Share Issuance
Valuation & Simulation

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Abstract

This paper shows how to measure the effect on shareholder value when a company issues new shares. Four different scenarios are covered: (1) New shares are issued in exchange for cash. (2) The company uses that cash to make an investment. (3) The company fully acquires another company using new shares as payment. There may also be synergies with the merged company. (4) The company only acquires a part of another company using newly issued shares as payment. The last two scenarios are also known as stock swaps. The formulas are different in all four scenarios. Simulations are used to give a complete overview of the possible outcomes and their probabilities.

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1 Introduction

When a company issues more shares, it dilutes the ownership of the current shareholders. This may be done for different reasons, e.g. to raise cash for the company's general use, or to repay some of its debt, or for the company to make a specific investment, or to acquire another company in whole or in part.

The question is whether this adds or destroys value for the current shareholders of the company that issues the shares, and for the other party that buys or receives the newly issued shares. The short answer is that it depends on the intrinsic value of the newly issued shares, compared to the cash payment or other value that the company receives in exchange for the new shares.

This paper derives the valuation formulas for different scenarios of a share issuance, and gives examples of how to use the formulas in practice. It is recommended that you read the entire paper, even though you may only be interested in the latter parts, because the first two valuation scenarios are easier to understand, and some of the formulas are also reused throughout the paper.

Especially some of the latter formulas are a bit complicated, but they have been made available as simulation models on the SimSim.Run web-site, so you can easily use them in practice. Example simulations are shown in this paper. Tutorial videos are also available.[1]

The only known precursor to this paper is found in section 4.8 of [Pedersen 2012] for the case of issuing new shares to fund an investment, which is also covered in section 4 below. Otherwise these formulas have apparently never been published anywhere else, perhaps because finance professors have believed for half a century that stock-markets are so-called "efficient" so that share-prices were believed to always equal the "risk-adjusted value" to long-term shareholders, so a share issuance would not affect the shareholder value, as the buyer and seller of the new shares would be exchanging the exact same value. But that is not a rational belief because the value typically depends on the company's future earnings which are uncertain. Once this "efficient market" belief is rejected we are able to derive proper valuation formulas for a share issuance, and by using simulations we get a complete overview of the possible impact to shareholder value.

1 https://www.youtube.com/@simsim-run

2 Intrinsic Value

To measure the valuation impact of a new share issuance, we first need to define the so-called intrinsic value of a company to its shareholders. We use subscripts to denote the two companies A and B. Formulas are identical for the two companies unless noted otherwise.

Let v_A (lower-case) be the intrinsic value of company A to its long-term shareholders, and similarly v_B for company B. This is the total amount and not per-share. It can be defined in different ways. The simplest is to consider the intrinsic value to be the company's excess cash that could be paid out as dividends now, plus the present value of all future earnings for eternity that could also be paid out as dividends:

$$v_A = Excess\ Cash_A + \sum_{t=1}^{\infty} \frac{Earnings_{A,t}}{(1+d)^t} \tag{1}$$

Let *TaxRateDividend* be the tax-rate for dividends. We could use different dividend tax-rates for the two companies, but the formulas would become unwieldy, and it is a reasonable simplification to use the same tax-rates, which also means they cancel out in the formulas below.

Let $Shares_A$ be the number of shares outstanding prior to the new share issuance. Then the per-share, after-tax intrinsic value is denoted as capital V_A and is:

$$V_A = v_A \cdot \frac{1 - TaxRateDividend}{Shares_A} \tag{2}$$

The market capitalization (or market-cap for short) of the company, is the total market-value of all the currently outstanding shares for the company:

$$MarketCap_A = Shares_A \cdot Share\ Price_A \tag{3}$$

Let $New\ Shares_A$ be the number of new shares issued in company A, and let $Issuance_A$ be the market-value of those new shares, so we have the following relations:

$$Issuance_A = New\ Shares_A \cdot Share\ Price_A \quad \Leftrightarrow \quad New\ Shares_A = \frac{Issuance_A}{Share\ Price_A} \tag{4}$$

The total number of shares after the new issuance is then:

$$Shares_A + New\ Shares_A = Shares_A + \frac{Issuance_A}{Share\ Price_A} = Shares_A \cdot \left(1 + \frac{Issuance_A}{MarketCap_A}\right) \tag{5}$$

Let W_A denote the intrinsic value of company A after the share issuance, which is per diluted share and adjusted for the dividend tax. Further below we will consider different scenarios for valuing a share issuance, and the exact definition of W_A depends on the scenario for the share issuance. But the following formulas are common in most scenarios.

2.1 Return on Intrinsic Value (ROIV)

The relative value of the share issuance is calculated as the ratio W_A / V_A between the intrinsic value after and before the share issuance. It can sometimes be reduced to a simple expression, depending on the definition of W_A. Subtracting 1 gives us the gain/loss ratio for the per-share intrinsic value to the current shareholders of the company. We call this for the Return on Intrinsic Value (ROIV):

$$ROIV_A = \frac{W_A}{V_A} - 1 \tag{6}$$

This and all other ROIV formulas are ill-defined when the intrinsic value V_A is zero or negative.

2.2 Return on Issuance (ROIS)

The ROIV ratio measures how much the per-share intrinsic value changed from the share issuance, but it does not measure whether that change was big or small compared to the issuance amount. For this we need another ratio that we call the Return on Issuance (ROIS), which measures the gain/loss of per-share intrinsic value relative to the issuance amount. It is defined as follows:

$$ROIS_A = \frac{W_A - V_A}{\frac{Issuance_A}{Shares_A} \cdot (1 - TaxRateDividend)} \tag{7}$$

The numerator $W_A - V_A$ is the net effect of the share issuance, as it is the difference between the per-share intrinsic value of the company with and without the share issuance.

Unlike the $ROIV_A$ ratio in Eq.(6), the $ROIS_A$ ratio in Eq.(7) can also be calculated for intrinsic values V_A that are zero or negative. But if V_A is positive then we can use Eq.(6) to rewrite the numerator:

$$ROIS_A = \frac{ROIV_A \cdot V_A}{\frac{Issuance_A}{Shares_A} \cdot (1 - TaxRateDividend)} \tag{8}$$

The denominator is the issuance amount adjusted for the number of shares prior to the new share issuance, as well as the dividend tax. This is done to make a fair comparison to V_A in the numerator. Then using the definition of V_A from Eq.(2), the ROIS formula can be reduced to the following:

$$ROIS_A = \frac{ROIV_A \cdot v_A}{Issuance_A} \tag{9}$$

2.3 ROIV vs. ROIS

It is important you understand the difference between the ROIV and ROIS ratios. The ROIV ratio measures the change in per-share intrinsic value to the original shareholders, while the ROIS ratio measures that change relative to the issuance amount. This probably sounds very abstract, but it is hopefully easier to understand from the examples further below.

3 Cash Payment

The first scenario that we consider, is when a company issues shares in exchange for a cash payment. This adds the issuance amount minus fees to the intrinsic value of the company, and it also increases the number of shares outstanding.

In this scenario, the intrinsic value W_A of company A after the share issuance, per diluted share and adjusted for dividend tax, is defined as follows:

$$W_A = \frac{v_A + Issuance_A - Fees_A}{Shares_A + New\ Shares_A} \cdot (1 - TaxRateDividend) \tag{10}$$

The fees are specified because they may be very large when issuing new shares, as opposed to making share buybacks where the fees are minimal in open-market trades. For example, the fees typically range between 5-10% of the issuance amount in Initial Public Offerings (IPO).

The $ROIV_A$ ratio measures the gain/loss of per-share intrinsic value to the current shareholders, which uses Eq.(6) with the W_A defined above, and that reduces to the following:

$$ROIV_A = \frac{W_A}{V_A} - 1 = \frac{1 + \dfrac{Issuance_A - Fees_A}{v_A}}{1 + \dfrac{New\ Shares_A}{Shares_A}} - 1 \tag{11}$$

Using Eqs.(3) and (4) the fraction between the number of new and old shares can also be written as:

$$ROIV_A = \frac{1 + \dfrac{Issuance_A - Fees_A}{v_A}}{1 + \dfrac{Issuance_A}{MarketCap_A}} - 1 \tag{12}$$

The $ROIS_A$ ratio also measures the gain/loss of intrinsic value to the current shareholders, but relative to the issuance amount instead of the intrinsic value. Using the original definition of $ROIS_A$ in Eq.(7) with the definition of W_A from Eq.(10) we get:

$$ROIS_A = \frac{\dfrac{v_A + Issuance_A - Fees_A}{1 + New\ Shares_A / Shares_A} - v_A}{Issuance_A} \tag{13}$$

The fraction between the number of new and old shares can again be rewritten as follows:

$$ROIS_A = \frac{\dfrac{v_A + Issuance_A - Fees_A}{1 + Issuance_A / MarketCap_A} - v_A}{Issuance_A} \tag{14}$$

Now consider the return on the issuance amount from the perspective of the buyer of the newly issued shares, which is denoted $ROIS_{Buyer}$ and defined as follows:

$$ROIS_{Buyer} = \frac{W_A}{\frac{Issuance_A}{New\ Shares_A} \cdot (1 - TaxRateDividend)} - 1 \tag{15}$$

The numerator is the per-share intrinsic value after the share issuance. The denominator is the amount that the buyer has paid for each of the newly issued shares, and adjusted for dividend tax like the numerator.

Using the definition of W_A from Eq.(10) and reducing the formula, we get the following:

$$ROIS_{Buyer} = \frac{v_A - MarketCap_A - Fees_A}{MarketCap_A + Issuance_A} \tag{16}$$

The $ROIS_{Buyer}$ ratio can be compared to Net Present Value ratios for other investments, to assess if those would be more profitable for the buyer than the newly issued shares in company A.

It can be proven that the share issuance in this scenario, is only a so-called "zero-sum game" where the current and new shareholders exchange the same value, when the fees are zero:

$$ROIS_A = -ROIS_{Buyer} \quad \Leftrightarrow \quad Fees_A = 0 \tag{17}$$

3.1 Example of Cash Payment

Let us consider an example where the stock is somewhat over-valued. Say the market-value of all the company's shares is $MarketCap_A = \$\ 10\ b$, but the intrinsic value of the company is only $v_A = \$\ 8\ b$. The company has $Shares_A = 10\ b$ outstanding and wants to issue $New\ Shares_A = 1\ b$ for the amount of $Issuance_A = \$\ 1\ b$ and they have to pay $Fees_A = \$\ 50\ m$.

To better understand this, we calculate the various numbers and ratios using both the general and specialized formulas for this particular share issuance scenario, so you can see how they compare. The dividend taxes are set to zero for convenience, because they cancel out in all the formulas.

The per-share intrinsic value to the original shareholders is calculated using Eq.(2):

$$V_A = \frac{v_A}{Shares_A} = \frac{\$\ 8\ b}{10\ b} = \$\ 0.8 \tag{18}$$

After the share issuance, the per-share intrinsic value to the original shareholders is calculated using Eq.(10) for this particular share issuance scenario:

$$W_A = \frac{v_A + Issuance_A - Fees_A}{Shares_A + New\ Shares_A} = \frac{\$\ 8\ b + \$\ 1\ b - \$\ 50\ m}{10\ b + 1\ b} \simeq \$\ 0.813636 \tag{19}$$

The gain/loss of intrinsic value to the original shareholders is calculated using the general Eq.(6):

$$ROIV_A = \frac{W_A}{V_A} - 1 = \frac{\$\ 0.813636}{\$\ 0.8} - 1 \simeq 1.7\ \% \tag{20}$$

We get the same result using the specialized formula for this share issuance scenario in Eq.(12):

$$ROIV_A = \frac{1 + \frac{Issuance_A - Fees_A}{v_A}}{1 + \frac{Issuance_A}{MarketCap_A}} - 1 = \frac{1 + \frac{\$\ 1\ b - \$\ 50\ m}{\$\ 8\ b}}{1 + \frac{\$\ 1\ b}{\$\ 10\ b}} - 1 \simeq 1.7\ \% \tag{21}$$

This may seem like a minor gain of only 1.7%, but remember that this is measured relative to the intrinsic value. To better understand the magnitude of the gain, we may calculate the $ROIS_A$ ratio, which measures the net effect of the share issuance, relative to the amount that was raised from the issuance of new shares. Using the general definition in Eq.(7) we get:

$$ROIS_A = \frac{W_A - V_A}{\frac{Issuance_A}{Shares_A}} = \frac{\$\ 0.813636 - \$\ 0.8}{\frac{\$\ 1\ b}{10\ b}} \simeq 13.6\ \% \tag{22}$$

Because the intrinsic value is positive we can also use Eq.(9) to get the same result:

$$ROIS_A = \frac{ROIV_A \cdot v_A}{Issuance_A} = \frac{1.7\ \% \cdot \$\ 8\ b}{\$\ 1\ b} \simeq 13.6\ \% \tag{23}$$

So although the number of shares is diluted, the original shareholders actually benefit from the share issuance, because the shares are over-valued compared to their intrinsic value.

Conversely, the buyer of the new shares will incur a significant loss on their invested amount, because they are paying more than the shares are intrinsically worth. This is measured by the $ROIS_{Buyer}$ ratio which can be calculated using the general formula in Eq.(15) as follows:

$$ROIS_{Buyer} = \frac{W_A}{\frac{Issuance_A}{New\ Shares_A}} - 1 = \frac{\$\ 0.813636}{\frac{\$\ 1\ b}{1\ b}} - 1 \simeq -18.6\ \% \tag{24}$$

We get the same result using the specialized formula for this share issuance scenario in Eq.(16):

$$ROIS_{Buyer} = \frac{v_A - MarketCap_A - Fees_A}{MarketCap_A + Issuance_A} = \frac{\$\ 8\ b - \$\ 10\ b - \$\ 50\ m}{\$\ 10\ b + \$\ 1\ b} \simeq -18.6\ \% \tag{25}$$

So the company's original shareholders gain $ROIS_A = 13.6\ \%$ while the buyers of the new shares lose $ROIS_{Buyer} = -18.6\ \%$ on the amount paid. This is not a "zero-sum game" because of the fees involved. If you repeat these calculations with $Fees_A = 0$ you should get $ROIS_A = -ROIS_{Buyer}$.

3.2 Simulation of Cash Payment

We typically don't know the exact intrinsic value of a company, because it is based on estimates of the future earnings. So we should repeat the calculations above with many different assumptions. But it is easier to run computer simulations. It is possible to enter the valuation formulas in an Excel spreadsheet and use simulation add-on packages, but they are old and clunky. The SimSim.Run web-site was made specifically for running this kind of simulation, so it can be done very easily there. We use the same input assumptions as above, except for the intrinsic value which is now a normal-distributed (bell-shaped) random variable with mean \$8b and standard deviation \$1b.

Figure 1 shows the simulated $ROIV_A$ ratio from Eq.(21), which measures the gain/loss of per-share intrinsic value for the company's original shareholders. Most of the simulation results are gains with an average of 1.88% and there is only a small probability of loss.

Figure 2 again shows the simulated $ROIV_A$ ratio from Eq.(21), but this time the current share-price (or equivalently the market-cap) is varied on the x-axis. This makes it easy to see how the $ROIV_A$ ratio would change when the current share-price changes, while the other inputs remain the same.

Figure 3 shows the simulated $ROIS_A$ ratio from Eq.(22), which also measures the gain/loss for the company's current shareholders, but it is relative to the issuance amount instead of the company's intrinsic value. This makes it easier to compare the gain/loss ratio to alternative investments. In this case the $ROIS_A$ ratio has mean 13.7% and std.dev. 9.1%, which is a decent rate of return.

Figure 4 again shows the simulated $ROIS_A$ ratio from Eq.(22), but with the current share-price being varied on the x-axis, to make it easy to see how different share-prices would affect the gain/loss to the company's current shareholders when making a new share issuance.

Figure 5 shows the simulated $ROIS_{Buyer}$ ratio from Eq.(25), which measures the gain/loss for the buyer of the newly issued shares. Most of the simulation results are losses with mean -18.7% and std.dev. 9.1%. So this would most likely be a terrible investment for the buyer of the new shares.

Figure 6 again shows the simulated $ROIS_{Buyer}$ ratio from Eq.(25), but with the current share-price being varied on the x-axis, to make it easy to see how different share-prices would affect the gain/loss to the buyer of the newly issued shares.

