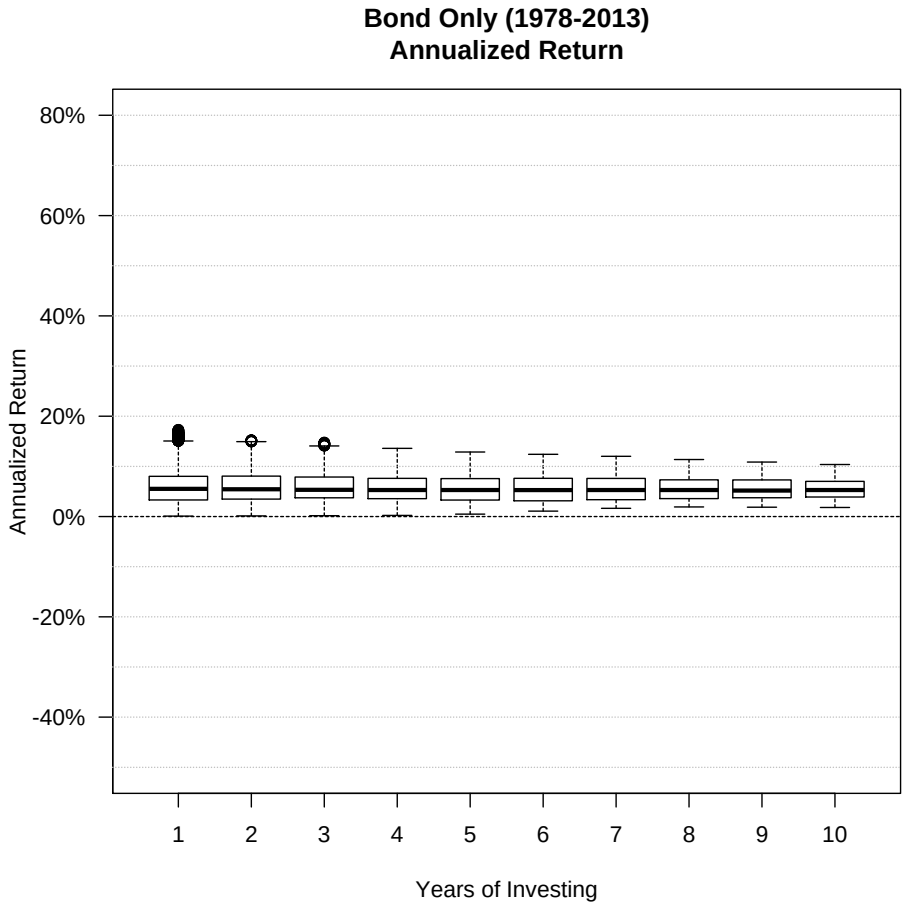

Figure 18: Annualized return statistics for bond-only investment.

Annualized Return for Bond-Only Investment										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	0.08%	3.3%	5.5%	5.8%	8.0%	17.3%	3.7%	0	-	0.75
2	0.13%	3.5%	5.4%	5.8%	8.1%	15.2%	3.6%	0	-	0.77
3	0.17%	3.7%	5.3%	5.8%	7.9%	14.8%	3.4%	0	-	0.75
4	0.22%	3.6%	5.3%	5.8%	7.6%	13.6%	3.2%	0	-	0.75
5	0.48%	3.3%	5.3%	5.8%	7.6%	12.9%	3.0%	0	-	0.76
6	1.08%	3.1%	5.3%	5.8%	7.6%	12.4%	2.8%	0	-	0.79
7	1.64%	3.4%	5.3%	5.8%	7.6%	12.0%	2.7%	0	-	0.86
8	1.91%	3.6%	5.3%	5.7%	7.3%	11.4%	2.6%	0	-	0.87
9	1.86%	3.8%	5.2%	5.7%	7.3%	10.9%	2.4%	0	-	0.90
10	1.80%	3.9%	5.3%	5.6%	7.0%	10.4%	2.3%	0	-	0.88

Table 1: Annualized return statistics for bond-only investment. Also shown is the probability of loss (i.e. the annualized return is less than zero), and the probability that the annualized return is less than that of a stock-only investment.

For example, if the investment period was 10 years then the minimum annualized return was 1.80% which occurred for the investment period starting March 12, 2003. The maximum annualized return for a 10-year investment period was about 10.4% and occurred for the period starting March 22, 1978. For a 10-year investment period the median annualized return was 5.3% and the mean was 5.6%, see Table 1.

The probability of loss is also listed in Table 1 and it was zero for all investment periods because the government of USA did not default on its bonds during the period of this data. The probability of loss may be greater than zero in later sections where a part of the portfolio is invested in the S&P 500 index.

The standard deviation of the annualized returns is 2.3% for 10-year investment periods. As discussed in section 2.6.1 the standard deviation (or equivalently, the variance) is commonly – but incorrectly – used as a risk-measure in finance. The standard deviation measures the spread of outcomes. In this case the average annualized return was 5.6% and the spread around this average as measured by the standard deviation was 2.3%. These two numbers alone do not signify the probability of loss or how great the losses were. As shown in Table 1, the standard deviation decreased for longer holding periods. The reason is that very high and very low bond yields only existed for shorter periods of time, so when the investment period gets longer, the annualized return gets closer to the average.

10. Stock-Only Investment

The previous section studied bond-only investments in which the entire portfolio is invested in US government bonds. The opposite strategy is a stock-only investment where the entire portfolio is invested in the S&P 500 stock-market index for a number of years and the dividends are also reinvested in the S&P 500. This is also known as the total return index because the dividends are reinvested in the S&P 500.

Bond-only investments are considered risk-free while stock-only investments have maximum exposure to the stock-market and hence maximum risk. Together the bond-only and stock-only investments form a benchmark for comparison to investment strategies that are partly invested in the S&P 500 index and partly in government bonds, so as to assess whether the investment strategy has attractive returns and risks.

10.1. Annualized Return

Figure 19 shows a box-plot of the annualized returns from investing in periods up to 10 years in the S&P 500 index and also reinvesting the dividends in the S&P 500. The same statistics are shown in Table 2.

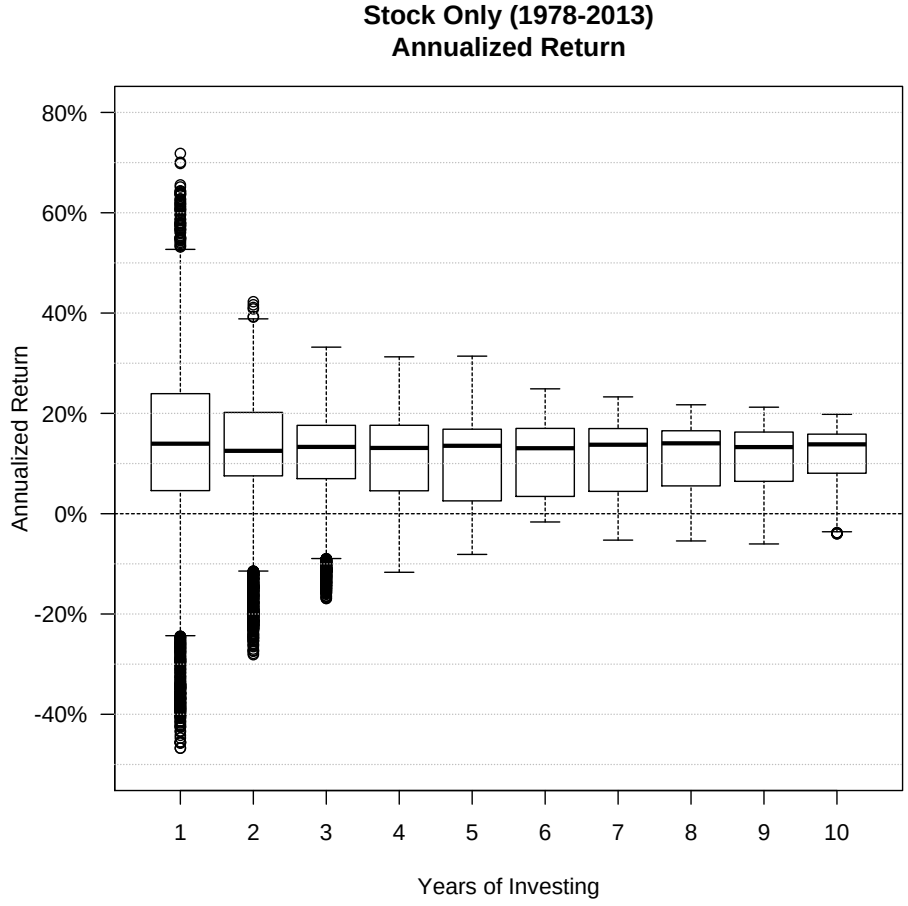

Figure 19: Annualized return statistics for stock-only investment.

Annualized Return for Stock-Only Investment										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(46.8%)	4.6%	14.0%	12.8%	23.9%	71.8%	17.3%	0.19	0.25	-
2	(28.1%)	7.5%	12.5%	12.0%	20.2%	42.2%	12.4%	0.14	0.23	-
3	(16.9%)	7.0%	13.3%	11.6%	17.6%	33.2%	10.5%	0.15	0.25	-
4	(11.7%)	4.6%	13.1%	11.4%	17.6%	31.3%	9.4%	0.18	0.25	-
5	(8.1%)	2.6%	13.5%	11.3%	16.8%	31.4%	8.4%	0.14	0.24	-
6	(1.7%)	3.4%	13.1%	11.3%	17.0%	24.9%	7.3%	0.04	0.21	-
7	(5.3%)	4.4%	13.8%	11.5%	17.0%	23.3%	6.7%	0.02	0.14	-
8	(5.4%)	5.6%	14.0%	11.5%	16.5%	21.7%	6.4%	0.04	0.13	-
9	(6.0%)	6.5%	13.3%	11.5%	16.3%	21.2%	6.1%	0.05	0.10	-
10	(4.0%)	8.1%	13.8%	11.5%	15.9%	19.8%	5.8%	0.07	0.12	-

Table 2: Annualized return statistics for stock-only investment. Also shown is the probability of loss (i.e. the annualized return is less than zero), and the probability that the annualized return is less than that of a bond-only investment.

For investment periods of one year the median return was 14.0% and the mean return was 12.8%. The minimum return was a negative (46.8%), that is, a loss of 46.8% which occurred in the year starting March 5, 2008, and consisted of a price change for the S&P 500 of (48.8%) and a dividend yield of 2% so the net loss was (46.8%). The maximum single-year return was 71.8% which occurred in the year starting March 9, 2009 and consisted of a price change of 68.6% and a dividend yield of 3.2%.

For investment periods of five years, the median annualized return was 13.5% and the mean was 11.3%. The minimum annualized return was (8.1%), that is, a loss of 8.1% for each of the five years starting March 5, 2004 thus giving a compounded loss of (34.5%) over five years. The maximum annualized return was 31.4%, that is, a gain of 31.4% for each of the five years starting August 11, 1982 thus giving a compounded gain of 292% over five years.

For investment periods of ten years, the median annualized return was 13.8% and the mean was 11.3%. The minimum annualized return was (4.0%), that is, a loss of (4.0%) for each of the ten years starting March 3, 1999 thus giving a compounded loss of (33.5%). The maximum annualized return was 19.8%, that is, a gain of 19.8% for each of the ten years starting August 23, 1990 thus giving a compounded gain of 509% over ten years.

Table 2 also shows the probability of loss, that is, the fraction of annualized returns that were negative. All investment periods up to 10 years had non-zero probabilities of loss. For example, for one year of investing in the S&P 500, the probability of loss was 0.19 (or 19%) and the greatest loss was (46.8%). For five years of investing, the probability of loss was 0.14 (or 14%) and the greatest annualized loss was (8.1%). For ten years of investing, the probability of loss was 0.07 (or 7%) and the greatest annualized loss was (4.0%).

Table 2 also shows the standard deviation of annualized returns. It should again be noted that the standard deviation is not a useful risk measure for investing. For example, the standard deviation is 6.7% for the annualized returns when the investment period was 7 years, while the standard deviation is slightly lower at 6.4% when the investment period was 8 years. So the standard deviation decreased but the actual risk increased as measured by the probability of loss which increased from 0.02 to 0.04 and the worst loss also increased in magnitude from (5.3%) to (5.4%). The standard deviation measures the spread of annualized returns, not the risk or magnitude of loss, see section 2.6.1 for more on this.

10.2. Comparison to Bond-Only Investment

The annualized return for stock-only and bond-only investments can be compared through the box-plots in Figure 18 and Figure 19, or through the statistics in Table 1 and Table 2.

The bond-only investments never experienced a loss while the stock-only investments experienced losses frequently, for example, 19% of one-year investments and 7% of ten-year investments experienced losses. The magnitude of the losses for stock-only investments could also be very large with as much as (46.8%) in a single year and an annualized loss of (4.0%) for the worst ten-year investment period, which compounds to a total loss of almost (34%) after ten years of investing.

Also shown in Table 2 is the probability (or more accurately, the frequency for this period) that the annualized return was less than that of a bond-only investment. For example, for one-year investments the probability was 0.25 (or 25%) that the return on a stock-only investment was lower than the return on a

bond-only investment. For ten-year investment periods the probability was 0.12 (or 12%) that the return on a stock-only investment was lower than the return on a bond-only investment.

Conversely, the annualized return was often much higher for stock-only than bond-only investments. After five years of investing, the median annualized return for stock-only investments was about 13-14% which was larger than the maximum annualized return for bond-only investments which was only 10-12%. The maximum return on a one-year stock-only investment was 71.8% while it was only 17.3% for a bond-only investment. The maximum annualized return on a ten-year stock-only investment was 19.8% while it was only 10.4% for a bond-only investment.

Figure 20 shows the total return of investing in the S&P 500 index and reinvesting all dividends back into the S&P 500 index. It is compared to the compounded return for bond-only investment, which reinvests the bond yields and is hence equivalent to the total return for the S&P 500. The data covers the period 1978-2013. The stock-only investment is clearly superior to bond-only investment in this 35-year period and it may be tempting to conclude that stock-only investments are generally superior to bond-only investments in the long-term. But recall that the stock-only investments frequently experienced large losses.

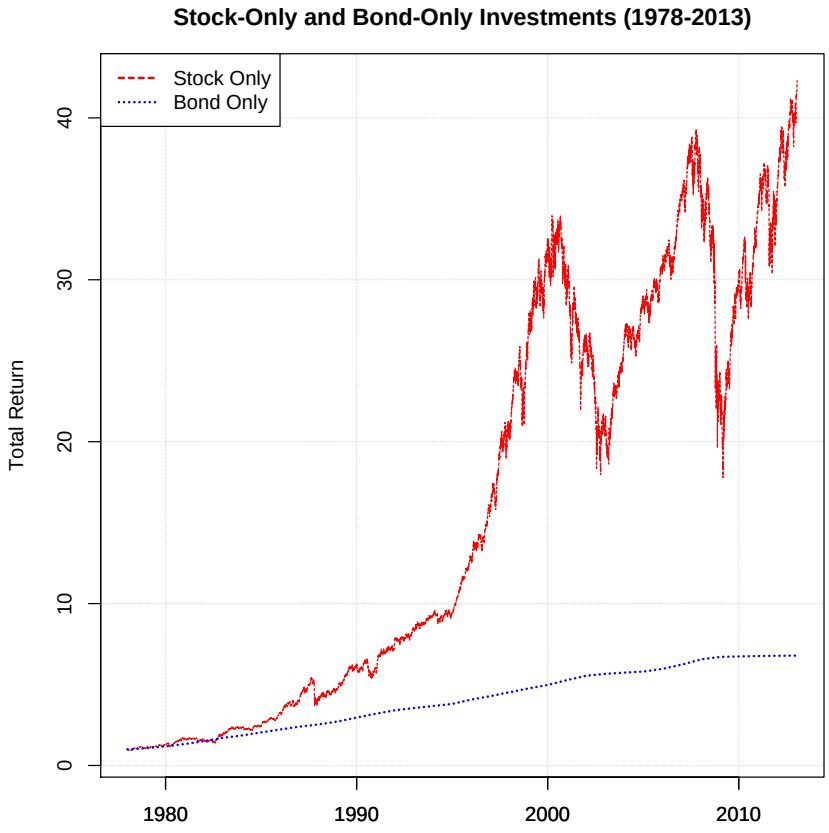

Figure 20: Total return of stock-only and bond-only investments for the period 1978-2013.

Figure 21 and Figure 22 compare the investment results for two 10-year investment periods, one starting August 23, 1990 in which the stock-only investment was superior and one starting March 3, 1999 in which the bond-only investment was superior. The superiority of either stock-only or bond-only investment clearly depends on the starting date and duration of the investment.

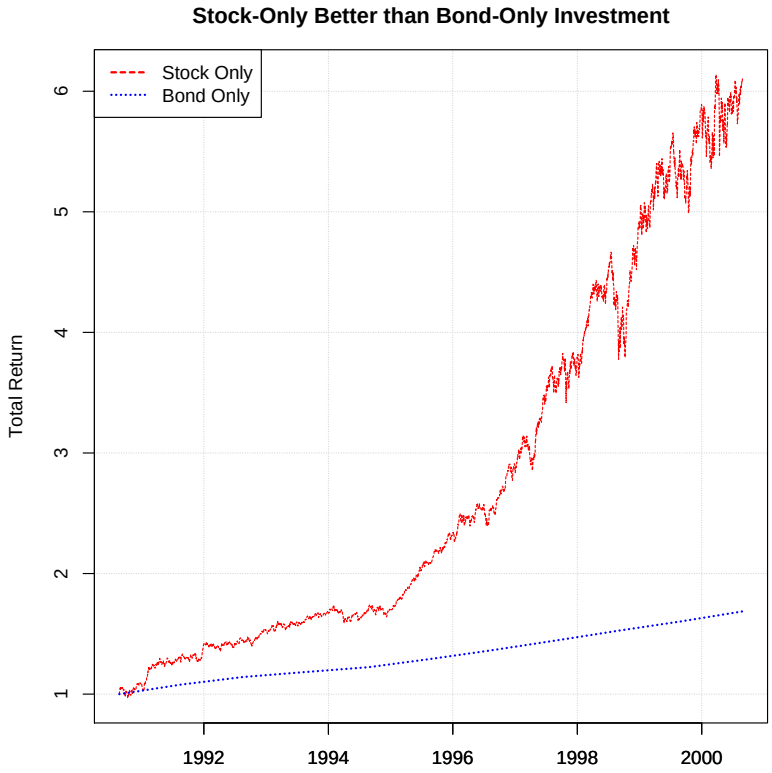

Figure 21: Total return of stock-only and bond-only investments for the 10-year period starting August 23, 1990.

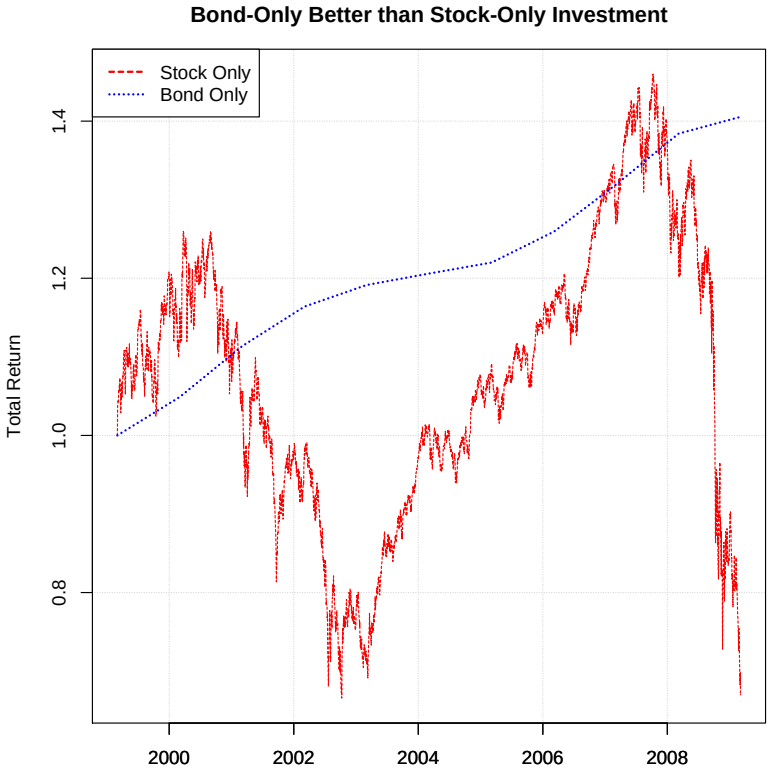

Figure 22: Total return of stock-only and bond-only investments for the 10-year period starting March 3, 1999.

11. Random Rebalancing

The previous sections studied investments exclusively in either US government bonds (aka. bond-only investments) or the S&P 500 stock-market index (aka. stock-only investments). An investment portfolio can consist partly of stock and bond investments. The stock-weight is a number between zero and one indicating how much of the portfolio is invested in stocks and the remainder of the portfolio is invested in government bonds.

The portfolio can be rebalanced annually using either the same weights or new weights each year. Different strategies are presented further below for selecting the portfolio weights. It is useful to compare the investment returns of a rebalancing strategy to stock-only and bond-only investments so as to assess if anything has been gained from the rebalancing strategy. Furthermore, it is useful to compare different strategies to random rebalancing in which the stock-weight is chosen randomly each year. This section compares random rebalancing to stock-only and bond-only investments.

11.1. Annualized Return

Figure 23 shows a box-plot of the annualized returns from random rebalancing during the period 1978-2013. This is also shown in Table 3. These are the results of selecting one set of random stock-weights and then calculating the annualized returns using those stock-weights. But there are an infinite number of possible combinations of random stock-weights, so the presented results do not cover all possible returns.

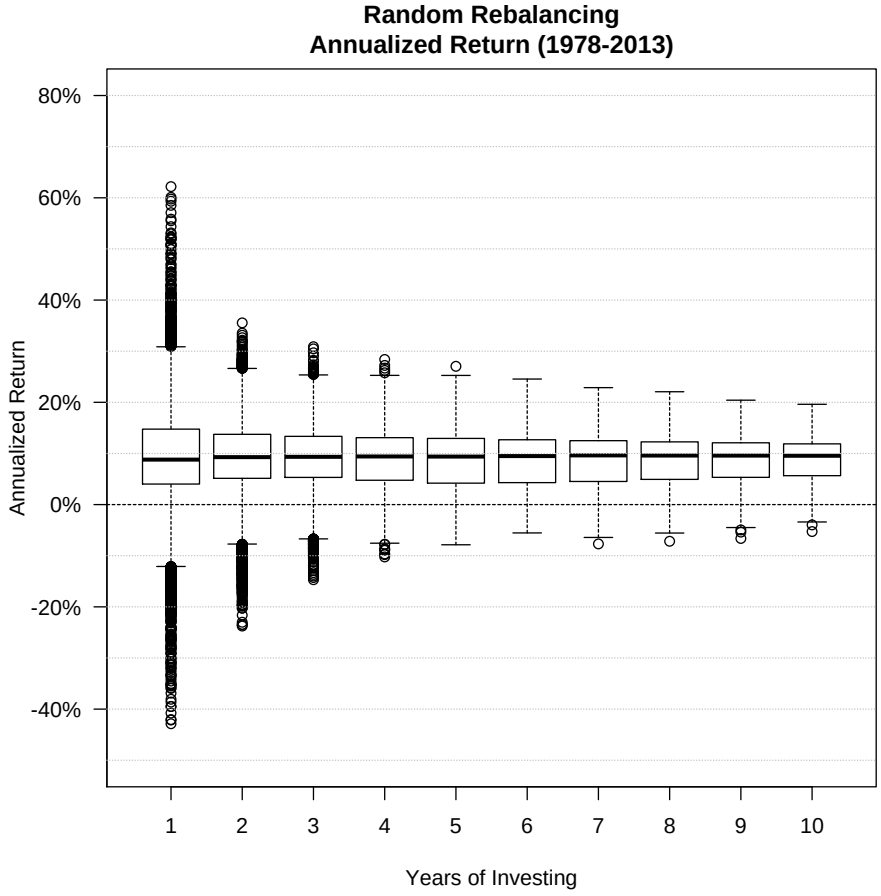

Figure 23: Annualized return statistics for random rebalancing.

Annualized Return for Random Rebalancing										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(42.9%)	4.0%	8.8%	9.3%	14.8%	62.2%	10.7%	0.12	0.25	0.74
2	(23.8%)	5.1%	9.3%	8.9%	13.7%	35.5%	7.8%	0.11	0.24	0.75
3	(14.7%)	5.3%	9.3%	8.8%	13.4%	30.9%	6.7%	0.11	0.24	0.74
4	(10.2%)	4.8%	9.4%	8.8%	13.1%	28.4%	6.1%	0.10	0.26	0.72
5	(7.9%)	4.2%	9.4%	8.7%	12.9%	27.0%	5.7%	0.08	0.24	0.73
6	(5.5%)	4.3%	9.5%	8.7%	12.7%	24.6%	5.1%	0.05	0.21	0.76
7	(7.7%)	4.5%	9.6%	8.7%	12.5%	22.9%	4.8%	0.03	0.18	0.78
8	(7.2%)	4.9%	9.6%	8.7%	12.3%	22.1%	4.6%	0.03	0.15	0.82
9	(6.6%)	5.3%	9.6%	8.7%	12.1%	20.4%	4.4%	0.03	0.14	0.84
10	(5.3%)	5.7%	9.6%	8.7%	11.9%	19.6%	4.2%	0.03	0.13	0.85

Table 3: Annualized return statistics for random rebalancing. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

For this particular choice of random stock-weights, the mean annualized return was 9.3% for one-year investment periods and this decreased to 8.7% for ten-year investment periods. The greatest one-year loss was (42.9%), while the lowest annualized return for ten-year investment periods was (5.3%), which gives a compounded loss of almost (42%) over ten years. The greatest one-year gain was 62.2%, while the highest annualized return for ten-year investment periods was 19.6%, which gives a compounded gain of 499% over ten years.

These are the results of only one choice of random stock-weights and it is actually possible to experience the full gains and losses of stock-only investments as shown in Table 2. So for one-year investment periods the greatest loss with random rebalancing could have been (46.8%) and the greatest gain could have been 71.8%, same as for stock-only investments.

Table 3 also shows the probability of loss for random rebalancing which was 0.12 (or 12%) for one-year investment periods, and gradually decreased for longer investment periods until it was 0.03 (or 3%) for ten-year investment periods.

11.2. Comparison to Bond-Only & Stock-Only Investments

Figure 24 shows the total return of random rebalancing compared to stock-only and bond-only investments for the full period 1978-2013, in which the rebalanced portfolio consistently performed better than the bond-only investment and worse than the stock-only investment.

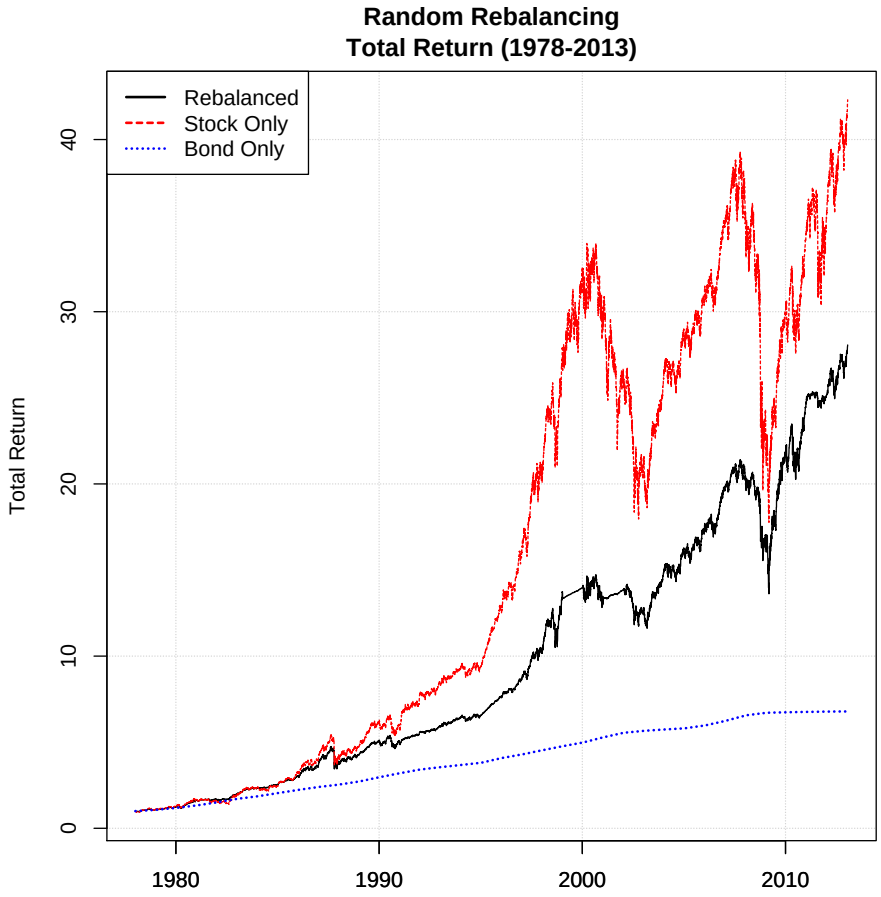

Figure 24: Total return of random rebalancing compared to stock-only and bond-only investments for the period 1978-2013.

But this long-term relationship is misleading as the mutual performance depends on the starting date and investment duration, as well as the random choice of stock-weights. Figure 25 shows an example of the random rebalancing performing better than both stock-only and bond-only investments when the starting date is March 4, 1999. Figure 26 shows an example of the random rebalancing performing worse than both stock-only and bond-only investments when the starting date is February 25, 1999, just days before.

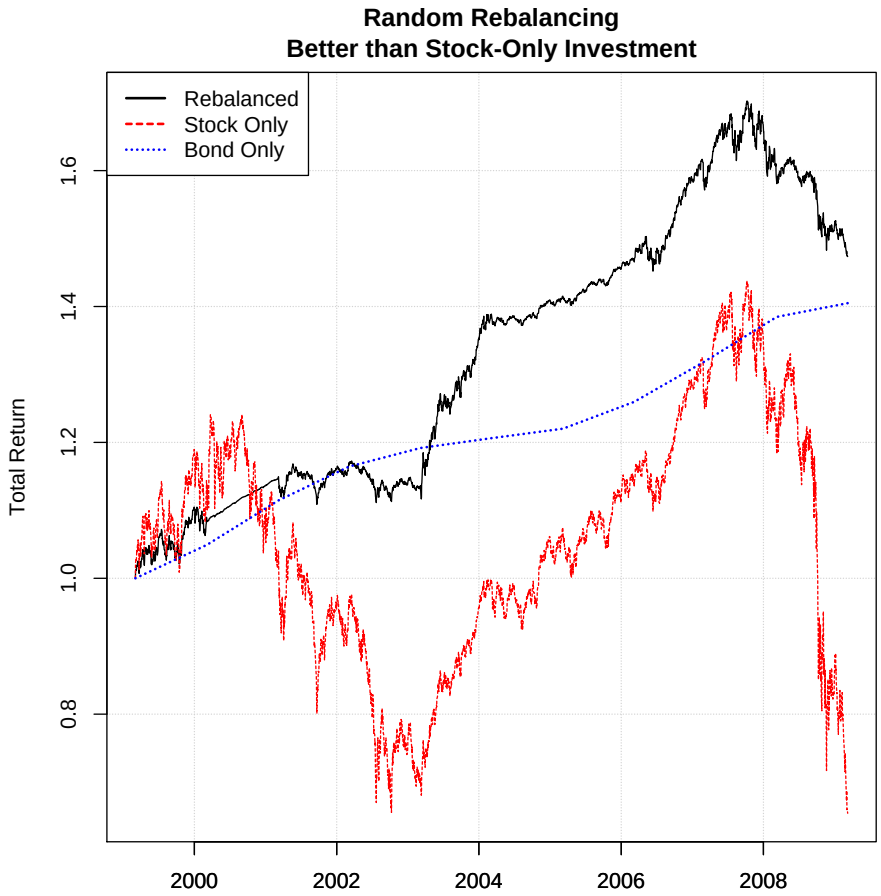

Figure 25: Total return of random rebalancing compared to stock-only and bond-only investments. The rebalanced investment is better than both stock-only and bond-only investments when the starting date is March 4, 1999.

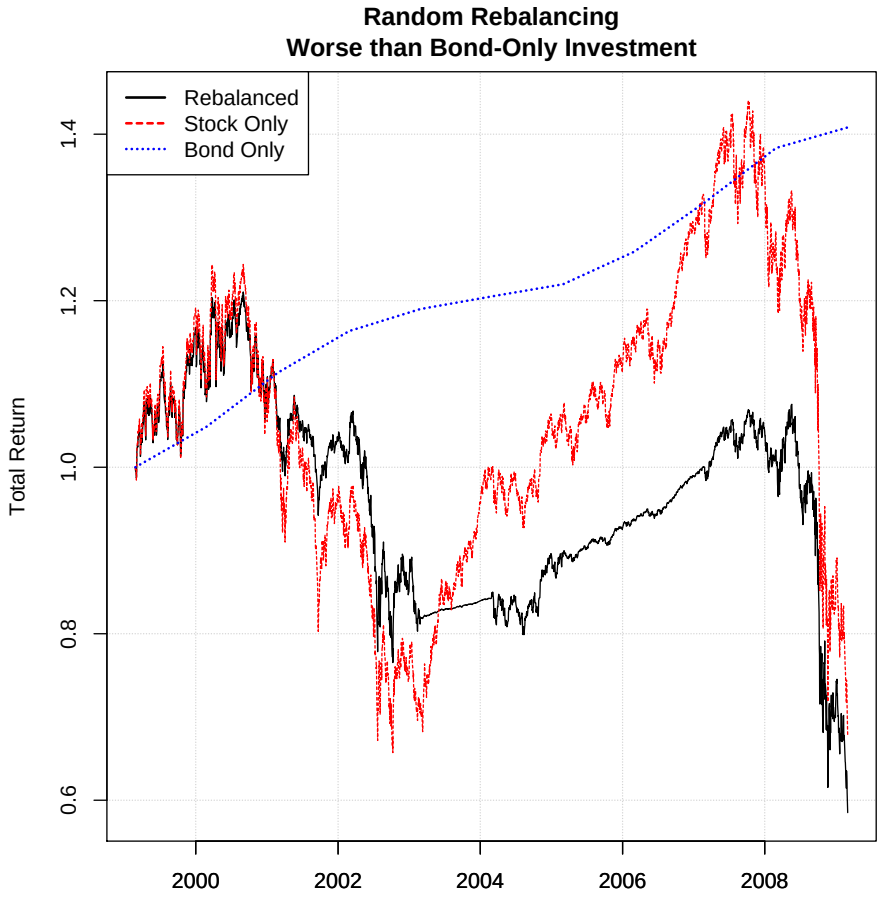

Figure 26: Total return of random rebalancing compared to stock-only and bond-only investments. The rebalanced investment is worse than both stock-only and bond-only investments when the starting date is February 25, 1999.

Table 3 shows that the probability of random rebalancing under-performing bond-only investments was 0.25 (or 25%) for one-year investment periods, and the probability gradually decreased for longer investment periods until it was 0.13 (or 13%) for ten-year investment periods.

The probability of random rebalancing under-performing stock-only investments was 0.74 (or 74%) for one-year investment periods and it was 0.85 (or 85%) for ten-year investment periods. The probabilities for intermediate investment periods were roughly in this range.

Table 1 shows the mean annualized return for bond-only investments was 5.8% for one-year investments and this gradually decreased to 5.6% for ten-year investments. Table 2 shows the mean annualized return for stock-only investments was 12.8% for one-year investments and this gradually decreased to 11.5% for ten-year investments periods. Table 3 shows the mean annualized return for random rebalancing was 9.3% for one-year investment periods and this gradually decreased to 8.7% for ten-year investment periods. So random rebalancing had a mean annualized return that was approximately the average of bond-only and stock-only investments.

11.3. Summary

Random rebalancing had a mean annualized return that was approximately the average of bond-only and stock-only investments. But random rebalancing may also experience the extreme gains and losses of stock-only investments. In order for this to happen, the randomly selected stock-weight must be equal or close to one. If the stock-weight is selected randomly from a uniform distribution between zero and one then the probability is low that the stock-weight is close to one.

So random rebalancing performed worse than stock-only investments on average but can experience the same extreme gains and losses of stock-only investments, although with lower probability.

12. Fixed Rebalancing

Sections 9 and 10 studied investments exclusively in either US government bonds (aka. bond-only investments) or the S&P 500 stock-market index (aka. stock-only investments). The bond-only investments had no losses in the period 1978-2013 while the stock-only investments generally had higher returns but sometimes also great losses. Whether bond-only or stock-only investments had superior returns depended on the starting date and duration of the investment period.

An attempt at getting some of the larger returns of the S&P 500 index, while avoiding its occasionally large losses, is to fix the allocation between the S&P 500 and US government bonds. Then after each year the portfolio is rebalanced to the same allocation. Taxes are ignored here and discussed in section 7.

The notation 25% stock / 75% bond, or just 25/75, means a fixed rebalancing strategy of 25% investment in the S&P 500 index and 75% investment in US government bonds.

12.1. 25% Stock, 75% Bond Rebalancing

First consider the rebalancing strategy with 25% stocks and 75% bonds. Figure 27 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 4.

Figure 27: Annualized return statistics for fixed rebalancing with 25% stock and 75% bond.

Annualized Return for Fixed Rebalancing with 25% Stock, 75% Bond										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(10.4%)	3.9%	7.6%	7.5%	11.2%	27.2%	5.5%	0.077	0.25	0.75
2	(4.1%)	4.2%	7.7%	7.5%	11.0%	17.4%	4.6%	0.069	0.20	0.76
3	(0.8%)	4.3%	7.9%	7.5%	10.9%	17.0%	4.2%	0.016	0.21	0.74
4	(0.1%)	3.5%	7.8%	7.5%	10.5%	15.8%	4.0%	0.0001	0.21	0.73
5	0.9%	3.3%	8.1%	7.4%	10.3%	15.5%	3.8%	0	0.18	0.73
6	2.1%	3.6%	8.0%	7.4%	10.1%	14.9%	3.5%	0	0.13	0.75
7	1.5%	3.6%	8.2%	7.4%	9.9%	14.4%	3.4%	0	0.09	0.83
8	1.5%	3.9%	8.0%	7.4%	9.7%	13.7%	3.2%	0	0.07	0.86
9	1.6%	4.4%	7.9%	7.4%	9.6%	13.3%	3.1%	0	0.09	0.89
10	2.1%	5.1%	8.1%	7.3%	9.2%	12.4%	2.9%	0	0.09	0.86

Table 4: Annualized return statistics for fixed rebalancing with 25% stock and 75% bond. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

For all investment periods the mean annualized return was about 7.3-7.5%, but the distributions are wider for shorter investment periods. For example, for one-year investment periods the minimum return of the fixed rebalancing strategy was a loss of (10.4%), while the maximum return was a gain of 27.2%. For ten-year investment periods the minimum annualized return was 2.1% giving a compounded gain of 23% over 10 years, while the maximum annualized return was 12.4% giving a compounded gain of about 222% over 10 years.

The probability of loss (that is, the annualized return is less than zero) gradually decreased from 0.077 (or 7.7%) for one-year investment periods to a probability of zero for investment periods of five years or more.

12.1.1. Weighted Statistics

The statistics for the fixed rebalancing strategy are not simply weighted from the statistics of the bond-only and stock-only investments. For example, for ten-year investment periods the mean annualized return was 11.5% for stock-only investments, see Table 2, while it was 5.6% for bond-only investments, see Table 1. Using the 25% stock / 75% bond weight would suggest a mean annualized return of 25% x 11.5% + 75% x 5.6% ≈ 7.08% for the rebalancing strategy while it was in fact slightly larger at 7.3%, see Table 4. This is because of the annual rebalancing.

12.1.2. Comparison to Bond-Only & Stock-Only Investments

Figure 28 shows the performance of the rebalancing strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the rebalancing strategy performed better than bond-only and worse than stock-only investments. But this long-term performance is misleading as the intermediate performance depends on the starting date and duration of the investment period.

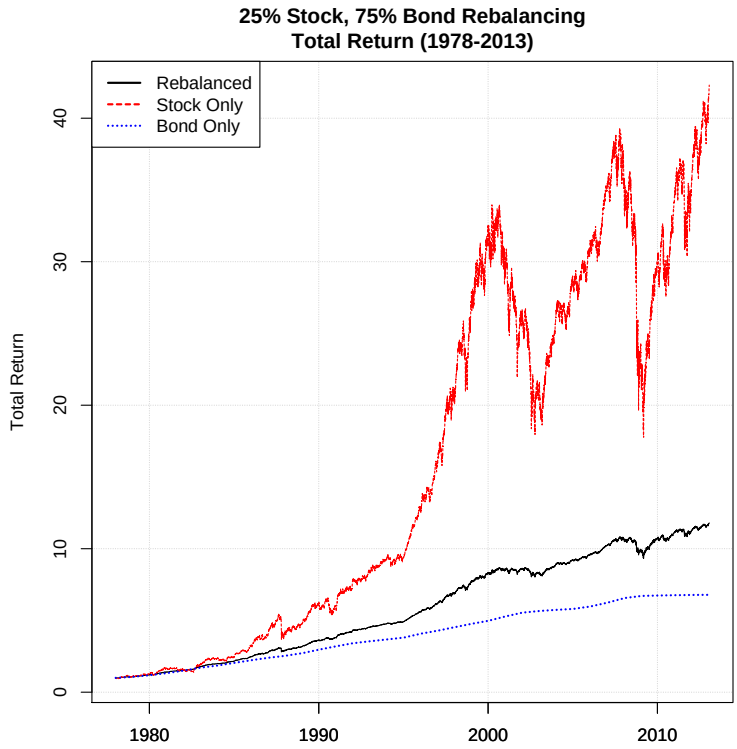

Figure 28: Total return of fixed rebalancing with 25% stock and 75% bond compared to stock-only and bond-only investments for the period 1978-2013.

Figure 29 and Figure 30 give examples of the rebalancing strategy performing both better and worse than bond-only and stock-only investments. The ten-year investment period that started on August 23, 1990 had the rebalanced portfolio perform consistently better than bond-only and worse than stock-only investments. But the ten-year investment period that started on February 25, 1999 had a more volatile relationship where the bond-only investment usually performed best and the stock-only investment performed worst.

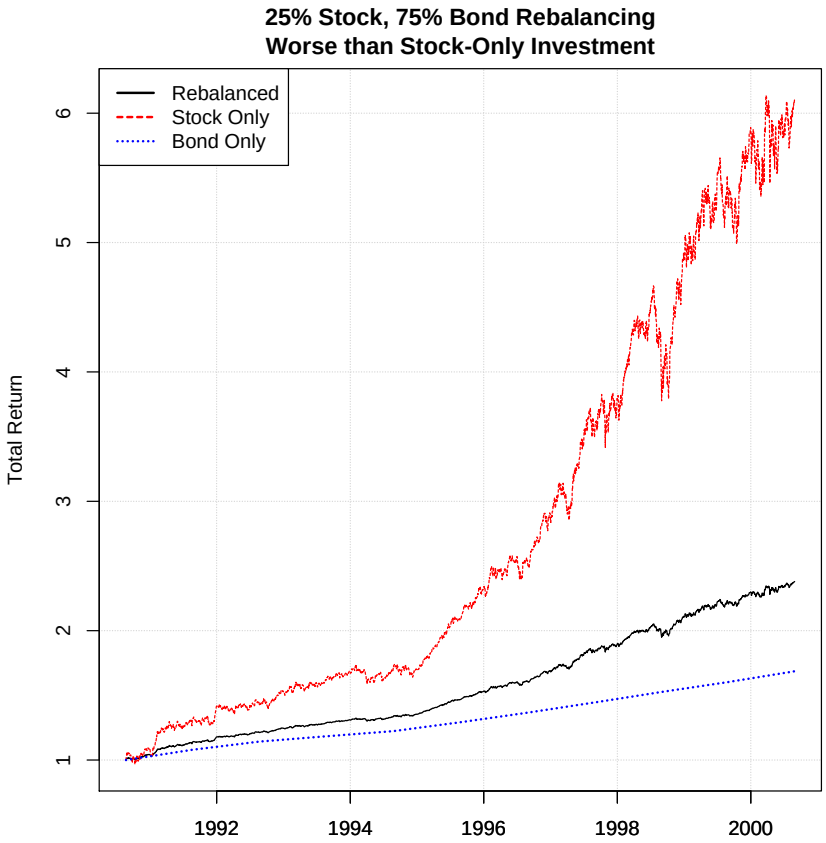

Figure 29: Total return of fixed rebalancing with 25% stock and 75% bond compared to stock-only and bond-only investments.

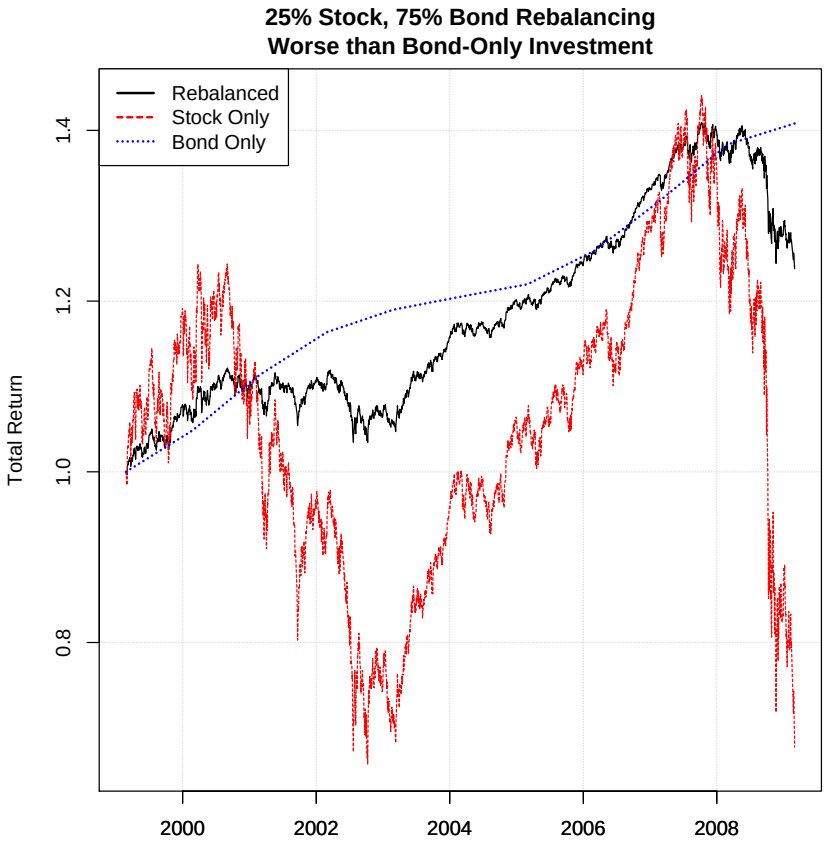

Figure 30: Total return of fixed rebalancing with 25% stock and 75% bond compared to stock-only and bond-only investments.

Table 4 shows the probability of the 25/75 rebalancing strategy under-performing a bond-only investment by having a lower annualized return. This probability was non-zero for all investment periods, which means that all investment periods had occurrences of bond-only investments that were superior to this rebalancing strategy. The probability was 0.25 (or 25%) for this rebalancing strategy to under-perform bond-only investments in the first year, and the probability gradually decreased to 0.09 (or 9%) for ten-year investment periods.

Also shown in Table 4 is the probability of the 25/75 rebalancing strategy under-performing a stock-only investment by having a lower annualized return. For one-year investment periods the probability was 0.75 (or 75%) and this gradually increased to 0.86 (or 86%) for ten-year investment periods.

So the rebalancing strategy with 25% stock and 75% bonds did not always avoid losses, it did not always outperform bond-only investments, but it did sometimes outperform stock-only investments.

12.2. 50% Stock, 50% Bond Rebalancing

Now consider a rebalancing strategy with 50% stocks and 50% bonds. Figure 31 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 5.

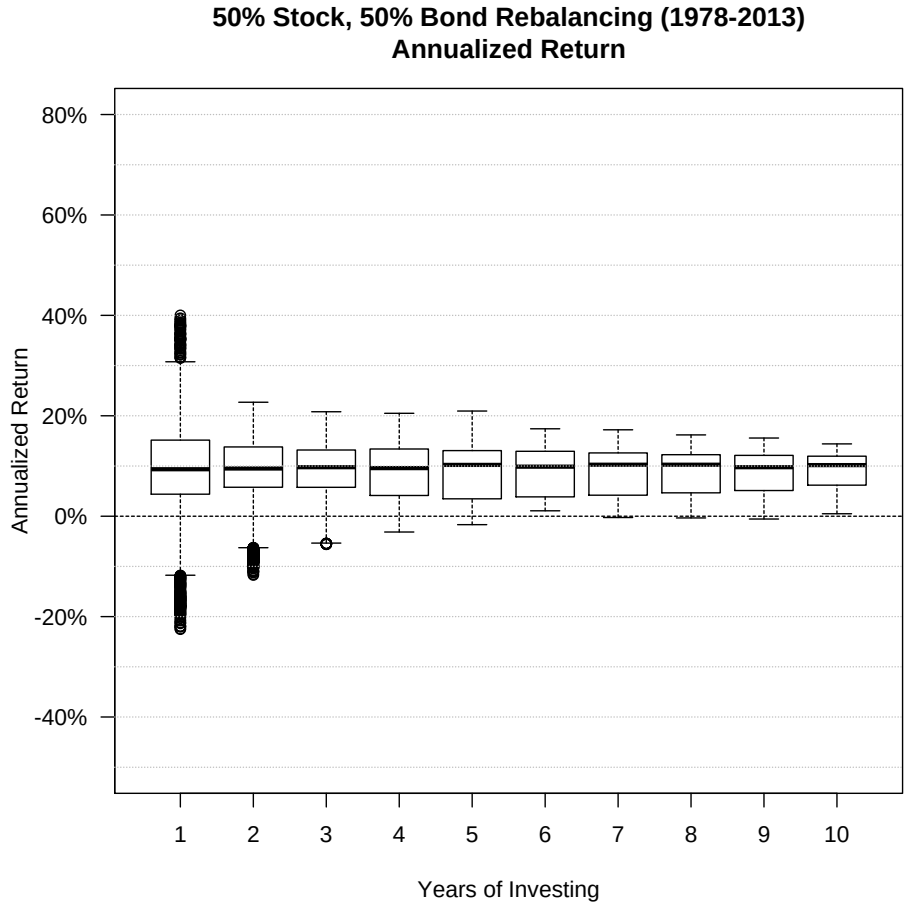

Figure 31: Annualized return statistics for fixed rebalancing with 50% stock and 50% bond.

Annualized Return for Fixed Rebalancing with 50% Stock, 50% Bond										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(22.5%)	4.4%	9.4%	9.3%	15.2%	40.0%	9.1%	0.13	0.25	0.75
2	(11.7%)	5.8%	9.5%	9.1%	13.8%	22.7%	6.8%	0.11	0.21	0.74
3	(5.6%)	5.8%	9.8%	9.0%	13.2%	20.8%	5.9%	0.11	0.22	0.73
4	(3.2%)	4.1%	9.6%	8.9%	13.4%	20.5%	5.5%	0.07	0.22	0.71
5	(1.7%)	3.5%	10.2%	8.9%	13.1%	21.0%	5.1%	0.004	0.20	0.73
6	1.1%	3.9%	9.9%	8.9%	12.9%	17.4%	4.6%	0	0.16	0.71
7	(0.3%)	4.2%	10.3%	8.9%	12.6%	17.2%	4.3%	0.0006	0.11	0.80
8	(0.3%)	4.7%	10.3%	8.9%	12.3%	16.2%	4.1%	0.002	0.08	0.85
9	(0.6%)	5.1%	9.8%	8.9%	12.1%	15.6%	3.9%	0.004	0.09	0.88
10	0.5%	6.2%	10.2%	8.9%	12.0%	14.4%	3.8%	0	0.09	0.85

Table 5: Annualized return statistics for fixed rebalancing with 50% stock and 50% bond. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

The mean annualized return is 9.3% for one-year investment periods and gradually decreases to 8.9% for ten-year investment periods, but the distributions are wider for shorter investment periods. For example, for one-year investment periods the minimum return is a loss of (22.5%) while the maximum return is a gain of 40.0%. For ten-year investment periods the minimum annualized return is 0.5% giving a compounded gain of 5.1% over ten years, while the maximum annualized return is 14.4% giving a compounded gain of 271%.

The probability of loss is non-zero for all investment periods except for six and ten years. For one-year investment periods the probability of loss is 0.13 (or 13%) and it gradually decreases towards zero for longer investment periods.

12.2.1. Comparison to Bond-Only & Stock-Only Investments

Figure 32 shows the performance of the rebalancing strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the rebalancing strategy performed better than bond-only and worse than stock-only investments. But this long-term performance is misleading as the intermediate performance depends on the starting date and duration of the investment period.

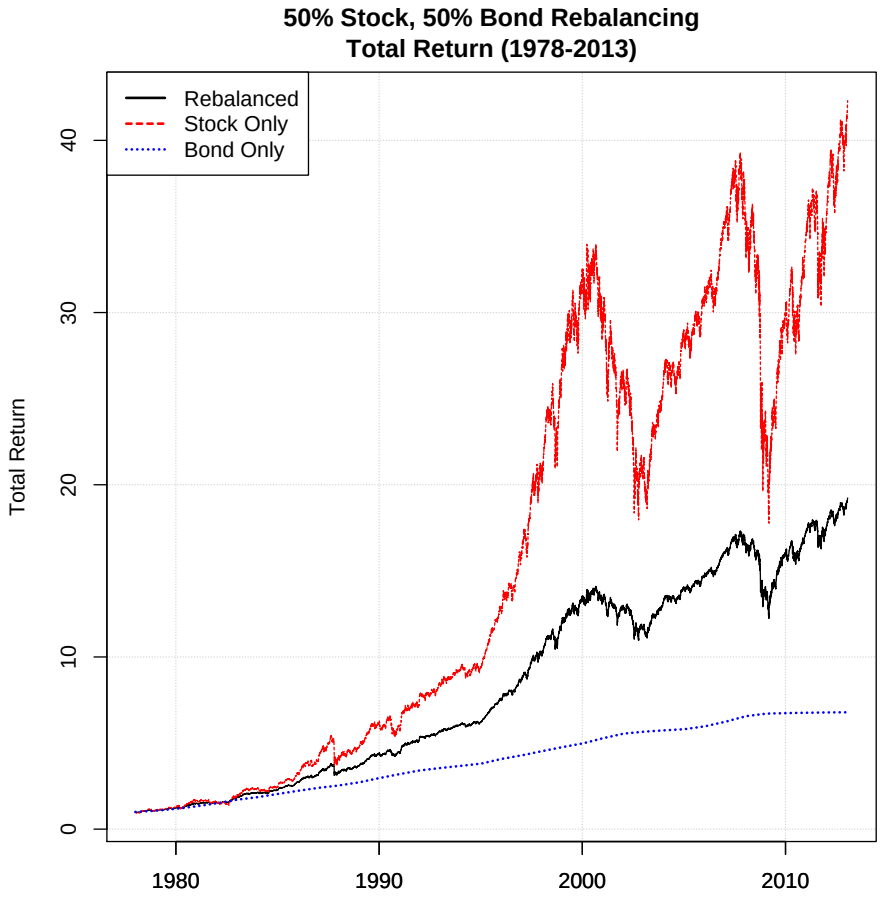

Figure 32: Total return of fixed rebalancing with 50% stock and 50% bond compared to stock-only and bond-only investments for the period 1978-2013.

Figure 33 and Figure 34 gives examples of the rebalancing strategy performing both better and worse than bond-only and stock-only investments. The ten-year investment period that started on August 23, 1990 had the rebalanced portfolio perform consistently between the bond-only and stock-only investments. But the ten-year investment period that started on February 25, 1999 had a more volatile relationship where the bond-only investment usually performed best and the stock-only investment performed worst.

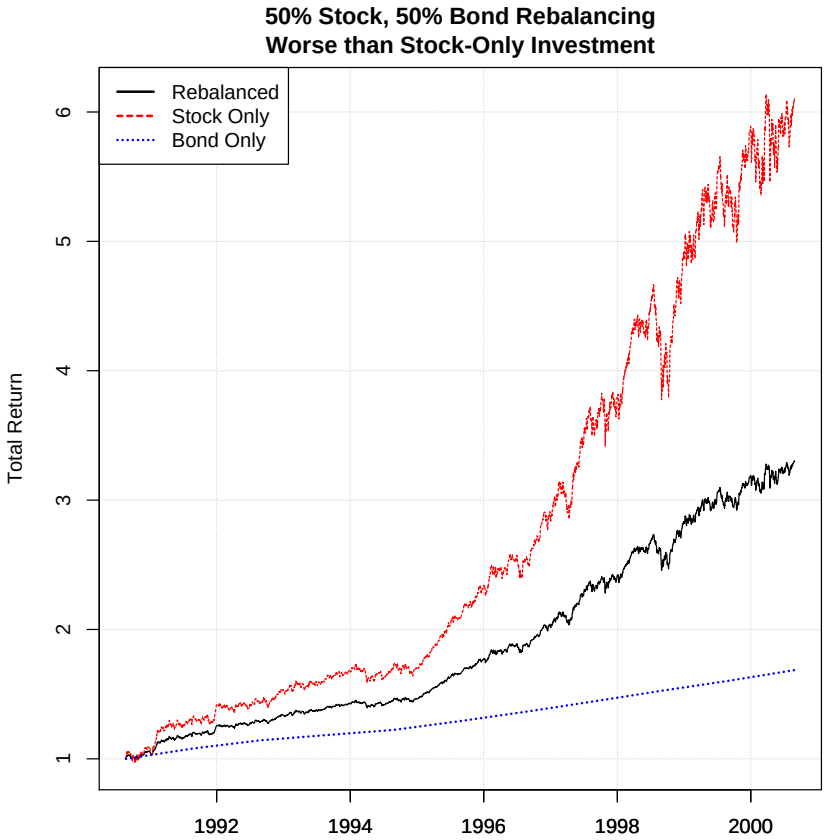

Figure 33: Total return of fixed rebalancing with 50% stock and 50% bond compared to stock-only and bond-only investments.

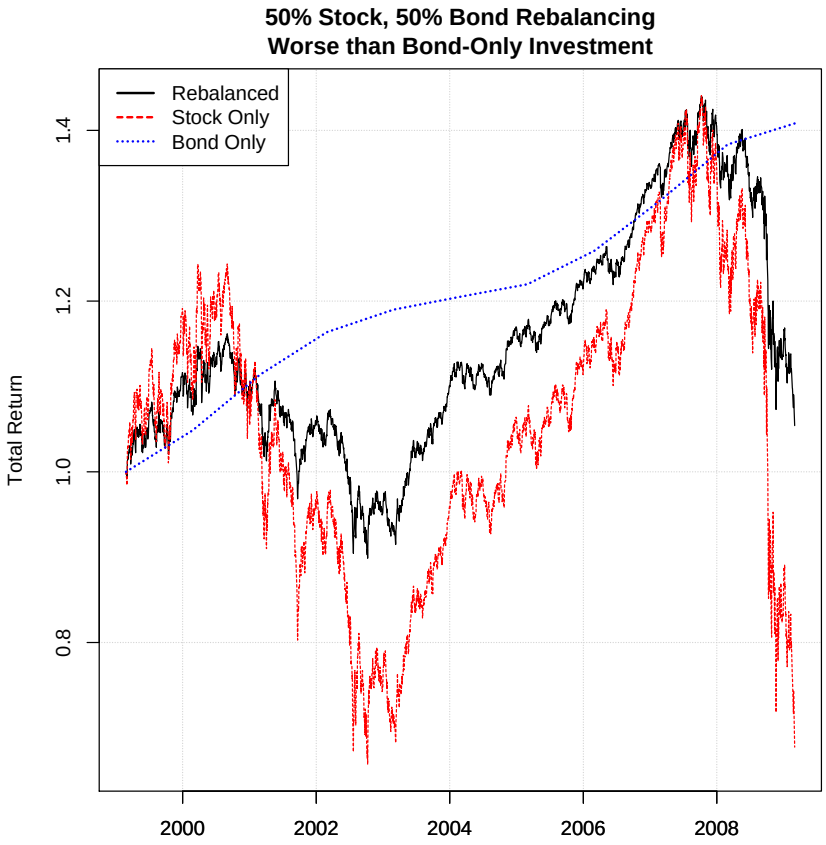

Figure 34: Total return of fixed rebalancing with 50% stock and 50% bond compared to stock-only and bond-only investments.

