The Value of Share Buybacks

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Abstract

A formal theory is presented for the valuation of share buybacks. Formulas are given for calculating the equilibrium and effect on shareholder value in different share buyback scenarios. The prevalent belief amongst scholars and practitioners is that share buybacks can substitute for dividend payouts as a way for companies to return capital to shareholders, possibly with a tax benefit to shareholders. This so-called substitution hypothesis is shown both theoretically and empirically to be a fallacy as share buybacks may significantly affect the value to shareholders. This is important because share buybacks are now substantially larger than dividend payouts, possibly as a result of the substitution hypothesis commonly believed to be valid. The presented theory is also applied in several case studies.

Executive Summary

- Share buybacks have an unpredictable effect on the share price.
- Share buybacks are generally not a reliable way of returning capital to shareholders.
- Share buybacks magnify any mispricing of the shares compared to the value to long-term shareholders, which is the company's excess cash plus the present value of future earnings that can be paid out as dividends.
- Buybacks of overpriced shares cause much greater losses to long-term shareholders compared to the gains from buying back underpriced shares.
- Share buybacks can be valued relative to alternative investments, acquisitions, restructurings, etc. to assess which is the most valuable to long-term shareholders.
- Share buybacks for offsetting the diluting impact of stock options should only be made when the shares are underpriced.
- Dividend tax is irrelevant to share buybacks if the future tax rate is constant or unknown.
- The greater the uncertainty about the value to long-term shareholders, the greater a margin of safety is required between the share price and the average value to long-term shareholders.
- Share buybacks increase the uncertainty of the value to long-term shareholders.
- The probability distribution of possible outcomes of a share buyback should be considered so as to properly assess the likely effect on shareholder value.
- Managers must inform shareholders about the share price limit below which shares may be bought back, and the source of cash. This information is needed for outsiders to value the share buybacks.

Nomenclature

v	Present value of all future dividends without share buyback, pre-tax, not per share.
V	Present value of all future dividends without share buyback, post-tax, per share.
w	Present value of all future dividends with share buyback, pre-tax, not per share.
W	Present value of all future dividends with share buyback, post-tax, per share.
$\mathbb{V}$	Stochastic variable for v.
$\mathbb{W}$	Stochastic variable for w.
k	Kilo, a factor 10^3
m	Million, a factor 10^6
b	Billion, a factor 10^9
∞	Infinity.
$Shares$	Number of shares outstanding.
$SharePrice$	Price per share.
$MarketCap$	Market capitalization (also written market-cap): $MarketCap = Shares \cdot SharePrice$
$Equity$	Capital supplied by shareholders and retained earnings.
$Earnings$	Earnings available for dividend payout, not per-share.
$Dividend$	Pre-tax dividend payout, not per-share.
$Buyback$	Amount used for share buyback.
$TaxDividend$	Tax-rate on dividend for an individual.
$TaxIncome$	Tax-rate on income for an individual.
$TaxBuyback$	Tax-rate for a company on its share buyback.
$TaxCompany$	Tax-rate for a company on its earnings.
$P/Book$	Price-to-Book ratio: $P/Book = MarketCap/Equity = SharePrice/(Equity/Shares)$
ROA	Return on Assets: $ROA = Earnings/Assets$
ROE	Return on Equity: $ROE = Earnings/Equity$
g	Annual growth rate used in valuation.
d	Discount rate used in valuation.
$=$	$a = b$ means that a equals b
$\geq$	$a \geq b$ means that a is greater than or equal to b
$\simeq$	$a \simeq b$ means that a is approximately equal to b
$\Rightarrow$	Implication: $A \Rightarrow B$ means that A implies B
$\Leftrightarrow$	Bi-implication: $A \Leftrightarrow B$ means that $A \Rightarrow B$ and $B \Rightarrow A$
$\cdot$	Multiplication: $a \cdot b$ means that a is multiplied by b
$\sum$	Summation: $\sum_{t=a}^{b} x_t = x_a + x_{a+1} + \cdots + x_b$
Π	Multiplication: $\prod_{t=a}^{b} x_t = x_a \cdot x_{a+1} \cdot \ldots \cdot x_b$
$Min(a, b)$	Minimum of a or b. Similarly for $Max(a, b)$
$\lim_{v \to c} f(v) = l$	Limit of the function f is l as v approaches c.
$\partial_v f$	Differential of the function f on the variable v.
$\int_a^b f(v)\, dv$	Definite integral on the range $[a, b]$ of the function f on the variable v.
$Min[\mathbb{V}]$	Minimum value that the stochastic variable $\mathbb{V}$ may take on.
$\Pr[\mathbb{V} = v]$	Probability of the stochastic variable $\mathbb{V}$ being equal to v.
$E[\mathbb{V}]$	Expected (or mean) value of the stochastic variable $\mathbb{V}$.
$Var[\mathbb{V}]$	Variance of the stochastic variable $\mathbb{V}$: $Var[\mathbb{V}] = E[(\mathbb{V} - E[\mathbb{V}])^2] = E[\mathbb{V}^2] - E[\mathbb{V}]^2$
$Stdev[\mathbb{V}]$	Standard deviation of the stochastic variable $\mathbb{V}$: $Stdev[\mathbb{V}] = \sqrt{Var[\mathbb{V}]}$

Introduction

1. Introduction

A company may use its cash earnings for investing in expansions, acquisitions of other companies, debt repayments, dividend payouts or share buybacks. The prevalent belief amongst scholars and practitioners is that share buybacks can substitute for dividends as a way for companies to return capital to shareholders, possibly with a tax benefit to shareholders depending on their individual tax rates for dividends and capital gains. Section 2 is a review of this and other hypotheses on share buybacks.

The substitution hypothesis is proven here to be a fallacy both theoretically and empirically, which is important because share buybacks have increased greatly in recent years, possibly due to the common belief that the substitution hypothesis was valid. During the year 2012, the companies in the S&P 500 stock market index had aggregate net income of about USD 780b, made dividend payouts of about USD 280b and share buybacks of almost USD 400b.[3] Section 8 shows that the companies in the S&P 500 index have had a tendency to increase share buybacks when the share price was high relative to the estimated value, and conversely decrease share buybacks when the share price was low relative to the estimated value, which is the opposite of what should have been done.