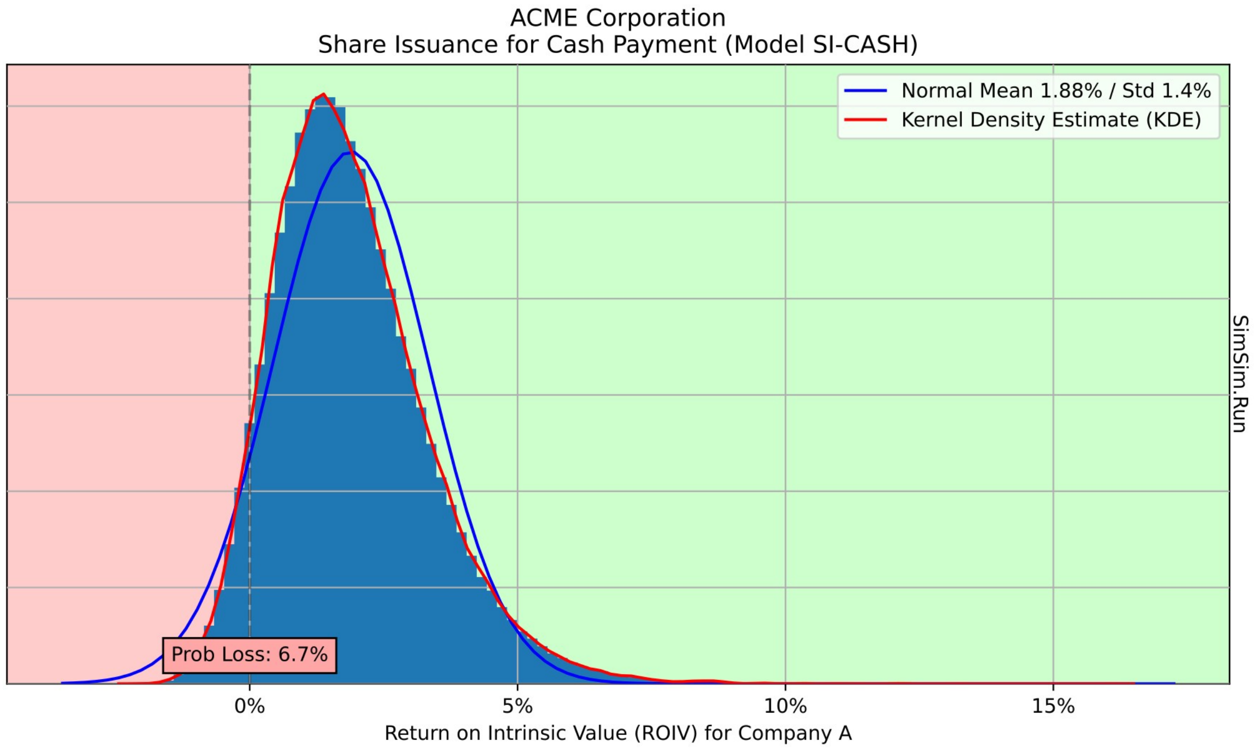

*Figure 1: Simulation of **ROIV**$_A$ in Eq.(21) using a random variable for the intrinsic value v_A.*

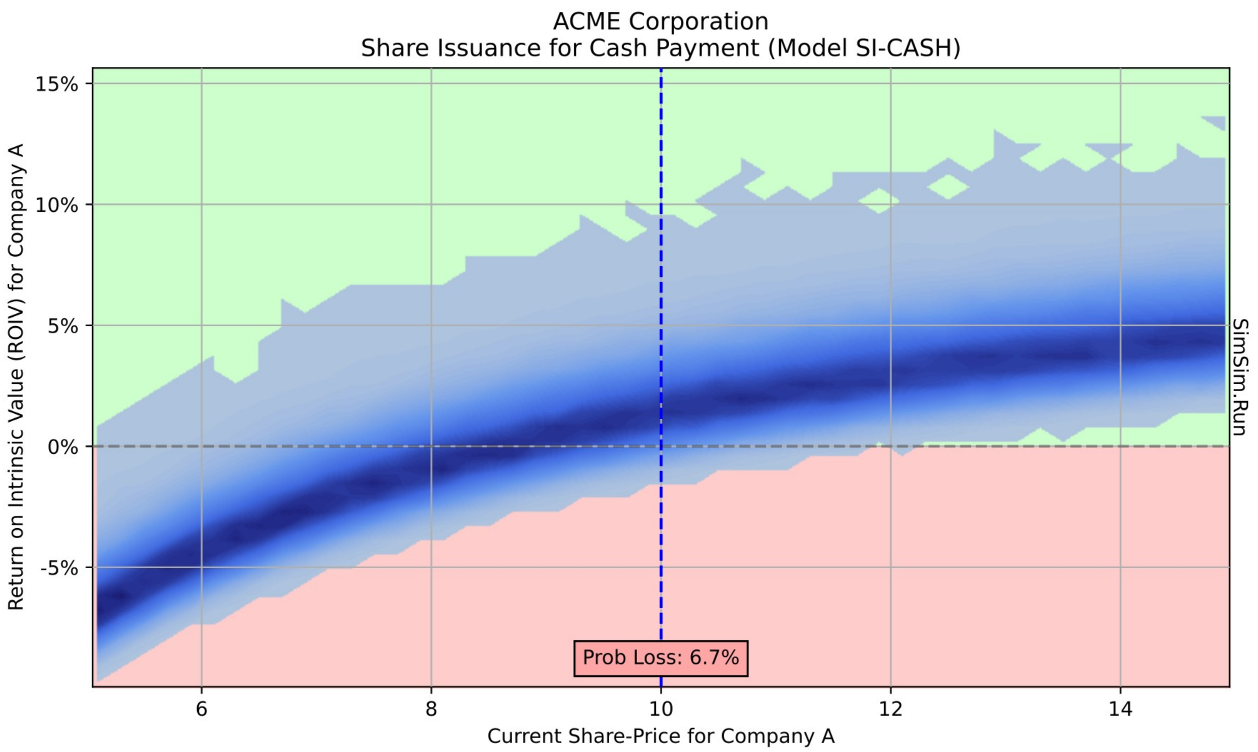

*Figure 2: Simulation of **ROIV**$_A$ in Eq.(21) using a random variable for the intrinsic value v_A, and varying the current share-price (or market-cap) on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 1 is the vertical dashed blue line here.*

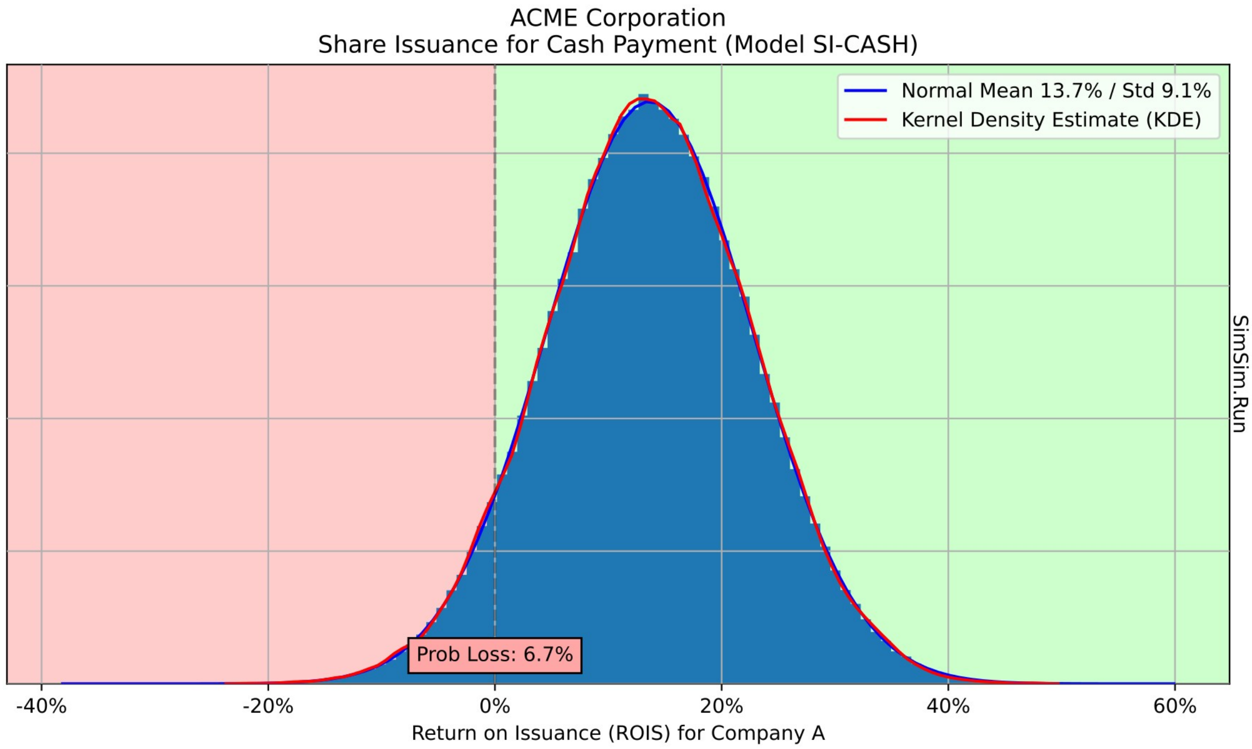

*Figure 3: Simulation of **ROIS**$_A$ in Eq.(22) using a random variable for the intrinsic value v_A.*

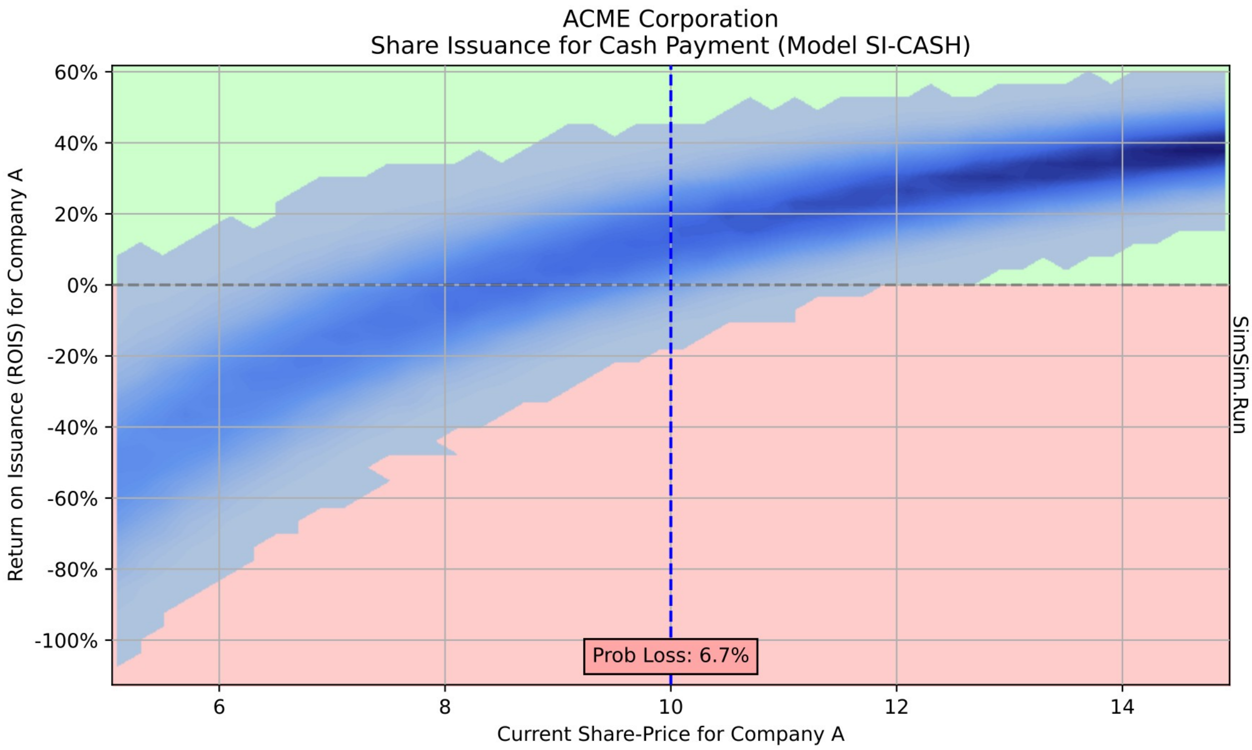

*Figure 4: Simulation of **ROIS**$_A$ in Eq.(22) using a random variable for the intrinsic value v_A, and varying the current share-price (or market-cap) on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 3 is the vertical dashed blue line here.*

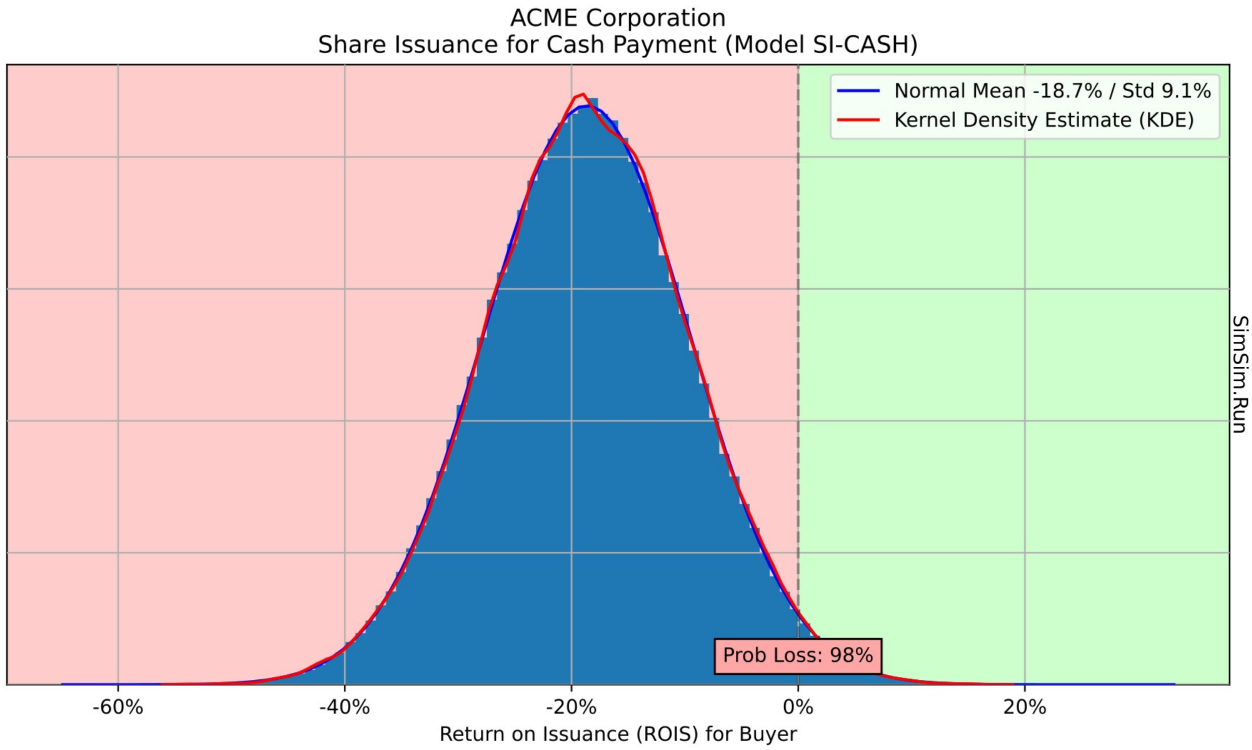

Figure 5: Simulation of $\textbf{ROIS}_{\textbf{Buyer}}$ *in Eq.(25) using a random variable for the intrinsic value* v_A*.*

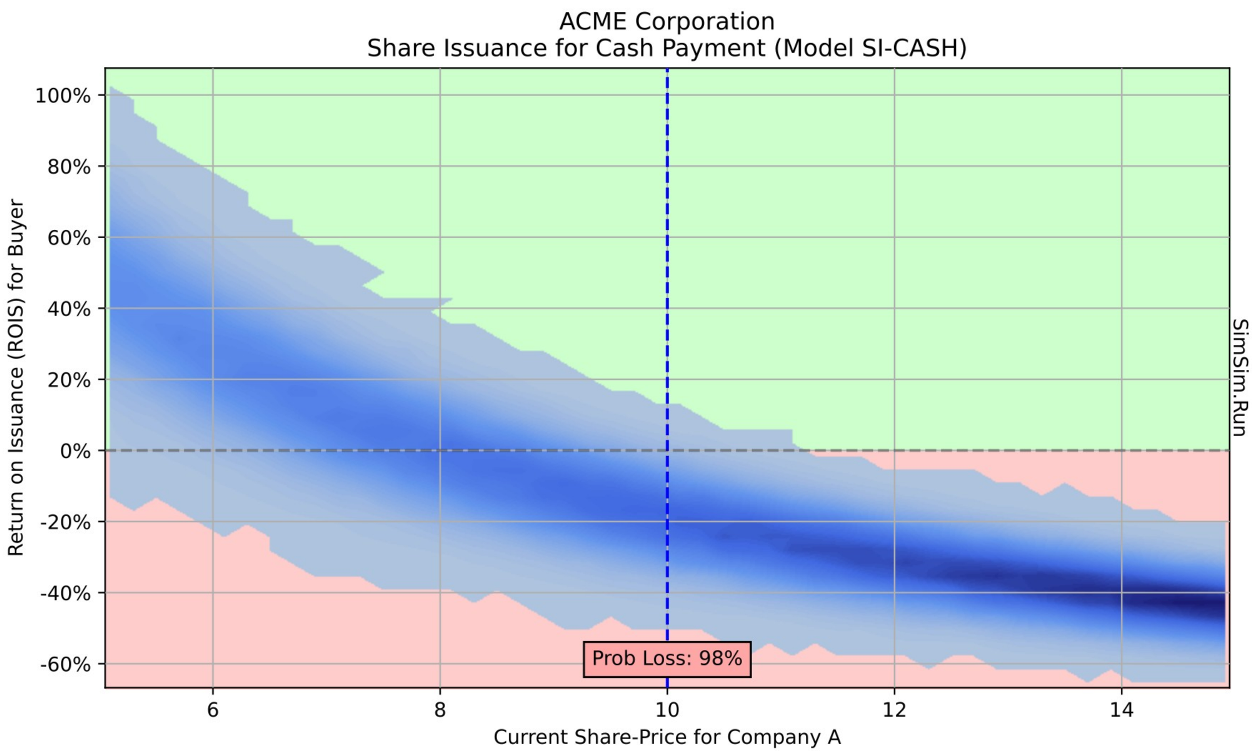

Figure 6: Simulation of $\textbf{ROIS}_{\textbf{Buyer}}$ *in Eq.(25) using a random variable for the intrinsic value* v_A*, and varying the current share-price (or market-cap) on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 5 is the vertical dashed blue line here.*

4 Investment

Now consider a scenario where the share issuance for company A is used to fund an investment that the company wants to make, such as the construction of a new factory. Let *Invest* denote the investment amount, and let *Return* denote the present value of the return on the investment.