Table 5 shows the probability of the annualized return from the rebalancing strategy being less than that of a bond-only investment. The probability was 0.25 (or 25%) for this rebalancing strategy to under-perform a bond-only investment in the first year, and the probability gradually decreased to 0.09 (or 9%) for ten-year investment periods.

Table 5 also shows the probability of the rebalancing strategy under-performing a stock-only investment by having a lower annualized return. The probability ranged between 0.71-0.88 (or 71-88%) depending on the duration of the investment periods.

12.2.2. Comparison to 25% Stock, 75% Bond Rebalancing

The annualized returns of the 25% stock / 75% bond and 50% stock / 50% bond rebalancing strategies can be compared in Table 4 and Table 5. Depending on the duration of the investment period, the mean return was 8.9-9.3% for the 50/50 strategy while it was only 7.3-7.5% for the 25/75 strategy. The gains and losses were also greater for the 50/50 strategy, with the largest one-year loss being (22.5%) and the largest one-year gain being 40.0%. Compare this to the 25/75 strategy whose largest one-year loss was (10.4%) and largest one-year gain was 27.2%. For ten-year investment periods the differences were smaller, where the 50/50 strategy had a minimum annualized return of 0.5% and the maximum was 14.4%, while the 25/75 strategy had a minimum annualized return of 2.1% and the maximum was 12.4%.

For one-year investments the probability of loss was 0.13 (or 13%) for the 50/50 strategy while the probability of loss was only 0.077 (or 7.7%) for the 25/75 strategy. For investment periods of five years or

more the 25/75 strategy had zero probability of loss, while the 50/50 strategy had occurrences of losses for all investment periods except for six and ten years.

The probabilities of the rebalancing strategies under-performing bond-only and stock-only investments were remarkably similar. The 50/50 strategy had only slightly higher probabilities than the 25/75 strategy for under-performing bond-only investments, and conversely, the 50/50 strategy had only slightly lower probabilities than the 25/75 strategy for under-performing stock-only investments.

So the 50% stock / 50% bond strategy had a larger mean annualized return than the 25% stock / 75% bond strategy, but the losses and gains could also be significantly larger, especially for shorter investment periods. The probability of loss was greater for the 50/50 strategy, but the probabilities of under-performing bond-only and stock-only investments were remarkably similar.

12.2.3. Comparison to Random Rebalancing

The 50% stock / 50% bond strategy had a mean annualized return of 9.3% for one-year investment periods and this decreased gradually to 8.9% for ten-year investment periods, see Table 5. This is comparable to the mean annualized return of random rebalancing which was 9.3% for one-year investment periods and gradually decreased to 8.7% for ten-year investment periods, see Table 3.

As discussed in section 11.3, random rebalancing can still experience the extreme gains and losses of stock-only investments because it is possible for the portfolio to be fully invested in stocks. But the 50% stock / 50% bond strategy only has half the gains and losses of stock-only investments plus half the gains from bond-only investments. Furthermore, as the investment period increases, the probability of loss becomes significantly smaller for the 50/50 strategy compared to random rebalancing, see Table 5 and Table 3.

The 50/50 rebalancing strategy therefore seems superior to random rebalancing.

12.3. 75% Stock, 25% Bond Rebalancing

Now consider a rebalancing strategy with 75% stocks and 25% bonds. Figure 35 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 6.

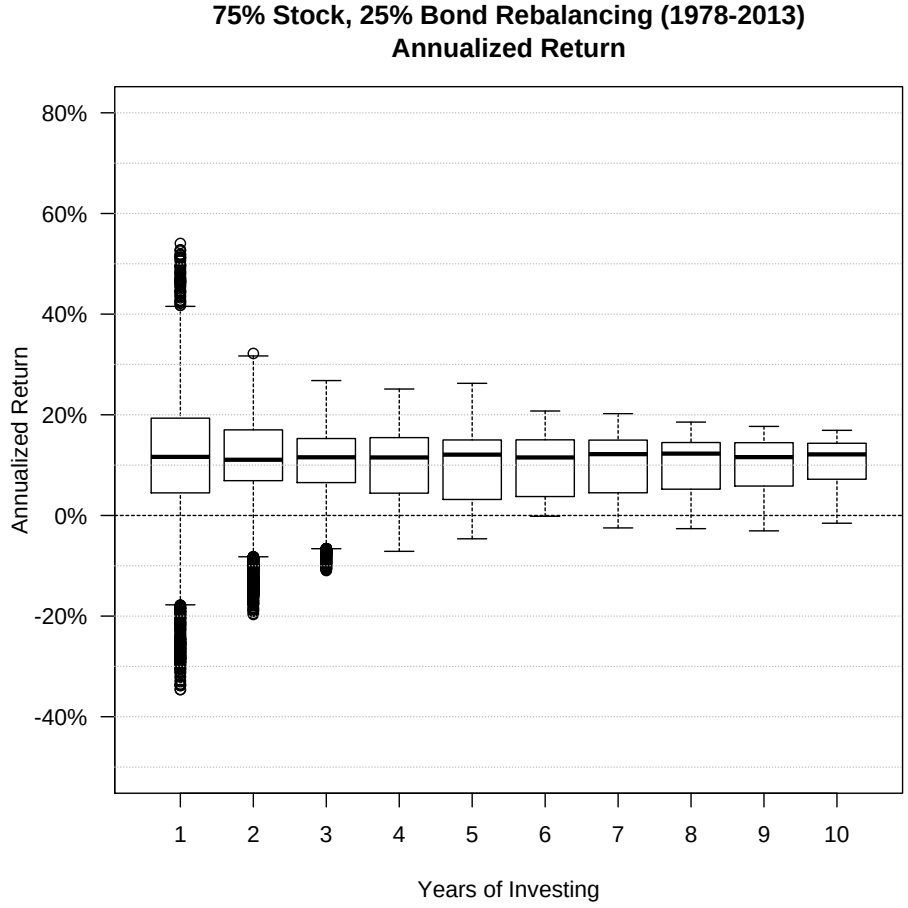

Figure 35: Annualized return statistics for fixed rebalancing with 75% stock and 25% bond.

Annualized Return for Fixed Rebalancing with 75% Stock, 25% Bond										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(34.6%)	4.5%	11.6%	11.0%	19.3%	54.0%	13.2%	0.18	0.25	0.75
2	(19.7%)	6.9%	11.1%	10.6%	17.0%	32.2%	9.5%	0.13	0.22	0.73
3	(10.9%)	6.5%	11.6%	10.4%	15.3%	26.8%	8.1%	0.14	0.24	0.72
4	(7.1%)	4.4%	11.5%	10.3%	15.5%	25.1%	7.3%	0.13	0.24	0.70
5	(4.6%)	3.2%	12.1%	10.2%	15.0%	26.3%	6.7%	0.07	0.22	0.72
6	(0.2%)	3.8%	11.5%	10.2%	15.0%	20.8%	5.9%	0.0007	0.18	0.69
7	(2.5%)	4.5%	12.2%	10.3%	15.0%	20.2%	5.5%	0.01	0.13	0.74
8	(2.6%)	5.2%	12.3%	10.3%	14.5%	18.6%	5.2%	0.03	0.11	0.83
9	(3.1%)	5.8%	11.6%	10.3%	14.5%	17.7%	5.0%	0.04	0.10	0.86
10	(1.5%)	7.2%	12.1%	10.3%	14.3%	16.9%	4.7%	0.02	0.11	0.84

Table 6: Annualized return statistics for fixed rebalancing with 75% stock and 25% bond. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

For a one-year investment period the mean annualized return is 11.0% which decreases to 10.3% for ten-year investment periods. The distributions are wider for shorter investment periods. For one-year investment periods the minimum return is (34.6%) and the maximum return is 54.0%. For ten-year investment periods the minimum annualized return is (1.5%) giving a compounded loss of (14%) over ten years, while the maximum annualized return is 16.9% giving a compounded gain of 377% over ten years.

The probability of loss is non-zero for all investment periods. For one-year investment periods the probability of loss is 0.18 (or 18%) and it decreases gradually to a probability of 0.02 (or 2%) for ten-year investment periods, although the probability of loss is smallest for six-year investment periods where it is only 0.0007 (or 0.07%).

12.3.1. Comparison to Bond-Only & Stock-Only Investments

Figure 36 shows the performance of the rebalancing strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the rebalancing strategy performed better than bond-only and worse than stock-only investments. Note that in 2009 after about 31 years of investing, the total return of this rebalancing strategy was almost equal to that of the stock-only investment because of the large decrease in the S&P 500 at that time. As usual, whether the rebalancing is better or worse than a stock-only or bond-only investment depends on the starting date and investment period.

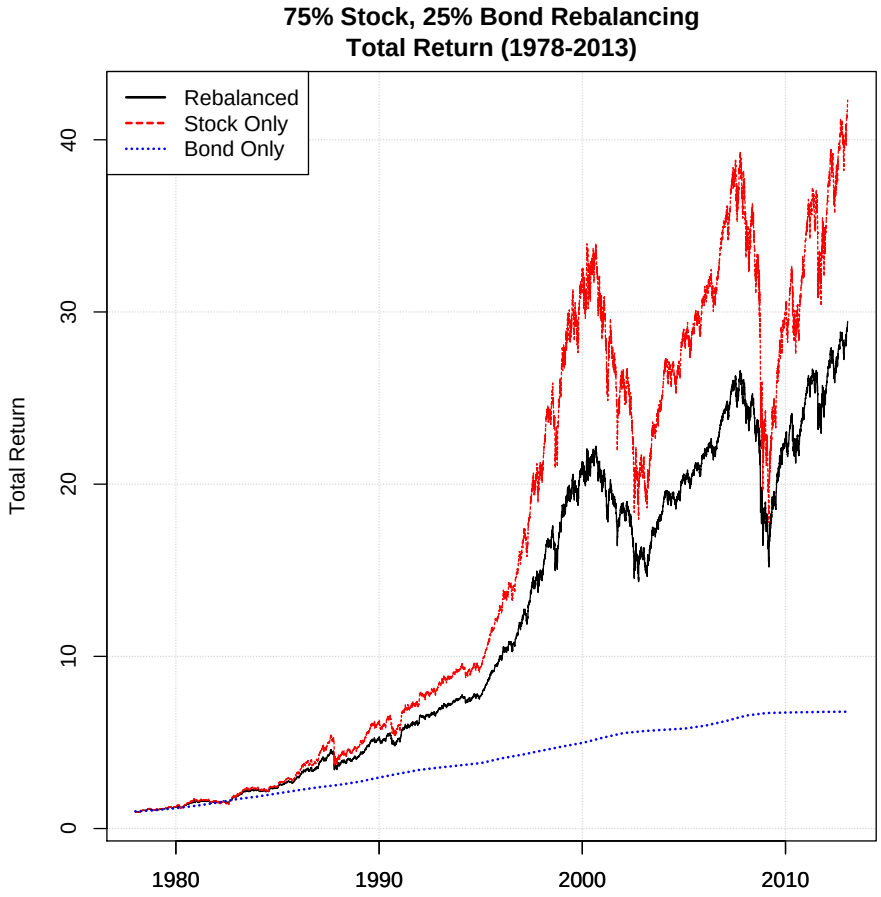

Figure 36: Total return of fixed rebalancing with 75% stock and 25% bond compared to stock-only and bond-only investments for the period 1978-2013.

Figure 37 and Figure 38 gives examples of the rebalancing strategy performing both better and worse than bond-only and stock-only investments. The ten-year investment period that started on August 23, 1990 had the rebalanced portfolio perform consistently better than the bond-only investment and somewhat worse than the stock-only investment. But in the ten-year investment period that started on February 25, 1999 the bond-only investment usually performed best and the stock-only investment usually performed worst.

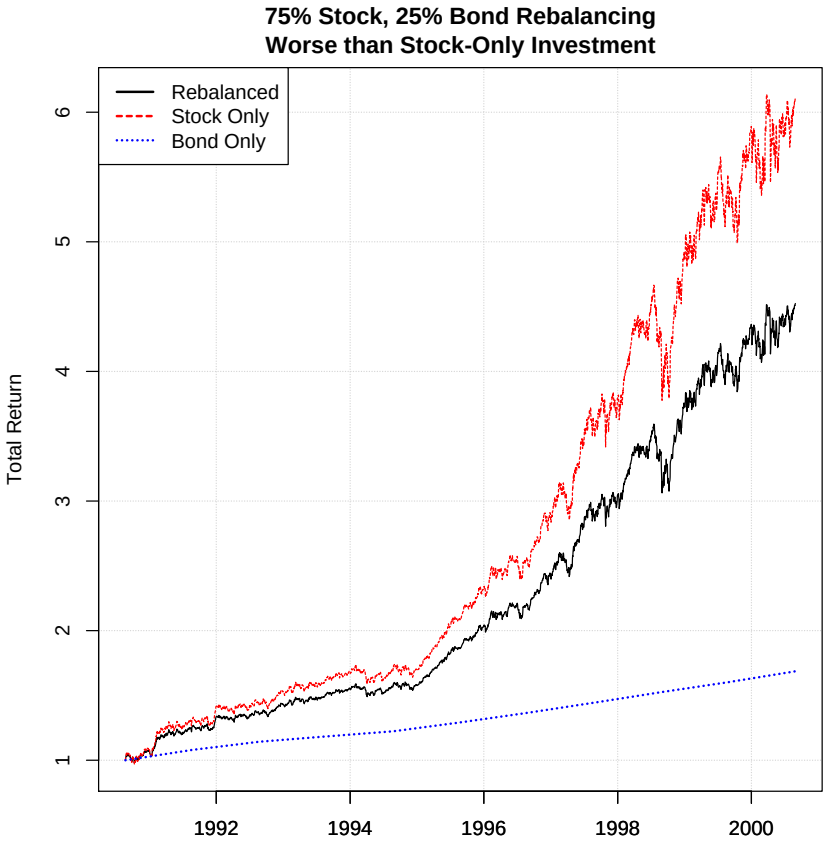

Figure 37: Total return of fixed rebalancing with 75% stock and 25% bond compared to stock-only and bond-only investments.

Table 6 shows the probability of the annualized return from the rebalancing strategy being less than that of a bond-only investment. The probability was 0.25 (or 25%) for this rebalancing strategy to under-perform a bond-only investment in the first year, and the probability gradually decreased to 0.11 (or 11%) for ten-year investment periods.

Table 6 also shows the probability of the annualized return from the rebalancing strategy being less than that of a stock-only investment. The probability for different investment periods ranged between 0.69-0.86 (or 69-86%) for this rebalancing strategy to under-perform stock-only investments.

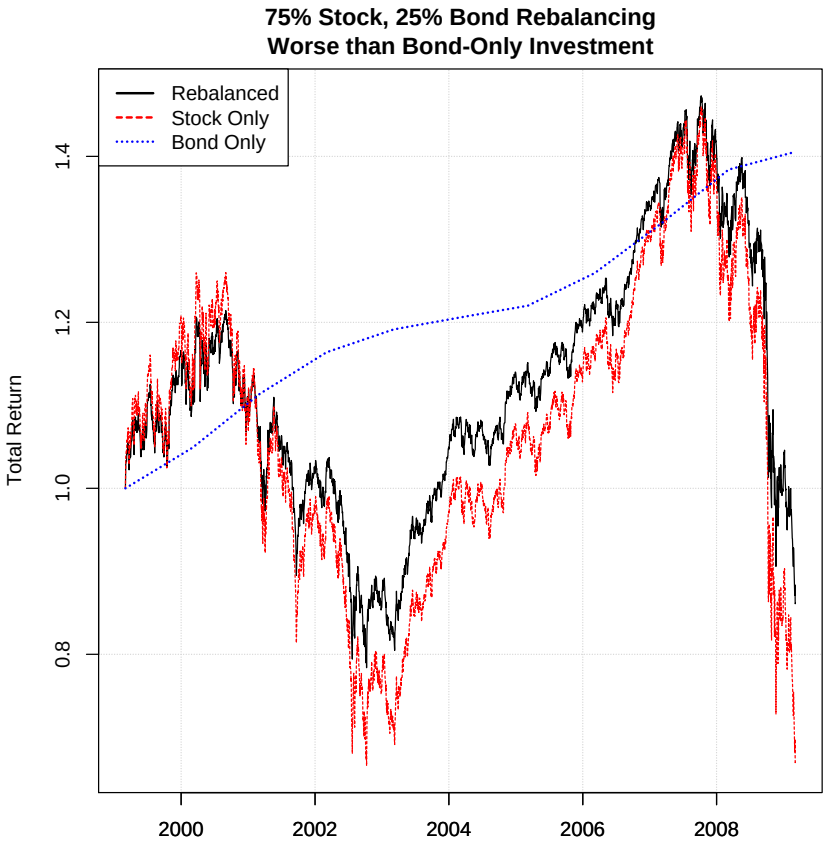

Figure 38: Total return of fixed rebalancing with 75% stock and 25% bond compared to stock-only and bond-only investments.

12.3.2. Comparison to 50% Stock, 50% Bond Rebalancing

The annualized returns of the 50% stock / 50% bond and 75% stock / 25% bond rebalancing strategies can be compared in Table 5 and Table 6. Depending on the duration of the investment period, the mean return was 10.3-11.0% for the 75/25 strategy while it was only 8.9-9.3% for the 50/50 strategy. The gains and losses were also greater for the 75/25 strategy, with the largest one-year loss being (34.6%) and the largest one-year gain being 54.0%. Compare this to the 50/50 strategy whose largest one-year loss was (22.5%) and largest one-year gain was 40.0%. For ten-year investment periods the differences are smaller although the 75/25 strategy still experienced losses with a minimum annualized return of (1.5%) and a maximum of 16.9%, while the 50/50 strategy had a minimum annualized return of 0.5% and a maximum of 14.4%

The 75/25 strategy had non-zero probability of loss for all investment periods. In comparison the probability of loss was significantly smaller for the 50/50 strategy. For example, for one-year investment periods the probability of loss was 0.18 (or 18%) for the 75/25 strategy while it was only 0.13 (or 13%) for the 50/50 strategy. For ten-year investment periods the probability of loss was 0.02 (or 2%) for the 75/25 strategy while it was zero for the 50/50 strategy.

The probabilities of the rebalancing strategies under-performing bond-only and stock-only investments were remarkably similar. The 75/25 strategy had only slightly higher probabilities than the 50/50 strategy for under-performing bond-only investments, and conversely, the 72/25 strategy had only slightly lower probabilities than the 50/50 strategy for under-performing stock-only investments.

So the 75% stock / 25% bond strategy had a larger mean annualized return than the 50% stock / 50% bond strategy, but the losses and gains could also be significantly larger, especially for shorter investment periods. The probability of loss was greater for the 75/25 strategy, but the probabilities of under-performing bond-only and stock-only investments were remarkably similar.

12.4. Summary

The statistics for the fixed rebalancing strategy are not simply weighted from the statistics of the bond-only and stock-only investments because of the annual rebalancing. When a larger portion of the portfolio was invested in the S&P 500 index rather than US government bonds, the mean annualized return increased along with the probability and magnitude of loss. However, the probabilities of under-performing bond-only and stock-only investments were remarkably similar amongst these three rebalancing strategies and it was the magnitude of the under-performance that differed.

13. Adaptive Rebalancing using P/Book

The previous section studied fixed rebalancing strategies which would allocate a predetermined fraction of the portfolio to the S&P 500 index and the remainder was allocated to US government bonds. After a year the portfolio was rebalanced to the same predetermined fraction. These fixed rebalancing strategies proved unable to avoid losses and also often under-performed bond-only and stock-only investments.

This section studies a simple rebalancing strategy in which the so-called P/Book ratio of the S&P 500 index is used in an attempt to better allocate the portfolio between the S&P 500 and US government bonds.

13.1. P/Book

The P/Book ratio is defined as the price per share divided by the book-value per share, also known as the equity-value per share:

$$P/Book = Price\ Per\ Share\ /\ Book\ Value\ Per\ Share$$

Eq. 13-1

The equity of a company is the amount of capital that shareholders have supplied directly to the company and indirectly through earnings that were retained by the company instead of being paid out as dividends to the shareholders (or used for share buybacks). The P/Book ratio therefore measures the market-price for a share relative to the accumulated shareholder capital.

13.1.1. Equity and Earnings

The earnings of companies are usually volatile but they are generally related to the amount of revenue, which in turn is related to the company's productive assets used for generating the revenue, such as land, buildings, machinery, etc. The assets are in turn related to the equity because a certain amount of equity is needed to fund a company's assets, otherwise the company would be entirely debt-funded and would have a high risk of bankruptcy in case the company's operating earnings were temporarily too low to pay interest on the debt.

These relations amongst revenue, earnings, assets and equity are stronger in some companies than others. The S&P 500 index consists of 500 large companies in USA. Although some of these companies can operate safely and profitably with very little equity, on average the 500 companies have a relation between revenue, earnings, assets and equity. So the equity of the S&P 500 is related to its future earnings that in turn determine the size of future dividend payouts, which is the fundamental source of value for long-term investors.

13.1.2. Historical P/Book

The share-price of the S&P 500 (or any company) is a complex mix of investor sentiment towards the future earnings and dividends combined with short-term price speculation. The P/Book ratio can be used to assess whether the S&P 500 is mispriced compared to its historical average. An advantage to using the P/Book ratio rather than the P/E ratio (share-price divided by earnings per share) is that the book-value is more stable than a single year's earnings.

Figure 2 shows the P/Book for the S&P 500 index for the period 1978-2013. The (arithmetic) mean P/Book was about 2.42 for this period but Figure 2 shows that the actual P/Book was rarely close to this average.

Figure 39 shows the histogram for the P/Book in this period which is useful for assessing the shape of the distribution.

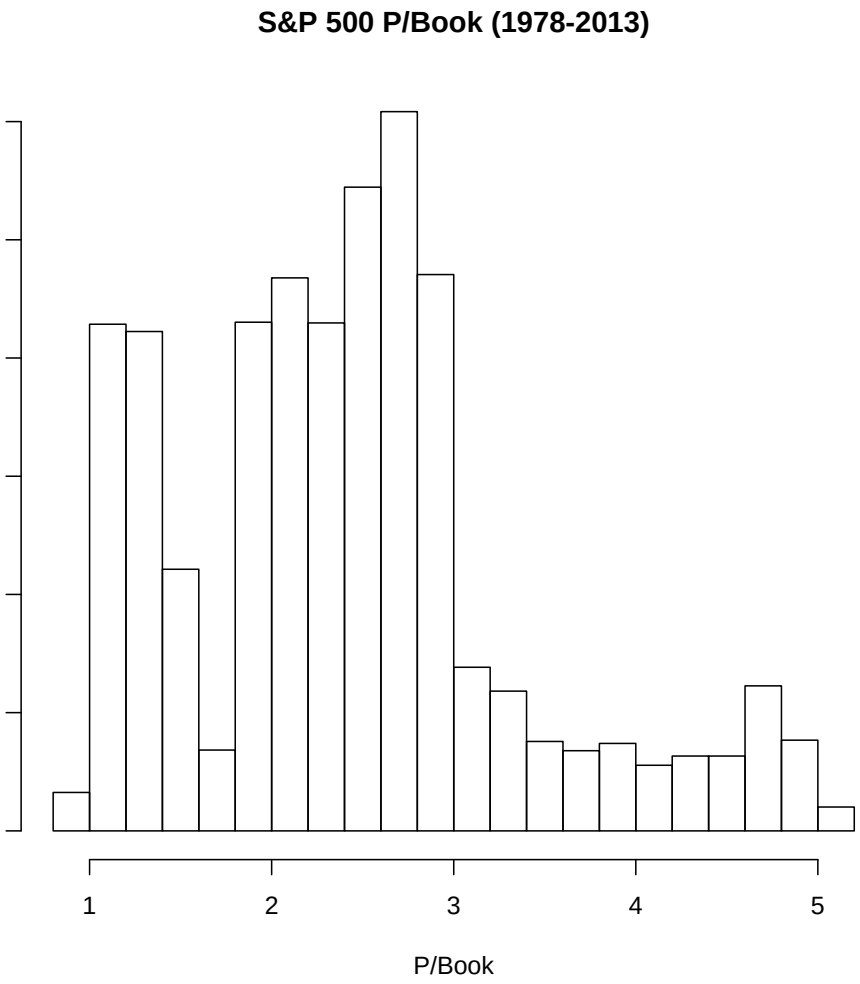

Figure 39: S&P 500 P/Book histogram for the data in Figure 2 covering the period 1978-2013.

Figure 40 shows the Cumulative Distribution Function (CDF) which is useful for determining how often the P/Book was below or above a certain level. For example, during 1978-2013 the P/Book was below 2 about 32% of the time and it was above 3 about 18% of the time.

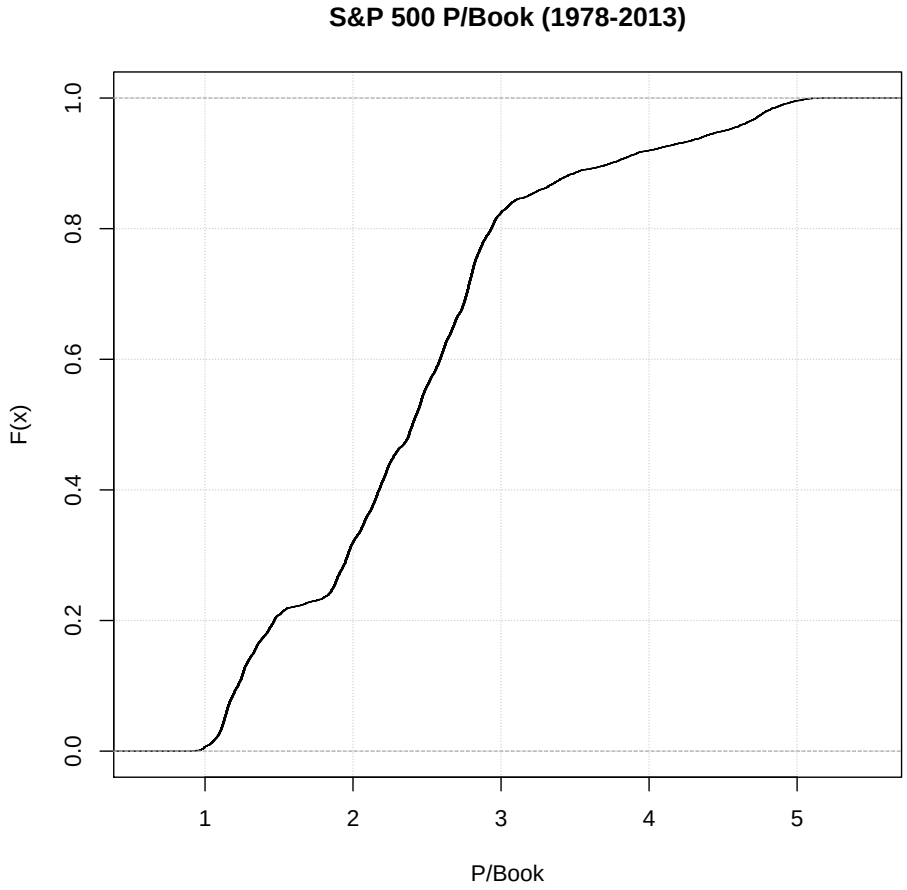

Figure 40: S&P 500 P/Book Cumulative Distribution Function (CDF) for the data in Figure 2 covering the period 1978-2013.

13.1.3. P/Book and Annualized Return

Figure 41 shows scatter-plots of the S&P 500 P/Book versus annualized return for investment periods of 1, 3, 6 and 10 years. There is no clear relation between the P/Book of the S&P 500 and its return for one year. But the relation becomes clearer for longer investment periods and for ten-year investments there appears to be a strong inverse relation between the P/Book and annualized return, so that a low P/Book results in a high annualized return over ten years and conversely a high P/Book results in a low annualized return.

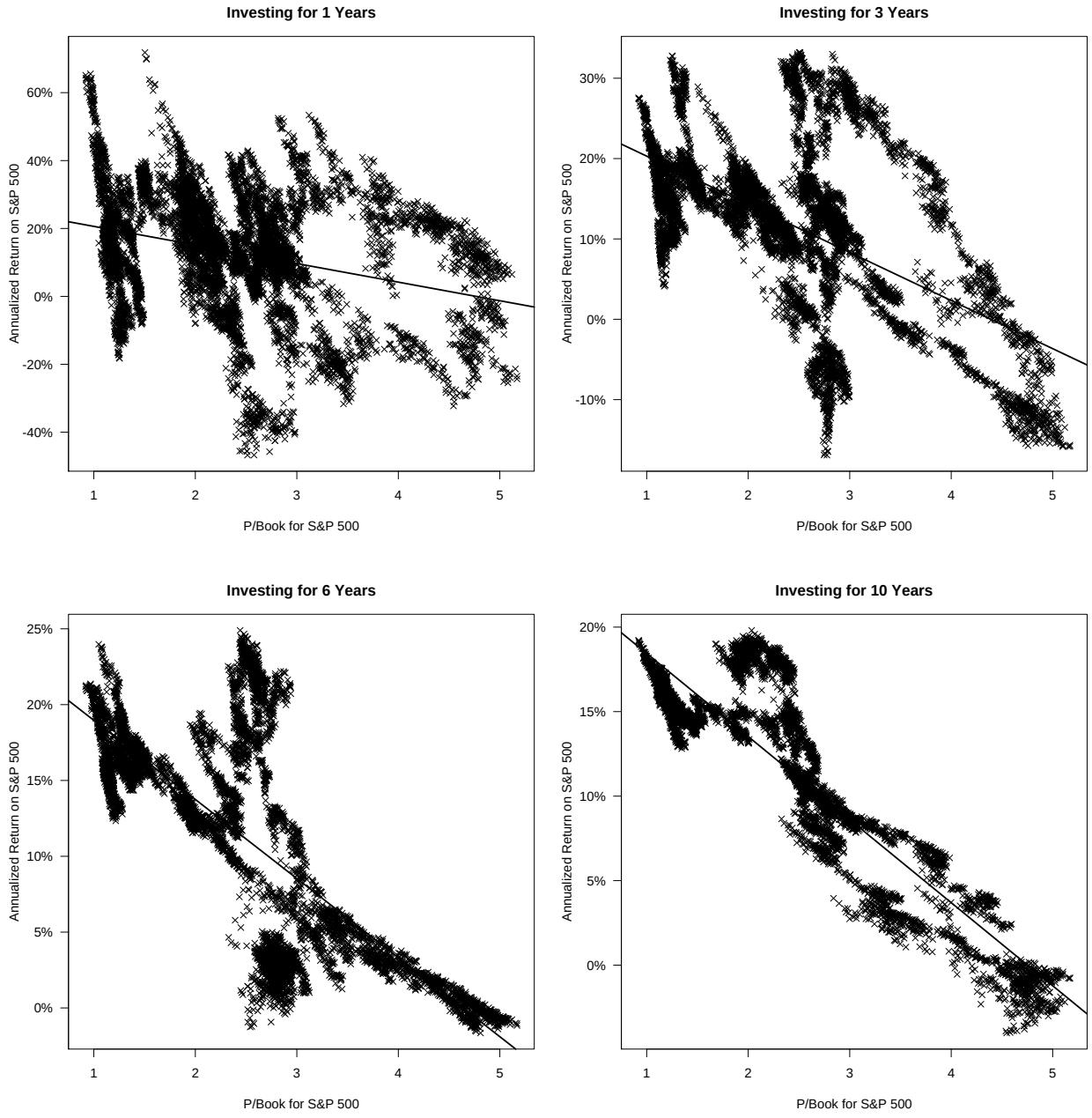

Figure 41: Scatter-plots of the S&P 500 P/Book versus annualized return for investment periods of 1, 3, 6 and 10 years. The plots have lines fitted whose coefficients of determination R^2 are 0.09, 0.30, 0.52, and 0.81, respectively.

As always in statistics, one should not conclude that a strong statistical relation is a causal relation. Low P/Book ratios for the S&P 500 may not always result in high annualized returns in the following ten years, and vice versa. But our choice is to either use fixed rebalancing between the S&P 500 and government bonds with the demonstrated flaws of such strategies, or we can try and use the P/Book of the S&P 500 to improve the rebalancing. This appears to be sensible because there is a logical relation between the equity and future earnings as discussed in section 13.1.1, and this is corroborated by a strong statistical relation.

13.1.4. Forecasting Annualized Return for 10 Years

The scatter-plot in Figure 41 for the P/Book versus annualized return of the S&P 500 for 10-year investment periods, has a high coefficient of determination $R^2 = 0.81$ and the adjusted R^2 is about the same because there are several thousand observations. The *p*-value is less than 2.2e-16 (i.e. very close to zero).

This suggests the relation is approximately linear and the formula is:

$$Annualized\ Return \approx 23.4\% - 4.9\% \times P/Book$$

Eq. 13-2

So the annualized return of the S&P 500 for ten year investment periods can be approximated using this formula given the P/Book. For example, if the P/Book of the S&P 500 is 1.5 then the annualized return of the S&P 500 for ten year investment periods is estimated to be about 16.05%. If the P/Book is 2.5 then the annualized return is estimated to be about 11.15%. If the P/Book is 3.5 then the annualized return is estimated to be about 6.25%. If the P/Book is 5 then the annualized return is estimated to be about (1.1%).

Estimation Error

The linear relation in Eq. 13-2 between the P/Book and annualized return of the S&P 500 is not perfect and the estimation errors (or residuals) can be quite large. The greatest residuals were (5.9%) and 6.7%, and the residual standard error was 2.5%. This potentially large inaccuracy should be kept in mind when forecasting the future returns of the S&P 500 using Eq. 13-2.

For example, if the P/Book is 2.5 then the annualized return of the S&P 500 over 10 years is estimated to be about 11.15% using the linear relation in Eq. 13-2. But the return could be as low as 5% or as high as 18% because of estimation error. And there is no guarantee that future returns are even within these margins of error as they are only known to hold for the period 1978-2013.

13.2. Methodology

The adaptive rebalancing strategies are based on simple formulas that use as input the P/Book of the S&P 500 and output stock-weights between zero and one for determining the fraction of the portfolio to invest in the S&P 500 with the remainder of the portfolio being invested in US government bonds.

The formulas have been devised using a proprietary computer program. The formulas are optimal in the sense that they maximize the annualized returns on average and under various risk constraints, e.g. that losses greater than a certain percentage are disallowed. The computer program considers investments starting on all dates during 1978-2013 and investment periods from one to ten years, as described in section 3.

13.3. Low Risk Strategy (Using P/Book)

The adaptive rebalancing strategies are named after their risk-level, that is, the magnitude of the largest losses experienced for any starting date and investment period up to ten years during 1978-2013. The low-risk strategy did not experience losses greater than (5%) during the period 1978-2013, as detailed below.

13.3.1. Stock Weight

The rebalancing is done annually and the fraction of the portfolio to invest in the S&P 500 is calculated using the formula:

$$Stock\ Weight = Limit(1.1 - 0.4 \times P/Book)$$

Eq. 13-3

The limit-function simply means that the stock-weight is limited between zero and one. For example, if P/Book is 3.5 then the result of the inner formula is $1.1 - 0.4 \times P/Book = 1.1 - 0.4 \times 3.5 = -0.3$ which is then limited between zero and one so the stock-weight is zero.

Because of this limiting, the stock-weight is one when the P/Book is 0.25 or lower, and the stock-weight is zero when the P/Book is 2.75 or higher. Between these two boundaries the stock-weight is determined by the inner formula which is linear and hence forms a straight line between the two boundaries. For example, if the P/Book is 2.4 then the stock-weight is $1.1 - 0.4 \times P/Book = 1.1 - 0.4 \times 2.4 = 0.14$, which means that 14% of the portfolio should be invested in the S&P 500 index when the P/Book is 2.4.

The bond-weight determines the amount of the portfolio to invest in US government bonds and it is simply the remainder of the portfolio after investing in the S&P 500 index:

$$Bond\ Weight = 1 - Stock\ Weight$$

Eq. 13-4

Continuing the above example where the P/Book was 2.4 and the resulting stock-weight was 0.14 (or 14%), the bond-weight is one minus the stock-weight which is 0.86 (or 86%).

Figure 42 shows the stock-weight for each day during the period 1978-2013 calculated using Eq. 13-3 with the P/Book data from Figure 2. About 69% of the days in this period had some part of the portfolio invested in the S&P 500 while the remaining 31% of the days had no investment in the S&P 500 because the stock-weight was zero. Note that the stock-weight is shown for each day but the investment strategy uses annual rebalancing, as demonstrated below.

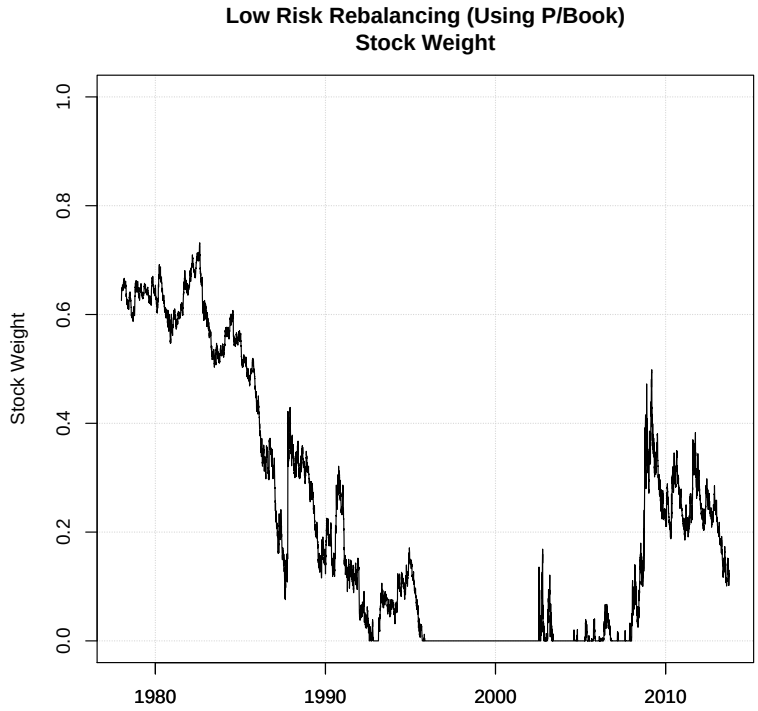

Figure 42: Stock-weight for adaptive low-risk rebalancing (using P/Book).

13.3.2. Annualized Returns

Figure 43 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 7.

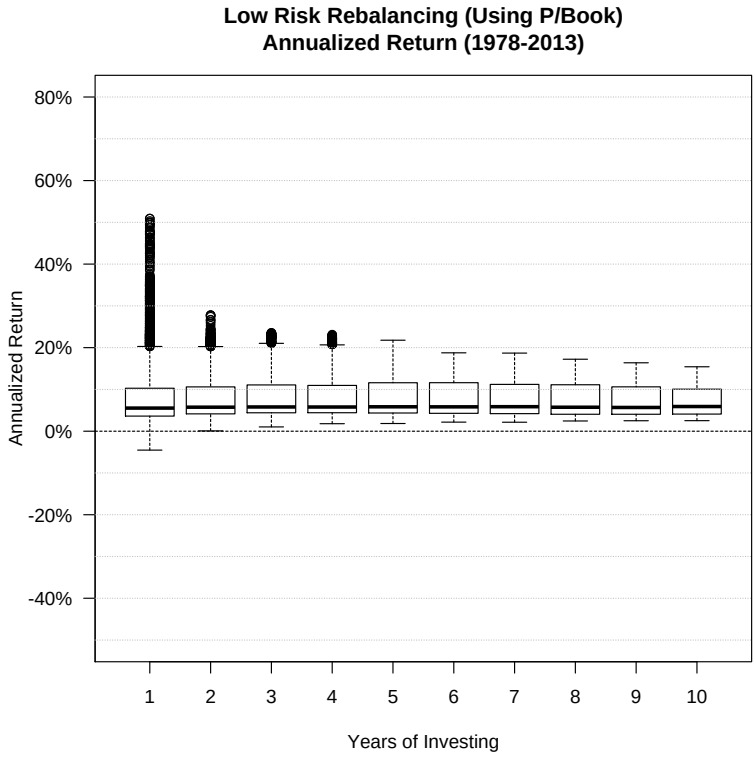

Figure 43: Annualized return statistics for adaptive low-risk rebalancing using the P/Book of the S&P 500 index.

Annualized Return for Low Risk Rebalancing (Using P/Book)										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(4.5%)	3.6%	5.5%	7.9%	10.3%	50.9%	7.2%	0.025	0.15	0.75
2	0.1%	4.2%	5.7%	7.8%	10.6%	27.8%	5.2%	0	0.07	0.72
3	1.0%	4.4%	5.8%	7.8%	11.1%	23.6%	4.7%	0	0.05	0.71
4	1.8%	4.4%	5.8%	7.7%	11.0%	23.1%	4.5%	0	0.04	0.68
5	1.8%	4.4%	5.8%	7.7%	11.6%	21.8%	4.5%	0	0.03	0.70
6	2.2%	4.3%	5.8%	7.6%	11.6%	18.8%	4.3%	0	0.03	0.69
7	2.2%	4.2%	5.9%	7.5%	11.2%	18.7%	4.2%	0	0.03	0.79
8	2.5%	4.1%	5.7%	7.4%	11.1%	17.2%	4.0%	0	0.03	0.85
9	2.5%	4.1%	5.7%	7.3%	10.6%	16.4%	3.8%	0	0.03	0.88
10	2.5%	4.1%	5.9%	7.3%	10.1%	15.4%	3.6%	0	0.03	0.85

Table 7: Annualized return statistics for adaptive low-risk rebalancing using the P/Book of the S&P 500 index. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

For a one-year investment period the mean annualized return is 7.9% which decreases to 7.3% for ten-year investment periods. The distributions are wider for shorter investment periods. For one-year investment periods the minimum return is a loss of (4.5%) and the maximum return is a gain of 50.9%. For ten-year investment periods the minimum annualized return is 2.5% giving a compounded gain of 28% over ten years, while the maximum annualized return is 15.4% giving a compounded gain of 319% over ten years.

The probability of loss was 0.025 (or 2.5%) for one-year investment periods and the probability was zero for all investment periods of 2 years or more.

13.3.3. Comparison to Bond-Only & Stock-Only Investments

Figure 44 shows the performance of the rebalancing strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the rebalancing strategy performed better than bond-only and worse than stock-only investments. But this mutual performance is misleading as it depends on the starting date and investment period.

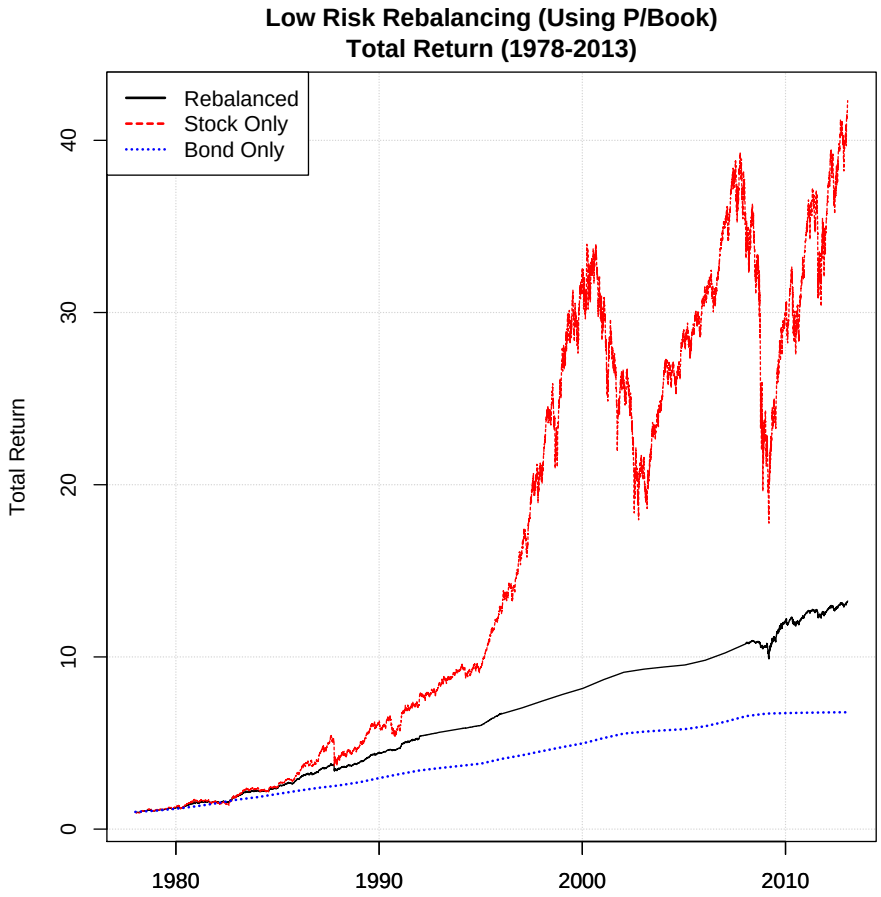

Figure 44: Total return of adaptive low-risk rebalancing (using P/Book) compared to stock-only and bond-only investments for the period 1978-2013.

Probability of Under-Performance

Table 7 shows the probabilities of the rebalanced portfolio under-performing bond-only investments. For one-year investment periods the probability was 0.15 (or 15%). The probability gradually decreased to about 0.03 (or 3%) for four years or more of investing.

Table 7 also shows the probabilities of the rebalanced portfolio under-performing stock-only investments. For one-year investment periods the probability was 0.75 (or 75%). The lowest probability was 0.68 (or 68%) which occurred for investment periods of 4 years. The highest probability was 0.88 (or 88%) which occurred for investment periods of 9 years.

Better than Bond-Only Investment

Figure 45 shows an example of the rebalancing strategy performing better than a bond-only investment. The investment period starts August 12, 1982. Table 8 shows that on this date the P/Book for the S&P 500 was 0.92 and using Eq. 13-3 gives a stock-weight of 0.73. Investing 0.73 (or 73%) of the portfolio in the S&P 500 and the rest in government bonds results in the portfolio growing to 1.51 (or 51% gain) in the first year until August 12, 1983. On this date the P/Book had increased to 1.41 and Eq. 13-3 now indicates that 0.54 (or 54%) of the portfolio should be invested in the S&P 500 with the remainder invested in government bonds. After the second year the rebalanced portfolio has a total return of 1.63 which means it has grown 63% over the two years. In comparison the bond-only portfolio had a total return of 1.24 (or 24% gain) and the stock-only had a total return of 1.65 (or 65% gain). For the ten-year period the rebalanced portfolio had a total return of 4.17 (or 317% gain), the bond-only investment had a total return of 2.27 (or 127% gain), and the stock-only investment had a total return of 5.80 (or 480% gain).

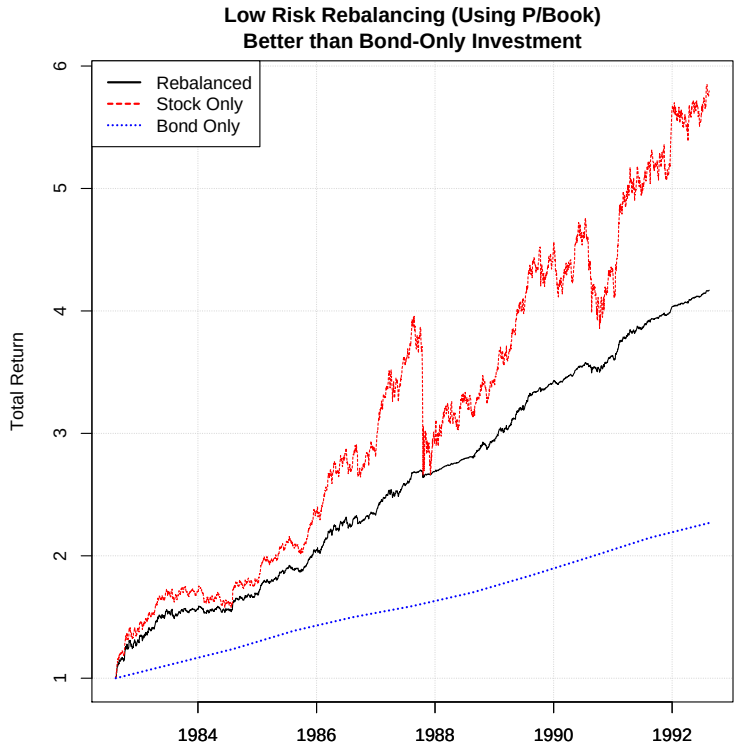

Figure 45: Total return of adaptive low-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

Low Risk Rebalancing (Using P/Book) Better than Bond-Only Investment											
Date	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
P/Book	0.92	1.41	1.37	1.51	1.95	2.54	1.88	2.38	2.20	2.46	2.75
Bond Yield	12.1%	10.8%	11.7%	8.1%	6.0%	6.9%	8.3%	8.2%	7.8%	5.7%	3.4%
Stock Weight	0.73	0.54	0.55	0.50	0.32	0.08	0.35	0.15	0.22	0.12	0
Rebalanced	1	1.51	1.63	1.89	2.30	2.68	2.81	3.32	3.55	3.91	4.17
Bond-Only	1	1.12	1.24	1.39	1.50	1.59	1.70	1.84	1.99	2.15	2.27
Stock-Only	1	1.65	1.75	2.08	2.82	3.92	3.17	4.33	4.31	5.16	5.80

Table 8: Total return of adaptive low-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 42.

Worse than Bond-Only Investment

Figure 46 shows an example of the rebalancing strategy performing worse than a bond-only investment. The starting date is June 30, 1999. Table 9 shows that the stock-weights were zero for the first seven years so the rebalanced portfolio was entirely invested in bonds during this period. In year 2006 only 0.03 (or 3%) of the portfolio was invested in the S&P 500. In year 2007 the stock-weight was zero again. This is because the stock-weight calculated using Eq. 13-3 is zero when the P/Book is 2.75 or higher and it was only in year 2006 that the P/Book was slightly lower at 2.67. Then again in 2008 the P/Book was lower at 2.37 giving a stock-weight of 0.15 (or 15%). The latest stock-weight shown in Table 9 is for year 2009 which is the stock-weight used in the portfolio allocation between June 30, 2009 and June 30, 2010, the results of which are not shown in Table 9 as it ends on June 30, 2009.

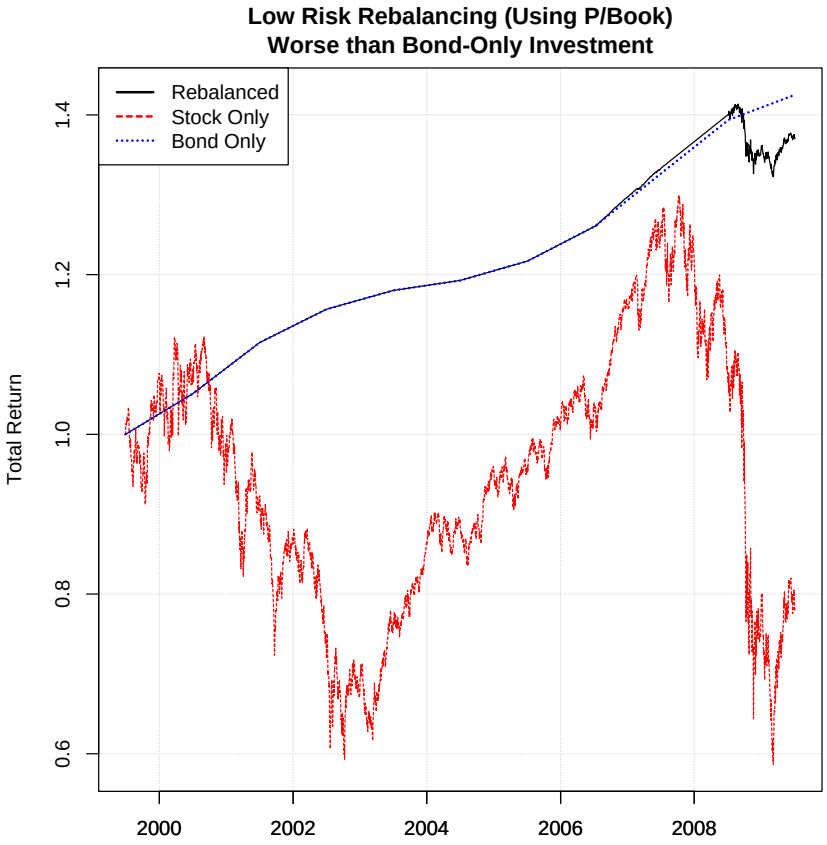

Figure 46: Total return of adaptive low-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

Low Risk Rebalancing (Using P/Book) Worse than Bond-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	4.97	4.65	3.58	2.88	2.87	2.91	2.76	2.67	2.95	2.37	1.80
Bond Yield	5.1%	6.1%	3.7%	2.0%	1.1%	2.0%	3.6%	5.3%	5.0%	2.3%	0.5%
Stock Weight	0	0	0	0	0	0	0	0.03	0	0.15	0.38
Rebalanced	1	1.05	1.11	1.16	1.18	1.19	1.22	1.26	1.33	1.40	1.37
Bond-Only	1	1.05	1.11	1.16	1.18	1.19	1.22	1.26	1.33	1.39	1.42
Stock-Only	1	1.07	0.92	0.72	0.77	0.88	0.96	1.03	1.26	1.06	0.77

Table 9: Total return of adaptive low-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 42.

For the ten-year investment period from June 30, 1999 to June 30, 2009 the rebalancing strategy had a total return of 1.37 while the bond-only investment had a total return of 1.42. It was in the last year that the rebalancing under-performed the bond-only investment. The reason is that the P/Book had decreased to 2.37 which meant that Eq. 13-3 gave a stock-weight of 0.15 but in the following year the S&P 500 decreased about (27%) net of dividends so the rebalanced portfolio experienced a loss of about (4.1%) resulting from the part of the portfolio invested in the S&P 500. This loss was only partially made up for by the 2.3% bond yield on the remaining part of the portfolio which was invested in government bonds between June 30, 2008 and June 30, 2009. The end result was that the rebalanced portfolio slightly under-performed a bond-only investment at the end of the ten year investment period.

Better than Stock-Only Investment

Figure 47 shows an example of the rebalancing strategy performing better than a stock-only investment. The starting date is February 24, 1999. Note that this is only about 4 months before the previous example.

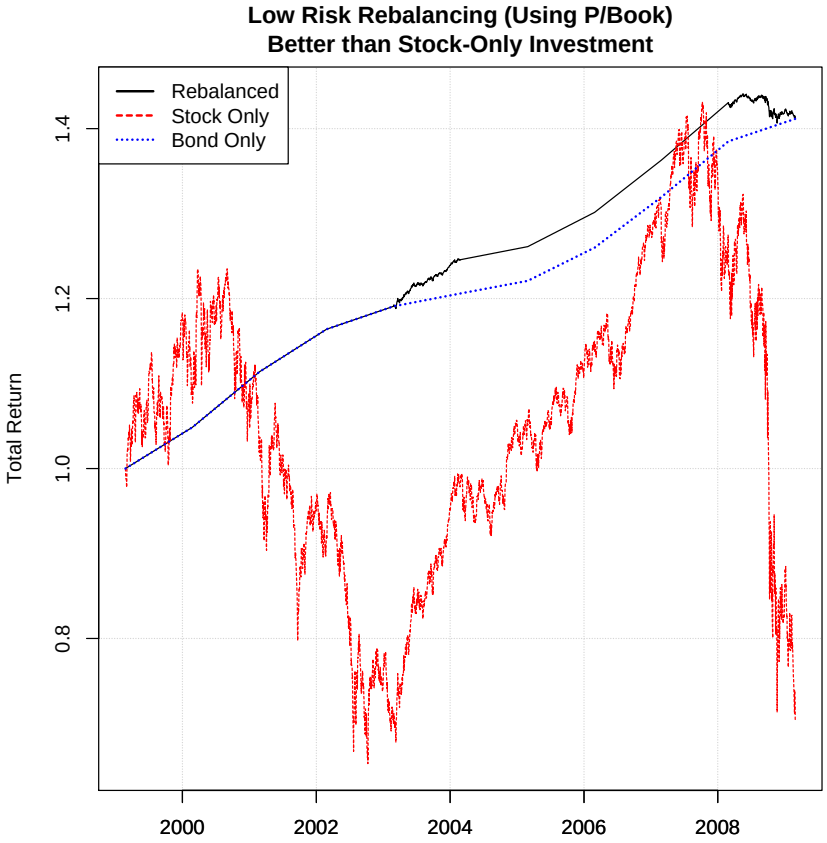

Figure 47: Left plot shows the total return of adaptive low-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

As shown in Table 10, the stock-weight is zero for the first four years of investing. This is again because the P/Book for the S&P 500 is above 2.75 which is the upper limit for the stock-weights calculated using Eq. 13-3. Then on February 24, 2003 the P/Book is only 2.54 which gives a stock-weight of 0.09, so 9% of the portfolio is invested in the S&P 500 in the following year and the remainder in government bonds. In the following four years the P/Book is again higher than 2.75 so the stock-weight is zero and the rebalanced portfolio is entirely invested in government bonds. But the small gain from investing a part of the portfolio

in the S&P 500 during 2003-2004 carries over so the rebalanced portfolio is slightly ahead until February 24, 2008 where the P/Book has again decreased to become 2.58 at which point 0.07 (or 7%) of the portfolio is allocated to the S&P 500, which then decreases about (47%) net of dividends in the following year, thus resulting in the rebalanced portfolio erasing the gain it had over the bond-only portfolio, but still performing about twice as well as the stock-only portfolio.

Low Risk Rebalancing (Using P/Book) Better than Stock-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	4.65	4.61	3.74	3.31	2.54	3.07	2.85	2.76	2.77	2.58	1.56
Bond Yield	4.9%	6.2%	4.5%	2.3%	1.3%	1.2%	3.2%	4.7%	5.0%	1.9%	0.7%
Stock Weight	0	0	0	0	0.09	0	0	0	0	0.07	0.48
Rebalanced	1	1.05	1.11	1.16	1.19	1.25	1.26	1.30	1.36	1.43	1.41
Bond-Only	1	1.05	1.11	1.16	1.19	1.21	1.22	1.26	1.32	1.38	1.41
Stock-Only	1	1.09	1.04	0.92	0.70	0.98	1.05	1.14	1.27	1.26	0.67

Table 10: Total return of adaptive low-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 42.

Worse than Stock-Only Investment

Figure 48 shows an example of the rebalancing strategy performing worse than a stock-only investment. The starting date is August 23, 1990.