A framework for sound logical reasoning about the value of share buybacks is presented here. The theory is derived from two axioms: (1) Share buybacks should always be made for the sake of the remaining shareholders rather than the selling shareholders. This ultimately means that share buybacks should be made for so-called eternal shareholders who own their shares for eternity and hence rely on dividends as the sole source of income from their shares. (2) The value of a company to its eternal shareholders is the present value of all future dividends, as argued by Williams [**1**]. The question is whether it is best for eternal shareholders to get a dividend payout now or a share buyback with its promise of increased future dividends per share. To answer this question, the two axioms are used with basic mathematics to derive valuation and equilibrium formulas for share buybacks in various scenarios.

Share buybacks are shown to be value neutral to eternal shareholders only when the market-cap (market capitalization, that is, the market value of all the company's shares) equals the excess cash and present value of future earnings available for dividend payout, and share buybacks magnify any mispricing as demonstrated in Figure 1 and Figure 2. Buying back overpriced shares causes much greater losses to eternal shareholders than the gains from buying back underpriced shares. Dividend taxes are irrelevant to the value of share buybacks if the future tax rate is constant or unknown.

Future earnings are generally not known in advance and must therefore be estimated when calculating their present value. Graham and Dodd suggested that investors should protect themselves from this uncertainty by having a margin of safety between the price they pay for a financial security and its expected value [**2**] [**3**]. The margin of safety principle applies equally well to share buybacks for increasing the probability that the value to eternal shareholders is increased from a share buyback.

[3] S&P Dow Jones Indices, Press Release, June 19, 2013:
http://eu.spindices.com/documents/index-news-and-announcements/20130619-sp-500-buyback.pdf

1.1. Treatise Overview

The treatise is structured as follows:

- Section 2 is a review of the prevalent hypotheses on share buybacks and how they relate to the theory developed here.
- Section 3 describes basic valuation theory.
- Section 4 presents the theory and formulas for valuation of share buybacks in various scenarios.
- Section 5 studies the non-linear properties of the relative value of share buybacks.
- Section 6 presents the theory and formulas for valuation of share buybacks under uncertainty.
- Section 7 is a review of the prevalent hypotheses on the selection of the discount rate used in valuation, along with suggestions for selecting the discount rate used in share buyback valuation.
- Section 8 analyses share buyback trends for the S&P 500 stock market index.
- Section 9 applies the theory of share buyback valuation to the analysis of real world companies.
- Section 10 gives suggestions for managers on how to decide whether to make debt repayments, investments, acquisitions, dividend payouts or share buybacks.
- Section 11 gives suggestions for investors on how to assess the valuation effect of share buybacks.
- Section 12 gives concluding remarks.

2. Review of Hypotheses

This section gives a review of the main hypotheses related to share buybacks and briefly discusses their relation to the theory of share buyback valuation presented in this treatise. More extensive surveys of the literature on share buybacks are compiled by Vermaelen [**4**], DeAngelo et al. [**5**], and Baker et al. [**6**].

2.1. Efficient Capital Markets

The Efficient Market Hypothesis (EMH) states that the participants in capital markets are so efficient at integrating new information that the price of a financial security "fully reflects" all available information. It is common in the literature to further assume market participants are rational and are capable of owning a financial security for eternity, and the market possesses some kind of aggregate intelligence which makes the price of a financial security identical to the long-term, risk-adjusted value of the underlying asset.

The idea of EMH has been the basis of much academic work and was formalized by Fama, see [**7**] for an early review and [**8**] for a newer review of the literature. EMH has been sought proven in various ways, e.g. by statistical tests of the correlation between the price of a financial security and the overall capital market surrounding the publication of news directly related to that financial security. EMH gained many followers in academia and serves as an assumption for much academic work as will be seen below, while the ideas of EMH were already discredited earlier by Graham and Dodd [**2**] [**3**] and Williams [**1**], as well as more recently by practitioners such as Buffett [**9**] and Klarman [**10**], and scholars such as Summers [**11**], Lo and MacKinlay [**12**], and Shiller [**13**] [**14**].

In an all-inclusive, pre-emptive defence of EMH, Fama [**7**] [**8**] asserted that the so-called joint hypothesis problem means EMH can only be tested jointly with economic models, which means contradictory empirical evidence cannot reveal if EMH is false or if the economic model is incorrect. Fama [**8**] further claims that this is in accordance with the scientific method; but it is decidedly not, because the opposite hypothesis that capital markets are not efficient, call it Non-EMH, can be proposed as well and it too will be un-testable because of the 'joint hypothesis problem'. So according to Fama's reasoning neither EMH nor its opposite Non-EMH would be testable. This means they would be unsuitable for scientific inquiry. In reality, however, EMH is testable in the sense that sound counter-arguments and conflicting empirical evidence should cause the confidence in EMH to decrease, just as with any other hypothesis concerning observable phenomena. Unfortunately, for nearly half a century finance scholars seem to have been biased towards supporting EMH rather than challenging it, while ignoring conflicting arguments and evidence.

EMH is ignored in the theory developed in this treatise because EMH is believed to be fundamentally unsound. As will be seen below, EMH also causes contradictions in the theory of finance. There is a multitude of reasons why market prices do not necessarily equal the value of the assets to long-term owners, a few of which are: Participants in capital markets have different tax rates so the return from assets will have different values to different participants. Market participants employ a wide variety of strategies, some of which are long-term strategies based on both quantitative and qualitative assessments of the assets, while other strategies try to predict and exploit short-term price changes from historical patterns and statistics. Some market participants use leverage (borrowed money) which means that a lower price, for whatever reason, may force a liquidation of the asset, which in turn may lead to over-supply in the market and hence a further decrease in price. Human emotions may also cause a too optimistic or pessimistic view of the future. Even if the price of an asset reflected all currently available

information, the future is unknown to everyone and market participants will generally have differing opinions on what might happen in the future. This means the price of an asset would at best reflect a consensus guess of what the future might bring, but each transaction price is determined by only one buyer and one seller and may not represent what everyone else believes the long-term value of the asset to be. Some companies have more uncertain futures than others and some market participants may have qualitative – but not necessarily informational – insights that gives them an advantage. The number of market participants and their capital is finite and must be allocated amongst a wide range of assets, all of which have varying degrees of uncertain futures and frequently updated information related directly or indirectly to both the asset value and uncertainty of this value. Such portfolio allocation is not an easy task. Even if an asset is identified as likely being mispriced, it may not be clear what part of one's portfolio, if any, should be sold so the newly discovered asset can be bought. Valuation of an asset based on its estimated return for eternity – which ultimately means its dividend payout over its lifetime, as will be seen below – is only relevant for an investor who can actually own the asset for eternity. Most market participants have a much shorter time horizon and will therefore be concerned to some extent about the price they think they might obtain in the future, that is, most market participants will engage in price speculation to some extent. Altogether this means there may be a considerable amount of time where the price of an asset can differ significantly from its value to a long-term owner.

