Comparison of U. S. Stock Indices

Magnus Erik Hvass Pedersen

Comparison of U.S. Stock Indices

Magnus Erik Hvass Pedersen
Hvass Laboratories Report HL-1503
First Edition September 30, 2015

Latest Revision

www.Hvass-Labs.org/books

Summary

This paper compares stock indices for USA: Large-Cap stocks (S&P 500), Mid-Cap stocks (S&P 400), and Small-Cap stocks (S&P 600). Between 1981 and 2015 the Large-Cap stocks returned about 11.3% per year on average, while the Mid-Cap stocks returned 14.0% per year. Between 1992 and 2015, the Small-Cap stocks returned 11.2% per year. But there were periods where each of these stock indices performed either best or worst. This paper studies more detailed performance statistics, including the average, best and worst investment periods, the correlation of returns, the historical probabilities of loss and performing worse than inflation and US government bonds, and the recovery times for these stock indices. The earnings and dividend growth is also studied and used to forecast the future returns on the stock indices.

About the Author

The author has a BSc degree in Computer Science and a PhD degree in Engineering Science. The author's previous work in finance includes a comprehensive theory on share buyback valuation, new models for financial Monte Carlo simulation, and strategies for investing in the S&P 500. The work is available at:

www.Hvass-Labs.Org

Copyright

Donations

This paper is made available for free so it can benefit as many people as possible. But the paper has been costly to produce as it required years of experience and months of research and writing. This work was funded entirely by the author. Donations are used towards these expenses and to assess whether there is interest in more books and papers like this.

If you found the paper useful then please donate. The average donation is $10 but a small donation is always better than no donation and even $1 is appreciated.

Click on the link to donate securely using PayPal.

www.Hvass-Labs.org/books

Contents

1. Introduction

Investing in individual companies is challenging because it requires insight into the future of the company's products, management, competitors, finances, etc. Some industries change so rapidly and dramatically that they are impossible to predict even for insiders. You can protect yourself against such risks by diversifying your investment in many different companies and industries.

The easiest and cheapest way of diversifying your investment is to invest in an index fund. The S&P 500 allows you to invest in 500 of the largest companies in USA. The S&P 500 is also called a Large-Cap stock-index. The S&P 400 allows you to invest in 400 mid-sized companies in USA and is called a Mid-Cap index. The S&P 600 allows you to invest in 600 small companies and is called a Small-Cap index.

The companies in these stock indices operate in a wide variety of industries including energy and utility, financial services, health care, information technology, heavy industry, manufacturers of consumer products, retailing, etc. Rather than having to assess the future of individual companies you now only have to assess the general future of companies in USA.

Between 1981 and 2015 the Large-Cap index returned about 11.3% per year on average through increases in share-price and reinvestment of dividends, while the Mid-Cap index returned 14.0% per year. Between 1992 and 2015, the Small-Cap index returned 11.2% per year. But there were periods where each of these stock indices performed either best or worst, so we need to consider more detailed statistics to properly assess their performance. That is the aim of this paper.

Statistics are given for holding periods between 1 and 10 years, where all possible investment periods between 1981 and 2015 are considered, not just the periods that coincide with calendar years. This means thousands of investment periods are considered. The paper studies the average, best and worst investment periods, the correlation of returns, the historical probabilities of loss and performing worse than inflation and US government bonds, and the recovery times for these stock indices. The earnings and dividend growth is also studied and used to forecast the future returns on the stock indices.

This paper should be useful to professional investors who manage very large portfolios, as well as small private investors who want more insight into the historical performance of US stock indices. Basic knowledge of investing and statistics is required. Beginners should first read my book dedicated to the S&P 500 [**1**] which explains all the basic concepts of index investing, but also contains information that may be useful to experienced investors.

1.1. Notation

Negative percentages are written as (x%) rather than –x%. For example, (43.2%) means –43.2%.

1.2. Video Talks

The main aspects of this paper are explained in video talks that can be viewed freely on the internet:

www.youtube.com/user/hvasslabs/playlists

2. US Stock Indices

This paper considers three broadly diversified stock indices for companies in USA. The stocks of large companies are represented by the S&P 500 index, while stocks of mid-sized companies are represented by the S&P 400 index, and stocks of small companies are represented by the S&P 600 index. These indices are mutually exclusive so that no stock is included in two indices.

2.1. Capitalization Weighted

These stock indices are weighted by the so-called float-adjusted market-capitalization of the companies. This means that each company occupies a portion of the index that is proportional to the company's number of shares available for public trading, multiplied by the price per share.

For example, the company Apple had about 5.7 billion shares outstanding on July 10, 2015 according to [**2**]. Assuming the number of shares was approximately the same a few months later on August 31, 2015 where the share-price was $112.76, gives a market capitalization (or market-cap) of about $643 billion.

2.2. Large-Cap Stocks (S&P 500)

As of August 31, 2015, the Large-Cap index known as the S&P 500 contained the stocks of 500 large US companies whose market-caps ranged between $2.4 billion for the smallest company and $643 billion for the largest company which was Apple. The average market-cap was $36.9 billion and the total market-cap for all 500 companies was $18,450 billion or about $18.5 trillion. This means the largest company in the S&P 500, Apple, with its market-cap of $643 billion accounted for about 3.5% of the entire S&P 500 index, while the smallest company with a market-cap of $2.4 billion only accounted for about 0.01% of the S&P 500 index. Data source [**3**].

2.3. Mid-Cap Stocks (S&P 400)

As of August 31, 2015, the Mid-Cap index known as the S&P 400 contained the stocks of 400 mid-sized US companies whose market-caps ranged between $736 million and $11.3 billion. The average market-cap was about $3.9 billion and the total market-cap for all 400 companies was $1,560 billion or about $1.6 trillion. This means the largest company in the S&P 400 (Church & Dwight Co.) with its market-cap of $11.3 billion accounted for about 0.7% of the entire S&P 400 index, while the smallest company with a market-cap of $736 million only accounted for about 0.05% of the S&P 400 index. Data source [**4**].

2.4. Small-Cap Stocks (S&P 600)

As of August 31, 2015, the Small-Cap index known as the S&P 600 contained the stocks of 600 small US companies whose market-caps ranged between $65 million and $4 billion. The average market-cap was almost $1.2 billion and the total market-cap for all 600 companies was almost $700 billion. This means the largest company in the S&P 600 (Toro Co.) with its market-cap of $4 billion accounted for about 0.6% of the entire S&P 600 index, while the smallest company with a market-cap of $65 million only accounted for about 0.01% of the S&P 600 index. Data source [**5**].

2.5. Criteria for Inclusion

A committee at Standard & Poor's (S&P) decides each quarter which companies to replace in these stock indices according to some guide-lines. All companies must be based in USA. At least 50% of the shares outstanding must be available for public trading. The shares must be frequently traded. The companies

must have positive as-reported earnings for the most recent quarter, as well as the most recent year. Proper balance between different business sectors is also sought when constructing the indices.

Some of these criteria are strictly enforced while others are subject to compromise. For example, during the financial crisis of 2008 and 2009, many of the companies in these indices reported negative earnings, so it would have been impossible to exclude them all from the stock indices, for violating the requirement that earnings must be positive.

2.6. Launch Dates

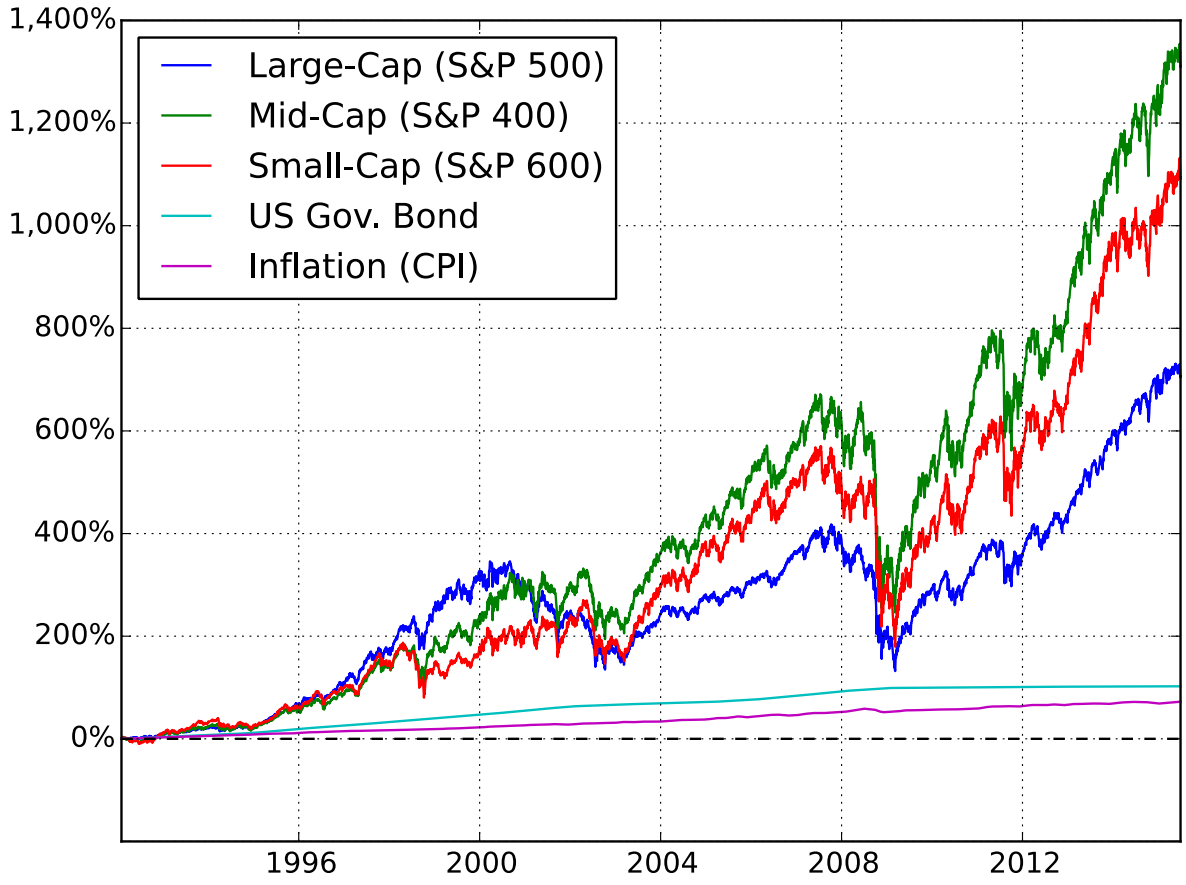
The Large-Cap index is the oldest of these three indices. It was launched in 1957 and both price and dividend data is available from this year. The other two indices are much younger. The Mid-Cap index was launched in June 1991, but the S&P researchers have re-constructed the price and dividend data all the way back to January 1981 as if the index already existed back then. The Small-Cap index was launched in October 1994, and its price and dividend data has been re-constructed back to January 1992. The older data is re-constructed according to the same principles as the newer index data, and should be valid for statistical analysis.

The following sections generally consider two time-periods: The shorter period between 1992 and 2015 where data is available for all three stock indices, and the longer period between 1981 and 2015 where data is only available for Large-Cap and Mid-Cap stocks.

3. Long-Term Returns

This section compares the three stock indices over several decades of investing. Figure 1 shows the total return for the Large-Cap, Mid-Cap and Small-Cap stock indices between January 1992 and June 2015. The total return for a stock index is the change in price-level for the index, with dividends reinvested in the index through the years and assuming there were no taxes. Also shown for comparison is the inflation as measured by the US Consumer Price Index (CPI), as well as the return that could have been earned simply by investing and reinvesting in US government bonds with 1-year maturity.

Figure 1: Total return for US Large-Cap, Mid-Cap and Small-Cap stocks between January 1992 and June 2015. Also shown is the inflation and compounded return on US Government Bonds. Data sources [6] [7] [8] [9] [10] [11].



Between January 1992 and June 2015 the total return was 707% for Large-Cap stocks, while Mid-Cap stocks returned 1,311% and Small-Cap stocks returned 1,094%. The corresponding annualized return was about 9.3% for Large-Cap stocks, while it was 12.0% for Mid-Cap stocks and 11.2% for Small-Cap stocks. Calculation of annualized returns is described in appendix 16.1.

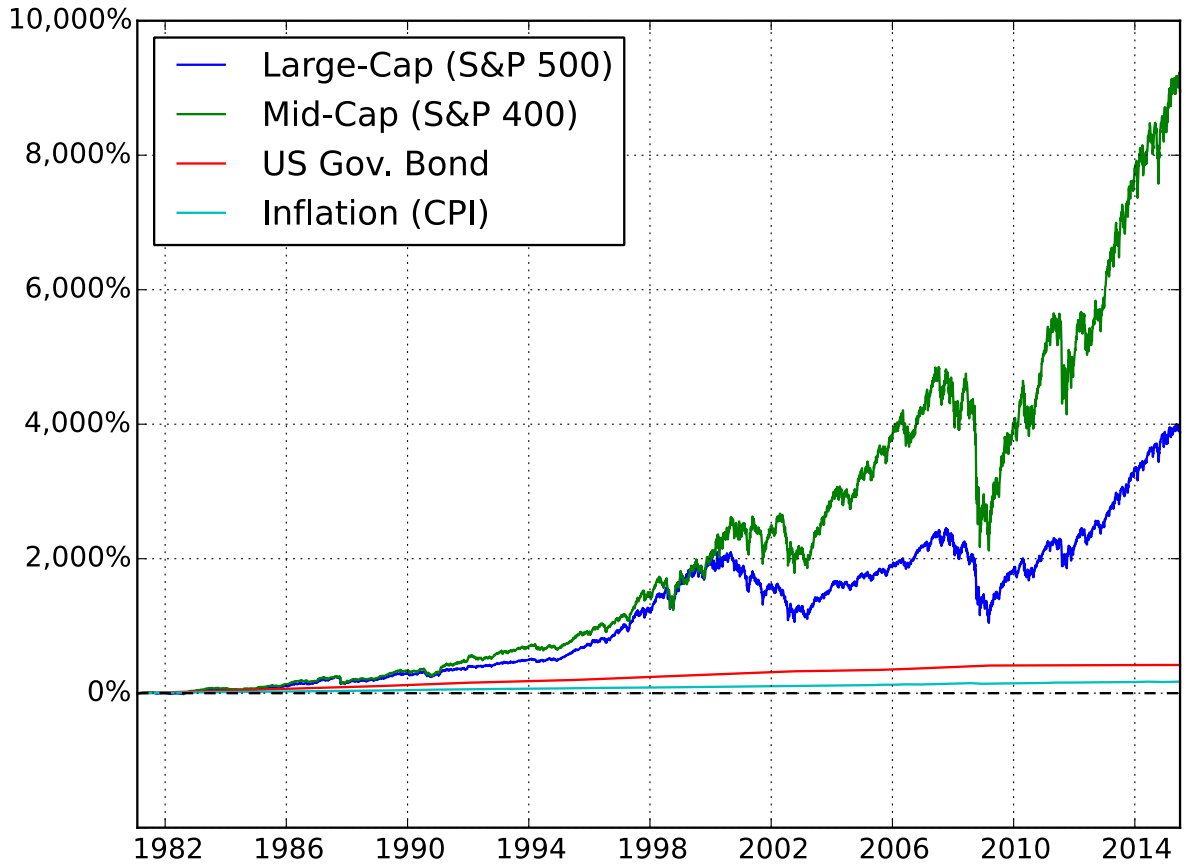
If you had invested $100 in Large-Cap stocks in January 1992 and held the investment until June 2015, and you had reinvested all dividends through the years without having to pay any taxes, then by June 2015 your investment would be worth $807 for a gain of $707. If you had instead invested $100 in Mid-Cap stocks then your gain would have been $1,311, and if you had invested $100 in Small-Cap stocks then your gain would have been $1,094.

3.1. Longer Data-Period

The data for Small-Cap stocks is only available from January 1992, while the data for Mid-Cap stocks is available from January 1981, and the data for Large-Cap stocks is available from 1957.

Figure 2 shows the total return on Large-Cap and Mid-Cap stocks for the longer data-period between January 1981 and June 2015. During this period the total return was 3,876% for Large-Cap stocks, while Mid-Cap stocks returned 8,950%. It is again assumed that dividends were reinvested and there were no taxes. The corresponding annualized returns were 11.3% for the Large-Cap stocks and 14.0% for the Mid-Cap stocks. Note that these were significantly higher than for the shorter data-period between 1992 and 2015, during which the annualized return was only 9.3% for Large-Cap stocks and 12.0% for Mid-Cap stocks.

Figure 2: Total return for US Large-Cap and Mid-Cap stocks between January 1981 and June 2015.



3.2. Which Is Best?

It would seem from Figure 1 and Figure 2 that Mid-Cap stocks were superior to Large-Cap and Small-Cap stocks. But consider two shorter periods where Mid-Cap stocks were inferior.

Figure 3 shows an example of Large-Cap stocks performing best over a 5-year period. Between April 1994 and 1999 the Large-Cap stocks had a total return of 233% with dividends reinvested, while the Mid-Cap stocks only returned 129% and the Small-Cap stocks only returned 71%. This corresponds to an annualized return of 27.2% for the Large-Cap stocks, 18.0% for the Mid-Cap stocks, and 11.3% for the Small-Cap stocks. During these 5 years US government bonds returned about 5.6% per year on average.

Figure 4 shows an example of Small-Cap stocks performing best over a 5-year period. Between September 2000 and 2005 the Large-Cap stocks lost (7.5%), while the Mid-Cap stocks returned 40.5% and the Small-Cap stocks returned 70%. This corresponds to an annualized loss of (1.6%) for the Large-Cap stocks, an annualized return of 7.0% for the Mid-Cap stocks, and an annualized return of 11.2% for the Small-Cap stocks. During these 5 years US government bonds returned about 2.7% per year on average.

This shows the necessity of comparing more statistics so as to properly assess which stock index performed best and in which way. Detailed performance statistics are studied in the following sections.

Figure 3: Total return of Large-Cap, Mid-Cap and Small-Cap stocks between April 1994 and 1999.