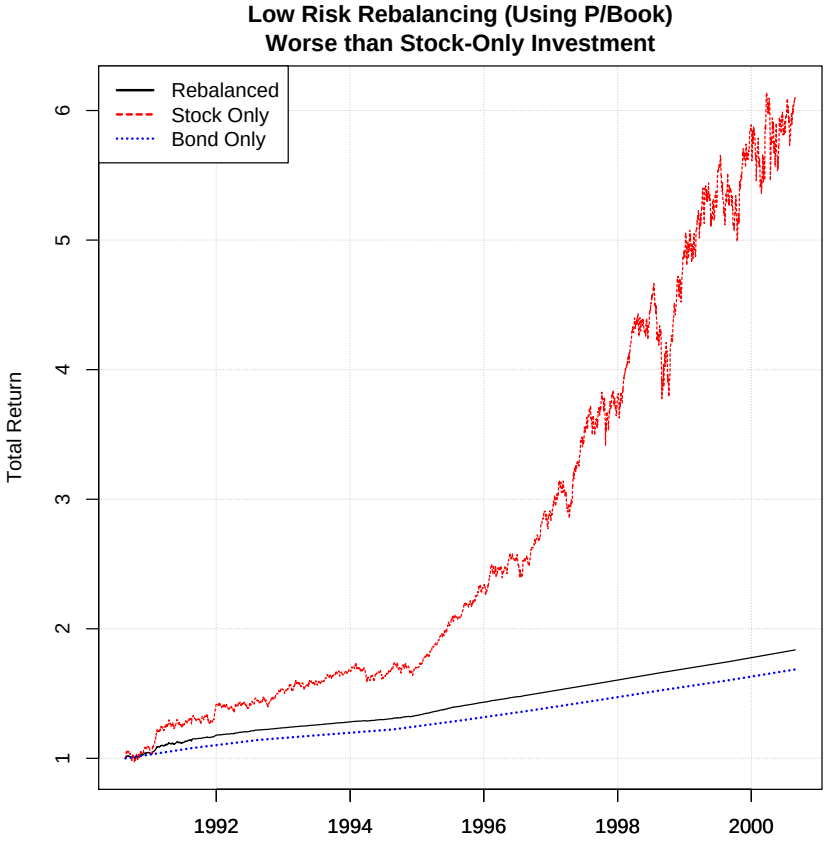

Figure 48: Total return of adaptive low-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

Table 11 shows the stock-weight was 0.29 in 1990 because the P/Book for the S&P 500 was 2.03. A year later the P/Book had increased to 2.51 and the stock-weight decreased to 0.09. In the following years the P/Book was between 2.52-2.69 so the stock-weight was low between 0.02-0.09. From 1996 and onwards the P/Book was higher than the limit of 2.75 for Eq. 13-3 so the stock-weights were zero for those years and hence the rebalanced portfolio was entirely allocated to government bonds. The result was that the rebalanced portfolio performed slightly better than the bond-only portfolio in the years 1990-1996 where the stock-weight was slightly above zero and the S&P 500 increased more than government bonds. After that the rebalanced portfolio was entirely allocated in bonds and the end result for the ten years was that the rebalanced portfolio had performed somewhat better than a bond-only investment but significantly worse than a stock-only investment, because the total return of the S&P 500 was about 509% for this period in which the rebalanced portfolio was mostly allocated to government bonds because of the high P/Book of the S&P 500.

Low Risk Rebalancing (Using P/Book) Worse than Stock-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.03	2.51	2.69	2.61	2.52	2.69	2.97	3.74	4.00	4.83	4.76
Bond Yield	8.0%	5.8%	3.6%	3.4%	5.6%	5.7%	5.8%	5.6%	5.0%	5.2%	6.3%
Stock Weight	0.29	0.09	0.02	0.06	0.09	0.02	0	0	0	0	0
Rebalanced	1	1.15	1.22	1.27	1.31	1.40	1.49	1.57	1.66	1.74	1.84
Bond-Only	1	1.08	1.14	1.18	1.22	1.29	1.37	1.44	1.52	1.60	1.69
Stock-Only	1	1.32	1.42	1.63	1.71	2.09	2.55	3.55	4.11	5.38	6.09

Table 11: Total return of adaptive low-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 42.

13.3.4. Comparison to Fixed Rebalancing

Depending on the duration of the investment, the low-risk adaptive rebalancing strategy had a mean annualized return between 7.3-7.9%, see Table 7. The fixed rebalancing strategy with 25% stock / 75% bond had somewhat similar mean annualized returns of 7.3-7.5%, see Table 4. But otherwise the risk and return characteristics were quite different for the two strategies.

The median annualized return was 5.5-5.9% for the adaptive strategy while it was considerably higher at 7.6-8.1% for the fixed 25/75 strategy.

The adaptive strategy only experienced losses for one-year investments periods with the probability of loss being 0.025 (or 2.5%) and the greatest loss was (4.5%), while the fixed 25/75 strategy had probability of loss 0.077 (or 7.7%) and the greatest loss was (10.4%). The fixed 25/75 strategy experienced losses for investment periods up to four years, with gradually decreasing probabilities and magnitudes of the losses. For ten-year investment periods the lowest annualized return was 2.5% for the adaptive strategy while it was 2.1% for the fixed 25/75 strategy.

The largest gain for one-year investment periods was 50.9% for the adaptive strategy while it was only 27.2% for the fixed 25/75 strategy. For ten-year investment periods the largest annualized return was 15.4% for the adaptive strategy while it was only 12.4% for the fixed 25/75 strategy.

Probability of Under-Performing Bond-Only and Stock-Only Investments

Table 7 shows the probability of the adaptive strategy under-performing bond-only and stock-only investments, and Table 4 shows the probabilities for the fixed 25/75 strategy. For one-year investment periods the adaptive strategy under-performed bond-only investments with probability 0.15 (or 15%) while the probability was 0.25 (or 25%) for the fixed 25/75 strategy. For investment periods of five years or more, the adaptive strategy had probability about 0.03 (or 3%) of under-performing bond-only investments, while the probabilities for the fixed 25/75 strategy gradually decreased to 0.09 (or 9%) for ten-year investment periods. For all investment periods, the adaptive strategy had significantly lower probability of under-performing bond-only investments than the fixed 25/75 strategy.

The probabilities for under-performing stock-only investments are remarkably similar, with the fixed 25/75 strategy having slightly higher probabilities than the adaptive strategy.

Standard Deviation

The standard deviation measures the spread around the mean of the annualized returns. For one-year investment periods the adaptive strategy had a standard deviation of 7.2% while the fixed 25/75 strategy had a standard deviation of only 5.5%. The standard deviation gradually decreases for longer investment periods and for ten-year investment periods it was 3.6% for the adaptive strategy and only 2.9% for the fixed 25/75 strategy. The adaptive strategy had a higher standard deviation than the fixed 25/75 strategy for all investment periods.

As previously discussed in section 2.6.1, the standard deviation (or equivalently, the variance) is often used as a measure of risk in finance. But in this example the adaptive strategy had consistently lower probability and magnitude of losses than the fixed 25/75 strategy, yet the standard deviation was higher for the adaptive strategy. The reason is that the adaptive strategy sometimes had much higher gains which gives a larger spread of the annualized returns and hence a greater standard deviation. So the standard deviation has once more been shown to be useless as a risk measure in finance.

13.4. Medium Risk Strategy (Using P/Book)

The medium-risk adaptive rebalancing strategy only experienced losses smaller than (14%) during the period 1978-2013. The strategy works by annually rebalancing the investment portfolio by investing a part of the portfolio in the S&P 500 index depending on its P/Book, and the remaining part of the portfolio is invested in US government bonds with one year maturity.

13.4.1. Stock Weight

The fraction of the portfolio invested in the S&P 500 index is called the stock-weight and calculated as:

$$\textit{Stock Weight} = \textit{Limit}(1.5 - 0.5 \times \textit{P/Book})$$

Eq. 13-5

The limit-function means that the stock-weight should be limited between zero and one. The stock-weight is one when the P/Book is 1 or lower, and the stock-weight is zero when the P/Book is 3 or higher. Between these two boundaries the stock-weight changes linearly. For example, if the P/Book is 2.4 then the stock-weight is 1.5 – 0.5 x P/Book = 1.5 – 0.5 x 2.4 = 0.3 so that 0.3 (or 30%) of the portfolio should be invested in the S&P 500 and the remaining 0.7 (or 70%) of the portfolio should be invested in US government bonds.

Figure 49 shows the stock-weight for each day during the period 1978-2013 calculated using Eq. 13-5 with the P/Book data from Figure 2. About 82% of the days in this period had some part of the portfolio invested in the S&P 500 index while the remaining 18% of the days only had investments in government bonds because the stock-weight was zero. Note that the stock-weight is shown for each day but the investment strategy uses annual rebalancing, as demonstrated below.

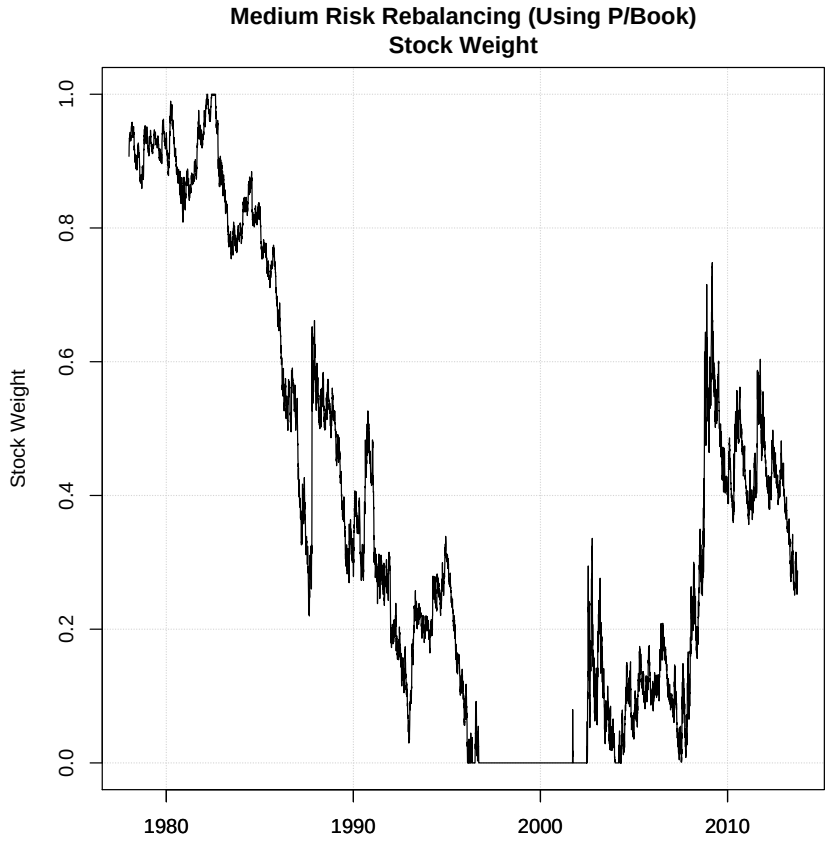

Figure 49: Stock-weight for adaptive medium-risk rebalancing (using P/Book).

13.4.2. Annualized Return

Figure 50 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 12.

For a one-year investment period the mean annualized return is 9.3% which decreases to 8.3% for ten-year investment periods. The distributions are wider for shorter investment periods. For one-year investment periods the greatest loss is (13.9%) and the greatest gain is 65.6%. For ten-year investment periods the minimum annualized return is 2.7% giving a compounded gain of 31% over ten years, while the maximum annualized return is 18.1% giving a compounded gain of 428% over ten years.

The probability of loss is 0.06 (or 6%) for one-year investment periods and the probability of loss gradually decreases until it is zero for all investment periods of 5 years or more.

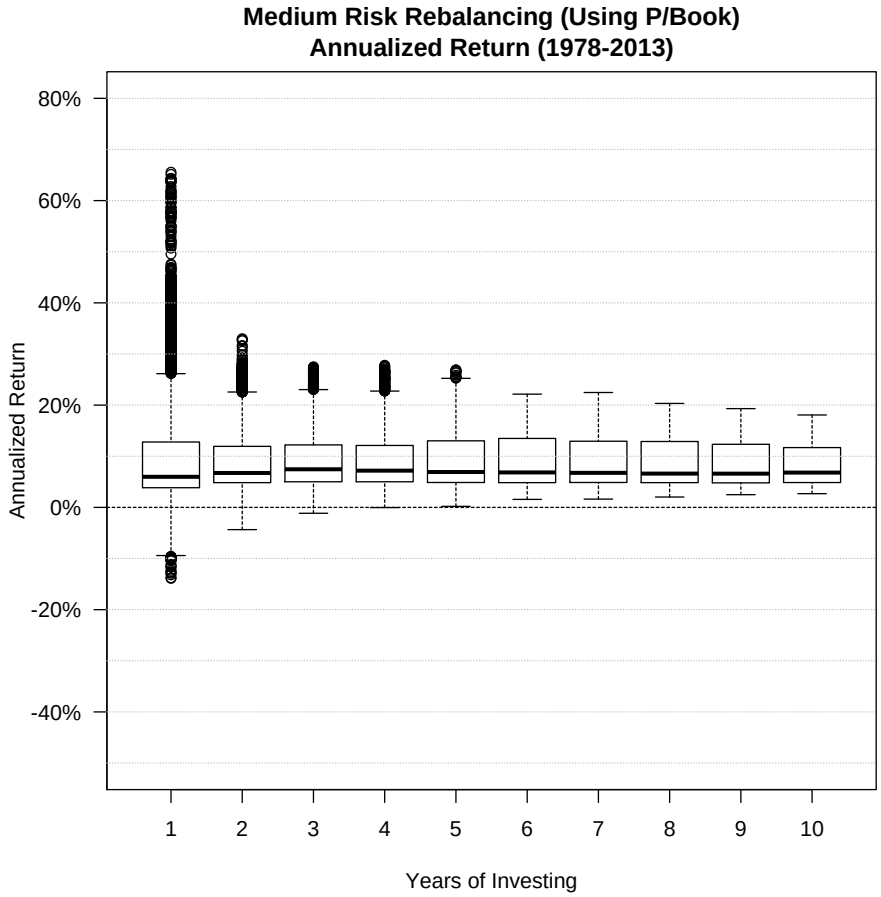

Figure 50: Annualized return statistics for adaptive medium-risk rebalancing using the P/Book of the S&P 500 index.

Annualized Return for Medium Risk Rebalancing (Using P/Book)										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(13.9%)	3.9%	6.0%	9.3%	12.8%	65.6%	9.7%	0.06	0.18	0.74
2	(4.3%)	4.8%	6.7%	9.0%	11.9%	33.0%	6.2%	0.02	0.09	0.68
3	(1.1%)	5.0%	7.5%	8.9%	12.2%	27.5%	5.4%	0.002	0.07	0.66
4	(0.004%)	5.0%	7.2%	8.8%	12.1%	27.8%	5.1%	0.0001	0.05	0.62
5	0.2%	4.9%	6.9%	8.8%	13.0%	26.9%	5.1%	0	0.03	0.60
6	1.6%	4.8%	6.8%	8.7%	13.5%	22.2%	4.8%	0	0.03	0.62
7	1.6%	4.9%	6.8%	8.6%	12.9%	22.5%	4.7%	0	0.03	0.69
8	2.0%	4.8%	6.6%	8.6%	12.9%	20.3%	4.6%	0	0.03	0.78
9	2.5%	4.8%	6.6%	8.4%	12.3%	19.3%	4.3%	0	0.03	0.84
10	2.7%	4.9%	6.8%	8.3%	11.7%	18.1%	4.1%	0	0.03	0.81

Table 12: Annualized return statistics for adaptive medium-risk rebalancing using the P/Book of the S&P 500 index. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

13.4.3. Comparison to Bond-Only & Stock-Only Investments

Figure 51 shows the performance of the rebalancing strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the rebalancing strategy performed better than bond-only and worse than stock-only investments. But this mutual performance is misleading as it depends on the starting date and investment period.

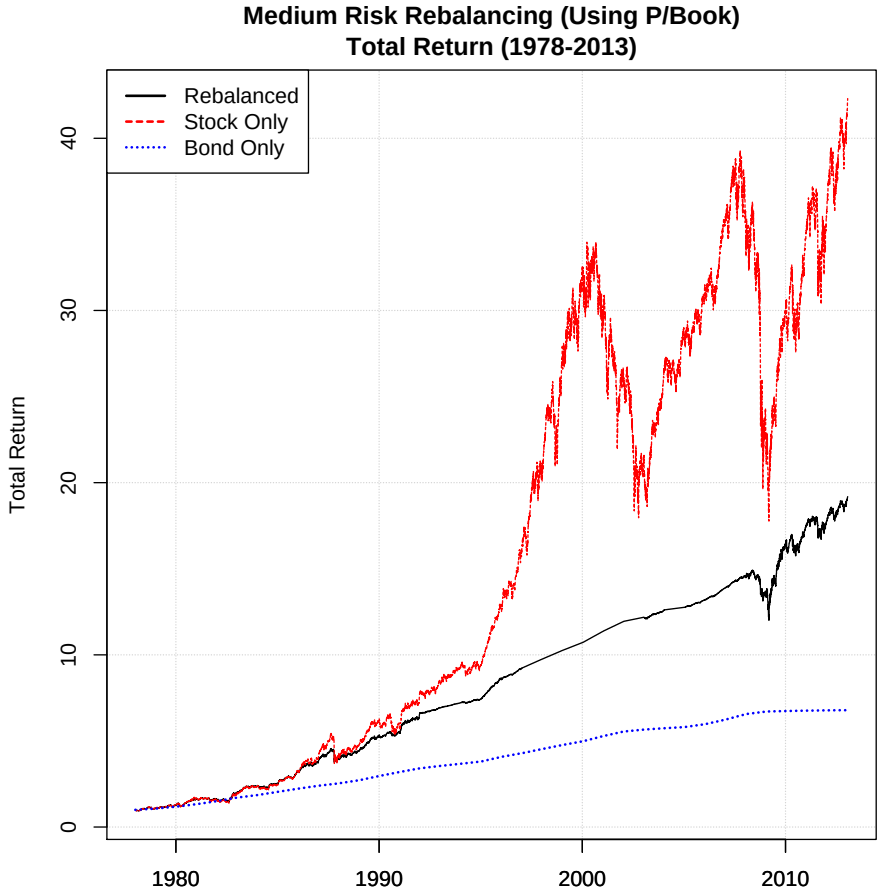

Figure 51: Total return of adaptive medium-risk rebalancing (using P/Book) compared to stock-only and bond-only investments for the period 1978-2013.

Probability of Under-Performance

Table 12 shows the probabilities of the rebalanced portfolio under-performing bond-only investments. For one-year investment periods the probability was 0.18 (or 18%). The probability gradually decreased to about 0.03 (or 3%) for five years or more of investing.

Table 12 also shows the probabilities of the rebalanced portfolio under-performing stock-only investments. For one-year investment periods the probability was 0.74 (or 74%). The lowest probability was 0.60 (or 60%) which occurred for investment periods of 5 years. The highest probability was 0.84 (or 84%) which occurred for investment periods of 9 years.

Better than Bond-Only Investment

Figure 52 shows an example of the rebalancing strategy performing better than a bond-only investment. The investment period starts August 12, 1982. Table 13 shows that the stock-weight was 1 on this date because the P/Book for the S&P 500 was low at 0.92. In the following years the P/Book increased and the stock-weight hence decreased. After ten years the rebalanced portfolio had a total return of 5.27 (or 427% gain), while the bond-only investment had a total return of 2.27 (or 127% gain), and the stock-only investment performed better than both with a total return of 5.80 (or 480% gain).

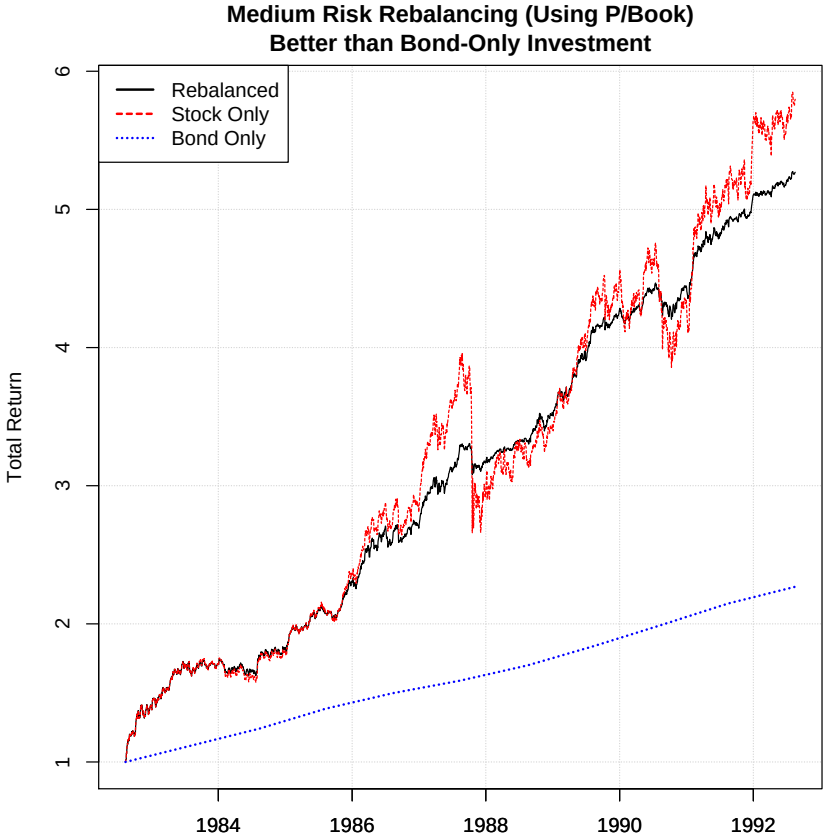

Figure 52: Total return of adaptive medium-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

Medium Risk Rebalancing (Using P/Book) Better than Bond-Only Investment											
Date	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
P/Book	0.92	1.41	1.37	1.51	1.95	2.54	1.88	2.38	2.20	2.46	2.75
Bond Yield	12.1%	10.8%	11.7%	8.1%	6.0%	6.9%	8.3%	8.2%	7.8%	5.7%	3.4%
Stock Weight	1.00	0.80	0.82	0.75	0.52	0.23	0.56	0.31	0.40	0.27	0.12
Rebalanced	1	1.65	1.76	2.07	2.67	3.29	3.32	4.13	4.35	4.90	5.27
Bond-Only	1	1.12	1.24	1.39	1.50	1.59	1.70	1.84	1.99	2.15	2.27
Stock-Only	1	1.65	1.75	2.08	2.82	3.92	3.17	4.33	4.31	5.16	5.80

Table 13: Total return of adaptive medium-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 49.

Worse than Bond-Only Investment

Figure 53 shows an example of the rebalancing strategy performing worse than a bond-only investment. The starting date is January 12, 1999.

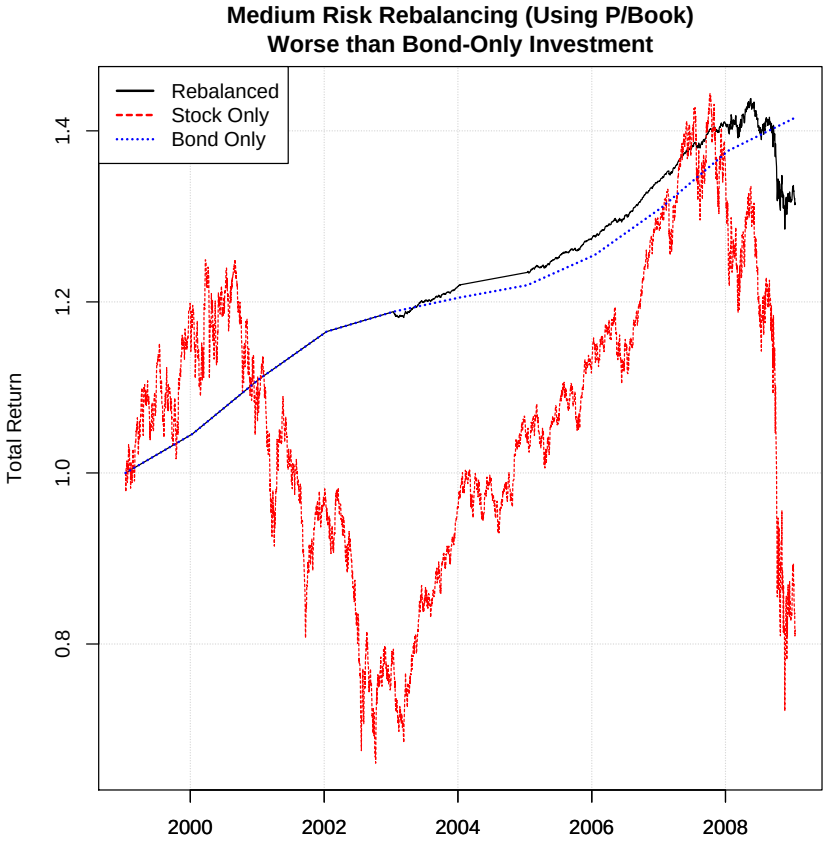

Figure 53: Total return of adaptive medium-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

Table 14 shows that the stock-weights were zero for the first four years so the rebalanced portfolio was entirely invested in bonds during this period. In year 2003 only 0.06 (or 6%) of the portfolio was invested in the S&P 500. In year 2004 the stock-weight was zero again. This is because the stock-weight calculated using Eq. 13-5 is zero when the P/Book is 3 or higher and it was only in year 2003 that the P/Book was slightly lower at 2.89. Then again in 2005 the P/Book was lower at 2.84 giving a stock-weight of 0.08 (or 8%), and so on.

Medium Risk Rebalancing (Using P/Book) Worse than Bond-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	4.64	4.92	4.01	3.37	2.89	3.07	2.84	2.81	2.83	2.52	1.79
Bond Yield	4.6%	6.2%	5.0%	2.0%	1.4%	1.2%	2.9%	4.4%	5.1%	2.8%	0.4%
Stock Weight	0	0	0	0	0.06	0	0.08	0.09	0.08	0.24	0.61
Rebalanced	1	1.05	1.11	1.16	1.19	1.22	1.23	1.28	1.34	1.40	1.30
Bond-Only	1	1.05	1.11	1.16	1.19	1.20	1.22	1.25	1.31	1.38	1.41
Stock-Only	1	1.17	1.09	0.95	0.79	0.98	1.04	1.15	1.30	1.24	0.77

Table 14: Total return of adaptive medium-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 49.

For the ten-year investment period from January 12, 1999 to January 12, 2009 the rebalancing strategy had a total return of 1.30 (or 30% gain) while the bond-only investment had a total return of 1.41 (or 41% gain). It was in the last year that the rebalancing under-performed the bond-only investment. The stock-only investment performed worse than both with a total return of 0.77 (or 23% loss).

Better than Stock-Only Investment

Figure 54 shows an example of the rebalancing strategy performing better than a stock-only investment. The starting date is February 24, 1999. This is only about 1.5 months after the previous example and the two examples are indeed similar but the rebalancing strategy performs even better compared to the stock-only investment.

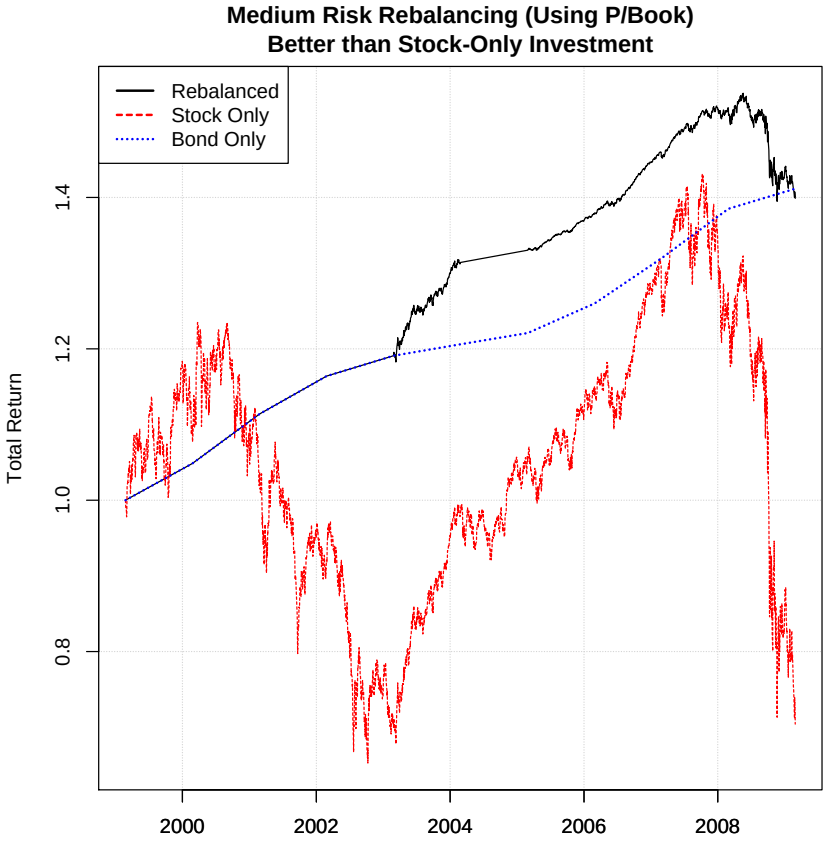

Figure 54: Total return of adaptive medium-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

Medium Risk Rebalancing (Using P/Book) Better than Stock-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	4.65	4.61	3.74	3.31	2.54	3.07	2.85	2.76	2.77	2.58	1.56
Bond Yield	4.9%	6.2%	4.5%	2.3%	1.3%	1.2%	3.2%	4.7%	5.0%	1.9%	0.7%
Stock Weight	0	0	0	0	0.23	0	0.08	0.12	0.11	0.21	0.72
Rebalanced	1	1.05	1.11	1.16	1.19	1.31	1.33	1.38	1.45	1.52	1.39
Bond-Only	1	1.05	1.11	1.16	1.19	1.21	1.22	1.26	1.32	1.38	1.41
Stock-Only	1	1.09	1.04	0.92	0.70	0.98	1.05	1.14	1.27	1.26	0.67

Table 15: Total return of adaptive medium-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 49.

As shown in Table 15, the total return for the rebalancing strategy was 1.39 (or 39% gain) while it was 0.67 (or 33% loss) for the stock-only investment. The bond-only investment performed slightly better than the rebalancing strategy with a total return of 1.41 (or 41% gain).

Worse than Stock-Only Investment

Figure 55 shows an example of the rebalancing strategy performing worse than a stock-only investment. The starting date is August 23, 1990.

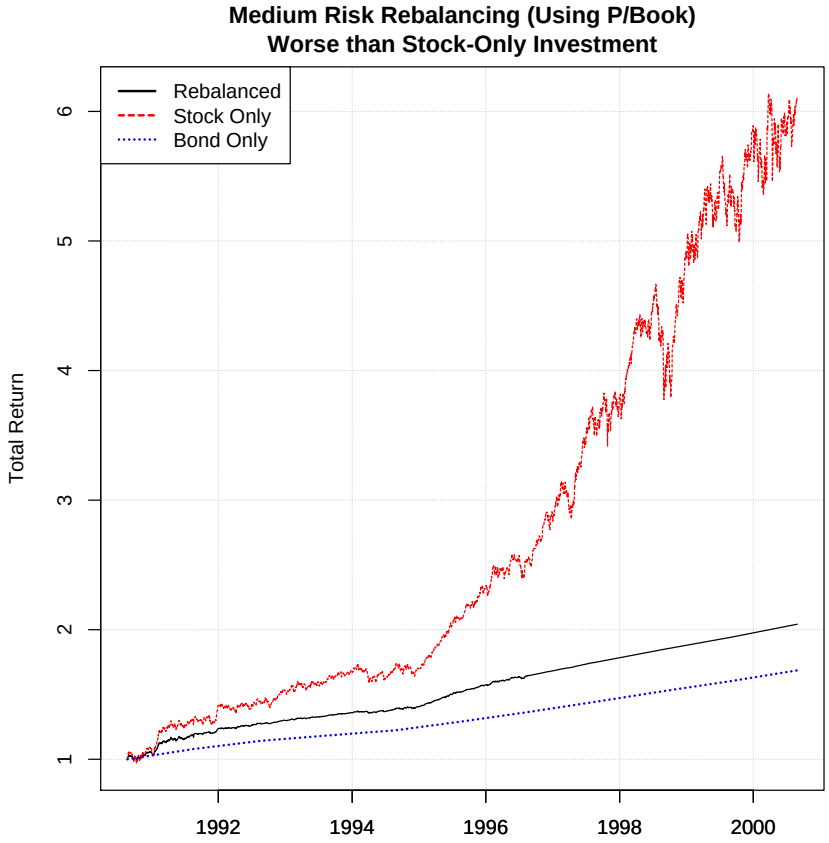

Figure 55: Total return of adaptive medium-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

Medium Risk Rebalancing (Using P/Book) Worse than Stock-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.03	2.51	2.69	2.61	2.52	2.69	2.97	3.74	4.00	4.83	4.76
Bond Yield	8.0%	5.8%	3.6%	3.4%	5.6%	5.7%	5.8%	5.6%	5.0%	5.2%	6.3%
Stock Weight	0.48	0.24	0.15	0.19	0.24	0.16	0.01	0	0	0	0
Rebalanced	1	1.20	1.27	1.34	1.39	1.52	1.65	1.75	1.85	1.94	2.04
Bond-Only	1	1.08	1.14	1.18	1.22	1.29	1.37	1.44	1.52	1.60	1.69
Stock-Only	1	1.32	1.42	1.63	1.71	2.09	2.55	3.55	4.11	5.38	6.09

Table 16: Total return of adaptive medium-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 49.

Table 16 shows that the stock-weight started at 0.48 in 1990 because the P/Book for the S&P 500 was 2.03. By 1997 the P/Book had increased to 3.74 which is above the upper limit of 3 for Eq. 13-5 so the stock-

weight is zero. In the following years the P/Book remained high so the stock-weight remained zero and the rebalanced portfolio was hence entirely invested in government bonds in those years. After the ten-year period the rebalancing strategy had a total return of 2.04 (or 104% gain) while the stock-only investment had a total return of 6.09 (or 509% gain). The bond-only investment performed somewhat worse than the rebalancing strategy with a total return of 1.69 (or 69% gain).

13.4.4. Comparison to Low-Risk Rebalancing

Table 12 shows the statistics for the annualized returns of the medium-risk strategy and Table 7 shows the statistics for the low-risk strategy. For one-year investment periods, the mean annualized return is 9.3% for the medium-risk strategy while it is only 7.9% for the low-risk strategy. The mean annualized returns gradually decrease with longer investment periods and for ten years it is 8.3% for the medium-risk strategy and only 7.3% for the low-risk strategy.

The greatest loss for a one-year investment period was (13.9%) for the medium-risk strategy, while it was only (4.5%) for the low-risk strategy. For investment periods of two years or more, the low-risk strategy did not experience any losses, while the medium-risk strategy experienced losses for investment periods up to four years. For ten-year investment periods the minimum annualized return was 2.7% for the medium-risk strategy, while it was slightly lower at 2.5% for the low-risk strategy.

The largest gain for a one-year investment period was 65.6% for the medium-risk strategy, while it was only 50.9% for the low-risk strategy. For ten-year investment periods the largest annualized return was 18.1% for the medium-risk strategy while it was only 15.4% for the low-risk strategy.

Probability of Loss

For one-year investment periods the probability of loss was 0.06 (or 6%) for the medium-risk strategy while it was only 0.025 (or 2.5%) for the low-risk strategy. For investment periods of two years or more, the low-risk strategy had zero probability of loss, while the medium-risk strategy had a gradually declining probability of loss and for investment periods of five years or more the probability was zero.

Probability of Under-Performing Bond-Only and Stock-Only Investments

For one-year investment periods the probability of under-performing bond-only investments was 0.18 (or 18%) for the medium-risk strategy while the probability was slightly lower at 0.15 (or 15%) for the low-risk strategy. The probabilities gradually decrease for longer investment periods and for five years or more the two strategies both had a probability of 0.03 (or 3%) of under-performing bond-only investments.

The probabilities for under-performing stock-only investments were often significantly lower for the medium-risk strategy compared to the low-risk strategy. For example, for five-year investment periods the probability was 0.60 (or 60%) for the medium-risk strategy to under-perform a stock-only investment, while the probability was 0.70 (or 70%) for the low-risk strategy. Although for some investment periods the difference was smaller, for example for one-year investment periods the probabilities were 0.74 and 0.75.

13.4.5. Comparison to Fixed Rebalancing

The risk and return characteristics for the medium-risk strategy are quite different from the fixed rebalancing strategies. Consider for example the fixed 50% stock / 50% bond strategy. Both these strategies had a mean annualized return of 9.3% for one-year investment periods, see Table 5 and Table 12. But for

longer investment periods the mean annualized return gradually decreased to 8.3% for the medium-risk strategy, while it decreased to 8.9% for ten-year investments using the fixed 50/50 strategy.

The median annualized return was between 6.0-7.5% for different investment periods using the medium-risk strategy, while it was significantly higher at 9.4-10.2% for the fixed 50/50 strategy.

The greatest loss for one-year investment periods was (13.9%) for the medium-risk strategy while it was (22.5%) for the fixed 50/50 strategy. For ten-year investment periods the smallest annualized return was 2.7% for the medium-risk strategy while it was 0.5% for the fixed 50/50 strategy.

The greatest gain for one-year investment periods was 65.6% for the medium-risk strategy while it was 40.0% for the fixed 50/50 strategy. For ten-year investment periods the largest annualized return was 18.1% for the medium-risk strategy while it was only 14.4% for the fixed 50/50 strategy.

Probability of Loss

Table 5 and Table 12 also show the probabilities of loss. For one-year investment periods the medium-risk strategy had probability of loss 0.06 (or 6%) while the fixed 50/50 strategy had more than twice as high a probability of loss at 0.13 (or 13%). The probability of loss gradually decreases for longer investment periods and the medium-risk strategy had zero probability of loss for investment periods of five years or more, while the fixed 50/50 strategy had non-zero probability of loss for all investment periods except six and ten years.

Probability of Under-Performing Bond-Only and Stock-Only Investments

Table 5 and Table 12 also show the probabilities of under-performing bond-only and stock-only investments.

The probability of under-performing bond-only investments for one-year investment periods was 0.18 (or 18%) for the medium-risk strategy while the probability was 0.25 (or 25%) for the fixed 50/50 strategy. The probabilities gradually decrease for longer investment periods and the medium-risk strategy has a probability of 0.03 (or 3%) for investment periods of five years or more, while the fixed 50/50 strategy has a probability of 0.09 (or 9%) for ten-year investment periods. For all investment periods the probability of under-performing bond-only investments is significantly greater for the fixed 50/50 strategy than for the medium-risk strategy.

The probability of under-performing stock-only investments for one-year investment periods is 0.74 (or 74%) for the medium-risk strategy while it is only slightly higher at 0.75 (or 75%) for the fixed 50/50 strategy. For some of the longer investment periods the probabilities are more different, for example, for five-year investment periods the probability was 0.60 (or 60%) for the medium-risk strategy, while it was 0.73 (or 73%) for the fixed 50/50 strategy. For all investment periods the fixed 50/50 strategy has a higher probability than the medium-risk strategy for under-performing stock-only investments.

13.5. High Risk Strategy (Using P/Book)

The low- and medium-risk strategies were devised so as to limit losses while maximizing the average annualized return if the strategies had been employed during the period 1978-2013. The high-risk strategy has no limitations on losses and is devised solely for maximizing the average annualized return.

13.5.1. Stock Weight

The strategy uses annual rebalancing as usual, with the stock-weight determining the part of the portfolio to invest in the S&P 500 index depending on its P/Book, and the remaining part of the portfolio is invested in US government bonds with one year maturity. The stock-weight is calculated as follows:

$$Stock\ Weight = Limit(2.8 - 0.6 \times P/Book)$$

Eq. 13-6

The stock-weight is one when the P/Book is 3 or lower, and the stock-weight is zero when the P/Book is 4.67 or higher.

Figure 56 shows the stock-weight for each day during the period 1978-2013 calculated using Eq. 13-6 with the P/Book data from Figure 2. About 96% of the days in this period had some part of the portfolio invested in the S&P 500 index and about 82% of the days had a full investment in the S&P 500 index. Note that the stock-weight is shown for each day but the investment strategy uses annual rebalancing.

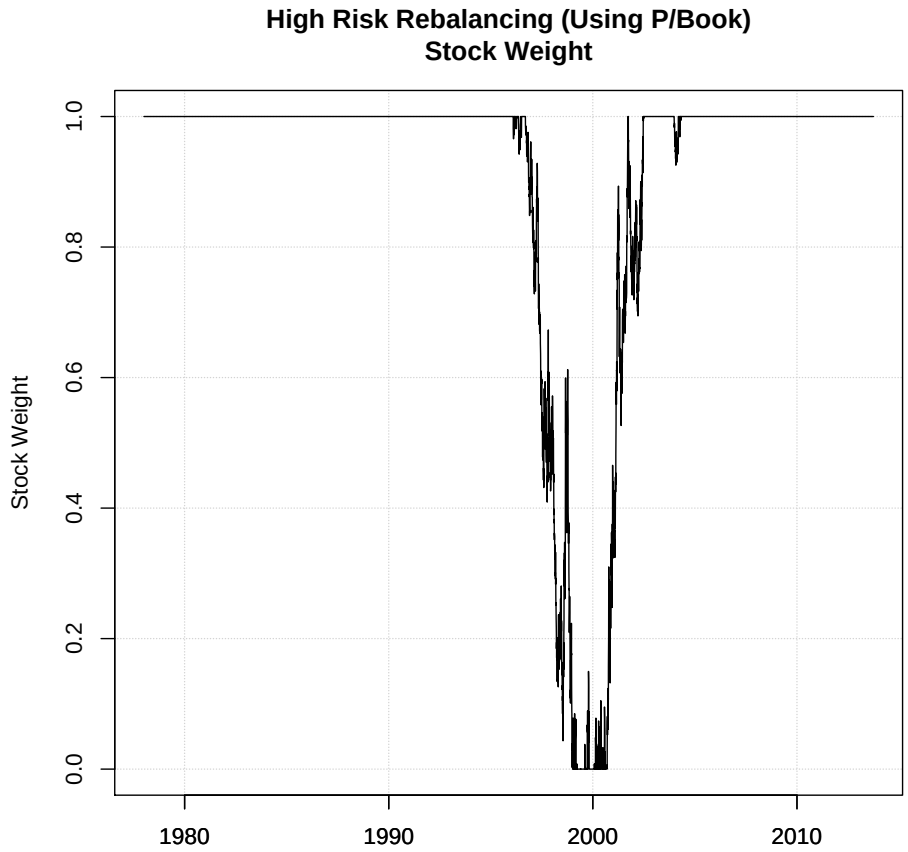

Figure 56: Stock-weight for adaptive high-risk rebalancing (using P/Book).

13.5.2. Annualized Return

Figure 57 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 17.

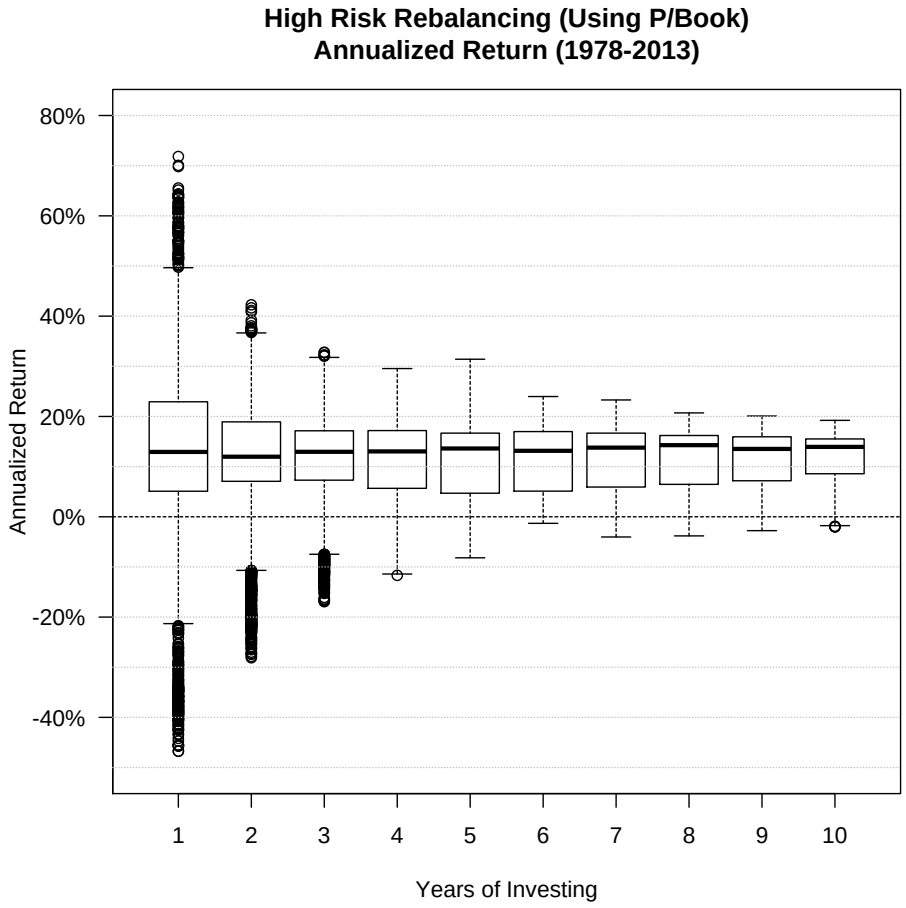

Figure 57: Annualized return statistics for adaptive high-risk rebalancing using the P/Book of the S&P 500 index.

Annualized Return for High Risk Rebalancing (Using P/Book)										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(46.8%)	5.1%	12.9%	12.9%	22.9%	71.8%	16.0%	0.16	0.23	0.10
2	(28.1%)	7.1%	12.0%	12.1%	18.9%	42.3%	10.8%	0.10	0.21	0.12
3	(16.9%)	7.3%	13.0%	11.8%	17.2%	32.8%	8.9%	0.11	0.22	0.13
4	(11.7%)	5.7%	13.0%	11.6%	17.2%	29.6%	7.9%	0.12	0.21	0.14
5	(8.2%)	4.7%	13.6%	11.5%	16.7%	31.4%	7.2%	0.05	0.18	0.14
6	(1.3%)	5.1%	13.2%	11.6%	17.0%	24.0%	6.2%	0.002	0.09	0.15
7	(4.0%)	5.9%	13.8%	11.8%	16.7%	23.3%	5.8%	0.02	0.03	0.15
8	(3.8%)	6.5%	14.3%	11.9%	16.2%	20.7%	5.5%	0.03	0.05	0.16
9	(2.8%)	7.2%	13.5%	11.9%	16.0%	20.0%	5.2%	0.02	0.07	0.16
10	(2.0%)	8.6%	13.9%	11.9%	15.5%	19.2%	4.9%	0.01	0.07	0.16

Table 17: Annualized return statistics for adaptive high-risk rebalancing using the P/Book of the S&P 500 index. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

For a one-year investment period the mean annualized return is 12.9% which decreases to 11.9% for ten-year investment periods. The distributions are wider for shorter investment periods. For one-year investment periods the greatest loss is (46.8%) and the greatest gain is 71.8%. For ten-year investment periods the minimum annualized return is (2.0%) giving a compounded loss of (18.3%) over ten years, while the maximum annualized return is 19.2% giving a compounded gain of 479% over ten years.

The probability of loss is 0.16 (or 16%) for one-year investment periods and the probability of loss gradually decreases until it is 0.01 (or 1%) for ten-year investment periods.

13.5.3. Comparison to Stock-Only Investment

The high-risk adaptive rebalancing strategy can be compared to stock-only investments because the high-risk strategy is usually fully invested in the S&P 500 index. Table 17 shows the statistics for the annualized return of the adaptive strategy and Table 2 shows it for the stock-only strategy.

The mean annualized return for all investment periods is slightly higher for the adaptive strategy than for the stock-only strategy. For one-year investment periods the adaptive strategy has a mean return of 12.9% while the stock-only strategy has 12.8%. This gradually decreases for longer investment periods and for ten years the adaptive strategy has a mean annualized return of 11.9% while the stock-only strategy has 11.5%.

The greatest losses are identical for the two strategies for investment periods up to four years, after which the adaptive strategy has lower losses than the stock-only strategy. For ten-year investment periods the greatest annualized loss is (2.0%) for the adaptive strategy while it is (4.0%) for the stock-only strategy.

The largest gains are identical for the two strategies for one-year investment periods after which the stock-only strategy has slightly higher maximum gains. For ten-year investment periods the greatest annualized return is 19.2% for the adaptive strategy while it is slightly higher at 19.8% for the stock-only strategy.

Probability of Loss

Table 17 and Table 2 also show the probability of loss for different investment periods. For the adaptive strategy the probability of loss is 0.16 (or 16%) for one-year investment periods, while the probability of loss is somewhat higher at 0.19 (or 19%) for the stock-only strategy. The probabilities gradually decrease for longer investment periods. For ten-year investment periods the adaptive strategy has probability of loss 0.01 (or 1%) while the stock-only strategy has probability of loss 0.07 (or 7%).

13.5.4. Comparison to Bond-Only & Stock-Only Investment

Figure 58 shows the performance of the rebalancing strategy compared to bond-only and stock-only investments for the period 1978-2013. Until the late 1990's the rebalancing strategy was fully invested in the S&P 500 index so the two returns are identical. When the P/Book of the S&P 500 index was very high around year 2000, the rebalancing strategy shifted the portfolio to government bonds. This caused the rebalancing strategy to under-perform the S&P 500 around year 2000 but gave a better performance afterwards because the losses of the S&P 500 in the years following 2000 were smaller for the rebalancing strategy than for the S&P 500.

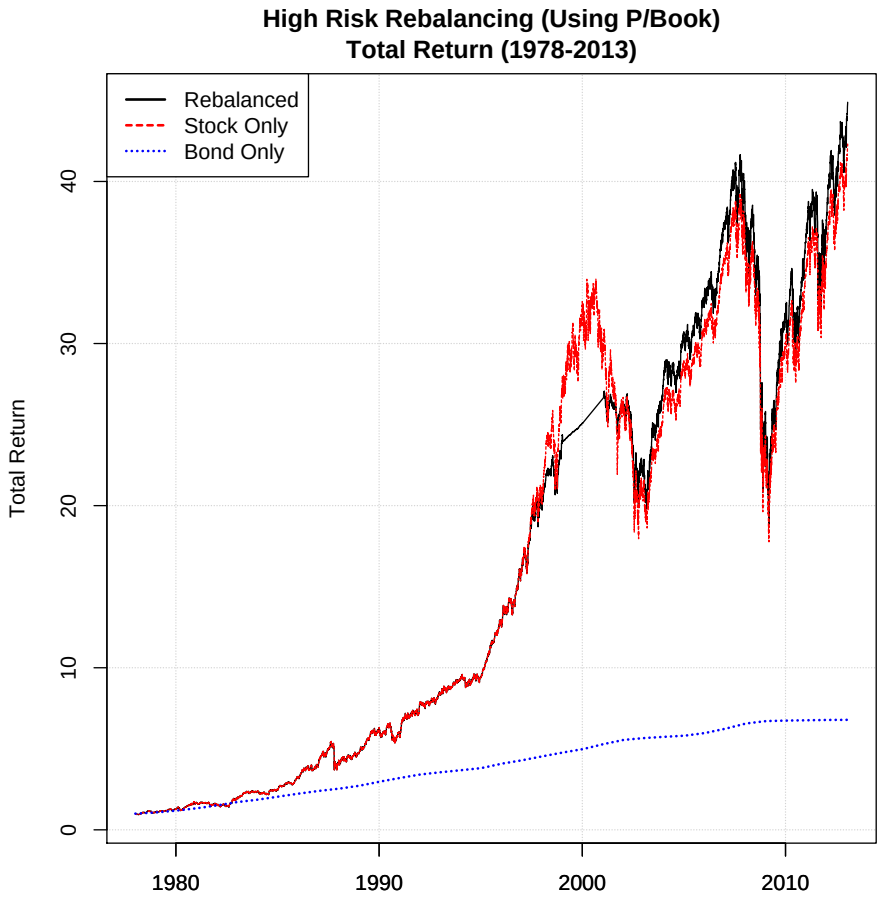

Figure 58: Total return of adaptive high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments for the period 1978-2013.

Probability of Under-Performance

Table 17 shows the probabilities of the rebalanced portfolio under-performing bond-only investments. For one-year investment periods the probability was 0.23 (or 23%). The probability gradually decreased to 0.07 (or 7%) for nine years or more of investing.

Table 17 also shows the probabilities of the rebalanced portfolio under-performing stock-only investments. For one-year investment periods the probability was 0.10 (or 10%). The probability gradually increased to 0.16 (or 16%) for ten-year investment periods.

Better than Bond-Only Investment

Figure 59 shows an example of the rebalancing strategy performing better than a bond-only investment. The investment period starts December 4, 1987. Table 18 shows that the stock-weight was 1 on this date as well as the following years until 1995 because the P/Book for the S&P 500 was lower than 3, which is the limit below which the stock-weight calculated using Eq. 13-6 always gives a stock-weight of 1. In 1996 the P/Book had increased to 3.19 so the stock-weight decreased to 0.89 meaning that only 89% of the portfolio was invested in the S&P 500 index and the remaining 11% of the portfolio was invested in US government bonds with one year maturity. After ten years the rebalanced portfolio had a total return of 5.57 (or 457% gain), while the bond-only investment had a total return of 1.80 (or 80% gain). The stock-only investment performed better than both with a total return of 5.69 (or 469% gain).

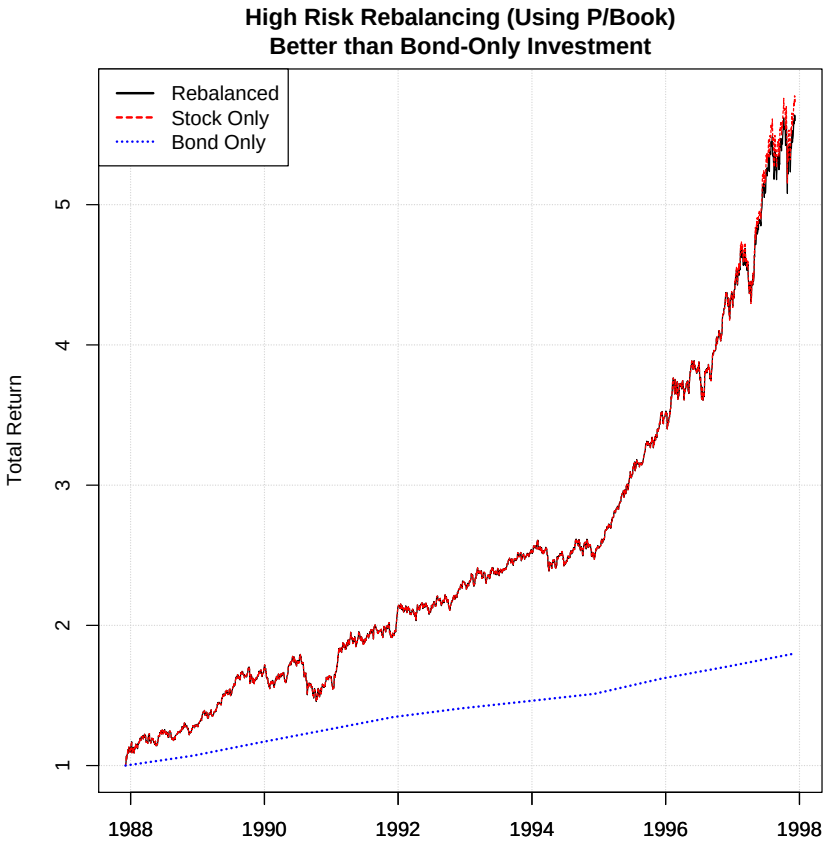

Figure 59: Total return of adaptive high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

High Risk Rebalancing (Using P/Book) Better than Bond-Only Investment											
Date	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
P/Book	1.68	1.97	2.37	2.16	2.39	2.91	2.59	2.32	2.87	3.19	3.90
Bond Yield	7.0%	8.8%	7.8%	7.3%	4.5%	3.7%	3.6%	7.2%	5.4%	5.4%	5.5%
Stock Weight	1	1	1	1	1	1	1	1	1	0.89	0.46
Rebalanced	1	1.28	1.66	1.63	1.93	2.29	2.51	2.47	3.50	4.32	5.57
Bond-Only	1	1.07	1.16	1.25	1.35	1.41	1.46	1.51	1.62	1.71	1.80
Stock-Only	1	1.28	1.66	1.63	1.93	2.29	2.51	2.47	3.50	4.32	5.69

Table 18: Total return of adaptive high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 56.

Worse than Bond-Only Investment

Figure 60 shows an example of the rebalancing strategy performing worse than a bond-only investment. The starting date is February 25, 1999. Table 19 shows that the stock-weights were low in the first few years of investing and then gradually increased to a stock-weight close to or equal one from year 2003 and onwards. For the ten-year investment period the rebalancing strategy had a total return of 0.82 (or 18% loss) while the bond-only investment had a total return of 1.41 (or 41% gain). It was in the last year that the rebalancing under-performed the bond-only investment. The stock-only investment performed worse than both with a total return of 0.67 (or 33% loss).

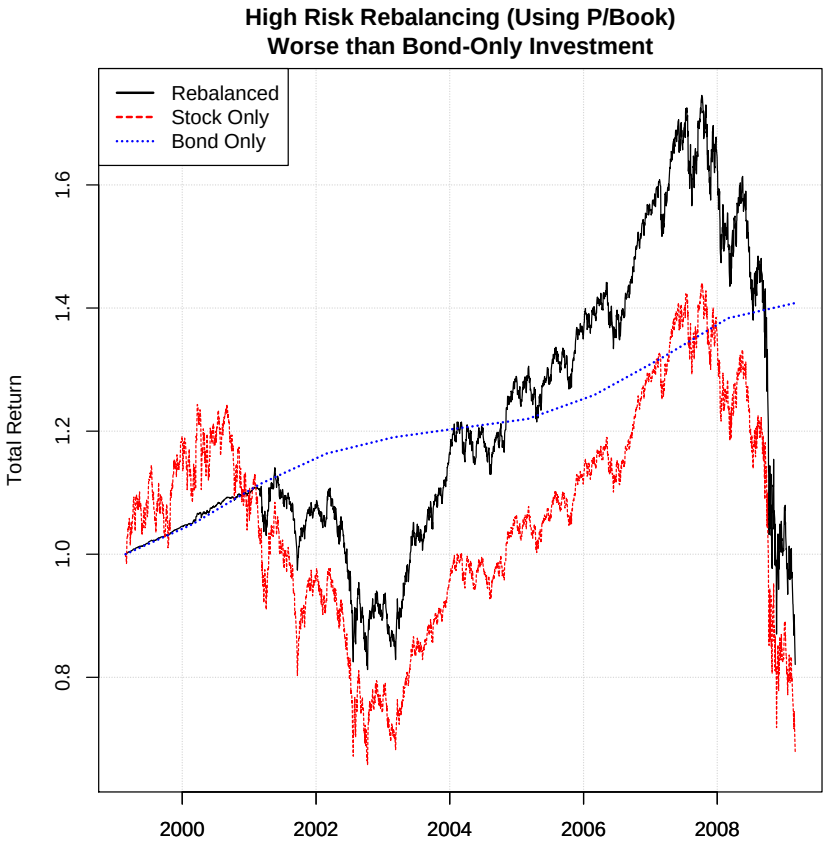

Figure 60: Total return of adaptive high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

High Risk Rebalancing (Using P/Book) Worse than Bond-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	4.62	4.54	3.70	3.32	2.57	3.07	2.86	2.79	2.77	2.51	1.55
Bond Yield	4.9%	6.2%	4.5%	2.3%	1.3%	1.2%	3.2%	4.7%	5.0%	1.8%	0.7%
Stock Weight	0.03	0.08	0.58	0.81	1	0.96	1	1	1	1	1
Rebalanced	1	1.05	1.11	1.06	0.87	1.20	1.29	1.40	1.55	1.50	0.82
Bond-Only	1	1.05	1.11	1.16	1.19	1.20	1.22	1.26	1.32	1.38	1.41
Stock-Only	1	1.08	1.04	0.93	0.71	0.99	1.06	1.15	1.28	1.24	0.67

Table 19: Total return of adaptive high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 56.