2.2. Dividend Substitution

Several closely related hypotheses state that share buybacks can substitute for dividends as a means of transferring capital from a company to its shareholders, because a share buyback results in an increase in share-price over time, a so-called capital gain. Depending on the relation between dividend tax and capital gains tax there might be a tax advantage to either a dividend or share buyback. This so-called substitution hypothesis assumes either EMH holds or the share price is a constant multiple of e.g. earnings per share or equity per share (book-value per share), see e.g. Bierman and West [**15**], and Elton and Gruber [**16**]. A broader hypothesis by Miller and Modigliani [**17**] states that overall dividend policy is irrelevant to a company's value to shareholders, assuming EMH holds and there are no transaction costs and taxes. Many papers seem to assume the substitution hypothesis is true *a priori* and neglect to explicitly state the required conditions such as EMH being assumed to hold, see e.g. Woods and Brigham [**18**], Grullon and Michaely [**19**], as well as several of the papers cited below on other hypotheses of share buybacks.

According to the substitution hypothesis, the company has a range of possible values to its shareholders depending on the company's future choice of dividends or share buybacks, and the tax rates on dividends and capital gains for the individual shareholders. But the market-cap of the company is only a single value which cannot be multiple values simultaneously, so it is clearly not possible for the market-cap to equal the value of the company to all shareholders. This means EMH cannot hold. As the substitution hypothesis holds if EMH holds, but the substitution hypothesis means EMH does not hold, a contradiction has arisen which the literature does not seem to address.

The substitution hypothesis is apparently the reason share buybacks are commonly called 'payouts' or 'returns' to shareholders, but this is incorrect terminology which only adds to the confusion about valuation of share buybacks. In a share buyback it is the selling shareholder who receives the 'payout' which thereby

terminates the share ownership, while the remaining shareholders have only a hypothetical promise of a future increase in share price as their 'payout' from the company.

The substitution hypothesis is shown to be an obvious fallacy in the case studies in section 9, and section 4 formalizes the conditions under which a share buyback increases or decreases value to the remaining shareholders.

2.3. Signalling

A number of so-called signalling hypotheses have been proposed, which state that corporate managers use dividends and share buybacks to signal information to shareholders, see e.g. Ross [**20**], Bhattacharya [**21**], Kalay [**22**], Miller and Rock [**23**], Ofer and Thakor [**24**], Williams [**25**], and Grullon and Michaely [**26**]. Common to these hypotheses seems to be an assumption that EMH holds so an unexpected change in dividend or share buyback is regarded as information – a signal – sent by a company's management to its shareholders, which then causes a change in valuation and market-cap of the company. These hypotheses use various mathematical models and empirical studies which, however, seem to be inconsistent and inconclusive.

For dividends, the argument seems to be that managers are reluctant to decrease future dividends as that is believed to result in a decrease in market-cap which is considered undesirable. So an increase in dividend may signal that managers believe the company's earnings prospects have increased permanently. Since dividends are costly compared to, say, a press release which merely states that management believes the company's future prospects have improved, the signal of a dividend change is considered more reliable. However, a dividend increase can also signal that the company has no more investment opportunities available, which can instead be interpreted as bad news by shareholders as it may indicate the company can no longer grow its earnings. Conversely, a decrease in dividend could just as well be caused by new investment opportunities for which the company needs the funds, as it could be caused by a temporary or permanent decrease in earning power. So it is unclear how a change in dividend by itself should serve as a signal of what the future brings and hence affect the valuation of a company.

Assuming EMH holds so the company's market-cap reflects all publicly known information about the company's earnings prospects, a share buyback can be interpreted as a signal that the market-cap is currently too low because management has inside information about the company's earnings prospects. However, as there are several conflicting hypotheses about the valuation of share buybacks, some of these hypotheses must be wrong. This means a company's management may be relying on a valuation model of share buybacks which is incorrect, rather than having private information about the company's earnings prospects. Unless management makes clear to the shareholders what assumptions and valuation models management uses, the shareholder must correctly guess this in order to accurately interpret the signal of the share buyback, which is obviously impossible.

To the practitioner the signalling hypothesis may seem particularly odd. It seems superfluous that management should have to resort to various forms of signalling or other devices of indirect, non-verbal communication with shareholders. If management believes the company is worth more to long-term shareholders than its market-cap, and the company has cash available for making a share buyback, there is no reason for management not to openly communicate this along with supporting arguments to

shareholders. This allows shareholders to consider the facts and reasoning of management, and in case of disagreement either take shareholder action against management or sell their shares. This also allows shareholders who consider selling their shares to redo their valuation of the company with management's insights and reconsider selling their shares. A good example of a company's management openly communicating to shareholders in this manner is given in section 9.1.

2.4. Stock Options

Stock options give employees the future right to purchase shares of the company at a predetermined price, typically the market-price of shares at the time the options are awarded. Stock options are awarded as part of the compensation to employees and are supposed to align the interests of employees and shareholders.

Several empirical studies find a strong correlation between an increase in stock options and an increase in share buybacks, see e.g. Lambert et al. [**27**], Fenn and Liang [**28**], Kahle [**29**], and Bens et al. [**30**]. It appears managers are assuming the substitution hypothesis holds so the increase in share buybacks does not affect the value to shareholders. Further assuming the future share price will be a constant multiple of earnings per share means a reduction in the number of shares as a result of share buybacks will increase the share price even though the total earnings are unchanged. Under these assumptions share buybacks would increase the expected value of the stock options while being value neutral to shareholders, and maybe even have a tax-advantage to shareholders. However, this is incorrect reasoning as will be shown both theoretically and empirically in this treatise. Section 4.11 gives the conditions for a share buyback to be value neutral to eternal shareholders upon exercising of stock options.