In this scenario, the intrinsic value W_A of company A after the share issuance, per diluted share and adjusted for dividend tax, is defined as follows:

$$W_A = \frac{v_A + Issuance_A - Fees_A - Invest + Return}{Shares_A + New\ Shares_A} \cdot (1 - TaxRateDividend) \tag{26}$$

Assume that $Invest = Issuance_A - Fees_A$ so we have:

$$W_A = \frac{v_A + Return}{Shares_A + New\ Shares_A} \cdot (1 - TaxRateDividend) \tag{27}$$

The $ROIV_A$ ratio measures the gain/loss of intrinsic value to the original shareholders, which is:

$$ROIV_A = \frac{W_A}{V_A} - 1 = \frac{1 + \frac{Return}{v_A}}{1 + \frac{New\ Shares_A}{Shares_A}} - 1 \tag{28}$$

The fraction between the number of new and old shares can again be rewritten as follows:

$$ROIV_A = \frac{1 + \frac{Return}{v_A}}{1 + \frac{Issuance_A}{MarketCap_A}} - 1 \tag{29}$$

The $ROIS_A$ ratio also measures the gain/loss to the original shareholders, but relative to the issuance amount instead of the intrinsic value. Using the definition of $ROIS_A$ from Eq.(7) with the definition of W_A from Eq.(27) we get:

$$ROIS_A = \frac{\frac{v_A + Return}{1 + New\ Shares_A / Shares_A} - v_A}{Issuance_A} \tag{30}$$

The fraction between the number of new and old shares can again be rewritten as follows:

$$ROIS_A = \frac{\frac{v_A + Return}{1 + Issuance_A / MarketCap_A} - v_A}{Issuance_A} \tag{31}$$

The $ROIS_{Buyer}$ ratio measures the gain/loss on the issuance amount from the perspective of the buyer of the newly issued shares. Using the original definition from Eq.(15) with W_A from Eq.(27) we get:

$$ROIS_{Buyer} \quad = \quad \frac{v_A + Return}{MarketCap_A + Issuance_A} - 1 \tag{32}$$

It can be proven that the share issuance is only a "zero-sum game" when the issuance amount exactly equals the return on the investment:

$$ROIS_A = -\, ROIS_{Buyer} \qquad \Leftrightarrow \qquad Issuance_A = Return \tag{33}$$

4.1 Example of Investment

Let us consider an example where the stock is somewhat under-valued. Say the market-value of all the company's shares is $MarketCap_A = \$\ 10\ b$, but the company's intrinsic value is $v_A = \$\ 12\ b$. The company has $Shares_A = 10\ b$ outstanding and wants to issue $New\ Shares_A = 1\ b$ for the amount of $Issuance_A = \$\ 1\ b$ including fees. The company makes an investment for the money it raises, and the present value of the return on the investment is $Return = \$\ 1.3\ b$.

To better understand this, we calculate the various numbers and ratios using both the general and specialized formulas for this particular share issuance scenario, so you can see how they compare. The dividend taxes are set to zero for convenience, because they cancel out in all the formulas.

The per-share intrinsic value to the original shareholders is calculated using Eq.(2):

$$V_A \quad = \quad \frac{v_A}{Shares_A} \quad = \quad \frac{\$\ 12\ b}{10\ b} \quad = \quad \$\ 1.2 \tag{34}$$

After the share issuance, the per-share intrinsic value to the original shareholders is calculated using Eq.(27) for this particular share issuance scenario:

$$W_A \quad = \quad \frac{v_A + Return}{Shares_A + New\ Shares_A} \quad = \quad \frac{\$\ 12\ b + \$\ 1.3\ b}{10\ b + 1\ b} \quad \simeq \quad \$\ 1.20909 \tag{35}$$

The gain/loss of intrinsic value to the original shareholders is calculated using the general Eq.(6):

$$ROIV_A \quad = \quad \frac{W_A}{V_A} - 1 \quad = \quad \frac{\$\ 1.20909}{\$\ 1.2} - 1 \quad \simeq \quad 0.76\ \% \tag{36}$$

We get the same result using the specialized formula for this share issuance scenario in Eq.(29):

$$ROIV_A \quad = \quad \frac{1 + \dfrac{Return}{v_A}}{1 + \dfrac{Issuance_A}{MarketCap_A}} - 1 \quad = \quad \frac{1 + \dfrac{\$\ 1.3\ b}{\$\ 12\ b}}{1 + \dfrac{\$\ 1\ b}{\$\ 10\ b}} - 1 \quad \simeq \quad 0.76\ \% \tag{37}$$

This may seem like a negligible gain, but this is relative to the intrinsic value. To better understand the magnitude of the gain, we may calculate the $ROIS_A$ ratio, which measures the net effect of the share issuance, relative to the issuance amount. Using the general definition in Eq.(7) we get:

$$ROIS_A \quad = \quad \frac{W_A - V_A}{\dfrac{Issuance_A}{Shares_A}} \quad = \quad \frac{\$\ 1.20909 - \$\ 1.2}{\dfrac{\$\ 1\ b}{10\ b}} \quad \simeq \quad 9.1\ \% \tag{38}$$

We can also use Eq.(31) that is specialized for this share issuance scenario, to get the same result:

$$ROIS_A \quad = \quad \frac{\dfrac{v_A + Return}{1 + Issuance_A / MarketCap_A} - v_A}{Issuance_A} \quad = \quad \frac{\dfrac{\$\ 12\ b + \$\ 1.3\ b}{1 + \$\ 1\ b / \$\ 10\ b} - \$\ 12\ b}{\$\ 1\ b} \simeq 9.1\ \% \tag{39}$$

So although the number of shares is diluted through a new share issuance, at a share-price that is lower than the intrinsic value, the original shareholders of company A still benefit from the share issuance, because the return on the investment is greater than the loss from issuing new shares at prices that are below their intrinsic value.

The buyer of the new shares gains both from the shares being under-valued, and from the return on the company's investment. This is measured by the $ROIS_{Buyer}$ ratio in Eq.(32):

$$ROIS_{Buyer} \quad = \quad \frac{v_A + Return}{MarketCap_A + Issuance_A} - 1 \quad = \quad \frac{\$\ 12\ b + \$\ 1.3\ b}{\$\ 10\ b + \$\ 1\ b} - 1 \quad \simeq \quad 20.9\ \% \tag{40}$$

So the company's original shareholders gain $ROIS_A = 9.1\ \%$ while the buyer of the new shares gain $ROIS_{Buyer} = 20.9\ \%$ measured relative to the amount paid for the new share issuance. Both parties gain from the share issuance in this example, even though the company's shares are under-valued, because the company's original shareholders gain even more from the return on the investment.

This is only a "zero-sum game" when $Issuance_A = Return$. If you repeat the calculations above with such values then you should get $ROIS_A = -\ ROIS_{Buyer}$.

4.2 Simulation of Investment

We typically don't know the exact values for the company's intrinsic value and the return on the investment, because they are estimated from future values that are uncertain. Instead of repeating the above calculations with many different input values, it is much easier to use computer simulations. The following plots were produced on the SimSim.Run web-site. We use the same input as above, except for the intrinsic value v_A which is a normal-distributed value with mean $12b and std.dev. $1b, while *Return* is a normal-distributed value with mean $1.3b and std.dev. $0.1b.

Figure 7 shows the simulated $ROIV_A$ ratio from Eq.(37), which measures the gain/loss of per-share intrinsic value for the company's original shareholders. About 76% of the simulation results are gains with an average of 0.8%.

Figure 8 again shows the simulated $ROIV_A$ ratio from Eq.(37), but this time the current share-price (or equivalently the market-cap) is varied on the x-axis. This makes it easy to see how the $ROIV_A$ ratio would change when the current share-price changes, while the other inputs remain the same. This shows the share-price should be above $14 for all the simulated $ROIV_A$ ratios to be gains.

Figure 9 shows the simulated $ROIS_A$ ratio from Eq.(39), which also measures the gain/loss for the company's current shareholders, but it is relative to the issuance amount instead of the company's intrinsic value. This makes it easier to compare the gain/loss ratio to alternative investments. In this case the $ROIS_A$ ratio has mean 9.1% and std.dev. 13%, which is a decent rate of return on average, but the distribution has a large spread so there is a 24% chance of loss for the current shareholders.

Figure 10 again shows the simulated $ROIS_A$ ratio from Eq.(39), but with the current share-price being varied on the x-axis, to make it easy to see how different share-prices would affect the gain/loss to the company's current shareholders when making a new share issuance. This again shows the share-price should be above $14 for all the simulated $ROIS_A$ ratios to be gains.

Figure 11 shows the simulated $ROIS_{Buyer}$ ratio from Eq.(40), which measures the gain/loss for the buyer of the newly issued shares. Most of the simulation results are gains with mean 20.9% and std.dev. 9.1%. There is a tiny probability of loss. So this would most likely be a great investment for the buyer of the newly issued shares.

Figure 12 again shows the simulated $ROIS_{Buyer}$ ratio from Eq.(40), but with the current share-price being varied on the x-axis, to make it easy to see how different share-prices would affect the gain/loss to the buyer of the newly issued shares.

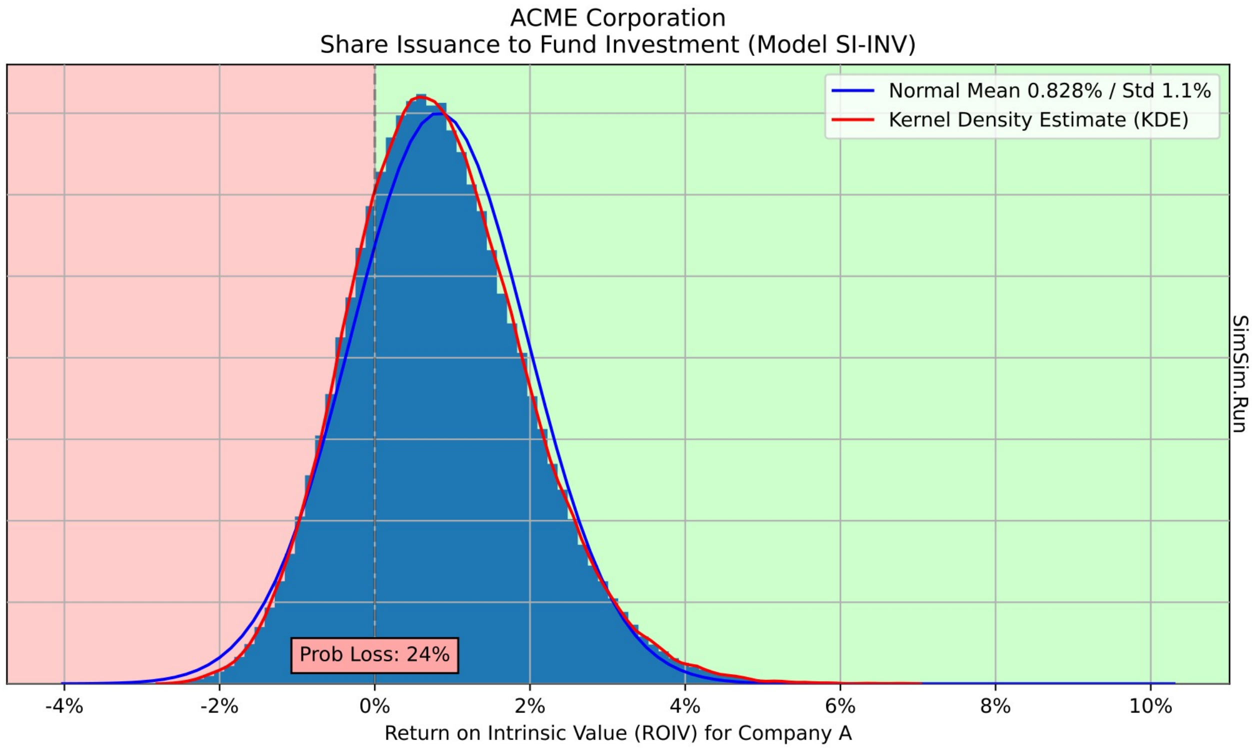

Figure 7: Simulation of $\mathbf{ROIV}_A$ *in Eq.(37) using random variables for the intrinsic value* v_A *and the Return.*

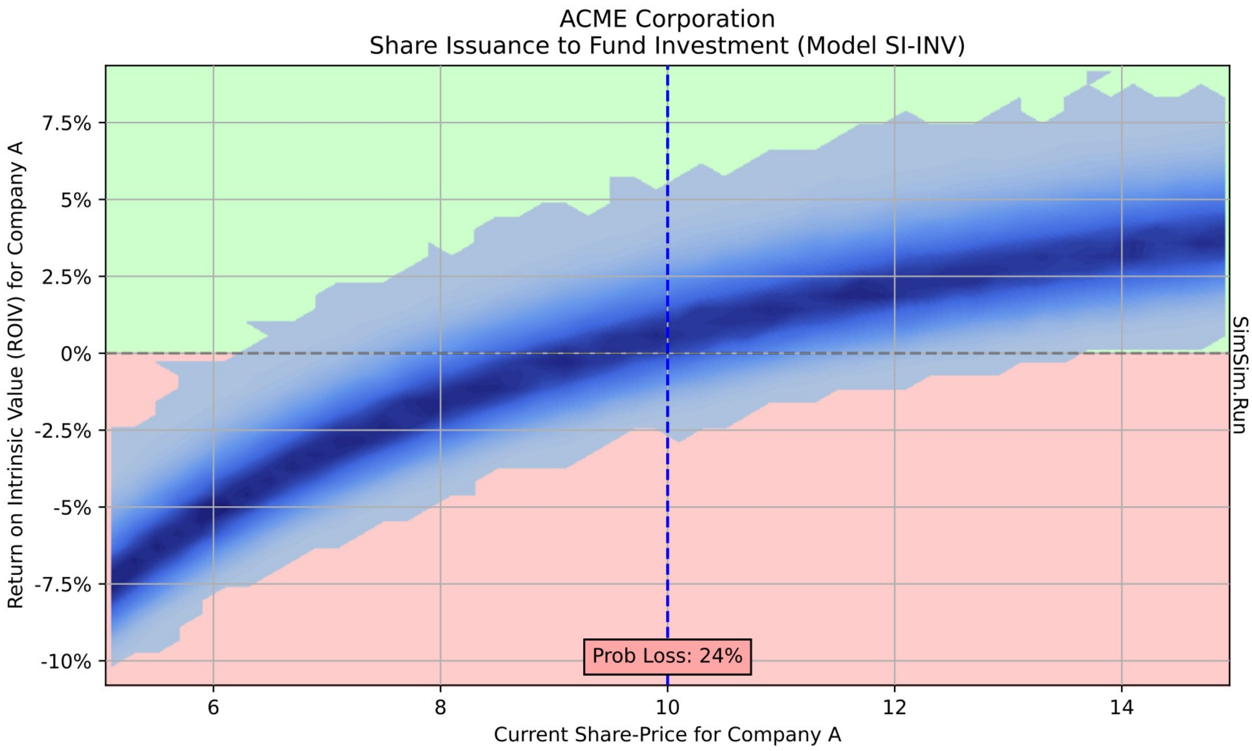

Figure 8: Simulation of $\mathbf{ROIV}_A$ *in Eq.(37) using random variables for the intrinsic value* v_A *and the Return, and varying the current share-price (or market-cap) on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 7 is the vertical dashed blue line here.*