Better than Stock-Only Investment

Figure 61 shows another example of the rebalancing strategy performing better than a stock-only investment. The starting date is March 22, 2000. Table 20 shows the total return for the rebalancing strategy was 1.44 (or 44% gain) while it was 0.94 (or 6% loss) for the stock-only investment. The bond-only investment had a total return of 1.35 (or 35% gain).

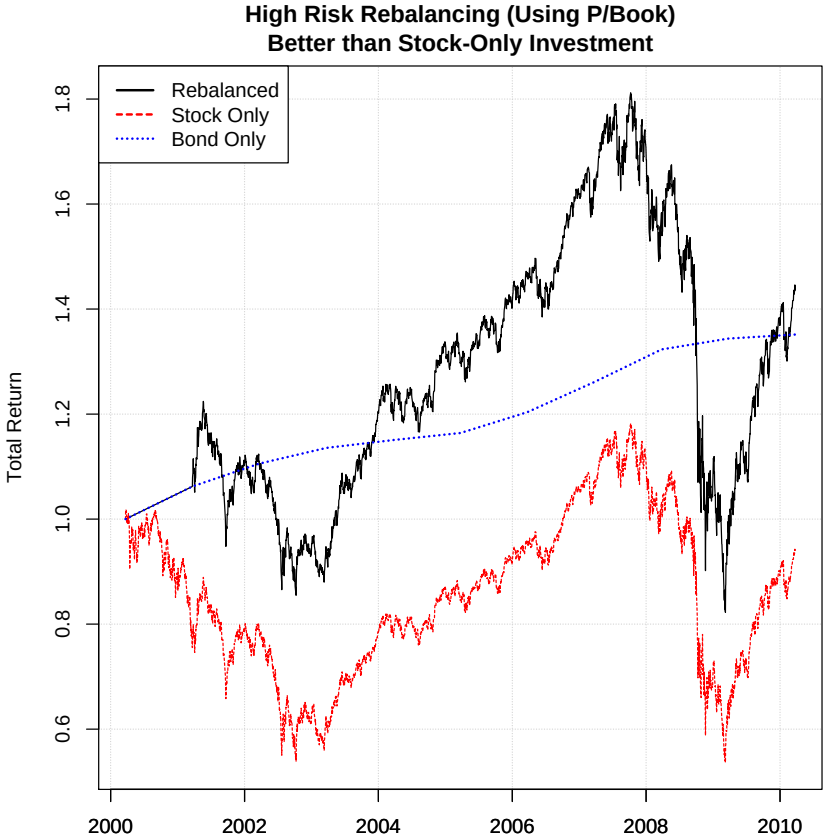

Figure 61: Total return of adaptive high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

High Risk Rebalancing (Using P/Book) Better than Stock-Only Investment											
Date	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
P/Book	5.08	3.24	3.45	2.63	2.90	2.75	2.79	2.83	2.53	1.85	2.20
Bond Yield	6.2%	4.1%	2.7%	1.3%	1.2%	3.4%	4.8%	4.9%	1.6%	0.6%	0.4%
Stock Weight	0	0.86	0.73	1	1	1	1	1	1	1	1
Rebalanced	1	1.06	1.11	0.93	1.19	1.30	1.47	1.65	1.57	1.01	1.44
Bond-Only	1	1.06	1.11	1.14	1.15	1.16	1.20	1.26	1.32	1.34	1.35
Stock-Only	1	0.76	0.79	0.60	0.77	0.85	0.96	1.07	1.02	0.66	0.94

Table 20: Total return of adaptive high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 56.

Worse than Stock-Only Investment

Figure 62 shows an example of the rebalancing strategy performing worse than a stock-only investment. The starting date is March 20, 1990. Table 21 shows that the stock-weight was 1 until year 1995 where it started to decrease because of the increasing P/Book for the S&P 500 index, and in 1999 the stock-weight was zero. After the ten-year period the rebalancing strategy had a total return of 4.18 (or 318% gain) while the stock-only investment had a total return of 5.57 (or 457% gain). The bond-only investment had a total return of 1.71 (or 71% gain).

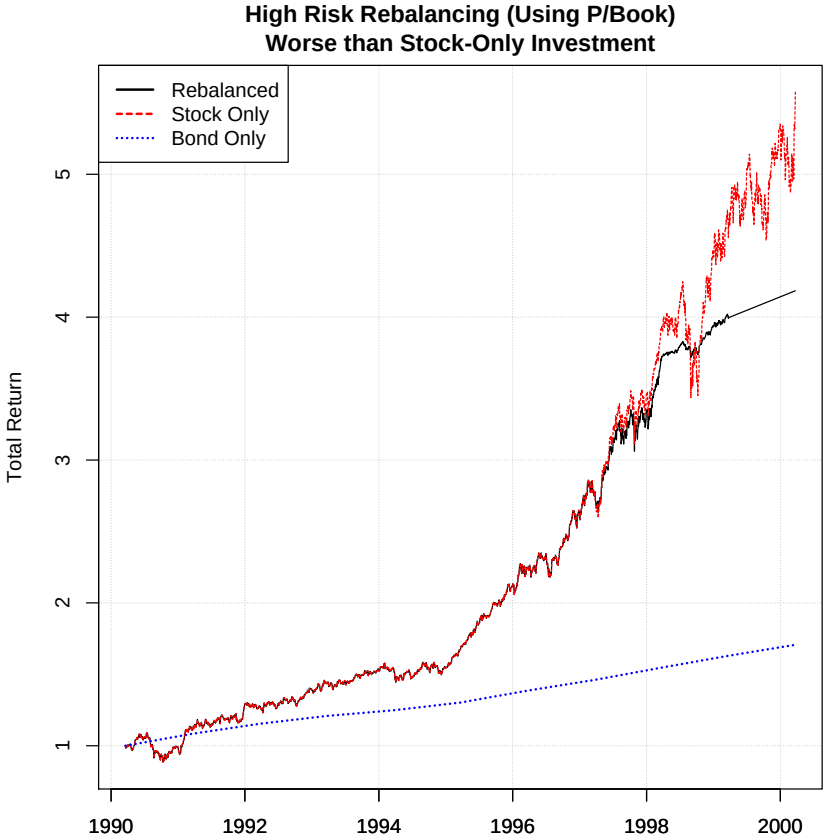

Figure 62: Total return of adaptive high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

High Risk Rebalancing (Using P/Book) Worse than Stock-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.30	2.38	2.62	2.61	2.61	2.49	3.01	3.35	4.38	4.68	5.17
Bond Yield	8.4%	6.4%	4.7%	3.3%	4.3%	6.4%	5.4%	5.9%	5.4%	4.7%	6.3%
Stock Weight	1	1	1	1	1	1	0.99	0.79	0.17	0	0
Rebalanced	1	1.11	1.28	1.44	1.54	1.67	2.24	2.78	3.73	4.00	4.18
Bond-Only	1	1.08	1.15	1.21	1.25	1.30	1.39	1.46	1.55	1.63	1.71
Stock-Only	1	1.11	1.28	1.44	1.54	1.67	2.24	2.78	3.94	4.58	5.57

Table 21: Total return of adaptive high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 56.

13.5.5. Comparison to Medium-Risk Rebalancing

Table 17 shows the statistics for the annualized returns of the high-risk strategy and Table 12 shows the statistics for the medium-risk strategy. For one-year investment periods, the mean annualized return is 12.9% for the high-risk strategy while it is only 9.3% for the medium-risk strategy. The mean annualized returns decrease somewhat with longer investment periods and for ten years it is 11.9% for the high-risk strategy and only 8.3% for the medium-risk strategy.

The greatest loss for a one-year investment period was (46.8%) for the high-risk strategy, while it was only (13.9%) for the medium-risk strategy. For investment periods of five years or more, the medium-risk strategy did not experience any losses, while the high-risk strategy experienced losses for all investment periods up to ten years. For ten-year investment periods the minimum annualized return was (2.0%) for the high-risk strategy, while it was 2.7% for the medium-risk strategy.

The largest gain for a one-year investment period was 71.8% for the high-risk strategy, while it was only 65.6% for the medium-risk strategy. For ten-year investment periods the largest gain was 19.2% for the high-risk strategy while it was only 18.1% for the medium-risk strategy.

Probability of Loss

For one-year investment periods the probability of loss was 0.16 (or 16%) for the high-risk strategy while it was only 0.06 (or 6%) for the medium-risk strategy. For investment periods of five years or more, the medium-risk strategy had zero probability of loss, while the high-risk strategy had generally declining but non-zero probability of loss for all investment periods, with the probability being 0.01 (or 1%) for ten-year investment periods.

Probability of Under-Performing Bond-Only and Stock-Only Investments

For one-year investment periods the probability of under-performing bond-only investments was 0.23 (or 23%) for the high-risk strategy while the probability was only 0.18 (or 18%) for the medium-risk strategy. The probabilities generally decrease for longer investment periods and for five years or more the medium-risk strategy had a probability of 0.03 (or 3%) of under-performing bond-only investments, while the probability was 0.07 (or 7%) for ten-year investment periods with the high-risk strategy.

The probabilities of under-performing stock-only investments were far lower for the high-risk strategy compared to the medium-risk strategy. For one-year investment periods the probability was 0.10 (or 10%) for the high-risk strategy, while the probability was 0.74 (or 74%) for the medium-risk strategy. For ten-year investment periods the probability was 0.16 (or 16%) for the high-risk strategy while it was 0.81 (or 81%) for the medium-risk strategy.

13.6. Medium-High Risk Composite Strategy (Using P/Book)

The previous rebalancing strategies for low-, medium- and high-risk can be combined to achieve strategies with intermediate risk levels. In the following example, the composite strategy is a 50% medium-risk and 50% high-risk strategy, meaning that the stock-weights are calculated by scaling the stock-weights of the medium-risk strategy in Figure 49 by 0.5 and adding the stock-weights for the high-risk strategy in Figure 56 which are also scaled by 0.5. Figure 63 shows the resulting stock-weights for the medium-high risk strategy for each day during the period 1978-2013. About 96% of the days in this period had some part of the portfolio invested in the S&P 500 index. Note that the stock-weight is shown for each day but the investment strategy uses annual rebalancing, as demonstrated below.

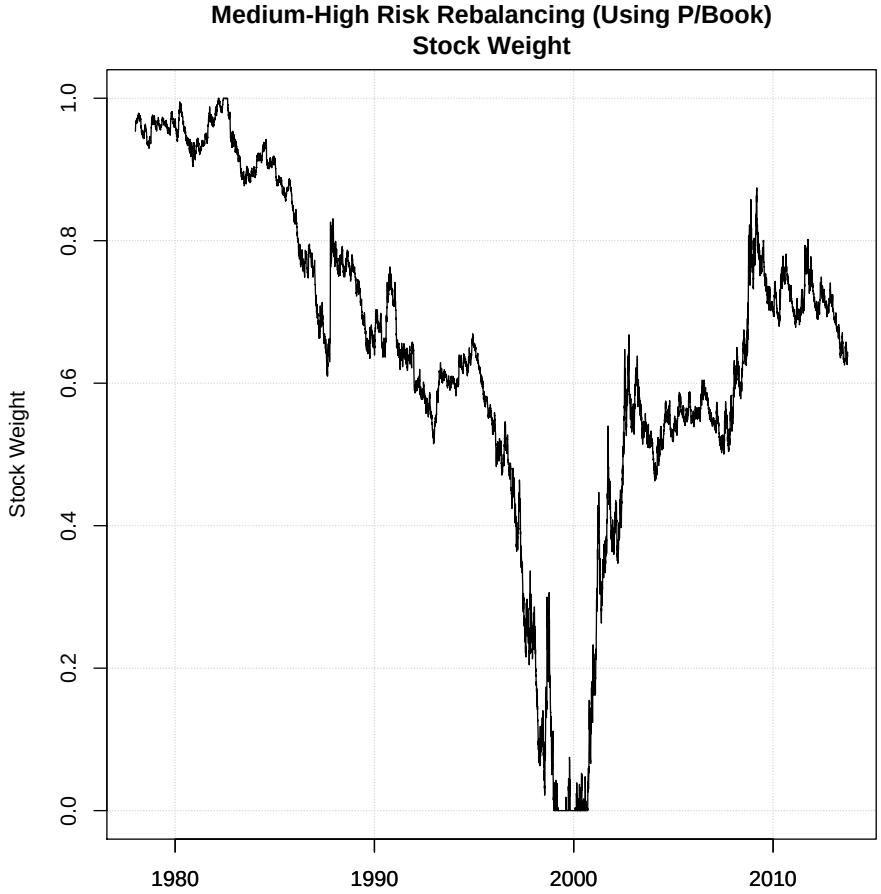

Figure 63: Stock-weight for adaptive medium-high-risk rebalancing (using P/Book). Calculated as 50% of the stock-weights for medium-risk rebalancing in Figure 49 plus 50% of the stock-weights for high-risk rebalancing in Figure 56.

13.6.1. Annualized Return

Figure 64 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 22.

For a one-year investment period the mean annualized return is 11.1% which decreases to 10.2% for ten-year investment periods. The distributions are wider for shorter investment periods. For one-year investment periods the greatest loss is (28.6%) and the greatest gain is 65.5%. For ten-year investment periods the minimum annualized return is 0.9% giving a compounded gain of 9% over ten years, while the maximum annualized return is 18.8% giving a compounded gain of 460% over ten years.

The probability of loss is 0.14 (or 14%) for one-year investment periods and the probability of loss gradually decreases until it is zero or nearly zero for investment periods of six years or more.

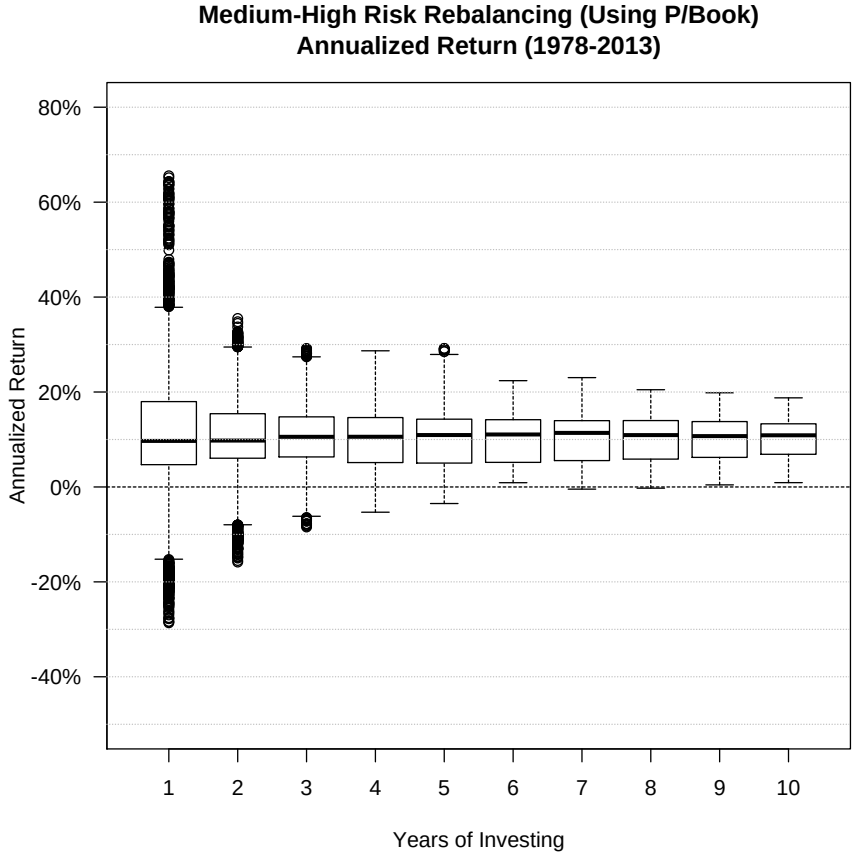

Figure 64: Annualized return statistics for adaptive medium-high-risk rebalancing using the P/Book of the S&P 500 index.

Annualized Return for Medium-High Risk Rebalancing (Using P/Book)										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(28.6%)	4.7%	9.7%	11.1%	18.0%	65.5%	12.1%	0.14	0.23	0.74
2	(15.8%)	6.1%	9.8%	10.6%	15.4%	35.5%	7.7%	0.08	0.18	0.67
3	(8.4%)	6.3%	10.6%	10.5%	14.8%	29.2%	6.3%	0.05	0.19	0.65
4	(5.3%)	5.1%	10.6%	10.4%	14.6%	28.7%	5.8%	0.03	0.15	0.60
5	(3.5%)	5.0%	10.9%	10.3%	14.3%	29.2%	5.5%	0.01	0.10	0.59
6	0.9%	5.2%	11.1%	10.3%	14.2%	22.4%	5.0%	0	0.05	0.58
7	(0.5%)	5.6%	11.4%	10.3%	14.0%	23.0%	4.8%	0.0008	0.03	0.62
8	(0.3%)	5.9%	10.9%	10.3%	14.0%	20.5%	4.6%	0.0007	0.03	0.73
9	0.4%	6.2%	10.7%	10.3%	13.8%	19.9%	4.3%	0	0.04	0.78
10	0.9%	6.9%	10.9%	10.2%	13.3%	18.8%	4.1%	0	0.04	0.80

Table 22: Annualized return statistics for adaptive medium-high-risk rebalancing using the P/Book of the S&P 500 index. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

13.6.2. Comparison to Bond-Only & Stock-Only Investment

Figure 65 shows the performance of the rebalancing strategy compared to bond-only and stock-only investments for the period 1978-2013. The rebalancing strategy performs better than the bond-only investment and worse than the stock-only investment. Note that the S&P 500 index had large losses during 2000-2003 and 2008-2009 where the rebalancing strategy almost reached the stock-only strategy. But these long-term results are misleading because the rebalancing strategy sometimes performed better than the stock-only investment and sometimes it performed worse than the bond-only investment, depending on the starting date and duration of the investment period.

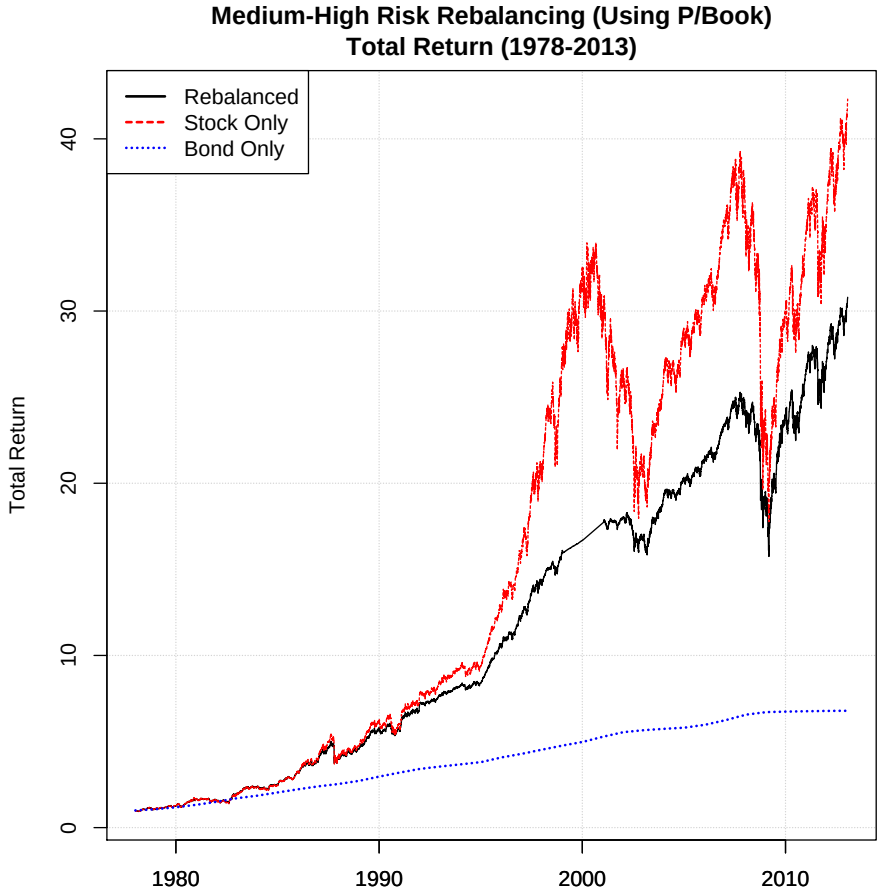

Figure 65: Total return of adaptive medium-high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments for the period 1978-2013.

Probability of Under-Performance

Table 22 shows the probabilities of the rebalanced portfolio under-performing bond-only investments. For one-year investment periods the probability was 0.23 (or 23%). The probability gradually decreased to 0.04 (or 4%) for nine years or more of investing.

Table 22 also shows the probabilities of the rebalanced portfolio under-performing stock-only investments. For one-year investment periods the probability was 0.74 (or 74%). For six-year investment periods the probability was only 0.58 (or 58%), and for ten-year investment periods the probability was 0.80 (or 80%).

Better than Bond-Only Investment

Figure 66 shows an example of the rebalancing strategy performing better than a bond-only investment. The investment period starts August 12, 1982. Table 23 shows that the stock-weights were especially high in the first four years because the P/Book was low. After ten years the rebalanced portfolio had a total return of 5.59 (or 459% gain), while the bond-only investment had a total return of 2.27 (or 127% gain). The stock-only investment performed better than both with a total return of 5.80 (or 480% gain).

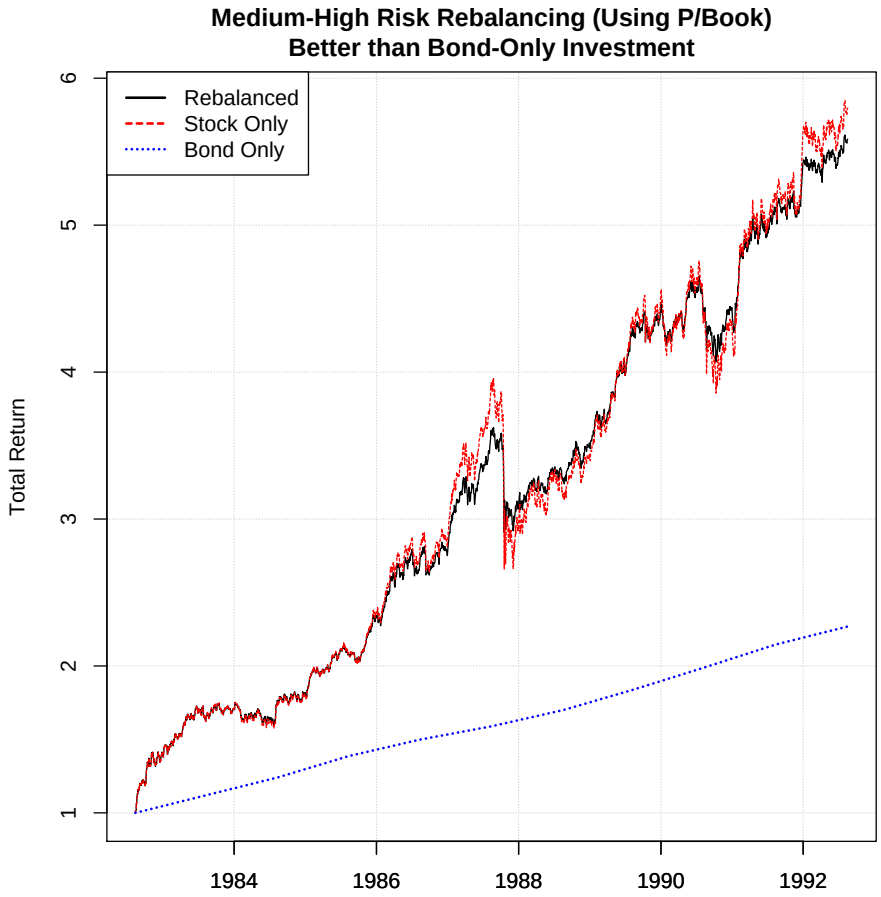

Figure 66: Total return of adaptive medium-high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

Medium-High Risk Rebalancing (Using P/Book) Better than Bond-Only Investment											
Date	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
P/Book	0.92	1.41	1.37	1.51	1.95	2.54	1.88	2.38	2.20	2.46	2.75
Bond Yield	12.1%	10.8%	11.7%	8.1%	6.0%	6.9%	8.3%	8.2%	7.8%	5.7%	3.4%
Stock Weight	1	0.90	0.91	0.87	0.76	0.61	0.78	0.65	0.70	0.63	0.56
Rebalanced	1	1.65	1.76	2.07	2.74	3.60	3.27	4.27	4.38	5.08	5.59
Bond-Only	1	1.12	1.24	1.39	1.50	1.59	1.70	1.84	1.99	2.15	2.27
Stock-Only	1	1.65	1.75	2.08	2.82	3.92	3.17	4.33	4.31	5.16	5.80

Table 23: Total return of adaptive medium-high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 63.

Worse than Bond-Only Investment

Figure 67 shows an example of the rebalancing strategy performing worse than a bond-only investment. The starting date is March 3, 1999. Table 24 shows that the stock-weights were low in the first few years of investing and then gradually increased to about 0.6 for the remaining years. For the ten-year investment period the rebalancing strategy had a total return of 1.09 (or 9% gain) while the bond-only investment had a total return of 1.40 (or 40% gain). It was in the last year that the rebalancing strategy under-performed the bond-only investment. The stock-only investment performed worse than both with a total return of 0.66 (or 34% loss).

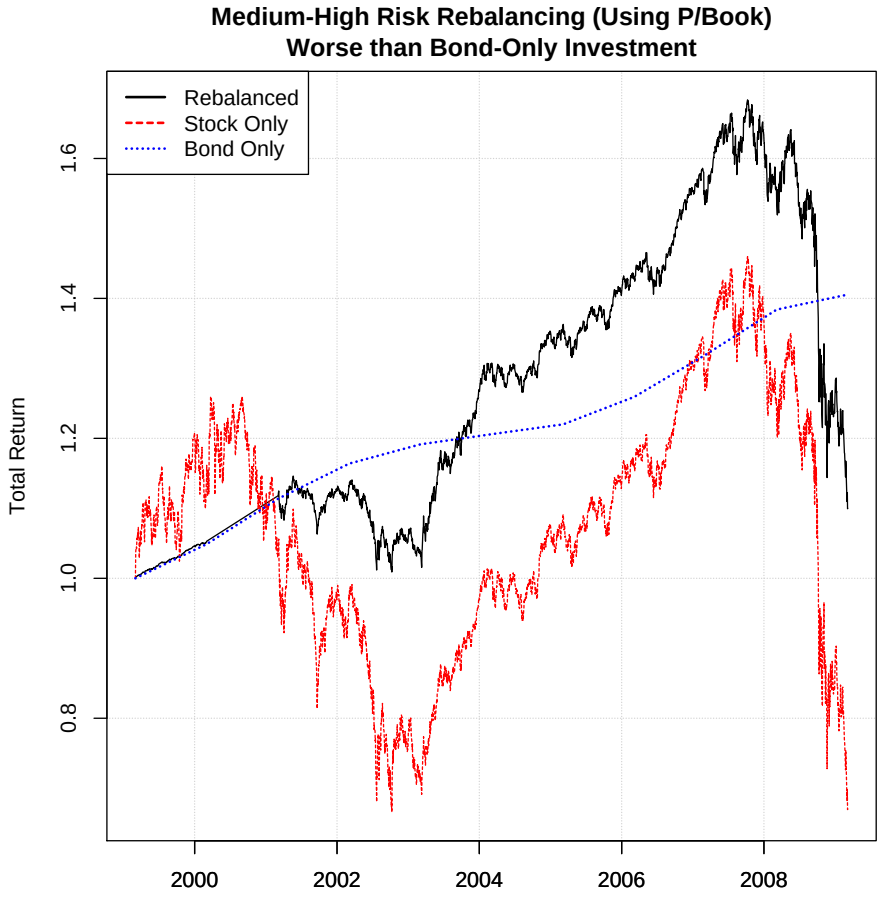

Figure 67: Total return of adaptive medium-high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

Medium-High Risk Rebalancing (Using P/Book) Worse than Bond-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	4.55	4.79	3.64	3.43	2.54	3.09	2.89	2.75	2.74	2.44	1.50
Bond Yield	4.9%	6.2%	4.5%	2.4%	1.2%	1.2%	3.2%	4.8%	4.9%	1.6%	0.7%
Stock Weight	0.04	0	0.31	0.37	0.61	0.47	0.53	0.56	0.56	0.64	0.87
Rebalanced	1	1.05	1.12	1.13	1.04	1.31	1.36	1.43	1.54	1.53	1.09
Bond-Only	1	1.05	1.11	1.16	1.19	1.21	1.22	1.26	1.32	1.38	1.40
Stock-Only	1	1.16	1.04	0.97	0.72	1.01	1.09	1.16	1.28	1.22	0.66

Table 24: Total return of adaptive medium-high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 63.

Better than Stock-Only Investment

Figure 68 shows another example of the rebalancing strategy performing better than a stock-only investment. The starting date is March 3, 2000 which is a year after the previous example. Table 25 shows the total return for the rebalancing strategy was 1.69 (or 69% gain) while it was 0.98 (or 2% loss) for the stock-only investment. The bond-only investment had a total return of 1.35 (or 35% gain).

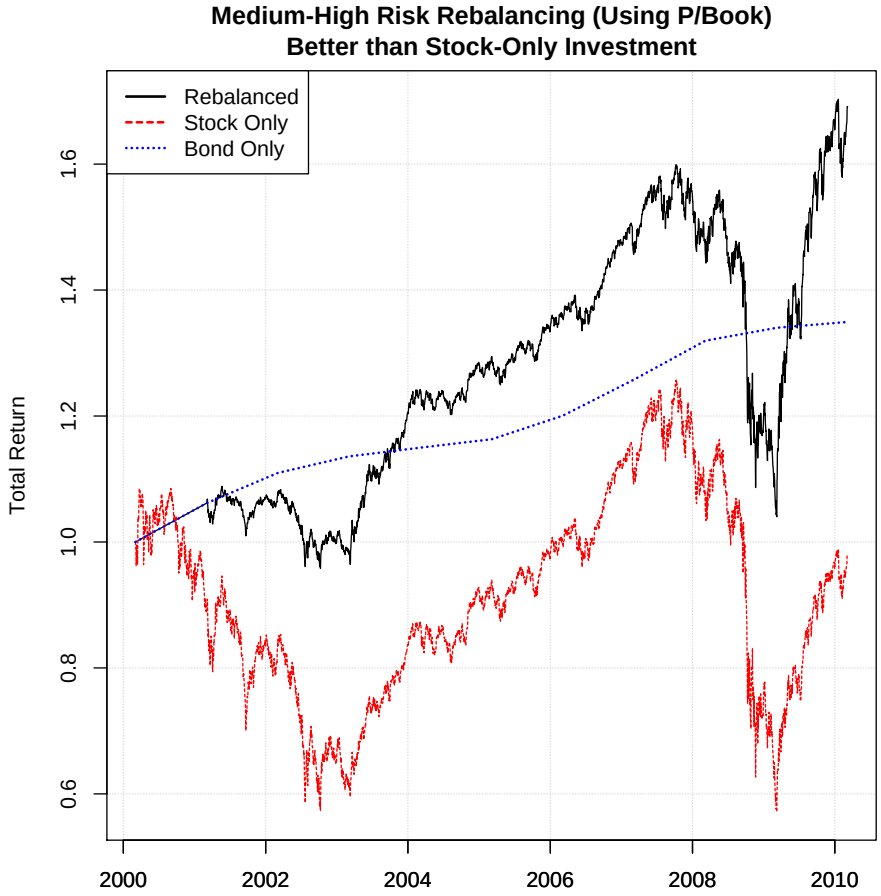

Figure 68: Total return of adaptive medium-high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

Medium-High Risk Rebalancing (Using P/Book) Better than Stock-Only Investment											
Date	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
P/Book	4.79	3.64	3.43	2.54	3.09	2.89	2.75	2.74	2.44	1.50	2.17
Bond Yield	6.2%	4.5%	2.4%	1.2%	1.2%	3.2%	4.8%	4.9%	1.6%	0.7%	0.4%
Stock Weight	0	0.31	0.37	0.61	0.47	0.53	0.56	0.56	0.64	0.87	0.71
Rebalanced	1	1.06	1.07	0.99	1.24	1.29	1.35	1.47	1.46	1.04	1.69
Bond-Only	1	1.06	1.11	1.14	1.15	1.16	1.20	1.26	1.32	1.34	1.35
Stock-Only	1	0.89	0.84	0.62	0.87	0.94	1.00	1.11	1.05	0.57	0.98

Table 25: Total return of adaptive medium-high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 63.

Worse than Stock-Only Investment

Figure 69 shows an example of the rebalancing strategy performing worse than a stock-only investment. The starting date is March 20, 1990. Table 26 shows that the stock-weight was 0.67 in the first year and around 0.6 for the following years, and then decreased to 0.09 in 1998 and zero in 1999 because the P/Book became very high. After the ten-year period the rebalancing strategy had a total return of 2.89 (or 189% gain) while the stock-only investment had a total return of 5.57 (or 457% gain). The bond-only investment only had a total return of 1.71 (or 71% gain).

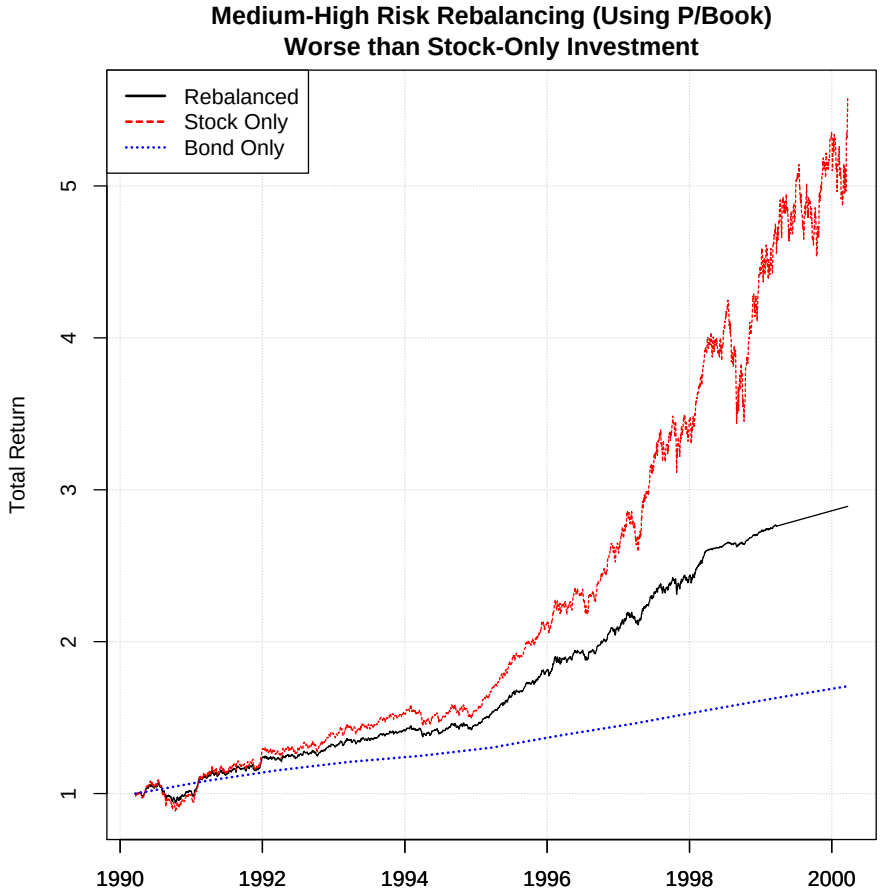

Figure 69: Total return of adaptive medium-high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments.

Medium-High Risk Rebalancing (Using P/Book) Worse than Stock-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.30	2.38	2.62	2.61	2.61	2.49	3.01	3.35	4.38	4.68	5.17
Bond Yield	8.4%	6.4%	4.7%	3.3%	4.3%	6.4%	5.4%	5.9%	5.4%	4.7%	6.3%
Stock Weight	0.67	0.65	0.59	0.60	0.60	0.63	0.50	0.39	0.09	0	0
Rebalanced	1	1.10	1.24	1.35	1.43	1.52	1.89	2.16	2.60	2.76	2.89
Bond-Only	1	1.08	1.15	1.21	1.25	1.30	1.39	1.46	1.55	1.63	1.71
Stock-Only	1	1.11	1.28	1.44	1.54	1.67	2.24	2.78	3.94	4.58	5.57

Table 26: Total return of adaptive medium-high-risk rebalancing (using P/Book) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2 and associated stock-weight from Figure 63.

13.6.3. Comparison to Medium-Risk Rebalancing

Table 22 shows the statistics for the annualized returns of the composite medium-high-risk strategy and Table 12 shows the statistics for the medium-risk strategy. For one-year investment periods, the mean annualized return is 11.1% for the medium-high-risk strategy while it is only 9.3% for the medium-risk strategy. The mean annualized returns decrease somewhat with longer investment periods and for ten years it is 10.2% for the medium-high-risk strategy and only 8.3% for the medium-risk strategy.

The greatest loss for a one-year investment period was (28.6%) for the medium-high-risk strategy, while it was only (13.9%) for the medium-risk strategy. For investment periods of five years or more, the medium-risk strategy did not experience any losses, while the medium-high-risk strategy experienced losses for all investment periods except six, nine and ten years. For ten-year investment periods the minimum annualized return was 0.9% for the medium-high-risk strategy, while it was 2.7% for the medium-risk strategy.

The largest gain for a one-year investment period was 65.5% for the medium-high-risk strategy, while it was almost the same at 65.6% for the medium-risk strategy. For ten-year investment periods the largest annualized return was 18.8% for the medium-high-risk strategy while it was only 18.1% for the medium-risk strategy.

Probability of Loss

For one-year investment periods the probability of loss was 0.14 (or 14%) for the medium-high-risk strategy while it was only 0.06 (or 6%) for the medium-risk strategy. For investment periods of five years or more, the medium-risk strategy had zero probability of loss, while the medium-high-risk strategy had zero or nearly zero probability of loss for investment periods of six years or more.

Probability of Under-Performing Bond-Only and Stock-Only Investments

For one-year investment periods the probability of under-performing bond-only investments was 0.23 (or 23%) for the medium-high-risk strategy while the probability was only 0.18 (or 18%) for the medium-risk strategy. The probabilities generally decrease for longer investment periods and for five years or more the medium-risk strategy had a probability of 0.03 (or 3%) of under-performing bond-only investments, while the probability was 0.04 (or 4%) for ten-year investment periods with the medium-high-risk strategy.

The probabilities for under-performing stock-only investments were similar for the two strategies with the medium-high-risk strategy having slightly lower probabilities than the medium-risk strategy.

13.6.4. Comparison to Fixed Rebalancing

The mean annualized returns for the medium-high-risk adaptive rebalancing strategy are similar to those of the fixed rebalancing strategy with 75% stock / 25% bond, see Table 22 and Table 6. For one-year investment periods the medium-high-risk strategy had mean annualized return of 11.1% while it was 11.0% for the fixed 75/25 strategy. The mean annualized returns gradually decrease and for ten-year investment periods it was 10.2% for the medium-high-risk strategy while it was 10.3% for the fixed 75/25 strategy. But the risk and return characteristics are otherwise somewhat different for the two strategies.

The median annualized return was between 9.7-11.4% for different investment periods using the medium-high-risk strategy, while it was somewhat higher at 11.1-12.1% for the fixed 75/25 strategy.

The greatest loss for one-year investment periods was (28.6%) for the medium-high-risk strategy while it was (34.6%) for the fixed 75/25 strategy. For ten-year investment periods the lowest annualized return was 0.9% for the medium-high-risk strategy while it was (1.5%) for the fixed 75/25 strategy.

The greatest gain for one-year investment periods was 65.5% for the medium-high-risk strategy while it was only 54.0% for the fixed 75/25 strategy. For ten-year investment periods the largest annualized return was 18.8% for the medium-high-risk strategy while it was only 16.9% for the fixed 75/25 strategy.

Probability of Loss

Table 22 and Table 6 also show the probabilities of loss. For one-year investment periods the medium-high-risk strategy had a probability of loss of 0.14 (or 14%) while the fixed 75/25 strategy had slightly higher probability of loss at 0.18 (or 18%). The probability of loss gradually decreases for longer investment periods and the medium-high-risk strategy had zero or nearly zero probability of loss for investment periods of six years or more, while the fixed 75/25 strategy had non-zero probability of loss for all investment periods, with the probability being 0.02 (or 2%) for ten-year investment periods.

Probability of Under-Performing Bond-Only and Stock-Only Investments

Table 22 and Table 6 also show the probabilities of under-performing bond-only and stock-only investments.

The probability of under-performing bond-only investments for one-year investment periods was 0.23 (or 23%) for the medium-high-risk strategy while the probability was 0.25 (or 25%) for the fixed 75/25 strategy. The probabilities gradually decrease for longer investment periods and for ten-year investment periods the medium-high-risk strategy has a probability of 0.04 (or 4%) while the fixed 75/25 strategy has a probability of 0.11 (or 11%). For all investment periods the probability of under-performing bond-only investments is significantly lower for the medium-high-risk strategy than for the fixed 75/25 strategy.

The probabilities of under-performing stock-only investments are always lower for the medium-high-risk strategy than for the fixed 75/25 strategy. For example, for five-year investment periods the medium-high-risk strategy has a probability of 0.59 (or 59%) while it is 0.72 (or 72%) for the fixed 75/25 strategy.

13.7. Summary

This section presented adaptive rebalancing strategies which use the P/Book of the S&P 500 to allocate the portfolio between the S&P 500 and US government bonds. The strategies avoided investing in the S&P 500 when it had a very high P/Book around year 2000 which saved the strategies from experiencing the ensuing losses. However, the stock-market crash in 2008-2009 was not due to the S&P 500 being over-priced like in year 2000, so the adaptive strategies were invested wholly or partly during that period.

Different strategy variants were presented for different levels of risk, where risk is defined as the probability and magnitude of loss. The strategies were tested for all starting dates and investment periods up to ten years during 1978-2013. It was found that the mean and median annualized returns increased with the risk of loss. All the strategies, including the low-risk strategy, experienced occasional under-performance compared to bond-only investments. All the strategies, except the high-risk strategy, experienced frequent under-performance compared to stock-only investments.

The adaptive strategies had several advantages over the fixed rebalancing strategies. For strategies with similar mean annualized returns, the adaptive strategies had much lower probability and magnitude of loss, and significantly lower probability of under-performing bond-only and stock-only investments, when compared to the fixed rebalancing strategies. Furthermore, the adaptive strategies sometimes experienced greater gains than were possible with the fixed rebalancing strategies, because the adaptive strategies were sometimes fully invested in the S&P 500 if the P/Book was sufficiently low.

14. Adaptive Rebalancing using P/Book & Bond Yield

The previous section studied adaptive rebalancing strategies that use the P/Book of the S&P 500 index to determine the portfolio allocation between the S&P 500 and government bonds. This section extends it by also using the government bond yield to determine the portfolio allocation. The idea is that a higher yield makes the government bond an increasingly attractive investment for the same P/Book of the S&P 500 and more of the portfolio should hence be allocated to government bonds.

14.1. Low Risk Strategy (Using P/Book & Bond Yield)

This section analyses the performance of the low-risk adaptive strategy that uses both the P/Book and bond yield when rebalancing the portfolio. The strategy's worst one-year loss was about (5%) during the period 1978-2013.

14.1.1. Stock Weight

The rebalancing is done annually and the fraction of the portfolio to invest in the S&P 500 index is calculated using the formula:

$$Stock\ Weight = Limit(2.6 - 1 \times P/Book - 5 \times Bond\ Yield)$$

Eq. 14-1

The P/Book is for the S&P 500 index and the bond yield is for US government bonds with one-year maturity.

As usual, the limit-function simply means that the stock-weight is limited between zero and one. For example, if P/Book is 3.5 and the bond yield is 2.7% then the result of the inner formula is:

$$2.6 - 1 \times P/Book - 5 \times Bond\ Yield = 2.6 - 1 \times 3.5 - 5 \times 2.7\% = -1.035$$

which is then limited between zero and one so the stock-weight is zero. Note that the bond yield is expressed as a percentage which is calculated as:

$$2.6 - 1 \times 3.5 - 5 \times \mathbf{2.7\%} = 2.6 - 1 \times 3.5 - 5 \times \mathbf{0.027} = -1.035$$

The stock-weight varies linearly with both the P/Book and bond yield so its boundaries depend on both. For example, holding the bond yield fixed at 1.2% means the stock-weight is zero when the P/Book is greater than 2.54, and the stock-weight is one when the P/Book is less than 1.54. If instead the bond yield is 2.7% then the stock-weight is zero when the P/Book is greater than 2.465, and the stock-weight is one when the P/Book is less than 1.465.

Consider another example where the P/Book is 1.8 and the bond yield is 3% (or 0.03), then the stock-weight is calculated from Eq. 14-1 as follows:

$$Stock\ Weight = Limit(2.6 - 1 \times P/Book - 5 \times Bond\ Yield) = Limit(2.6 - 1.8 - 5 \times 0.03) = 0.65$$

That is, 0.65 (or 65%) of the portfolio should be invested in the S&P 500 according to this strategy.

If instead the bond yield had been 7% (or 0.07) then the stock-weight would be:

$$Stock\ Weight = Limit(2.6 - 1 \times P/Book - 5 \times Bond\ Yield) = Limit(2.6 - 1.8 - 5 \times 0.07) = 0.45$$

So now only 0.45 (or 45%) of the portfolio should be invested in the S&P 500 because the bond yield is higher at 7% and hence more attractive than a bond yield of only 3%.

Figure 70 shows the stock-weight for each day during the period 1978-2013 calculated using Eq. 14-1 with the P/Book data from Figure 2 and the government bond yields from Figure 3. About 47% of the days in this period had some part of the portfolio invested in the S&P 500 while the remaining 53% of the days had no investment in the S&P 500 because the stock-weight was zero. Note that the stock-weight is shown for each day but the investment strategy uses annual rebalancing.

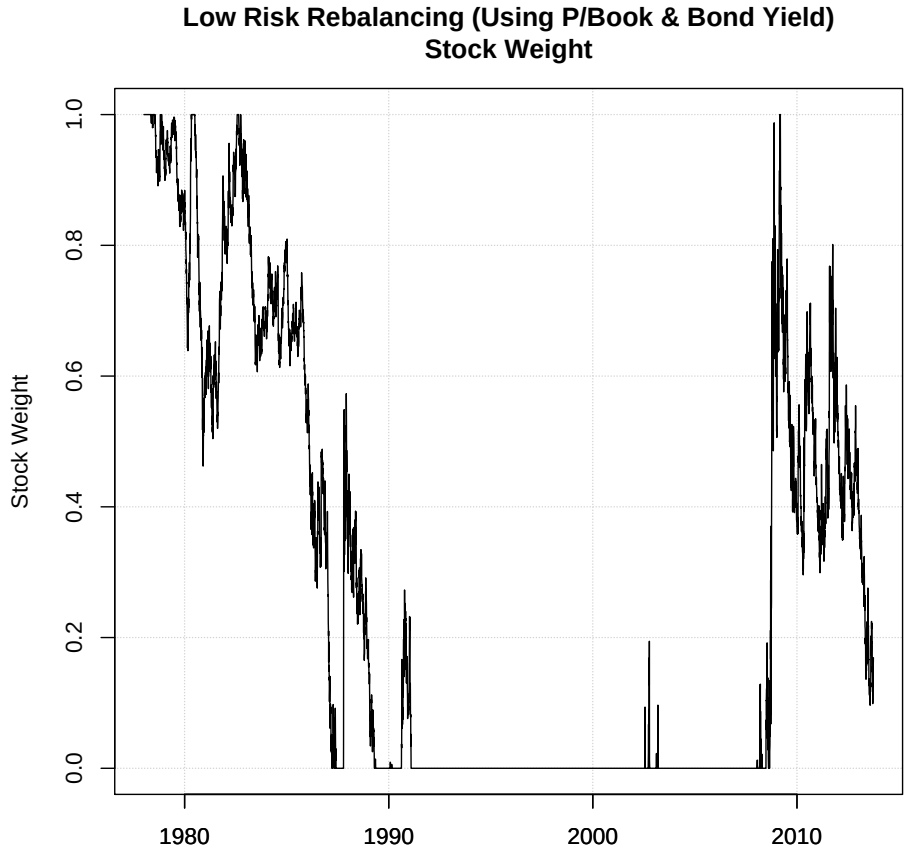

Figure 70: Stock-weight for adaptive low-risk rebalancing (using P/Book & Bond Yield).

Bond Weight

The bond-weight determines the amount of the portfolio to invest in US government bonds and it is simply the remainder of the portfolio after investing in the S&P 500 index:

$$Bond\ Weight = 1 - Stock\ Weight$$

Eq. 14-2

14.1.2. Annualized Return

Figure 71 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 27.

For one-year investment periods the mean annualized return was 8.9% which gradually decreased to 7.6% for ten-year investment periods. Only one-year investment periods experienced losses. The greatest loss

was (4.5%) and the probability of loss was 0.015 (or 1.5%). For ten-year investment periods the lowest annualized return was 2.6% giving a compounded gain of 29% over ten years. The greatest one-year gain was 71.8% and it was 18.0% for ten-year investment periods giving a compounded gain of 423% over ten years.

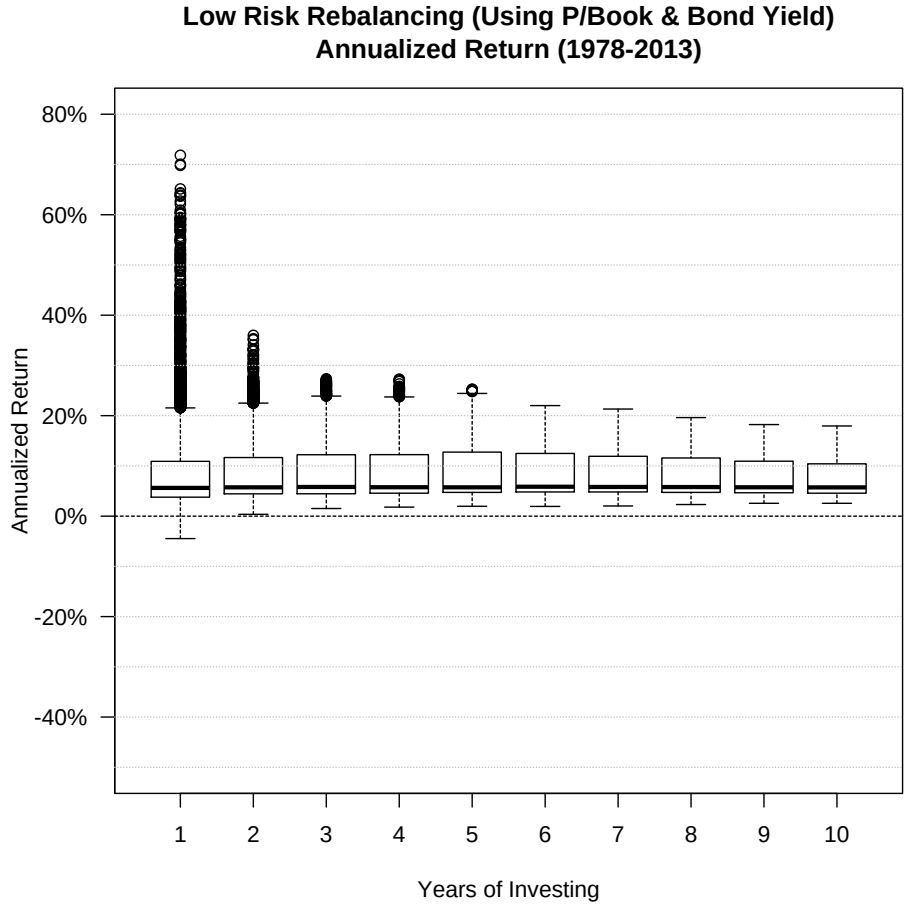

Figure 71: Annualized return statistics for adaptive low-risk rebalancing using the P/Book of the S&P 500 index and the yield on US government bonds.

Annualized Return for Low Risk Rebalancing (Using P/Book & Bond Yield)										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(4.5%)	3.8%	5.6%	8.9%	10.9%	71.8%	9.0%	0.015	0.09	0.72
2	0.4%	4.4%	5.7%	8.7%	11.7%	36.0%	6.3%	0	0.04	0.69
3	1.5%	4.5%	5.8%	8.6%	12.2%	27.3%	5.6%	0	0.02	0.67
4	1.8%	4.6%	5.8%	8.5%	12.2%	27.3%	5.4%	0	0.02	0.62
5	2.0%	4.7%	5.7%	8.4%	12.8%	25.3%	5.2%	0	0.01	0.59
6	1.9%	4.8%	5.9%	8.2%	12.5%	22.0%	5.0%	0	0.01	0.61
7	2.0%	4.8%	5.8%	8.0%	11.9%	21.3%	4.8%	0	0.01	0.66
8	2.3%	4.7%	5.8%	7.9%	11.6%	19.6%	4.6%	0	0.01	0.75
9	2.6%	4.7%	5.7%	7.8%	10.9%	18.2%	4.4%	0	0.01	0.81
10	2.6%	4.6%	5.7%	7.6%	10.4%	18.0%	4.1%	0	0.01	0.81

Table 27: Annualized return statistics for adaptive low-risk rebalancing using the P/Book of the S&P 500 index and the yield on US government bonds. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

14.1.3. Comparison to Bond-Only & Stock-Only Investments

Figure 72 shows the performance of the rebalancing strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the rebalancing strategy performed better than bond-only and worse than stock-only investments. But this mutual performance is misleading as it depends on the starting date and investment period, as shown below.

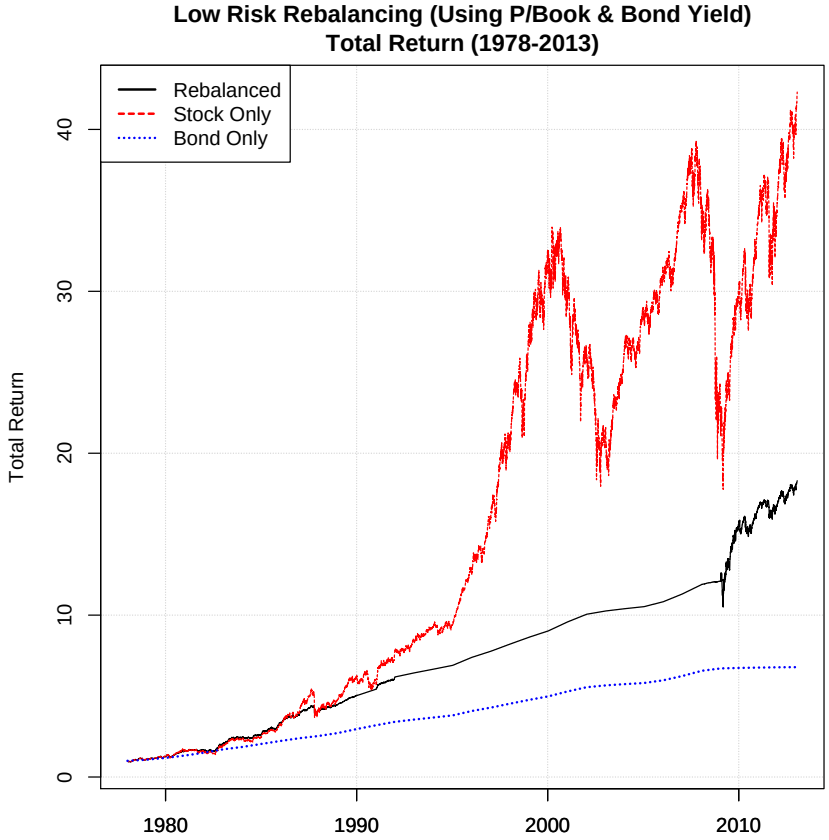

Figure 72: Total return of adaptive low-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments for the period 1978-2013.

Probability of Under-Performance

Table 27 shows the probabilities of the rebalanced portfolio under-performing bond-only investments. For one-year investment periods the probability was 0.09 (or 9%). The probability gradually decreased to about 0.01 (or 1%) for five years or more of investing.

Table 27 also shows the probabilities of the rebalanced portfolio under-performing stock-only investments. For one-year investment periods the probability was 0.72 (or 72%). The lowest probability was 0.59 (or 59%) which occurred for investment periods of 5 years. The highest probability was 0.81 (or 81%) which occurred for investment periods of 9 and 10 years.

Better than Bond-Only Investment

Figure 73 shows an example of the rebalancing strategy performing better than a bond-only investment. The investment period starts August 19, 1982. Table 28 shows that the stock-weight was one on this date so the entire portfolio was invested in the S&P 500. This is because the P/Book was low. In the following years the P/Book increased so the stock-weight decreased. For the ten-year period the rebalanced portfolio had a total return of 4.70 (or 370% gain), the bond-only investment had a total return of 2.23 (or 123% gain), and the stock-only investment had a total return of 5.33 (or 433% gain).

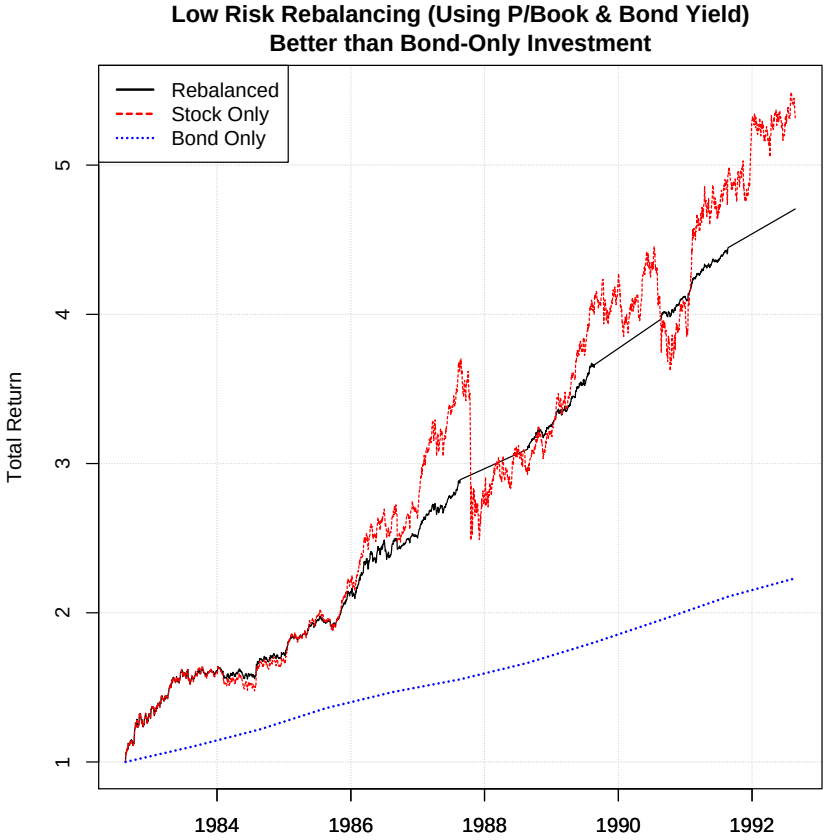

Figure 73: Total return of adaptive low-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

Low Risk Rebalancing (Using P/Book & Bond Yield) Better than Bond-Only Investment											
Date	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
P/Book	0.98	1.42	1.39	1.52	1.98	2.56	1.85	2.37	2.03	2.51	2.69
Bond Yield	10.3%	10.4%	11.8%	7.9%	5.7%	7.0%	8.3%	8.4%	8.0%	5.8%	3.6%
Stock Weight	1	0.66	0.62	0.68	0.34	0	0.33	0	0.17	0	0
Rebalanced	1	1.57	1.69	1.95	2.48	2.89	3.09	3.66	3.97	4.44	4.70
Bond-Only	1	1.10	1.22	1.36	1.47	1.55	1.66	1.80	1.95	2.11	2.23
Stock-Only	1	1.57	1.67	1.96	2.67	3.69	2.93	4.04	3.74	4.95	5.33

Table 28: Total return of adaptive low-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 70.