In the theory of agency costs, managers are considered agents for shareholders with conflicting interests which gives rise to so-called agency costs, see e.g. Jensen [**31**]. Like most others, Jensen assumed the substitution hypothesis holds for share buybacks and dividends, so share buybacks can be used to transfer capital from the company to its shareholders, thus decreasing agency costs by limiting the amount of excess capital under management's control. This is incorrect as will be shown in this treatise. Instead share buybacks may increase agency costs when it is done to offset the dilutive impact of stock options and the share price is too high. This is ironic because stock options were intended to align the interests of employees and shareholders.

2.5. Takeover Defence

A takeover is the purchase of a company by another company or entity. A hostile takeover is when the management of the target company does not agree to the sale, so shares must be purchased in the stock market or through a so-called tender offer directly to shareholders so as to circumvent the company's management.

The hypothesis that hostile takeovers can be deterred by share buybacks has been studied by several researchers. For example, Bagnoli et al. [**32**] use the signalling hypothesis to develop a model in which managers buying back shares thereby also signal inside information which leads to an increase in market-cap thus making the company less attractive as a takeover target. Bagwell [**33**] develops another model assuming shareholders have different valuations of the company so a share buyback removes shareholders with lower valuations and the remaining shareholders have higher valuations for which they are willing to sell their shares, thus making a takeover unattractive. Billett and Xue [**34**] model takeover probability from

various factors such as company-size, leverage, price-to-book ratio, return on assets, etc., and then measure the statistical correlation of the modelled takeover probability with observed share buybacks, where a significant relation is found.

However, the hypothesis that there is a causal effect between share buybacks and takeover prevention is a logical fallacy. As will be shown in section 4, shares should only be bought back when the price is a bargain, otherwise the value to shareholders will decrease. Contrary to the assumption of many researchers, the case studies in section 9 show that the share price following a buyback frequently does not increase and may just as well decrease, thus making the company more attractive as a takeover target. Section 9.8 gives an example of a company that would become more attractive as a takeover and liquidation target as a result of a share buyback.

In case of synergistic effects between the target company and its acquirer, the value of the target company will be higher to that particular acquirer than its value to other shareholders. So the acquirer may be willing to pay a higher price for the company than other shareholders. If a share buyback is made at such higher prices it will decrease the value of the company to its existing shareholders while increasing the value to the acquirer. The company's market-cap must be higher than the value to the acquirer before the share buyback will decrease value to the acquirer, but this will also magnify the decrease in value to existing shareholders.

Share buybacks should not be used to deter hostile takeovers because the effect is uncertain and decreases value of the company to its existing shareholders if the share price is too high. Section 10 gives advice to managers on deciding when to make share buybacks regardless of the possibility of a hostile takeover.

2.6. Catering

The catering hypothesis has been proposed more recently, see e.g. Baker and Wurgler [**35**], where various statistical correlations are demonstrated on empirical data, such as a significant relation between companies paying dividends and their market-price relative to book-value. Furthermore, the relation shifts during the observation period 1962-2000. As an explanation, the authors suggest that the investment public changes their desire for dividends over time and managers cater to this in order to maximize the share price of their companies.

However, it is a fallacy to conclude from such a statistical correlation that a company's management tries to maximize the share price. The converse could just as well be true; that management chooses a dividend payout when the share price is high compared to book-value. In general, a company's management should not be concerned with maximizing the share price, firstly because setting the share price is beyond management's control once they fulfil their obligation of candid communication to shareholders, secondly because a too high share price will eventually be realized as such and then decrease and disappoint shareholders.

Section 10 gives advice to managers on how to decide between reinvestment, dividends and share buybacks, which should ideally be decided in cooperation with the company's shareholders as they are the beneficiaries. Although it may be economically preferable to reinvest in the company, its shareholders may have other needs for the funds and hence prefer a dividend. In this sense, management should indeed cater to shareholders.

2.7. Statistical Analysis

A large number of academic papers on the subject of share buybacks use statistical analysis, mainly regression analysis, to seek for correlations between observable variables such as the amount of share buybacks and number of stock options granted. Some of the papers cited above use such a statistical approach and a small selection of recent papers is briefly reviewed here.

Chen and Wang [**36**] find that companies which are financially constrained while making share buybacks have a tendency to have both their operating performance and share-price underperform those of their peers in the years following a share buyback. Babenko et al. [**37**] find correlation between insider share purchases prior to share buybacks and abnormal share-price increase afterwards. Bhargava [**38**] finds positive correlation between granting of stock options and share buybacks, as well as negative correlation between share buybacks and long-term investments and spending for research and development. Bonaime et al. [**39**] find that share buybacks have a tendency to occur when share prices are high relative to equity value and sales per share. For companies in the United Kingdom, Crawford and Wang [**40**] find that companies buying back shares at low prices relative to the equity value per share have a tendency to experience greater increase in share-price after two years.

Statistical analyses such as these have several limitations. First, an abnormal change in share-price relative to that of peer companies is commonly used to measure whether a share buyback was successful, that is, whether the shareholders experienced an abnormal capital gain during a period ranging from a few days up to a few years. But many other factors may contribute to abnormal changes in share-price which are impossible to fully separate in statistical analysis. Furthermore, as argued in this treatise, share buybacks should be made for the sake of eternal shareholders who only rely on dividends as their source of income from owning the shares, so a change in share-price following a share buyback is irrelevant. Second, although statistical analysis may find correlation between ex-post observations it does not reveal the causal relationship between share buybacks and their effect on the value to shareholders, which must be known if future share buybacks are to be properly executed by managers and properly understood by investors. A sound theoretical framework is needed for this.

This treatise provides such a theoretical framework that enables managers to make rational share buyback decisions and investors to assess the effect on the value of the company. Statistical analysis is used briefly in section 8 to show the historical trends of share buybacks and dividends in relation to earnings for a large group of companies in the aggregate, and for specific companies in some of the case studies in section 9.

2.8. Manager Surveys & Interviews

Surveys and interviews with financial managers were conducted by Brav et al. [**41**] to assess managers' reasoning for making dividend payouts and share buybacks, and how their reasoning relates to the hypotheses above. A total of 384 financial managers participated in the survey, of which 23 were interviewed in-depth. The survey covers both public and private, and paying and non-paying companies. The surveyed companies had average revenues of USD 11b and the 23 companies of the in-depth interviews had average revenues of USD 36b. The number and size of the companies and their broad selection means the survey can be considered a representative sample and statistical tests can be made.