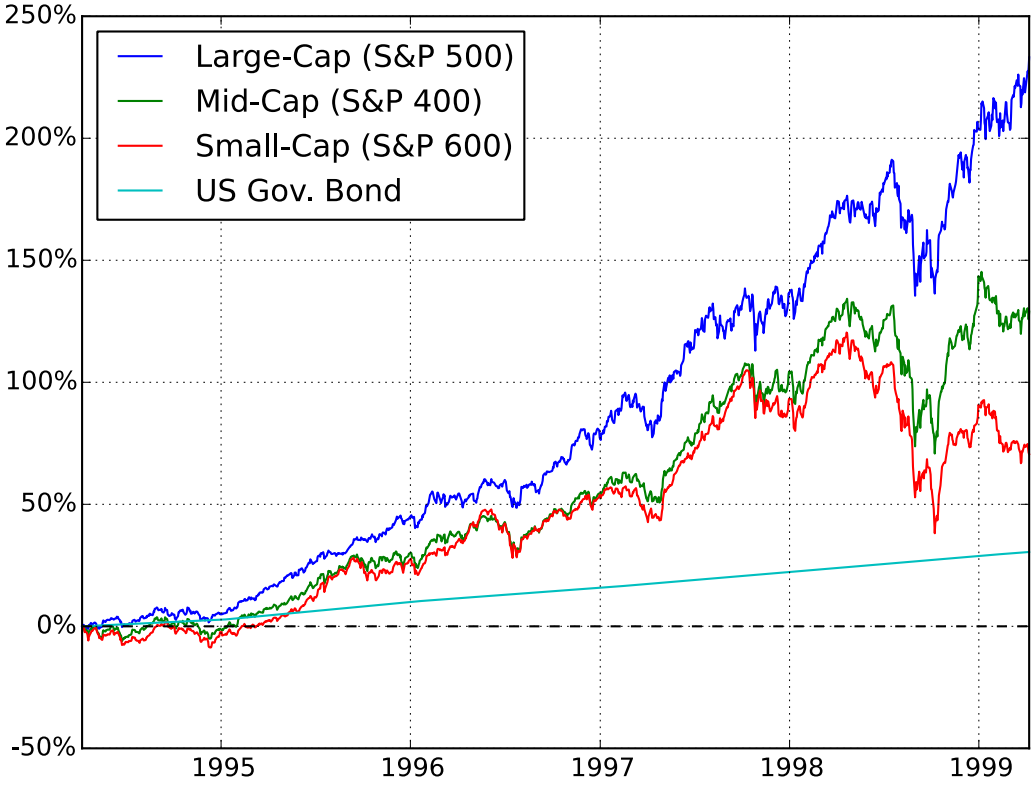
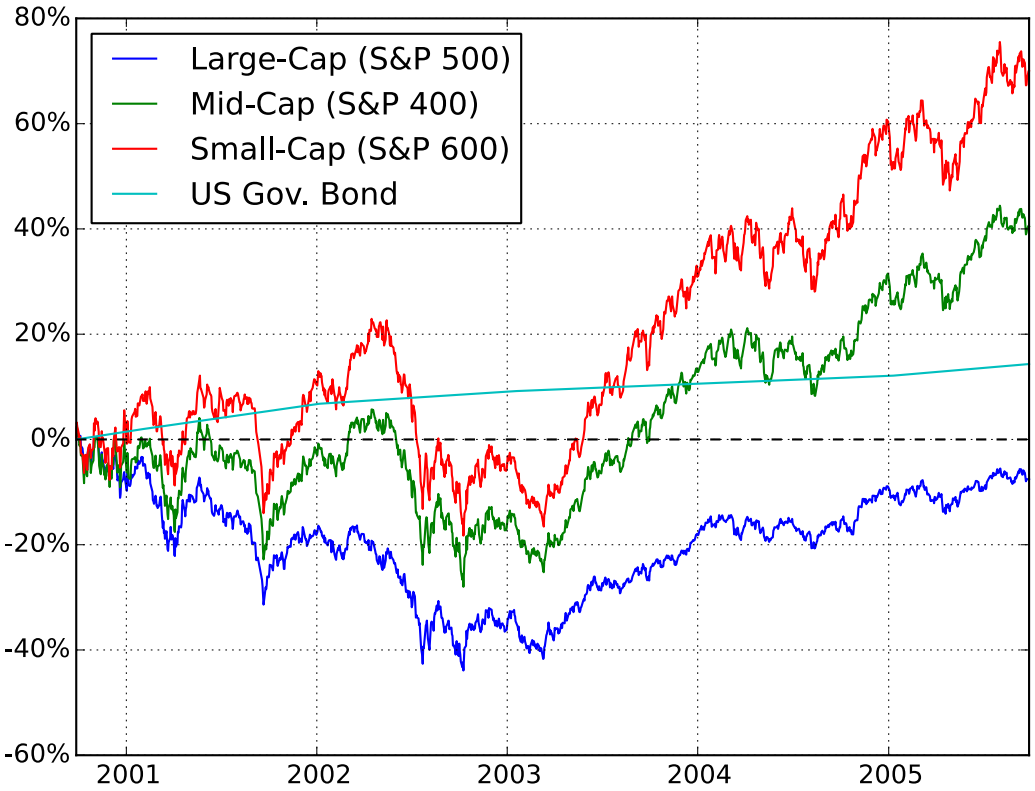


Figure 4: Total return of Large-Cap, Mid-Cap and Small-Cap stocks between September 2000 and 2005.



4. Statistics for Annualized Returns

This section studies some basic statistics for the annualized returns of Large-Cap, Mid-Cap and Small-Cap stocks for different investment periods.

4.1. Calculation Method

The statistics are calculated from the total returns shown in Figure 1 and Figure 2, by considering the total return for all investment periods of a given duration and calculating the annualized returns from these, and then calculating statistics for the annualized returns.

The starting date in Figure 1 is January 31, 1992 so we consider all investment periods starting on this date and lasting either one year, two years, and so on up to ten years. The next trading day for which we have data is February 3, 1992 but the data has been interpolated for all non-trading days as well (weekends, holidays, etc.) to make the data easier to work with. This means the next date is February 1, 1992 for which a new set of investment periods are considered, lasting either one year, two years, and up to ten years.

The last one-year period started on June 30, 2014 because the data ends on June 30, 2015. Similarly, the last 10-year period being considered started on June 30, 2005 and ended on June 30, 2015.

Between January 31, 1992 and June 30, 2015 there were a total of almost 8,200 periods of one-year duration. If the data had not been interpolated then there would only have been about 5,600 one-year periods. Similarly, there were a total of 4,900 periods of 10-year duration between January 31, 1992 and June 30, 2015, and if the data had not been interpolated then there would only have been about 3,400 periods of 10-year duration. Although we are considering many more investment periods due to the interpolation of data for non-trading days, it should give reasonably accurate performance statistics.

4.2. Average Return

Table 1 shows basic statistics for the annualized return in all investment periods of 1 up to 10 years between 1992 and 2015. For one-year investment periods, the mean (or average) annualized return was 11.1% for Large-Cap stocks, while it was 13.8% for Mid-Cap stocks, and 13.1% for Small-Cap stocks.

For longer investment periods the average annualized returns decreased and it was only 6.4% for Large-Cap stocks over 10-year periods. This means that when considering all 10-year investment periods between January 1992 and June 2015, the return on Large-Cap stocks was only 6.4% per year on average. Compare this to the higher average annualized return of 10.8% for Mid-Cap stocks and 9.8% for Small-Cap stocks. So for investment periods of 10 years, Mid-Cap stocks returned an average of 4.4% more per year than Large-Cap stocks, while Small-Cap stocks returned 3.4% more per year than Large-Cap stocks.

4.3. Worst Returns

The averages are very limited statistics that cannot tell us e.g. if there were losses and how big they were. For this we use the other statistics in Table 1 which reveal more about the returns of these stock indices.

For one-year investment periods, the worst loss was (47.4%) for Large-Cap stocks, while it was (49.2%) for Mid-Cap stocks, and (48.1%) for Small-Cap stocks. These losses occurred in the stock-market crash around year 2008 and 2009.

For 10-year investment periods the worst annualized loss was (4.5%) for Large-Cap stocks, which corresponds to a net loss of (36.9%) over ten years. There were no losses for Mid-Cap stocks in any 10-year period between 1992 and 2015, but the lowest annualized return was 2.2% which corresponds to a gain of 24.3% over ten years. There also were no losses for Small-Cap stocks in any 10-year periods, and the lowest annualized return was 2.0% which is a gain of 21.9% over ten years. These were the worst 10-year periods for the three stock indices and the periods all ended in early March 2009 which was the bottom of a large stock-market crash. The reason that Large-Cap stocks performed worse than Mid-Cap and Small-Cap stocks, was that Large-Cap stocks were severely overvalued in the so-called Dot-Com Bubble around year 1999, which apparently did not affect Mid-Cap and Small-Cap stocks as much.

4.4. Best Returns

Also shown in Table 1 are the best annualized returns for the stock indices. For one-year investment periods the best return was 72.1% for Large-Cap stocks, while it was 94.0% for Mid-Cap stocks, and 98.0% for Small-Cap stocks. These extremely large gains all occurred in the year following March 2009 which was the bottom of a large stock-market crash.

For 10-year investment periods the best annualized return was 13.4% for Large-Cap stocks, while it was 16.3% for Mid-Cap stocks, and 14.8% for Small-Cap stocks. These were the best 10-year periods for the three stock indices. For Large-Cap and Mid-Cap stocks they occurred between 1992 and 2002, while the best 10-year period for Small-Cap stocks was between 1994 and 2004.

4.5. Quartiles

The quartiles in Table 1 reveal many other interesting aspects about the returns on the stock indices. Although Large-Cap stocks sometimes experienced losses after 10 years of investing, the 1st quartile shows that at least 75% of the 10-year investment periods between 1992 and 2015 actually had positive returns greater than 3.3%. Combined with the 3rd quartile we know that half of the annualized returns on Large-Cap stocks were between 3.3% and 9.4% after 10 years of investing. For Mid-Cap stocks this range was much higher between 8.2% and 13.8%, while the range for Small-Cap stocks was between 7.9% and 11.8%.

4.6. Risk

In the academic literature, financial risk is usually measured from the standard deviation of investment returns (or equivalently the variance). The standard deviation is a simple statistical measure of spread. Several Nobel Prizes have even been awarded to theories that are based on the assumption that the standard deviation is a good measure of financial risk. But this is a gross misunderstanding of both statistics and finance, as can be demonstrated with a small example.

Table 1 shows that the standard deviation was 5.2% for annualized returns after 6 years of investing in Mid-Cap stocks. Then for 7-year investment periods the standard deviation for annualized returns dropped to 4.3%. Finance professors commonly interpret a lower standard deviation as a lower financial risk. But the quartiles show that there were losses in some 7-year investment periods with the worst annualized loss being (2.8%), while there were no losses for 6-year investment periods. So the risk of loss was greater for 7-year investment periods than it was for 6-year periods, even though the standard deviation was lower for 7-year investment periods.

The standard deviation measures the spread of outcomes. A small standard deviation means there was a small spread of outcomes, and conversely a large standard deviation means there was a large spread of outcomes. The standard deviation does not reveal whether there were losses, how big those losses were, or how often losses occurred. The standard deviation also does not reveal whether a stock index performed worse than inflation, government bonds, or other stock indices. More detailed statistics are needed if we want to answer these questions, which is done in some of the sections below.

4.7. Longer Data-Period

The period between 1992 and 2015 experienced three bull-markets and two market crashes of historic proportions, which may have distorted the performance statistics compared to longer data-periods. Table 2 shows the performance statistics for the longer period between 1981 and 2015, for which data is only available for Large-Cap and Mid-Cap stocks, but not Small-Cap stocks.

For Large-Cap stocks the average annualized return was 10.7% for all 10-year investment periods between 1981 and 2015, compared to an average of only 6.4% for all 10-year periods between 1992 and 2015.

For Mid-Cap stocks the average annualized return was 13.7% for all 10-year investment periods between 1981 and 2015, compared to an average of only 10.8% for all 10-year periods between 1992 and 2015.

The great market volatility with three bull-markets and two collapses between 1992 and 2015 contributed to Large-Cap and Mid-Cap stocks performing significantly worse than their longer-term averages.

4.8. Summary

This section studied basic performance statistics for the three stock indices. It was shown that the average annualized return generally decreased for longer investment periods. It was also shown that Large-Cap stocks generally performed significantly worse than Mid-Cap and Small-Cap stocks.

These basic performance statistics are useful for an overview, but we still do not know the probability of the stock indices having losses, and the probability of performing worse than inflation and government bonds, and if the stock indices performed in sync with each other. This is studied in the following sections.

Table 1: Annualized return for Large-Cap, Mid-Cap and Small-Cap stocks.
Statistics are shown for all investment periods from 1 to 10 years between January 1992 and June 2015.

Large-Cap (S&P 500, 1992-2015)							
Years of Investing	**Mean**	**Stdev**	**Min**	**1st Qrt.**	**Median**	**3rd Qrt.**	**Max**
1	11.1%	17.7%	(47.4%)	4.4%	13.4%	22.5%	72.1%
2	10.3%	14.3%	(28.9%)	2.8%	12.0%	20.8%	42.5%
3	9.8%	12.3%	(17.2%)	0.2%	12.1%	17.8%	33.5%
4	9.2%	10.9%	(11.8%)	(1.1%)	8.6%	17.3%	31.5%
5	8.7%	9.6%	(8.2%)	0.2%	6.1%	17.0%	28.5%
6	7.9%	7.9%	(1.7%)	2.2%	4.0%	13.2%	25.1%
7	7.1%	6.3%	(5.7%)	3.1%	5.0%	8.1%	21.9%
8	6.8%	5.2%	(5.7%)	3.7%	6.2%	9.0%	20.9%
9	6.5%	4.5%	(6.1%)	4.5%	7.0%	9.3%	16.8%
10	6.4%	4.2%	(4.5%)	3.3%	7.7%	9.4%	13.4%

Mid-Cap (S&P 400, 1992-2015)							
Years of Investing	**Mean**	**Stdev**	**Min**	**1st Qrt.**	**Median**	**3rd Qrt.**	**Max**
1	13.8%	17.6%	(49.2%)	4.7%	15.5%	24.3%	94.0%
2	12.9%	11.9%	(29.2%)	8.3%	14.2%	20.3%	56.7%
3	12.5%	9.0%	(18.0%)	7.2%	14.3%	18.7%	36.5%
4	12.1%	7.5%	(10.9%)	6.2%	13.0%	18.5%	31.2%
5	11.9%	6.6%	(6.6%)	6.7%	11.5%	17.6%	29.8%
6	11.5%	5.2%	0.5%	7.2%	10.5%	16.1%	26.1%
7	11.0%	4.3%	(2.8%)	8.2%	10.2%	13.4%	19.9%
8	10.9%	4.0%	(1.6%)	8.3%	11.2%	13.5%	20.2%
9	10.8%	3.6%	(0.7%)	8.9%	10.7%	13.9%	17.9%
10	10.8%	3.2%	2.2%	8.2%	10.8%	13.8%	16.3%

Small-Cap (S&P 600, 1992-2015)							
Years of Investing	**Mean**	**Stdev**	**Min**	**1st Qrt.**	**Median**	**3rd Qrt.**	**Max**
1	13.1%	18.3%	(48.1%)	3.1%	14.6%	24.6%	98.0%
2	11.9%	12.0%	(31.9%)	6.2%	13.1%	19.8%	56.7%
3	11.4%	9.2%	(20.2%)	6.6%	12.9%	17.2%	37.2%
4	10.9%	7.4%	(13.0%)	6.1%	12.0%	16.5%	32.0%
5	10.6%	6.4%	(7.7%)	4.2%	11.4%	15.0%	31.8%
6	10.0%	4.6%	0.4%	5.8%	10.7%	12.9%	26.8%
7	9.6%	3.3%	(3.2%)	7.4%	9.4%	12.2%	16.2%
8	9.7%	2.9%	(1.3%)	7.8%	9.7%	11.7%	16.5%
9	9.7%	2.7%	(1.3%)	8.3%	9.7%	11.8%	15.6%
10	9.8%	2.5%	2.0%	7.9%	10.1%	11.8%	14.8%

Table 2: Annualized return for Large-Cap and Mid-Cap stocks.
Statistics are shown for all investment periods from 1 to 10 years between January 1981 and June 2015.

Large-Cap (S&P 500, 1981-2015)							
Years of Investing	**Mean**	**Stdev**	**Min**	**1st Qrt.**	**Median**	**3rd Qrt.**	**Max**
1	13.0%	17.5%	(47.4%)	4.8%	14.5%	24.3%	72.1%
2	12.4%	12.8%	(28.9%)	7.8%	13.2%	21.2%	42.5%
3	11.8%	10.8%	(17.2%)	6.2%	13.7%	18.0%	33.5%
4	11.5%	9.6%	(11.8%)	4.2%	13.7%	17.9%	31.5%
5	11.3%	8.7%	(8.2%)	2.2%	13.4%	17.4%	31.8%
6	10.9%	7.5%	(1.7%)	3.3%	12.3%	17.3%	25.1%
7	10.6%	6.7%	(5.7%)	4.3%	11.1%	16.4%	23.7%
8	10.7%	6.3%	(5.7%)	5.3%	10.8%	16.2%	22.2%
9	10.7%	6.1%	(6.1%)	6.2%	11.4%	15.9%	21.5%
10	10.7%	5.9%	(4.5%)	7.4%	11.2%	15.3%	19.9%

Mid-Cap (S&P 400, 1981-2015)							
Years of Investing	**Mean**	**Stdev**	**Min**	**1st Qrt.**	**Median**	**3rd Qrt.**	**Max**
1	15.5%	18.2%	(49.2%)	4.4%	16.8%	26.4%	94.0%
2	14.7%	11.1%	(29.2%	9.5%	15.3%	22.0%	56.7%
3	14.1%	8.2%	(18.0%)	10.8%	15.2%	19.4%	36.5%
4	13.9%	6.9%	(10.9%)	9.5%	15.5%	18.9%	31.2%
5	13.8%	6.2%	(6.6%)	9.4%	14.5%	18.6%	29.8%
6	13.5%	5.0%	0.5%	9.7%	14.7%	17.4%	26.1%
7	13.3%	4.6%	(2.8%)	9.4%	13.7%	17.2%	24.2%
8	13.4%	4.4%	(1.6%)	10.0%	13.8%	17.2%	21.4%
9	13.5%	4.3%	(0.7%)	10.1%	14.2%	17.0%	21.2%
10	13.7%	4.1%	2.2%	10.5%	14.5%	16.7%	21.8%

5. Probability of Loss

This section studies the historical probability of loss for different investment periods. It is assumed that dividends are reinvested and there were no taxes. For example, if you had invested $100 in either Large-Cap, Mid-Cap or Small-Cap stocks on any date between 1992 and 2010, and held on to the investment for five years while reinvesting the dividends tax-free; what was the probability that the investment would be worth less than $100 after five years? This question can be answered from the following tables.

Table 3 shows the probability of loss for Large-Cap, Mid-Cap and Small-Cap stocks between 1992 and 2015. These probabilities are calculated by counting the number of investment periods that resulted in a loss and divide by the total number of investment periods of a given duration. There were about 8,200 one-year investment periods between January 1992 and June 2015. Of these one-year periods, 19.2% showed a loss on Large-Cap stocks, while 18.4% showed a loss on Mid-Cap stocks, and 20.7% showed a loss on Small-Cap stocks according to Table 3. For two-year investment periods the probability of loss increased to 22.5% for Large-Cap stocks, while the probability of loss decreased to 13.2% for Mid-Cap stocks and 15.0% for Small-Cap stocks. The probability of loss continued to increase to 30.3% for Large-Cap stocks with investment periods of 4 years, where the probability of loss decreased to only 5.9% for Mid-Cap stocks and 9.9% for Small-Cap stocks. For investment periods of 6 years or more the probability of loss was zero or nearly zero for Mid-Cap and Small-Cap stocks, while the probability of loss remained high for Large-Cap stocks and was 14.0% for 10-year investment periods.