Worse than Bond-Only Investment

Figure 74 shows an example of the rebalancing strategy performing worse than a bond-only investment. The starting date is March 11, 1999. Table 29 shows that the P/Book for the S&P 500 was high in most of these years ranging from 4.80 in 1999 and decreasing to 2.41 in 2008. Hence the stock-weight was zero for all these years except 2008 where the P/Book was 2.41 and the bond yield was 1.3% which gives a stock-weight of 0.13 (or 13%) as calculated using Eq. 14-1. Between 2008-2009 the stock-only investment decreased about (37%) which meant a loss to the rebalanced portfolio of (4.8%) as 13% of the portfolio was invested in the S&P 500. This loss was not made up by the remainder of the portfolio invested in bonds as they only yielded 1.3% thus giving a return of about 1.1% from the part of the portfolio that was invested in bonds. The net loss for the rebalanced portfolio was (3.7%) thus bringing the total return for the ten-year period below that of a bond-only investment, but significantly better than a stock-only investment.

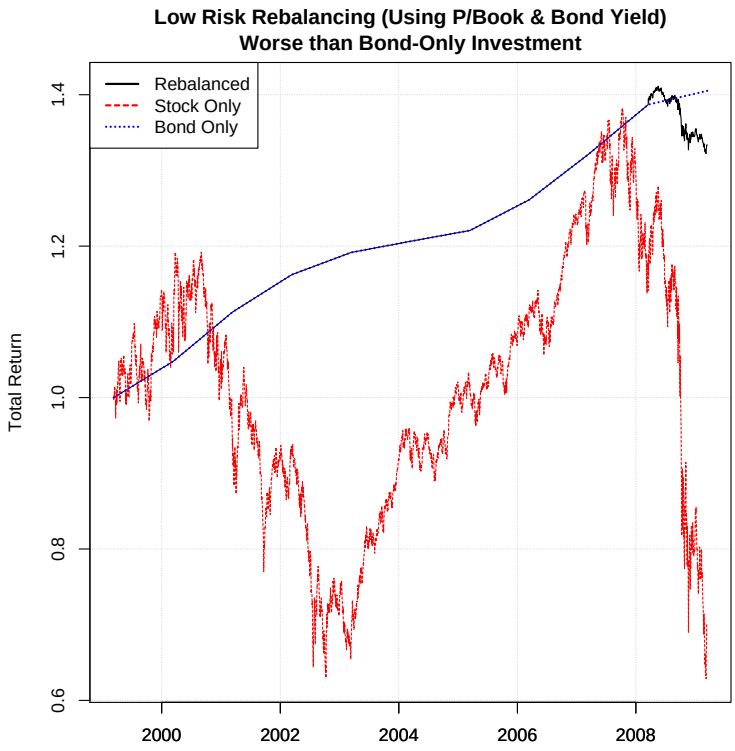

Figure 74: Total return of adaptive low-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

Low Risk Rebalancing (Using P/Book & Bond Yield) Worse than Bond-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	4.80	4.69	3.49	3.46	2.54	2.95	2.82	2.80	2.74	2.41	1.73
Bond Yield	4.8%	6.2%	4.4%	2.5%	1.2%	1.2%	3.3%	4.8%	4.9%	1.3%	0.7%
Stock Weight	0	0	0	0	0	0	0	0	0	0.13	0.84
Rebalanced	1	1.05	1.11	1.16	1.19	1.21	1.22	1.26	1.32	1.39	1.34
Bond-Only	1	1.05	1.11	1.16	1.19	1.21	1.22	1.26	1.32	1.39	1.40
Stock-Only	1	1.08	0.95	0.92	0.68	0.92	1.01	1.12	1.22	1.14	0.72

Table 29: Total return of adaptive low-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 70.

Better than Stock-Only Investment

Figure 75 shows another example of the rebalancing strategy performing better than a stock-only investment. The starting date is February 29, 2000, which is less than a year following the previous example.

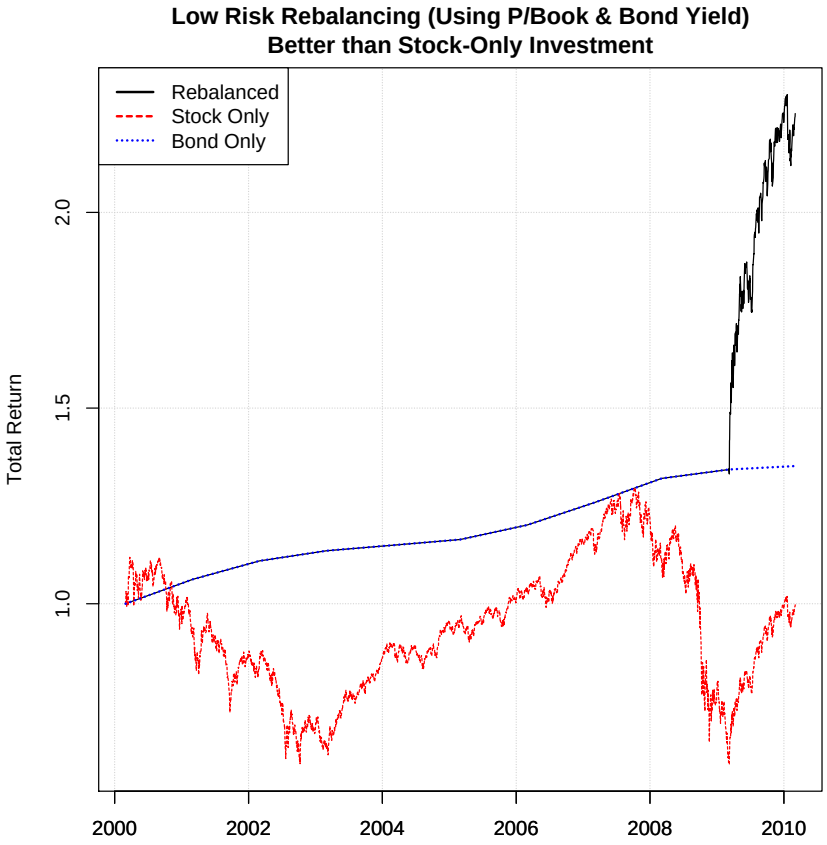

Figure 75: Left plot shows the total return of adaptive low-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

Low Risk Rebalancing (Using P/Book & Bond Yield) Better than Stock-Only Investment											
Date	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
P/Book	4.65	3.65	3.38	2.56	3.08	2.86	2.78	2.71	2.51	1.52	2.17
Bond Yield	6.2%	4.5%	2.3%	1.3%	1.3%	3.2%	4.8%	4.9%	1.7%	0.7%	0.4%
Stock Weight	0	0	0	0	0	0	0	0	0.0002	1	0.42
Rebalanced	1	1.06	1.11	1.14	1.15	1.16	1.20	1.26	1.32	1.34	2.28
Bond-Only	1	1.06	1.11	1.14	1.15	1.16	1.20	1.26	1.32	1.34	1.35
Stock-Only	1	0.92	0.85	0.64	0.90	0.96	1.04	1.13	1.12	0.59	1.01

Table 30: Total return of adaptive low-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 70.

Table 30 shows the P/Book for the S&P 500 was high for most of these years ranging from 4.65 in 2000 to 1.52 in 2009. Hence the stock-weight was practically zero until 2009 where the stock-weight became one. So for the first nine years the rebalanced portfolio was invested entirely in government bonds giving a total return of 34% until 2009. In several of these years the stock-only investment had large losses, first during

2000-2003 due to a severe overvaluation of stocks with the P/Book of the S&P 500 being 4.65 in year 2000. Then a large crash of almost (50%) occurred between 2008 and 2009. This brought the P/Book of the S&P 500 down to 1.52. The bond yield also decreased from 6.2% in 2000 to 0.7% in 2009. This caused the rebalancing strategy to shift from bonds to a full investment in the S&P 500. In the following year the S&P 500 gained about 71% including dividends, thus giving the rebalanced portfolio a total return of 2.28 (or 128% gain) for the ten-year investment period, while the bond-only investment had a total return of 1.35 (or 35% gain), and the stock-only investment had a total return of 1.01 (or 1% gain).

Worse than Stock-Only Investment

Figure 76 shows an example of the rebalancing strategy performing worse than a stock-only investment. The starting date is August 23, 1990. Table 31 shows the P/Book ranged from 2.03 in year 1990 to 4.76 in 2000. The bond yield ranged from 8.0% in 1990 to a low of 3.6% in 1993 and then gradually increasing to 6.3% in 2000. Because the bond yield was deemed attractive compared to the P/Book of the S&P 500 by this low-risk rebalancing strategy, the portfolio was invested entirely in government bonds for all years except the first where the P/Book was somewhat low at 2.03 thus giving a stock-weight of 0.17, meaning that 17% of the portfolio was invested in the S&P 500 and 83% was invested in bonds in that year.

For the ten-year investment period the rebalancing strategy had a total return of 1.75 (or 75% gain), which was slightly higher than the total return of the bond-only investment which was 1.69 (or 69% gain) because the rebalanced portfolio had a small investment in the S&P 500 during the first year. But this ten-year period was a great bull-market for the S&P 500 where the P/Book valuation increased from 2.03 in 1990 to 4.76 in 2000, so the stock-only investment had a total return of 6.09 (or 509% gain).

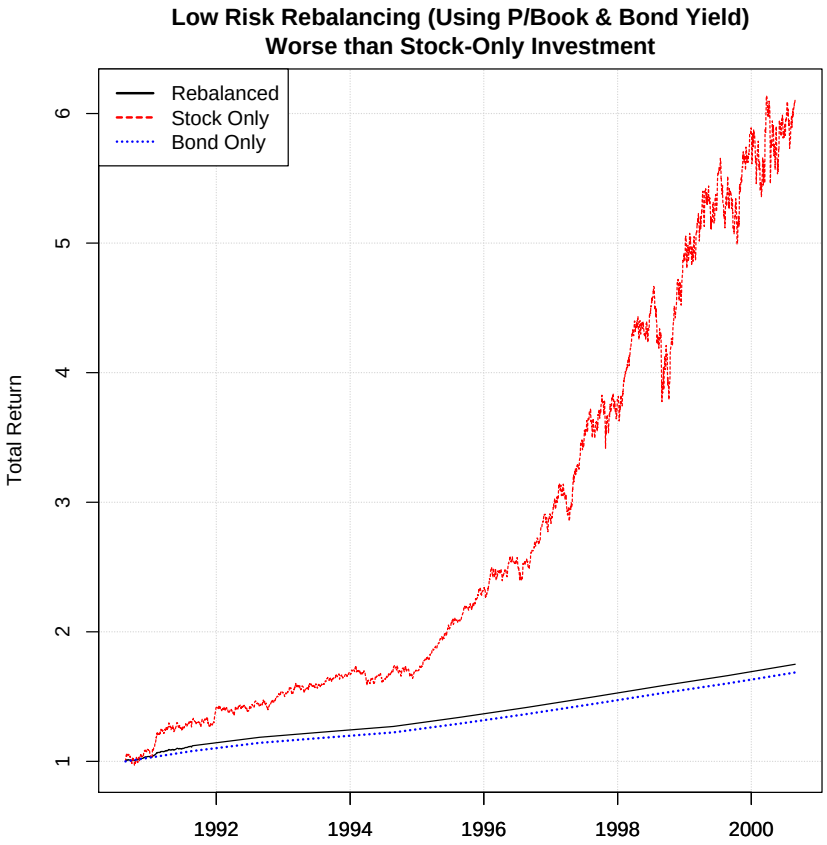

Figure 76: Total return of adaptive low-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

Low Risk Rebalancing (Using P/Book & Bond Yield) Worse than Stock-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.03	2.51	2.69	2.61	2.52	2.69	2.97	3.74	4.00	4.83	4.76
Bond Yield	8.0%	5.8%	3.6%	3.4%	5.6%	5.7%	5.8%	5.6%	5.0%	5.2%	6.3%
Stock Weight	0.17	0	0	0	0	0	0	0	0	0	0
Rebalanced	1	1.12	1.19	1.23	1.27	1.34	1.42	1.50	1.58	1.66	1.75
Bond-Only	1	1.08	1.14	1.18	1.22	1.29	1.37	1.44	1.52	1.60	1.69
Stock-Only	1	1.32	1.42	1.63	1.71	2.09	2.55	3.55	4.11	5.38	6.09

Table 31: Total return of adaptive low-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 70.

14.1.4. Comparison to P/Book Strategy

For easier reference, the strategy that adapts the stock-weight using both the P/Book and the bond yield is referred to as the 'bond-yield strategy', while the strategy that uses only the P/Book is referred to as the 'P/Book strategy'. This section compares the performance of the two strategies.

Stock Weight

The stock-weight for the bond-yield strategy during the period 1978-2013 is shown in Figure 70 and it is shown for the P/Book strategy in Figure 42. The bond-yield strategy had more extreme stock-weights than the P/Book strategy, with the stock-weights being zero for longer periods and conversely also being closer to one in some years while the stock-weight for the P/Book strategy was always below 0.7.

Annualized Return

Table 27 shows the annualized returns for the bond-yield strategy and Table 7 shows it for the P/Book strategy. The median annualized returns were quite similar and were about 5.6-5.9% depending on investment period.

The mean annualized returns were higher for the bond-yield strategy especially for shorter investment periods, because the maximum annualized returns were greater for that strategy. For example, for one year investment periods the bond-yield strategy had a maximum return of 71.8% while the P/Book strategy only had 50.9%. This is because the stock-weight for the bond-yield strategy would sometimes be close or equal to one so the rebalanced portfolio was entirely invested in the S&P 500, while the P/Book strategy had a maximum stock-weight around 0.7.

Probability of Loss

Both strategies had zero probability of loss for investment periods of two years or more, but the bond-yield strategy had a lower probability of loss about 0.015 (or 1.5%) while it was about 0.025 (or 2.5%) for the P/Book strategy.

Probability of Under-Performing Bond-Only and Stock-Only Investments

The probability of under-performing bond-only investments for one-year investment periods was 0.09 (or 9%) for the bond-yield strategy and it was 0.15 (or 15%) for the P/Book strategy. The probability gradually decreased for longer investment periods and for ten years the bond-yield strategy had probability 0.01 (or 1%) while the P/Book strategy had probability 0.03 (or 3%). So the P/Book strategy had significantly greater risk of under-performing bond-only investments.

The probabilities of under-performing stock-only investments were somewhat lower for the bond-yield strategy than for the P/Book strategy. For example, for one-year investment periods the bond-yield strategy had probability 0.72 of under-performing stock-only investments while the probability was 0.75 for the P/Book strategy. For five-year investment periods the bond-yield strategy had probability 0.59 while it was 0.70 for the P/Book strategy.

14.2. Medium Risk Strategy (Using P/Book & Bond Yield)

This section analyses the performance of the medium-risk adaptive strategy that uses both the P/Book and bond yield when rebalancing the portfolio. The strategy's worst one-year loss was about (15%) during the period 1978-2013.

14.2.1. Stock Weight

The rebalancing is done annually and the fraction of the portfolio to invest in the S&P 500 index is calculated using the formula:

$$Stock\ Weight = Limit(3.1 - 1.1 \times P/Book - 5 \times Bond\ Yield)$$

Eq. 14-3

The P/Book is for the S&P 500 index and the bond yield is for US government bonds with one-year maturity.

The limit-function simply means that the stock-weight is limited between zero and one. The stock-weight varies with both the P/Book and bond yield so its boundaries depend on both. For example, holding the bond yield fixed at 1.2% means the stock-weight is zero when the P/Book is greater than about 2.76, and the stock-weight is one when the P/Book is less than about 1.85. If instead the bond yield is 2.7% then the stock-weight is zero when the P/Book is greater than about 2.70, and the stock-weight is one when the P/Book is less than about 1.79.

Consider another example where the P/Book is 1.8 and the bond yield is 3% (or 0.03), then the stock-weight is calculated from Eq. 14-3 as follows:

$$Stock\ Weight = Limit(3.1 - 1.1 \times P/Book - 5 \times Bond\ Yield) = Limit(3.1 - 1.1 \times 1.8 - 5 \times 0.03) = 0.97$$

That is, 0.97 (or 97%) of the portfolio should be invested in the S&P 500 according to this strategy.

If instead the bond yield had been 7% (or 0.07) then the stock-weight would be:

$$Stock\ Weight = Limit(3.1 - 1.1 \times P/Book - 5 \times Bond\ Yield) = Limit(3.1 - 1.1 \times 1.8 - 5 \times 0.07) = 0.77$$

So now only 0.77 (or 77%) of the portfolio should be invested in the S&P 500 because the bond yield is higher at 7% and hence more attractive than a bond yield of only 3%.

Figure 77 shows the stock-weight for each day during the period 1978-2013 calculated using Eq. 14-3 with the P/Book data from Figure 2 and the government bond yields from Figure 3. About 63% of the days in this period had some part of the portfolio invested in the S&P 500 while the remaining 37% of the days had no investment in the S&P 500 because the stock-weight was zero. Note that the stock-weight is shown for each day but the investment strategy uses annual rebalancing.

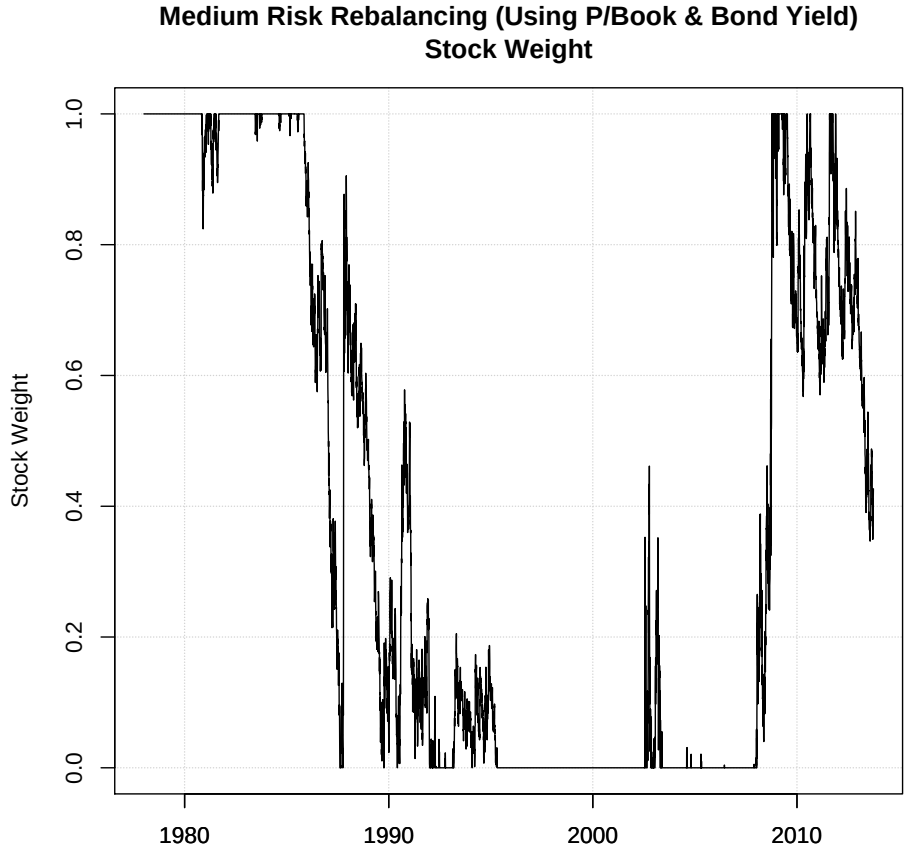

Figure 77: Stock-weight for adaptive medium-risk rebalancing (using P/Book & Bond Yield).

14.2.2. Annualized Return

Figure 78 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 32.

For one-year investment periods the mean annualized return was 10.0% which gradually decreased to 8.5% for ten-year investment periods. The median annualized return was 5.8% for one-year investment periods and this increased to 6.6% for ten-year investment periods.

For one-year investment periods the probability of loss was 0.05 (or 5%) which gradually decreased to zero probability of loss for investment periods of six years or more. The greatest loss was (15.3%) for one-year investment periods. The lowest annualized return was 2.3% for ten-year investment periods thus giving a compounded gain of 26% over ten years. The greatest one-year gain was 71.8% and the greatest annualized return was 20.1% for ten-year investment periods thus giving a compounded gain of 524% over ten years.

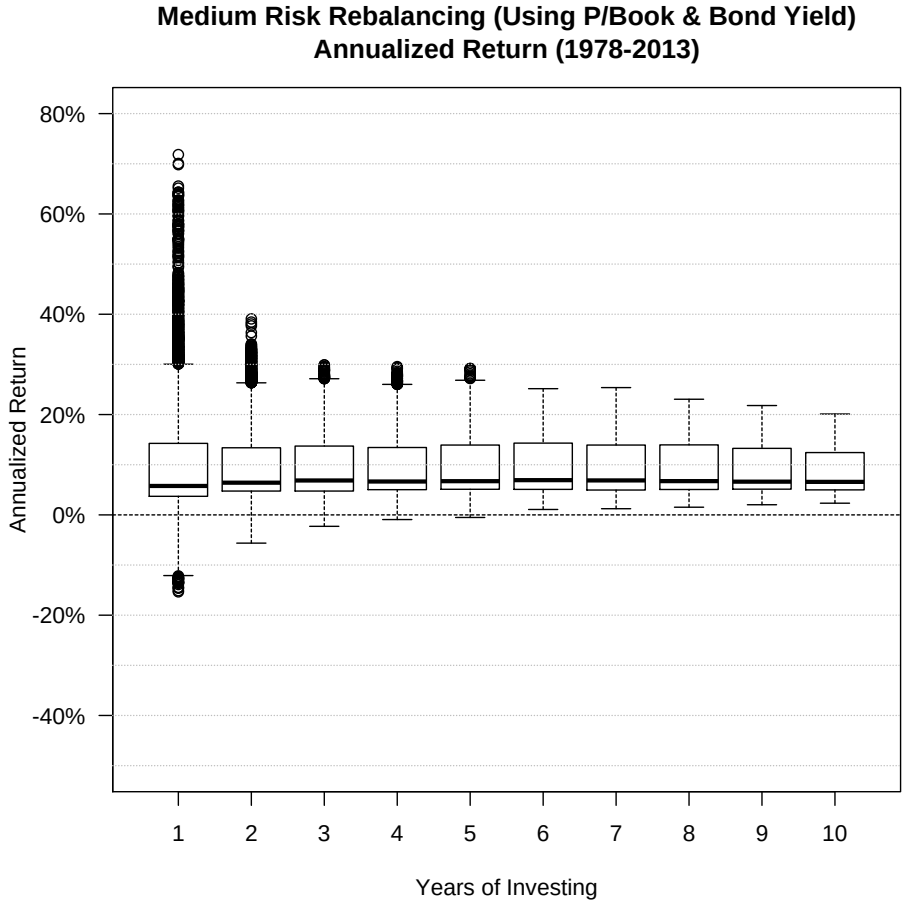

Figure 78: Annualized return statistics for adaptive medium-risk rebalancing using the P/Book of the S&P 500 index and the yield on US government bonds.

Annualized Return for Medium Risk Rebalancing (Using P/Book & Bond Yield)										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(15.3%)	3.7%	5.8%	10.0%	14.3%	71.8%	11.1%	0.05	0.14	0.58
2	(5.6%)	4.7%	6.4%	9.6%	13.4%	39.1%	7.3%	0.01	0.08	0.56
3	(2.3%)	4.8%	6.9%	9.5%	13.7%	29.9%	6.2%	0.002	0.07	0.55
4	(0.9%)	5.0%	6.7%	9.3%	13.4%	29.6%	5.9%	0.0008	0.05	0.53
5	(0.5%)	5.1%	6.7%	9.2%	13.9%	29.2%	5.7%	0.0007	0.03	0.51
6	1.1%	5.1%	6.9%	9.0%	14.3%	25.2%	5.4%	0	0.03	0.52
7	1.2%	5.0%	6.9%	8.9%	13.9%	25.4%	5.3%	0	0.03	0.56
8	1.5%	5.1%	6.7%	8.8%	14.0%	23.1%	5.1%	0	0.03	0.64
9	2.0%	5.1%	6.6%	8.7%	13.3%	21.8%	4.9%	0	0.03	0.72
10	2.3%	5.0%	6.6%	8.5%	12.4%	20.1%	4.6%	0	0.03	0.72

Table 32: Annualized return statistics for adaptive medium-risk rebalancing using the P/Book of the S&P 500 index and the yield on US government bonds. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

14.2.3. Comparison to Bond-Only & Stock-Only Investments

Figure 79 shows the performance of the rebalancing strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the rebalancing strategy performed better than bond-only and worse than stock-only investments. But this mutual performance is misleading as it depends on the starting date and investment duration, as shown below.

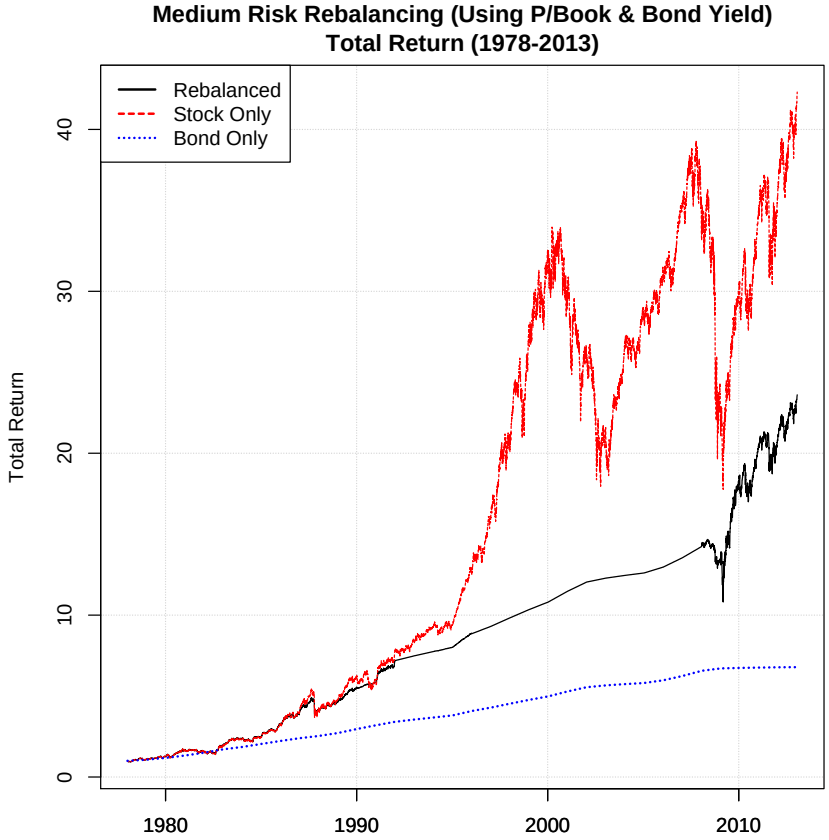

Figure 79: Total return of adaptive medium-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments for the period 1978-2013.

Probability of Under-Performance

Table 32 shows the probabilities of the rebalanced portfolio under-performing bond-only investments. For one-year investment periods the probability was 0.14 (or 14%). The probability gradually decreased to about 0.03 (or 3%) for five years or more of investing using this strategy.

Table 32 also shows the probabilities of the rebalanced portfolio under-performing stock-only investments. For one-year investment periods the probability was 0.58 (or 58%). The lowest probability was 0.51 (or 51%) which occurred for investment periods of 5 years. The highest probability was 0.72 (or 72%) which occurred for investment periods of 9 and 10 years.

Better than Bond-Only Investment

Figure 80 shows an example of the rebalancing strategy performing better than both bond-only and stock-only investments. The investment period starts August 19, 1982. Table 33 shows that the stock-weight was one or nearly one for the first four years so the entire portfolio was invested in the S&P 500 in these years. From 1986 and onwards the P/Book for the S&P 500 increased and hence the stock-weight decreased so

the portfolio was invested less in the S&P 500 and more in government bonds. For the entire ten-year period the rebalanced portfolio had a total return of 6.25 (or 525% gain), the bond-only investment had a total return of 2.23 (or 123% gain), and the stock-only investment had a total return of 5.33 (or 433% gain).

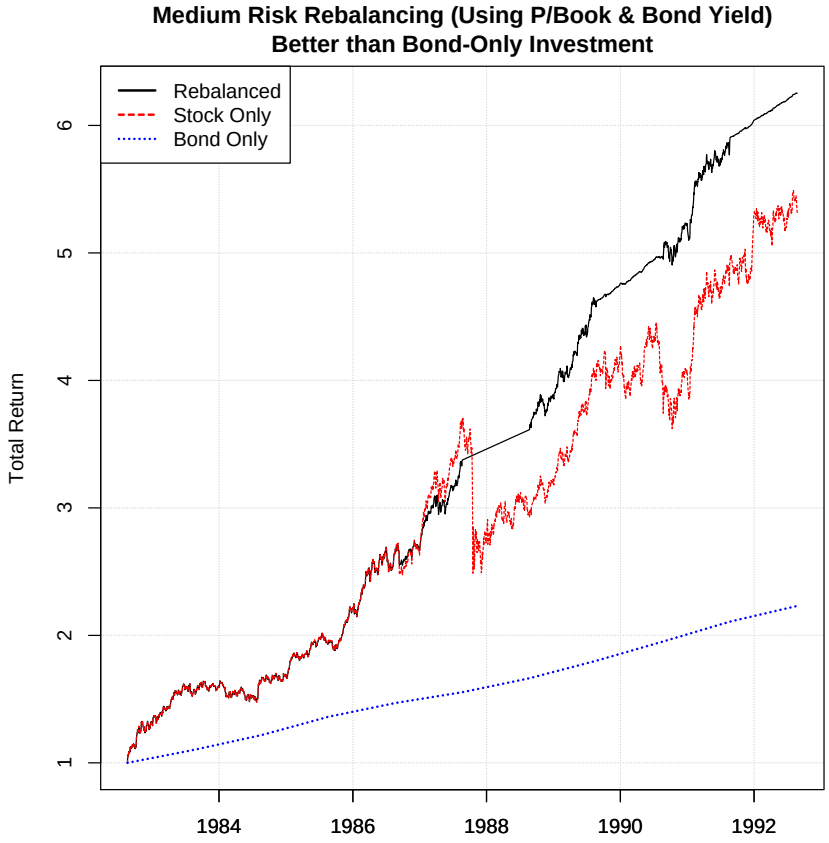

Figure 80: Total return of adaptive medium-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

Medium Risk Rebalancing (Using P/Book & Bond Yield) Better than Bond-Only Investment											
Date	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
P/Book	0.98	1.42	1.39	1.52	1.98	2.56	1.85	2.37	2.03	2.51	2.69
Bond Yield	10.3%	10.4%	11.8%	7.9%	5.7%	7.0%	8.3%	8.4%	8.0%	5.8%	3.6%
Stock Weight	1	1	0.98	1	0.64	0	0.65	0.07	0.46	0.04	0
Rebalanced	1	1.57	1.67	1.96	2.67	3.37	3.61	4.61	4.95	5.90	6.25
Bond-Only	1	1.10	1.22	1.36	1.47	1.55	1.66	1.80	1.95	2.11	2.23
Stock-Only	1	1.57	1.67	1.96	2.67	3.69	2.93	4.04	3.74	4.95	5.33

Table 33: Total return of adaptive medium-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 77.

Worse than Bond-Only Investment

Figure 81 shows an example of the rebalancing strategy performing worse than a bond-only investment. The starting date is July 2, 1999. Table 34 shows that the P/Book for the S&P 500 was very high at 5.03 in the first year and then gradually decreased to 1.81 in 2009. The rebalancing strategy therefore allocated the entire portfolio to government bonds and it was first in 2008 where the P/Book had decreased to 2.35

that 40% of the portfolio was invested in the S&P 500. Because the S&P 500 continued to decrease greatly during 2008-2009, the rebalanced portfolio which was partly invested in the S&P 500 also experienced a loss and hence under-performed the bond-only investment. But the rebalanced portfolio still performed better than the stock-only investment. For the ten-year investment period the total return of the rebalanced portfolio was 1.26 (or 26% gain), while the total return was 1.42 (or 42% gain) for the bond-only investment and 0.76 (or 24% loss) for the stock-only investment.

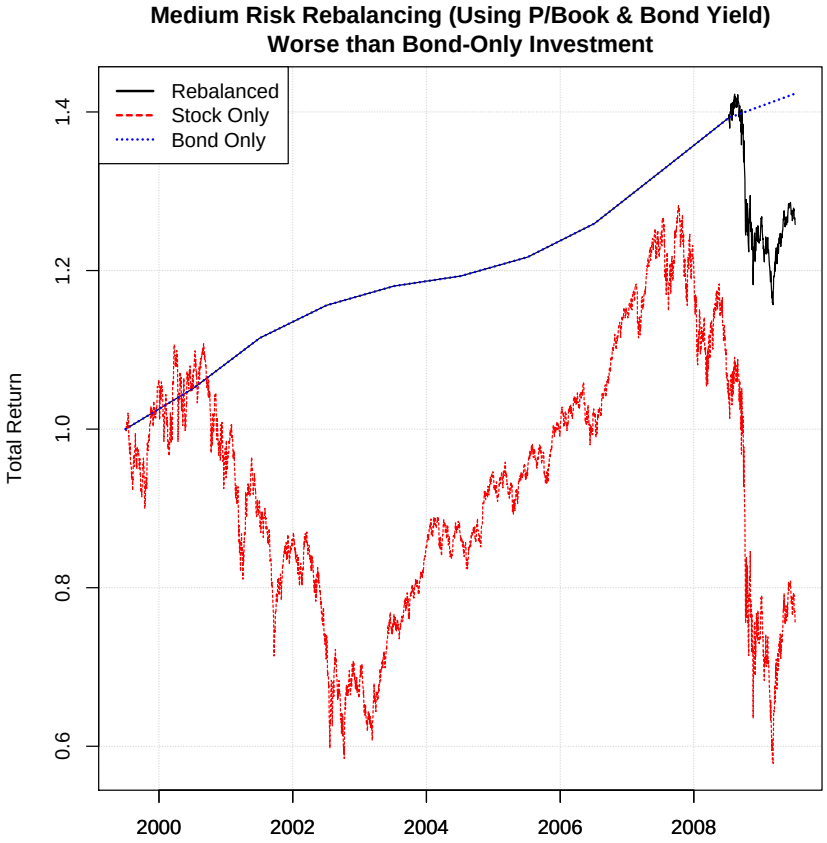

Figure 81: Total return of adaptive medium-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

Medium Risk Rebalancing (Using P/Book & Bond Yield) Worse than Bond-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	5.03	4.62	3.53	3.00	2.90	2.89	2.74	2.66	2.96	2.35	1.81
Bond Yield	5.1%	6.1%	3.7%	2.1%	1.1%	2.0%	3.5%	5.3%	5.0%	2.2%	0.5%
Stock Weight	0	0	0	0	0	0	0	0	0	0.40	1
Rebalanced	1	1.05	1.11	1.16	1.18	1.19	1.22	1.26	1.32	1.39	1.26
Bond-Only	1	1.05	1.11	1.16	1.18	1.19	1.22	1.26	1.32	1.39	1.42
Stock-Only	1	1.05	0.90	0.74	0.76	0.86	0.94	1.01	1.25	1.04	0.76

Table 34: Total return of adaptive medium-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 77.

Better than Stock-Only Investment

Figure 82 shows an example of the rebalancing strategy performing better than both bond-only and stock-only investments. The starting date is February 24, 2000, which is less than a year following the previous example where the rebalancing strategy performed worse than the bond-only investment.

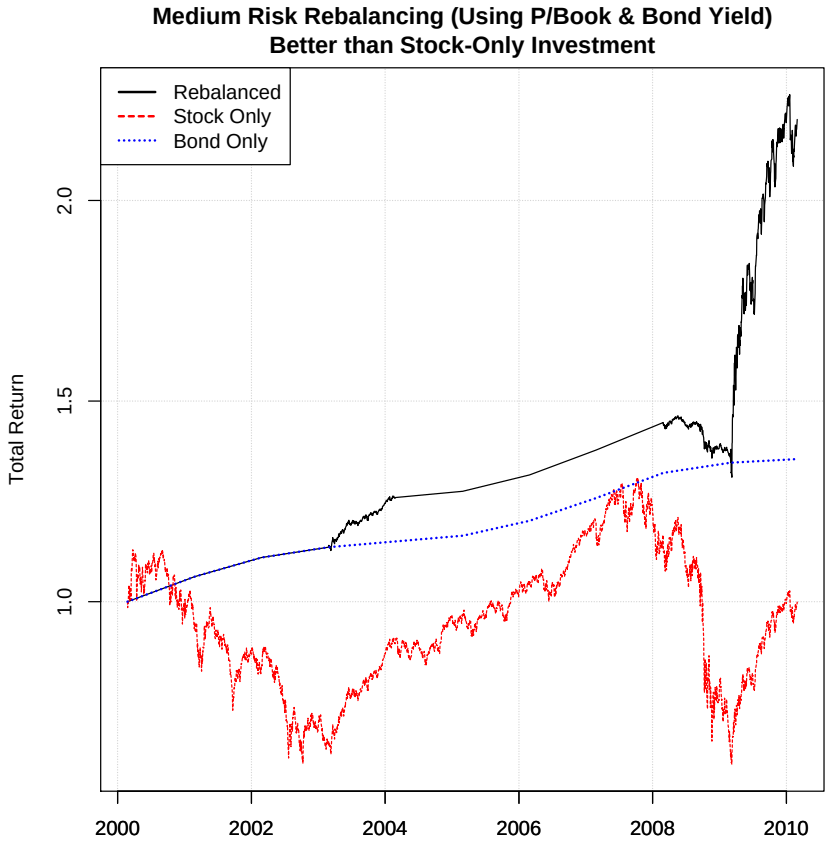

Figure 82: Total return of adaptive medium-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

Medium Risk Rebalancing (Using P/Book & Bond Yield) Better than Stock-Only Investment											
Date	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
P/Book	4.61	3.74	3.31	2.54	3.07	2.85	2.76	2.77	2.58	1.56	2.13
Bond Yield	6.2%	4.5%	2.3%	1.3%	1.2%	3.2%	4.7%	5.0%	1.9%	0.7%	0.3%
Stock Weight	0	0	0	0.25	0	0	0	0	0.17	1	0.74
Rebalanced	1	1.06	1.11	1.14	1.26	1.27	1.32	1.38	1.45	1.36	2.21
Bond-Only	1	1.06	1.11	1.14	1.15	1.16	1.20	1.26	1.32	1.35	1.35
Stock-Only	1	0.95	0.84	0.64	0.90	0.96	1.04	1.16	1.15	0.62	1.00

Table 35: Total return of adaptive medium-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 77.

Table 35 shows that the P/Book for the S&P 500 was high for most of these years ranging from 4.61 in 2000 to 2.58 in 2008 and then the P/Book decreased greatly to 1.56 in 2009. Hence the stock-weight was zero for

most of these years except 2003 where the stock-weight was 0.25 so that 25% of the portfolio was invested in the S&P 500 in that year. For the ten-year investment period the total return of the rebalancing strategy was 2.21 (or 121% gain), while the total return was 1.35 (or 35% gain) for the bond-only investment and 1.00 (no change) for the stock-only investment.

Worse than Stock-Only Investment

Figure 83 shows an example of the rebalancing strategy performing worse than a stock-only investment. The starting date is August 27, 1990.

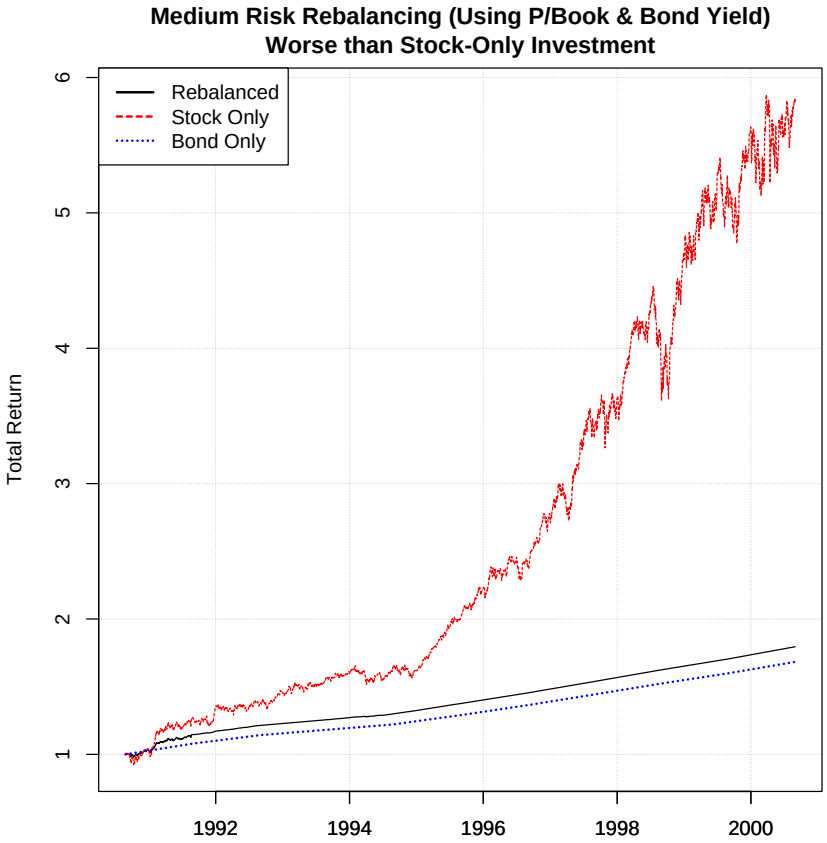

Figure 83: Total return of adaptive medium-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

Medium Risk Rebalancing (Using P/Book & Bond Yield) Worse than Stock-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.13	2.51	2.70	2.61	2.55	2.69	2.93	3.68	3.67	4.73	4.78
Bond Yield	7.9%	5.8%	3.5%	3.4%	5.6%	5.7%	5.8%	5.6%	5.0%	5.3%	6.2%
Stock Weight	0.36	0.05	0	0.06	0.01	0	0	0	0	0	0
Rebalanced	1	1.15	1.21	1.26	1.30	1.37	1.45	1.54	1.62	1.70	1.79
Bond-Only	1	1.08	1.14	1.18	1.22	1.29	1.36	1.44	1.52	1.60	1.68
Stock-Only	1	1.26	1.37	1.56	1.65	2.00	2.40	3.34	3.61	5.04	5.86

Table 36: Total return of adaptive medium-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 77.

Table 36 shows the stock-weight was 0.36 in the first year and was close to or equal zero for the remaining years. This was because the P/Book for the S&P 500 was typically above 2.5 and the bond yield was about 3-8% during this period, so the rebalancing strategy deemed it too risky to invest in the S&P 500 because historically these conditions had resulted in losses greater than (15%) which is the maximum loss tolerated by this medium-risk strategy. This period was different though, as it was a ten-year bull market where the P/Book of the S&P 500 went from 2.13 in 1990 to 4.78 in 2000 and the total return of the S&P 500 was 5.86 (or 486% gain) while the rebalancing strategy was mostly invested in bonds and only had a total return of 1.79 (or 79% gain) which was only slightly better than the bond-only investment whose total return was 1.68 (or 68% gain).

14.2.4. Comparison to Low-Risk Strategy

This section compares the medium and low risk strategies that adapt using both the P/Book and bond yield.

Stock Weight

Figure 77 shows the stock-weights for the medium-risk strategy during the period 1978-2013 and Figure 70 shows it for the low-risk strategy. The two strategies have similar tendencies for their stock-weights but the medium-risk strategy generally has larger stock-weights and also has non-zero stock-weights for more years during the period 1990-2008, so the medium-risk strategy generally invests more in the S&P 500 than the low-risk strategy.

Annualized Return

Table 32 shows the annualized returns for the medium-risk strategy and Table 27 shows it for the low-risk strategy.

The medium-risk strategy had median annualized return ranging between 5.8-6.9% depending on the duration of the investment period. This was slightly higher than the low-risk strategy which ranged between 5.6-5.9%.

The mean annualized return was somewhat higher for the medium-risk strategy where it was 10.0% for one-year investment periods and decreased gradually to 8.5% for ten-year investment periods. For the low-risk strategy the mean annualized return was 8.9% for one-year investment periods and decreased gradually to 7.6% for ten-year investment periods.

The greatest gain for one-year investment periods was 71.8% for both strategies. For longer investment periods the medium-risk strategy was somewhat better, for example the annualized return was 20.1% for ten-year investment periods while it was only 18.0% for the low-risk strategy.

The greatest loss for one-year investment periods was (4.5%) for the low-risk strategy while it was (15.3%) for the medium-risk strategy. The low-risk strategy did not have losses for longer investment periods but the medium-risk strategy experienced a maximum loss of (5.6%) for two-year investment periods and gradually decreasing losses for investment periods up to five years after which it did not experience losses.

Probability of Loss

The low-risk strategy had a probability of loss about 0.015 (or 1.5%) for one-year investment periods, and zero probability of loss for longer investment periods. The medium-risk strategy had a probability of loss

about 0.05 (or 5%) for one-year investment periods and this gradually decreased to zero probability of loss for investment periods of six years or more.

Probability of Under-Performing Bond-Only and Stock-Only Investments

The low-risk strategy had a lower probability of under-performing bond-only investments than the medium-risk strategy. For the low-risk strategy the probability was 0.09 (or 9%) for one-year investments and this decreased to 0.01 (or 1%) for investment periods of five years or more. The medium-risk strategy had a probability of under-performing bond-only investments of about 0.14 (or 14%) for one-year investment periods and this decreased to 0.03 (or 3%) for investment periods of five years or more.

The low-risk strategy had a higher probability of under-performing stock-only investments than the medium-risk strategy. For the low-risk strategy the probability ranged between 0.59-0.81 depending on the investment duration, while the probability ranged between 0.51-0.72 for the medium-risk strategy.

14.2.5. Comparison to P/Book Strategy

For easier reference, the strategy that adapts the stock-weight using both the P/Book and the bond yield is referred to as the 'bond-yield strategy', while the strategy that uses only the P/Book is referred to as the 'P/Book strategy'. This section compares the performance of the two strategies.

Stock Weight

The stock-weight for the bond-yield strategy is shown in Figure 77 for the period 1978-2013 and it is shown for the P/Book strategy in Figure 49. The bond-yield strategy had more extreme stock-weights than the P/Book strategy, with the stock-weights for the bond-yield strategy being closer to either zero or one.

Annualized Return

Table 32 shows the annualized returns for the bond-yield strategy and Table 12 shows it for the P/Book strategy. The median annualized returns for the bond-yield strategy ranged between 5.8-6.9% depending on the investment duration, while the range was slightly higher at 6.0-7.5% for the P/Book strategy.

The mean annualized returns were slightly higher for the bond-yield strategy especially for shorter investment periods. For one-year investment periods the mean annualized return was 10.0% and this decreased to 8.5% for ten-year investment periods. Compare this to the P/Book strategy whose mean annualized return was 9.3% for one-year investment periods and 8.3% for ten-year investment periods.

The greatest one-year loss was (15.3%) for the bond-yield strategy while it was only (13.9%) for the P/Book strategy. The greatest one-year gain was 71.8% for the bond-yield strategy while it was only 65.6% for the P/Book strategy.

Probability of Loss

The P/Book strategy had probability of loss 0.06 (or 6%) for one-year investment periods, while the bond-yield strategy had probability of loss 0.05 (or 5%). These probabilities gradually decreased and the P/Book strategy had zero probability of loss for investment periods of five years or more, while the bond-yield strategy had zero probability of loss for investment periods of six years or more.

Probability of Under-Performing Bond-Only and Stock-Only Investments

The probability of under-performing bond-only investments was somewhat lower for the bond-yield strategy than for the P/Book strategy. The bond-yield strategy had probability 0.14 (or 14%) of under-

performing bond-only investments in one-year investment periods while the P/Book strategy had probability 0.18 (or 18%). These gradually decreased for longer investment periods and for five years or more both strategies had probability 0.03 (or 3%) of under-performing bond-only investments.

The probability of the bond-yield strategy under-performing stock-only investments ranged between 0.51-0.72 depending on the investment duration. This was significantly lower than the P/Book strategy where the probability ranged between 0.60-0.84.

14.3. High Risk Strategy (Using P/Book & Bond Yield)

This section analyses the performance of the high-risk adaptive strategy that uses both the P/Book and bond yield when rebalancing the portfolio. The strategy's worst one-year loss was almost (47%) during the period 1978-2013.

14.3.1. Stock Weight

The rebalancing is done annually and the fraction of the portfolio to invest in the S&P 500 index is calculated using the formula:

$$Stock\ Weight = Limit(6 - 1.2 \times P/Book - 5 \times Bond\ Yield)$$

Eq. 14-4

The P/Book is for the S&P 500 index and the bond yield is for US government bonds with one-year maturity.

The limit-function simply means that the stock-weight is limited between zero and one. The stock-weight varies with both the P/Book and bond yield so its boundaries depend on both. For example, holding the bond yield fixed at 1.2% means the stock-weight is zero when the P/Book is greater than 4.95, and the stock-weight is one when the P/Book is less than about 4.12. If instead the bond yield is 7% then the stock-weight is zero when the P/Book is greater than about 4.71, and the stock-weight is one when the P/Book is less than about 3.88.

Consider another example where the P/Book is 4.5 and the bond yield is 3% (or 0.03), then the stock-weight is calculated from Eq. 14-4 as follows:

$$Stock\ Weight = Limit(6 - 1.2 \times P/Book - 5 \times Bond\ Yield) = Limit(6 - 1.2 \times 4.5 - 5 \times 0.03) = 0.45$$

That is, 0.45 (or 45%) of the portfolio should be invested in the S&P 500 according to this strategy.

If instead the bond yield had been 7% (or 0.07) then the stock-weight would be:

$$Stock\ Weight = Limit(6 - 1.2 \times P/Book - 5 \times Bond\ Yield) = Limit(6 - 1.2 \times 4.5 - 5 \times 0.07) = 0.25$$

So now only 0.25 (or 25%) of the portfolio should be invested in the S&P 500 because the bond yield is higher at 7% and hence more attractive than a bond yield of only 3%.

Figure 84 shows the stock-weight for each day during the period 1978-2013 calculated using Eq. 14-4 the P/Book data from Figure 2 and the government bond yields from Figure 3. About 98% of the days in this period had some part of the portfolio invested in the S&P 500 while the remaining 2% of the days had no investment in the S&P 500 because the stock-weight was zero. Note that the stock-weight is shown for each day but the investment strategy uses annual rebalancing.

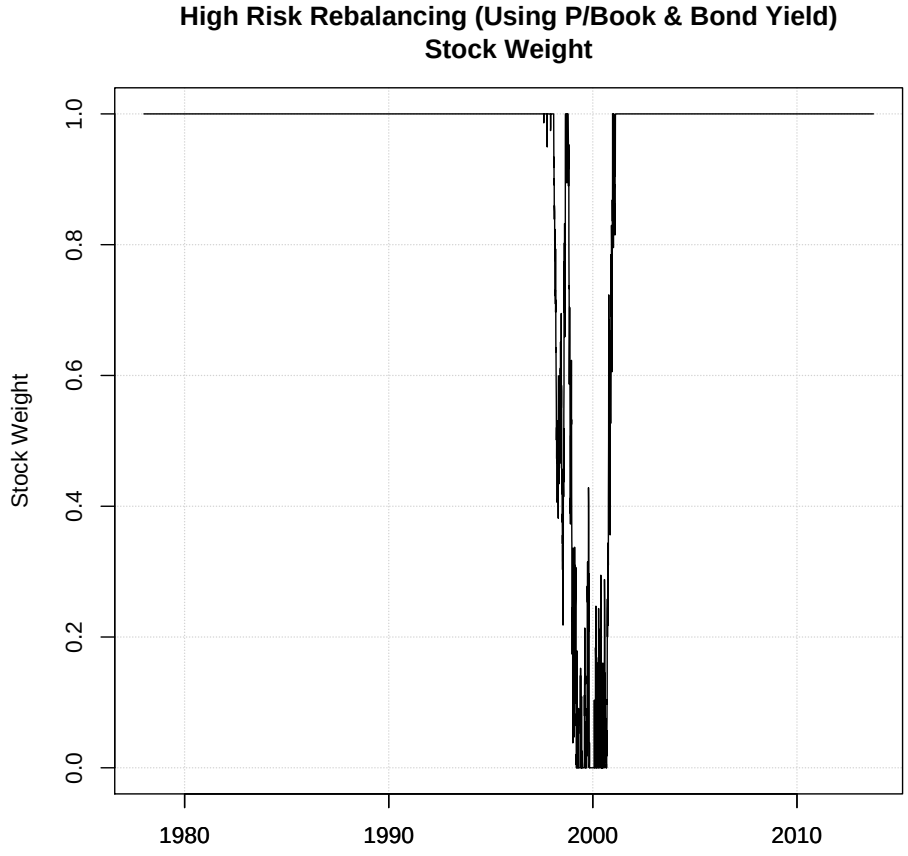

Figure 84: Stock-weight for adaptive high-risk rebalancing (using P/Book & Bond Yield

14.3.2. Annualized Return

Figure 85 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 37.

For one-year investment periods the mean annualized return was 13.1% which gradually decreased to 11.9% for ten-year investment periods. The median annualized return was 13.5% for one-year investment periods and it was 14.1% for ten-year investment periods.

All investment periods had non-zero probability of loss. For one-year investment periods the probability of loss was 0.17 (or 17%) which gradually decreased to about 0.02-0.03 (or 2-3%) for investment periods of seven years or more. The greatest loss was (46.8%) for one-year investment periods and it was (3.4%) for ten-year investment periods thus giving a compounded loss of about (31%) over ten years. The greatest gain was 71.8% for one-year investment periods and the greatest annualized return was 19.5% for ten-year investment periods thus giving a compounded gain of 494% over ten years.

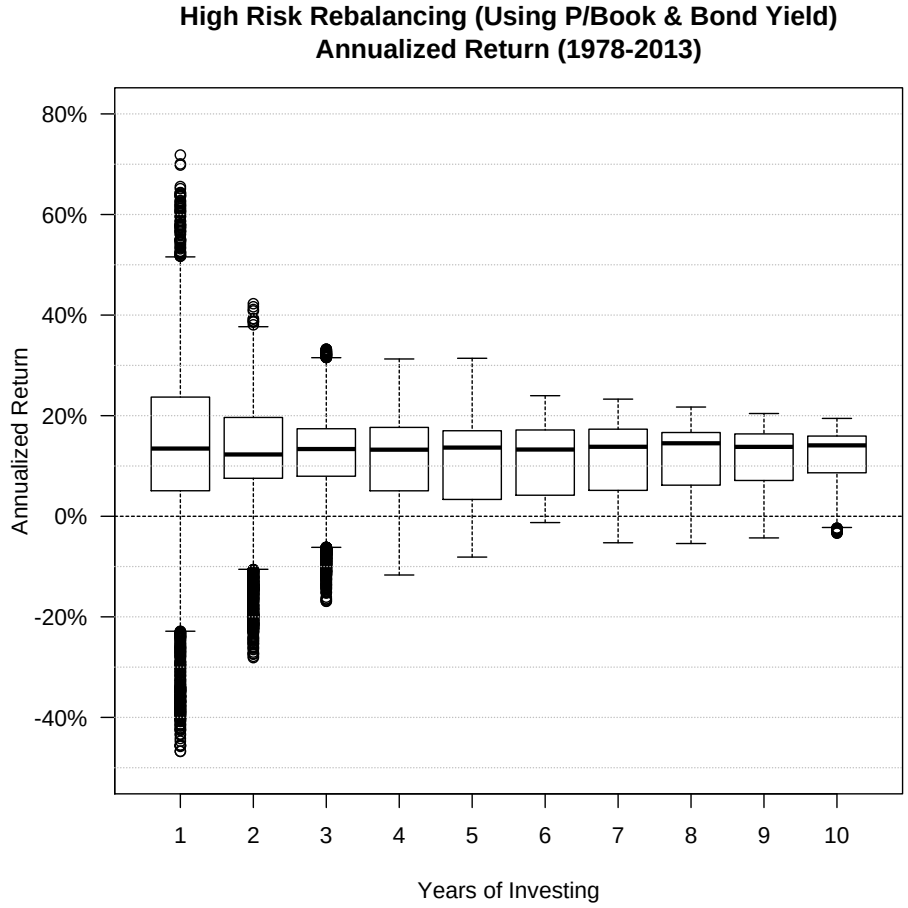

Figure 85: Annualized return statistics for adaptive high-risk rebalancing using the P/Book of the S&P 500 index and the yield on US government bonds.

Annualized Return for High Risk Rebalancing (Using P/Book & Bond Yield)										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(46.8%)	5.1%	13.5%	13.1%	23.7%	71.8%	16.6%	0.17	0.24	0.05
2	(28.1%)	7.6%	12.3%	12.2%	19.7%	42.3%	11.6%	0.12	0.22	0.05
3	(16.9%)	8.0%	13.4%	11.9%	17.4%	33.2%	9.7%	0.14	0.23	0.05
4	(11.7%)	5.1%	13.2%	11.7%	17.7%	31.3%	8.7%	0.15	0.24	0.05
5	(8.1%)	3.4%	13.7%	11.6%	17.0%	31.4%	7.8%	0.06	0.22	0.06
6	(1.2%)	4.2%	13.3%	11.7%	17.2%	24.0%	6.8%	0.002	0.14	0.06
7	(5.3%)	5.1%	13.8%	11.9%	17.3%	23.3%	6.3%	0.02	0.07	0.06
8	(5.4%)	6.2%	14.5%	11.9%	16.7%	21.7%	6.0%	0.03	0.08	0.06
9	(4.3%)	7.1%	13.8%	12.0%	16.4%	20.4%	5.7%	0.03	0.09	0.06
10	(3.4%)	8.7%	14.1%	11.9%	16.0%	19.5%	5.4%	0.02	0.11	0.07

Table 37: Annualized return statistics for adaptive high-risk rebalancing using the P/Book of the S&P 500 index and the yield on US government bonds. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

14.3.3. Comparison to Bond-Only & Stock-Only Investments

Figure 86 shows the performance of the rebalancing strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the rebalancing strategy performed slightly better than the stock-only investment because the rebalancing strategy invested partially in government bonds around year 2000 where the S&P 500 was in a bubble of very high P/Book ratios. But this long-term performance is misleading as the mutual performance depends on the starting date and investment duration, as shown below.