The survey generally found that the beliefs of managers do not correspond to the hypotheses above. In general, the surveyed managers consider maintaining a historical level of dividend payout as important as the company's investment decision because the managers fear a decrease in dividend will be viewed by shareholders as a signal of permanent impairment of the company's earning power and hence cause a large decrease in share price. Managers also believe they have more flexibility in changing the level of share buybacks and therefore prefer making share buybacks instead of paying dividends. Indeed, 77% of the surveyed companies that have not paid dividends in the previous 3 years answered they may never initiate a dividend payout, while 56% may never initiate share buybacks. This is an absurd proposition because the managers must then believe their companies can grow indefinitely with an eternal need for accumulating capital. That 77% of the managers may never initiate a dividend payout also shows the managers misunderstand that a dividend payout is the only certain way of distributing capital to shareholders and share buybacks must only be made under beneficial circumstances which are not guaranteed to occur at all. The share price relative to its 'true value' is important to 86% of the surveyed managers, but considering that 76% of the managers answer they will buy back shares so as to increase the earnings-per-share and 68% answer they will buy back shares to offset the diluting impact of stock options, it appears the managers are unaware of how a share buyback actually affects the 'true value' of the company to its shareholders, which will be detailed in this treatise.

2.9. Relative Value Formula

A formula for measuring the effect of a share buyback on the intrinsic shareholder value was derived by Mauboussin [**42**]. The formula is easy to miss as it is placed in the back of that paper (Appendix C), its notation is awkward, the formula is apparently not used in that paper, and contrary to what the formula shows, the paper concludes that share buybacks and dividends are mathematically equivalent means of returning capital to shareholders. The formula actually reduces to Eq. 4-7 in this treatise, and is crucial in understanding the effect a share buyback has on shareholder value. This treatise is not influenced by [**42**].

2.10. Textbooks

Textbooks are condensed versions of the research literature thus providing students with the theoretical framework they will use as future practitioners. So textbooks are important.

Textbooks on corporate finance typically only briefly mention share buybacks and assume EMH holds so the substitution and signalling hypotheses also hold, see e.g. Brealey et al. [**43**], Ross et al. [**44**], Moles et al. [**45**], Koller et al. [**46**], Ehrhardt and Brigham [**47**], Damodaran [**48**], and Tirole [**49**].

Only few specialized textbooks on dividends and share buybacks exist, see e.g. Lease et al. [**50**] who briefly mention the above share buyback hypotheses, while Baker et al. [**6**] have a more extensive treatment, as does Vermaelen [**4**] who also makes a rare criticism of EMH but does not develop the theory of share buyback valuation in the absence of EMH which is the aim of this treatise.

Theory

3. Valuation Basics

This section describes the basics of valuing the shares of a company.

3.1. Eternal Shareholders

An eternal shareholder is defined here as owning shares of a company for eternity, thus deriving value from the shares solely through the receipt of dividends because capital gains (or losses) will never be realized.

The value of share buybacks to eternal shareholders is being considered for several reasons, most importantly because share buybacks should be made when they increase value to the remaining shareholders rather than the selling shareholders, which ultimately means that the share buybacks should increase value to eternal shareholders. Furthermore, the value to eternal shareholders is easier to assess than the value to shareholders who own their shares for only a limited period. Such temporary shareholders derive value from their shares both through the receipt of dividends as well as capital gains, and capital gains are inherently difficult to predict because they rely on the market price of the shares which can fluctuate greatly and unpredictably.

These reasons provide the logical foundation for focusing on the value to eternal shareholders but it should also be noted that shareholders who could potentially own their shares for eternity do actually exist in the form of pension funds, insurance companies, holding companies, governments, etc.

The theory of share buyback valuation can be considered the purest form of valuation theory because the holding period of the investment is eternity so there is no speculation regarding a future selling price.

3.2. Value to Eternal Shareholders

In Williams' theory of investment value [**1**], the value of a company to its eternal shareholders is defined as the present value of all future dividends.

The present value of the dividend for future year t is the amount that would have to be invested today with an annual rate of return d, also called the discount rate, so as to compound into becoming $Dividend_t$ after t years:

$$Present\ Value\ of\ Dividend_t \cdot (1+d)^t = Dividend_t \Leftrightarrow Present\ Value\ of\ Dividend_t = \frac{Dividend_t}{(1+d)^t}$$

Eq. 3-1

Let v denote the pre-tax present value of all future dividends to eternal shareholders, that is, prior to dividend tax, and not per share. Assume the discount rate d remains the same forever and whose selection is described in section 7. The present value v is then:

$$v = \sum_{t=0}^{\infty} Present\ Value\ of\ Dividend_t = \sum_{t=0}^{\infty} \frac{Dividend_t}{(1+d)^t}$$

Eq. 3-2

3.3. Dividends & Earnings

The value of a company to its eternal shareholders is defined in terms of dividends, however, many companies only payout a fraction of their earnings as dividends and make share buybacks or retain the remaining earnings. Valuation formulas must take this into account by considering the potential for dividend payouts instead.

Let $Excess\ Cash$ denote the company's excess cash holdings that can be paid out as dividends to shareholders now. Let $Earnings_t$ denote the company's earnings for year t which can be paid out as dividends to shareholders, that is, $Earnings_t$ is a cash measure and not an accounting measure as will be discussed later. The earnings for the current year are denoted $Earnings_1$ which is available for dividend payout a year from now and should therefore be discounted by $1 + d$, and so forth. Since the excess cash is separated in the valuation formula, any interest income from that cash must be deducted in $Earnings_t$.

Assuming the company's excess cash and future earnings are all paid out as dividends, the value to eternal shareholders is:

$$v = Excess\ Cash + \sum_{t=1}^{\infty} \frac{Earnings_t}{(1+d)^t}$$

Eq. 3-3

The valuation formulas for retained earnings are given in section 3.8 below.

3.3.1. Actual & Estimated Value

It is important to distinguish between actual and estimated values for v. The actual value v is being used in the valuation formulas below and the condition for a share buyback to increase or decrease value to eternal shareholders depends on the actual value v. However, future earnings are typically not known in advance so the actual value v is unknown and it must be estimated so as to form an opinion whether a share buyback is likely to increase or decrease value to eternal shareholders. If the estimate for v proves to be wrong in the future then it affects the value of share buybacks being made now. It should be clear from the context whether actual or estimated values are being used.