Table 3: Probability of loss for Large-Cap, Mid-Cap and Small-Cap stocks.

Probability of Loss (1992-2015)										
Years of Investing	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Large-Cap (S&P 500)	19.2%	22.5%	24.6%	30.3%	23.2%	6.5%	3.8%	7.2%	9.3%	14.0%
Mid-Cap (S&P 400)	18.4%	13.2%	12.5%	5.9%	2.6%	0%	0.6%	0.3%	0.1%	0%
Small-Cap (S&P 600)	20.7%	15.0%	11.8%	9.9%	4.2%	0%	0.6%	0.2%	0.2%	0%

Table 4 shows the probability of loss for only Large-Cap and Mid-Cap stocks between 1981 and 2015. For one-year investment periods the probability of loss on Large-Cap stocks was 19.0% while the probability of loss on Mid-Cap stocks was 19.8%. The probability of loss generally decreased for longer investment periods, although the Large-Cap stocks still had a 4.0% probability of loss for 6-year periods while Mid-Cap stocks had zero or nearly zero probability of loss for investment periods of 6 years or more.

Table 4: Probability of loss for Large-Cap and Mid-Cap stocks.

Probability of Loss (1981-2015)										
Years of Investing	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Large-Cap (S&P 500)	19.0%	14.9%	16.0%	19.4%	14.6%	4.0%	2.3%	4.2%	5.3%	7.7%
Mid-Cap (S&P 400)	19.8%	8.7%	8.1%	3.8%	1.6%	0%	0.3%	0.2%	0.08%	0%

The conclusion is that losses were increasingly unlikely for longer investment periods. Mid-Cap and Small-Cap stocks rarely had losses after 6 years of investing, while Large-Cap stocks sometimes had losses for even 10-year periods. This was possibly because of the severe overvaluation and ensuing crash of Large-Cap stocks around year 2000, which did not affect Mid-Cap and Small-Cap stocks as much, see Figure 5 further below.

It is important to note that these are historical probabilities (or frequencies) which may not hold in the future. It is possible for Mid-Cap and Small-Cap stocks to experience losses after 10 years of investing in the future, if these stock indices should become severely overvalued as the Large-Cap stocks were around year 2000. But losses on the original amount invested should become less likely for longer investment periods, provided the earnings of the underlying companies will grow over time, so that overvalued stock-prices will eventually be earned back through earnings growth. This is discussed in more detail in sections 12 and 13.

6. Compared to Inflation

The previous section studied the probabilities of nominal losses, that is, if you had invested $100 then what was the probability that your investment was worth less than $100 after, say, 5 years of investing.

This section studies the probabilities of inflation-adjusted losses, that is, if you had invested $100 and held the investment for 5 years during which the inflation was 10%, then what was the probability your investment was worth less than $110 after those 5 years? Such questions can be answered from the following tables.

Table 5 shows the probabilities of Large-Cap, Mid-Cap and Small-Cap stocks performing worse than inflation for different investment periods between 1992 and 2015. It is assumed that dividends are reinvested and there were no taxes. Note that the probabilities of inflation-adjusted losses are significantly greater than the probabilities of nominal losses in Table 3, especially for longer investment periods. For example, for one-year investment periods the probability was 22.3% that Large-Cap stocks did not match inflation, while the probability was 22.2% for Mid-Cap stocks and 24.1% for Small-Cap stocks. Large-Cap stocks frequently performed worse than inflation even after several years of investing. For 5-year investment periods the probability was 40.2% that Large-Cap stocks failed to match inflation and for 10-year investment periods the probability was 20.3%. Conversely, Mid-Cap and Small-Cap stocks almost always performed better than inflation when investing for 6 years or more, and for 10 years of investing the probability of performing worse than inflation was only 0.1% for Mid-Cap and Small-Cap stocks.

Table 5: Probability of performing worse than inflation when investing in Large-Cap, Mid-Cap and Small-Cap stocks.

Probability of Under-Performing Inflation (1992-2015)										
Years of Investing	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Large-Cap (S&P 500)	22.3%	25.1%	29.5%	34.9%	40.2%	27.5%	20.1%	17.4%	18.5%	20.3%
Mid-Cap (S&P 400)	22.2%	15.2%	15.2%	10.2%	5.3%	0.3%	3.3%	3.4%	1.5%	0.1%
Small-Cap (S&P 600)	24.1%	18.5%	14.9%	15.1%	13.7%	0.6%	2.8%	1.5%	1.1%	0.1%

Table 6 shows the probability of performing worse than inflation for Large-Cap and Mid-Cap stocks between 1981 and 2015. For one-year investment periods the probability of performing worse than inflation was 22.4% for Large-Cap stocks and 23.6% for Mid-Cap stocks. For 6 years or more of investing, Mid-Cap stocks rarely performed worse than inflation, while Large-Cap stocks performed worse than inflation in 11.2% of all 10-year investment periods between 1981 and 2015.

Table 6: Probability of performing worse than inflation when investing in Large-Cap and Mid-Cap stocks.

Probability of Under-Performing Inflation (1981-2015)										
Years of Investing	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Large-Cap (S&P 500)	22.4%	16.9%	19.8%	22.3%	25.2%	16.9%	12.1%	10.2%	10.5%	11.2%
Mid-Cap (S&P 400)	23.6%	10.9%	10.2%	6.5%	3.3%	0.2%	2.0%	2.0%	0.9%	0.06%

The conclusion is that Large-Cap stocks frequently failed to match inflation, even for longer investment periods, while Mid-Cap and Small-Cap stocks rarely ever failed to match inflation when investing for 6 years or more. Keep in mind that these are historical statistics and may not hold in the future.

7. Compared to US Government Bonds

The previous two sections studied the historical probabilities of nominal and inflation-adjusted losses when investing in Large-Cap, Mid-Cap and Small-Cap stock indices. This section studies the probabilities of the stock indices performing worse than what could have been earned simply by investing and reinvesting in US government bonds with one-year maturity. It is assumed that there were no taxes on stocks and bonds.

Table 7 shows the probabilities of Large-Cap, Mid-Cap and Small-Cap stocks under-performing investments in US government bonds. Note that these probabilities are slightly higher than the probabilities of under-performing inflation as shown in Table 5, which is to be expected because the bond yields are typically somewhat higher than the inflation. For example, for 5-year investment periods the probability of under-performing US government bonds was 40.6% for Large-Cap stocks, while the probability was only 7.7% for Mid-Cap stocks and 17.5% for Small-Cap stocks. For 6 years or more of investing, the Mid-Cap and Small-Cap stocks rarely ever under-performed US government bonds, while Large-Cap stocks under-performed the bonds in 22.7% of all 10-year investment periods between 1992 and 2015.

Table 7: Probability of under-performing US Government Bonds when investing in Large-Cap, Mid-Cap and Small-Cap stocks.

Probability of Under-Performing US Gov. Bonds (1992-2015)										
Years of Investing	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Large-Cap (S&P 500)	22.3%	26.3%	30.7%	35.8%	40.6%	36.2%	25.0%	22.3%	18.9%	22.7%
Mid-Cap (S&P 400)	22.7%	16.6%	16.9%	13.6%	7.7%	0.5%	3.5%	4.8%	4.2%	0.4%
Small-Cap (S&P 600)	24.7%	21.3%	18.7%	17.1%	17.5%	1.7%	3.3%	2.8%	2.1%	0.4%

Table 8 shows the probabilities of Large-Cap and Mid-Cap stocks performing worse than US government bonds between 1981 and 2015. For one-year investment periods the probabilities for Large-Cap and Mid-Cap stocks are similar at around 24-25%. But for longer investment periods the probabilities are very different. For longer investment periods the Mid-Cap stocks perform increasingly better and for investment periods of 6 years or more the Mid-Cap stocks rarely ever under-performed US government bonds. Conversely, Large-Cap stocks frequently under-performed US government bonds and the probability of under-performance was 25.4% for 5-year investment periods and 12.5% for 10-year investment periods.

Table 8: Probability of under-performing US Government Bonds when investing in Large-Cap and Mid-Cap stocks.

Probability of Under-Performing US Gov. Bonds (1981-2015)										
Years of Investing	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Large-Cap (S&P 500)	24.2%	20.2%	22.3%	23.0%	25.4%	22.2%	15.0%	13.0%	10.7%	12.5%
Mid-Cap (S&P 400)	24.7%	15.0%	12.4%	8.8%	4.8%	0.3%	2.1%	2.8%	2.4%	0.2%

The conclusion is that the Large-Cap, Mid-Cap and Small-Cap indices were about equally likely to under-perform US government bonds in any given year, but for longer investment periods their performances were very different. Mid-Cap and Small-Cap stocks performed increasingly better for longer investment periods. When investing for 6 years or more, Mid-Cap and Small-Cap stocks rarely ever under-performed US government bonds. Large-Cap stocks frequently under-performed US government bonds even for 5 and 10-year investment periods. Keep in mind that these are historical statistics and may not hold in the future.

8. Compared to Other Stocks

The previous two sections studied the probabilities of the three stock indices performing worse than inflation and US government bonds. This section studies the probabilities that the stock indices perform worse than each other.

Worse than Large-Cap

Table 9 shows the historical probabilities of Mid-Cap and Small-Cap stocks performing worse than Large-Cap stocks for different investment periods between 1992 and 2015. For one-year investment periods the probabilities were quite similar at around 42%. This means that in about 42% of all one-year periods between 1992 and 2015, the Mid-Cap and Small-Cap stocks performed worse than Large-Cap stocks. The probabilities decrease for longer investment periods. For 5-year investment periods the probability was 20.5% that Mid-Cap stocks performed worse than Large-Cap stocks, and the probability was 25.7% that Small-Cap stocks performed worse than Large-Cap stocks. For 10-year investment periods the probability was zero that Mid-Cap stocks performed worse than Large-Cap stocks, so there were no 10-year periods between 1992 and 2015 in which Mid-Cap stocks performed worse than Large-Cap stocks. The probability was 1.3% that Small-Cap stocks performed worse than Large-Cap stocks for 10-year investment periods, so this rarely ever happened as well.

Table 9: Probability of Mid-Cap stocks (S&P 400) and Small-Cap stocks (S&P 600) under-performing Large-Cap stocks (S&P 500).

Probability of Under-Performing Large-Cap Stocks (1992-2015)										
Years of Investing	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Mid-Cap < Large-Cap	41.7%	37.3%	31.5%	26.6%	20.5%	16.4%	12.6%	4.7%	0.02%	0%
Small-Cap < Large-Cap	42.8%	39.2%	36.3%	31.6%	25.7%	22.9%	19.7%	15.2%	12.9%	1.3%

Small-Cap Worse than Mid-Cap

The above table would indicate that Mid-Cap stocks performed somewhat better than Small-Cap stocks. To confirm this, Table 10 compares Mid-Cap stocks directly to Small-Cap stocks. For one-year investment periods the probability was 50.5% that Small-Cap stocks performed worse than Mid-Cap stocks, which means there was about equal chance that either Small-Cap or Mid-Cap stocks performed best after a single year of investing. This was also the case for investment periods up to 4 years after which Mid-Cap stocks frequently performed better than Small-Cap stocks. For investment periods of 5 years the probability was 62.2% that Small-Cap stocks performed worse than Mid-Cap stocks, while the probability was 75.1% for 10-year investment periods. So if you had invested in Mid-Cap stocks in any 10-year period between 1992 and 2015 then there was 75.1% chance that your return would have been higher than if you had invested in Small-Cap stocks. Furthermore, Table 9 shows there was 100% chance that your investment in Mid-Cap stocks returned more than Large-Cap stocks after 10 years of investing.

Table 10: Probability of Small-Cap stocks (S&P 600) under-performing Mid-Cap stocks (S&P 400).

Probability of Small-Cap Stocks Under-Performing Mid-Cap Stocks (1992-2015)										
Years of Investing	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Small-Cap < Mid-Cap	50.5%	48.2%	51.0%	56.0%	62.2%	68.8%	76.3%	80.6%	82.8%	75.1%

Longer Data-Period

Table 11 shows the probabilities of Mid-Cap stocks performing worse than Large-Cap stocks for the longer data-period between 1981 and 2015. The probabilities are somewhat similar to those for the shorter data-period between 1992 and 2015 shown in Table 9 above. For one-year investment periods the probability was 39.3% that Mid-Cap stocks performed worse than Large-Cap stocks. The probabilities gradually decreased for longer investment periods. For 5-year investment periods the probability was 26.1% that Mid-Cap stocks performed worse than Large-Cap stocks, and for 10-year investment periods the probability was 7.2%. Note that the latter probability was not zero for this longer data-period between 1981 and 2015, while it was zero for the data-period between 1992 and 2015; although a probability of 7.2% still means that Mid-Cap stocks only rarely performed worse than Large-Cap stocks after 10 years of investing.

Table 11: Probability of Mid-Cap stocks (S&P 400) under-performing Large-Cap stocks (S&P 500).

Probability of Mid-Cap Stocks Under-Performing Large-Cap Stocks (1981-2015)										
Years of Investing	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Mid-Cap < Large-Cap	39.3%	37.6%	34.2%	31.2%	26.1%	25.6%	19.1%	14.4%	10.4%	7.2%

Summary

The conclusion is that Mid-Cap and Small-Cap stocks have historically performed better than Large-Cap stocks. For one-year investment periods the probability was almost 60% that Mid-Cap and Small-Cap stocks performed better than Large-Cap stocks. For longer investment periods the probability was even higher and for 10-year investment periods Mid-Cap and Small-Cap stocks were almost always better than Large-Cap stocks. Furthermore, there was about 50% chance that Mid-Cap stocks were better than Small-Cap stocks in any given year, but after 10 years of investing the probability was about 75% that Mid-Cap stocks were better than Small-Cap stocks. So the overall conclusion is that Mid-Cap stocks have mostly performed better than both Small-Cap and Large-Cap stocks for longer investment periods.

Keep in mind that these are historical probabilities which may not hold in the future.

9. Correlation

This section studies the correlation between stock indices. A correlation coefficient of 1 means two stock indices are perfectly correlated so they have high or low returns in perfect synchronization. Conversely, a correlation coefficient of -1 means one stock index always has a high return when the other index has a low return, and vice versa. A correlation coefficient of zero means there is no linear relationship between the returns of the two stock indices.

Table 12 shows the correlation coefficients between Large-Cap, Mid-Cap and Small-Cap stocks for different investment periods between 1992 and 2015. These correlation coefficients are calculated for the annualized total returns of the stock indices. All these correlation coefficients are very high. For ten-year investment periods the correlation coefficient is almost 1 for all three stock indices, which means there is almost perfect synchronization of the returns. This means that whenever one stock index had a high return after 10 years of investing, then the other two indices also had high returns, and vice versa for low returns. But it does not mean that the returns were identical, merely that the returns were almost always high or low for the same 10-year periods.

Table 12: Correlation coefficients between Large-Cap, Mid-Cap and Small-Cap stocks.

Correlation Between Stocks (1992-2015)										
Years of Investing	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Mid-Cap vs. Large-Cap	0.89	0.88	0.89	0.90	0.90	0.88	0.86	0.88	0.91	0.94
Small-Cap vs. Large-Cap	0.79	0.77	0.76	0.77	0.75	0.71	0.67	0.76	0.85	0.92
Small-Cap vs. Mid-Cap	0.94	0.93	0.91	0.90	0.91	0.89	0.90	0.95	0.97	0.97

Table 13 shows the correlation coefficients between Large-Cap and Mid-Cap stocks for the longer data-period between 1981 and 2015. These correlation coefficients are very similar but slightly higher than for the shorter data-period between 1992 and 2015 shown in Table 12.

Table 13: Correlation coefficients between Mid-Cap and Large-Cap stocks.

Correlation Between Mid-Cap and Large-Cap Stocks (1981-2015)										
Years of Investing	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Mid-Cap vs. Large-Cap	0.90	0.88	0.89	0.90	0.90	0.91	0.91	0.93	0.95	0.97

The conclusion is that Large-Cap, Mid-Cap and Small-Cap stock indices have had highly correlated returns. For 10-year investment periods the correlation was nearly perfect, which means the stock indices have mostly had high or low returns for the same 10-year periods. This does not mean that the returns have been equal as Mid-Cap stocks have previously been shown to out-perform both Small-Cap and Large-Cap stocks for most 10-year periods. But the near-perfect correlation shows that whenever Mid-Cap stocks have had a high return then so has Small-Cap and Large-Cap stocks, and vice versa for low returns.

Keep in mind that these are historical correlations which may not hold in the future.

10. Recovery Times

This section studies the time it takes for a stock index to recover from losses. Only the first recovery is considered here and subsequent declines and recoveries are ignored.

Table 14 shows the historical probability of recovering from losses within a given time period. For example, 62.3% of all losses on the Large-Cap stock index S&P 500 were recovered within 7 calendar days (not trading days), while 85.8% of all losses were recovered within a calendar month, and 96.9% of all losses were recovered within a calendar year. The probabilities of recovering for Mid-Cap and Small-Cap stocks were similar.

Table 14 shows these historical probabilities for all three stock indices for the period 1992-2015, while Table 15 only shows the probabilities for Large-Cap and Mid-Cap stocks but for the longer period 1981-2015. The probabilities are similar.

Table 14: Probability of recovering from losses within a given period of time for Large-Cap, Mid-Cap and Small-Cap stocks between 1992 and 2015.

Probability of Recovering From Losses Within Given Period						
Period	**7 Days**	**1 Month**	**3 Months**	**6 Months**	**1 Year**	**2 Years**
Large-Cap (S&P 500)	62.3%	85.8%	93.9%	95.9%	96.9%	97.8%
Mid-Cap (S&P 400)	58.9%	84.5%	92.8%	96.5%	98.4%	99.8%
Small-Cap (S&P 600)	57.8%	83.1%	92.1%	96.6%	98.1%	99.7%

Table 15: Probability of recovering from losses within a given period of time for Large-Cap and Mid-Cap stocks between 1981 and 2015.