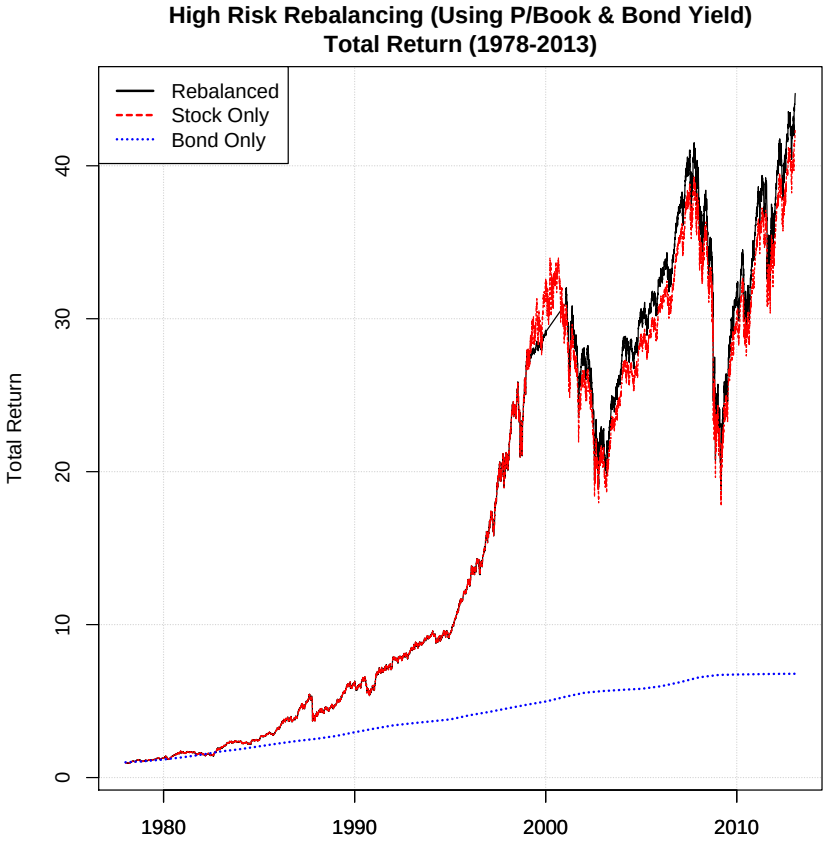

Figure 86: Total return of adaptive high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments for the period 1978-2013.

Probability of Under-Performance

Table 37 shows the probabilities of the rebalanced portfolio under-performing bond-only investments. For one-year investment periods the probability was 0.24 (or 24%). The lowest probability was 0.07 (or 7%) which occurred for seven-year investment periods.

Table 37 also shows the probabilities of the rebalanced portfolio under-performing stock-only investments. For one-year investment periods the probability was 0.05 (or 5%) and this gradually increased to 0.07 (or 7%) for ten-year investment periods.

Better than Bond-Only Investment

Figure 87 shows an example of the rebalancing strategy performing better than a bond-only investment. The investment period starts September 27, 1990. Table 38 shows that the stock-weight was one until year 1999 when the P/Book for the S&P 500 had increased to 4.57 so the stock-weight was only 0.25. For the entire ten-year period the rebalanced portfolio had a total return of 5.60 (or 460% gain), the bond-only investment had a total return of 1.66 (or 66% gain), and the stock-only investment had a total return of 5.88 (or 488% gain).

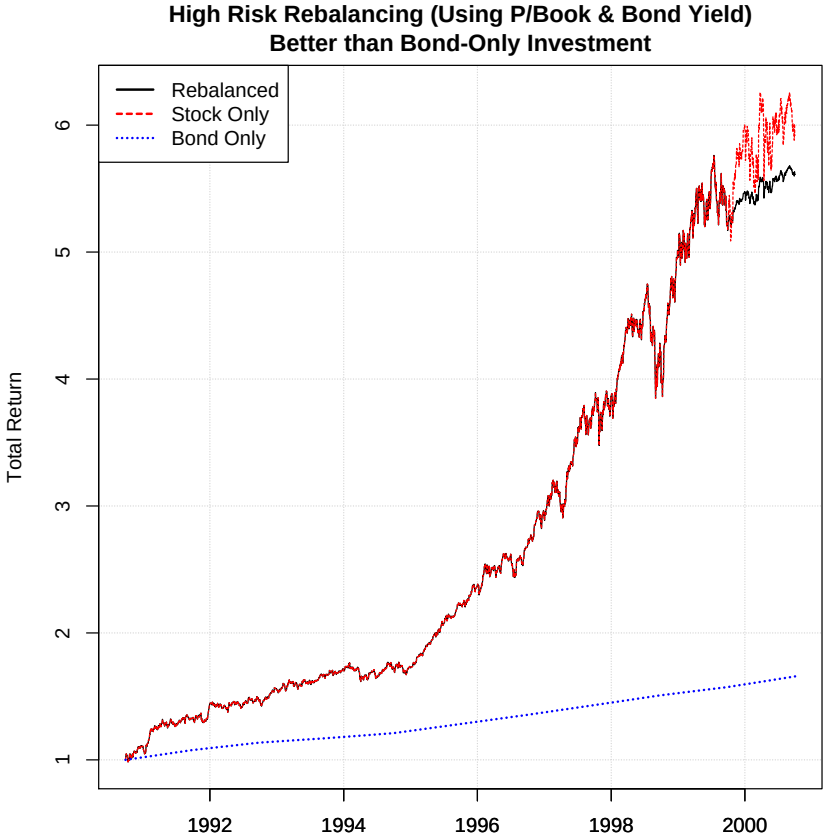

Figure 87: Total return of adaptive high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

High Risk Rebalancing (Using P/Book & Bond Yield) Better than Bond-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	1.99	2.45	2.74	2.60	2.46	2.78	3.05	3.88	3.75	4.57	4.46
Bond Yield	7.7%	5.4%	3.1%	3.4%	6.0%	5.7%	5.7%	5.4%	4.3%	5.3%	6.1%
Stock Weight	1	1	1	1	1	1	1	1	1	0.25	0.34
Rebalanced	1	1.32	1.47	1.67	1.72	2.23	2.68	3.78	3.97	5.23	5.60
Bond-Only	1	1.08	1.14	1.17	1.21	1.28	1.35	1.43	1.51	1.57	1.66
Stock-Only	1	1.32	1.47	1.67	1.72	2.23	2.68	3.78	3.97	5.23	5.88

Table 38: Total return of adaptive high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 84.

Worse than Bond-Only Investment

Figure 88 shows an example of the rebalancing strategy performing worse than a bond-only investment. The starting date is February 25, 1999. Table 39 shows that the stock-weight was 0.22-0.25 in the first two years and the stock-weight was one for the remaining years. For the ten-year period the total return of the rebalancing strategy was 0.71 (or 29% loss), while the total return was 1.41 (or 41% gain) for the bond-only investment and 0.67 (or 33% loss) for the stock-only investment.

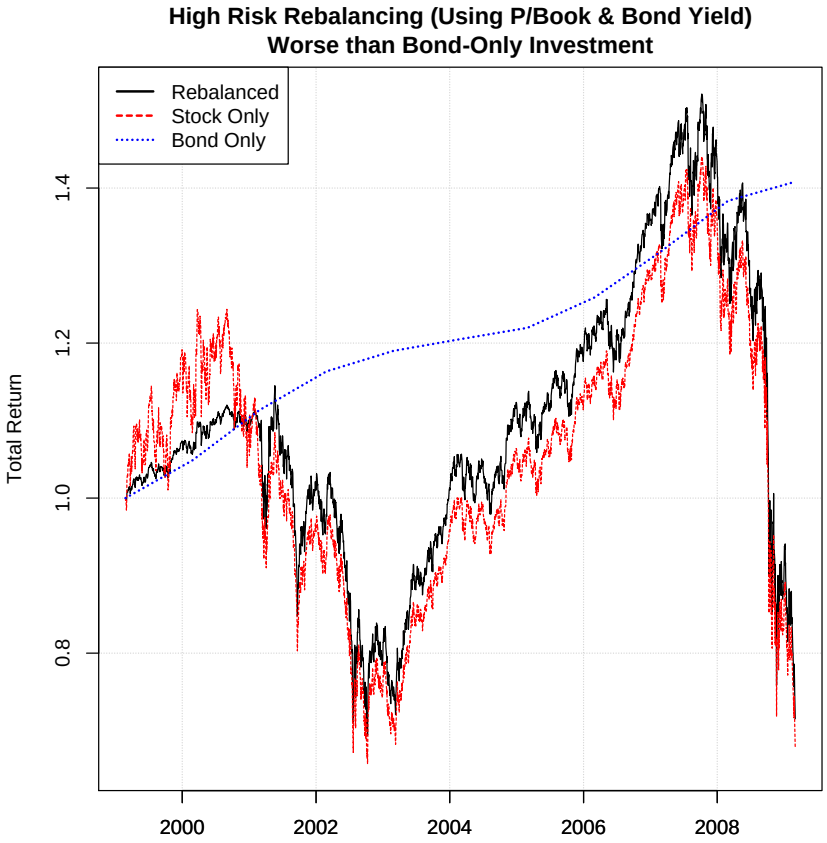

Figure 88: Total return of adaptive high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

High Risk Rebalancing (Using P/Book & Bond Yield) Worse than Bond-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	4.62	4.54	3.70	3.32	2.57	3.07	2.86	2.79	2.77	2.51	1.55
Bond Yield	4.9%	6.2%	4.5%	2.3%	1.3%	1.2%	3.2%	4.7%	5.0%	1.8%	0.7%
Stock Weight	0.22	0.25	1	1	1	1	1	1	1	1	1
Rebalanced	1	1.06	1.09	0.98	0.75	1.05	1.12	1.22	1.35	1.31	0.71
Bond-Only	1	1.05	1.11	1.16	1.19	1.20	1.22	1.26	1.32	1.38	1.41
Stock-Only	1	1.08	1.04	0.93	0.71	0.99	1.06	1.15	1.28	1.24	0.67

Table 39: Total return of adaptive high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 84.

Better than Stock-Only Investment

Figure 89 shows an example of the rebalancing strategy performing better than both bond-only and stock-only investments. The starting date is September 11, 2000. Table 40 shows that the stock-weight was one for all years except the first where it was 0.07 because the P/Book was very high at 4.68. For the ten-year period the total return of the rebalancing strategy was 1.34 (or 34% gain), while the total return was 1.33 (or 33% gain) for the bond-only investment and 0.92 (or 8% loss) for the stock-only investment.

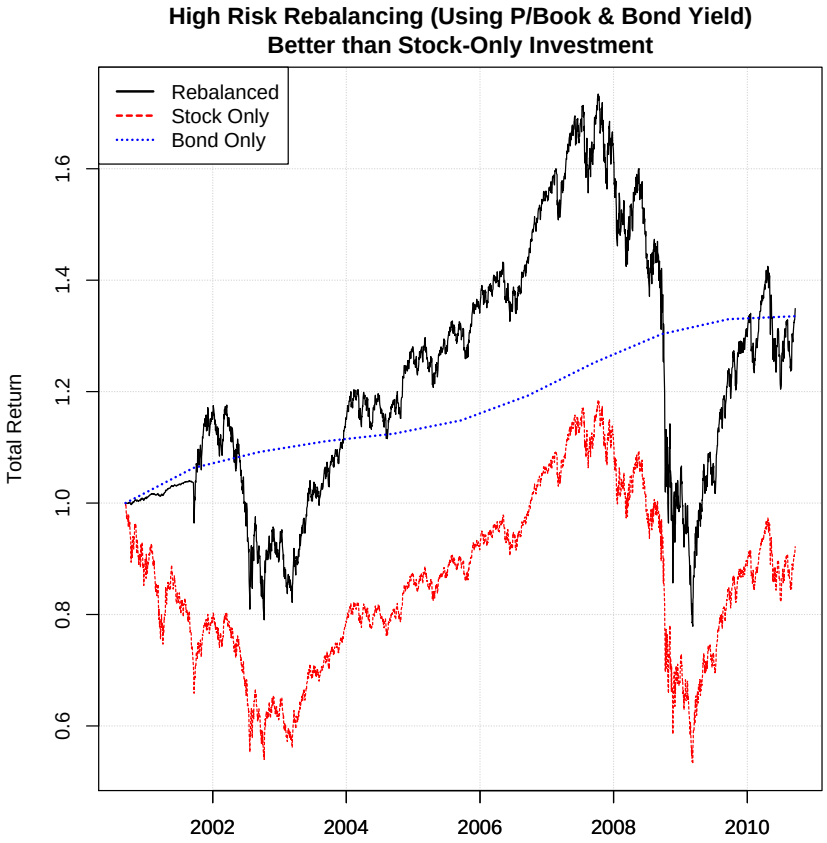

Figure 89: Total return of adaptive high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

High Risk Rebalancing (Using P/Book & Bond Yield) Better than Stock-Only Investment											
Date	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
P/Book	4.68	3.05	2.61	2.92	2.82	2.79	2.69	2.92	2.43	2.14	2.02
Bond Yield	6.2%	2.7%	1.8%	1.2%	2.1%	3.9%	5.0%	4.1%	2.1%	0.4%	0.3%
Stock Weight	0.07	1	1	1	1	1	1	1	1	1	1
Rebalanced	1	1.04	0.89	1.06	1.18	1.31	1.43	1.69	1.42	1.23	1.34
Bond-Only	1	1.06	1.09	1.11	1.12	1.15	1.19	1.25	1.30	1.33	1.33
Stock-Only	1	0.71	0.61	0.72	0.81	0.90	0.98	1.16	0.97	0.84	0.92

Table 40: Total return of adaptive high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 84.

Worse than Stock-Only Investment

Figure 90 shows an example of the rebalancing strategy performing worse than a stock-only investment. The starting date is March 21, 1990. Table 41 shows the stock-weight was one for the first eight years after which the P/Book increased so much that the stock-weight decreased gradually to zero, thus causing the portfolio to be increasingly invested in government bonds rather than the S&P 500. For the ten-year period the total return of the rebalancing strategy was 4.66 (or 366% gain), while the total return was 1.71 (or 71% gain) for the bond-only investment and 5.59 (or 459% gain) for the stock-only investment.

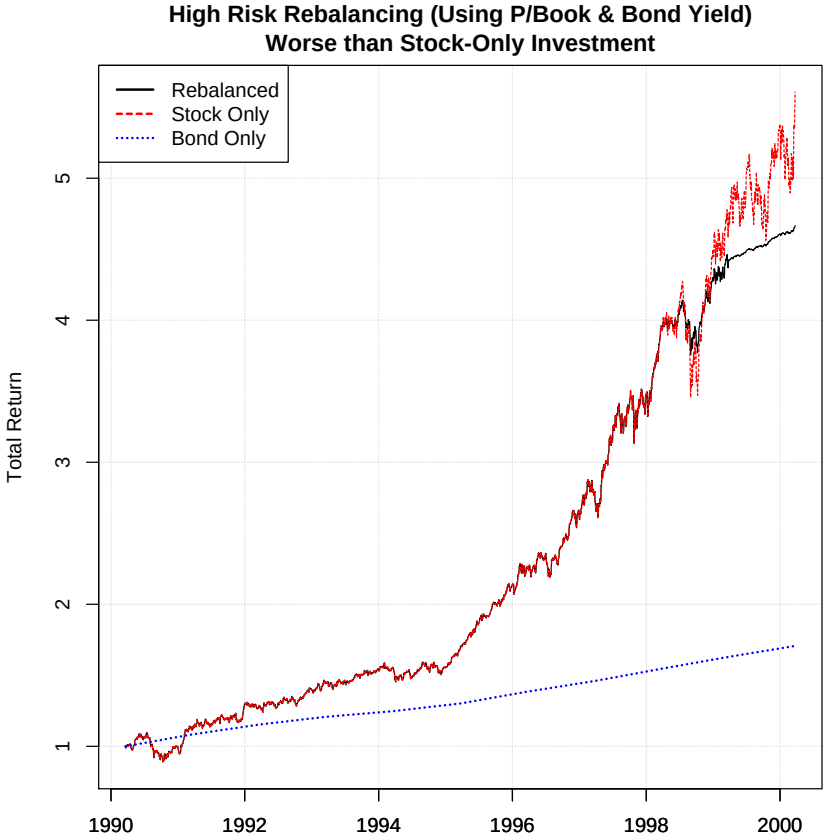

Figure 90: Total return of adaptive high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

High Risk Rebalancing (Using P/Book & Bond Yield) Worse than Stock-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.29	2.38	2.61	2.61	2.61	2.49	3.01	3.34	4.37	4.75	5.15
Bond Yield	8.4%	6.4%	4.7%	3.3%	4.3%	6.4%	5.4%	5.9%	5.4%	4.8%	6.3%
Stock Weight	1	1	1	1	1	1	1	1	0.49	0.06	0
Rebalanced	1	1.11	1.28	1.44	1.55	1.68	2.25	2.79	3.95	4.42	4.66
Bond-Only	1	1.08	1.15	1.21	1.25	1.30	1.38	1.46	1.55	1.63	1.71
Stock-Only	1	1.11	1.28	1.44	1.55	1.68	2.25	2.79	3.95	4.68	5.59

Table 41: Total return of adaptive high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 84.

14.3.4. Comparison to Medium-Risk Strategy

This section compares the performance of the high and medium risk strategies that adapt the stock-weight using both P/Book and bond yield.

Stock Weight

Figure 84 shows the stock-weights for the high-risk strategy during the period 1978-2013 and Figure 77 shows it for the medium-risk strategy. The stock-weight for the high-risk strategy is one for all years except around year 2000, this means the high-risk strategy was fully invested in the S&P 500 index in most years except when the P/Book of the S&P 500 was very high around year 2000. In comparison, the medium-risk strategy had little or no investment in the S&P 500 during most of the period 1995-2008.

Annualized Return

Table 37 shows the annualized returns for the high-risk strategy and Table 32 shows it for the medium-risk strategy.

The high-risk strategy had median annualized return of 13.5% for one-year investment periods which increased to 14.1% for ten-year investment periods. This was much higher than the medium-risk strategy whose median annualized return ranged between 5.8-6.9% depending on the investment period.

The mean annualized return was also much higher for the high-risk strategy where it was 13.1% for one-year investment periods and decreased to 11.9% for ten-year investment periods. For the medium-risk strategy the mean annualized return was 10.0% for one-year investment periods and decreased gradually to 8.5% for ten-year investment periods.

The greatest one-year gain was 71.8% for both strategies. For some of the longer investment periods the medium-risk strategy was slightly better, for example the greatest annualized return was 20.1% for ten-year investment periods while it was only 19.5% for the high-risk strategy.

The greatest one-year loss was (15.3%) for the medium-risk strategy while it was (46.8%) for the high-risk strategy. The medium-risk strategy did not have losses for investment periods of six years or more, but the high-risk strategy experienced losses for all investment periods with the greatest annualized loss for ten-year investment periods being (3.4%) which gives a compounded loss of (29%) over ten years.

Probability of Loss

The medium-risk strategy had probability of loss about 0.05 (or 5%) for one-year investment periods, and zero probability of loss for investment periods of six years or more. The high-risk strategy had probability of loss about 0.17 (or 17%) for one-year investment periods and this gradually decreased to 0.02 (or 2%) for investment periods of ten years.

Probability of Under-Performing Bond-Only and Stock-Only Investments

The high-risk strategy had a much higher probability of under-performing bond-only investments. For one-year investment periods the probability was 0.24 (or 24%) while it was only 0.14 (or 14%) for the medium-risk strategy. The probabilities decreased for longer investment periods so the medium-risk strategy had probability 0.03 (or 3%) for investment periods of five years or more, while the high-risk strategy had probability 0.22 (or 22%) for five-year investment periods and 0.11 (or 11%) for ten-year investment periods.

Conversely, the high-risk strategy had much lower probability of under-performing stock-only investments, with probabilities ranging between 0.05-0.07 (or 5-7%) depending on the investment duration, while the probabilities for the medium-risk strategy ranged between 0.51-0.72 (or 51-72%).

14.3.5. Comparison to P/Book Strategy

For easier reference, the strategy that adapts the stock-weight using both the P/Book and the bond yield is referred to as the 'bond-yield strategy', while the strategy that uses only the P/Book is referred to as the 'P/Book strategy'. This section compares the performance of the two strategies.

Stock Weight

The stock-weight for the bond-yield strategy is shown in Figure 84 for the period 1978-2013 and it is shown for the P/Book strategy in Figure 56. Both plots show a wedge around year 2000 where the stock-weight went down from one and close to zero, but the wedge is narrower for the bond-yield strategy which also has somewhat higher stock-weights than the P/Book strategy around year 2000.

Annualized Return

Table 37 shows the annualized returns for the bond-yield strategy and Table 17 shows it for the P/Book strategy. The median annualized returns for the bond-yield strategy ranged between 12.3-14.5% depending on the investment duration, while the range was slightly lower at 12.0-14.3% for the P/Book strategy.

The mean annualized returns were slightly higher for the bond-yield strategy. For one-year investment periods the mean annualized return was 13.1% and this decreased to 11.9% for ten-year investment periods. Compare this to the P/Book strategy whose mean annualized return was 12.9% for one-year investment periods and 11.9% for ten-year investment periods.

For both strategies, the greatest one-year loss was (46.8%) and the greatest one-year gain was 71.8%. For longer investment periods the bond-yield strategy experienced somewhat greater losses than the P/Book strategy, for example for ten-year investment periods the greatest annualized loss was (3.4%) for the bond-yield strategy while it was only (2.0%) for the P/Book strategy. Conversely, the bond-yield strategy experienced slightly greater gains than the P/Book strategy for some investment periods, for example for ten-year investment periods the greatest annualized return of the bond-yield strategy was 19.5% while it was 19.2% for the P/Book strategy.

Probability of Loss

The bond-yield strategy had probability of loss 0.17 (or 17%) for one-year investment which decreased to 0.02 (or 2%) for ten-year investment periods. This was slightly higher than for the P/Book strategy whose probability of loss ranged from 0.16 (or 16%) for one-year investment periods to 0.01 (or 1%) for ten-year investment periods.

Probability of Under-Performing Bond-Only and Stock-Only Investments

The probability of under-performing bond-only investments was slightly higher for the bond-yield strategy than for the P/Book strategy. For one-year investment periods the bond-yield strategy had probability 0.24 (or 24%) while it was 0.23 (or 23%) for the P/Book strategy. For ten-year investment periods the bond-yield strategy had probability 0.11 (or 11%) while it was 0.07 (or 7%) for the P/Book strategy.

The probability of the bond-yield strategy under-performing stock-only investments ranged between 0.05-0.07 (or 5-7%) which was less than half that of the P/Book strategy which ranged between 0.10-0.16 (or 10-16%).

14.4. Medium-High Risk Composite Strategy (Using P/Book & Bond Yield)

The previous rebalancing strategies for low-, medium- and high-risk can be combined into strategies with intermediate risk levels. In the following example, the composite strategy is a 50% medium-risk and 50% high-risk strategy, meaning that the stock-weights are calculated by scaling the stock-weights of the medium-risk strategy in Figure 77 by 0.5 and adding the stock-weights for the high-risk strategy in Figure 84 which are also scaled by 0.5. Figure 91 shows the resulting stock-weights for the medium-high risk strategy for each day during the period 1978-2013. About 98% of the days in this period had some part of the portfolio invested in the S&P 500 index. Note that the stock-weight is shown for each day but the investment strategy uses annual rebalancing.

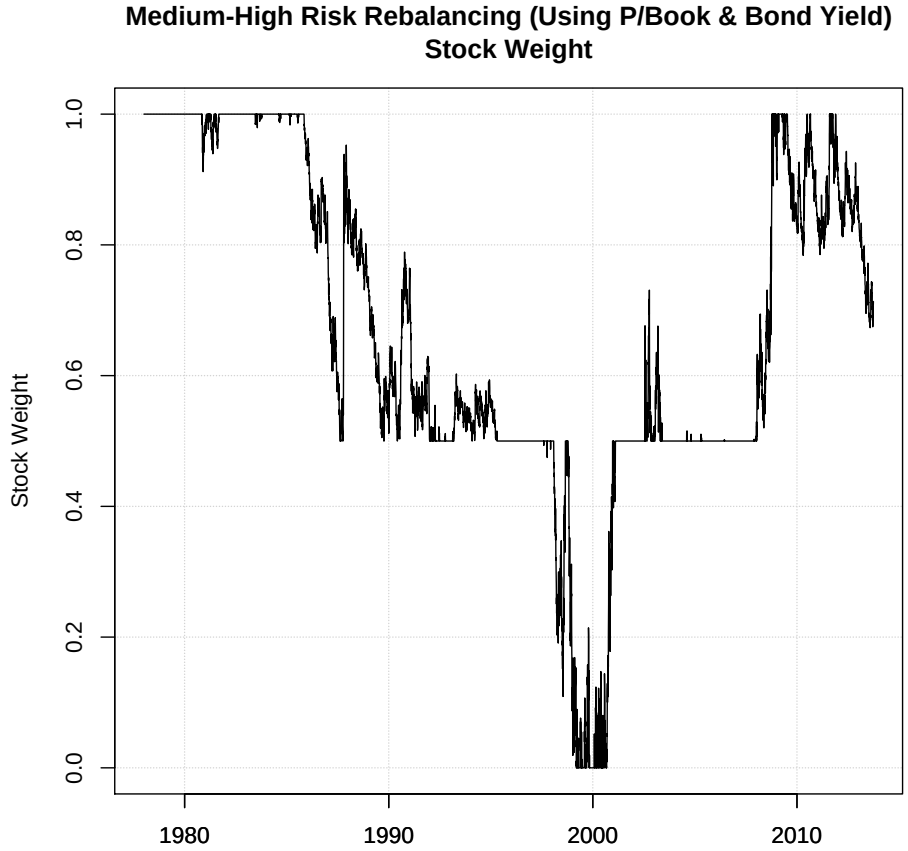

Figure 91: Stock-weight for adaptive medium-high-risk rebalancing (using P/Book & Bond Yield). Calculated as 50% of the stock-weights for medium-risk rebalancing in Figure 77 plus 50% of the stock-weights for high-risk rebalancing in Figure 84.

14.4.1. Annualized Return

Figure 92 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 42.

For one-year investment periods the mean annualized return was 11.5% which gradually decreased to 10.4% for ten-year investment periods. The median annualized return was 10.0% for one-year investment periods and this increased to 10.9% for ten-year investment periods.

For one-year investment periods the probability of loss was 0.15 (or 15%) which gradually decreased to less than 0.01 (or 1%) for investment periods of five years or more. The greatest one-year loss was (30.0%). For ten-year investment periods the lowest annualized return was 0.3% thus giving a compounded gain of 3% over ten years. The greatest one-year gain was 71.8% and the greatest annualized return for ten-year investment periods was 19.7% which gives a compounded gain of 504% over ten years.

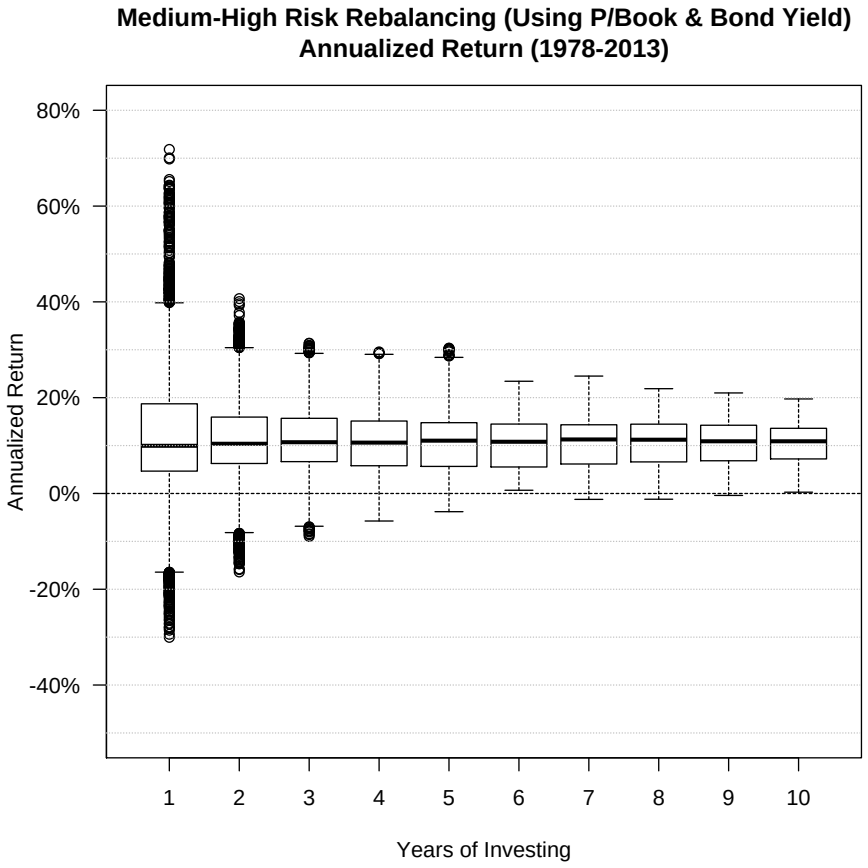

Figure 92: Annualized return statistics for adaptive medium-high-risk rebalancing using the P/Book of the S&P 500 index and the yield on US government bonds.

Annualized Return for Medium-High Risk Rebalancing (Using P/Book & Bond Yield)										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(30.0%)	4.7%	10.0%	11.5%	18.7%	71.8%	12.8%	0.15	0.24	0.58
2	(16.4%)	6.3%	10.4%	11.0%	15.9%	40.7%	8.2%	0.08	0.18	0.55
3	(9.0%)	6.7%	10.7%	10.8%	15.7%	31.4%	6.7%	0.07	0.19	0.53
4	(5.8%)	5.8%	10.6%	10.7%	15.1%	29.6%	6.2%	0.03	0.17	0.51
5	(3.8%)	5.7%	11.0%	10.6%	14.8%	30.3%	5.7%	0.01	0.13	0.49
6	0.7%	5.5%	10.8%	10.5%	14.5%	23.4%	5.2%	0	0.07	0.49
7	(1.2%)	6.2%	11.3%	10.6%	14.3%	24.5%	5.0%	0.003	0.04	0.50
8	(1.2%)	6.6%	11.2%	10.6%	14.5%	21.9%	4.8%	0.005	0.04	0.57
9	(0.4%)	6.8%	10.9%	10.5%	14.2%	21.0%	4.6%	0.001	0.05	0.66
10	0.3%	7.2%	10.9%	10.4%	13.6%	19.7%	4.3%	0	0.05	0.69

Table 42: Annualized return statistics for adaptive medium-high-risk rebalancing using the P/Book of the S&P 500 index and the yield on US government bonds. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

14.4.2. Comparison to Bond-Only & Stock-Only Investments

Figure 93 shows the performance of the rebalancing strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the rebalancing strategy performed nearly as well as the stock-only investment for long periods while dampening the large swings of the S&P 500 around year 2000 and 2008. But this long-term performance is misleading as the mutual performance depends on the starting date and investment duration, as shown below.

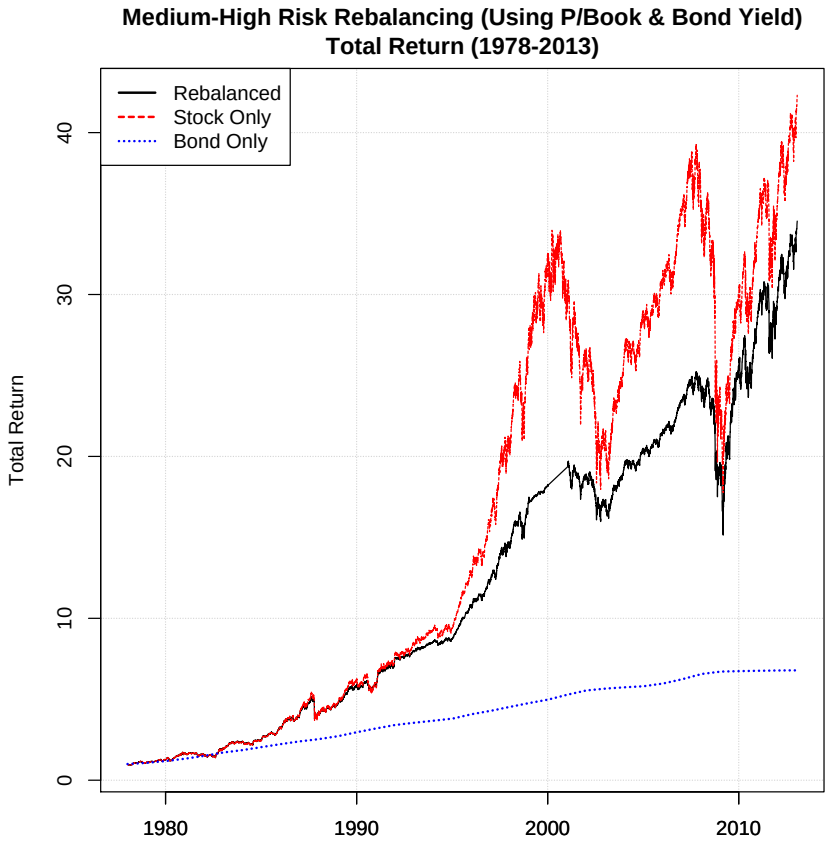

Figure 93: Total return of adaptive medium-high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments for the period 1978-2013.

Probability of Under-Performance

Table 42 shows the probabilities of the rebalanced portfolio under-performing bond-only investments. For one-year investment periods the probability was 0.24 (or 24%). The lowest probability was 0.04 (or 4%) which occurred for seven and eight year investment periods.

Table 42 also shows the probabilities of the rebalanced portfolio under-performing stock-only investments which ranged between 0.49-0.69 (or 49-69%) depending on the investment duration.

Better than Bond-Only Investment

Figure 94 shows an example of the rebalancing strategy performing better than both bond-only and stock-only investments. The investment period starts August 12, 1982. Table 43 shows that the stock-weight was one for the first four years after which it decreased. For the ten-year period the rebalanced portfolio had a total return of 6.06 (or 506% gain), the bond-only investment had a total return of 2.27 (or 127% gain), and the stock-only investment had a total return of 5.80 (or 480% gain).

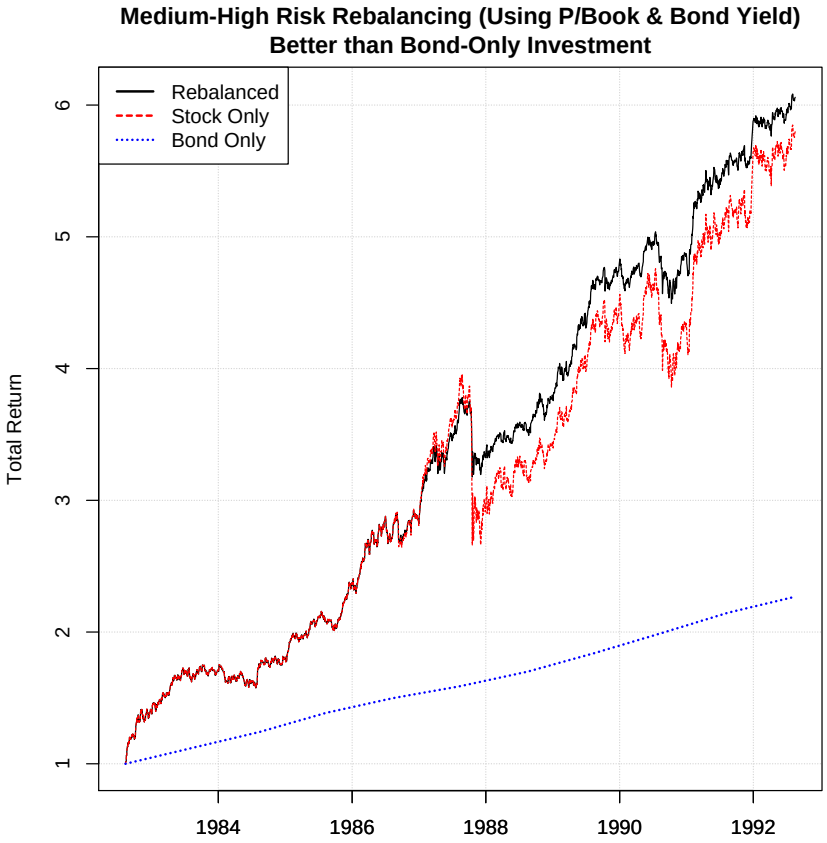

Figure 94: Total return of adaptive medium-high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

Medium-High Risk Rebalancing (Using P/Book & Bond Yield) Better than Bond-Only Investment											
Date	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
P/Book	0.92	1.41	1.37	1.51	1.95	2.54	1.88	2.38	2.20	2.46	2.75
Bond Yield	12.1%	10.8%	11.7%	8.1%	6.0%	6.9%	8.3%	8.2%	7.8%	5.6%	3.4%
Stock Weight	1	1	1	1	0.83	0.5	0.81	0.53	0.64	0.56	0.5
Rebalanced	1	1.65	1.75	2.08	2.82	3.76	3.53	4.63	4.80	5.54	6.06
Bond-Only	1	1.12	1.24	1.39	1.50	1.59	1.70	1.84	1.99	2.15	2.27
Stock-Only	1	1.65	1.75	2.08	2.82	3.92	3.17	4.33	4.31	5.16	5.80

Table 43: Total return of adaptive medium-high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 91.

Worse than Bond-Only Investment

Figure 95 shows an example of the rebalancing strategy performing worse than a bond-only investment. The starting date is March 3, 1999. Table 44 shows that for the first nine years the rebalancing strategy performed best, but in the last year from 2008 to 2009 the S&P 500 had a great loss of about (46%) and the rebalancing strategy had invested 0.67 (or 67%) of its portfolio in the S&P 500 so it experienced a significant loss of almost (30%) on the portfolio. For the entire ten-year period the total return of the rebalancing strategy was 1.03 (or 3% gain), while the total return was 1.40 (or 40% gain) for the bond-only investment and 0.66 (or 34% loss) for the stock-only investment.

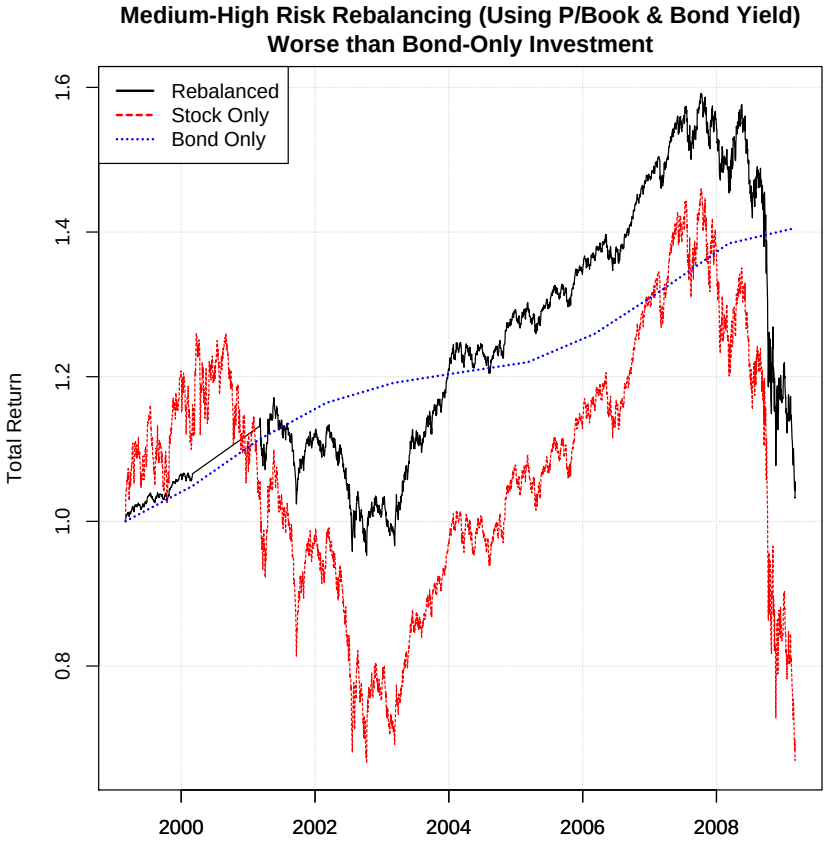

Figure 95: Total return of adaptive medium-high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

Medium-High Risk Rebalancing (Using P/Book & Bond Yield) Worse than Bond-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	4.55	4.79	3.64	3.43	2.54	3.09	2.89	2.75	2.74	2.44	1.50
Bond Yield	4.9%	6.2%	4.5%	2.4%	1.2%	1.2%	3.2%	4.8%	4.9%	1.6%	0.7%
Stock Weight	0.15	0	0.5	0.5	0.62	0.5	0.5	0.5	0.5	0.67	1
Rebalanced	1	1.07	1.13	1.12	0.99	1.25	1.30	1.36	1.47	1.47	1.03
Bond-Only	1	1.05	1.11	1.16	1.19	1.21	1.22	1.26	1.32	1.38	1.40
Stock-Only	1	1.16	1.04	0.97	0.72	1.01	1.09	1.16	1.28	1.22	0.66

Table 44: Total return of adaptive medium-high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 91.

Better than Stock-Only Investment

Figure 96 shows an example of the rebalancing strategy performing better than both bond-only and stock-only investments. The starting date is April 3, 2000 which is only about a year after the previous example in which the rebalancing strategy performed worse than a bond-only investment. Table 45 shows that for the ten-year period the total return of the rebalancing strategy was 1.68 (or 68% gain), while the total return was 1.35 (or 35% gain) for the bond-only investment and 0.95 (or 5% loss) for the stock-only investment.

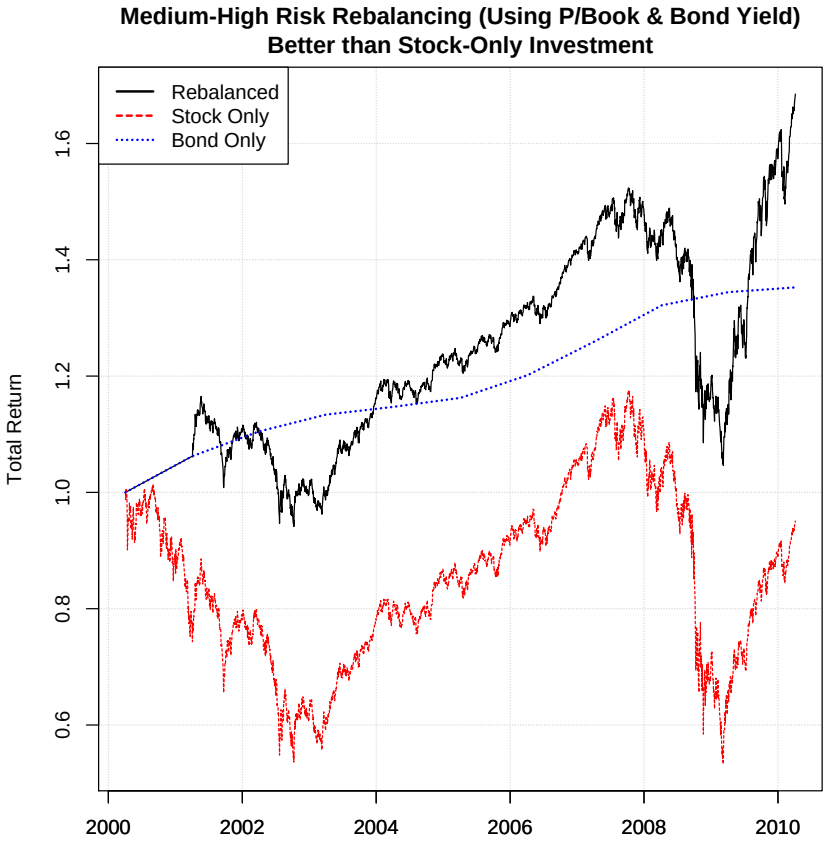

Figure 96: Total return of adaptive medium-high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

Medium-High Risk Rebalancing (Using P/Book & Bond Yield) Better than Stock-Only Investment											
Date	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
P/Book	5.08	3.19	3.38	2.66	3.05	2.76	2.80	2.83	2.59	1.80	2.22
Bond Yield	6.2%	4.0%	2.6%	1.2%	1.3%	3.3%	4.8%	4.9%	1.7%	0.6%	0.5%
Stock Weight	0	0.5	0.5	0.56	0.5	0.5	0.5	0.5	0.59	1	0.82
Rebalanced	1	1.06	1.10	1.00	1.19	1.23	1.33	1.44	1.45	1.13	1.68
Bond-Only	1	1.06	1.10	1.13	1.15	1.16	1.20	1.26	1.32	1.34	1.35
Stock-Only	1	0.75	0.77	0.61	0.81	0.85	0.96	1.07	1.04	0.64	0.95

Table 45: Total return of adaptive medium-high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 91.

Worse than Stock-Only Investment

Figure 97 shows an example of the rebalancing strategy performing worse than a stock-only investment. The starting date is March 21, 1990. Table 46 shows the stock-weight was slightly above 0.5 for the first eight years after which the P/Book for the S&P 500 increased substantially so the stock-weight decreased towards zero. For the ten-year period the total return of the rebalancing strategy was 2.93 (or 193% gain), while the total return was 1.71 (or 71% gain) for the bond-only investment and 5.59 (or 459% gain) for the stock-only investment.

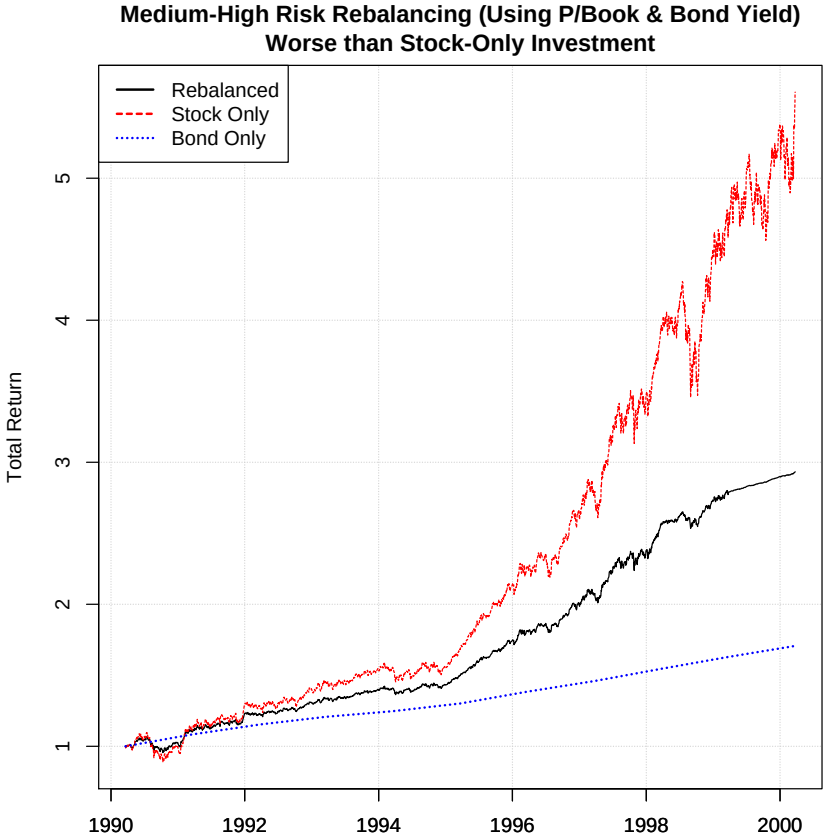

Figure 97: Total return of adaptive medium-high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments.

Medium-High Risk Rebalancing (Using P/Book & Bond Yield) Worse than Stock-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.29	2.38	2.61	2.61	2.61	2.49	3.01	3.34	4.37	4.75	5.15
Bond Yield	8.4%	6.4%	4.7%	3.3%	4.3%	6.4%	5.4%	5.9%	5.4%	4.8%	6.3%
Stock Weight	0.58	0.58	0.5	0.53	0.51	0.52	0.5	0.5	0.24	0.03	0
Rebalanced	1	1.10	1.23	1.33	1.41	1.50	1.81	2.07	2.57	2.79	2.93
Bond-Only	1	1.08	1.15	1.21	1.25	1.30	1.38	1.46	1.55	1.63	1.71
Stock-Only	1	1.11	1.28	1.44	1.55	1.68	2.25	2.79	3.95	4.68	5.59

Table 46: Total return of adaptive medium-high-risk rebalancing (using P/Book & Bond Yield) compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 91.

14.4.3. Comparison to Medium-Risk Strategy

This section compares the performance of the medium and medium-high risk strategies.

Annualized Return

Table 42 shows the annualized returns for the medium-high risk strategy and Table 32 shows it for the medium-risk strategy.

The medium-high risk strategy had median annualized ranging between 10.0-11.3% depending on the investment duration. This was much higher than the medium-risk strategy whose median annualized return ranged between 5.8-6.9%.

The mean annualized return was also higher for the medium-high risk strategy where it was 11.5% for one-year investment periods and decreased to 10.4% for ten-year investment periods. For the medium-risk strategy the mean annualized return was 10.0% for one-year investment periods and decreased gradually to 8.5% for ten-year investment periods.

The greatest one-year gain was 71.8% for both strategies. For some of the longer investment periods the medium-risk strategy was slightly better, for example the greatest annualized return was 20.1% for ten-year investment periods while it was only 19.7% for the medium-high risk strategy.

The greatest one-year loss was (15.3%) for the medium-risk strategy while it was (30.0%) for the medium-high risk strategy. The medium-risk strategy did not have losses for investment periods of six years or more, while the medium-high risk strategy had probabilities of loss that were less than 0.01 (or 1%) for longer investment periods. The lowest annualized return was 2.3% for ten-year investment periods using the medium-risk strategy thus giving a compounded gain of 26% over ten years. For the medium-high risk strategy the lowest annualized return was 0.3% thus giving a compounded gain of 3% over ten years.

Probability of Loss

The medium-risk strategy had probability of loss about 0.05 (or 5%) for one-year investment periods, and zero probability of loss for investment periods of six years or more. The medium-high risk strategy had a probability of loss about 0.15 (or 15%) for one-year investment periods and this gradually decreased to zero for investment periods of ten years.

Probability of Under-Performing Bond-Only and Stock-Only Investments

The medium-high risk strategy had a much higher probability of under-performing bond-only investments. For one-year investment periods the probability was 0.24 (or 24%) while it was only 0.14 (or 14%) for the medium-risk strategy. The probabilities decreased for longer investment periods so the medium-high risk strategy had probability 0.05 (or 5%) for ten-year investment periods, while the medium-risk strategy had probability 0.03 (or 3%) for investment periods of five years or more.

The medium-high and medium-risk strategies had quite similar probabilities of under-performing stock-only investments, approximately ranging between 0.5-0.7 (or 50-70%) depending on the investment duration.

14.4.4. Comparison to P/Book Strategy

For easier reference, the strategy that adapts the stock-weight using both the P/Book and the bond yield is referred to as the 'bond-yield strategy', while the strategy that uses only the P/Book is referred to as the 'P/Book strategy'. This section compares the performance of the two strategies.

Stock Weight

The stock-weight for the bond-yield strategy is shown in Figure 91 for the period 1978-2013 and it is shown for the P/Book strategy in Figure 63. The two strategies have similar tendencies with stock-weights near zero around year 2000 and high stock-weights around year 1980 and 2010, but the bond-yield strategy has more extreme stock-weights that are often closer to one.

Annualized Return

Table 42 shows the annualized returns for the bond-yield strategy and Table 22 shows it for the P/Book strategy. The median annualized returns for the bond-yield strategy ranged between 10.0-11.3% depending on the investment duration, while the range was 9.7-11.4% for the P/Book strategy.

The mean annualized returns were slightly higher for the bond-yield strategy. For one-year investment periods the mean annualized return was 11.5% and this decreased to 10.4% for ten-year investment periods. Compare this to the P/Book strategy whose mean annualized return was 11.1% for one-year investment periods and 10.2% for ten-year investment periods.

The greatest one-year loss was (30.0%) for the bond-yield strategy while it was slightly better at (28.6%) for the P/Book strategy. The greatest one-year gain was 71.8% for the bond-yield strategy while it was slightly worse at 65.5% for the P/Book strategy. For longer investment periods the bond-yield strategy experienced slightly greater losses than the P/Book strategy, for example for five-year investment periods the greatest annualized loss was (3.8%) for the bond-yield strategy while it was only (3.5%) for the P/Book strategy. Conversely, the bond-yield strategy experienced slightly higher gains than the P/Book strategy for some investment periods, for example for ten-year investment periods the greatest annualized return of the bond-yield strategy was 19.7% while it was 18.8% for the P/Book strategy.

Probability of Loss

The bond-yield strategy had probability of loss 0.15 (or 15%) for one-year investment which decreased to zero probability of loss for ten-year investment periods. The P/Book strategy had slightly lower probability of loss ranging from 0.14 (or 14%) for one-year investment periods to zero for ten-year investment periods.

Probability of Under-Performing Bond-Only and Stock-Only Investments

The probability of under-performing bond-only investments was slightly higher for the bond-yield strategy ranging from 0.24 to 0.05 depending on investment period, while it ranged from 0.23 to 0.04 for the P/Book strategy.

The probability of the bond-yield strategy under-performing stock-only investments was significantly lower for the bond-yield strategy where it ranged between 0.49-0.69 (or 49-69%) depending on investment duration, while the P/Book strategy ranged between 0.58-0.80 (or 58-80%).

14.5. Summary

This section presented adaptive rebalancing strategies which use both the P/Book of the S&P 500 and the yield on US government bonds to allocate the portfolio between the S&P 500 and government bonds. These strategies perform slightly differently compared to the strategies from section 13 that only use the P/Book to determine the portfolio allocation.

The strategies which also use the bond yield in the stock-weight formulas have stock-weights that are generally more extreme and closer to either zero or one. So these strategies were more frequently either fully invested in the S&P 500 or fully invested in government bonds.

The median annualized returns for the two kinds of strategies were typically quite similar but the mean annualized returns were slightly higher for the strategies that also employ the government bond yield in making the portfolio allocation, because those strategies were more invested in the S&P 500 during the periods of its most extreme gains. But those strategies would also sometimes perform worse.

The low-risk strategy that used both the P/Book and bond yield in determining the portfolio allocation had significant advantages in terms of lower probability of loss and lower probability of under-performing bond-only investments when compared to the low-risk strategy that only used the P/Book in its stock-weight formula. However, there were no clear and consistent advantages of this kind for the medium- and high-risk strategies, in fact, there were sometimes disadvantages.

The probability of under-performing stock-only investments was significantly lower for the strategies that used both the P/Book and bond yield in determining the portfolio allocation and this advantage was especially clear for the high-risk strategy.

Overall, although there are some advantages to using the government bond yield in determining the portfolio allocation, the advantages are not consistent, they are not always significant, and sometimes there are also disadvantages.

15. Stop-Loss

The previous sections presented investment strategies that historically have had limited losses. But the strategies would sometimes be fully invested in the S&P 500 and hence fully exposed to its potential losses. Historically those losses had been limited when the allocation was done according to the stock-weight formulas, but this does not guarantee that the losses will also be limited in the future.

To ensure that losses are limited in the future, it is necessary to sell the investment in the S&P 500 when the price decreases below a so-called stop-price. However, there are several practical problems in doing this, relating to the selection of the stop-price and deciding when to reinvest after having sold the stock.

The following sections study stop-loss strategies based on annual rebalancing. At the beginning of each year the stock-weight is determined from a formula much like in the previous sections. According to the stock-weight, the portfolio is allocated between the S&P 500 and US government bonds with one-year maturity. The bond investment is held for one year but the investment in the S&P 500 may be sold during the year if the price becomes lower than the stop-price, which is selected as a percentage below the purchase price at the beginning of the year. If the price of the S&P 500 increases again, then the stock is repurchased at the same stop-price where the stock was previously sold. After each year the portfolio is rebalanced using a new stock-weight and a new stop-price is determined. So the repurchase of the S&P 500 is either done at the stop-price or after one year when the portfolio is rebalanced. The stock-weight does not change during a year even though the S&P 500 may be bought and sold a number of times. It is the stock-weight from the beginning of the year that is always used.

Hypothetically this ensures that the annual loss will not exceed the stop-price. However, it may not be possible to sell and repurchase the S&P 500 precisely at the stop-price, so there may be some 'frictional cost' at each sell and buy. If the stock crosses the stop-price numerous times during a year then the frictional cost at each sell and buy may cause significantly lower annual returns than a buy-and-hold strategy, or possibly even losses. Although some friction is taken into account in the performance statistics for these stop-loss strategies, it remains that an essential factor in successfully executing these strategies is to sell and repurchase close to the stop-price.

15.1. Stop-Loss without Penalty

This stop-loss strategy has the stop-price set at the purchase price of the S&P 500 and it is assumed that the selling and repurchasing can be done exactly at this stop-price without any frictional costs. Although this is unlikely to be possible in reality, it gives a benchmark case for use in performance comparisons.

In the following, the strategy is sometimes referred to as a rebalancing strategy and sometimes as a stop-loss strategy because it uses both annual rebalancing and a stop-loss during the year.

15.1.1. Stock Weight

The stock-weight determines the part of the portfolio to invest in the S&P 500 and the remainder is invested in US government bonds. The stock-weight remains the same throughout a year even though the stock may be sold and repurchased several times as the stop-price is crossed.

The stock-weight formula is:

$$Stock\ Weight = Limit(6 - 1.05 \times P/Book - 5 \times Bond\ Yield)$$

Eq. 15-1

The P/Book is for the S&P 500 index and the bond yield is for US government bonds with one-year maturity.

The limit-function simply means that the stock-weight is limited between zero and one. The stock-weight varies with both the P/Book and bond yield so its boundaries depend on both. For example, holding the bond yield fixed at 1.2% means the stock-weight is zero when the P/Book is greater than 5.65, and the stock-weight is one when the P/Book is less than about 4.70. If instead the bond yield is 7% then the stock-weight is zero when the P/Book is greater than about 5.38, and the stock-weight is one when the P/Book is less than about 4.43.

Figure 98 shows the stock-weight for each day during the period 1978-2013 calculated using Eq. 15-1 with the P/Book data from Figure 2 and the government bond yields from Figure 3. All of the days in this period had some part of the portfolio invested in the S&P 500. Note that the stock-weight is shown for each day but the investment strategy uses annual rebalancing.

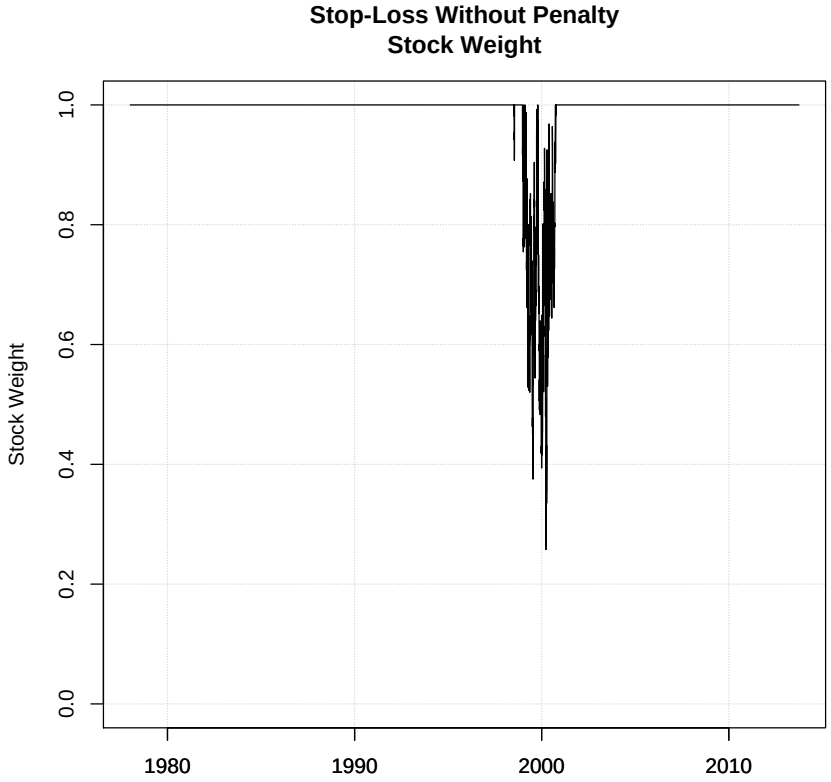

Figure 98: Stock-weight for stop-loss strategy without penalty.