3.4. Excess Cash

Excess cash is treated specially in valuation for several reasons. Excess cash can readily be paid out as dividends to shareholders. Both the company and its shareholders can invest such cash in e.g. low-risk government bonds. So unlike assets such as production plants which must be owned and operated directly by the company, cash is an asset which can generate income to shareholders even if the company separates it from its operating assets, which is easily done through dividend payouts.

One reason for a company to accumulate cash is that the company's management may be uncertain of how much cash is needed for future investments and acquisitions, for example, if the industry is experiencing rapid technological development and management does not want to be in a future situation where they need to raise capital at possibly unfavourable market-prices for the company's shares.

When a company accumulates cash for years instead of paying it out as dividends to shareholders, it raises the question of what that cash is worth to eternal shareholders. If the company and its shareholders are taxed identically then the value to eternal shareholders is identical whether the company first invests the cash in e.g. low-risk government bonds and then later makes a dividend payout, or if the company first makes a dividend payout which the shareholder then invests in a similar manner. The order of taxation is indifferent because of the commutative property of multiplication. But if the company is taxed differently than some shareholders then the present value of the company's excess cash to those shareholders will depend on whether the company or the shareholders make the investment. This makes it impossible to determine a lower bound for the present value of a company's excess cash holdings to the eternal shareholders.

Because companies should generally only retain cash in excess of their needs for limited periods of time, it is assumed here that excess cash is immediately paid out as dividends, used for share buybacks, or retained in the company for a limited period of time and invested in e.g. low-risk government bonds, where the post-tax return to shareholders is assumed to approximately equal the return if shareholders had invested directly themselves. This allows for the excess cash to be separated in the valuation formulas.

Note, however, that the individual shareholder may have better investment opportunities elsewhere and therefore choose to sell their shares at a lower price than the value to eternal shareholders so as to use the cash for other investments. This is one example of a situation that can lead to a significant difference between the market-price of a company's shares and their value to eternal shareholders.

3.5. Debt

If the company has debt whose interest payments are deemed too large and unsustainable there are two possibilities to decrease the risk of bankruptcy. First, the company can make a recapitalization by issuing more shares and use the proceeds to repay debt. The valuation formulas for this are given in section 4.9. Second, the company can use future earnings to repay debt (recall that earnings is defined here as a cash measure), which means the calculation of v should be adjusted by subtracting the debt repayments and adding the future interest payments that will no longer be made.

Companies with excessive debt should first repay debt to a sustainable level before contemplating share buybacks, so it will be assumed here that companies have a debt burden that can be sustained indefinitely.

3.6. Growth

Future earnings are normally not known in advance so they will have to be estimated according to what can reasonably be predicted about the future of the company.

Some companies can grow their revenue and earnings without increasing their assets. For example, a company making computer software can sell additional user licenses with a negligible increase in productive assets because computer software is easy and cheap to replicate, so growth in revenue and earnings mainly depends on customer demand rather than the amount of assets. This means growth estimation for such companies must consider future customer demand, which can be difficult to assess. Instead, valuation can be reversed by calculating the growth required for the value to equal the market-cap, and a qualitative assessment can then be made whether such growth is likely.

3.6.1. Growth Starts Next Year

A simple growth model assumes earnings have an annual growth rate g for eternity starting next year. The start earnings are denoted $Earnings$ and equal the current year's earnings denoted $Earnings_1$ which are available for dividend payout a year from now. For year t the earnings are:

$$Earnings_t = Earnings \cdot (1+g)^{t-1}$$

Eq. 3-4

Assuming excess cash and earnings are paid out as dividends, the value to eternal shareholders is:

$$v = Excess\ Cash + \sum_{t=1}^{\infty} Earnings \cdot \frac{(1+g)^{t-1}}{(1+d)^t} = Excess\ Cash + \frac{Earnings}{d-g}$$

Eq. 3-5

This follows from the properties of the so-called geometric series, see Eq. 13-2.

The annual growth-rate required for the value to eternal shareholders to be greater than the market-cap is:

$$MarketCap < Excess\ Cash + \frac{Earnings}{d-g} \Leftrightarrow g > d - \frac{Earnings}{MarketCap - Excess\ Cash}$$

Eq. 3-6

3.6.2. Growth Starts in Current Year

If instead the earnings growth starts in the current year $t = 1$, the earnings for year t are:

$$Earnings_t = Earnings \cdot (1+g)^t$$

Eq. 3-7

The value to eternal shareholders is then:

$$v = Excess\ Cash + \sum_{t=1}^{\infty} Earnings \cdot \left(\frac{1+g}{1+d}\right)^t = Excess\ Cash + Earnings \cdot \frac{1+g}{d-g}$$

Eq. 3-8

For this value to be greater than the market-cap, the required growth rate is:

$$MarketCap < Excess\ Cash + Earnings \cdot \frac{1+g}{d-g} \Leftrightarrow g > \frac{d - \frac{Earnings}{MarketCap - Excess\ Cash}}{1 + \frac{Earnings}{MarketCap - Excess\ Cash}}$$

Eq. 3-9

3.7. Value Yield

Value yield is defined here as the discount rate which makes the market-cap equal to the value to eternal shareholders:

$$d = Value\ Yield \Leftrightarrow MarketCap = v$$

Eq. 3-10

The value yield is the rate of return a shareholder would get from buying shares at the given share-price (market-cap) and owning the shares for eternity. The value yield is also called the Internal Rate of Return (IRR) in the literature but that may be confused with Return on Equity (ROE) and is not used here.

If v is well-behaved as a function of d (see section 3.7.3), the value v increases when the discount rate d decreases, and vice versa, so the following also holds:

$$d < Value\ Yield \Leftrightarrow MarketCap < v$$

Eq. 3-11

This may be easier to understand when the notation makes clear that the value to eternal shareholders v is a function of the discount rate d by writing the value as $v(d)$, thus giving:

$$MarketCap = v(Value\ Yield)$$

$$d < Value\ Yield \Leftrightarrow MarketCap < v(d)$$

Eq. 3-12

3.7.1. Actual & Estimated Value Yield

As noted in section 3.3.1, we must distinguish between actual and estimated values for v, so we must also distinguish between actual and estimated value yields because they are defined from v. The actual value yield is typically not known in advance because it depends on future earnings which are rarely known in advance. The condition for a share buyback to increase or decrease value to eternal shareholders depends on the actual value yield. In practice, however, we must usually estimate the value yield so as to form an opinion whether a share buyback is likely to increase or decrease value to eternal shareholders. It should be clear from the context whether actual or estimated value yields are being considered.