Probability of Recovering From Losses Within Given Period						
Period	**7 Days**	**1 Month**	**3 Months**	**6 Months**	**1 Year**	**2 Years**
Large-Cap (S&P 500)	61.0%	85.8%	94.3%	96.3%	97.5%	98.5%
Mid-Cap (S&P 400)	58.0%	83.8%	92.5%	96.3%	98.5%	99.8%

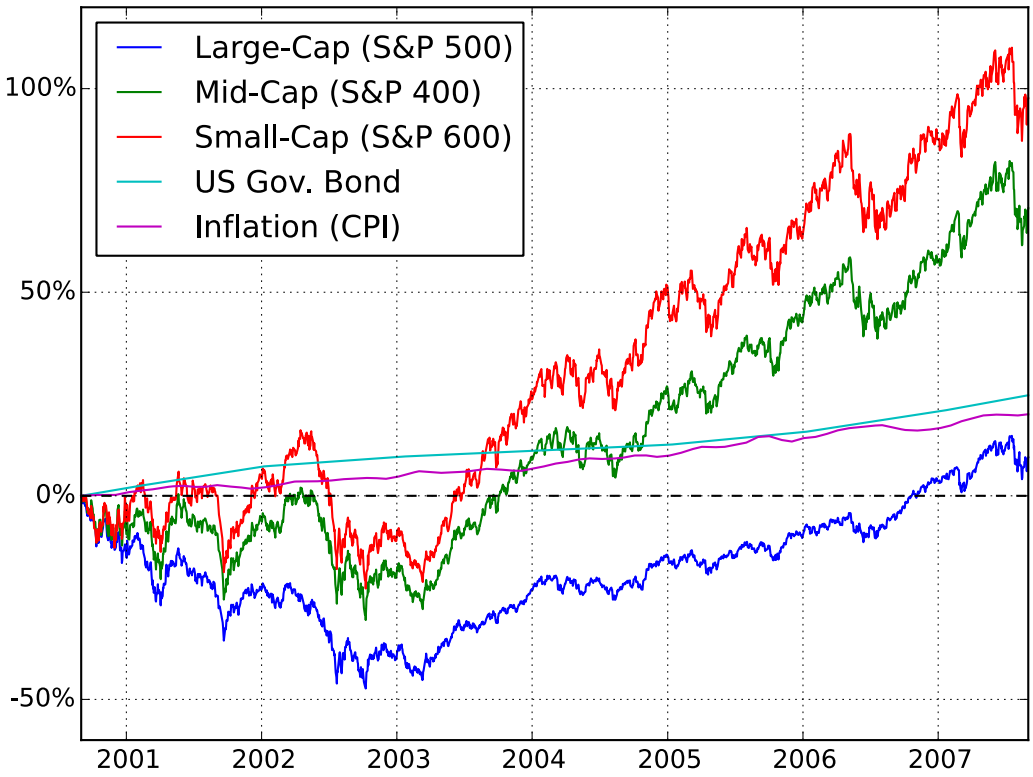
So historically, 97-99% of the losses on these stock indices were recovered within a year, but occasionally the stock-markets crashed which resulted in recovery times that exceeded a few years.

Also keep in mind that the statistics in Table 14 and Table 15 only show the time to recover the first time but the stock indices often decrease again after their first recovery. To get a better understanding of recovery times you should also consider Table 3 and Table 4 which show the probability of loss for different investment periods, where Mid-Cap and Small-Cap stocks have occasionally experienced losses after 9 years, and Large-Cap stocks experienced losses in almost 8% of all 10-year periods between 1981 and 2015.

Worst Large-Cap Crash

Figure 5 shows the worst crash for Large-Cap stocks which started in September 2000 and took more than 6 years to recover, provided the dividends were reinvested and there were no taxes, otherwise it would have taken even longer to recover. It also took longer to make up for the inflation or what could have been earned from investing in US government bonds. During this period the Mid-Cap and Small-Cap stocks only experienced about half the loss of the Large-Cap stocks and they also recovered much faster.

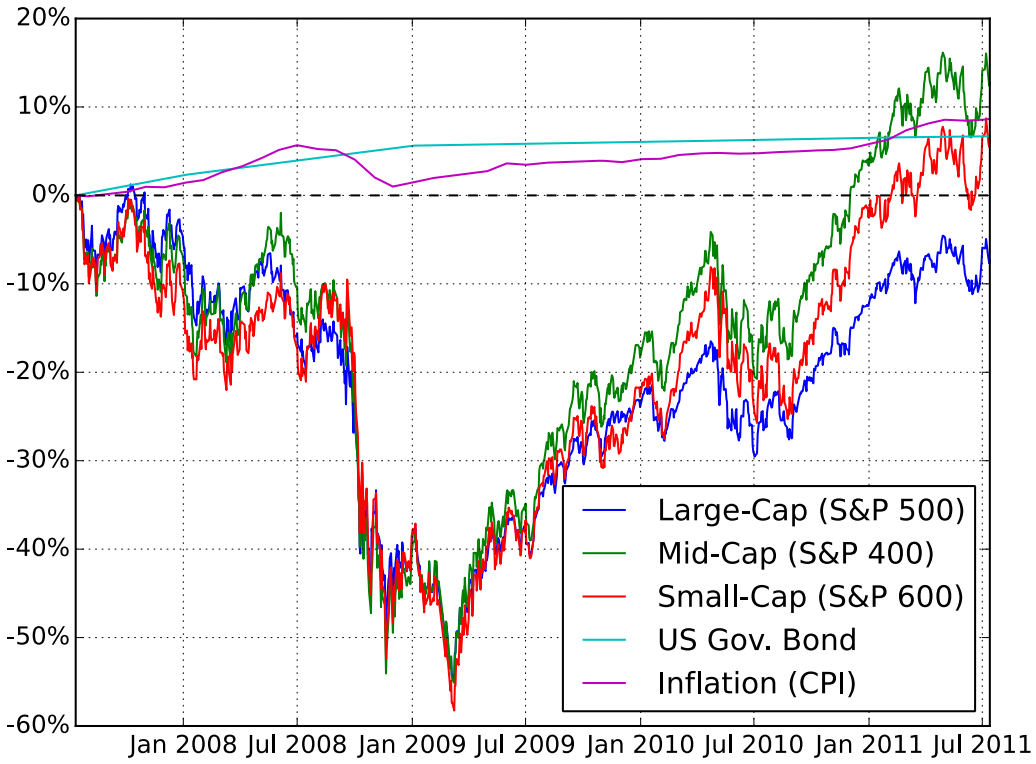
Figure 5: Longest crash and recovery for Large-Cap stocks started in September 2000.



Worst Mid-Cap and Small-Cap Crash

Figure 6 shows the worst crash for Mid-Cap and Small-Cap stocks which slowly started in July 2007. The markets collapsed in late 2008 and reached bottom in March 2009. The three stock indices almost moved in sync during this period. But towards the end of 2009 the Mid-Cap and Small-Cap stocks started to recover slightly faster and by the end of 2010 and beginning of 2011 they had both recovered, well before the Large-Cap stocks recovered.

Figure 6: Longest crash and recovery for Mid-Cap and Small-Cap stocks started in July 2007.



11. Rebalancing

The three stock indices can be combined into a single investment portfolio that is rebalanced annually. The portfolio can be divided equally so that 1/3 of the portfolio is invested in the Large-Cap stock index, 1/3 is invested in Mid-Cap stocks, and 1/3 is invested in Small-Cap stocks. Each year the portfolio is rebalanced back to these allocations. If e.g. Large-Cap stocks have gained and Small-Cap stocks have lost in one year, then we sell some of Large-Cap stocks and buy more of the Small-Cap stocks to bring the portfolio back to the desired allocation.

The idea behind such rebalancing is to take advantage of the volatility for the different stock indices, so as to both stabilize and increase the return of the entire portfolio, when compared to the individual stock indices. However, this only works if the individual stock indices move out of sync relative to each other. But it was shown in section 9 that the returns of the three stock indices were highly correlated when investing for a year or more, which means that the stock indices would mostly have either gains or losses simultaneously. So there was no benefit to portfolio rebalancing between the three stock indices, and the entire portfolio should be invested in the stock index that usually had the best returns, which was shown in the previous sections to have been the Mid-Cap stock index.

11.1. Statistics for Annualized Returns

Nevertheless, it may be of interest to see the performance statistics of rebalancing between the three stock indices. The rebalancing is done annually and each stock index occupies 1/3 of the portfolio.

Table 16 shows the basic statistics for the annualized returns of such portfolio rebalancing. These results can be compared to those of the individual stock indices in Table 1. As can be seen, the performance of the rebalanced portfolio lies somewhere between the three individual stock indices, as would be expected.

Table 16: Annualized return for rebalancing evenly between Large-Cap, Mid-Cap and Small-Cap stocks. Statistics are shown for all investment periods from 1 to 10 years between January 1992 and June 2015.

Rebalancing Between Large-Cap, Mid-Cap and Small-Cap (1992-2015)							
Years of Investing	**Mean**	**Stdev**	**Min**	**1st Qrt.**	**Median**	**3rd Qrt.**	**Max**
1	12.7%	17.1%	(47.5%)	4.1%	14.4%	23.1%	88.0%
2	11.7%	12.1%	(30.0%)	6.3%	13.2%	19.8%	52.0%
3	11.3%	9.6%	(18.4%)	5.3%	13.8%	18.0%	34.4%
4	10.9%	8.2%	(11.9%)	3.7%	11.3%	17.9%	29.7%
5	10.5%	7.2%	(7.5%)	4.2%	9.7%	17.1%	29.0%
6	9.9%	5.6%	0.4%	5.5%	8.0%	14.5%	25.4%
7	9.4%	4.4%	(3.8%)	6.7%	8.2%	10.6%	19.1%
8	9.3%	3.9%	(2.8%)	6.9%	8.9%	11.3%	19.1%
9	9.2%	3.5%	(2.7%)	7.4%	9.0%	11.6%	16.5%
10	9.2%	3.3%	0.0%	6.6%	9.5%	11.7%	14.9%

11.2. Probability of Under-Performance

Table 17 shows the probability that the rebalanced portfolio had a loss or performed worse than other investments. For example, the probability of loss for one-year investment periods was 18.6%, while the probability of performing worse than inflation was 22.8%, and the probability of performing worse than US government bonds was 23.6%. The probability of loss was very low after 6 years or more of investing, although the rebalanced portfolio would sometimes under-perform inflation and US government bonds.

Table 17: Rebalancing between Large-Cap, Mid-Cap and Small-Cap stocks. Shown are the probabilities of having a loss and under-performing inflation and US Government Bonds, and the individual indices.

Probabilities of Rebalancing Under-Performing Other Investments (1992-2015)										
Years of Investing	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Probability of Loss	18.6%	15.0%	16.6%	9.8%	3.6%	0%	1.1%	1.4%	1.0%	0%
Worse than Inflation	22.8%	19.2%	18.8%	19.8%	13.8%	1.0%	3.9%	5.2%	6.2%	2.9%
Worse than Gov. Bonds.	23.6%	21.4%	20.1%	22.7%	17.9%	2.0%	4.2%	5.6%	7.0%	6.0%
Worse than Large-Cap	42.5%	38.2%	33.2%	27.0%	21.8%	18.6%	15.6%	11.3%	1.3%	0%
Worse than Mid-Cap	62.1%	65.2%	69.7%	75.3%	83.7%	89.3%	94.1%	99.4%	100%	100%
Worse than Small-Cap	55.9%	58.0%	57.6%	57.2%	58.5%	59.8%	65.6%	67.2%	69.4%	77.2%

Table 17 also shows the probability that the rebalanced portfolio performed worse than the individual stock indices. For one-year investment periods, the probability was 42.5% that the rebalanced portfolio performed worse than Large-Cap stocks. The probability decreased for longer investment periods until it was zero for 10-year periods, which means there were no 10-year investment periods between 1992 and 2015 in which the rebalanced portfolio performed worse than Large-Cap stocks.

The probability was 62.1% that the rebalanced portfolio performed worse than Mid-Cap stocks for one-year investment periods, and the probability gradually increased to 100% for 9 and 10-year investment periods, which means the rebalanced portfolio performed worse than Mid-Cap stocks in all 9 and 10-year periods between 1992 and 2015.

The probability was 55.9% that the rebalanced portfolio performed worse than Small-Cap stocks for one-year investment periods, and the probability gradually increased to 77.2% for 10-year investment periods.

12. Earnings & Dividends

The previous sections showed that Mid-Cap stocks had usually performed better than Small-Cap stocks, and they had both performed much better than Large-Cap stocks. For investment periods of 10 years, Mid-Cap stocks typically outperformed Large-Cap stocks by about 3% (percentage points) per year, which is a large difference when compounded over many years. The question is what has caused this large performance difference and whether a similar difference can be expected in the future.

In the short-term the price of a stock index can be very volatile because of the speculative nature of stock-markets, but in the long-term we would expect the price to grow similarly to the earnings and dividends, because the earnings and dividends ultimately determine the value of a stock to its long-term investors.

12.1. Large-Cap Stocks

Figure 7 compares the growth in share-price, earnings and dividends for the Large-Cap index between December 1981 and May 2015. During these nearly 34 years, the price-growth was about 1,622% which corresponds to 8.9% per year, while the earnings only grew 555% corresponding to 5.8% per year, and the dividends grew 525% corresponding to 5.6% per year. So the earnings and dividends grew almost the same amount during this period, but the price grew about 3% (percentage points) more each year, which compounded into a very large difference over 34 years. This might suggest that Large-Cap stocks are now over-priced because the price has grown much more than the earnings and dividends seem to justify.

However, the stocks could have been under-priced at the beginning of the period, which could have contributed to the excess price-growth during that period. To assess whether this was the case, Figure 8 shows the so-called earnings yield and dividend yield for the Large-Cap index, which are calculated from the earnings and dividends for the trailing 12 months, divided by the daily share-price. Also shown for comparison is the yield on US government bonds with 1-year maturity.

Figure 8 shows that the earnings yield was nearly 14% at the beginning of the period in 1982 and then fluctuated and gradually decreased towards an average of about 5% between 1992 and 2015. This suggests that Large-Cap stocks were indeed under-priced at the beginning of the period and this likely contributed to the price-growth being greater than the earnings and dividend growth.

A simple explanation why Large-Cap stocks were priced so low in 1982 is that inflation had been very high in the 1970's and in 1982 the government bond yield was very high at around 14%. This made it more attractive to invest in government bonds so stock-prices were correspondingly low.

If we consider the Large-Cap stocks for the shorter period between December 1995 and May 2015, then the price only grew about 6.5% per year, while the earnings grew 5.7% per year and the dividends grew 5.8% per year. Although the price still grew about 0.8% (percentage point) more per year than the earnings and dividends, it confirms that the unusually low stock-prices in the early 1980's were likely a major cause of the excess price-growth for Large-Cap stocks in the 1980's.

12.2. Mid-Cap Stocks

Figure 9 compares the growth in share-price, earnings and dividends for the Mid-Cap index between December 1981 and May 2015. During these nearly 34 years, the price-growth was about 3,951% which corresponds to 11.7% per year, while the earnings only grew 910% corresponding to 7.2% per year, and the

dividends grew 989% corresponding to 7.4% per year. So the earnings and dividends grew almost the same amount during this period, but the price grew nearly 4.5% (percentage points) more each year, which compounded into a very large difference over 34 years.

This growth rate difference of about 4.5% (percentage points) between the price and earnings of Mid-Cap stocks was even larger than the 3% difference for Large-Cap stocks. As was the case for Large-Cap stocks, a significant cause of this excess price-growth for Mid-Cap stocks seems to be the under-priced stocks in the early 1980's. This is corroborated by Figure 10 which shows that the earnings yield of Mid-Cap stocks was around 14% in 1982 and then fluctuated and gradually decreased to 4-5% between 1992 and 2015.

But there is still a significant difference in growth-rates for the price and earnings when considering the shorter period between December 1995 and May 2015, during which the Mid-Cap stocks had a price growth of 10.5% per year, while the earnings only grew 7.7% per year and dividends grew 9.6% per year. This suggests that Mid-Cap stocks may have had excess price-growth of maybe 1-3% (percentage points) per year compared to what might have been justified by the actual earnings and dividend growth. There is uncertainty about the difference in growth-rates because it depends on the choice of start- and end-dates.

12.3. Small-Cap Stocks

Figure 11 compares the growth in share-price, earnings and dividends for the Small-Cap index between December 1995 and May 2015 because data for this index is only available for this shorter period. During these nearly 20 years, the price-growth was about 488% which corresponds to 9.6% per year, while the earnings only grew 383% corresponding to 8.5% per year, and the dividends grew 690% corresponding to 11.2% per year. So the price grew about 1.1% (percentage points) more each year than the earnings, but the price also grew about 1.6% (percentage points) less each year than the dividends. So it is unclear whether the price has grown more or less than is warranted by the increase in earnings and dividends.

Figure 12 shows the earnings and dividend yield for Small-Cap stocks between 1995 and 2015. During these nearly 20 years, the earnings yield has fluctuated around 3% on average, with the maximum being 5.2% in July 2006 and the minimum being (5.3%) in March 2009 where the Small-Cap stocks had negative earnings. The dividend yield was almost 1% on average for this 20-year period, with the minimum being 0.6% which occurred in September 2000, and the maximum dividend yield being 2.5% which occurred in March 2009 during a market-crash in which the dividend was first maintained although the index-price fell dramatically.

Overall, it is unclear from this data whether the price of Small-Cap stocks has grown more or less than warranted by the growth in earnings and dividends, because the price has grown more than the earnings but less than the dividends.

12.4. Summary

Both Large-Cap and Mid-Cap stocks seem to have had greater price-growth than justified by the growth in earnings and dividends between 1981 and 2015. A significant part of the excess price-growth was apparently caused by severe under-pricing of stocks in the early 1980's. For the shorter period between 1995 and 2015, the Large-Cap stocks have had slightly higher growth in share-prices compared to earnings and dividends, while Mid-Cap stocks apparently had more excess price-growth. The data was inconclusive for Small-Cap stocks because the price had grown more than earnings but less than dividends.

Figure 7: Large-Cap growth of share-price, earnings and dividends between 1981 and 2015.

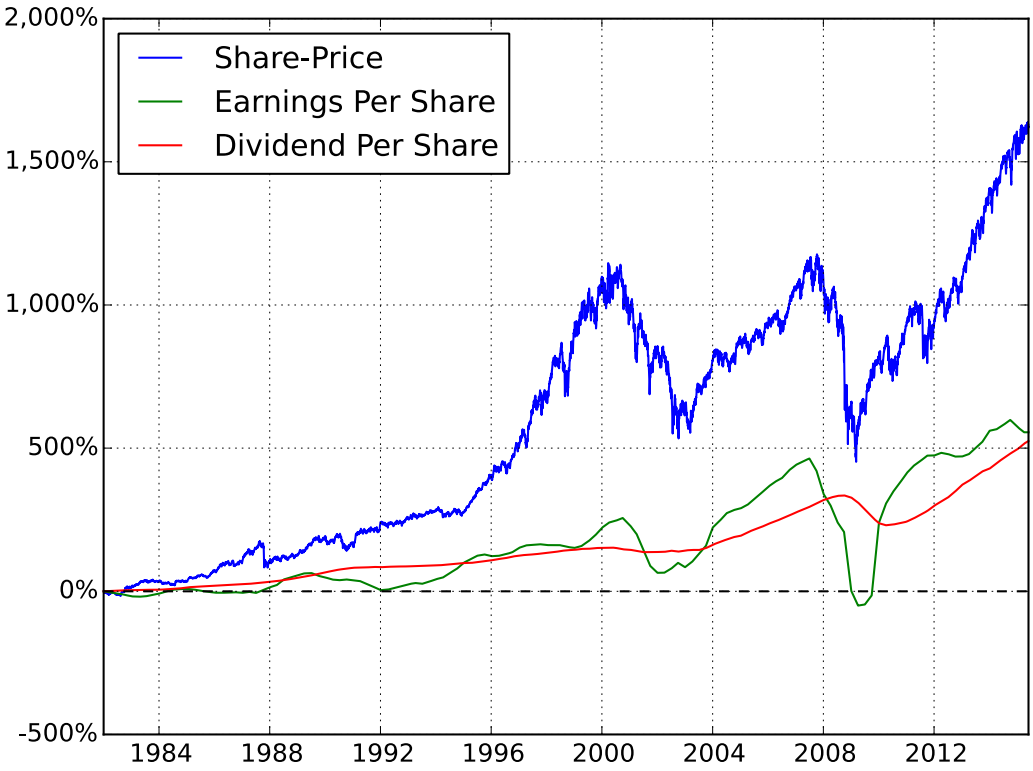


Figure 8: Large-Cap earnings yield, dividend yield, and US government bond yield.