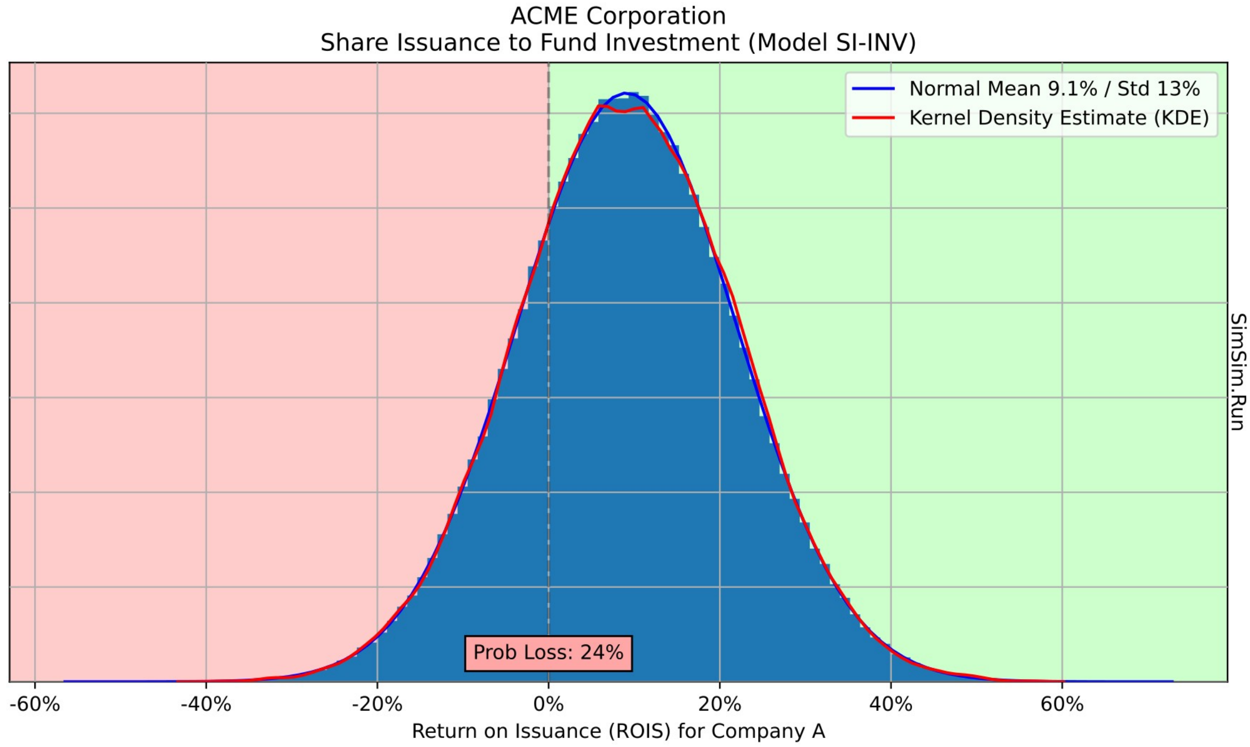

*Figure 9: Simulation of **ROIS**$_A$ in Eq.(39) using random variables for the intrinsic value v_A and the Return.*

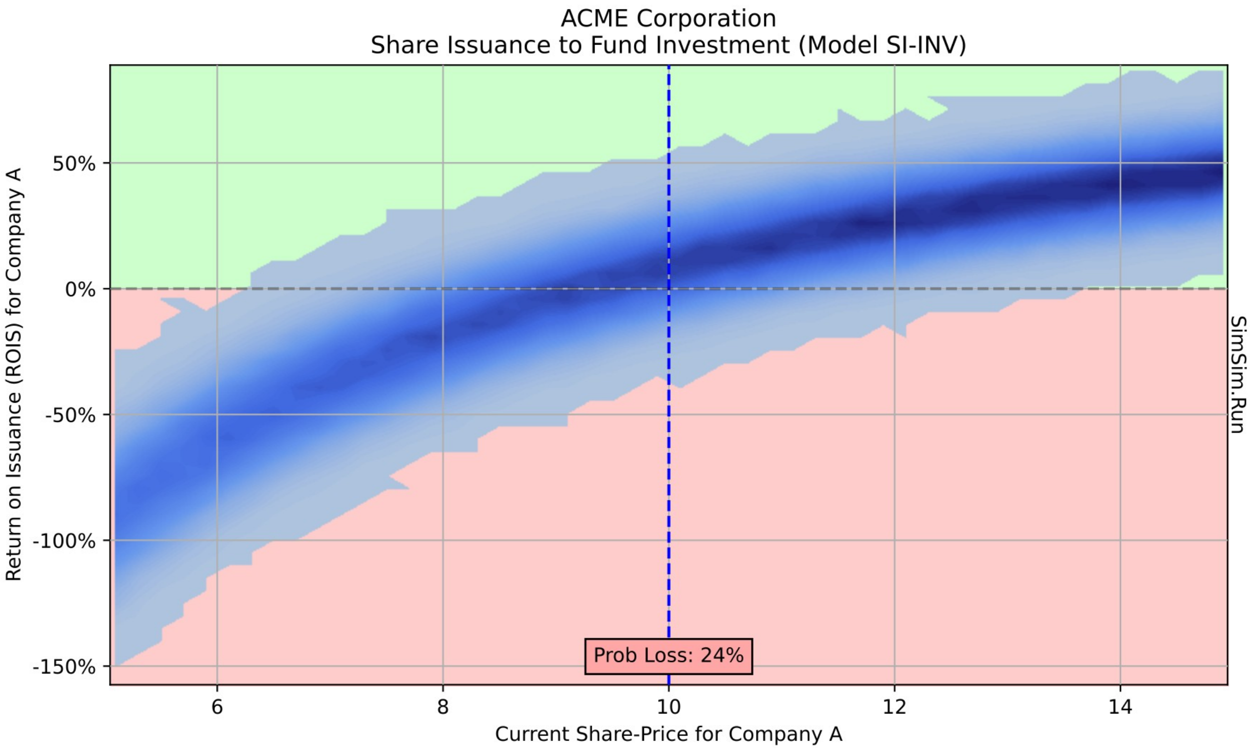

*Figure 10: Simulation of **ROIS**$_A$ in Eq.(39) using random variables for the intrinsic value v_A and the Return, and varying the current share-price (or market-cap) on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 9 is the vertical dashed blue line here.*

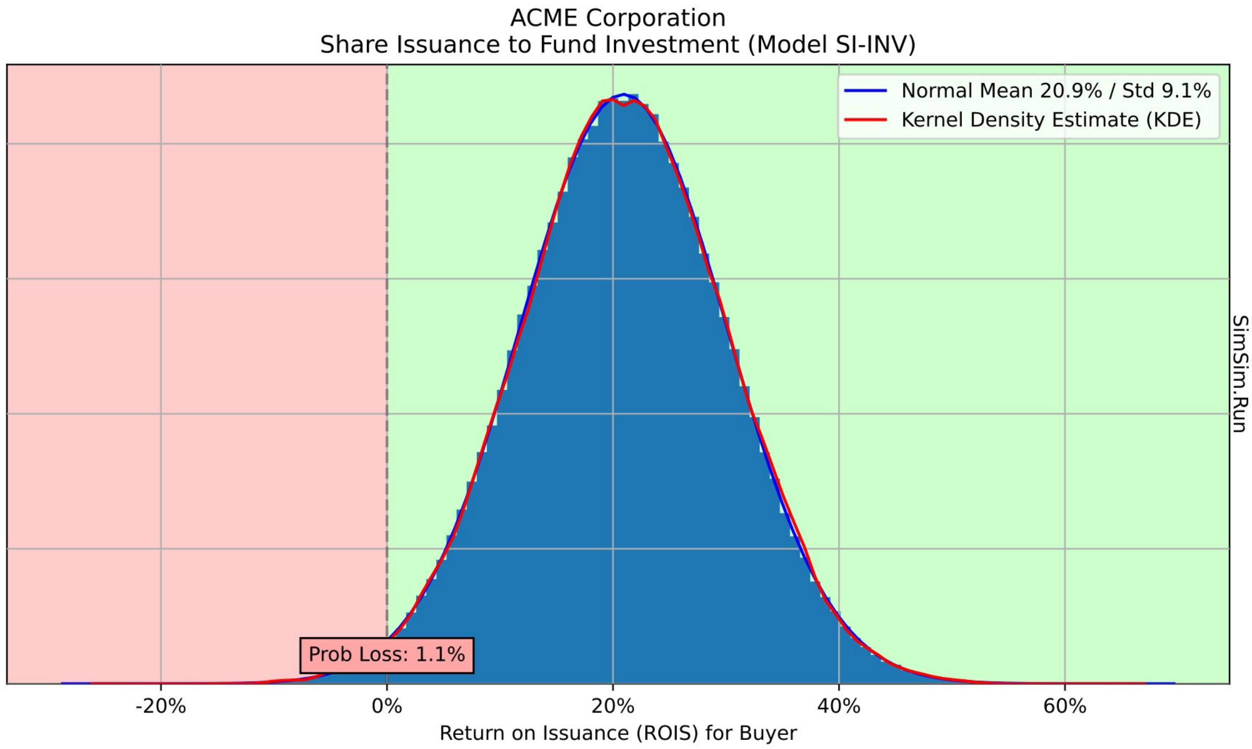

Figure 11: Simulation of $\boldsymbol{ROIS_{Buyer}}$ in Eq.(40) using random variables for the intrinsic value v_A and the Return.

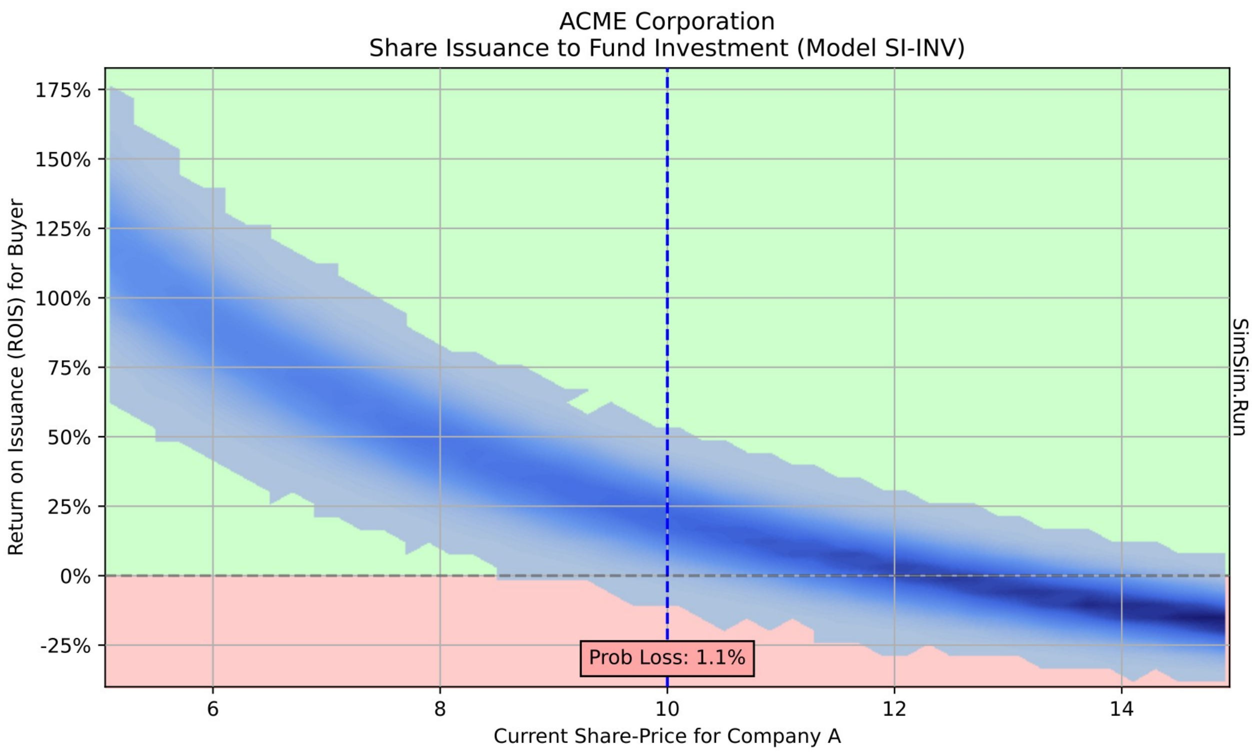

Figure 12: Simulation of $\boldsymbol{ROIS_{Buyer}}$ in Eq.(40) using random variables for the intrinsic value v_A and the Return, and varying the current share-price (or market-cap) on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 11 is the vertical dashed blue line here.

5 Full Acquisition

Now consider a scenario with two companies A and B, where company A acquires company B in full, and pays for the acquisition with newly issued shares in company A. This is also called a Stock Swap because the shareholders of company B exchange all their shares for newly issued shares in company A. The question is whether the shareholders in either company will gain or lose from this exchange.

The Swap Ratio is how many new shares in company A are issued and exchanged for each share in company B:

$$Swap\ Ratio = \frac{New\ Shares_A}{Shares_B} \tag{41}$$

The issuance amount for company A is defined in Eq.(4) and is the market-value for the new shares in company A. If the stock swap is done according to current market-values for the shares in both companies, then the issuance amount must also equal the entire market-cap of company B as defined in Eq.(3) (with subscript B instead). Combining these two formulas and reducing, the swap ratio can also be written as the ratio between the two share-prices:

$$Swap\ Ratio = \frac{Share\ Price_B}{Share\ Price_A} \tag{42}$$

This is useful if you want to use a swap ratio with current market-values for the shares of the two companies. But this does NOT mean the shareholders of the two companies have equal chance of gain or loss. Because that also depends on the intrinsic values of the two companies relative to their market-values, as well as the earnings synergies that arise from the merger. That is why the simulation model allows you to set another swap ratio, than the one implied by the share-prices.

Let *Synergy* denote the present value of the earnings synergies that arise from the merger of the two companies. This could be the present value of all future savings from closing redundant factories.

In this scenario, the intrinsic value of the merged company A after the stock swap, per diluted share and adjusted for dividend tax, is defined as follows:

$$W_A = \frac{v_A + v_B + Synergy - Fees_A}{Shares_A + New\ Shares_A} \cdot (1 - TaxRateDividend) \tag{43}$$

For the original shareholders in company A, the $ROIV_A$ ratio measures their gain/loss of intrinsic value, which reduces to the following:

$$ROIV_A = \frac{W_A}{V_A} - 1 = \frac{1 + \dfrac{v_B + Synergy - Fees_A}{v_A}}{1 + \dfrac{New\ Shares_A}{Shares_A}} - 1 \tag{44}$$

The fraction between the number of new and old shares in the merged company A can be written as:

$$ROIV_A \quad = \quad \frac{1 + \dfrac{v_B + Synergy - Fees_A}{v_A}}{1 + \dfrac{Issuance_A}{MarketCap_A}} - 1 \tag{45}$$

But to calculate the relative value of the stock swap for the original shareholders of company B, we need a slightly different definition of its intrinsic value. So let the per-share, after-tax intrinsic value of company B prior to the stock swap be denoted $\hat{V}_B$ with a so-called "hat" marker, because it is calculated differently by using the number of newly issued shares in company A, rather than the original number of shares in company B:

$$\hat{V}_B \quad = \quad v_B \cdot \frac{1 - TaxRateDividend}{New\ Shares_A} \tag{46}$$

For the original shareholders in company B, the $ROIV_B$ ratio is calculated using $\hat{V}_B$, so we make a proper comparison of the number of shares in the merged company. The formula reduces to:

$$ROIV_B \quad = \quad \frac{W_A}{\hat{V}_B} - 1 \quad = \quad \frac{1 + \dfrac{v_A + Synergy - Fees_A}{v_B}}{1 + \dfrac{Shares_A}{New\ Shares_A}} - 1 \tag{47}$$

The fraction between the number of old and new shares in company A can again be rewritten as:

$$ROIV_B \quad = \quad \frac{1 + \dfrac{v_A + Synergy - Fees_A}{v_B}}{1 + \dfrac{MarketCap_A}{Issuance_A}} - 1 \tag{48}$$

The $ROIS_A$ ratio for company A is the same as Eq.(7), but for the original shareholders of company B it is defined using $\hat{V}_B$ and $New\ Shares_A$ to make a proper comparison to the per-share numbers in the merged company:

$$ROIS_B \quad = \quad \frac{W_A - \hat{V}_B}{\dfrac{Issuance_A}{New\ Shares_A} \cdot \left(1 - TaxRateDividend\right)} \tag{49}$$

which can be reduced to the following:

$$ROIS_B \quad = \quad \frac{ROIV_B \cdot v_B}{Issuance_A} \tag{50}$$

It can be proven that this is only a "zero-sum game" when the fees equal the earnings synergies:

$$ROIS_A = -ROIS_B \quad \Leftrightarrow \quad Fees_A = Synergy \tag{51}$$

5.1 Example of Full Acquisition

Let us consider an example where the stock of company A is somewhat under-valued, and the stock of company B is somewhat over-valued. Say the market-cap and intrinsic value of company A is:

$$MarketCap_A = \$\ 10\ b \qquad v_A = \$\ 12\ b \tag{52}$$

For company B the numbers are:

$$MarketCap_B = \$\ 1\ b \qquad v_B = \$\ 800\ m \tag{53}$$

The present value of the earnings synergies from the merger is $200m. The fees are $50m. We will make the stock swap at the current market-value for the two companies, so the newly issued shares in company A must have the same market-value as all the shares in company B:

$$Issuance_A = MarketCap_B = \$\ 1\ b \tag{54}$$

The gain/loss of intrinsic value to the original shareholders of company A is found using Eq.(45):

$$ROIV_A = \frac{1 + \dfrac{v_B + Synergy - Fees_A}{v_A}}{1 + \dfrac{Issuance_A}{MarketCap_A}} - 1 = \frac{1 + \dfrac{\$\ 800\ m + \$\ 200\ m - \$\ 50\ m}{\$\ 12\ b}}{1 + \dfrac{\$\ 1\ b}{\$\ 10\ b}} - 1 \simeq -1.9\ \% \tag{55}$$

The gain/loss of intrinsic value to the original shareholders of company B is found using Eq.(48):

$$ROIV_B = \frac{1 + \dfrac{v_A + Synergy - Fees_A}{v_B}}{1 + \dfrac{MarketCap_A}{Issuance_A}} - 1 = \frac{1 + \dfrac{\$\ 12\ b + \$\ 200\ m - \$\ 50\ m}{\$\ 800\ m}}{1 + \dfrac{\$\ 10\ b}{\$\ 1\ b}} - 1 \simeq 47.2\ \% \tag{56}$$

There is a very large difference in ROIV ratios for the two companies, because these ratios measure the change in intrinsic values for the two companies, and company A has 15x higher intrinsic value than company B. If we instead consider the ROIS ratios from Eqs.(9) and (50), then we compare the change in shareholder value relative to the issuance amount. In this example we get more similar numbers for the two companies, but with opposite signs:

$$ROIS_A = \frac{ROIV_A \cdot v_A}{Issuance_A} = \frac{-1.9\ \% \cdot \$\ 12\ b}{\$\ 1\ b} \simeq -22.7\ \% \tag{57}$$

$$ROIS_B = \frac{ROIV_B \cdot v_B}{Issuance_A} = \frac{47.2\ \% \cdot \$\ 800\ m}{\$\ 1\ b} \simeq 37.7\ \% \tag{58}$$

This means the shareholders of company A have lost -22.7% on the issuance amount, while the shareholders of company B have gained 37.7%. This is because the shares of company A were somewhat under-valued, while the shares of company B were somewhat over-valued.