3.7.2. Earnings Growth & Earnings Yield

If all excess cash and earnings are paid out to shareholders as dividends, and earnings are assumed to grow by the rate g every year for eternity starting next year, the value yield is found by inserting $d = Value\ Yield$ and rearranging Eq. 3-5:

$$MarketCap = Excess\ Cash + \frac{Earnings}{Value\ Yield - g} \Leftrightarrow Value\ Yield = \frac{Earnings}{MarketCap - Excess\ Cash} + g$$

Eq. 3-13

The case of $g = 0$ is called the earnings yield:

$$Earnings\ Yield = \frac{Earnings}{MarketCap - Excess\ Cash}$$

Eq. 3-14

3.7.3. Existence of Value Yield

The value yield is not always well-defined. For example, in case $Earnings \geq 0$ and $MarketCap \leq Excess\ Cash$ the value yield is not well-defined because the excess cash is assumed to be paid out as dividends immediately and hence covers the entire price of all the company's shares, which causes the formula for the value yield to become negative although the return to eternal shareholders is positive.

The so-called Cauchy-Hadamard theorem states that the power series used in the calculation of present value, converges if the earnings growth does not exceed exponential growth indefinitely, which is a reasonable assumption for earnings growth in the real world. According to that theorem, the present value exists for some continuous range of discount rates. In case the power series is finite, meaning all earnings are zero after some year, then the present value exists for all choices of discount rate except $d = -100\%$. In case all earnings are non-negative then the value yield exists, that is, the discount rate can be chosen so as to make the present value equal the market-cap, because the present value decreases continuously and monotonically from a limit of infinity when $d = -100\%$ towards zero as the discount rate increases.

The value yield may not be well-defined when some of the earnings are negative. Consider for example the case where positive earnings are followed by negative earnings and then zero earnings for eternity, which may be the pattern of a company going bankrupt. The present value does not change monotonically with a changing discount rate because the positive earnings may dominate the present value when the discount rate is sufficiently high and the negative earnings may dominate the present value when the discount rate is sufficiently low. This means there is a maximum present value when varying the discount rate. If the market-cap is higher than this maximum present value then the value yield is not well-defined. However, this does not matter in share buyback valuation because the present value is then known to be less than the market-cap which is sufficient. An example of this is given in section 9.2.

The value yield for general valuation formulas which cannot be analytically inverted can instead be found using a numerical root-finding method. Although it is beyond the scope of this treatise to give details on how to use such a method, example results are given in section 9.2.

3.8. Retained Earnings

The company's equity is the capital supplied directly by shareholders as well as the accumulation of retained earnings. Earnings are retained for the purpose of investing in new assets that can grow future earnings. The question is whether this increases value to eternal shareholders who could have received a dividend payout instead.

3.8.1. Investment & Return

First consider the general formulas for valuing investments that a company makes. Let v denote the company's value to eternal shareholders when the company makes no additional investments, and let v' denote the value with an investment, so v' equals v minus the investment amount plus the return on that

investment, which is the present value of the additional earnings attributable to shareholders arising from that investment:

$$v' = v - Invest + Return$$

Eq. 3-15

This increases value to eternal shareholders when the return is greater than the investment:

$$v' > v \Leftrightarrow Return > Invest$$

Eq. 3-16

3.8.2. Constant Rate of Return

Now assume the investment has a constant rate of return each year for eternity which is dubbed Return on Investment (ROI). The present value of the return is then calculated using Eq. 13-2 with zero growth:

$$Return = \sum_{t=1}^{\infty} \frac{ROI \cdot Invest}{(1+d')^t} = Invest \cdot \frac{ROI}{d'}$$

Eq. 3-17

The discount rate d' is selected according to the risk of the return differing from that expected, so d' equals the discount rate d used for calculating v if the risk is similar to that of the earnings arising from the company's existing assets.

The value to eternal shareholders is:

$$v' = v - Invest + Return = v + Invest \cdot \left(\frac{ROI}{d'} - 1\right)$$

Eq. 3-18

This increases value to eternal shareholders when the rate of return is greater than the discount rate:

$$v' > v \Leftrightarrow ROI > d'$$

Eq. 3-19

3.8.3. Return on Equity

The formulas above will now be given in terms of the Return on Equity (ROE) which is defined as a year's earnings divided by the equity at the beginning of the year:

$$ROE = \frac{Earnings}{Equity}$$

Eq. 3-20

Similarly, Return on Assets (ROA) is a year's earnings divided by the assets at the beginning of the year:

$$ROA = \frac{Earnings}{Assets}$$

Eq. 3-21

If excess cash is treated separately in valuation, then assets and equity should not include excess cash and the earnings should not include any interest income from the excess cash. Similarly for other assets which may be valued separately, such as marketable financial securities which are not required for the operations of the company.

For a company whose earnings are directly related to its assets so that ROA is constant, and if the company maintains a fixed capital structure, then the company will increase its assets and liabilities by the same rate as its equity upon retaining earnings. As a result ROE will also be constant.

Assuming ROE is constant and all excess cash and earnings are paid out as dividends, meaning the company does not make any additional investments, the value to eternal shareholders is:

$$v = Excess\ Cash + \sum_{t=1}^{\infty} \frac{Earnings}{(1+d)^t} = Excess\ Cash + \frac{Earnings}{d}$$

Eq. 3-22

If the company makes an investment whose annual rate of return is ROE and whose risk is similar to that of the company's existing earnings so the same discount rate d is used, the present value of the return is derived from Eq. 3-17:

$$Return = \sum_{t=1}^{\infty} \frac{ROE \cdot Invest}{(1+d)^t} = Invest \cdot \frac{ROE}{d}$$

Eq. 3-23

The value of the company to eternal shareholders is then:

$$v' = v - Invest + Return = v + Invest \cdot \left(\frac{ROE}{d} - 1\right)$$

Eq. 3-24

This increases value to eternal shareholders when:

$$v' > v \Leftrightarrow ROE > d$$

Eq. 3-25

This condition also holds for investments made in the future where the difference between the investment and the return is merely discounted further so as to calculate the present value now. If the investment is made k years from now the value of the company is:

$$v' = v + \frac{Return - Invest}{(1+d)^k}$$

Eq. 3-26

The additional discounting is irrelevant for the investment to increase value to eternal shareholders:

$$v' > v \Leftrightarrow Return > Invest \Leftrightarrow ROE > d$$

Eq. 3-27

The reason companies should not retain all earnings for investing in new productive assets even though they have sufficiently high historical ROE, is that investment in new assets and hence increased production must match an increase in customer demand for the company's products, otherwise the ROA and hence ROE will decrease in the future.