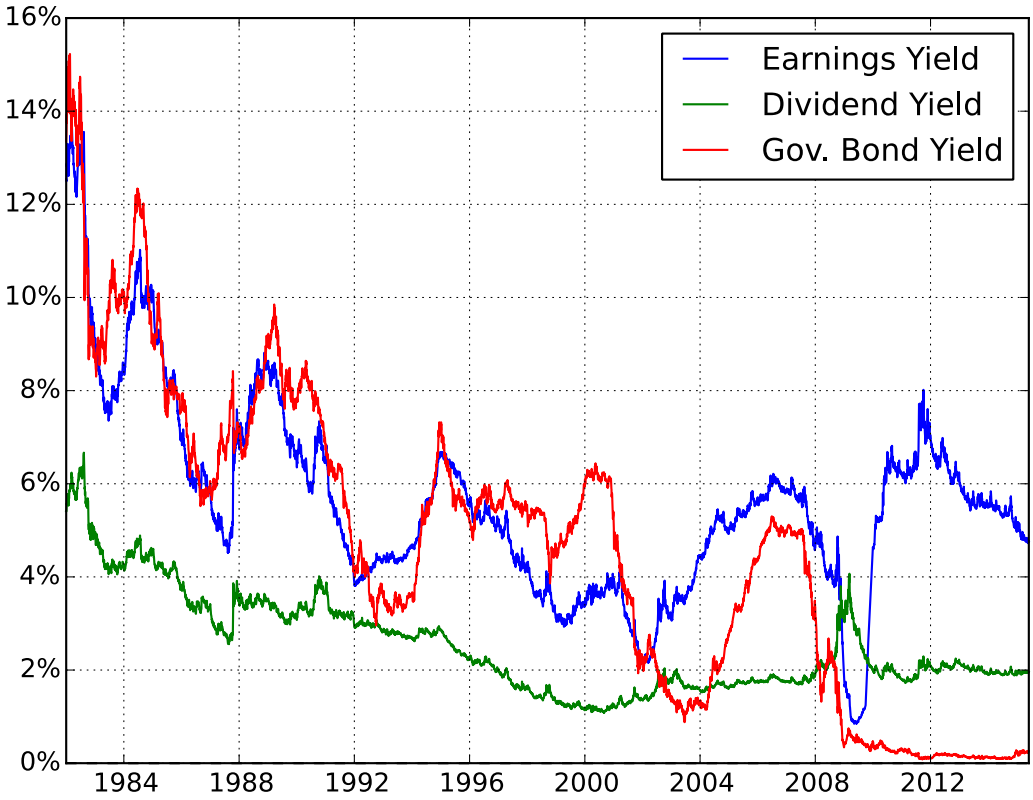


Figure 9: Mid-Cap growth of share-price, earnings and dividends between 1981 and 2015.

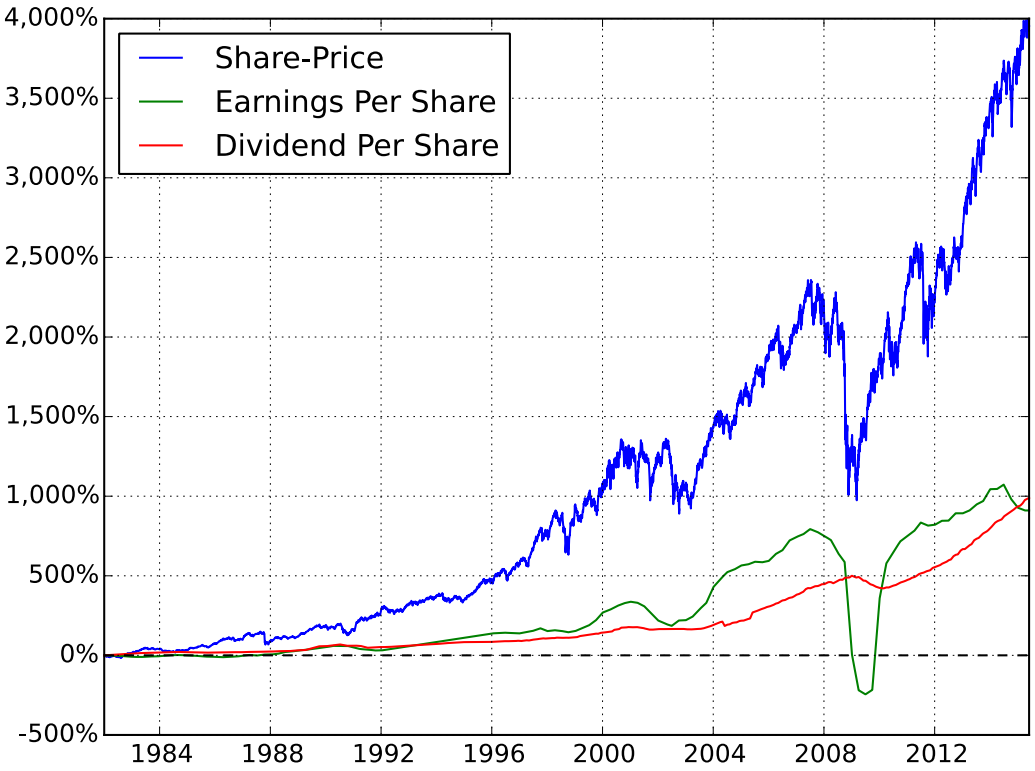


Figure 10: Mid-Cap earnings yield, dividend yield, and US government bond yield.

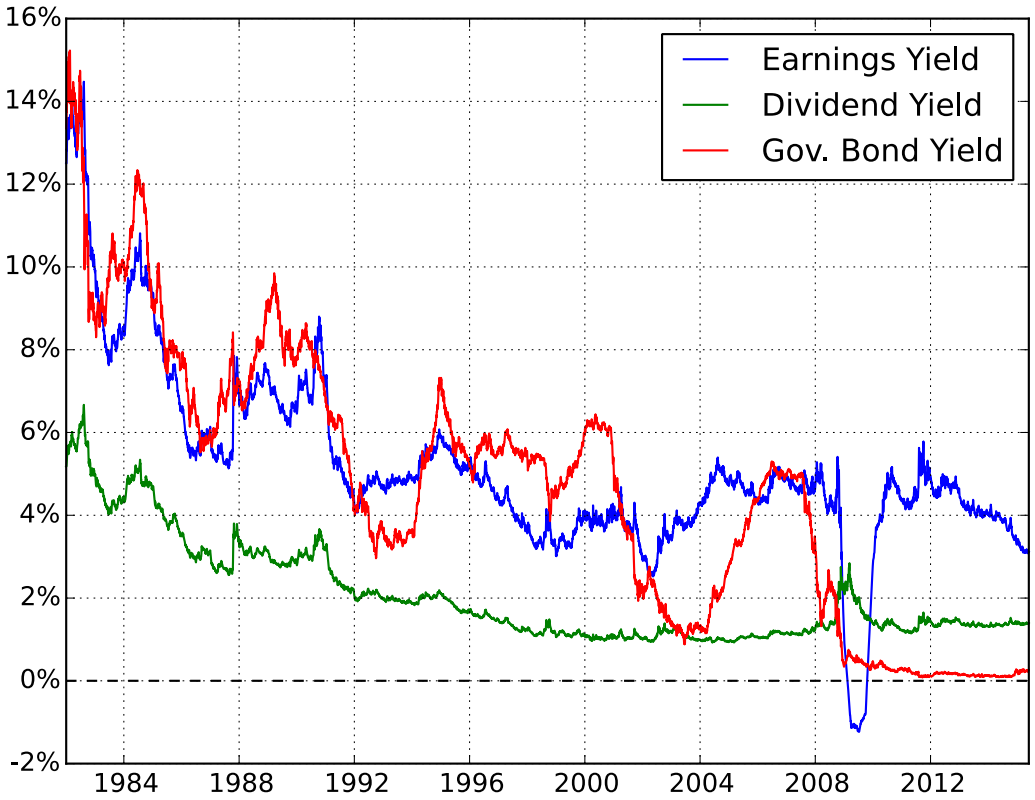


Figure 11: Small-Cap growth of share-price, earnings and dividends between 1995 and 2015.

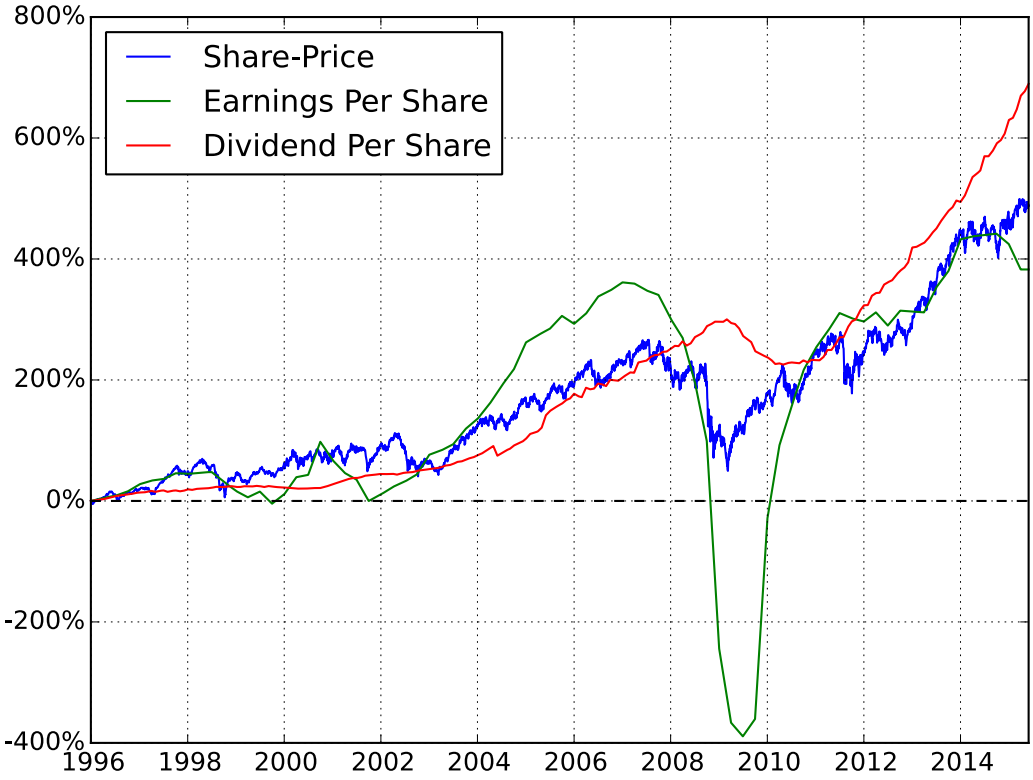
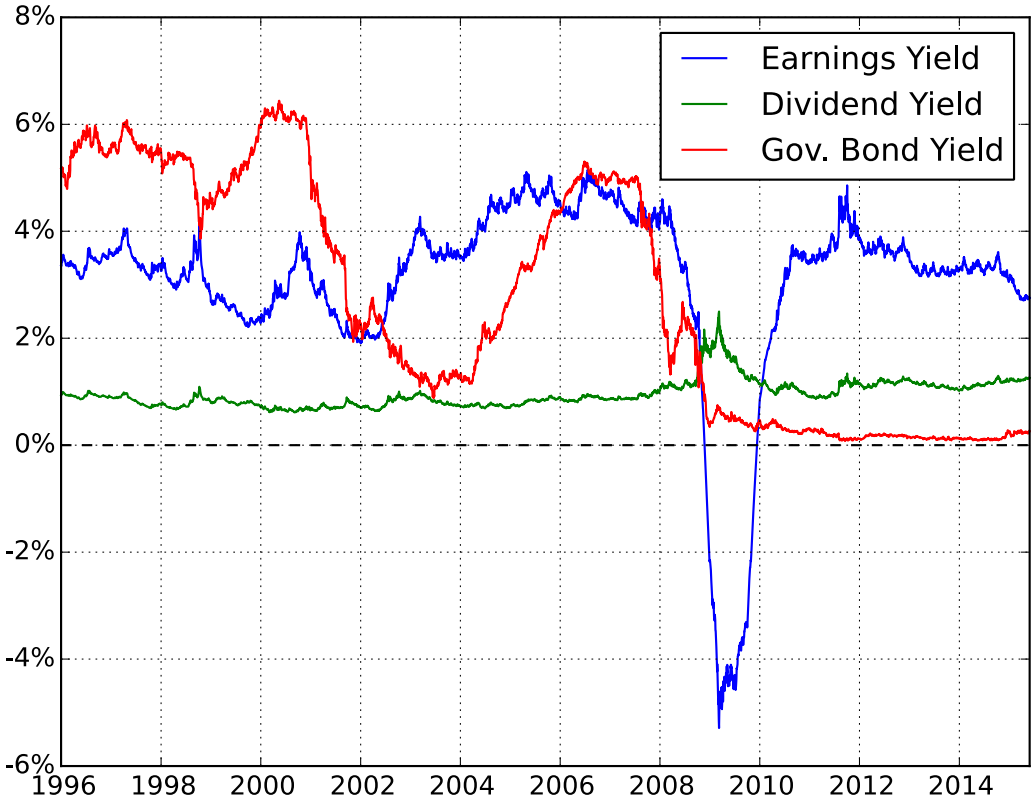


Figure 12: Small-Cap earnings yield, dividend yield, and US government bond yield.



13. Forecasting Future Returns

It was shown through numerous statistics that Mid-Cap stocks historically performed much better than Large-Cap stocks in terms of higher average return, lower probability of loss, etc. But the previous section showed that the price-returns of both Large-Cap and Mid-Cap stocks have been greater than seemingly justified by the growth in earnings and dividends between 1981 and 2015. A significant cause of the excess price-growth appears to have been very low stock-prices in the early 1980's. This leaves the question whether Mid-Cap and Small-Cap stocks can be expected to perform better than Large-Cap stocks in the future, and what the returns might be for these stock indices.

13.1. Historical Growth-Rates

The historical growth-rates will be used to forecast the future returns on the stock indices. Table 18 summarizes the growth-rates from section 12 for the share-price, earnings and dividends for the Large-Cap, Mid-Cap and Small-Cap indices. Two periods are considered, the first period is between 1981 and 2015, and the second period is between 1995 and 2015.

Table 18: Growth-rates for price, earnings and dividends of Large-Cap, Mid-Cap and Small-Cap stocks. These growth-rates are calculated from the end-points of the data.

	1981-2015			1995-2015		
	Price	Earnings	Dividend	Price	Earnings	Dividend
Large-Cap (S&P 500)	8.9%	5.8%	5.6%	6.5%	5.7%	5.8%
Mid-Cap (S&P 400)	11.7%	7.2%	7.4%	10.5%	7.7%	9.6%
Small-Cap (S&P 600)	-	-	-	9.6%	8.5%	11.2%

The table shows that the growth-rates depend on the chosen period. This is because the price, earnings and dividends do not grow steadily each year, and their cycles of growth and decline may also be offset from each other. The growth-rates could be calculated for all intermediate periods and statistics could be shown in tables much like those for the annualized returns in Table 1, but this would result in a large number of tables which might be more confusing than informative.

Curve-Fitted Growth Rates

Another way of estimating the growth-rates is to fit an exponential curve to the data. This should dampen the distorting effect from choosing a particular period, because the growth-rate is calculated from the entire data-range rather than just the end-points. The curve-fitted growth-rates are shown in Table 19. These growth-rates are slightly different from those in Table 18, but the tendencies are mostly similar. An exception is that between 1995 and 2015, Large-Cap stocks had price-growth of only 4.2% per year while the earnings grew 5.8% and dividends grew 5.2%.

Table 19: Growth-rates for price, earnings and dividends of Large-Cap, Mid-Cap and Small-Cap stocks. These growth-rates are calculated by fitting exponential curves to the data.

	1981-2015			1995-2015		
	Price	Earnings	Dividend	Price	Earnings	Dividend
Large-Cap (S&P 500)	8.3%	5.8%	5.1%	4.2%	5.8%	5.2%
Mid-Cap (S&P 400)	11.5%	7.6%	7.0%	9.6%	8.3%	9.2%
Small-Cap (S&P 600)	-	-	-	8.8%	8.3%	10.6%

13.2. Historical Growth Continues

We will now assume that the historical growth-rates continue in the future and see what this leads to.

Large-Cap Stocks

On May 31, 2015 the price of the Large-Cap stock index S&P 500 was $2,107 while the earnings for the previous 12 months were almost $101 which gives an earnings yield of $101 / $2,107 ≈ 4.8%. The dividends for the previous 12 months were about $41 so the dividend yield was $41 / $2,107 ≈ 1.9%.

Now assume that the historical growth rates continue in the future. According to Table 18 and Table 19 the price of the Large-Cap index grew between 8.3-8.9% per year between 1981 and 2015, depending on how the growth-rate is calculated. Let us make a compromise and assume the future price growth is 8.5% per year, while the earnings are assumed to grow 5.8% per year and dividends grow 5.4% per year.

If these growth-rates continue for 10 years, then the price would grow from $2,107 in May 2015 to $4,764 in 2025. In year 2035 the price would have grown to $10,771 and in year 2045 it would be $24,353.

Similarly, if the earnings continue to grow at 5.8% per year, then the earnings would grow from $101 in 2015 to $177 in year 2025, to $312 in year 2035, and to $548 in year 2045.

If the dividends continue to grow at 5.4% per year, then the dividends would grow from $41 in 2015 to $69 in year 2025, to $117 in year 2035, and to $199 in year 2045.

The corresponding earnings and dividend yields are found by dividing the forecasted earnings and dividends by the forecasted prices. So in year 2025 the earnings yield would be $177 / $4,764 ≈ 3.7%, while it would be $312 / $10,771 ≈ 2.9% in year 2035, and $548 / $24,353 ≈ 2.3% in year 2045.

The forecasted dividend yield would be $69 / $4,764 ≈ 1.4% in year 2025, while it would be $117 / $10,771 ≈ 1.1% in year 2035, and $199 / $24,353 ≈ 0.8% in year 2045.