The earnings synergies were not enough to counter-balance the fees and mis-valuations that caused losses for the original shareholders of company A.

This is only a "zero-sum game" when the fees equal the earnings synergies. If you repeat the calculations above with $Fees_A = Synergy$ then you should get $ROIS_A = -ROIS_B$.

5.2 Simulation of Full Acquisition

We typically don't know the exact intrinsic values of the two companies and their earnings synergy, because they are estimated from future values that are uncertain. Instead of repeating the above calculations with many different input values, it is much easier to use computer simulations. The following plots were produced on the SimSim.Run web-site. We use the same input as above, except for the intrinsic value v_A which is normal-distributed with mean \$12b and std.dev. \$1b, and the intrinsic value v_B which is normal-distributed with mean \$800m and std.dev \$100m, and *Synergy* which is normal-distributed with mean \$200m and std.dev. \$20m.

Figure 13 shows the simulated $ROIV_A$ ratio from Eq.(55), which measures the gain/loss of per-share intrinsic value for company A's current shareholders. About 96% of the simulation results are losses with an average of -1.84%.

Figure 14 again shows the simulated $ROIV_A$ ratio from Eq.(55), but this time the Swap Ratio is varied on the x-axis. This makes it easy to see how the $ROIV_A$ ratio would change with different Swap Ratios, while the other inputs remain the same. This shows the Swap Ratio should probably be below 0.5 for most of the simulated $ROIV_A$ ratios to be gains.

Figure 15 shows the simulated $ROIV_B$ ratio from Eq.(56), which measures the gain/loss of per-share intrinsic value for company B's current shareholders. About 99.8% of the simulation results are gains with an average of 49.4%.

Figure 16 shows the simulated $ROIV_B$ ratio from Eq.(56) with varying Swap Ratios on the x-axis.

Figure 17 shows the simulated $ROIS_A$ ratio from Eq.(57), which also measures the gain/loss for company A's current shareholders, but it is relative to the issuance amount instead of the company's intrinsic value. This makes it easier to compare the gain/loss ratio to alternative investments. In this case the $ROIS_A$ ratio has a 96% probability of loss with an average of -22.7%. So the stock swap would most likely be a horrible deal for the current shareholders of company A.

Figure 18 shows the simulated $ROIS_A$ ratio from Eq.(57) with varying Swap Ratios on the x-axis. This again shows the Swap Ratio should probably be below 0.5 for most $ROIS_A$ ratios to be gains.

Figure 19 shows the simulated $ROIS_B$ ratio from Eq.(58), which measures the gain/loss for company B's current shareholders relative to the issuance amount. These are nearly all gains with a mean of 37.7%. So the deal would most likely be a big gain for the current shareholders of company B.

Figure 20 shows the simulated $ROIS_B$ ratio from Eq.(58) with varying Swap Ratios on the x-axis. Swap Ratios down to 0.8 would probably still be a good deal for the shareholders of company B.

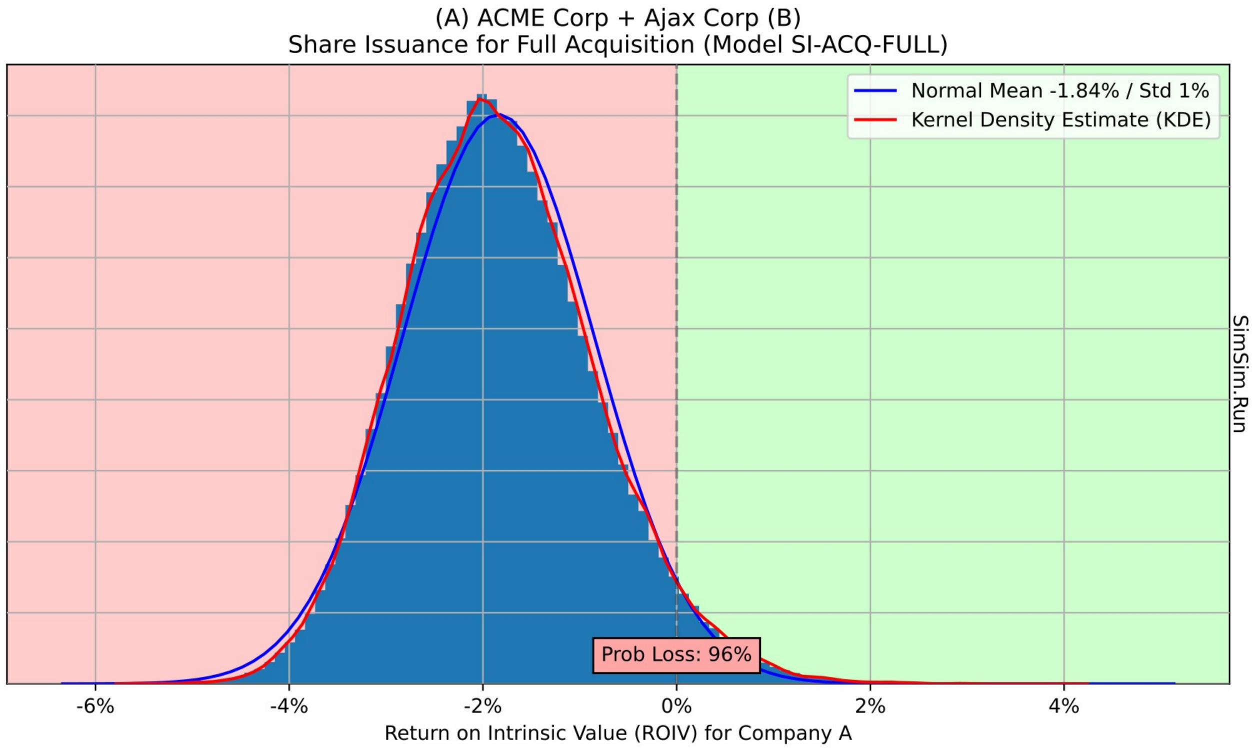

Figure 13: Simulation of ***ROIV***

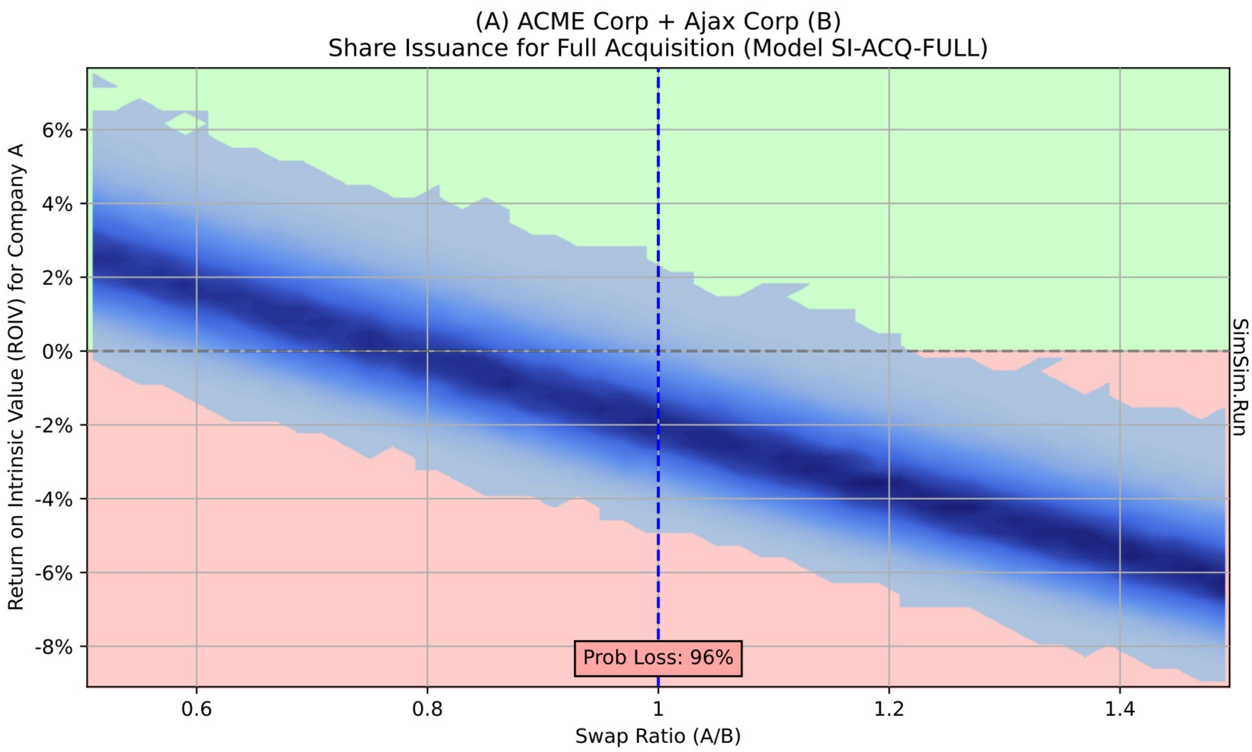

Figure 14: Simulation of $\mathbf{ROIV}_A$ *in Eq.(55) using random variables for the intrinsic values* v_A, v_B, *and Synergy, and varying the Swap Ratio on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 13 is the vertical dashed blue line here.*

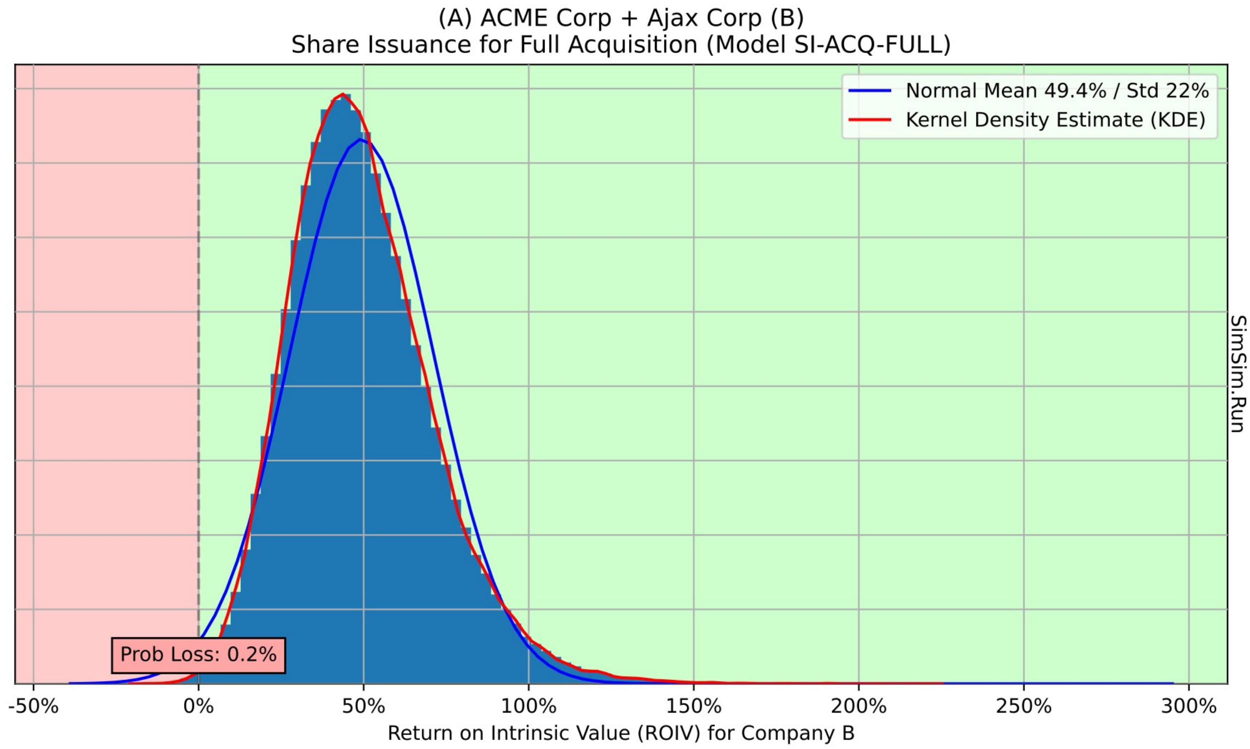

Figure 15: Simulation of $\boldsymbol{ROIV_B}$ *in Eq.(56) using random variables for the intrinsic values* v_A, v_B, *and Synergy.*

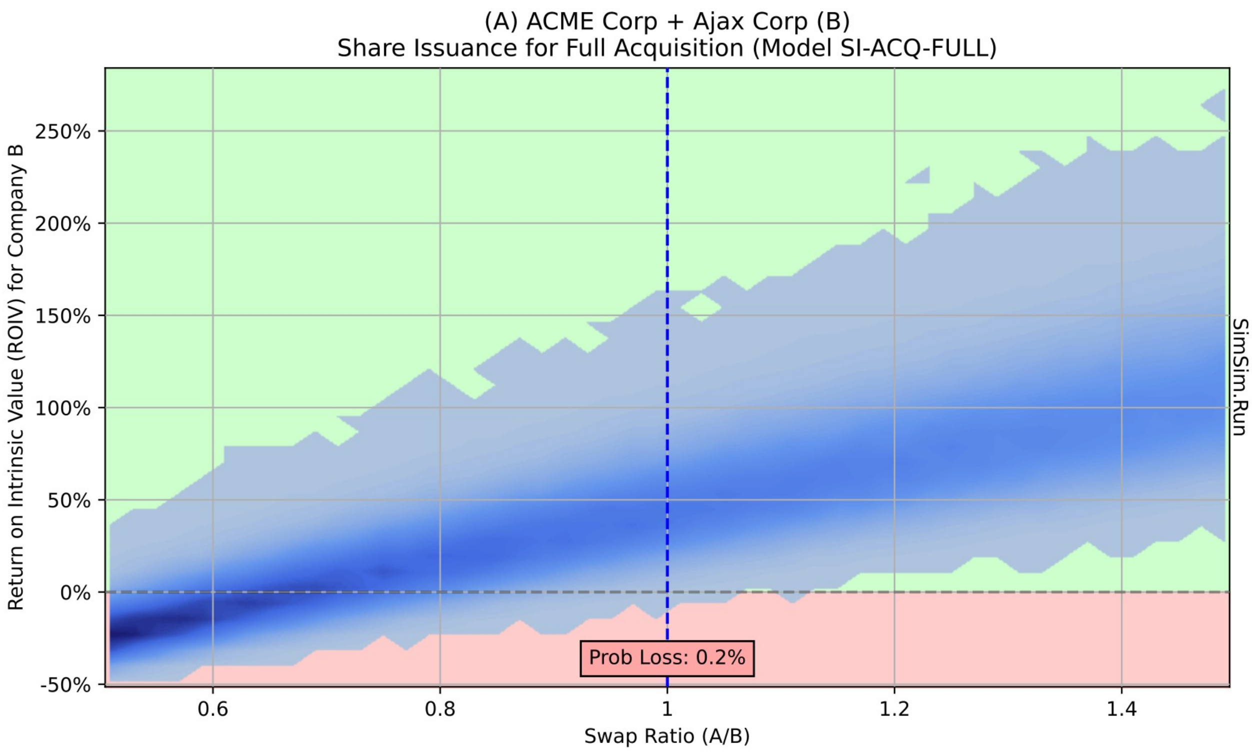

Figure 16: Simulation of $\boldsymbol{ROIV_B}$ *in Eq.(56) using random variables for the intrinsic values* v_A, v_B, *and Synergy, and varying the Swap Ratio on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 15 is the vertical dashed blue line here.*