15.1.2. Annualized Return

Figure 99 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 47. For one-year investment periods the mean annualized return was 15.5% which gradually decreased to 14.9% for ten-year investment periods. The median annualized return was 13.8% for one-year investment periods and this gradually increased to 15.5% for ten-year investment periods.

All investment periods had zero probability of loss because the stop-loss is assumed to occur precisely at the purchase price from the beginning of each year. The greatest gain was 71.8% for one-year investment periods and the greatest annualized return was 22.3% for ten-year investment periods thus giving a compounded gain of 649% over ten years.

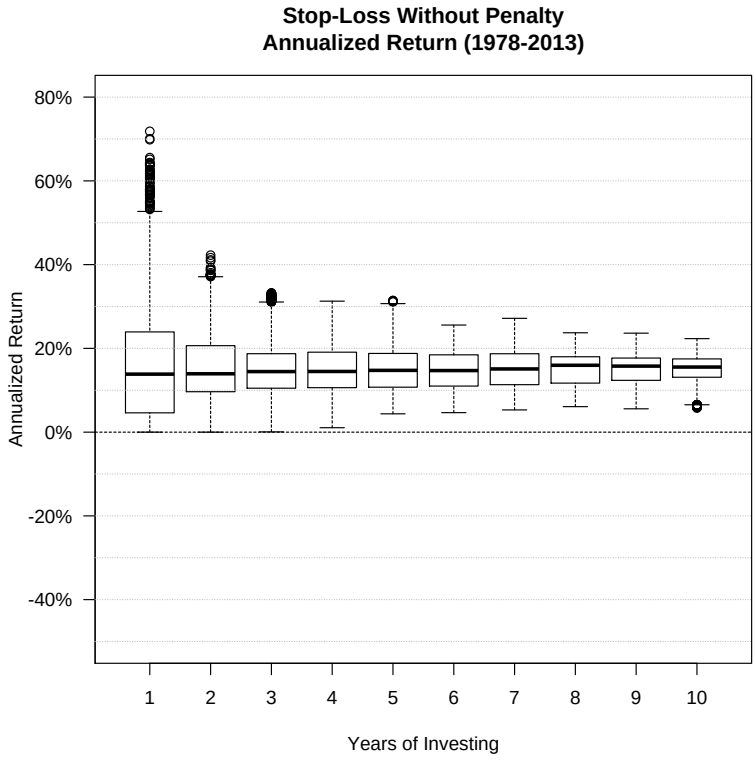

Figure 99: Annualized return statistics for stop-loss strategy without penalty.

Annualized Return for Stop-Loss Without Penalty										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	0.0%	4.6%	13.8%	15.5%	23.9%	71.8%	12.7%	0	0.25	0.02
2	0.0%	9.6%	13.9%	15.1%	20.6%	42.3%	8.3%	0	0.12	0.03
3	0.1%	10.5%	14.5%	15.0%	18.7%	33.2%	6.8%	0	0.06	0.03
4	1.1%	10.6%	14.5%	15.0%	19.1%	31.3%	6.1%	0	0.02	0.03
5	4.4%	10.7%	14.8%	15.0%	18.8%	31.4%	5.4%	0	0	0.03
6	4.7%	11.0%	14.7%	14.9%	18.5%	25.6%	4.7%	0	0	0.03
7	5.3%	11.3%	15.1%	15.0%	18.7%	27.2%	4.3%	0	0	0.03
8	6.1%	11.7%	16.0%	15.0%	18.0%	23.7%	4.0%	0	0	0.03
9	5.6%	12.4%	15.8%	14.9%	17.7%	23.6%	3.7%	0	0	0.03
10	5.8%	13.1%	15.5%	14.9%	17.5%	22.3%	3.4%	0	0	0.03

Table 47: Annualized return statistics for stop-loss strategy without penalty. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

15.1.3. Comparison to Bond-Only & Stock-Only Investments

Figure 100 shows the performance of the stop-loss strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the stop-loss strategy performed about 3 times better than the stock-only investment because the stop-loss strategy avoided the great losses of the S&P 500 during the years 2000-2003 and 2008-2009.

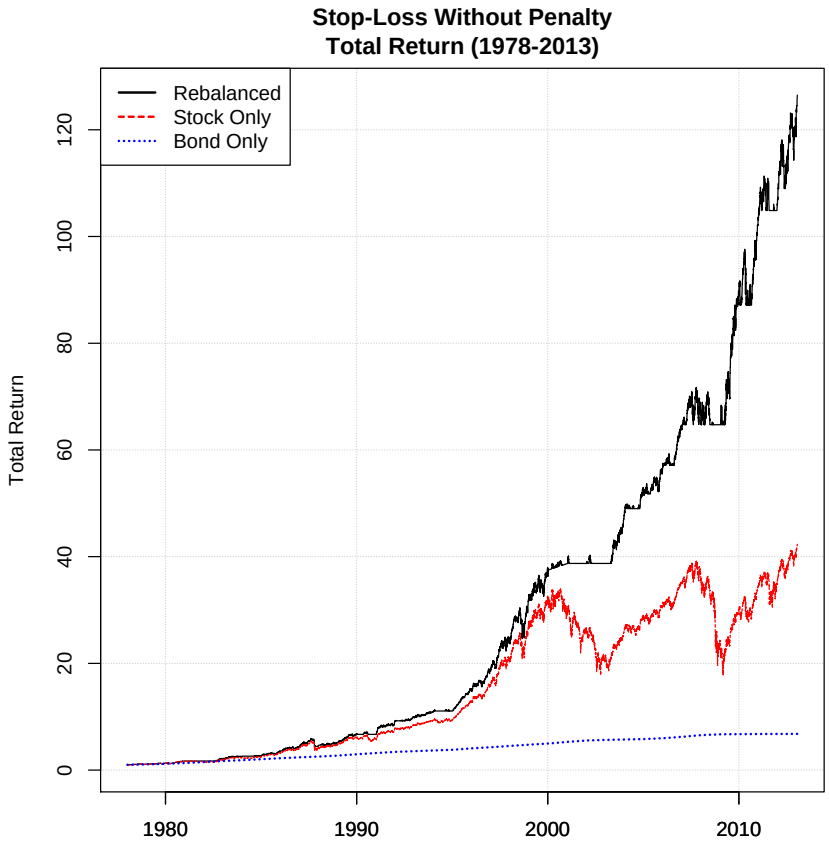

Figure 100: Total return of stop-loss strategy without penalty compared to stock-only and bond-only investments for the period 1978-2013.

Probability of Under-Performance

Table 47 shows the probabilities of the rebalanced portfolio under-performing bond-only investments. For one-year investment periods the probability was 0.25 (or 25%), while the probability was zero for investment periods of five years or more.

Table 47 also shows the probabilities of the rebalanced portfolio under-performing stock-only investments, which was 0.02-0.03 (or 2-3%) for all investment periods.

Better than Stock-Only Investment

Figure 101 shows an example of the stop-loss (aka. rebalancing) strategy performing better than both bond-only and stock-only investments. The starting date is March 3, 2000. Table 48 shows that the stock-weight was one for all years except the first where it was 0.67 because the P/Book was very high at 4.79. For the ten-year period the total return of the stop-loss strategy was 3.15 (or 215% gain), while the total return was 1.35 (or 35% gain) for the bond-only investment and 0.98 (or 2% loss) for the stock-only investment.

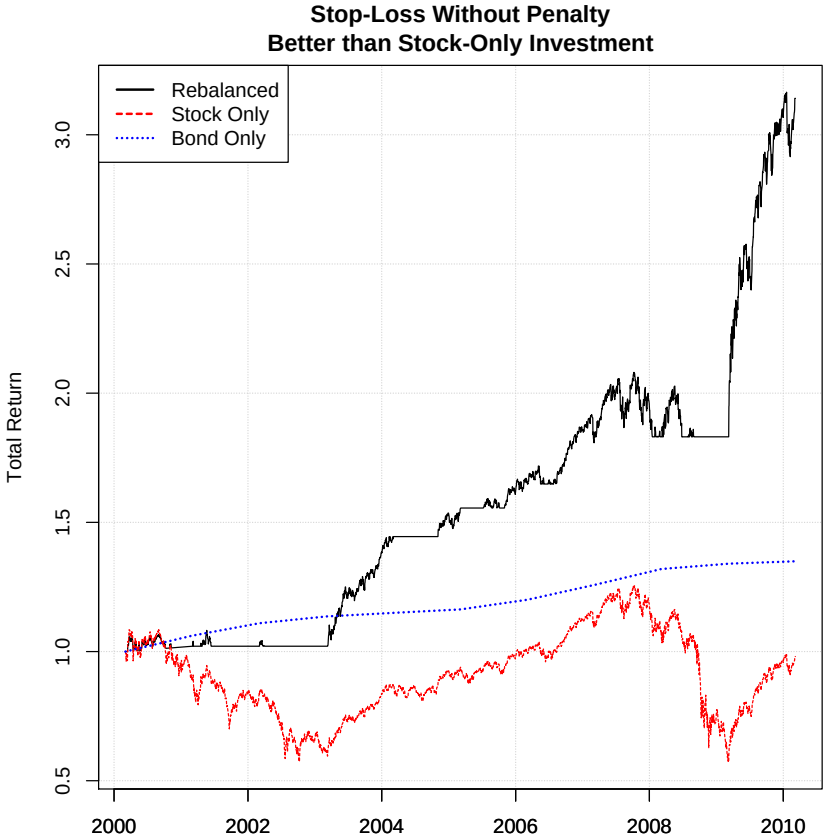

Figure 101: Total return of stop-loss strategy without penalty compared to stock-only and bond-only investments.

Stop-Loss Without Penalty Better than Stock-Only Investment											
Date	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
P/Book	4.79	3.64	3.43	2.54	3.09	2.89	2.75	2.74	2.44	1.50	2.17
Bond Yield	6.2%	4.5%	2.4%	1.2%	1.2%	3.2%	4.8%	4.9%	1.6%	0.7%	0.4%
Stock Weight	0.67	1	1	1	1	1	1	1	1	1	1
Rebalanced	1	1.02	1.02	1.02	1.44	1.56	1.65	1.83	1.83	1.83	3.15
Bond-Only	1	1.06	1.11	1.14	1.15	1.16	1.20	1.26	1.32	1.34	1.35
Stock-Only	1	0.89	0.84	0.62	0.87	0.94	1.00	1.11	1.05	0.57	0.98

Table 48: Total return of stop-loss strategy without penalty compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 98.

Worse than Stock-Only Investment

Figure 102 shows an example of the stop-loss strategy performing worse than a stock-only investment. The starting date is April 3, 1990. Table 49 shows the stock-weight was one for the first nine years after which the P/Book increased so much that the stock-weight decreased to 0.58, thus causing almost half the portfolio to be invested in government bonds rather than exclusively in the S&P 500. For the ten-year period the total return of the stop-loss strategy was 5.29 (or 429% gain), while the total return was 1.71 (or 71% gain) for the bond-only investment and 5.46 (or 446% gain) for the stock-only investment.

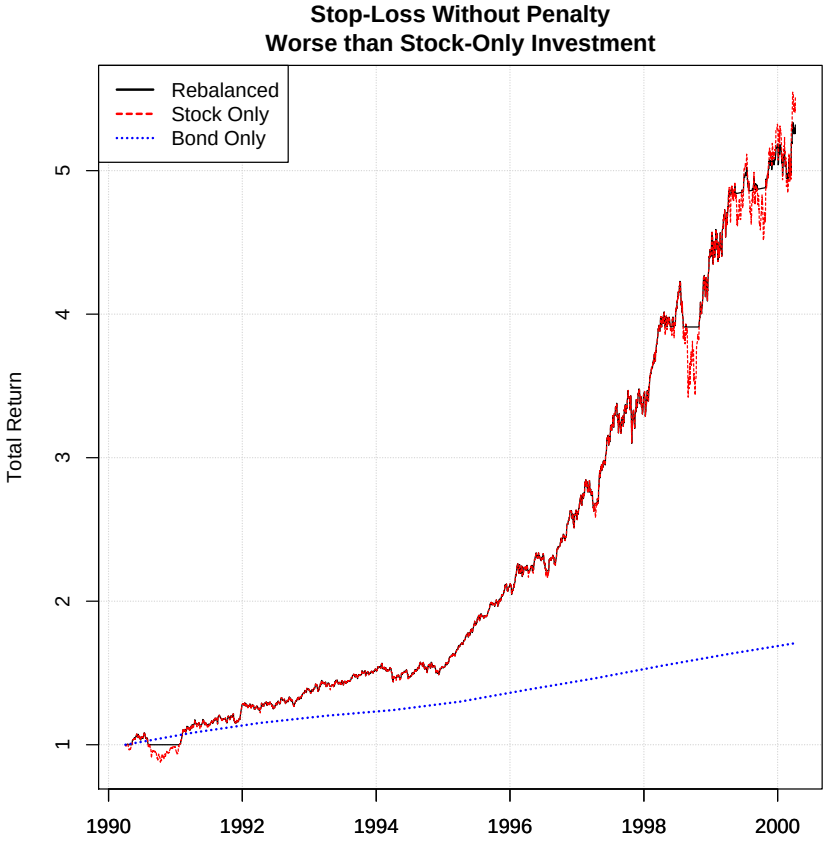

Figure 102: Total return of stop-loss strategy without penalty compared to stock-only and bond-only investments.

Stop-Loss Without Penalty Worse than Stock-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.31	2.45	2.57	2.54	2.49	2.52	2.97	3.24	4.36	4.94	5.05
Bond Yield	8.3%	6.3%	4.4%	3.3%	4.7%	6.4%	5.7%	6.0%	5.3%	4.6%	6.1%
Stock Weight	1	1	1	1	1	1	1	1	1	0.58	0.39
Rebalanced	1	1.14	1.24	1.41	1.47	1.70	2.21	2.68	3.91	4.83	5.29
Bond-Only	1	1.08	1.15	1.20	1.24	1.30	1.38	1.46	1.55	1.63	1.71
Stock-Only	1	1.14	1.24	1.41	1.47	1.70	2.21	2.68	3.91	4.83	5.46

Table 49: Total return of stop-loss strategy without penalty compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 98.

15.2. Penalty

The stop-loss strategy in the previous section had its stop-price set at the purchase price and it was assumed that the S&P 500 could be sold and repurchased at exactly that price. This meant there would never be any losses from an investment in the S&P 500. In reality, however, there are frictional costs in executing a stop-loss strategy. Although trading fees and commissions have become negligible today, it may not be possible to sell and repurchase stocks at exactly the stop-price due to price-jumps especially between trading days. This frictional cost may cause the stop-loss strategy to under-perform a buy-and-hold investment in the S&P 500.

15.2.1. Penalty per Transaction

A simple way of modelling such frictional costs is to penalize each buy and sell transaction whenever the stock-price crosses the stop-price. Daily price data for the S&P 500 is used here so the actual number of stop-price crossings during intra-day trading is likely much greater than the numbers used here. This can be taken into account by increasing the penalty per transaction.

During 2014 the daily trading volume for the SPY ETF, which tracks the S&P 500, was about 100 million shares on average, amounting to about USD 20 billion at a share-price of USD 200 (which corresponds to a price of USD 2000 for the S&P 500 index). So the SPY ETF can accommodate very large transactions on a daily basis. The question is how close to a desired stop-price a transaction can be expected to be.

During the period between January 1957 and January 2015, the probability was about 0.52 that the S&P 500 index increased or decreased less than 0.5% in one day, that is, slightly more than half of the days in this 57 year period had daily price returns less than ±0.5%. The probability was about 0.79 for daily price returns less than ±1%, and the probability was about 0.95 for daily price returns less than ±2%.

Considering the very high volume of SPY shares and that the daily price returns were less than ±0.5% for about half the days during the past 57 years, it seems reasonable to assume that the average frictional costs could be kept below 0.5% per transaction, which is used here as the penalty per transaction.

15.2.2. Penalty Formula

The penalty is applied by multiplying a penalty factor with the annual return from a stop-loss strategy. The penalty factor is compounding in the number of stop-price crossings. If the penalty per transaction is a loss of (0.5%) then the penalty factor is calculated as follows:

$$Penalty = (1 - 0.5\%)^{Number\ of\ Stop\ Price\ Crossings} = 0.995^{Number\ of\ Stop\ Price\ Crossings}$$

Eq. 15-2

For example, if a year has 15 stop-price crossings then the penalty factor is:

$$Penalty = 0.995^{Number\ of\ Stop\ Price\ Crossings} = 0.995^{15} \approx 0.928$$

So if the stop-loss strategy had a return of 5%, then the penalized return would be a loss of (2.6%):

$$Penalized\ Return = Penalty \times (1 + Return) - 1 = 0.928 \times (1 + 5\%) - 1 \approx (2.6\%)$$

If there were only 4 stop-price crossings during the year then the penalty factor would have been 0.98 and hence the penalized return would instead have been a gain of 2.9%.

15.2.3. Number of Stop-Price Crossings

Figure 103 shows histograms for the number of stop-price crossings when the stop-price was either equal to the purchase price or below the purchase price by (2%), (5%) or (15%). These histograms are different from each other because they really show conditional probability distributions. In the first case where the stop-price equals the purchase-price we are merely counting how many times the purchase-price is crossed in the following year. But in the other cases the S&P 500 must first decrease to the stop-price which is below the purchase price, and then we count the number of times the stop-price is crossed during the year.

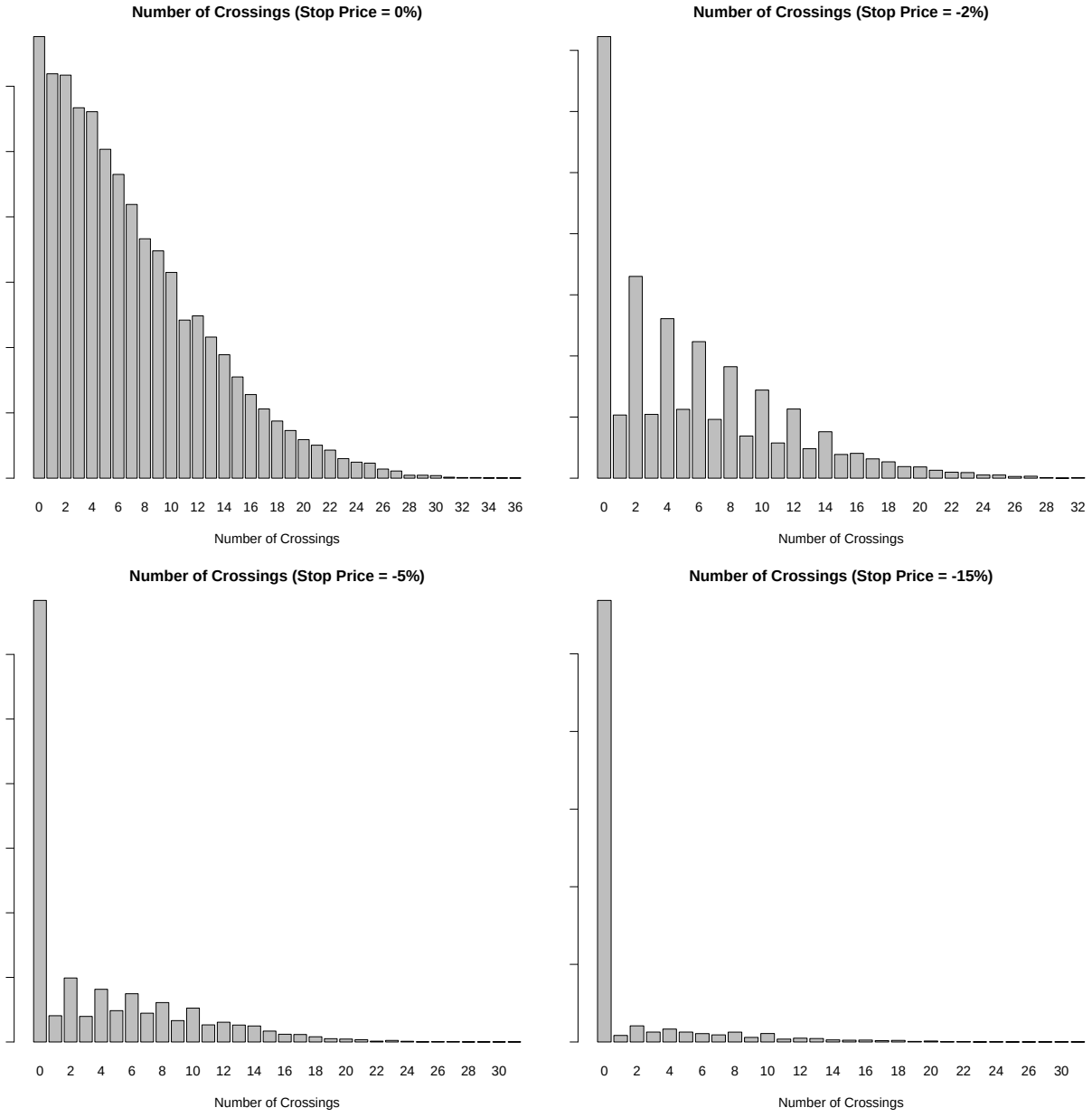

Figure 103: Histograms for the number of annual stop-price crossings for the S&P 500 when the stop-price is (0%), (2%), (5%) or (15%) below the purchase price. Data is for the period 1957-2014.

Table 50 shows in tabular form the Cumulative Distribution Functions (CDF) for the number of stop-price crossings that correspond to the histograms in Figure 103. For example, the probability was 0.09 (or 9%) that there were zero crossings of the purchase price during the following year, while the probability was 0.49 (or 49%) that there were 5 or fewer crossings of the purchase price.

Stop Price	Probability of Number of Crossings Being Less Than or Equal To													
	0	1	2	3	4	5	6	7	8	9	10	15	20	25
(0%)	0.09	0.18	0.27	0.34	0.42	0.49	0.56	0.62	0.67	0.72	0.76	0.91	0.97	0.994
(2%)	0.25	0.29	0.40	0.44	0.53	0.57	0.64	0.68	0.74	0.77	0.82	0.93	0.98	0.997
(5%)	0.47	0.50	0.57	0.60	0.65	0.69	0.74	0.77	0.82	0.84	0.88	0.96	0.993	0.999
(15%)	0.79	0.80	0.83	0.85	0.87	0.89	0.91	0.92	0.94	0.95	0.96	0.99	0.998	0.999

Table 50: Probability of the number of annual stop-price crossings being less than or equal to different numbers. This essentially shows the Cumulative Distribution Function (CDF). The S&P 500 data is for the period 1957-2014.

For a stop-price that was (2%) lower than the purchase price, there was a probability of 0.25 (or 25%) that there were zero crossings of the stop-price during the following year. The probability was 0.57 (or 57%) that there were 5 or fewer crossings of the stop-price.

For a stop-price that was (5%) lower than the purchase price, there was a probability of 0.47 (or 47%) that there were zero crossings of the stop-price during the following year. The probability was 0.69 (or 69%) that there were 5 or fewer crossings of the stop-price.

For a stop-price that was (15%) lower than the purchase price, there was a probability of 0.79 (or 79%) that there were zero crossings of the stop-price during the following year. The probability was 0.89 (or 89%) that there were 5 or fewer crossings of the stop-price.

So as the stop-price becomes lower relative to the purchase price, the number of stop-price crossings is reduced. This means the frictional cost of a stop-loss strategy can be lowered by decreasing the stop-price relative to the purchase price, because fewer trades will likely have to be executed. However, a lower stop-price also allows greater losses to occur from a decline in the price of the S&P 500. So the choice becomes a balance between direct losses from the S&P 500 declining to a lower stop-price, or indirect losses from the frictional costs of repeatedly trading at a higher stop-price. The conclusion is that the closer to a desired stop-price an investor can successfully execute trades, the higher the stop-price should be.

15.3. Stop-Loss with Penalty

This stop-loss strategy has the stop-price set at the purchase price of the S&P 500, and the selling and repurchasing has frictional costs as described in section 15.2.

15.3.1. Stock Weight

The stock-weight determines the part of the portfolio to invest in the S&P 500 index and the remainder is invested in US government bonds. The stock-weight remains the same throughout a year even though the stock may be sold and repurchased several times as the stop-price is crossed. The stock-weight formula is:

$$\textit{Stock Weight} = \textit{Limit}(6 - 1.17 \times \textit{P/Book} - 5 \times \textit{Bond Yield})$$

Eq. 15-3

The P/Book is for the S&P 500 index and the bond yield is for US government bonds with one-year maturity.

The limit-function simply means that the stock-weight is limited between zero and one. The stock-weight varies with both the P/Book and bond yield so its boundaries depend on both. For example, holding the bond yield fixed at 1.2% means the stock-weight is zero when the P/Book is greater than 5.08, and the stock-weight is one when the P/Book is less than about 4.22. If instead the bond yield is 7% then the stock-weight is zero when the P/Book is greater than about 4.83, and the stock-weight is one when the P/Book is less than about 3.97.

Figure 104 shows the stock-weight for each day during the period 1978-2013 calculated using Eq. 15-3 with the P/Book data from Figure 2 and the government bond yields from Figure 3. Almost 99% of the days in this period had some part of the portfolio invested in the S&P 500 index. Note that the stock-weight is shown for each day but the investment strategy uses annual rebalancing.

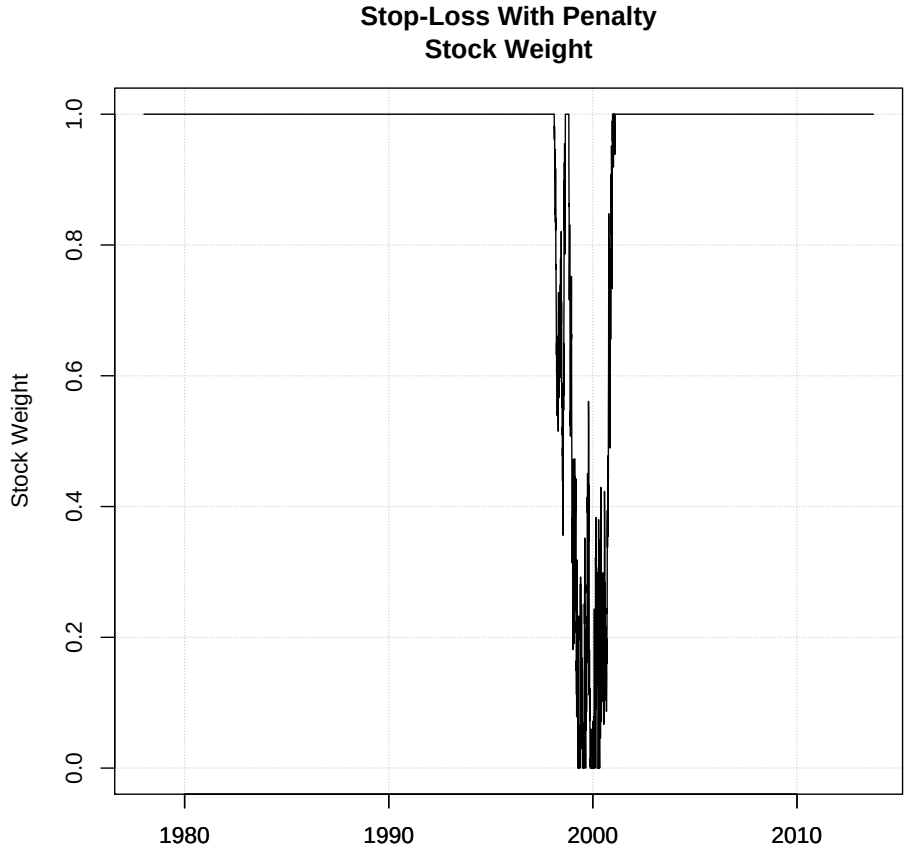

Figure 104: Stock-weight for stop-loss strategy with penalty.

15.3.2. Annualized Return

Figure 105 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 51.

For one-year investment periods the mean annualized return was 11.5% which gradually decreased to 10.9% for ten-year investment periods. The median annualized return was 9.5% for one-year investment periods and this gradually increased to 11.7% for ten-year investment periods.

For one-year investment periods the probability of loss was 0.25 (or 25%) which gradually decreased to zero for investment periods of six years or more. The greatest one-year loss was (14.0%) and the lowest

annualized return was 1.3% for ten-year investment periods thus giving a compounded gain of about 14% over ten years. The greatest one-year gain was 71.8% and the greatest annualized return was 19.3% for ten-year investment periods thus giving a compounded gain of 484% over ten years.

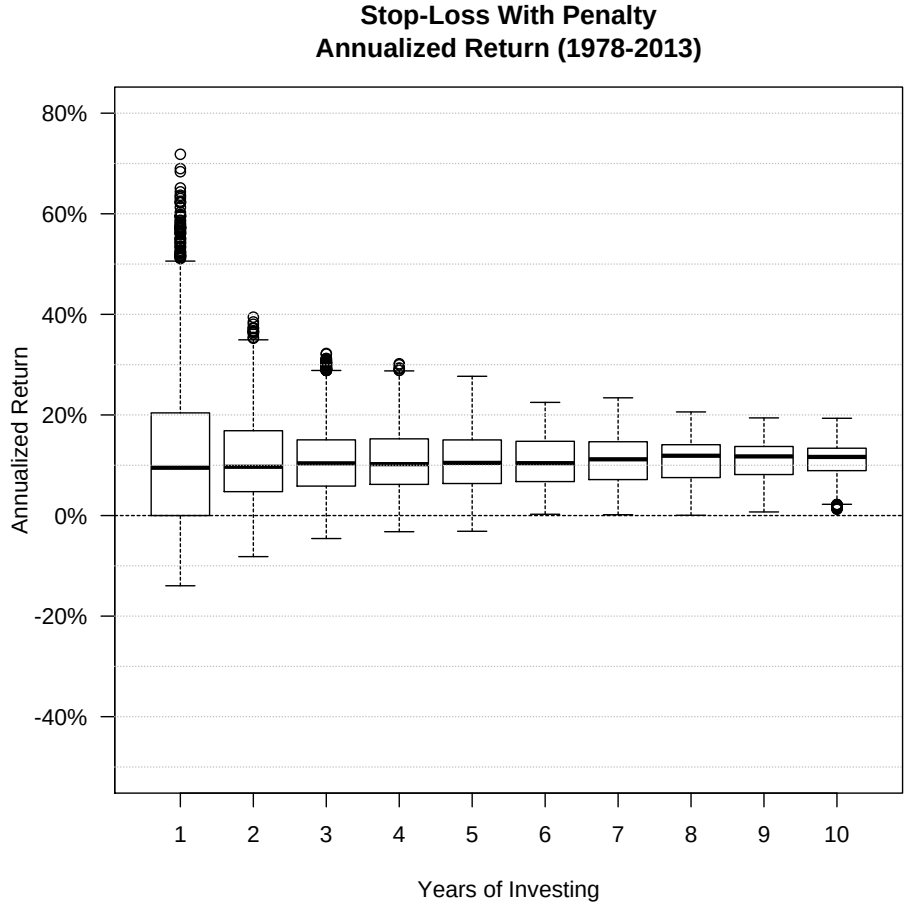

Figure 105: Annualized return statistics for stop-loss strategy with penalty.

Annualized Return for Stop-Loss With Penalty										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(14.0%)	0.0%	9.5%	11.5%	20.4%	71.8%	13.6%	0.25	0.38	0.77
2	(8.2%)	4.8%	9.7%	11.1%	16.9%	39.4%	8.7%	0.08	0.30	0.75
3	(4.6%)	5.9%	10.4%	11.0%	15.1%	32.2%	7.1%	0.03	0.25	0.69
4	(3.2%)	6.2%	10.2%	11.0%	15.3%	30.2%	6.2%	0.01	0.19	0.64
5	(3.1%)	6.4%	10.5%	10.9%	15.0%	27.7%	5.6%	0.005	0.12	0.60
6	0.3%	6.8%	10.4%	10.9%	14.8%	22.5%	4.8%	0	0.06	0.58
7	0.2%	7.2%	11.2%	10.9%	14.7%	23.4%	4.5%	0	0.05	0.57
8	0.1%	7.5%	11.9%	11.0%	14.1%	20.6%	4.1%	0	0.04	0.57
9	0.7%	8.2%	11.8%	11.0%	13.7%	19.4%	3.7%	0	0.03	0.57
10	1.3%	8.9%	11.7%	10.9%	13.4%	19.3%	3.4%	0	0.02	0.57

Table 51: Annualized return statistics for stop-loss strategy with penalty. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

15.3.3. Comparison to Bond-Only & Stock-Only Investments

Figure 106 shows the performance of the stop-loss (aka. rebalancing) strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the stop-loss strategy performed a little worse than the stock-only investment. In particular, the stop-loss strategy avoided the peak and subsequent decline of the S&P 500 during the years 2000-2003 and again during 2004-2009. But this long-term performance is misleading as the mutual performance depends on the starting date and investment duration, as shown below.

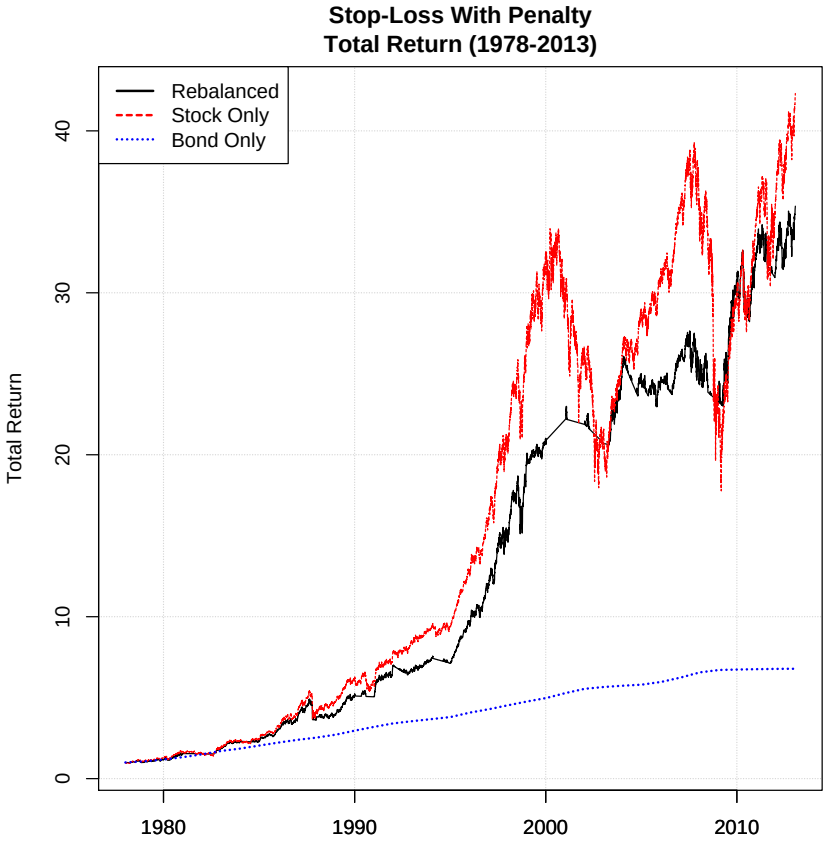

Figure 106: Total return of stop-loss strategy with penalty compared to stock-only and bond-only investments for the period 1978-2013.

Probability of Under-Performance

Table 51 shows the probabilities of the stop-loss strategy under-performing bond-only investments. For one-year investment periods the probability was 0.38 (or 38%) and the probability gradually declined to 0.02 (or 2%) for ten-year investment periods.

Table 51 also shows the probabilities of the stop-loss strategy under-performing stock-only investments. For one-year investment periods the probability was 0.77 (or 77%) and this gradually decreased to 0.57 (or 57%) for ten-year investment periods.

Better than Bond-Only Investment

Figure 107 shows an example of the stop-loss strategy performing better than a bond-only investment. The investment period starts November 7, 1990. Table 52 shows that the stock-weight was one until year 1998 when the P/Book for the S&P 500 had increased to 4.23 so the stock-weight was only 0.83. In 1999 the P/Book was 4.89 so the stock-weight was almost zero. For the entire ten-year period the stop-loss (aka. rebalancing) strategy had a total return of 5.03 (or 403% gain), the bond-only investment had a total return of 1.66 (or 66% gain), and the stock-only investment had a total return of 5.46 (or 446% gain).

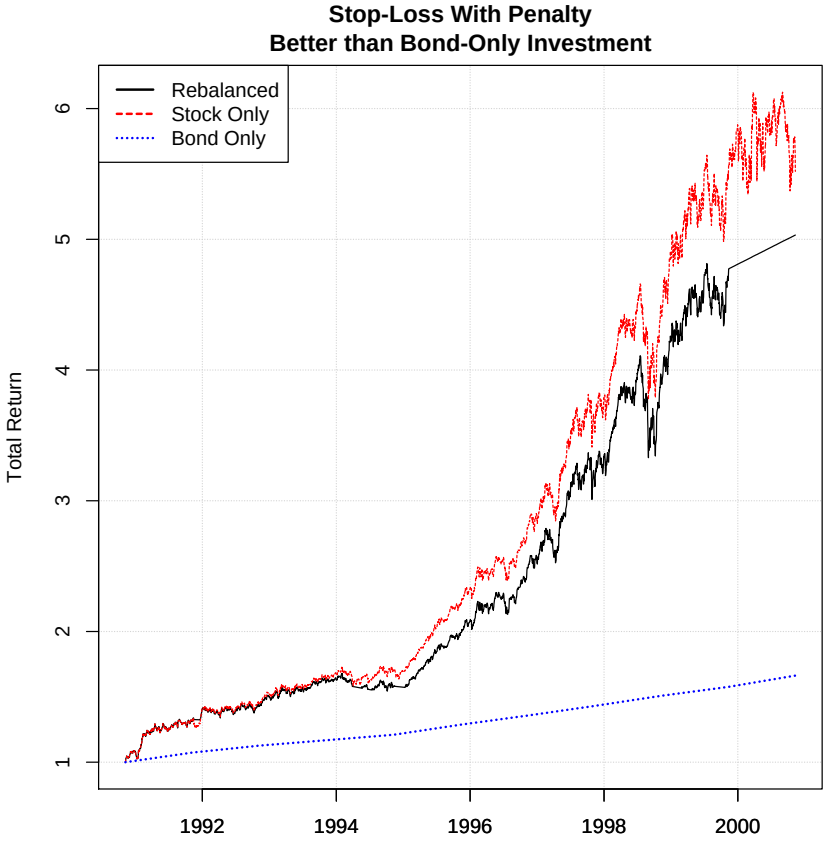

Figure 107: Total return of stop-loss strategy with penalty compared to stock-only and bond-only investments.

Stop-Loss With Penalty Better than Bond-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.01	2.49	2.77	2.57	2.45	2.79	3.16	3.65	4.23	4.89	4.19
Bond Yield	7.4%	5.0%	3.7%	3.5%	6.4%	5.4%	5.4%	5.4%	4.5%	5.5%	6.1%
Stock Weight	1	1	1	1	1	1	1	1	0.83	0.002	0.79
Rebalanced	1	1.33	1.43	1.62	1.58	1.99	2.49	3.11	3.89	4.77	5.03
Bond-Only	1	1.07	1.13	1.17	1.21	1.29	1.36	1.43	1.51	1.58	1.66
Stock-Only	1	1.33	1.45	1.64	1.71	2.22	2.79	3.52	4.41	5.57	5.46

Table 52: Total return of stop-loss strategy with penalty compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 104.

Worse than Bond-Only Investment

Figure 108 shows an example of the stop-loss strategy performing worse than a bond-only investment. The starting date is January 12, 1999. Table 53 shows that the stock-weight was 0.34 in the first year, zero in the second year and one for the remaining years. For the ten-year period the total return of the stop-loss strategy was 1.13 (or 13% gain), while the total return was 1.41 (or 41% gain) for the bond-only investment and 0.77 (or 23% loss) for the stock-only investment.

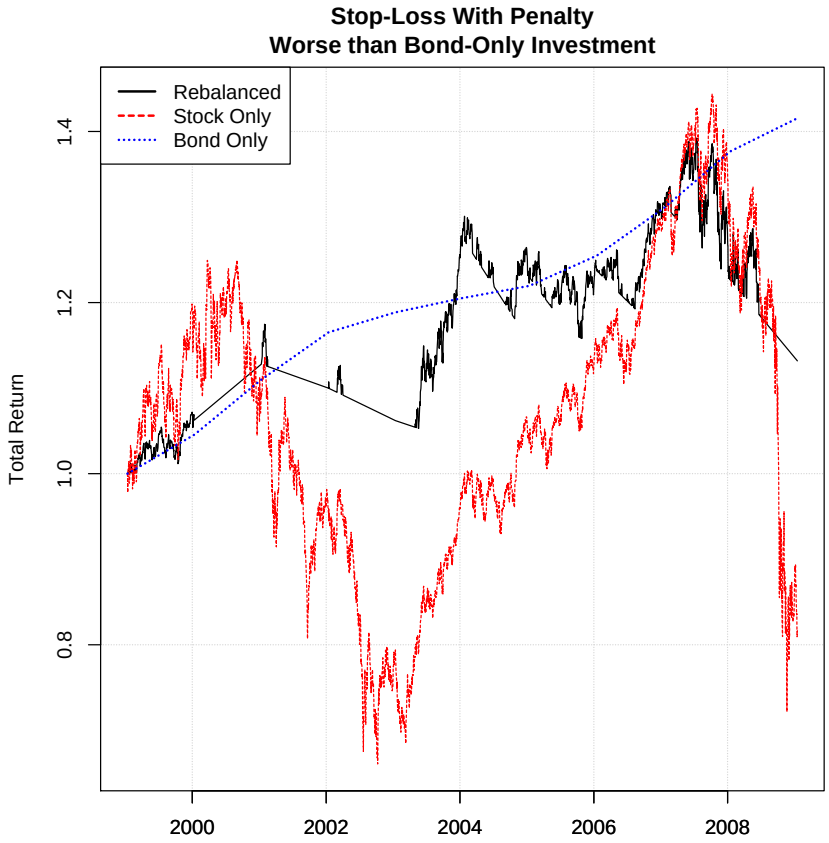

Figure 108: Total return of stop-loss strategy with penalty compared to stock-only and bond-only investments.

Stop-Loss With Penalty Worse than Bond-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	4.64	4.92	4.01	3.37	2.89	3.07	2.84	2.81	2.83	2.52	1.79
Bond Yield	4.6%	6.2%	5.0%	2.0%	1.4%	1.2%	2.9%	4.4%	5.1%	2.8%	0.4%
Stock Weight	0.34	0	1	1	1	1	1	1	1	1	1
Rebalanced	1	1.06	1.13	1.10	1.06	1.28	1.23	1.24	1.32	1.23	1.13
Bond-Only	1	1.05	1.11	1.16	1.19	1.20	1.22	1.25	1.31	1.38	1.41
Stock-Only	1	1.17	1.09	0.95	0.79	0.98	1.04	1.15	1.30	1.24	0.77

Table 53: Total return of stop-loss strategy with penalty compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 104.

Better than Stock-Only Investment

Figure 109 shows an example of the stop-loss strategy performing better than both bond-only and stock-only investments. The starting date is October 6, 1998 and note how this is just a few months prior to the previous example where the stop-loss strategy performed worse than the bond-only investment. Table 54 shows that the stock-weight was one for all years except the second and third where it was 0.22 and 0.55 because the P/Book was very high at 4.71 and 4.40. For the ten-year period the total return of the stop-loss strategy was 2.99 (or 199% gain), while the total return was 1.43 (or 43% gain) for the bond-only investment and 1.10 (or 10% loss) for the stock-only investment.

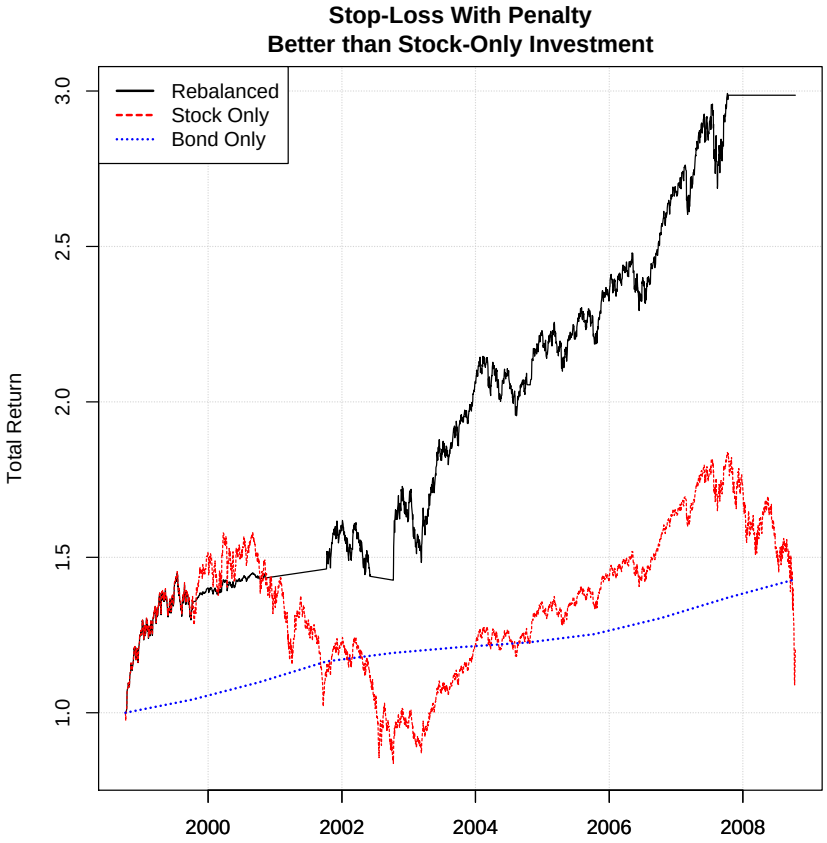

Figure 109: Total return of stop-loss strategy with penalty compared to stock-only and bond-only investments.

Stop-Loss With Penalty Better than Stock-Only Investment											
Date	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
P/Book	3.74	4.71	4.40	3.11	2.33	2.94	2.78	2.66	2.76	2.98	1.80
Bond Yield	4.2%	5.4%	6.1%	2.4%	1.6%	1.2%	2.2%	4.1%	5.0%	4.3%	1.1%
Stock Weight	1	0.22	0.55	1	1	1	1	1	1	1	1
Rebalanced	1	1.36	1.43	1.46	1.43	1.94	2.06	2.19	2.56	2.99	2.99
Bond-Only	1	1.04	1.10	1.16	1.19	1.21	1.23	1.25	1.30	1.37	1.43
Stock-Only	1	1.36	1.47	1.12	0.84	1.14	1.25	1.34	1.57	1.83	1.10

Table 54: Total return of stop-loss strategy with penalty compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 104.

Worse than Stock-Only Investment

Figure 110 shows an example of the stop-loss strategy performing worse than a stock-only investment. The starting date is February 5, 1990. Table 55 shows the stock-weight was one for the first nine years after which the P/Book increased so much that the stock-weight decreased almost to zero, thus causing the portfolio to be increasingly invested in government bonds rather than the S&P 500. For the ten-year period the total return of the stop-loss strategy was 2.76 (or 176% gain), while the total return was 1.67 (or 67% gain) for the bond-only investment and 5.32 (or 432% gain) for the stock-only investment.

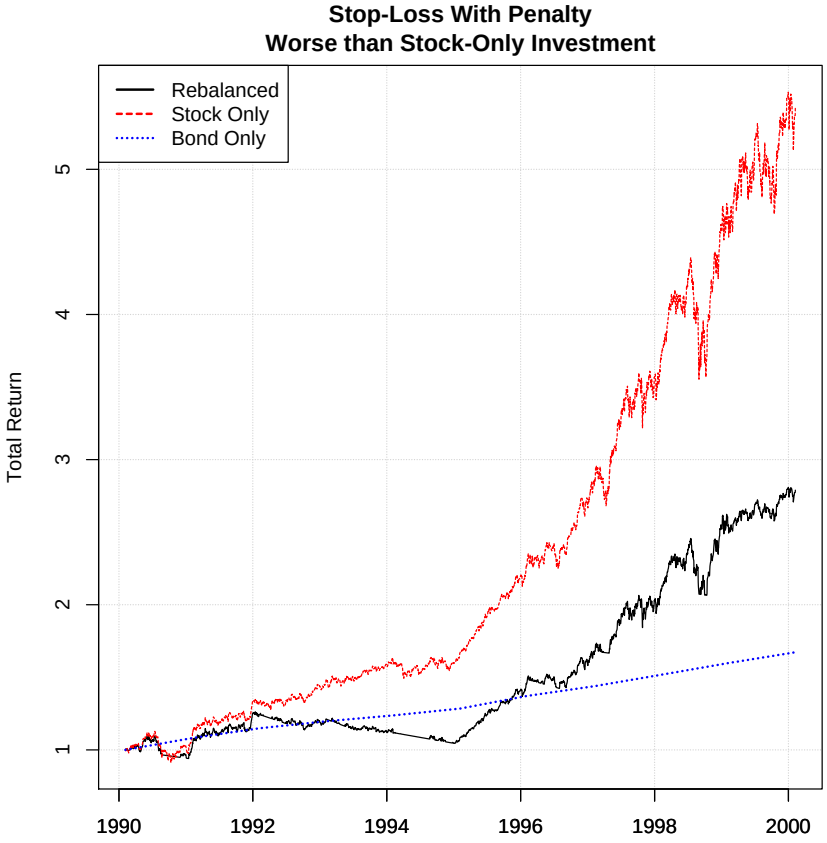

Figure 110: Total return of stop-loss strategy with penalty compared to stock-only and bond-only investments.

Stop-Loss With Penalty Worse than Stock-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.24	2.29	2.62	2.81	2.62	2.45	3.00	3.34	4.03	4.53	4.82
Bond Yield	8.2%	6.2%	4.2%	3.4%	3.8%	6.8%	4.9%	5.5%	5.3%	4.7%	6.2%
Stock Weight	1	1	1	1	1	1	1	1	1	0.47	0.05
Rebalanced	1	1.03	1.24	1.21	1.12	1.09	1.49	1.68	2.11	2.50	2.76
Bond-Only	1	1.08	1.15	1.20	1.24	1.28	1.37	1.44	1.52	1.60	1.67
Stock-Only	1	1.09	1.33	1.48	1.60	1.67	2.31	2.86	3.72	4.53	5.32

Table 55: Total return of stop-loss strategy with penalty compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 104.

15.3.4. Comparison to Non-Penalized Strategy

This section compares the performance of the penalized and non-penalized stop-loss strategies.

Annualized Return

Table 51 shows the annualized returns for the penalized strategy and Table 47 shows it for the non-penalized strategy.

The penalized strategy had median annualized return of 9.5% for one-year investment periods which increased to 11.7% for ten-year investment periods. This was significantly lower than the non-penalized strategy whose median annualized return was 13.8% for one-year investment periods and 15.5% for ten-year investment periods.

The mean annualized return was also significantly lower for the penalized strategy where it was 11.5% for one-year investment periods and decreased to 10.9% for ten-year investment periods. For the non-penalized strategy the mean annualized return was 15.5% for one-year investment periods and decreased gradually to 14.9% for ten-year investment periods.

The greatest one-year gain was 71.8% for both strategies. For longer investment periods the penalized strategy was somewhat worse, for example its greatest annualized return was 19.3% for ten-year investment periods while it was 22.3% for the non-penalized strategy.

The non-penalized strategy had no losses while the greatest one-year loss for the penalized strategy was (14.0%) and this gradually decreased in magnitude until there were no losses for investment periods of six years of more. The lowest annualized return of the penalized strategy for ten-year investment periods was 1.3% which gives a compounded gain of 14% over ten years. For the non-penalized strategy the lowest annualized return was 5.8% which gives a compounded gain of 76% over ten years.

Probability of Loss

The non-penalized strategy had zero probability of loss for all investment periods, while the penalized strategy had probability of loss about 0.25 (or 25%) for one-year investment periods, which gradually decreased to zero probability of loss for investment periods of six years or more.

Probability of Under-Performing Bond-Only and Stock-Only Investments

The penalized strategy had a much higher probability of under-performing bond-only investments. For one-year investment periods the probability was 0.38 (or 38%) while it was only 0.25 (or 25%) for the non-penalized strategy. The probabilities decreased for longer investment periods so the penalized strategy had probability 0.02 (or 2%) for ten-year investment periods, while the non-penalized strategy had zero probability of under-performing bond-only investments for periods of five years or more.

The penalized strategy had even greater probability of under-performing stock-only investments. For one-year investment periods the probability was 0.77 (or 77%) which gradually decreased to 0.57 (or 57%) for investment periods of seven years or more. Compare this to the non-penalized strategy whose probability of under-performing stock-only investments was 0.02-0.03 (or 2-3%) for all investment periods.

15.4. Low Risk Stop-Loss

The low-risk stop-loss strategy uses annual rebalancing between the S&P 500 and US government bonds with one-year maturity. The part of the portfolio invested in the S&P 500 may be sold during the year if the price becomes lower than (5%) below the purchase price at the beginning of the year. If the price of the S&P 500 again becomes above this stop-price then the shares in the S&P 500 are repurchased. The selling and repurchasing may not be possible exactly at the stop-price which causes frictional costs as described in section 15.2 and which is taken into account in the following performance analysis.

15.4.1. Stock Weight

The stock-weight determines the part of the portfolio to invest in the S&P 500 and the remainder is invested in US government bonds. The stock-weight remains the same throughout a year even though the stock may be sold and repurchased several times as the stop-price is crossed. The stock-weight formula is:

$$Stock\ Weight = Limit(1.45 - 0.38 \times P/Book - 2.58 \times Bond\ Yield)$$

Eq. 15-4

The P/Book is for the S&P 500 index and the bond yield is for US government bonds with one-year maturity.

The limit-function simply means that the stock-weight is limited between zero and one. The stock-weight varies with both the P/Book and bond yield so its boundaries depend on both. For example, holding the bond yield fixed at 1.2% means the stock-weight is zero when the P/Book is greater than 3.73, and the stock-weight is one when the P/Book is less than about 1.10. If instead the bond yield is 7% then the stock-weight is zero when the P/Book is greater than about 3.34, and the stock-weight is one when the P/Book is less than about 0.71.

Consider another example where the P/Book is 1.8 and the bond yield is 3% (or 0.03), then the stock-weight is calculated from Eq. 15-4 as follows:

$$Stock\ Weight = Limit(1.45 - 0.38 \times P/Book - 2.58 \times Bond\ Yield) = Limit(1.45 - 0.38 \times 1.8 - 2.58 \times 0.03) = 0.69$$

That is, 0.69 (or 69%) of the portfolio should be invested in the S&P 500 according to this strategy.

If instead the bond yield had been 7% (or 0.07) then the stock-weight would be:

$$Stock\ Weight = Limit(1.45 - 0.38 \times P/Book - 2.58 \times Bond\ Yield) = Limit(1.45 - 0.38 \times 1.8 - 2.58 \times 0.07) = 0.59$$

So now only 0.59 (or 59%) of the portfolio should be invested in the S&P 500 because the bond yield is higher at 7% and hence more attractive than a bond yield of only 3%.

Figure 111 shows the stock-weight for each day during the period 1978-2013 calculated using Eq. 15-4 with the P/Book data from Figure 2 and the government bond yields from Figure 3. Almost 89% of the days in this period had some part of the portfolio invested in the S&P 500 because the stock-weight was non-zero. The period around year 2000 had zero stock-weights because the P/Book for the S&P 500 was very high. Note that the stock-weight is shown for each day but the investment strategy uses annual rebalancing.

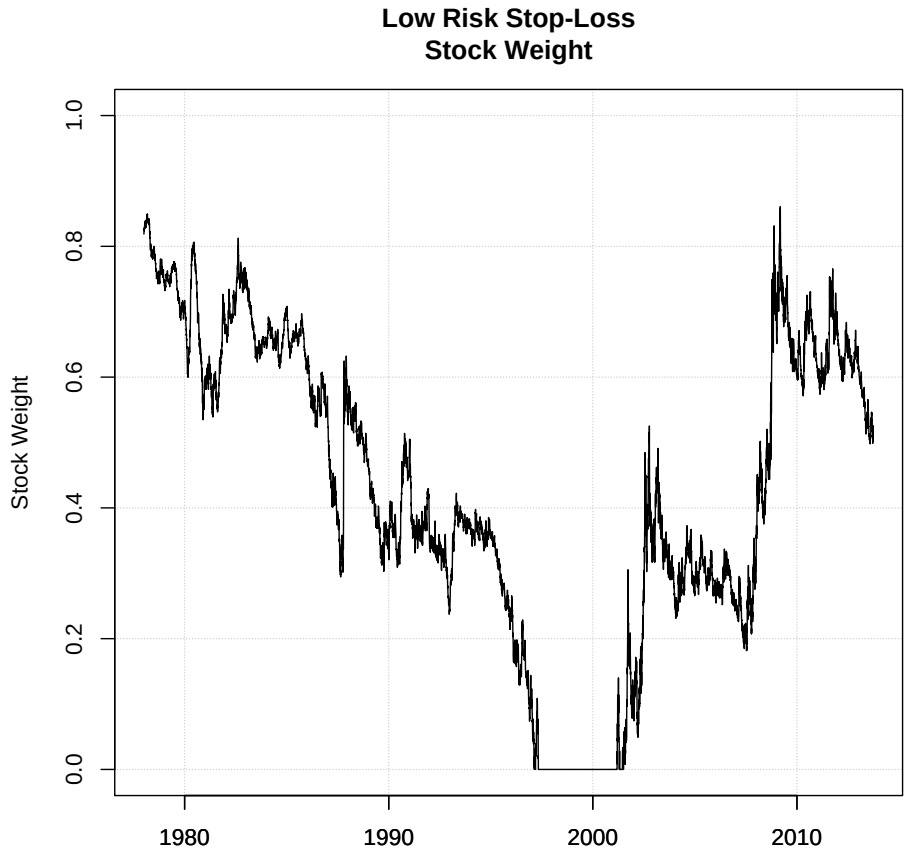

Figure 111: Stock-weight for low-risk stop-loss strategy.

15.4.2. Annualized Return

Figure 112 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 56.

For one-year investment periods the mean annualized return was 9.3% which gradually decreased to 8.6% for ten-year investment periods. The median annualized return was 6.8% for one-year investment periods and this increased to 7.7% for ten-year investment periods.

For one-year investment periods the probability of loss was 0.06 (or 6%) which decreased to zero probability of loss for investment periods of three years or more. The greatest one-year loss was (5.0%) and the lowest annualized return was 3.2% for ten-year investment periods thus giving a compounded gain of about 37% over ten years. The greatest one-year gain was 61.9% and the greatest annualized return was 17.5% for ten-year investment periods thus giving a compounded gain of 402% over ten years.