So if the Large-Cap index continues its historical growth-rates until year 2045, then the earnings yield would become 2.3% and the dividend yield would become 0.8% in year 2045. According to Figure 8, such a low dividend yield has only been experienced around the Dot-Com bubble in year 2000, while such a low earnings yield has only been experienced in year 2002 during the ensuing crash of the Dot-Com bubble, as well as during the crash around year 2009. So these would be abnormally low earnings and dividend yields.

Another way to see that the forecasted earnings and dividend yields are likely much too low, is to compare them to US government bonds whose yield has normally ranged between 1-6%. The bond yield is currently near zero which is kept low in an effort to improve the US economy, while the bond yield was more than 14% in the early 1980's after the US experienced great inflation. But decades into the future, the bond yield can probably be expected to be around 5% on average for US government bonds with 1-year maturity.

If you can buy a government bond with a guaranteed return of 5%, then why would you want to buy a stock-index whose earnings yield is only 2.3% and dividend yield is only 0.8%? The dividend on Large-Cap stocks is not even guaranteed even though it would be much lower than the return on government bonds. The price of the Large-Cap stocks may also fluctuate wildly in addition to the low dividend yield. So an earnings yield of 2.3% and a dividend yield of 0.8% would seem much too low. This means the assumption that Large-Cap prices, earnings and dividends will continue their historical growth-rates is likely wrong.

Mid-Cap Stocks

On May 31, 2015 the price of the Mid-Cap stock index (S&P 400) was $1,525 while the earnings for the previous 12 months were almost $48 which gives an earnings yield of $48 / $1,525 ≈ 3.1%. The dividends for the previous 12 months were about $21 so the dividend yield was $21 / $1,525 ≈ 1.4%.

Now assume that the historical growth rates continue in the future. According to Table 18 and Table 19 the price of the Mid-Cap index grew between 11.5-11.7% per year between 1981 and 2015, depending on how the growth-rate is calculated. Let us make a compromise and assume the future price growth is 11.6% per year, while the earnings are assumed to grow 7.4% per year and dividends grow 7.2% per year.

If the price continues to grow at 11.6% per year, then the price would grow from $1,525 in 2015 to $41,039 after 30 years in 2045. Similarly, if the earnings continue to grow 7.4% per year, then the earnings would grow from $48 in 2015 to $409 in year 2045. If the dividends continue to grow 7.2% per year, then they would grow from $21 in 2015 to $169 in year 2045.

The corresponding earnings yield in year 2045 would be the forecasted earnings divided by the forecasted price, which is $409 / $41,039 ≈ 1.0%, while the dividend yield would be $169 / $41,039 ≈ 0.4%.

If we assume the future bond yield is 5% on average for US government bonds with 1-year maturity, then why would anyone want to buy Mid-Cap stocks with an earnings yield of only 1% and a dividend yield of only 0.4%? According to Figure 10, the lowest dividend yield for Mid-Cap stocks was about 1% between 1982 and 2015, while the earnings yield has almost tracked the US government bond yield, except during the most recent years in which the bond yield has been kept artificially low. The conclusion must be that the price of Mid-Cap stocks will have to grow less than the historical 11.6% per year, or the earnings and dividends will have to grow more than the historical 7-7.5% per year.

Small-Cap Stocks

On May 31, 2015 the price of the Small-Cap stock index (S&P 600) was $713 while the earnings for the previous 12 months were almost $20 which gives an earnings yield of $20 / $713 ≈ 2.8%. The dividends for the previous 12 months were about $9 so the dividend yield was $9 / $713 ≈ 1.3%.

Now assume that the historical growth rates continue in the future. According to Table 18 and Table 19 the price of the Small-Cap index grew between 8.8-9.6% per year between 1995 and 2015, depending on how the growth-rate is calculated. Let us make a compromise and assume the future price growth is 9.2% per year, while the earnings are assumed to grow 8.4% per year and dividends grow 10.9% per year.

If the price continues to grow at 9.2% per year, then the price would grow from $713 in 2015 to $9,995 after 30 years in 2045. Similarly, if the earnings continue to grow 8.4% per year, then the earnings would grow from $20 in 2015 to $225 in year 2045. If the dividends continue to grow 10.9% per year, then they would grow from $9 in 2015 to $200 in year 2045.

Note that the dividends of $200 in year 2045 would be almost as big as the earnings of $225. If these growth-rates continue for another 10 years, then the earnings would be $504 in year 2055 while the dividends would be $564. The only way for the dividends to exceed the earnings is if the companies in the Small-Cap index either pay the dividends from cash reserves that have been built up in prior years, or if the

companies borrow money to pay the dividends. Neither case is sustainable in the long-term so the dividend growth-rate cannot exceed the earnings growth-rate indefinitely as was assumed here.

Furthermore, the earnings yield in year 2045 would be the forecasted earnings divided by the forecasted price, which is $225 / $9,995 ≈ 2.3%, while the dividend yield would be $200 / $9,995 ≈ 2.0%. But according to Figure 12 the historical earnings yield was about 3% on average between 1995 and 2015, with a large negative earnings yield occurring around year 2009 during a large stock-market crash. Otherwise the lowest earnings yield was around 2% which occurred around year 2002. The historical dividend yield was about 1% on average and only exceeded 2% during the crash in year 2009.

Altogether, it seems unlikely that the historical growth-rates for the price, earnings and dividends of Small-Cap stocks will continue for several decades into the future.

13.3. Equal Growth-Rates

It was shown above that the historical growth-rates of the stock indices had been significantly different for their prices, earnings and dividends. One cause of this disparity was apparently that the stocks had been priced very low in the early 1980's compared to their earnings and dividends at the time. The low stock prices were caused by high yields on US government bonds which in turn were caused by high inflation in the 1970's. The mispricing was gradually corrected and this resulted in the stock-prices growing much more than the earnings and dividends in the following years.

The previous section showed what would eventually happen if the historical growth-rates were to continue for decades into the future. It would result in earnings and dividend yields that would be very low in comparison to their historical averages and the expected future yield on US government bonds. For Small-Cap stocks it would also result in the dividends eventually exceeding the earnings by an ever-growing margin, which would be impossible. So it seems unlikely that the growth-rates will continue to diverge as much as they have done in the past decades.

This section instead considers what would happen if the price and dividends of a stock index grow at the same rate as the earnings. In the long-term we would expect this to happen, because the earnings ultimately determine how much can be paid out in dividends to shareholders and therefore also determine the value of a stock to long-term shareholders.

Large-Cap Growth-Rate

According to Table 18 and Table 19, the earnings of the Large-Cap stock index grew about 5.8% per year on average between 1981 and 2015. We will assume that this growth-rate continues indefinitely into the future and that both the dividends and price of the Large-Cap index will grow at the same rate in the long-term, although in reality the earnings and dividend, and especially the price may fluctuate in any given year.

Dividend Yield

On May 31, 2015 the price of the Large-Cap stock index (S&P 500) was $2,107 while the dividends for the previous 12 months were about $41 so the dividend yield was about $41 / $2,107 ≈ 1.9%. This uses the dividend from the previous 12 months and we really want to calculate the dividend yield using the dividend we would receive in the year between May 31, 2015 and 2016. This is being written in September 2015, so we will have to estimate this future dividend. The earnings were assumed to grow at 5.8% per year and the dividend was assumed to grow at the same rate as the earnings. So the dividend for the year between May

31, 2015 and 2016 is estimated to be (1 + 5.8%) x $41 ≈ $43.38 which means the dividend yield on May 31, 2015 would be $43.38 / $2,107 ≈ 2.06% which is rounded up to 2.1%. The formula for the dividend yield is:

Eq. 13-1

$$\textit{Dividend Yield} = (1 + \textit{Growth-Rate}) \times \textit{Dividend} / \textit{Share-Price} = (1 + 5.8\%) \times \$41 / \$2{,}107 \approx 2.1\%$$

When using the following formulas to estimate the future return on the Large-Cap index, you should first calculate the dividend yield using this formula with the current price for the Large-Cap index (S&P 500) and its dividend for the prior year.

Total Return

The total return of a stock index consists of the change in price plus the dividend. The price is assumed to grow the same as the earnings which is 5.8% per year, and the dividend yield is estimated to be 2.1% on May 31, 2015. So the total return for Large-Cap stocks bought on May 31, 2015 would be:

Eq. 13-2

$$\textit{Total Return} = \textit{Price Growth} + \textit{Dividend Yield} = 5.8\% + 2.1\% \approx 7.9\%$$

This estimated total return of 7.9% is for long-term investing of maybe several decades during which the earnings, dividends and price of the Large-Cap index are all assumed to grow 5.8% per year on average. But the index can be very volatile, so the short-term return may differ greatly from this long-term estimate.

Inflation-Adjusted Return

During the 55 years between 1960 and 2015, the earnings of the Large-Cap stock index grew 6.4% per year while the price grew 6.8% per year on average. If the future earnings, price and dividends of the Large-Cap index were to grow at 6.4% per year, then the total return would be 8.5% instead of only 7.9% as estimated in Eq. 13-2 for a growth-rate of 5.8% per year. Which one of these two estimates is most accurate?

One reason that the growth-rates were different for the two periods 1960-2015 and 1981-2015, is simply because the inflation was different during the two periods. Inflation was about 3.9% per year on average between 1960 and 2015, which means the earnings of Large-Cap stocks only grew about 2.5% per year when adjusted for inflation. Between 1981 and 2015 the inflation was only about 2.9% per year on average, so the earnings growth of 5.8% was actually 2.9% per year when adjusted for inflation. The difference in inflation-adjusted growth-rates was about 0.4% per year for these two periods, but the inflation-adjusted growth-rate was higher between 1981 and 2015 than it was for the longer period between 1960 and 2015.

We should adjust for the expected inflation when estimating the future return of a stock index. If we assume the future price-growth is 5.8% which is the average annual earnings-growth of Large-Cap stocks between 1981 and 2015, and we then use the average inflation-rate of 2.9% during that period, then the inflation-adjusted total return of Large-Cap stocks is estimated to be:

Eq. 13-3

$$\textit{Inflation-Adjusted Total Return} = \textit{Price Growth} + \textit{Dividend Yield} - \textit{Inflation Rate} = 5.8\% + 2.1\% - 2.9\% \approx 5\%$$

An inflation-adjusted return of 5% per year corresponds to a doubling in purchasing power every 14 years.

Mid-Cap

According to Table 18 and Table 19, the earnings of the Mid-Cap stock index grew about 7.4% per year on average between 1981 and 2015, while the growth-rate was slightly higher at 8% per year on average between 1995 and 2015, depending on how the growth-rate is calculated. Let us make a compromise and say the earnings growth-rate was 7.7% per year on average. We will assume that this growth-rate continues indefinitely into the future and that both the dividends and price of the Mid-Cap index will grow at the same rate in the long-term, although the dividend and especially the price may fluctuate in any given year.

On May 31, 2015 the price of the Mid-Cap index (S&P 400) was $1,525 while the dividends for the previous 12 months were about $21 so the dividend yield was about $21 / $1,525 ≈ 1.4%. This uses the dividend from the previous 12 months and we really want to calculate the dividend yield using the dividend we would receive in the year between May 31, 2015 and 2016. The earnings were assumed to grow at 7.7% per year and the dividend was assumed to grow at the same rate as the earnings. So the dividend for the year between May 31, 2015 and 2016 is estimated to be (1 + 7.7%) x $21 ≈ $22.62 which means the dividend yield on May 31, 2015 would be $22.62 / $1,525 ≈ 1.5%. The formula is:

Eq. 13-4

$$\textit{Dividend Yield} = (1 + \textit{Growth-Rate}) \times \textit{Dividend} / \textit{Share-Price} = (1 + 7.7\%) \times \$21 / \$1{,}525 \approx 1.5\%$$

For Mid-Cap stocks bought on May 31, 2015 where the dividend yield is estimated to be 1.5% and the price-growth is assumed to be 7.7%, the total return is therefore estimated to be:

Eq. 13-5

$$\textit{Total Return} = \textit{Price Growth} + \textit{Dividend Yield} = 7.7\% + 1.5\% \approx 9.2\%$$

Inflation was about 2.9% per year on average between 1981 and 2015. Assuming this inflation-rate continues in the future gives the following inflation-adjusted estimate of the total return of Mid-Cap stocks:

Eq. 13-6

$$\textit{Inflation-Adjusted Total Return} = \textit{Price Growth} + \textit{Dividend Yield} - \textit{Inflation Rate} = 7.7\% + 1.5\% - 2.9\% \approx 6.3\%$$

An inflation-adjusted return of 6.3% per year corresponds to a doubling in purchasing power about every 11 years, which is 3 years less than the estimated time for Large-Cap stocks.

Small-Cap

According to Table 18 and Table 19, the earnings of the Small-Cap stock index grew about 8.4% per year on average between 1995 and 2015. We will assume that this growth-rate continues indefinitely into the future and that both the dividends and price of the Small-Cap index will grow at the same rate in the long-term, although the dividend and especially the price may fluctuate in any given year.

On May 31, 2015 the price of the Small-Cap stock index (S&P 600) was $713 while the dividends for the previous 12 months were about $9 so the dividend yield was about $9 / $713 ≈ 1.3%. This uses the dividend from the previous 12 months and we really want to calculate the dividend yield using the dividend that we would receive in the year between May 31, 2015 and 2016. The earnings were assumed to grow at 8.4% per year and the dividend was assumed to grow at the same rate as the earnings. So the dividend for the

year between May 31, 2015 and 2016 is estimated to be (1 + 8.4%) x $9 ≈ $9.76 which means the dividend yield on May 31, 2015 would be $9.76 / $713 ≈ 1.4%. The formula is:

Eq. 13-7

$$\textit{Dividend Yield} = (1 + \textit{Growth-Rate}) \times \textit{Dividend} / \textit{Share-Price} = (1 + 8.4\%) \times \$9 / \$713 \approx 1.4\%$$

For Small-Cap stocks bought on May 31, 2015 where the dividend yield is estimated to be 1.4% and the price-growth is assumed to be 8.4%, the total return is therefore estimated to be:

Eq. 13-8

$$\textit{Total Return} = \textit{Price Growth} + \textit{Dividend Yield} = 8.4\% + 1.4\% \approx 9.8\%$$

Inflation was only about 2.3% per year on average between 1995 and 2015, which is the same period as the annual growth-rate is taken from. Assuming this inflation-rate continues in the future gives the following inflation-adjusted estimate of the total return on Small-Cap stocks:

Eq. 13-9

$$\textit{Inflation-Adjusted Total Return} = \textit{Price Growth} + \textit{Dividend Yield} - \textit{Inflation Rate} = 8.4\% + 1.4\% - 2.3\% \approx 7.5\%$$

An inflation-adjusted return of 7.5% per year corresponds to a doubling in purchasing power every 9.5 years, which is 4.5 years less than the estimated time for Large-Cap stocks, and about 1.5 years less than for Mid-Cap stocks.

Summary

Table 20 summarizes the estimated total return for the stock indices. The growth-rates are the historical averages while the dividend yields are calculated for May 31, 2015 and should be recalculated with current data using Eq. 13-1 for Large-Cap stocks, Eq. 13-4 for Mid-Cap stocks, and Eq. 13-7 for Small-Cap stocks.

The inflation-adjusted total return is also shown in Table 20 along with the assumed inflation rates. Note that the inflation rate for Small-Cap stocks is considerably lower than for Large-Cap and Mid-Cap stocks. This is because the historical growth-rate for Small-Cap stocks is for the period between 1995 and 2015 which had an inflation-rate of about 2.3% per year, while the growth-rates for Large-Cap and Mid-Cap stocks are for the period between 1981 and 2015 which had an inflation-rate of about 2.9% per year.

If these growth- and inflation-rates continue in the future, and the earnings, dividends and prices of the stock indices all grow at these rates, then Large-Cap stocks would return 5.0% per year after inflation, while Mid-Cap stocks would return 6.3% per year, and Small-Cap stocks would return 7.5% per year.

Table 20: Estimated total return for Large-Cap, Mid-Cap and Small-Cap stocks. The dividend yield is calculated for May 31, 2015 and should be recalculated with current data.

	Growth-Rate	Dividend Yield	Total Return	Inflation	Inflation-Adjusted Total Return
Large-Cap (S&P 500)	5.8%	2.1%	7.9%	2.9%	5.0%
Mid-Cap (S&P 400)	7.7%	1.5%	9.2%	2.9%	6.3%
Small-Cap (S&P 600)	8.4%	1.4%	9.8%	2.3%	7.5%

13.4. Share Buybacks

Instead of paying a dividend, a company can buy back and cancel some of its own shares. This reduces the number of shares outstanding. US companies have significantly increased their share buybacks since the mid 1990's and share buybacks are now much greater than dividends. In 2014, the Large-Cap companies had earnings of $909 billion, paid dividends of $350 billion and spent $553 billion on share buybacks [**12**].

The common belief is that a share buyback substitutes for a dividend payout, because a share buyback results in a corresponding increase in the earnings per share as well as the share-price itself. There may also be a tax advantage for shareholders whose tax on capital gains is lower than their tax on dividends. But this belief is incorrect and share buybacks can be very destructive to shareholder value, as shown below.

Buy High & Sell Low

Companies have had a tendency to buy back shares when the share-price was high rather than low. For example, in year 2007 the Large-Cap companies had earnings of $587 billion, paid dividends of $247 billion and spent a total of $589 billion on share buybacks. The combined amount spent on dividends and share buybacks was $836 billion which was much larger than the earnings. This means the companies borrowed money to fund the large share buybacks in 2007.

Then a financial crisis started in 2008 and the stock-market reached bottom in March 2009. For the full year 2009 the Large-Cap companies had earnings of $449 billion (24% less than in 2007), paid dividends of $196 billion (21% less than in 2007) and made share buybacks for only $138 billion (77% less than in 2007).

In year 2007 the share-price for the Large-Cap index was as high as $1,550 while it fell below $680 at the bottom of the crash in 2009. The Large-Cap companies spent $589 billion on share buybacks in 2007 when the share-price was high. A large portion of that share buyback was funded with borrowed money. Then in 2009 when the price of the Large-Cap index had dropped (56%) compared to its price-level two years earlier in 2007, the Large-Cap companies reduced their share buybacks (77%) to only $138 billion.

The destruction of shareholder value was actually much greater, because the Large-Cap companies issued new shares for $461 billion in 2008 and $241 billion in 2009 during the stock-market crash where share-prices were very low. In 2007 when the companies made large share buybacks for borrowed money and share-prices were high, the companies only issued new shares for $158 billion, see Figure 14.

In effect, the Large-Cap companies bought their own shares at high prices and for borrowed money in 2007, and then sold even more shares at very low prices in 2008 and 2009 to repay the debt. This was reckless and could easily have been avoided. See [**13**] for a detailed treatment of share buyback valuation.

Forecasted Return

When forecasting the return on Large-Cap stocks, it was assumed that the earnings would continue to grow at 5.8% per year, which was the historical average growth-rate for Large-Cap earnings between 1981 and 2015. The future dividend and share-price was assumed to grow at the same rate as the earnings. The dividend yield on May 31, 2015 was calculated to be 2.1%. The sum of the growth-rate and the dividend yield is 7.9% and this is the forecasted total return on the Large-Cap index under these assumptions.