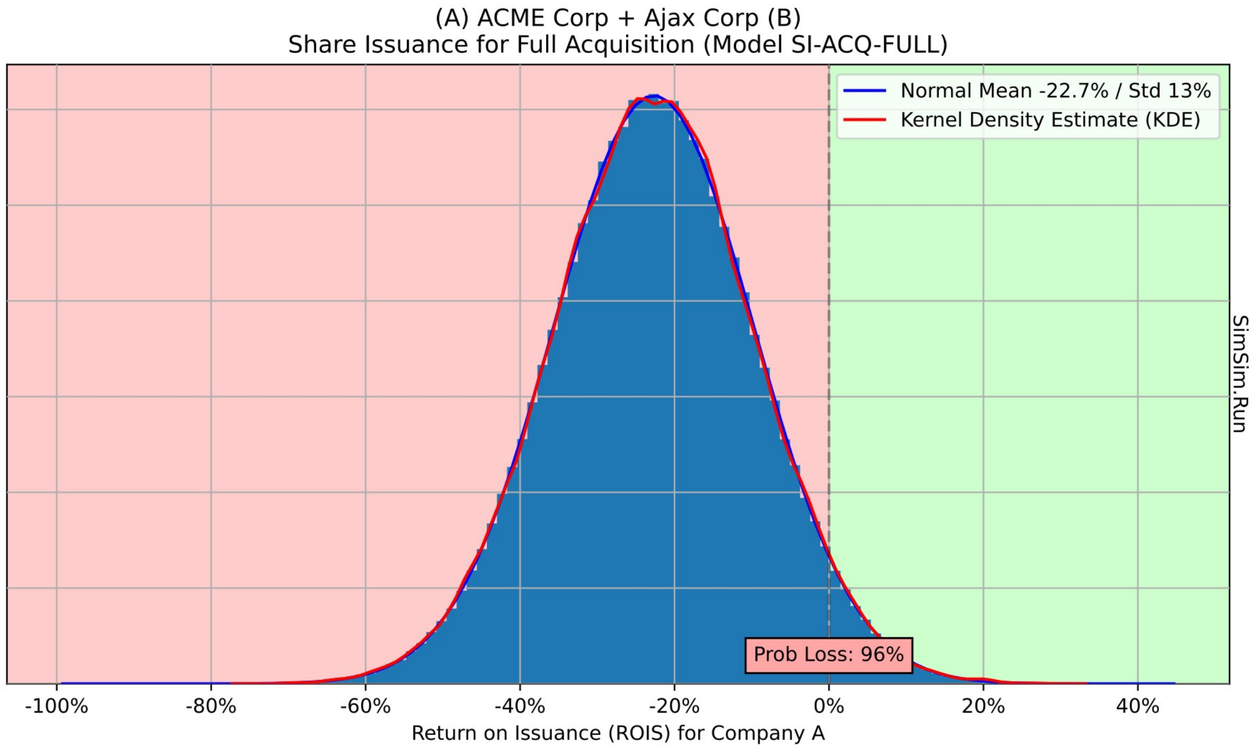

*Figure 17: Simulation of **ROIS**$_A$ in Eq.(57) using random variables for the intrinsic values v_A, v_B, and Synergy.*

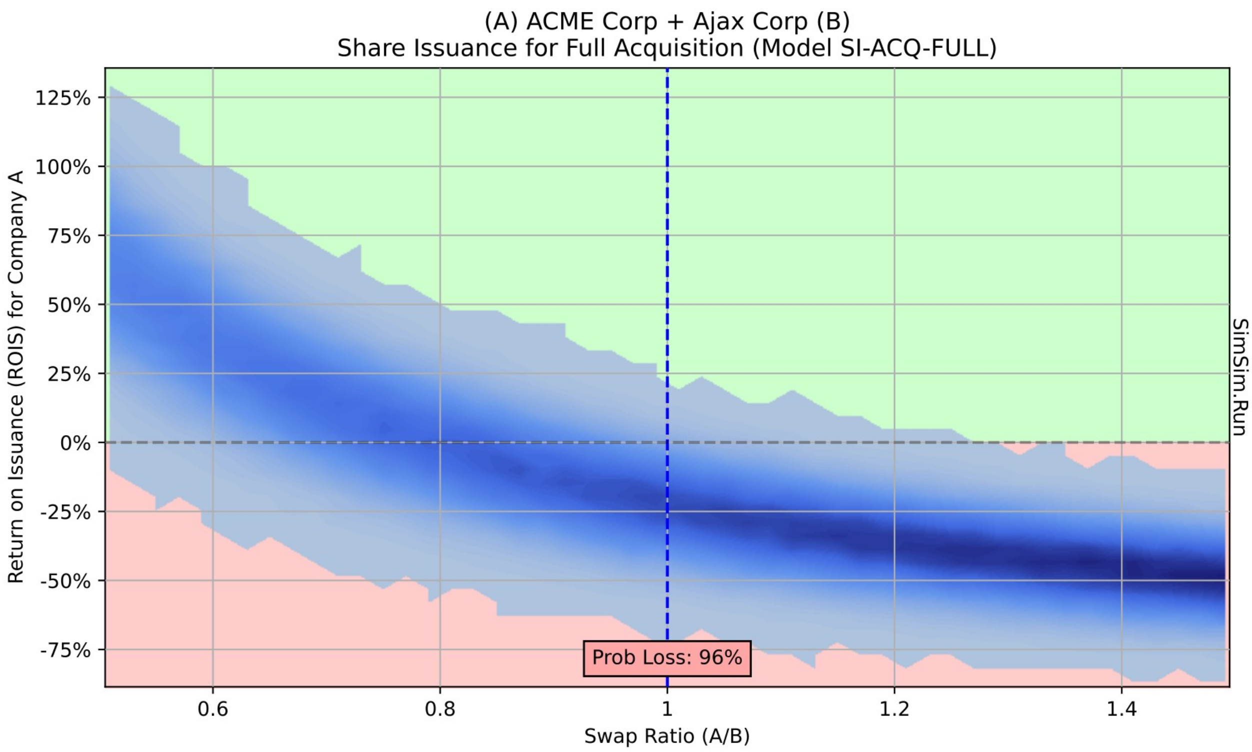

*Figure 18: Simulation of **ROIS**$_A$ in Eq.(57) using random variables for the intrinsic values v_A, v_B, and Synergy, and varying the Swap Ratio on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 17 is the vertical dashed blue line here.*

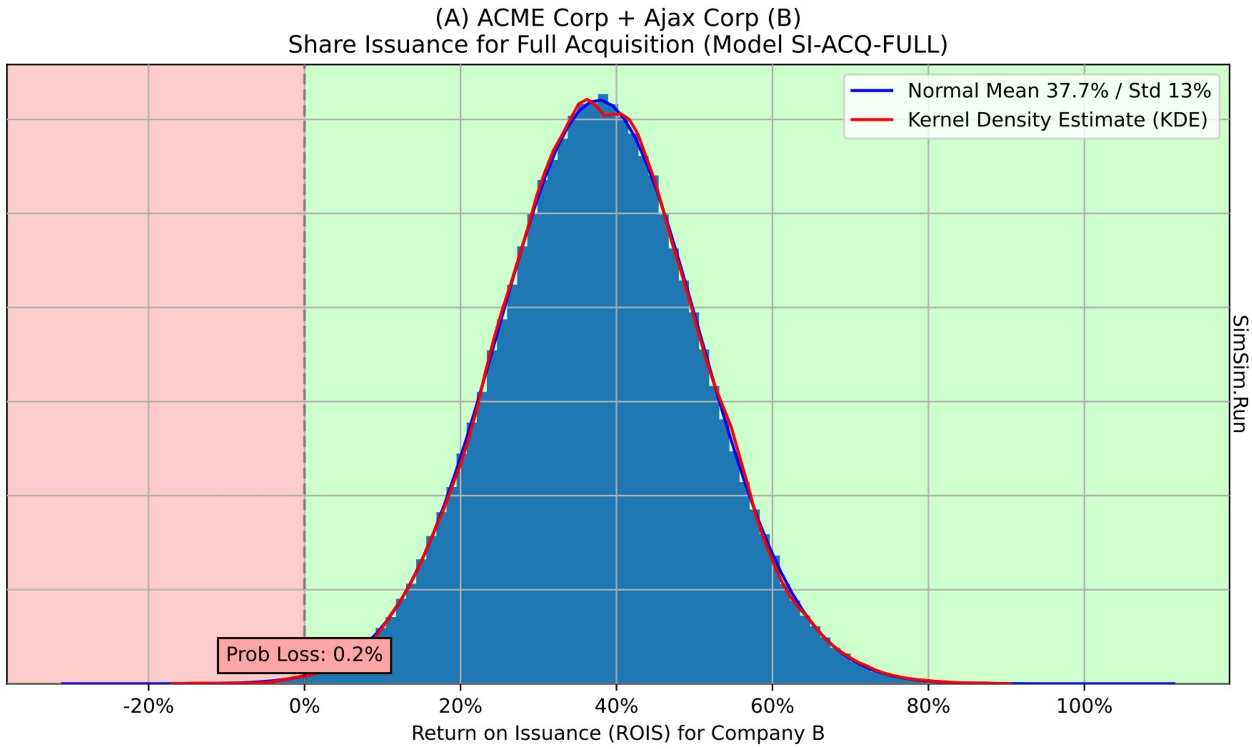

*Figure 19: Simulation of **ROIS$_B$** in Eq.(58) using random variables for the intrinsic values v_A, v_B, and Synergy.*

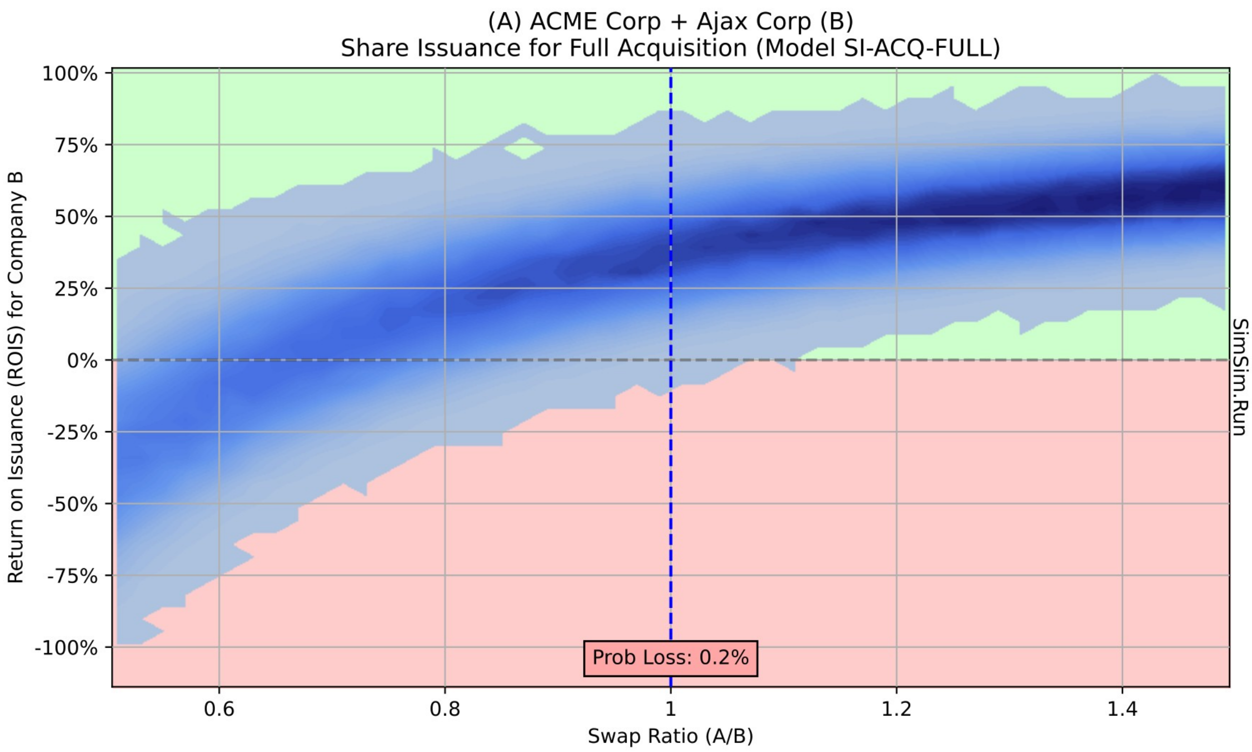

*Figure 20: Simulation of **ROIS$_B$** in Eq.(58) using random variables for the intrinsic values v_A, v_B, and Synergy, and varying the Swap Ratio on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 19 is the vertical dashed blue line here.*

6 Partial Acquisition

Now consider a scenario with two companies A and B that both issue new shares and swap them, so each company owns a part of the other. The question is whether the shareholders in either company will gain or lose from this exchange.

The Swap Ratio is how many new shares in company A are exchanged for each new share in company B:

$$Swap\ Ratio = \frac{New\ Shares_A}{New\ Shares_B} \tag{59}$$

If the stock swap is done according to current market-values for the shares in the two companies, then we must have $Issuance_A = Issuance_B$, and using the definition from Eq.(4), it means the swap ratio can again be written as the ratio between the two share-prices as in Eq.(42). This is useful if you want to use a swap ratio with current market-values for the shares of the two companies. But this does NOT mean the shareholders of the two companies have equal chance of gain or loss. Because that also depends on the intrinsic values of the two companies relative to their market-values. That is why the simulation model allows you to set another swap ratio, than the one implied by the current share-prices.

Let $\Delta\ Shares$ (the Greek letter "Delta") denote the number of new shares in a company relative to its total number of shares after the issuance:

$$\Delta\ Shares_A = \frac{New\ Shares_A}{Shares_A + New\ Shares_A} \tag{60}$$

$$\Delta\ Shares_B = \frac{New\ Shares_B}{Shares_B + New\ Shares_B} \tag{61}$$

Let w_A denote the intrinsic value of company A after the stock swap, but not per-share, and before dividend tax. This consists of the company's intrinsic value before the share issuance v_A, plus the part of company B that is owned by company A after the stock swap and written as $w_B \cdot \Delta\ Shares_B$, minus the fees for company A:

$$w_A = v_A + w_B \cdot \Delta\ Shares_B - Fees_A \tag{62}$$

Similarly for company B, the intrinsic value after the stock swap, but not per-share and before dividend tax, is:

$$w_B = v_B + w_A \cdot \Delta\ Shares_A - Fees_B \tag{63}$$

The two formulas above are defined in terms of each other. That is, w_A is defined in terms of w_B, which is then defined in terms of w_A again. This seems like an impossible circular definition, but it can actually be solved by inserting one formula into the other and reducing.

The results are:

$$w_A = \frac{v_A + (v_B - Fees_B) \cdot \Delta\ Shares_B - Fees_A}{1 - \Delta\ Shares_A \cdot \Delta\ Shares_B} \tag{64}$$

$$w_B = \frac{v_B + (v_A - Fees_A) \cdot \Delta\ Shares_A - Fees_B}{1 - \Delta\ Shares_A \cdot \Delta\ Shares_B} \tag{65}$$

We can now calculate the per-share, after-tax intrinsic values after the stock swap:

$$W_A = w_A \cdot \frac{1 - TaxRateDividend}{Shares_A + New\ Shares_A} \tag{66}$$

$$W_B = w_B \cdot \frac{1 - TaxRateDividend}{Shares_B + New\ Shares_B} \tag{67}$$

The ROIV ratios measure the gain/loss of intrinsic value to the current shareholders of each company. We do not expand these formulas further as they would become very long:

$$ROIV_A = \frac{W_A}{V_A} - 1 = \frac{w_A / v_A}{1 + \frac{New\ Shares_A}{Shares_A}} - 1 \tag{68}$$

$$ROIV_B = \frac{W_B}{V_B} - 1 = \frac{w_B / v_B}{1 + \frac{New\ Shares_B}{Shares_B}} - 1 \tag{69}$$

The fraction between the number of new and old shares can again be rewritten as follows:

$$ROIV_A = \frac{w_A / v_A}{1 + \frac{Issuance_A}{MarketCap_A}} - 1 \tag{70}$$

$$ROIV_B = \frac{w_B / v_B}{1 + \frac{Issuance_B}{MarketCap_B}} - 1 \tag{71}$$

For the original shareholders in each company, the ROIS ratios are the same as Eq.(7), with the respective subscripts for company A and B.

In the other scenarios above, we determined whether they were "zero-sum games" by comparing their ROIS ratios. That was possible because the two ROIS ratios were calculated with the same issuance amount. But in this scenario we use Eq.(7) to calculate the ROIS ratios with different issuance amounts for the two companies – they may be equal but they don't have to be.