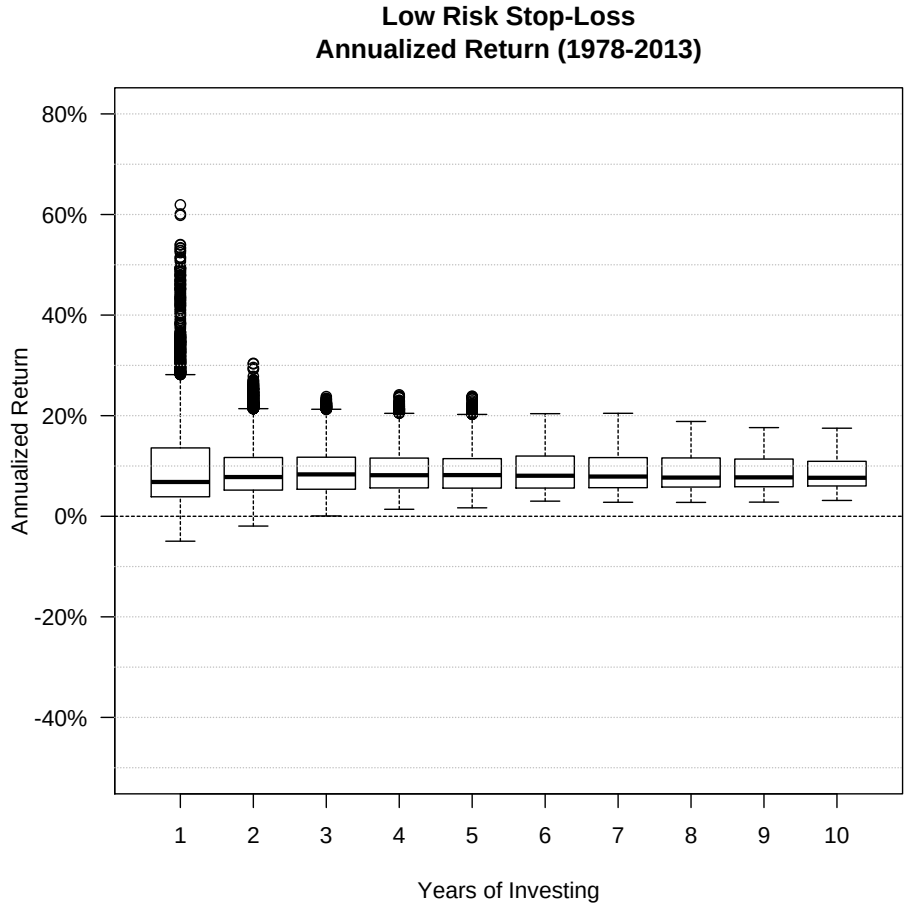

Figure 112: Annualized return statistics for low-risk stop-loss strategy.

Annualized Return for Low Risk Stop-Loss										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(5.0%)	3.9%	6.8%	9.3%	13.6%	61.9%	8.4%	0.06	0.25	0.78
2	(1.9%)	5.2%	7.8%	9.1%	11.7%	30.4%	5.4%	0.02	0.16	0.74
3	0.1%	5.4%	8.3%	9.0%	11.7%	23.8%	4.4%	0	0.13	0.71
4	1.4%	5.6%	8.2%	9.0%	11.6%	24.2%	4.2%	0	0.10	0.68
5	1.7%	5.6%	8.2%	8.9%	11.5%	23.9%	4.1%	0	0.07	0.67
6	3.0%	5.6%	8.1%	8.8%	12.0%	20.4%	3.8%	0	0.02	0.66
7	2.8%	5.7%	7.9%	8.8%	11.7%	20.5%	3.7%	0	0.002	0.68
8	2.8%	5.8%	7.7%	8.7%	11.6%	18.9%	3.6%	0	0.0009	0.74
9	2.8%	5.9%	7.7%	8.7%	11.4%	17.6%	3.4%	0	0.0009	0.80
10	3.2%	6.0%	7.7%	8.6%	10.9%	17.5%	3.2%	0	0.0009	0.82

Table 56: Annualized return statistics for low-risk stop-loss strategy. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

15.4.3. Comparison to Bond-Only & Stock-Only Investments

Figure 113 shows the performance of the stop-loss (aka. rebalancing) strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the stop-loss strategy consistently performed better than the bond-only investment and worse than the stock-only investment. But this long-term performance is misleading as the mutual performance depends on the starting date and investment duration, as shown below.

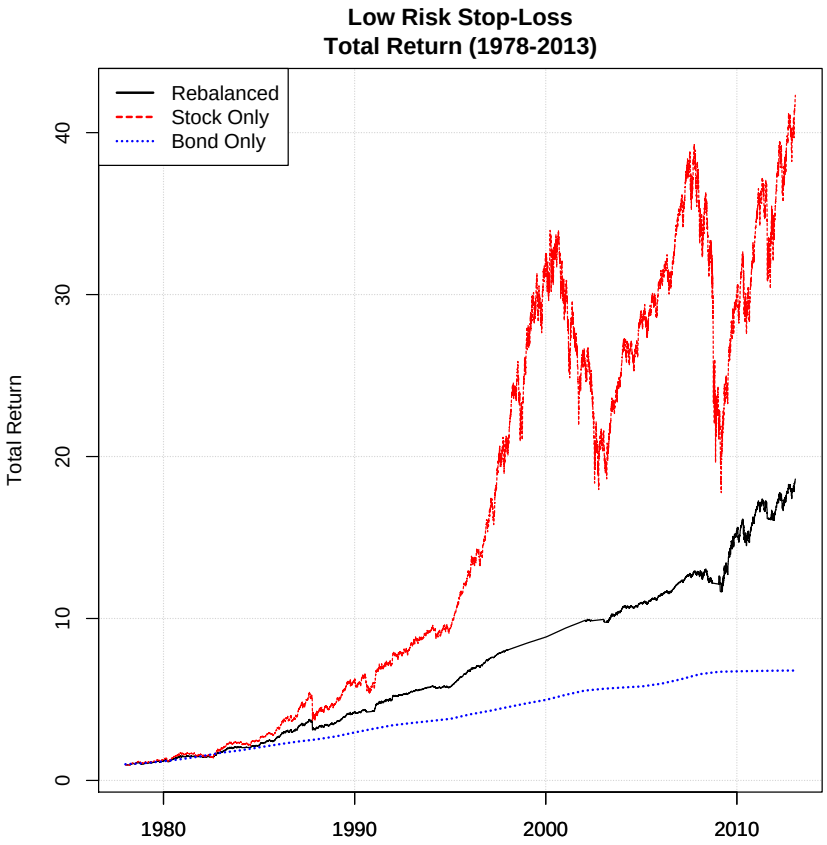

Figure 113: Total return of low-risk stop-loss strategy compared to stock-only and bond-only investments for the period 1978-2013.

Probability of Under-Performance

Table 56 shows the probabilities of the stop-loss strategy under-performing bond-only investments. For one-year investment periods the probability was 0.25 (or 25%) and the probability gradually declined to 0.0009 (or 0.09%) for ten-year investment periods.

Table 56 also shows the probabilities of the stop-loss strategy under-performing stock-only investments. For one-year investment periods the probability was 0.78 (or 78%) and it was 0.82 (or 82%) for ten-year investment periods.

Better than Bond-Only Investment

Figure 114 shows an example of the stop-loss strategy performing better than a bond-only investment. The investment period starts August 19, 1982. Table 57 shows that the stock-weight varied between 0.30-0.81 during this period. For the entire ten-year period the stop-loss (aka. rebalancing) strategy had a total return of 4.57 (or 357% gain), the bond-only investment had a total return of 2.23 (or 123% gain), and the stock-only investment had a total return of 5.33 (or 433% gain).

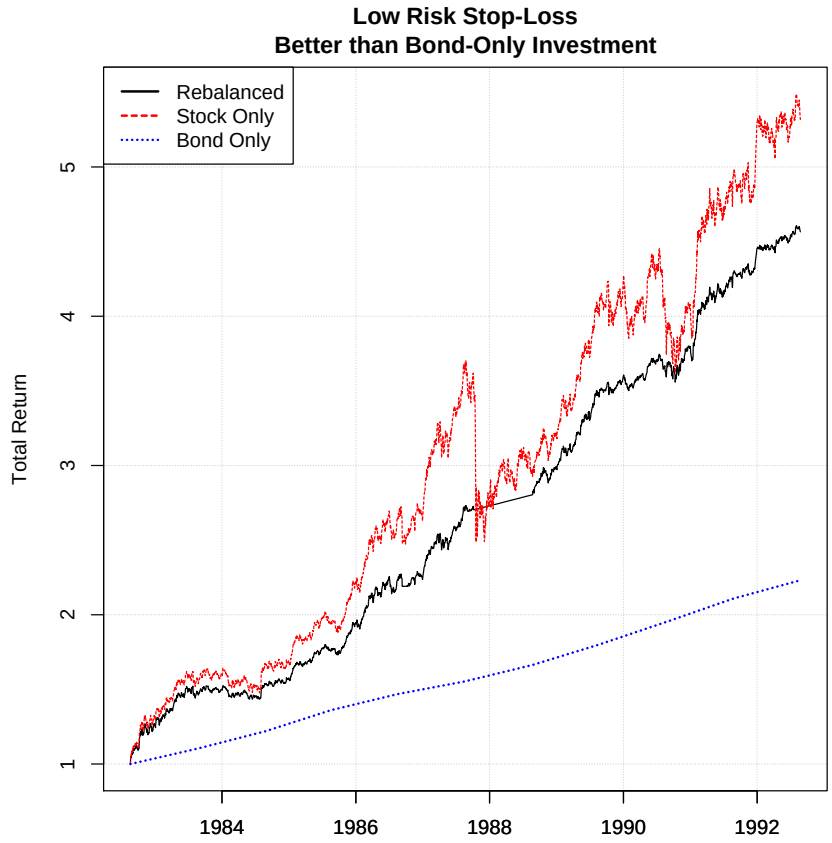

Figure 114: Total return of the low-risk stop-loss strategy compared to stock-only and bond-only investments.

Low Risk Stop-Loss Better than Bond-Only Investment											
Date	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
P/Book	0.98	1.42	1.39	1.52	1.98	2.56	1.85	2.37	2.03	2.51	2.69
Bond Yield	10.3%	10.4%	11.8%	7.9%	5.7%	7.0%	8.3%	8.4%	8.0%	5.8%	3.6%
Stock Weight	0.81	0.64	0.62	0.67	0.55	0.30	0.53	0.33	0.47	0.34	0.34
Rebalanced	1	1.48	1.54	1.77	2.25	2.73	2.80	3.48	3.59	4.29	4.57
Bond-Only	1	1.10	1.22	1.36	1.47	1.55	1.66	1.80	1.95	2.11	2.23
Stock-Only	1	1.57	1.67	1.96	2.67	3.69	2.93	4.04	3.74	4.95	5.33

Table 57: Total return of the low-risk stop-loss strategy compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 104.

Worse than Bond-Only Investment

Figure 115 shows an example of the stop-loss strategy performing worse than a bond-only investment. The starting date is August 18, 1999. Table 58 shows that the stock-weight was zero the first two years because the P/Book for the S&P 500 was more than 4.7 which is very high. The P/Book decreased in the following years and the stock-weight hence increased and was around 0.3 for several of these years, meaning that approximately 30% of the portfolio was typically invested in the S&P 500. For the ten-year period the total return of the stop-loss strategy was 1.39 (or 39% gain), while the total return was slightly higher at 1.41 (or 41% gain) for the bond-only investment and only 0.92 (or 8% loss) for the stock-only investment.

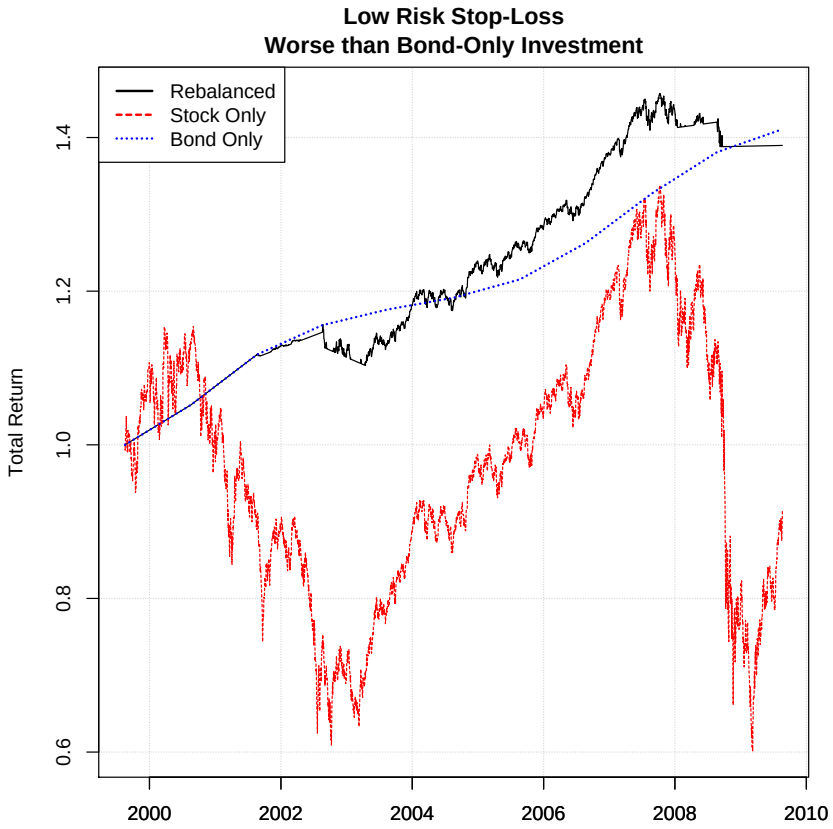

Figure 115: Total return of the low-risk stop-loss strategy compared to stock-only and bond-only investments.

Low Risk Stop-Loss Worse than Bond-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	4.78	4.71	3.42	2.82	2.86	2.78	2.78	2.68	2.81	2.48	2.08
Bond Yield	5.2%	6.2%	3.4%	1.8%	1.3%	2.0%	3.9%	5.1%	4.1%	2.2%	0.4%
Stock Weight	0	0	0.06	0.33	0.33	0.34	0.29	0.30	0.28	0.45	0.65
Rebalanced	1	1.05	1.12	1.15	1.13	1.19	1.26	1.32	1.43	1.42	1.39
Bond-Only	1	1.05	1.12	1.16	1.18	1.19	1.21	1.26	1.33	1.38	1.41
Stock-Only	1	1.13	0.90	0.73	0.79	0.89	1.00	1.09	1.25	1.13	0.92

Table 58: Total return of the low-risk stop-loss strategy compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 104.

Better than Stock-Only Investment

Figure 116 shows an example of the stop-loss strategy performing better than both bond-only and stock-only investments. The starting date is March 3, 2000 and note how this is just a half year after the previous example where the stop-loss strategy performed worse than the bond-only investment. Table 59 shows that the stock-weight was zero or close to zero for the first three years because the P/Book was high. Then the P/Book decreased and hence the stock-weight increased. In 2009 the P/Book was 1.50 so the stock-weight was 0.86. For the ten-year period the total return of the stop-loss strategy was 2.40 (or 140% gain), while the total return was 1.35 (or 35% gain) for the bond-only investment and 0.98 (or 2% loss) for the stock-only investment.

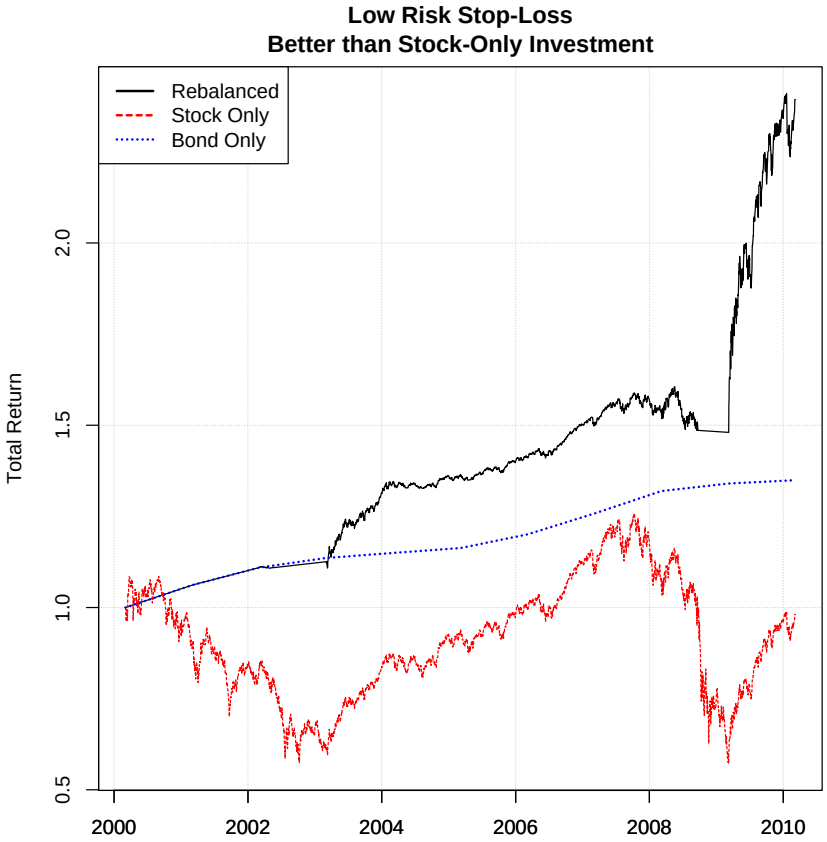

Figure 116: Total return of the low-risk stop-loss strategy compared to stock-only and bond-only investments.

Low Risk Stop-Loss Better than Stock-Only Investment											
Date	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
P/Book	4.79	3.64	3.43	2.54	3.09	2.89	2.75	2.74	2.44	1.50	2.17
Bond Yield	6.2%	4.5%	2.4%	1.2%	1.2%	3.2%	4.8%	4.9%	1.6%	0.7%	0.4%
Stock Weight	0	0	0.09	0.45	0.24	0.27	0.28	0.28	0.48	0.86	0.62
Rebalanced	1	1.06	1.11	1.13	1.34	1.36	1.41	1.50	1.53	1.48	2.40
Bond-Only	1	1.06	1.11	1.14	1.15	1.16	1.20	1.26	1.32	1.34	1.35
Stock-Only	1	0.89	0.84	0.62	0.87	0.94	1.00	1.11	1.05	0.57	0.98

Table 59: Total return of the low-risk stop-loss strategy compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 104.

Worse than Stock-Only Investment

Figure 117 shows an example of the stop-loss strategy performing worse than a stock-only investment. The starting date is August 27, 1990. Table 60 shows the stock-weight gradually decreased from 0.44 in the first year to zero after eight years because the P/Book increased greatly during this period. For the ten-year period the total return of the stop-loss strategy was 2.13 (or 113% gain), while the total return was 1.68 (or 68% gain) for the bond-only investment and 5.86 (or 486% gain) for the stock-only investment.

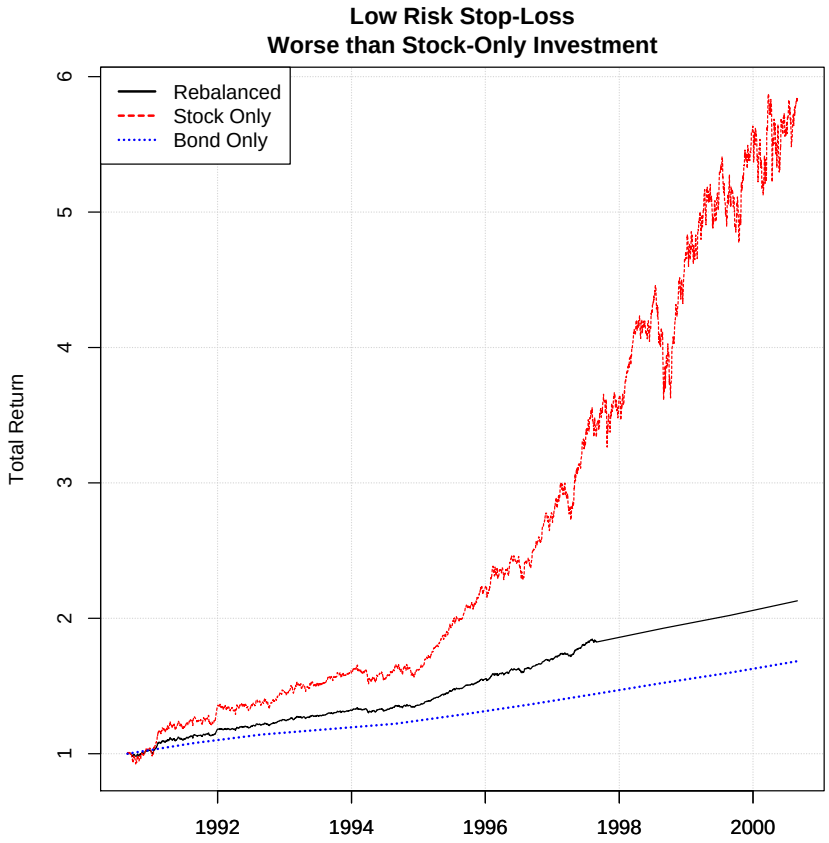

Figure 117: Total return of the low-risk stop-loss strategy compared to stock-only and bond-only investments.

Low Risk Stop-Loss Worse than Stock-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.13	2.51	2.70	2.61	2.55	2.69	2.93	3.68	3.67	4.73	4.78
Bond Yield	7.9%	5.8%	3.5%	3.4%	5.6%	5.7%	5.8%	5.6%	5.0%	5.3%	6.2%
Stock Weight	0.44	0.35	0.33	0.37	0.34	0.28	0.19	0	0	0	0
Rebalanced	1	1.14	1.21	1.30	1.35	1.48	1.63	1.82	1.93	2.02	2.13
Bond-Only	1	1.08	1.14	1.18	1.22	1.29	1.36	1.44	1.52	1.60	1.68
Stock-Only	1	1.26	1.37	1.56	1.65	2.00	2.40	3.34	3.61	5.04	5.86

Table 60: Total return of the low-risk stop-loss strategy compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 104.

15.4.4. Comparison to Non-Stop-Loss Strategy

This section compares the performance of the low-risk stop-loss strategy to the non-stop-loss strategy from section 14.1.

Stock-Weight

The stock-weight for the stop-loss strategy during the period 1978-2013 is shown in Figure 111 and it is shown for the non-stop-loss strategy in Figure 70. The stop-loss strategy had non-zero stock-weight more often and was hence invested in the S&P 500 more often than the non-stop-loss strategy. However, the non-stop-loss strategy had more extreme stock-weights that were either closer to one or zero.

Annualized Return

Table 56 shows the annualized returns for the stop-loss strategy and Table 27 shows it for the non-stop-loss strategy.

The stop-loss strategy had median annualized return that ranged between 6.8-8.3% depending on the investment period, while it ranged between 5.6-5.9% for the non-stop-loss strategy.

The mean annualized return was slightly higher for the stop-loss strategy. For one-year investment periods it was 9.3% and it gradually decreased to 8.6% for ten-year investment periods. The non-stop-loss strategy had mean annualized return 8.9% for one-year investment periods which gradually decreased to 7.6% for ten-year investment periods.

Gain & Loss

The greatest one-year gain was 61.9% for the stop-loss strategy while it was 71.8% for the non-stop-loss strategy. For ten-year investment periods the highest annualized return was 17.5% for the stop-loss strategy while it was 18.0% for the non-stop-loss strategy.

The greatest one-year loss was (5.0%) for the stop-loss strategy while it was (4.5%) for the non-stop-loss strategy. The stop-loss strategy also experienced losses for two-year investment periods where the lowest annualized return was (1.9%) giving a compounded loss of (3.8%), while the non-stop-loss strategy did not experience losses for investment periods of two years or more.

The stop-loss strategy could have always been fully invested in the S&P 500, but this would sometimes cause losses of (5.0%) for multiple years in a row which was sought avoided when developing this strategy.

Probability of Loss

The stop-loss strategy had probability of loss 0.06 (or 6%) for one-year investment periods, probability of loss 0.02 (or 2%) for two-year investment periods, and zero probability of loss for longer investment periods. The non-stop-loss strategy had probability of loss 0.015 (or 1.5%) for one-year investment periods and zero probability of loss for longer investment periods.

Historical vs. Future Losses

When comparing these historical losses it is important to note that the non-stop-loss strategy was sometimes fully invested in the S&P 500 and hence fully exposed to its potential losses. Although the historically greatest loss was (4.5%) for the non-stop-loss strategy, much greater losses could be experienced in the future if the S&P 500 should experience a great loss at an already low P/Book, which is when that strategy would be heavily invested in the S&P 500.

The stop-loss strategy limits the magnitude of losses by selling the S&P 500 investment once it decreases below (5%) of the purchase price, and then repurchasing the shares when the price is above the stop-price again. But the S&P 500 index may linger around the stop-price which causes repeated selling and repurchasing of the shares. Although the transaction costs may be negligible, it may not be possible to buy and sell precisely at the stop-price which causes frictional costs in the form of small compounding losses. This is another reason why the stop-loss strategy is not fully invested in the S&P 500.

Probability of Under-Performing Bond-Only and Stock-Only Investments

The stop-loss strategy had a much higher probability of under-performing bond-only investments for shorter investment periods. For one-year investment periods the probability was 0.25 (or 25%) while it was only 0.09 (or 9%) for the non-stop-loss strategy. The probabilities decreased for longer investment periods so the stop-loss strategy had probability 0.0009 (or 0.09%) for ten-year investment periods, while the non-stop-loss strategy had probability 0.01 (or 1%) of under-performing bond-only investments.

The stop-loss strategy had slightly higher probability of under-performing stock-only investments. For one-year investment periods the probability was 0.78 (or 78%) while it was only 0.72 (or 72%) for the non-stop-loss strategy. For ten-year investment periods the stop-loss strategy had probability 0.82 (or 82%) while it was 0.81 (or 81%) for the non-stop-loss strategy.

15.5. Medium Risk Stop-Loss

The medium-risk stop-loss strategy uses annual rebalancing between the S&P 500 and US government bonds with one-year maturity. The part of the portfolio invested in the S&P 500 may be sold during the year if the price becomes lower than (15%) below the purchase price at the beginning of the year. If the price of the S&P 500 again becomes above this stop-price then the shares in the S&P 500 are repurchased. The selling and repurchasing may not be possible exactly at the stop-price which causes frictional costs as described in section 15.2 and which is taken into account in the following performance analysis.

15.5.1. Stock Weight

The stock-weight determines the part of the portfolio to invest in the S&P 500 and the remainder is invested in US government bonds. The stock-weight remains the same throughout a year even though the stock may be sold and repurchased several times as the stop-price is crossed. The stock-weight formula is:

$$\textit{Stock Weight} = \textit{Limit}(1.93 - 0.44 \times \textit{P/Book} - 3.14 \times \textit{Bond Yield})$$

Eq. 15-5

The limit-function simply means that the stock-weight is limited between zero and one. The stock-weight varies with both the P/Book and bond yield so its boundaries depend on both. For example, holding the bond yield fixed at 1.2% means the stock-weight is zero when the P/Book is greater than 4.30, and the stock-weight is one when the P/Book is less than about 2.03. If instead the bond yield is 7% then the stock-weight is zero when the P/Book is greater than about 3.89, and the stock-weight is one when the P/Book is less than about 1.61.

Consider another example where the P/Book is 1.8 and the bond yield is 3% (or 0.03), then the stock-weight is calculated from Eq. 15-5 as follows:

Stock Weight = Limit(1.93 – 0.44 x P/Book – 3.14 x Bond Yield) = Limit(1.93 – 0.44 x 1.8 – 3.14 x 0.03) = Limit(1.04) = 1

That is, 100% of the portfolio should be invested in the S&P 500 according to this strategy.

If instead the bond yield had been 7% (or 0.07) then the stock-weight would be:

Stock Weight = Limit(1.93 – 0.44 x P/Book – 3.14 x Bond Yield) = Limit(1.93 – 0.44 x 1.8 – 3.14 x 0.07) = 0.92

So now only 0.92 (or 92%) of the portfolio should be invested in the S&P 500 because the bond yield is higher at 7% and hence more attractive than a bond yield of only 3%.

Figure 118 shows the stock-weight for each day during the period 1978-2013 calculated using Eq. 15-5 with the P/Book data from Figure 2 and the government bond yields from Figure 3. Almost 92% of the days in this period had some part of the portfolio invested in the S&P 500 index. The period around year 2000 had zero stock-weights because the P/Book for the S&P 500 was very high during these years. Note that the stock-weight is shown for each day but the investment strategy uses annual rebalancing.

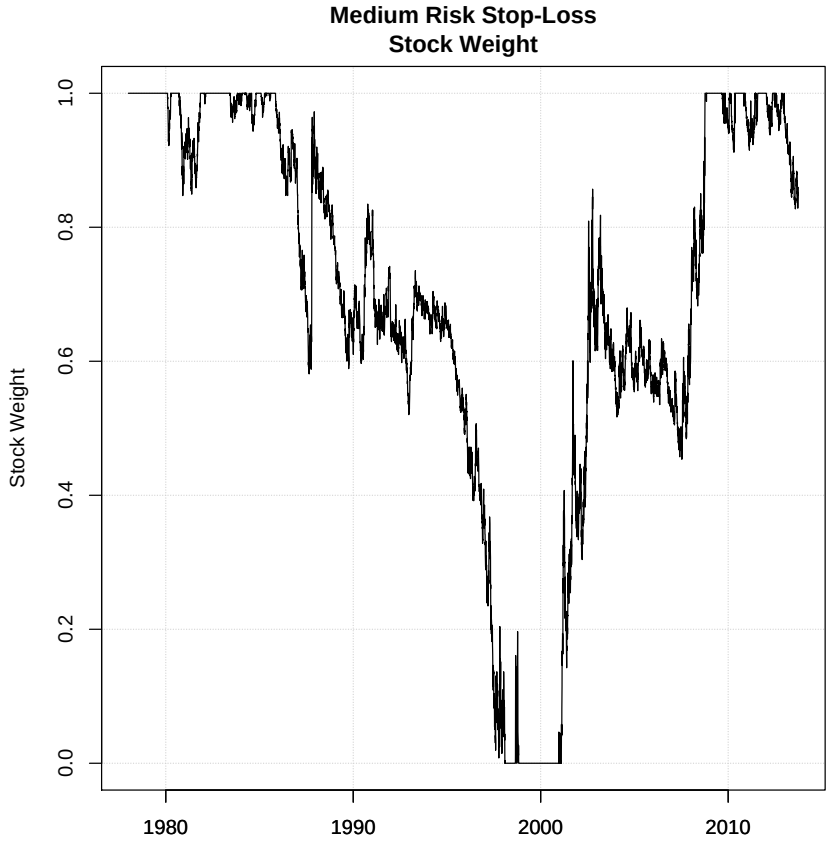

Figure 118: Stock-weight for medium-risk stop-loss strategy.

15.5.2. Annualized Return

Figure 119 shows the annualized returns of this strategy for investment periods up to 10 years. The statistics are also shown in Table 61.

For one-year investment periods the mean annualized return was 11.8% which gradually decreased to 10.5% for ten-year investment periods. The median annualized return was 9.9% for one-year investment periods and this increased to 10.4% for ten-year investment periods.

For one-year investment periods the probability of loss was 0.15 (or 15%) which gradually decreased to zero for investment periods of five years or more. The greatest loss was (15.2%) for one-year investment periods and the lowest annualized return was 2.2% for ten-year investment periods thus giving a compounded gain of about 24% over ten years. The greatest one-year gain was 71.8% and the greatest annualized return was 20.1% for ten-year investment periods thus giving a compounded gain of 524% over ten years.

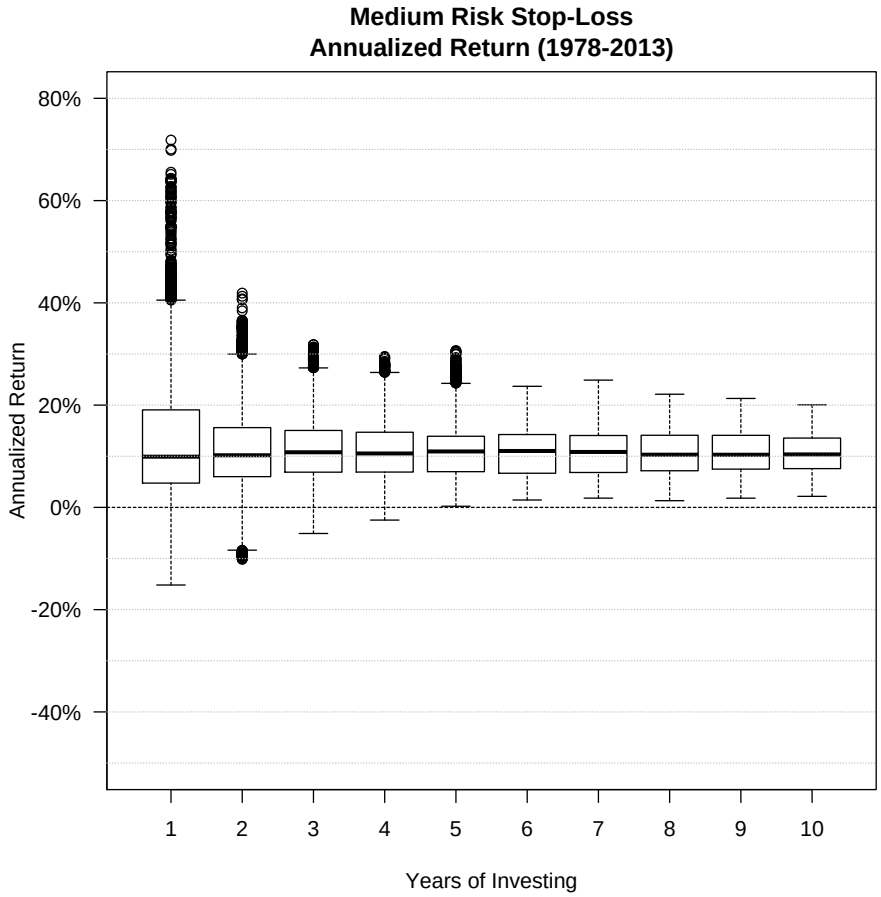

Figure 119: Annualized return statistics for medium-risk stop-loss strategy.

Annualized Return for Medium Risk Stop-Loss										
Years of Investing	Min	1st Qrt.	Median	Mean	3rd Qrt.	Max	Stdev	Probability of Loss	Probability < Bond-Only	Probability < Stock-Only
1	(15.2%)	4.8%	9.9%	11.8%	19.1%	71.8%	12.6%	0.15	0.23	0.60
2	(10.2%)	6.0%	10.2%	11.2%	15.6%	41.9%	7.7%	0.06	0.17	0.62
3	(5.1%)	6.9%	10.8%	11.0%	15.0%	31.8%	6.1%	0.03	0.16	0.63
4	(2.5%)	6.9%	10.5%	10.9%	14.7%	29.5%	5.5%	0.01	0.13	0.61
5	0.2%	7.0%	10.9%	10.8%	13.9%	30.7%	5.1%	0	0.08	0.57
6	1.4%	6.7%	11.0%	10.7%	14.3%	23.7%	4.6%	0	0.04	0.55
7	1.8%	6.8%	10.8%	10.8%	14.1%	24.9%	4.5%	0	0.01	0.57
8	1.3%	7.2%	10.3%	10.7%	14.1%	22.1%	4.3%	0	0.01	0.62
9	1.8%	7.5%	10.3%	10.6%	14.1%	21.3%	4.0%	0	0.02	0.69
10	2.2%	7.6%	10.4%	10.5%	13.6%	20.1%	3.8%	0	0.02	0.72

Table 61: Annualized return statistics for medium-risk stop-loss strategy. Also shown is the probability of loss (i.e. the annualized return is less than zero), the probability that the annualized return is less than that of a bond-only investment, and the probability that the annualized return is less than that of a stock-only investment.

15.5.3. Comparison to Bond-Only & Stock-Only Investments

Figure 120 shows the performance of the stop-loss (aka. rebalancing) strategy compared to bond-only and stock-only investments for the period 1978-2013. In this period of 35 years the stop-loss strategy performed almost as well as the S&P 500 except for the two large peaks around year 2000 and 2008 where the S&P 500 greatly increased and then greatly decreased, while the stop-loss strategy was less invested in the S&P 500 during these periods and hence experienced smaller gains and subsequently smaller losses. But this long-term performance is misleading as the mutual performance depends on the starting date and investment duration, as shown below.

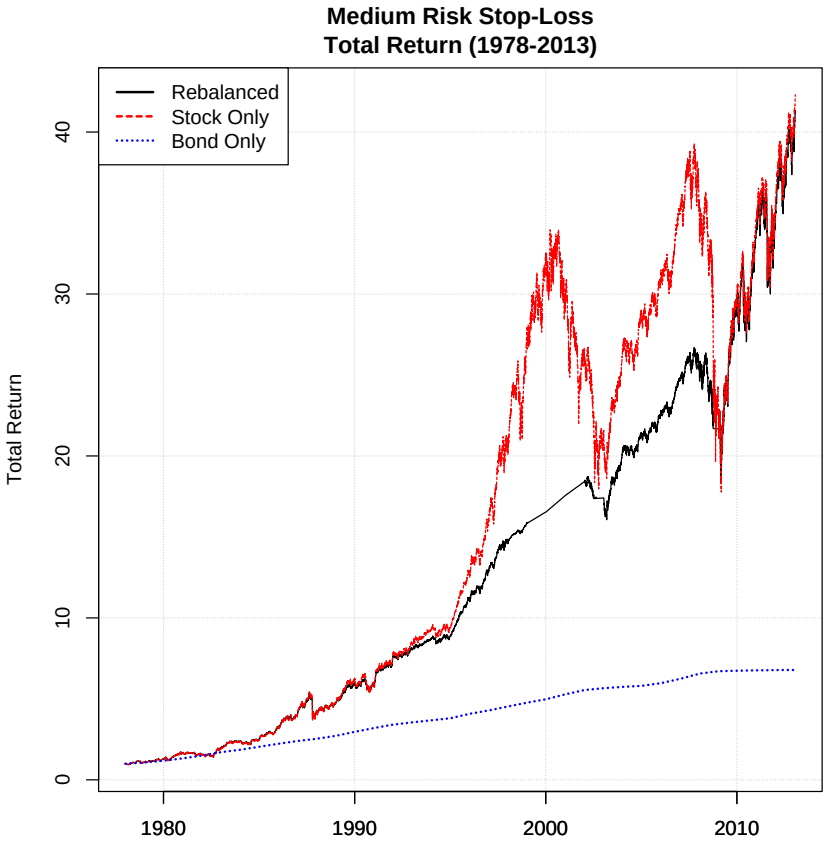

Figure 120: Total return of medium-risk stop-loss strategy compared to stock-only and bond-only investments for the period 1978-2013.

Probability of Under-Performance

Table 61 shows the probabilities of the stop-loss strategy under-performing bond-only investments. For one-year investment periods the probability was 0.23 (or 23%) and the probability gradually declined to 0.02 (or 2%) for ten-year investment periods.

Table 61 also shows the probabilities of the stop-loss strategy under-performing stock-only investments. For one-year investment periods the probability was 0.60 (or 60%) and it was 0.72 (or 72%) for ten-year investment periods.

Better than Bond-Only Investment

Figure 121 shows an example of the stop-loss strategy performing better than both the bond-only and stock-only investments. The investment period starts March 3, 2003. Table 62 shows that the stock-weight varied between 0.53-1.00 during this period, so the portfolio always had a substantial investment in the S&P 500 during this period. For the entire ten-year period the stop-loss (aka. rebalancing) strategy had a total return of 3.31 (or 231% gain), the bond-only investment had a total return of 1.20 (or 20% gain), and the stock-only investment had a total return of 2.27 (or 127% gain).

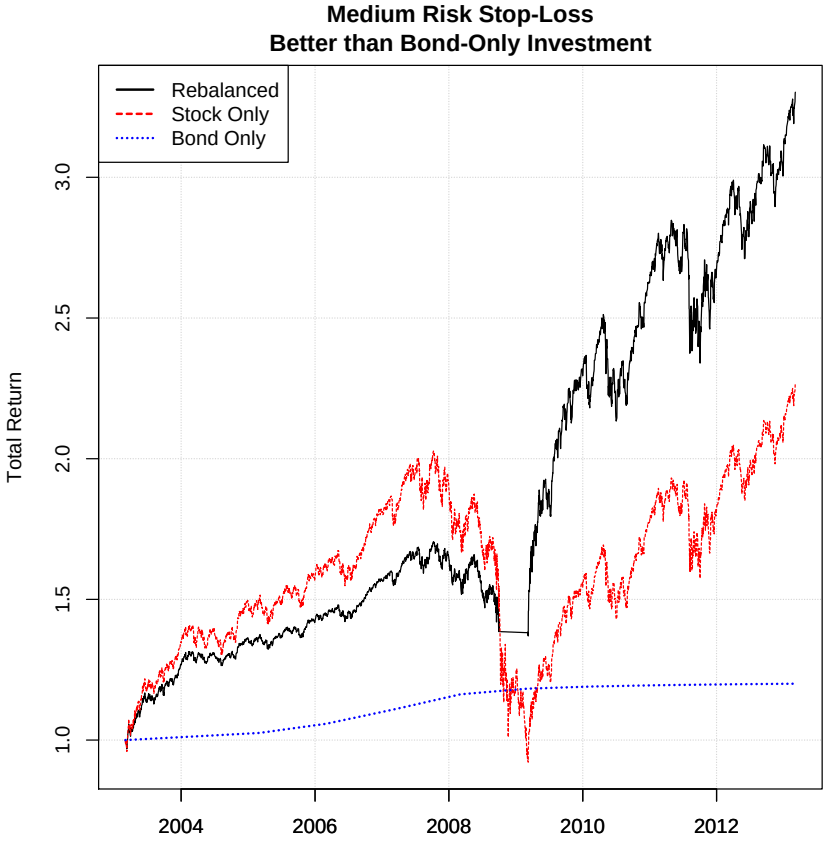

Figure 121: Total return of the medium-risk stop-loss strategy compared to stock-only and bond-only investments.

Medium Risk Stop-Loss Better than Bond-Only Investment											
Date	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
P/Book	2.56	3.08	2.86	2.78	2.71	2.51	1.52	2.17	2.22	2.15	-
Bond Yield	1.3%	1.3%	3.2%	4.8%	4.9%	1.7%	0.7%	0.4%	0.3%	0.2%	-
Stock Weight	0.77	0.53	0.57	0.56	0.59	0.77	1	0.97	0.95	0.98	-
Rebalanced	1	1.31	1.37	1.45	1.55	1.57	1.38	2.35	2.74	2.85	3.31
Bond-Only	1	1.01	1.03	1.06	1.11	1.16	1.18	1.19	1.19	1.20	1.20
Stock-Only	1	1.40	1.50	1.62	1.76	1.75	0.93	1.58	1.85	1.95	2.27

Table 62: Total return of the medium-risk stop-loss strategy compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 118.

Worse than Bond-Only Investment

Figure 122 shows an example of the stop-loss strategy performing worse than a bond-only investment but better than a stock-only investment. The starting date is July 6, 1999. Table 63 shows that the stock-weight was zero the first two years because the P/Book for the S&P 500 was close to 5 which is very high. The P/Book decreased in the following years and the stock-weight hence increased to finally become 1 in 2009. For the ten-year period the total return of the stop-loss strategy was 1.24 (or 24% gain), while the total return was 1.42 (or 42% gain) for the bond-only investment and 0.76 (or 24% loss) for the stock-only investment.

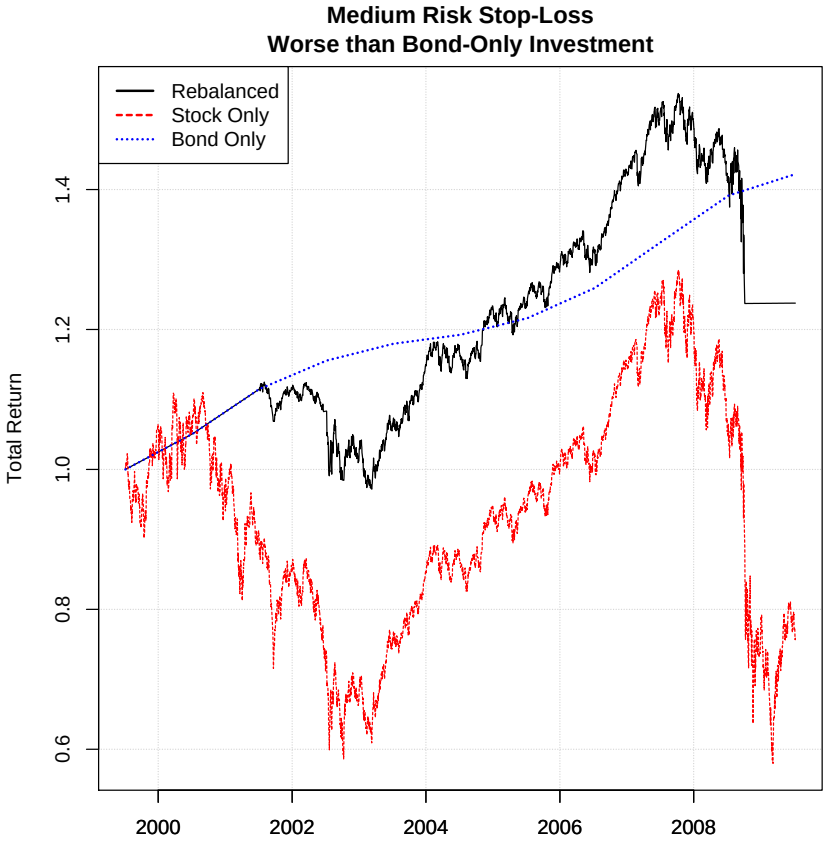

Figure 122: Total return of the medium-risk stop-loss strategy compared to stock-only and bond-only investments.

Medium Risk Stop-Loss Worse than Bond-Only Investment											
Date	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
P/Book	5.02	4.65	3.45	2.96	2.91	2.86	2.77	2.66	2.92	2.37	1.80
Bond Yield	5.1%	6.1%	3.6%	2.1%	1.1%	2.0%	3.5%	5.3%	5.0%	2.2%	0.5%
Stock Weight	0	0	0.30	0.56	0.62	0.61	0.60	0.59	0.49	0.82	1
Rebalanced	1	1.05	1.11	1.08	1.07	1.16	1.24	1.31	1.50	1.41	1.24
Bond-Only	1	1.05	1.11	1.16	1.18	1.19	1.22	1.26	1.33	1.39	1.42
Stock-Only	1	1.06	0.88	0.73	0.77	0.86	0.96	1.02	1.23	1.05	0.76

Table 63: Total return of the medium-risk stop-loss strategy compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 118.

Better than Stock-Only Investment

Figure 123 shows another example of the stop-loss strategy performing better than both bond-only and stock-only investments. The starting date is March 3, 2000. Table 64 shows that the stock-weight was zero in the first year because the P/Book was very high, but the P/Book then decreased in the following years and hence the stock-weight increased. In 2009 the P/Book was 1.50 so the stock-weight was 1. For the ten-year period the total return of the stop-loss strategy was 2.44 (or 144% gain), while the total return was 1.35 (or 35% gain) for the bond-only investment and 0.98 (or 2% loss) for the stock-only investment.

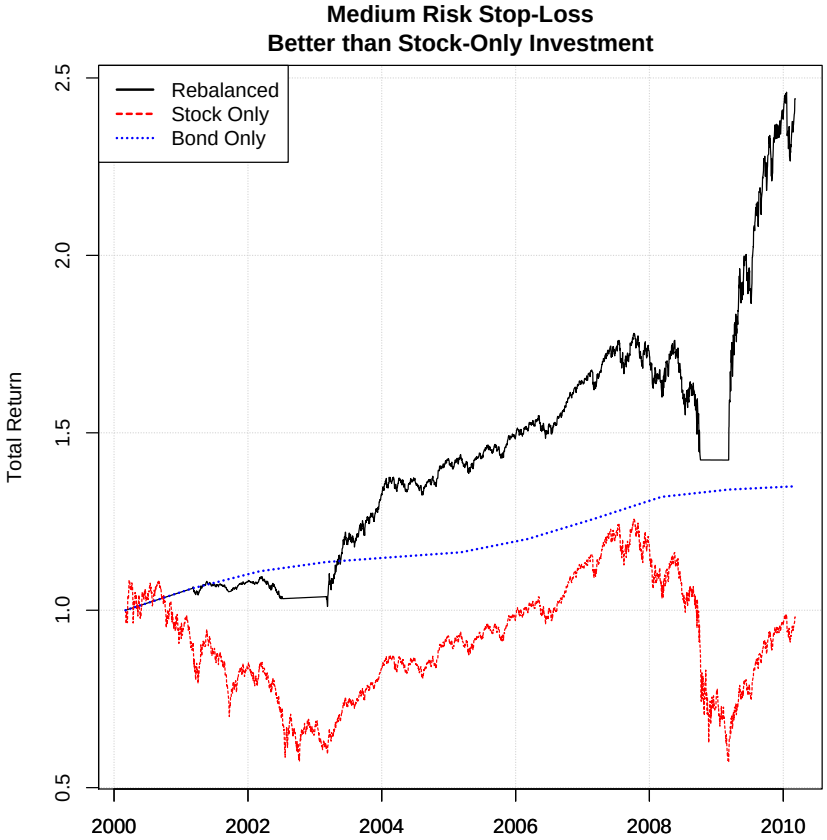

Figure 123: Total return of the medium-risk stop-loss strategy compared to stock-only and bond-only investments.

Medium Risk Stop-Loss Better than Stock-Only Investment											
Date	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
P/Book	4.79	3.64	3.43	2.54	3.09	2.89	2.75	2.74	2.44	1.50	2.17
Bond Yield	6.2%	4.5%	2.4%	1.2%	1.2%	3.2%	4.8%	4.9%	1.6%	0.7%	0.4%
Stock Weight	0	0.19	0.35	0.77	0.53	0.56	0.57	0.57	0.81	1	0.97
Rebalanced	1	1.06	1.09	1.04	1.37	1.44	1.51	1.63	1.62	1.42	2.44
Bond-Only	1	1.06	1.11	1.14	1.15	1.16	1.20	1.26	1.32	1.34	1.35
Stock-Only	1	0.89	0.84	0.62	0.87	0.94	1.00	1.11	1.05	0.57	0.98

Table 64: Total return of the medium-risk stop-loss strategy compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 118.

Worse than Stock-Only Investment

Figure 124 shows an example of the stop-loss strategy performing worse than a stock-only investment. The starting date is August 23, 1990. Table 65 shows the stock-weight gradually decreased from 0.78 in the first year to zero at the end of the period because the P/Book increased greatly during this period. For the ten-year period the total return of the stop-loss strategy was 2.99 (or 199% gain), while the total return was 1.69 (or 69% gain) for the bond-only investment and 6.09 (or 509% gain) for the stock-only investment.

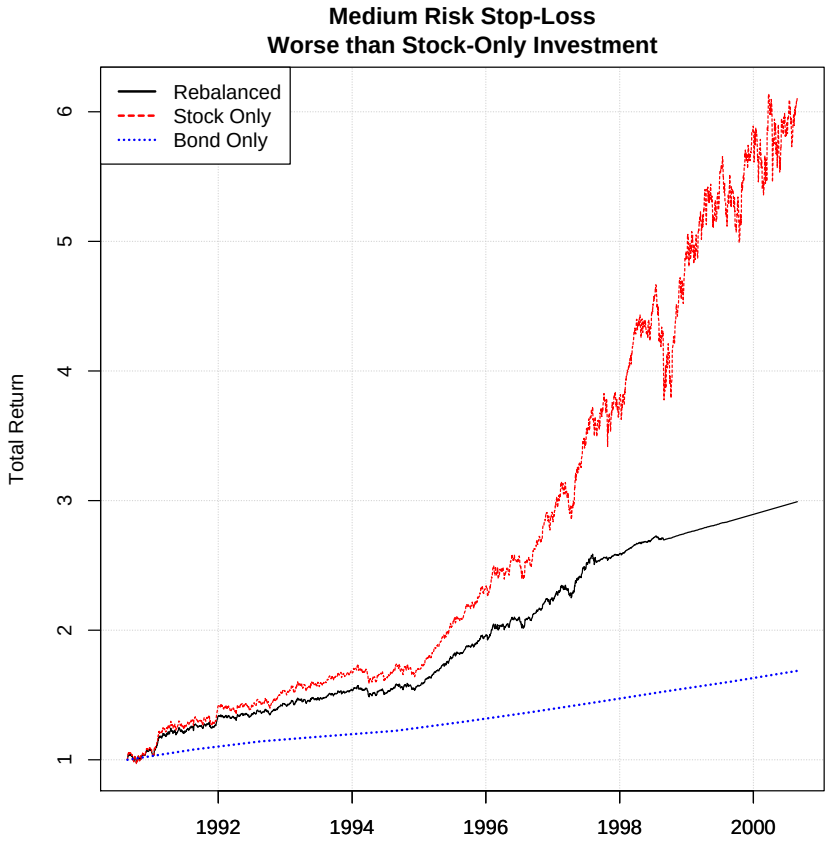

Figure 124: Total return of the medium-risk stop-loss strategy compared to stock-only and bond-only investments.

Medium Risk Stop-Loss Worse than Stock-Only Investment											
Date	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
P/Book	2.03	2.51	2.69	2.61	2.52	2.69	2.97	3.74	4.00	4.83	4.76
Bond Yield	8.0%	5.8%	3.6%	3.4%	5.6%	5.7%	5.8%	5.6%	5.0%	5.2%	6.3%
Stock Weight	0.78	0.64	0.63	0.68	0.65	0.57	0.44	0.11	0.01	0	0
Rebalanced	1	1.27	1.36	1.50	1.57	1.83	2.09	2.53	2.70	2.84	2.99
Bond-Only	1	1.08	1.14	1.18	1.22	1.29	1.37	1.44	1.52	1.60	1.69
Stock-Only	1	1.32	1.42	1.63	1.71	2.09	2.55	3.55	4.11	5.38	6.09

Table 65: Total return of the medium-risk stop-loss strategy compared to stock-only and bond-only investments. Also shown is the P/Book of the S&P 500 from Figure 2, the government bond yield from Figure 3, and the associated stock-weight from Figure 118.

15.5.4. Comparison to Non-Stop-Loss Strategy

This section compares the performance of the medium-risk stop-loss strategy to the non-stop-loss strategy from section 14.2.

Stock-Weight

The stock-weight for the stop-loss strategy during the period 1978-2013 is shown in Figure 118 and it is shown for the non-stop-loss strategy in Figure 77. The stop-loss strategy had non-zero stock-weight more often and was hence invested in the S&P 500 more often than the non-stop-loss strategy. However, the non-stop-loss strategy had more extreme stock-weights that were either closer to one or zero.

Annualized Return

Table 61 shows the annualized returns for the stop-loss strategy and Table 32 shows it for the non-stop-loss strategy.

The stop-loss strategy had median annualized return that ranged between 9.9-11.0% depending on the investment period, while it ranged between 5.8-6.9% for the non-stop-loss strategy.

The mean annualized return was also higher for the stop-loss strategy. For one-year investment periods it was 11.8% and gradually decreased to 10.5% for ten-year investment periods. The non-stop-loss strategy had mean annualized return 10.0% for one-year investment periods which gradually decreased to 8.5% for ten-year investment periods.

Gain & Loss

The greatest one-year gain was 71.8% for both the stop-loss and non-stop-loss strategies. The highest annualized returns were quite similar for longer investment periods and for ten-year periods they were identical at 20.1% which gives a compounded gain of 524% over ten years.

For one-year investment periods, the greatest loss was (15.2%) for the stop-loss strategy while it was (15.3%) for the non-stop-loss strategy. The stop-loss strategy experienced losses for investment periods up to four years, while the non-stop-loss strategy experienced losses up to five years.

The stop-loss strategy could have always been fully invested in the S&P 500, but this would sometimes cause losses of (15%) for several years in a row which was sought avoided when developing this strategy.

Probability of Loss

The stop-loss strategy had probability of loss 0.15 (or 15%) for one-year investment periods which gradually decreased to zero probability for investment periods of five years or more. The non-stop-loss strategy generally had a much lower probability of loss which was 0.05 (or 5%) for one-year investment periods and this decreased to zero probability of loss for investment periods of six years or more.

Historical vs. Future Losses

When comparing these historical losses it is important to note that the non-stop-loss strategy was sometimes fully invested in the S&P 500 and hence fully exposed to its potential losses. Although the historically greatest loss was (15.3%) for the non-stop-loss strategy, much greater losses could be experienced in the future if the S&P 500 should experience a great loss at an already low P/Book, which is when that strategy would be heavily invested in the S&P 500.

The stop-loss strategy limits the magnitude of losses by selling the S&P 500 investment once it decreases below (15%) of the purchase price, and then repurchasing the shares when the price is above the stop-price again. But the S&P 500 may linger around the stop-price which causes repeated selling and repurchasing of the shares. Although the transaction costs may be negligible, it may not be possible to buy and sell precisely at the stop-price which causes frictional costs in the form of small compounding losses. This is another reason that the stop-loss strategy is not fully invested in the S&P 500.

Probability of Under-Performing Bond-Only and Stock-Only Investments

The stop-loss strategy had a much higher probability of under-performing bond-only investments for shorter investment periods. For one-year investment periods the probability was 0.23 (or 23%) while it was only 0.14 (or 14%) for the non-stop-loss strategy. The probabilities decreased for longer investment periods so the stop-loss strategy had probability 0.02 (or 2%) for ten-year investment periods, while the non-stop-loss strategy had probability 0.03 (or 3%) of under-performing bond-only investments.

The stop-loss strategy had slightly higher probability of under-performing stock-only investments. Depending on the investment period the probability ranged between 0.55-0.72 (or 55-72%), while it ranged between 0.51-0.72 (or 51-72%) for the non-stop-loss strategy.

15.6. Summary

This section studied investment strategies that adapted the allocation between the S&P 500 index and US government bonds using the P/Book and bond yield, and also used stop-loss trading to limit the losses on the part of the portfolio invested in the S&P 500. Hypothetically, such stop-loss trading can be used to avoid losses altogether, but realistically there are some frictional costs associated with repeatedly selling and repurchasing shares of the S&P 500 around a given stop-price.

The frictional cost was modelled here as a compounding loss where each individual trade had a loss of (0.5%). It was assumed that a maximum of one trade was done per day. The number of trades and hence the frictional costs were larger for stop-prices closer to the purchase price, because then the S&P 500 was more likely to cross the stop-price repeatedly. As the stop-price became lower relative to the purchase price, the frictional costs would decrease significantly, but then the direct loss from the S&P 500 would also be potentially greater.

In order to successfully execute these stop-loss strategies it is imperative that the frictional costs are kept very low, especially if multiple intra-day trades are executed as the stop-price is crossed.

Historically, the non-stop-loss strategies had similar losses as the stop-loss strategies for low and medium risk levels, but the future losses of the non-stop-loss strategies are potentially much greater because the non-stop-loss strategies are sometimes fully invested in the S&P 500 index and hence fully exposed to its potential losses without the safety of a stop-price. Although the historical losses were of similar magnitude for these strategies, the stop-loss strategies actually had much higher historical probabilities of loss. Similarly, the stop-loss strategies had much higher probability of under-performing bond-only investments for investment periods up to a few years. The probabilities of under-performing stock-only investments were somewhat similar for the stop-loss and non-stop-loss strategies. But the stop-loss strategies did have significantly higher median and mean annualized returns than the non-stop-loss strategies.

Several modifications to these strategies are possible. For example, the stop-price could be a so-called trailing-stop which would monitor the S&P 500 price-level and adjust the stop-price to a percentage below the maximum price seen thus far, rather than having the stop-price as a percentage below the purchase price at the beginning of the year. The rebalancing could also be done monthly or quarterly rather than annually. It is not obvious whether such modifications would be an advantage because the frictional costs might also increase, so these and other modifications could be the subject of future research.

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