But the companies in the Large-Cap index now make share buybacks that exceed their dividend payouts. If the companies were to stop buying back shares and instead use the money for dividends, then the dividend

yield would increase by as much as 2% (percentage points) or more. This would increase the forecasted total return to maybe 9.9% if the growth-rate is still assumed to be 5.8%. But would the growth-rate have to decrease by the same amount because share buybacks would no longer contribute to growth?

Figure 13 shows the growth in earnings and earnings-per-share for the Large-Cap stocks between December 1989 and December 2014. The combined earnings for the Large-Cap companies were about $154 billion in year 1989 while they had grown to about $909 billion in 2014. This corresponds to an average growth of about 7.4% per year. Compare this to the earnings-per-share which grew from about $23 in 1989 to $102 in 2014, corresponding to about 6.1% growth per year. So the earnings-per-share grew about 1.3% (percentage points) less each year than the actual earnings of the Large-Cap companies.

Figure 14 shows that the companies started to increase their share buybacks in the mid 1990's. Comparing Figure 14 to Figure 13 shows that the divergence between earnings and earnings-per-share widened whenever share buybacks increased. This suggests share buybacks had a negative effect on the growth in earnings-per-share, which seems counter-intuitive and is the exact opposite of what is commonly believed.

It is possible that the divergence between earnings and earnings-per-share is caused by something else than share buybacks. For example, stock options could cause dilution in excess of the number of shares bought back, if there were many stock options and share buybacks were comparatively small. But the "fair value" of stock options has been subtracted from corporate earnings since 2005 by law, so both the earnings and earnings-per-share should be adjusted for stock options since then, yet share buybacks and the divergence between earnings and earnings-per-share have greatly increased since then.

Whatever the explanation may be for the divergence between earnings and earnings-per-share, it is absurd that US companies spend more than $500 billion per year on share buybacks, because of a romantic fantasy that share buybacks will translate into real returns for shareholders, even though it has never been proven and there is evidence from numerous researchers that share buybacks often destroy shareholder value.

If companies were to stop making share buybacks altogether, then the dividend yield for Large-Cap stocks would increase maybe 2% (percentage points) or more. It is also possible that the growth-rate in earnings-per-share would remain the same. This would result in the future return on Large-Cap stocks to be maybe 2% (percentage points) higher than the 7.9% that was forecasted, for a total return of maybe 9.9% per year.

Mid-Cap & Small-Cap

Data for share buybacks is apparently only available for the Large-Cap index and not for the Mid-Cap and Small-Cap indices, so it is unknown how share buybacks may have affected the return on those two indices.

Figure 13: Growth in earnings and earnings-per-share for Large-Cap stocks. Data source [14].

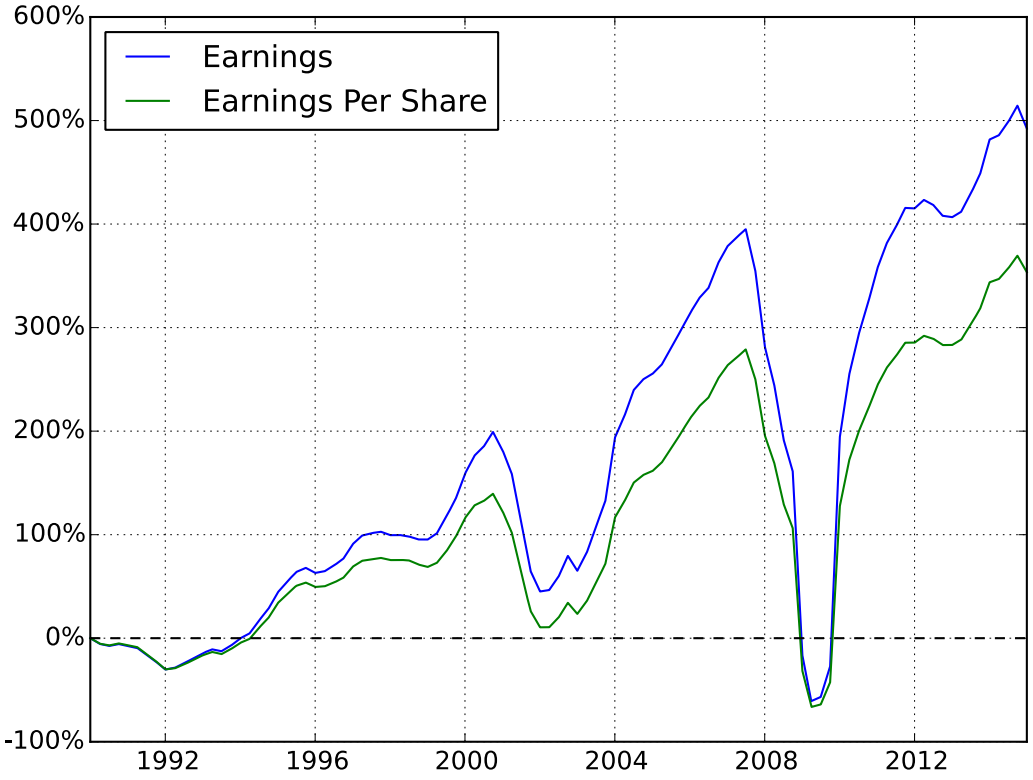
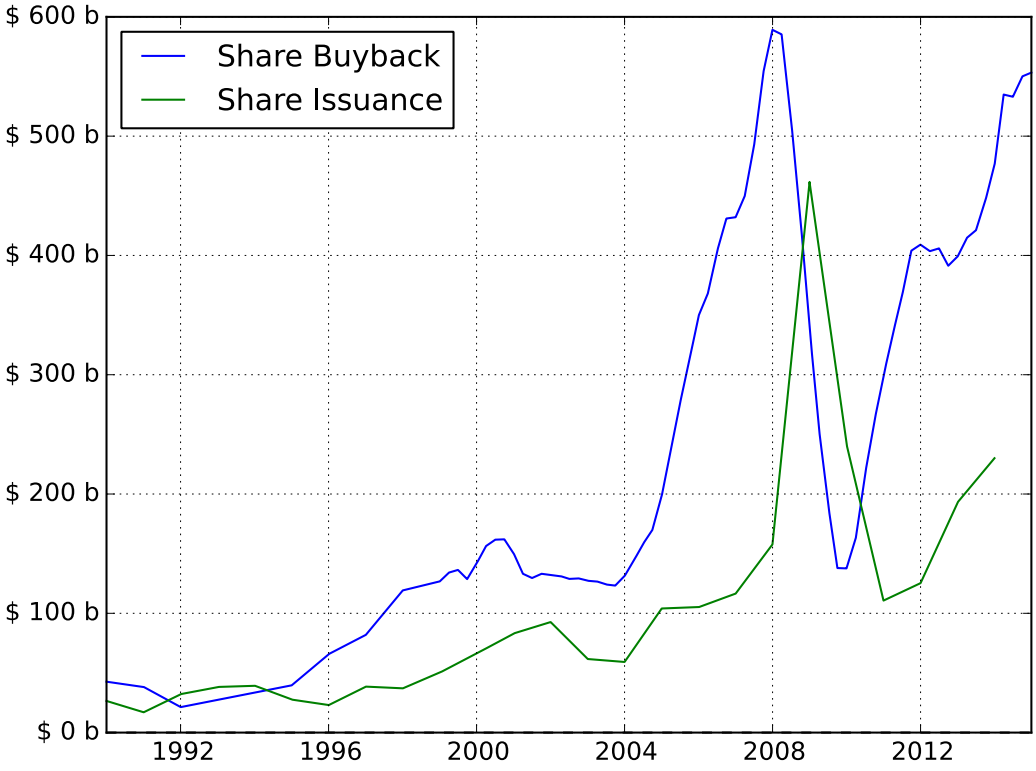


Figure 14: Annual amounts of share buyback and issuance for Large-Cap companies. Data source [15].



13.5. Forecasting Using P/Book

In the previous sections, the historical earnings growth was added to the current dividend yield as a simple method of forecasting the future return of a stock index, under the assumption that earnings growth would be constant and the price and dividend would grow at the same rate.

There are other methods of forecasting the future return on stock indices. One such method was proposed in my other book [**16**] (section 13.1 on page 80) and uses the historical relationship between the P/Book ratio of a stock index and its return in the following years.

Book-Value vs. Return

The P/Book ratio measures the price of a stock-index in relation to its so-called book-value, which is an accounting measure of how much capital shareholders have supplied to the companies in the stock-index, either directly through the issuance of new shareholder capital, or indirectly through retained earnings.

The future earnings of a company are related to the company's assets, and the assets are related to the company's book-value. So the P/Book ratio relates the current price of the companies to their future earnings, and this in turn relates to the future dividends and share-prices. The relationship is stronger for longer investment periods because the short-term randomness of stock-prices becomes less influential.

Formula

The following formula forecasts the annualized return for 10 years of investing in the Large-Cap index (S&P 500), assuming dividends are reinvested and there are no taxes:

Eq. 13-10

$$\textit{Annualized Return} = 23.4\% - 4.9\% \times \textit{P/Book}$$

The formula does not predict exactly what happens during the next 10 years, and there is considerable uncertainty about the forecasted return because there could be a bull-market or a crash in 10 years, which is unpredictable. The formula estimates the average return of a wide range of outcomes.

The book-value is apparently only available for the Large-Cap index (S&P 500) and does not seem to be available for the Mid-Cap (S&P 400) and Small-Cap (S&P 600) indices, so this type of forecasting formula currently only exists for the Large-Cap index.

Example

The formula requires the P/Book ratio, which is the current price of the Large-Cap index divided by the book-value. We will use data for May 31, 2015 so the forecasted return can be compared to the other forecast that was made above. On May 31, 2015 the price of the Large-Cap stock index S&P 500 was $2,107. The book-value per share was $729.29 on March 31, 2015 and it probably grew very little if anything in the following two months until May 31, 2015, so we will use the book-value of $729.29 for that date as well. This means the P/Book ratio on May 31, 2015 was:

$$\textit{P/Book} = \textit{Price / Book-Value} = \$2{,}107 / \$729.29 \approx 2.89$$

Inserting this P/Book ratio into Eq. 13-10 gives us the forecasted return on the Large-Cap index:

$$\textit{Annualized Return} = 23.4\% - 4.9\% \times P/Book = 23.4\% - 4.9\% \times 2.89 \approx 9.2\%$$

That is, if we invested in the Large-Cap index on May 31, 2015 and hold the investment for 10 years until May 31, 2025, and we reinvest all dividends during those years without having to pay taxes, then the annualized return is estimated to be 9.2% per year on average.

Comparison

Compare this forecasted return of 9.2% per year to the 7.9% that was forecasted in Table 20 above. There is a large difference of 1.3% between these two forecasts and we might ask which is most accurate?

The forecasting methods are fundamentally different. One method assumes that the historical average earnings growth will continue indefinitely into the future, and that both the dividends and share-price will grow at exactly the same rate. Although the growth-rates may indeed converge over many decades of investing, it is not a realistic assumption in the short-term where the earnings and dividends may grow differently from each other, and the price may fluctuate wildly. The long-term growth-rates may also turn out to be different from the historical average as it would require continued economic growth through technological innovation, population growth, etc.

The other method uses the P/Book ratio to forecast 10-year returns of the Large-Cap index, and tries to make a good compromise that would have worked reasonably well at forecasting returns for all 10-year periods between 1978 and 2013. This takes a large number of historical scenarios into account, including periods of over- and under-valuation, bull-markets and crashes for the Large-Cap stocks. This might seem like the preferable forecasting method compared to the unrealistic assumption that the future is always perfectly stable and predictable. But there are also limitations to the P/Book forecasting method, because it is a compromise which cannot predict exactly what will happen and merely forecasts an average outcome.

So both forecasting methods are inaccurate and they merely serve as a rough guide to assessing the future returns on the Large-Cap stock index.

13.6. Summary

Between 1982 and 2015, Large-Cap stocks returned about 11.3% per year on average and Mid-Cap stocks returned about 14.0% per year. The high returns were partially due to very low share-prices in the early 1980's which caused share-prices to grow much more than the earnings and dividends in the following years. If these divergent growth-rates were to continue in the future, then it would eventually lead to the earnings and dividend yield being very low compared to their historical ranges as well as the expected future yield on US government bonds. So it seems unlikely that Large-Cap stocks would continue in the future to return 11.3% per year and Mid-Cap stocks would continue to return 14.0% per year.

It was then assumed that the historical average earnings growth would continue in the future and that the share-price and dividend would grow at the exact same rate. Large-Cap stocks were thus forecasted to return about 7.9% per year, Mid-Cap stocks would return 9.2% per year, and Small-Cap stocks would return 9.8% per year. Although the dividend yields were usually much lower for Mid-Cap and Small-Cap stocks, they have historically had significantly higher earnings growth than Large-Caps stocks. This results in the higher forecasted returns for Mid-Cap and Small-Cap stocks.

It was further noted that share buybacks are now much larger than dividend payouts for Large-Cap companies, but the share buybacks have often been destructive to shareholder value. For example, very large share buybacks were made in year 2007 at high share-prices and for borrowed money. The debt was then repaid in 2008 and 2009 by making even larger share issuances during a severe stock-market crash where the share-prices were very low. This was reckless and could easily have been avoided.

If Large-Cap companies would stop making share buybacks altogether and make dividend payouts instead, then it is possible that the growth-rates for earnings and dividends per-share would remain the same over time, but the increase in dividend yield would add perhaps 2% (percentage points) to the forecasted return, so Large-Cap stocks would return maybe 9.9% instead of only 7.9%.

A limitation of this forecasting method is the assumption that earnings, dividends and share-prices will grow at the same constant rate forever. This is an unrealistic assumption. Another forecasting method was therefore presented for Large-Cap stocks, which uses the so-called P/Book ratio to forecast the 10-year annualized return if dividends are reinvested and there are no taxes. The P/Book forecasting method takes many historical scenarios into account, including periods of over- and under-valuation, bull-markets and crashes. The forecast is an average outcome of all such events and cannot predict exactly whether Large-Cap stocks will be in a bull-market or in a crash 10 years into the future.

Neither of these forecasting methods is precise. Whether the future return is 7.9% or 9.9% or something else for Large-Cap stocks is of course impossible to predict. The main conclusion from the different scenarios and forecasting methods discussed in this section, is that it seems unlikely that the historical average return of 11.3% per year for Large-Cap stocks and 14.0% for Mid-Cap stocks will continue in the future. The future returns would seem to be considerably lower.

This also raises the question whether the many other historical statistics have any relevance in the future. For example, is the historical probability of loss described in section 5 a good estimate of the future probability of loss for these stock indices? And similarly for the historical probability of under-performing US government bonds as described in section 7, and even the historical probability that the stock indices perform better or worse than each other as described in section 8. These questions are unanswered.

14. How to Invest

This section briefly discusses some of the practical aspects of investing in these stock indices.

14.1. Portfolio Allocation

The so-called S&P 1500 index combines the Large-Cap, Mid-Cap and Small-Cap indices into one single index according to their market capitalizations. We will call it the All-Cap index.

On August 31, 2015, the total market capitalization of all companies in the Large-Cap index was about $18,450 billion, while it was $1,560 billion for the companies in the Mid-Cap index and $700 billion for the companies in the Small-Cap index. Altogether the market-cap for these 3 indices was about $20.7 trillion.

This means the Large-Cap index corresponded to about 89% of the All-Cap index, while the Mid-Cap index was 7.5% of the All-Cap index, and the Small-Cap index was only 3.5% of the All-Cap index.

If you invest in the All-Cap index then you have effectively allocated your portfolio according to these ratios, so the majority of your portfolio would be invested in the Large-Cap index, while only a small part of your portfolio would be invested in the Mid-Cap and Small-Cap indices.

Historically the Mid-Cap index has mostly performed better than the Small-Cap index, especially for longer investment periods, while they have both performed much better than the Large-Cap index with regard to the average return, the probability of loss, the probability of under-performing government bonds, etc. Although section 13 raised doubts about the historical performance continuing in the future, the Mid-Cap and Small-Cap stocks did have higher earnings growth historically, which would seem to merit that a significant part of the portfolio is still allocated to Mid-Cap and Small-Cap stocks, under the assumption that their future earnings growth will continue to be higher than for Large-Cap stocks.

The obvious solution would be to allocate one third of the portfolio to each of the stock indices and then rebalance annually. The historical statistics for such rebalancing were given in section 11, but it should again be noted that the future performance may be very different.

Another thing to consider is that Large-Cap stocks have usually had greater dividend yields than Mid-Cap and Small-Cap stocks, so if dividend income is important to you, then you may wish to allocate more of your portfolio to Large-Cap stocks.

Another advantage of investing in the Large-Cap index is that you are likely much more familiar with the companies in the index, with many household names ranging from Coca-Cola to Wal-Mart, Apple and Microsoft. Most people have probably never heard of the majority of the companies in the Mid-Cap and Small-Cap indices and during a stock-market crash you may be more comfortable knowing your investment is in large, well-known companies rather than smaller, unknown companies. So perhaps you will want to allocate 50% of your portfolio to Large-Cap stocks, 30% to Mid-Cap stocks and 20% to Small-Cap stocks.

There is no magic formula that guarantees the best return on your investment, so you should allocate your portfolio between the 3 indices according to your personal preferences.

14.2. Similar Indices

There are many different indices for US Large-Cap, Mid-Cap and Small-Cap stocks. This paper studied the indices created by Standard & Poor's (S&P 500, 400 and 600, respectively). Although the different indices are all created from the same universe of US stocks, there may be subtle differences in how individual stocks are selected. Section 2.5 listed the criteria for inclusion in the S&P indices. For example, one criterion is that companies must have had positive earnings in the most recent quarter and year. This is effectively a stock-screen that other indices may not apply. Conversely, other indices may be more broadly diversified e.g. one Small-Cap index covers more than 1800 stocks compared to only 600 for the S&P index.

It is unclear how these differences may affect the performance of different indices and it would be an interesting and important topic for future research.

14.3. Funds

Investing in stock indices can be done in different ways. You can invest in mutual funds from companies such as Vanguard that offers very low expense ratios which is of great importance for long-term investors, as demonstrated below.

Exchange Traded Funds

There are also so-called Exchange Traded Funds (ETFs) where you can invest in index funds through your stock-broker as if they were ordinary shares. Vanguard's ETF for Large-Cap stocks (S&P 500) trades under the ticker symbol VOO, while the company SPDR offers the ETF with the ticker symbol SPY, and the company Blackrock (aka. iShares) offers the ETF with the ticker symbol IVV.

There are also numerous ways to invest in Mid-Cap stocks (S&P 400), for example Vanguard's ETF trades under the ticker symbol IVOO, while SPDR's ETF trades under the symbol MDY, and the iShares ETF trades under the symbol IJH.

For Small-Cap stocks (S&P 600), Vanguard's ETF trades under the ticker symbol VIOO, while SPDR's ETF trades under the symbol SLY, and the iShares ETF trades under the symbol IJR.