So to determine if this kind of stock swap is also a "zero-sum game", we will instead compare the change of intrinsic value for the two companies. It can then be proven that this stock swap is also a "zero-sum game" when the fees are zero:

$$ROIV_A \cdot v_A = -ROIV_B \cdot v_B \quad \Leftrightarrow \quad Fees_A = Fees_B = 0 \tag{72}$$

6.1 Example of Partial Acquisition

Let us consider an example where the stock of company A is somewhat under-valued, and the stock of company B is somewhat over-valued. Say the intrinsic value of company A is $v_A = \$\ 12\ b$ and it is $v_B = \$\ 8\ b$ for company B. Both companies have the same number of shares and share-price e.g. $Shares = 1\ b$ and $SharePrice = \$\ 10$ so that $MarketCap = \$\ 10\ b$ for both companies. The fees involved are $Fees_A = Fees_B = \$\ 25\ m$.

We will make a stock swap at the current market-value for the two companies, so the newly issued shares of the two companies must have the same market-value. Because the share-prices are equal we know from Eq.(42) that the swap ratio is one.

We want to issue and swap $New\ Shares = 100\ m$ in each company, corresponding to the amount:

$$Issuance_A = Issuance_B = New\ Shares \cdot Share\ Price = 100\ m \cdot \$\ 10 = \$\ 1\ b \tag{73}$$

We first calculate the delta-values for the number of new shares in each company using Eqs.(60) and (61). In this example both companies have the same number of old and new shares, so we get:

$$\Delta\ Shares_A = \Delta\ Shares_B = \frac{New\ Shares}{Shares + New\ Shares} = \frac{100\ m}{1\ b + 100\ m} \simeq 0.0909 \tag{74}$$

We then use Eq.(64) to calculate the intrinsic value of company A after the share-issuance, not per-share, and before dividend tax:

$$\begin{aligned} w_A &= \frac{v_A + (v_B - Fees_B) \cdot \Delta\ Shares_B - Fees_A}{1 - \Delta\ Shares_A \cdot \Delta\ Shares_B} \\ &= \frac{\$\ 12\ b + (\$\ 8\ b - \$\ 25\ m) \cdot 0.0909 - \$\ 25\ m}{1 - 0.0909 \cdot 0.0909} \simeq \$\ 12.81\ b \end{aligned} \tag{75}$$

Similarly we use Eq.(65) for company B:

$$\begin{aligned} w_B &= \frac{v_B + (v_A - Fees_A) \cdot \Delta\ Shares_A - Fees_B}{1 - \Delta\ Shares_A \cdot \Delta\ Shares_B} \\ &= \frac{\$\ 8\ b + (\$\ 12\ b - \$\ 25\ m) \cdot 0.0909 - \$\ 25\ m}{1 - 0.0909 \cdot 0.0909} \simeq \$\ 9.14\ b \end{aligned} \tag{76}$$

The ROIV ratios for the original shareholders of the companies are found using Eqs.(70) and (71):

$$ROIV_A = \frac{w_A / v_A}{1 + \frac{Issuance_A}{MarketCap_A}} - 1 = \frac{\$\ 12.81\ b / \$\ 12\ b}{1 + \frac{\$\ 1\ b}{\$\ 10\ b}} - 1 \simeq -2.99\ \% \tag{77}$$

$$ROIV_B = \frac{w_B / v_B}{1 + \frac{Issuance_B}{MarketCap_B}} - 1 = \frac{\$\ 9.14\ b / \$\ 8\ b}{1 + \frac{\$\ 1\ b}{\$\ 10\ b}} - 1 \simeq 3.85\ \% \tag{78}$$

So the original shareholders of company A have lost -2.99% on the intrinsic value of their shares, while the original shareholders of company B have gained 3.85% on the intrinsic value of their shares. Although the companies swapped the same number of shares, to obtain the same ownership size in each other, the intrinsic values of the two companies were different, so their gain/loss from the stock swap are also different.

In this example, the two ROIV ratios are of similar scale, but with opposite signs. This is because the intrinsic values of the two companies are fairly close and the market-caps are identical. Otherwise the ROIV ratios could be very different for the two companies. This is because the ROIV ratios measure the change in intrinsic value as a result of the stock swap.

It is often useful to consider the ROIS ratios as well, which measure the change in intrinsic value relative to the issuance amount. The ROIS ratios are calculated using Eq.(9):

$$ROIS_A = \frac{ROIV_A \cdot v_A}{Issuance_A} = \frac{-2.99\ \% \cdot \$\ 12\ b}{\$\ 1\ b} \simeq -35.9\ \% \tag{79}$$

$$ROIS_B = \frac{ROIV_B \cdot v_B}{Issuance_B} = \frac{3.85\ \% \cdot \$\ 8\ b}{\$\ 1\ b} \simeq 30.8\ \% \tag{80}$$

For company A it means the stock swap had a similar effect as a $1b investment with a loss of -35.9% in present value. Conversely, for company B the stock swap had a similar effect as a $1b investment with a gain of 30.8% in present value. This is because the shares in company A were under-valued, and the shares in company B were over-valued. So the original shareholders of company A were issuing shares at a too low price, in exchange for new shares in company B that were issued at a too high price, compared to their intrinsic values.

This is only a "zero-sum game" when the fees are zero. If you repeat the calculations above with zero fees, then you should get that $ROIV_A \cdot v_A = -ROIV_B \cdot v_B$. In this example we could also have compared the ROIS ratios, because the issuance amount is the same for both companies, but that may not always be the case.

6.2 Simulation of Partial Acquisition

We typically don't know the exact intrinsic values of the two companies, because they are estimated from future earnings that are uncertain. Instead of repeating the above calculations with many different input values, it is much easier to use computer simulations. The following plots were produced on the SimSim.Run web-site. We use the same input assumptions as above, except for the intrinsic value v_A which is normal-distributed with mean \$12b and std.dev. \$1b, and the intrinsic value v_B is normal-distributed with mean \$8b and std.dev \$1b.

Figure 21 shows the simulated $ROIV_A$ ratio from Eq.(77), which measures the gain/loss of per-share intrinsic value for company A's original shareholders. Nearly all the simulation results are losses with an average of -2.95%.

Figure 22 again shows the simulated $ROIV_A$ ratio from Eq.(77), but this time the Swap Ratio is varied on the x-axis. This makes it easy to see how the $ROIV_A$ ratio would change with different Swap Ratios, while the other inputs remain the same. This shows the Swap Ratio should probably be below 0.4 for most of the simulated $ROIV_A$ ratios to be gains.

Figure 23 shows the simulated $ROIV_B$ ratio from Eq.(78), which measures the gain/loss of per-share intrinsic value for company B's original shareholders. Nearly all the simulation results are gains with an average of 4.05%.

Figure 24 shows the simulated $ROIV_B$ ratio from Eq.(78) with varying Swap Ratios on the x-axis.

Figure 25 shows the simulated $ROIS_A$ ratio from Eq.(79), which also measures the gain/loss for company A's current shareholders, but it is relative to the issuance amount instead of the company's intrinsic value. This makes it easier to compare the gain/loss ratio to alternative investments. In this case the $ROIS_A$ ratio has a 99.9% probability of loss with an average of -35.8%. So the stock swap would most likely be a horrible deal for the current shareholders of company A.

Figure 26 shows the simulated $ROIS_A$ ratio from Eq.(79) with varying Swap Ratios on the x-axis. This again shows the Swap Ratio should probably be below 0.4 for most $ROIS_A$ ratios to be gains.

Figure 27 shows the simulated $ROIS_B$ ratio from Eq.(80), which measures the gain/loss for company B's current shareholders relative to the issuance amount. These are nearly all gains with a mean of 30.8%. So the deal would most likely be a big gain for the current shareholders of company B.

Figure 28 shows the simulated $ROIS_B$ ratio from Eq.(80) with varying Swap Ratios on the x-axis. Swap Ratios down to 0.8 would probably still be a good deal for the shareholders of company B.

Note that the ROIS plots in Figure 26 and Figure 28 look quite different for the two companies. This is not a mistake. It is because the new number of shares in company B is held constant at $New\ Shares_B = 100\ m$, while the new number of shares in company A varies with the Swap Ratio. This means the issuance amount for company B is constant in the calculation of the $ROIS_B$ ratio, while the issuance amount varies in the calculation of the $ROIS_A$ ratio for company A. This tends to make the plot for company A look more curved, while the plot for company B looks more linear.

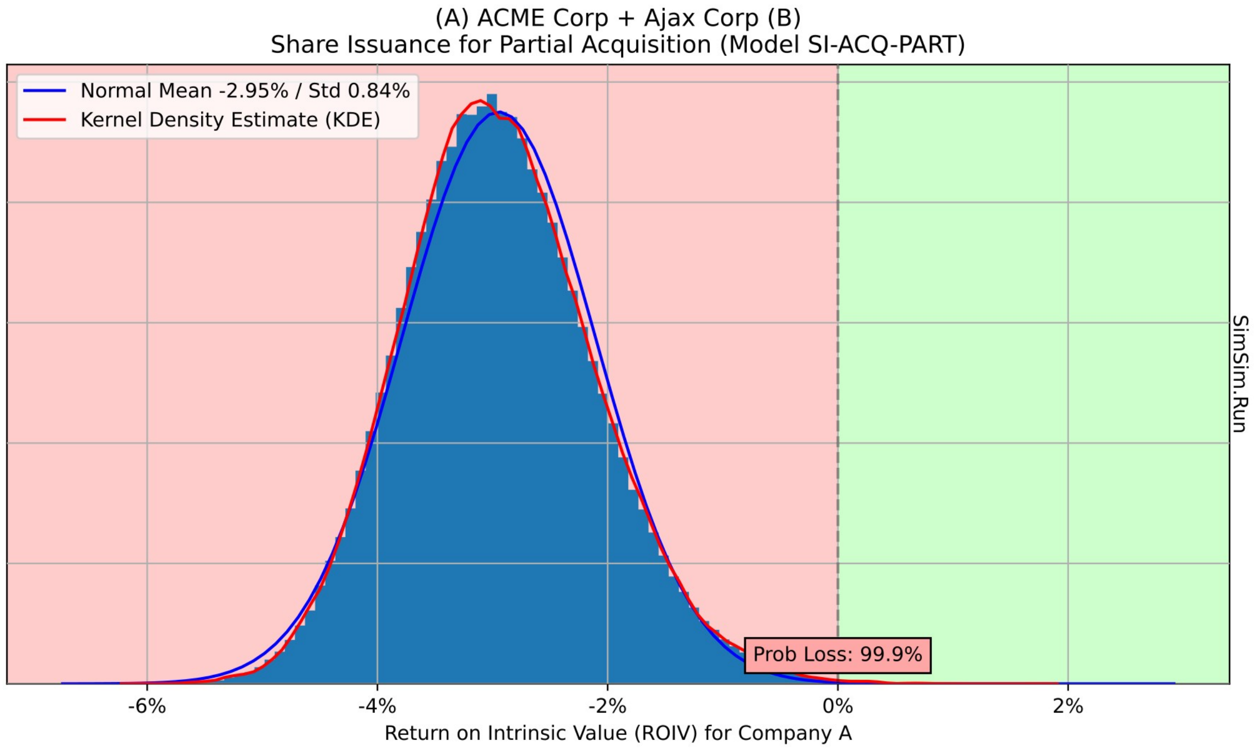

*Figure 21: Simulation of **ROIV**$_A$ in Eq.(77) using random variables for the intrinsic values v_A and v_B.*

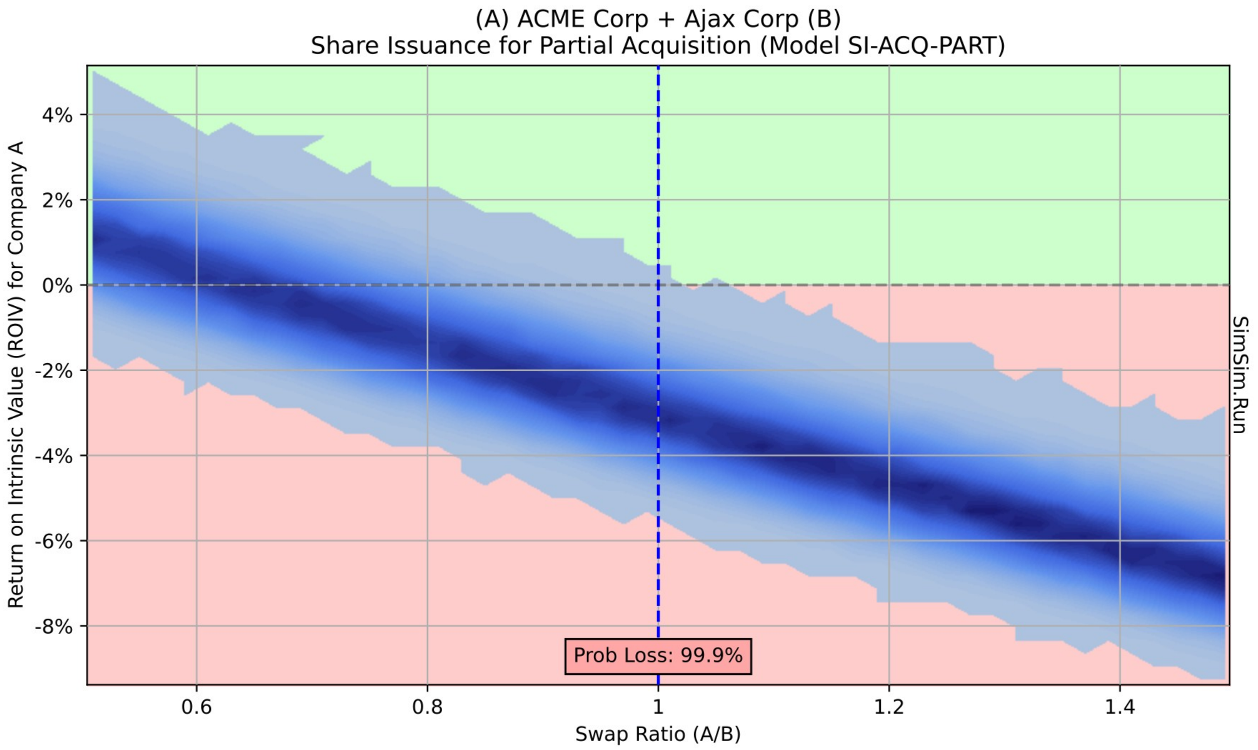

*Figure 22: Simulation of **ROIV**$_A$ in Eq.(77) using random variables for the intrinsic values v_A and v_B, and varying the Swap Ratio on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 21 is the vertical dashed blue line here.*

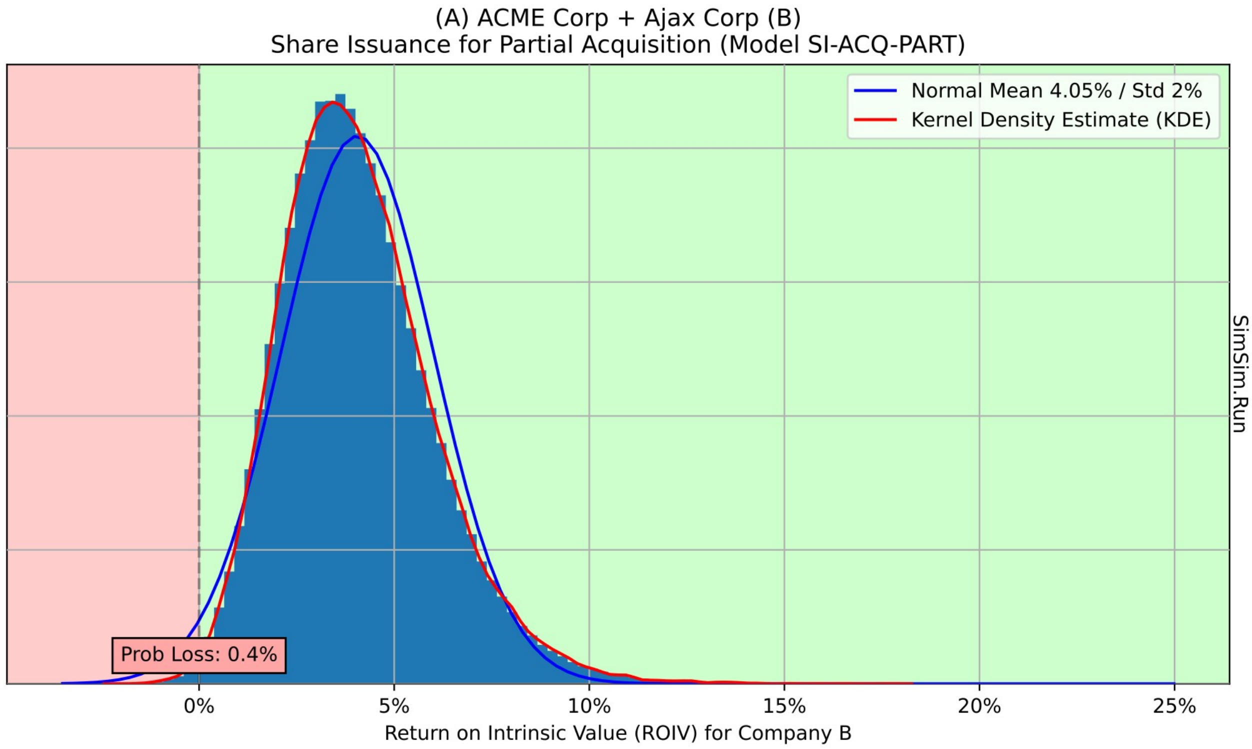

*Figure 23: Simulation of **ROIV**$_B$ in Eq.(78) using random variables for the intrinsic values v_A and v_B.*

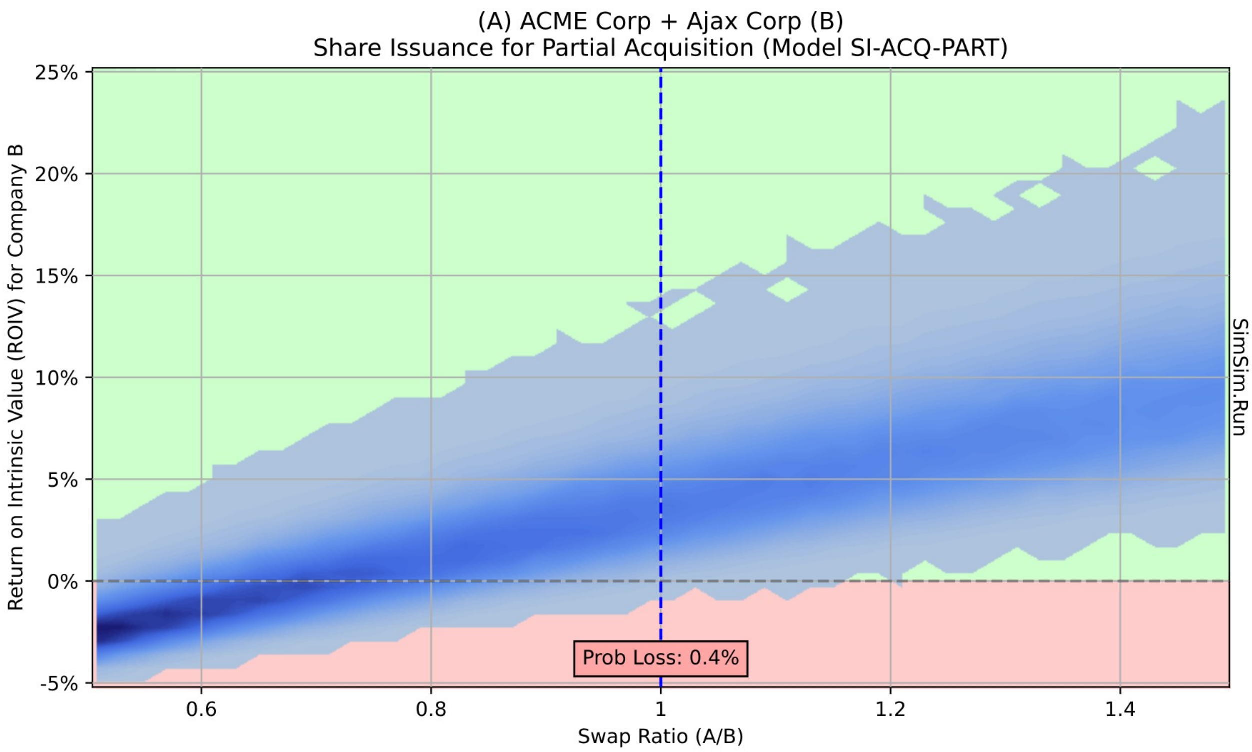

*Figure 24: Simulation of **ROIV**$_B$ in Eq.(78) using random variables for the intrinsic values v_A and v_B, and varying the Swap Ratio on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 23 is the vertical dashed blue line here.*

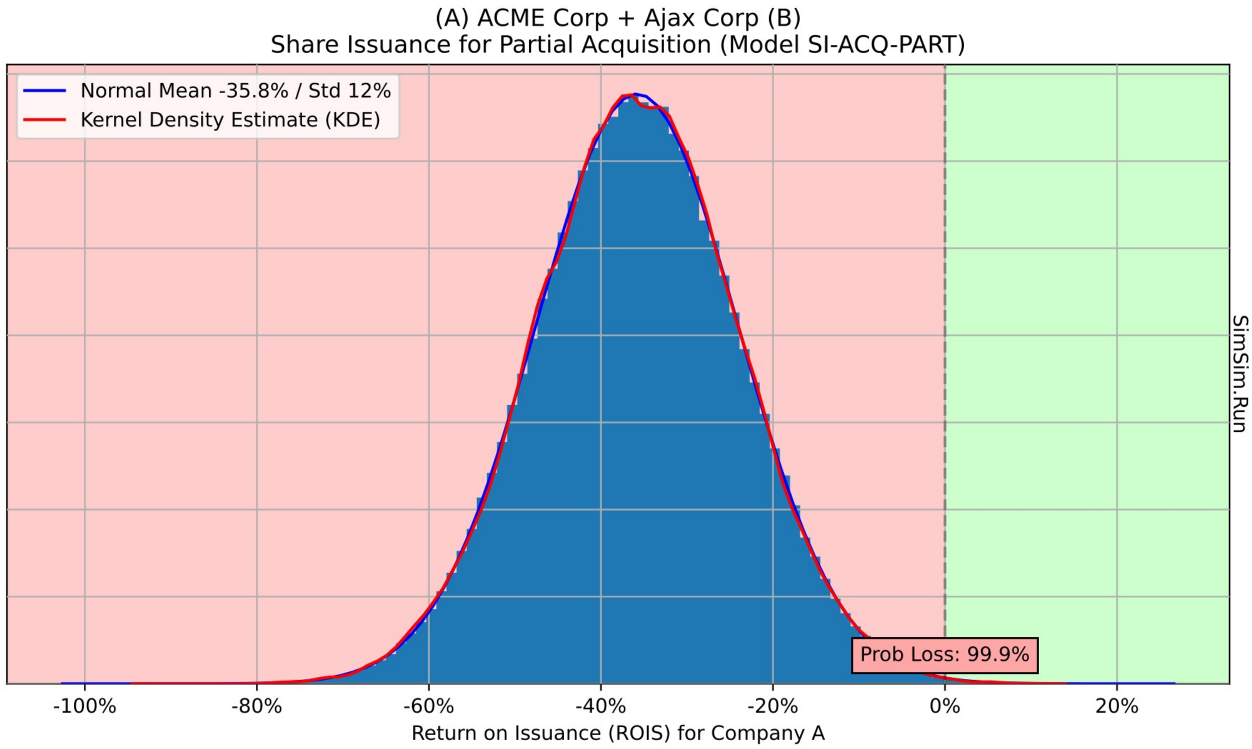

Figure 25: Simulation of ***ROIS$_A$*** *in Eq.(79) using random variables for the intrinsic values v_A and v_B.*

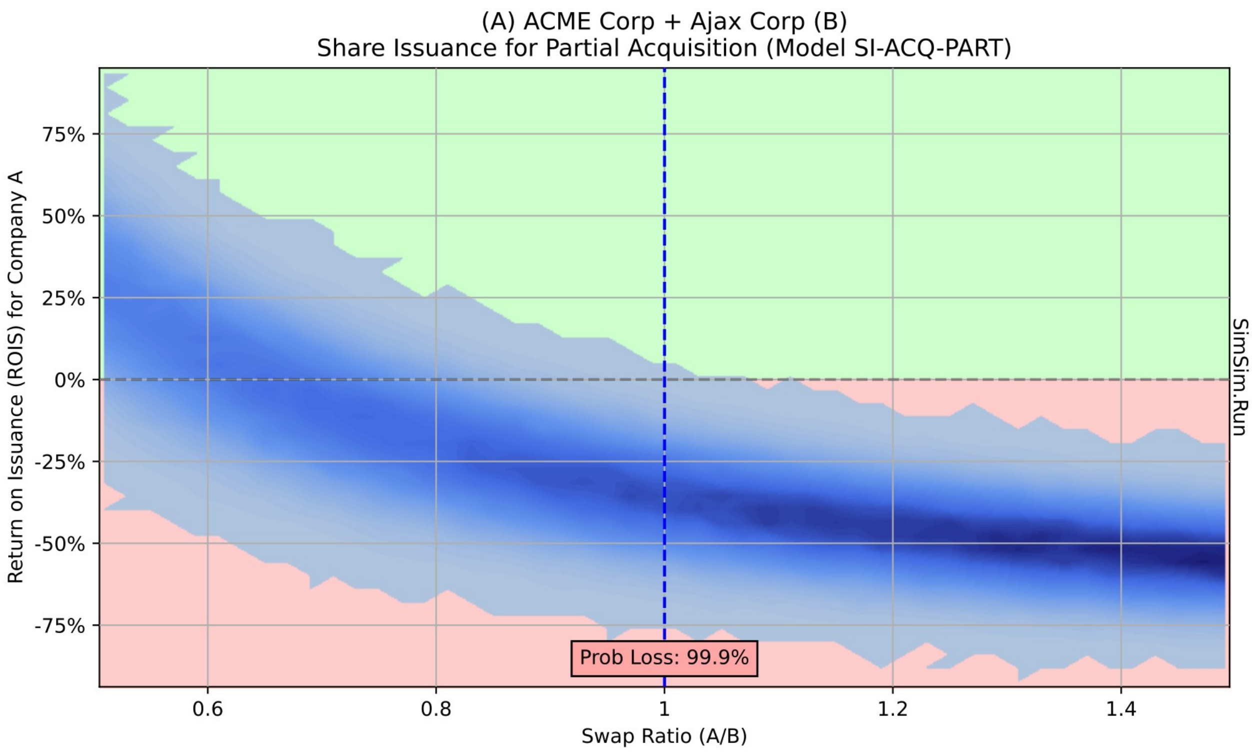

Figure 26: Simulation of ***ROIS$_A$*** *in Eq.(79) using random variables for the intrinsic values v_A and v_B, and varying the Swap Ratio on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 25 is the vertical dashed blue line here.*

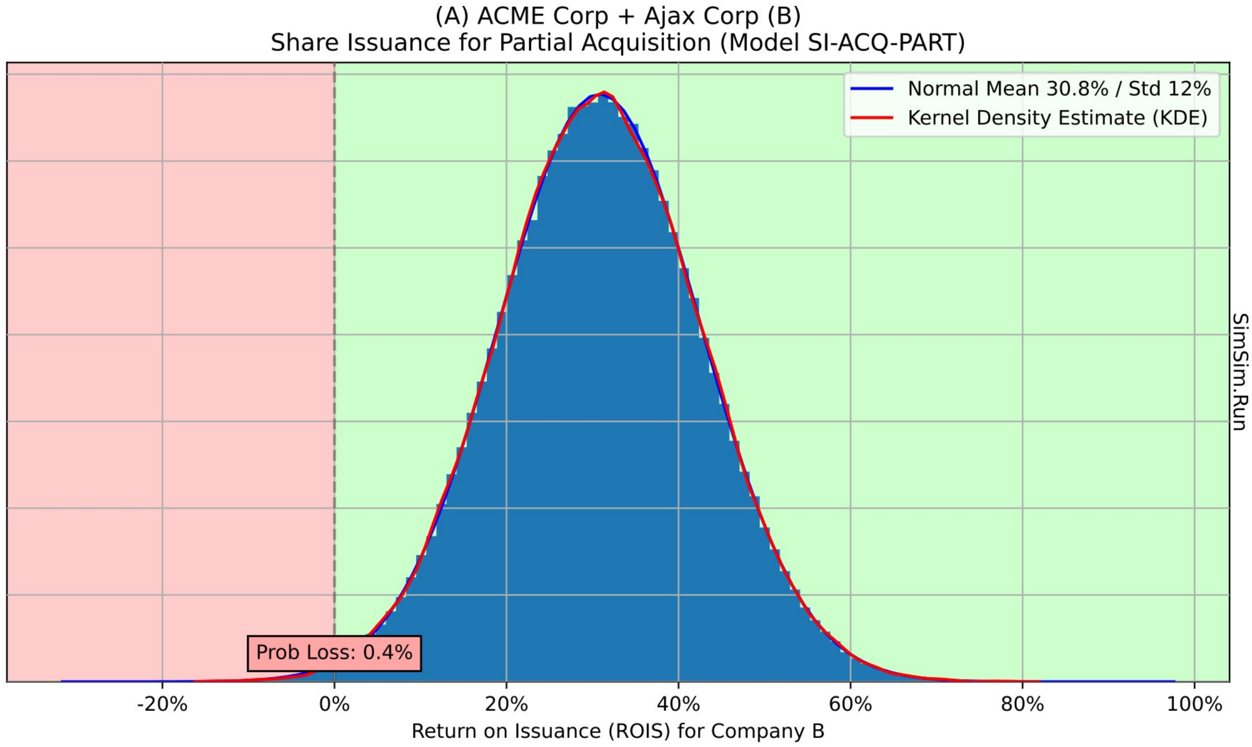

*Figure 27: Simulation of **ROIS$_B$** in Eq.(80) using random variables for the intrinsic values v_A and v_B.*

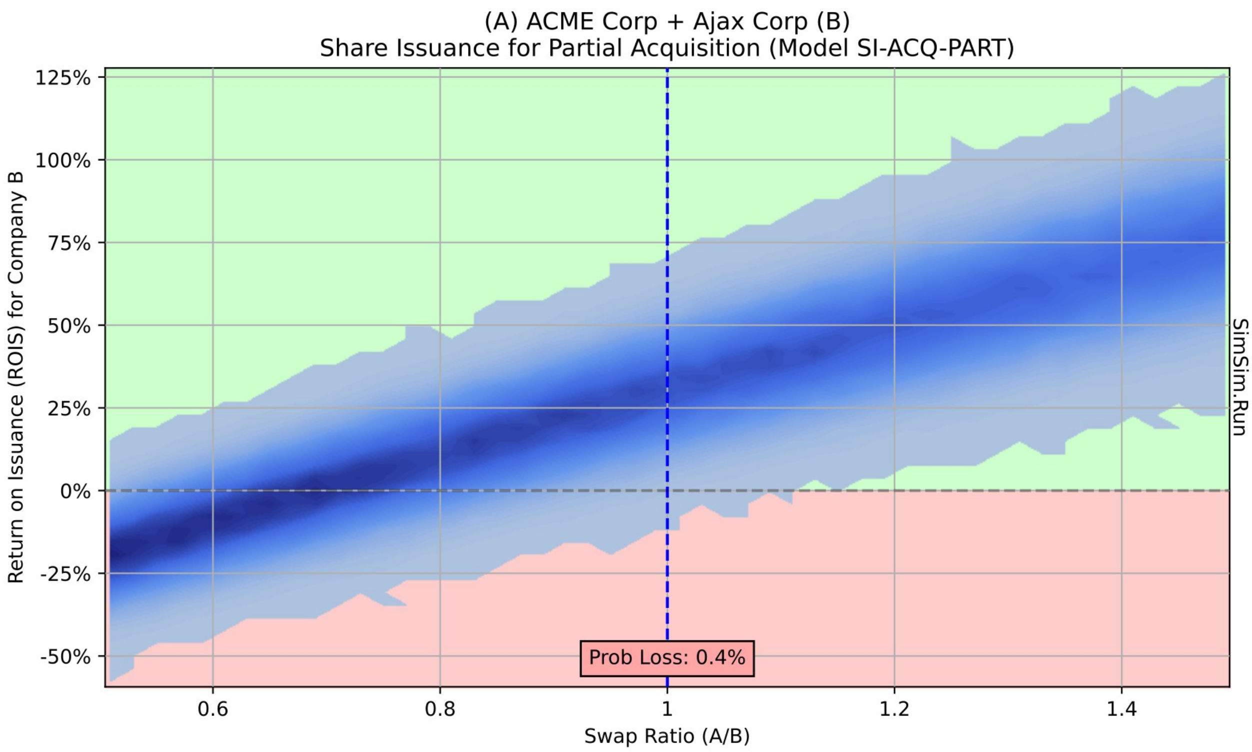

*Figure 28: Simulation of **ROIS$_B$** in Eq.(80) using random variables for the intrinsic values v_A and v_B, and varying the Swap Ratio on the x-axis. This is a 2D histogram where darker blues indicate more simulated values in a region. The histogram in Figure 27 is the vertical dashed blue line here.*

7 Conclusion

This paper showed how to measure the effect on shareholder value when a company issues new shares in different scenarios. The effect generally depends on the intrinsic value of the newly issued shares, compared to the cash payment or other value that the company receives for the new shares.

Companies should avoid issuing new shares when the shares are under-valued, because it incurs a loss to the current shareholders. This is especially true when the company is in financial distress and the share-price is extremely low. Companies should avoid this situation by not taking on excessive debt in the first place.

When a company's shares are significantly over-valued, it is beneficial for the current shareholders if the company issues new shares, even if the company doesn't have a specific purpose for the cash. For companies with large debt, this would be a good opportunity to use the cash to pay down debt.

In most scenarios, a share issuance is only a so-called "zero-sum game", where the combined gain/loss for the seller and buyer is zero, if the fees involved in the share issuance are also zero. The exception is when the company issues shares to fund an investment, where both the company's current and new shareholders can gain or lose together, depending on the return on the investment.

8 References

[Pedersen 2012] M.E.H. Pedersen, "The Value of Share Buybacks", 2012. [PDF]

My previous papers and books can all be downloaded through SSRN and GitHub.