Alternatives

There are numerous alternatives that invest in slightly different indices. For example, the stock-broker Charles Schwab offers its own ETFs which have expense ratios that are slightly lower than Vanguard's. Their ETF for Large-Cap stocks has the ticker symbol SCHX, while their ETF for Mid-Cap stocks has the symbol SCHM, and their ETF for Small-Cap stocks has the symbol SCHA. The ETFs invest in indices derived from the so-called Dow Jones US Total Stock Market Index. If you are a client of Charles Schwab then you can trade these ETFs commission-free so they would be a good alternative to Vanguard.

Some companies provide multiple ETFs that offer similar stock indices but from different index providers. For example, Vanguard provides both the ETF with symbol IVOO which invests in the S&P 400 Mid-Cap index, and the ETF with symbol VO which invests in the CRSP Mid-Cap index. As previously discussed, it is unclear whether there is any significant performance difference between indices that are so similar.

Liquidity Risks of ETFs

The Net Asset Value (NAV) is the value of the stocks actually owned by the ETF (plus cash and minus liabilities which are usually close to zero). The NAV can be different from the share-price of the ETF, but the difference is usually small because institutional investors can buy large blocks of the ETF shares and redeem them for the underlying stocks, which can then be sold for a profit if the combined market price of those stocks is higher than the ETF price. It is also possible to do the opposite trade and profit if the NAV is less than the ETF price.

Because of this, index-based ETFs tend to trade at prices that are very close to their NAV. An exception is when the shares of an ETF are illiquid and someone wants to sell a large number of shares. This happened on August 24, 2015. The actual S&P 400 index dropped about (5%) shortly after the market opened. But one ETF which trades under the ticker symbol MDYV and invests in about 300 of the 400 stocks in the S&P 400, dropped almost (50%) – fifty percent! – even though its NAV had only dropped about (5%). Another ETF which trades under the ticker symbol SCHM dropped about (30%). SCHM also uses a slightly different Mid-Cap index, but the underlying stocks are mostly the same as for the S&P 400.

After an hour or so, the prices of these two ETFs recovered and converged again with the S&P 400, as they should because they are mostly invested in the same underlying stocks. People who had bought these ETFs at the bottom would have gained almost 100% on MDYV and 40% on SCHM.

Why did this happen? We cannot know for certain. Perhaps someone had invested in these ETFs but did not understand that the ETFs actually represent ownership in the underlying stocks which had only dropped about (5%). Or perhaps the initial price-drop of (5%) triggered stop-loss orders or margin calls for someone who had made leveraged investments in these ETFs, and this set off a chain reaction of further price-drops and further stop-loss orders and margin calls. The trading was halted several times and perhaps the price would have dropped even lower otherwise.

The lesson to be learned is that you should not use stop-loss orders on ETFs with low trading volumes because the stop-loss orders may be executed far below the NAV in case of market turmoil. If you want to sell your ETF shares then you should do so manually with a normal limit-order, or use a stop-limit order.

14.4. Expense Ratios

One of the most important aspects of deciding which fund to invest in, is the fund's expense ratio which is deducted from the annual return on the index itself. There are various expenses in running an index fund, e.g. for buying and selling stocks, paying employees, accountants, marketing, etc. You would think the expenses were more or less the same for different index funds, but there are actually large differences and they become especially important when investing for many decades.

For example, assume you invest $10,000 in a Large-Cap index fund and the index itself returns 9% per year when dividends are reinvested and there are no taxes. If you hold the investment for 30 years then it would be worth $132,677 if there were no expenses. The lowest expense ratios for index funds are about 0.05% per year. So if the index returns 9% in the first year then the investment should be worth $10,900 but expenses of $5 must be deducted so the investment is only worth $10,895. This may seem like a small expense but it compounds over many years of investing, because you also lose the compounded return on that $5 as well. After 30 years of deducting 0.05% in expenses each year, the investment is only worth $130,863 which is $1,814 less than the $132,677 that the index actually returned.

The formula for calculating the investment's value after deducting expenses each year is:

Eq. 14-1

$$\textit{Investment Value After Expenses} = \textit{Invested Amount} \times (1 + \textit{Index Return} - \textit{Expense Ratio})^{\textit{Years}}$$

$$= \$10{,}000 \times (1 + 9\% - 0.05\%)^{30} = \$130{,}863$$

If the expense ratio had instead been 0.36%, which is the average expense ratio for Large-Cap index funds according to the research company Morningstar, then the investment's value would only be $120,141 after 30 years, which is $12,536 less than the index actually returned. There are index funds with even higher expense ratios. So it is very important to select an index fund with a very low expense ratio.

14.5. Monthly Investing

It is generally impossible to predict whether stock indices will rise or fall in the near future, but over long periods of time stocks have generally had a positive return, because the earnings have grown over time for the companies in the index. If you add to your investment on a monthly basis, then you will sometimes buy the stocks when they are much too expensive and sometimes when they are bargains compared to the future earnings, but on average you will get a good return if you also hold the investments for many years. Furthermore, if you start investing shortly before a stock-market crash and you continue adding to your investment in stock indices every month during the crash, then you will lower your average purchase price as stock-prices become lower, so you will also recover your investment faster than the stock-market itself.

A detailed example of monthly investing in Large-Cap stocks is given in my other book [**1**].

15. Summary

Historically the Mid-Cap index performed better than the Small-Cap index, and they both performed much better than the Large-Cap index on several performance statistics which are summarized in this section. It is unknown whether this performance will continue in the future.

Long-Term Returns

Between January 1992 and June 2015, Large-Cap stock had an annualized return of 9.3%, while Mid-Cap stocks had an annualized return of 12.0% and Small-Cap stocks had an annualized return of 11.2%. For the longer data-period between January 1981 and June 2015, Large-Cap stocks had an annualized return of 11.3% while Mid-Cap stocks had an annualized return of 14.0%. But there were periods where all three indices performed either best or worst so we also have to consider more detailed performance statistics.

One-Year Returns

Considering all one-year investment periods between 1992 and 2015, Large-Cap stocks had an average return of 11.1%, while Mid-Cap stocks had an average return of 13.8% and Small-Cap stocks had an average return of 13.1%. For all three stock indices, the worst one-year loss was about (48%), while the best one-year gain was 72.1% for Large-Cap stocks, 94.0% for Mid-Cap stocks, and 98.0% for Small-Cap stocks.

10-Year Returns

Considering all 10-year investment periods between 1992 and 2015, Large-Cap stocks had an average annualized return of 6.4%, while Mid-Cap stocks had 10.8%, and Small-Cap stocks had 9.8%.

The worst 10-year period for Large-Cap stocks had an annualized loss of (4.5%), while the worst 10-year periods for Mid-Cap and Small-Cap stocks both had annualized returns of about 2%.

The best 10-year period for Large-Cap stocks had an annualized return of 13.4%, while it was 16.3% for Mid-Cap stocks and 14.8% for Small-Cap stocks.

Probability of Loss

All three stock indices experienced losses in about 20% of all one-year periods between 1992 and 2015. Large-Cap stocks had losses in about 23% of all 5-year periods, while Mid-Cap stocks only had losses in about 3% of all 5-year periods, and Small-Cap stocks had losses in about 4% of all 5-year periods. When investing for 6 years or more, losses were very rare for Mid-Cap and Small-Cap stocks, while Large-Cap stocks had losses in 14% of all 10-year periods.

Considering the longer data-period between 1981 and 2015, Large-Cap and Mid-Cap stocks had losses in about 19-20% of all one-year periods. Large-Cap stocks had losses in about 15% of all 5-year periods, while Mid-Cap stocks had losses in less than 2% of all 5-year periods. Mid-Cap stocks experienced no losses for 10-year periods while Large-Cap stocks had losses in almost 8% of all 10-year periods.

Compared to Inflation and US Government Bonds

The historical probabilities of under-performing inflation and US government bonds showed similar tendencies as the probabilities of loss. All three stock indices had under-performed inflation and government bonds in almost 25% of all one-year investment periods. Mid-Cap and Small-Cap stocks rarely performed worse than inflation and government bonds after investing for 6 years or more, while Large-Cap stocks performed worse than inflation and government bonds in almost 13% of all 10-year periods.

Compared to Other Stocks

Mid-Cap and Small-Cap stocks performed worse than Large-Cap stocks in about 42% of all one-year periods between 1991 and 2015. For longer investment periods it became increasingly rare that Mid-Cap and Small-Cap stocks performed worse than Large-Cap stocks and there were no 10-year investment periods between 1992 and 2015 in which Mid-Cap stocks performed worse than Large-Cap stocks, while Small-Cap stocks performed worse than Large-Cap stock in about 1% of all those 10-year periods.

Small-Cap stocks performed worse than Mid-Cap stocks in about 50% of all one-year periods between 1992 and 2015, while it was about 75% of all 10-year periods.

For the longer data-period between 1981 and 2015, Mid-Cap stocks performed worse than Large-Cap stocks in about 39% of all one-year investment periods, while it was only about 7% of all 10-year periods.

Correlation

The annualized returns on the stock indices were highly correlated, and for 10-year investment periods the returns were nearly perfectly correlated. This does not mean that the returns on the stock indices were nearly identical, but merely that the stock indices mostly had high or low returns for the same periods.

Recovery Times

For all three stock indices, about 60% of all their losses were recovered within 7 calendar days, while 85% of all losses were recovered within one month, 96% of all losses were recovered within 6 months, and about 97-98% of all losses were recovered within one year. Mid-Cap and Small-Cap stocks recovered about 99.8% of all losses within two years, but Large-Cap stocks occasionally experienced losses that took significantly longer to recover and only about 98% of all Large-Cap losses were recovered within two years.

These statistics considered the time it took for the stock indices to recover from losses the first time and ignored any subsequent declines.

Earnings and Price Growth

Between 1981 and 2015 the Large-Cap index had average price-growth of about 8.3-8.9% per year, depending on how the growth-rate is calculated. Compare this to the growth in earnings-per-share which was only about 5.8% per year.

The Mid-Cap index also had a large disparity in growth-rates. The average price-growth was about 11.6% per year, while the growth in earnings-per-share was only about 7.4% per year between 1981 and 2015.

Earnings-data for the Small-Cap index is only available between 1995 and 2015, where the average price-growth was about 8.8-9.6% per year, while the earnings-per-share had a somewhat lower growth-rate of about 8.3-8.5% per year, depending on the calculation method.

During the shorter period between 1995 and 2015, the Large-Cap stocks had growth in earnings-per-share of about 5.8% per year on average, while its price grew 4.2-6.5% per year, depending on calculation method. The Mid-Cap index had earnings-per-share growth of about 7.7-8.3% per year and price-growth of 9.6-10.5% per year, depending on calculation method.

Forecasting Future Returns

Between 1981 and 2015 the Large-Cap stocks returned about 11.3% per year on average through increases in share-price and reinvestment of dividends, while the Mid-Cap stocks returned 14.0% per year. Between 1992 and 2015, Small-Cap stocks returned 11.2% per year. But the growth-rates for share-prices and earnings-per-share were quite different.

Between 1981 and 2015, the price-growth for Large-Cap stocks had exceeded its earnings-per-share growth by more than 2.5% (percentage points) per year. For Mid-Cap stocks the price-growth had exceeded the earnings-per-share growth by more than 3.9% (percentage points) per year. A part of this difference in growth-rates appears to have been the very low stock-prices in the early 1980's that were gradually corrected as the share-prices grew faster than the earnings in the following years. This suggests that the future returns on the stock indices will not be as high as their averages between 1981 and 2015.

Furthermore, if these historical growth-rates for earnings and prices were to continue in the future, then it would eventually lead to the earnings yield and dividend yield being very low compared to their historical ranges and averages, and also compared to the expected future yield on US government bonds.

So it seems more reasonable that the future price-growth will be closer to the earnings-growth. We then estimated the future return on the stock indices from their historical average earnings-growth plus the current dividend yield. On May 31, 2015 the dividend yield was estimated to be about 2.1% for Large-Cap stocks, while it was 1.5% for Mid-Cap stocks and 1.4% for Small-Cap stocks. Adding these to the long-term average earnings growth-rates of 5.8% for Large-Cap stocks, 7.7% for Mid-Cap stocks and 8.4% for Small-Cap stocks, resulted in the forecasted total returns being 7.9% for Large-Cap stocks, 9.2% for Mid-Cap stocks and 9.8% for Small-Cap stocks. These are the estimated returns for long-term investments assuming the historical growth-rates continue in the future, and that the price, earnings and dividends will grow at the same rate. Although these assumptions are unrealistic, especially in the short-term, the forecasts do suggest that the future returns on the stock indices will be significantly lower than their long-term historical averages.

However, it was noted that companies have greatly increased their share buybacks in recent years and they are now much larger than dividend payouts. But the Large-Cap companies have apparently made share buybacks that were destructive to shareholder value. For example, the companies made large share buybacks for borrowed money in year 2007 when the share-prices were high, and then during the stock-market crash of 2008 and 2009 the companies issued even more shares at much lower share-prices to repay the debt. This was reckless and could easily have been avoided.

If companies stop buying back shares altogether, then the dividend yield for Large-Cap companies might increase 2% (percentage points), while the growth in earnings-per-share would apparently remain the same because destructive share buybacks would be avoided. This means the total return on the Large-Cap stocks would increase from the forecasted 7.9% to maybe 9.9% per year.

Data on share buybacks was only available for Large-Cap stocks so it is unclear how Mid-Cap and Small-Cap stocks might be affected.

Portfolio Allocation

Although Mid-Cap stocks have mostly performed better than Small-Cap stocks and they have both performed much better than Large-Cap stocks, there is no guarantee that this will continue in the future.

If the future earnings-growth for Mid-Cap and Small-Cap stocks continue to exceed the earnings-growth of Large-Cap stocks by a few percentage points each year, then Mid-Cap and Small-Cap stocks should eventually perform better than Large-Cap stocks after several years or maybe a decade, even if Mid-Cap and Small-Cap stocks were to become relatively overvalued in the short-term. But it must again be stressed that there is no guarantee that Mid-Cap and Small-Cap stocks will have significantly greater earnings-growth than Large-Cap stocks in the future.

Furthermore, you may not be familiar with most of the companies in the Mid-Cap and Small-Cap indices, and during a stock-market crash you may be more comfortable knowing that your investment is in well-known companies such as Coca-Cola, Wal-Mart and Google, rather than companies you have never heard about.

So the historical performance advantage of Mid-Cap and Small-Cap stocks suggest that you should invest a large part of your portfolio in those two indices, while the familiarity of Large-Cap companies suggests that you should also keep a part of your portfolio in Large-Cap stocks. Because the future returns are inherently unpredictable, it is not possible to determine the best portfolio allocation. A simple solution would be to divide your portfolio evenly between the three indices, or perhaps overweigh Large-Cap stocks if you prefer the familiarity of those stocks, or overweigh Mid-Cap and Small-Cap stocks if you have a long time-horizon and believe those stocks will continue to have greater earnings-growth than Large-Cap companies.

It is furthermore suggested that you add to your investment on a monthly basis, regardless of whether there is currently a bull-market or a crash. If you invest in these indices on a monthly basis and keep the investment for many years or decades, then you should get a good return on average.

Godspeed!

16. Appendix

16.1. Calculating Annualized Returns

According to section 3, the total return for Large-Cap stocks was 707% between January 1992 and June 2015. What was the average return per year that compounded into this total return of 707% during these 23 years? This is called the annualized return and is calculated using the following formula. The formula requires a more accurate number of years in the calculation. The data-period was January 31, 1992 to June 30, 2015 which is about 23.4 years. So the annualized return is calculated as follows:

Eq. 16-1

$$Annualized\ Return = (1 + Total\ Return)^{1/Years} - 1 = (1 + 707\%)^{1/23.4} - 1 \approx 9.334\%$$

This means the Large-Cap stocks returned an average of about 9.334% per year during those 23.4 years. We can check that this is correct by compounding this return for 23.4 years to obtain a total return of 707%:

Eq. 16-2

$$Total\ Return = (1 + Annualized\ Return)^{Years} - 1 = (1 + 9.334\%)^{23.4} - 1 \approx 707\%$$

So the annualized return between January 1992 and June 2015 was about 9.3% for Large-Cap stocks, while it was 12.0% for Mid-Cap stocks and 11.2% for Small-Cap stocks. If you calculate the corresponding total return over 23.4 years then you should get approximately 1,311% for Mid-Cap stocks and 1,094% Small-Cap stocks, with minor differences due to rounding errors.

Bibliography

[1] M.E.H. Pedersen, "Layman's Guide to Investing in the S&P 500," Hvass Laboratories, HL-1502, 2015. [Online]. http://www.hvass-labs.org/books

[2] Apple Inc. Financial Report 10-Q for the Quarter Ending June 27, 2015. [Online]. http://www.sec.gov/Archives/edgar/data/320193/000119312515259935/d927922d10q.htm

[3] Fact Sheet for S&P 500 Large-Cap (August 31, 2015). [Online]. http://www.spindices.com/indices/equity/sp-500

[4] Fact Sheet for S&P 400 Mid-Cap (August 31, 2015). [Online]. http://www.spindices.com/indices/equity/sp-400

[5] Fact Sheet for S&P 600 Small-Cap (August 31, 2015). [Online]. http://www.spindices.com/indices/equity/sp-600

[6] Compustat, Dividend and Earnings Data for S&P 500, S&P 400, and S&P 600.

[7] Historical Prices for S&P 500 Large-Cap. [Online]. https://finance.yahoo.com/q/hp?s=%5EGSPC+Historical+Prices

[8] Historical Prices for S&P 400 Mid-Cap. [Online]. http://finance.yahoo.com/q/hp?s=%5EMID+Historical+Prices

[9] Historical Prices for S&P 600 Small-Cap. [Online]. http://ichart.finance.yahoo.com/table.csv?s=%5ESML&a=1&b=1&c=1902&d=07&e=4&f=2039&g=d&ignore=.csv

[10] US Federal Reserve. Government Bond Yields. [Online]. http://www.federalreserve.gov/releases/h15/data.htm

[11] Federal Reserve Bank of St. Louis (FRED). Consumer Price Index for All Urban Consumers: All Items (CPI-U). [Online]. https://research.stlouisfed.org/fred2/series/CPIAUCSL

[12] Share Buyback Data for S&P 500 (Retrieved September 2015). [Online]. http://www.spindices.com/documents/additional-material/sp-500-buyback.xlsx

[13] M.E.H. Pedersen, "The Value of Share Buybacks," Hvass Laboratories, Report HL-1201, 2012. [Online]. http://www.hvass-labs.org/people/magnus/publications/pedersen2012share-buyback.pdf

[14] Earnings and Earnings-Per-Share for S&P 500 (Note: Earnings = Earnings-Per-Share x Divisor). [Online]. http://www.spindices.com/documents/additional-material/sp-500-eps-est.xlsx

[15] Share Buyback and Issuance amounts supplied by Compustat / S&P Research Staff.

[16] M.E.H. Pedersen, "Strategies for Investing in the S&P 500," Hvass Laboratories, HL-1501, 2015. [Online]. http://www.hvass-labs.org/books/