3.8.4. Interpretation

In the above, an investment was said to increase value to eternal shareholders if the present value of the return is greater than the investment amount. Similarly, an investment may be said to be value neutral if the present value of the return equals the investment amount, which is commonly interpreted to mean that the investment does not add value for shareholders so there is no reason to make the investment. But that interpretation is incorrect.

The present value of an investment's future returns is merely a measure of the value relative to another investment or class of investments; see also the discussion in section 7.2. It is used to gauge whether an investment has a return that is sufficient compared to alternative investments with similar risk characteristics. The proper way of interpreting a value neutral investment is therefore that it provides a return that is identical to the return of alternative investments with similar risk characteristics.

If companies only made investments with above average returns, then the average would increase until only the company with the highest rate of return would continue to make investments.

It is therefore oversimplified to say that an investment increases value to eternal shareholders when the present value of the return is greater than the investment amount, as was done above. It would be more correct to say that the value to eternal shareholders is increased *relative* to the alternative investment which is used as the discount rate in the present value calculation. But this is rather cumbersome so the shorter statement is normally used and the reader should keep the proper interpretation in mind.

For example, if the discount rate in Eq. 3-17 is chosen to be $d' = 20\%$ and the annual rate of return on the investment is $ROI = 21\%$, then the present value of the return is only 5% more than the investment amount. But if government bonds yield, say, 5% and the stock market is expected to yield, say, 8% for long-term shareholders, then a 21% rate of return is very high in comparison but this would not show in the present value calculation if the discount rate is 20%. Conversely, if the investment return is 19% per year

and the discount rate is 20%, then the present value would be slightly less than the investment amount and the naive interpretation would be that the investment decreases value to shareholders, while in fact the return would still be very high compared to government bonds and expected stock market returns.

3.8.5. Value Yield

The value yield is the discount rate for which the market-cap is equal to the present value of the company. That is, the value yield is the rate of return a shareholder would get at the given market price of the company's shares and by holding the shares for eternity, see section 3.7.

When the company makes an investment, its present value is denoted v' which is calculated with the discount rate d, so the value yield satisfies:

$$d = Value\ Yield \Leftrightarrow MarketCap = v'$$

$$d < Value\ Yield \Leftrightarrow MarketCap < v'$$

Eq. 3-28

This may be clearer if the present value is written as a function of the discount rate $v'(d)$, so that:

$$MarketCap = v'(Value\ Yield)$$

$$d < Value\ Yield \Leftrightarrow MarketCap < v'(d)$$

Eq. 3-29

Assuming ROE is constant, the value yield when the company makes a single investment is derived by using this definition of value yield with the definition of v' from Eq. 3-22 and Eq. 3-24:

$$\begin{aligned} &MarketCap = v'(Value\ Yield) \\ &\Leftrightarrow MarketCap = Excess\ Cash + \frac{Earnings}{Value\ Yield} + Invest \cdot \left(\frac{ROE}{Value\ Yield} - 1\right) \\ &\Leftrightarrow Value\ Yield = \frac{Earnings + Invest \cdot ROE}{MarketCap - Excess\ Cash + Invest} \end{aligned}$$

Eq. 3-30

3.8.6. Earnings Yield

A special case of the value yield is when no investment is being made which is called the earnings yield:

$$d = Earnings\ Yield \Leftrightarrow MarketCap = v$$

$$d < Earnings\ Yield \Leftrightarrow MarketCap < v$$

Eq. 3-31

Note that market-cap is now compared to v rather than v' as was the case for the value yield, because no earnings are retained and reinvested in the company as all earnings are paid out as dividends.

The earnings yield is deduced from Eq. 3-22 or by setting $Invest = 0$ in Eq. 3-30:

$$\begin{aligned} & MarketCap = v \\ \Leftrightarrow\ & MarketCap = Excess\ Cash + \frac{Earnings}{Earnings\ Yield} \\ \Leftrightarrow\ & Earnings\ Yield = \frac{Earnings}{MarketCap - Excess\ Cash} \end{aligned}$$

Eq. 3-32

Special Case

Consider the special case where the company has no excess cash and for some factor a the market-cap is:

$$MarketCap = a \cdot Equity$$

By definition $Earnings = ROE \cdot Equity$, so the earnings yield for this case is:

$$Earnings\ Yield = \frac{Earnings}{MarketCap} = \frac{ROE \cdot Equity}{a \cdot Equity} = \frac{ROE}{a}$$

This can also be expressed as an inequality:

$$MarketCap < a \cdot Equity \Rightarrow Earnings\ Yield > \frac{ROE}{a}$$

Eq. 3-33

3.8.7. Comparators

Assuming ROE is constant, it can be determined if the market-cap is less than the value of the company making investments from retained earnings, by considering the value of the company without investments in relation to the market-cap and the ROE in relation to the discount rate. This follows from Eq. 3-25:

$$MarketCap < v \text{ and } ROE > d \Rightarrow MarketCap < v'$$

Eq. 3-34

This can also be written in terms of the earnings and value yield using Eq. 3-31 and Eq. 3-28:

$$Earnings\ Yield > d \text{ and } ROE > d \Rightarrow Value\ Yield > d$$

Eq. 3-35

This holds regardless of how often and how much of future earnings are retained and reinvested in the company, as long as the condition $ROE > d$ continues to hold. This is useful for determining if share buybacks should be made based solely on the earnings yield and ROE without having to forecast how large a part of future earnings will be retained and reinvested in the company.

Note, however, that bi-implication does not hold, which means there may be cases where e.g. $MarketCap > v$ or $ROE < d$ which still results in $MarketCap < v'$ because the returns from investing retained earnings will sufficiently increase the value to eternal shareholders.

The opposite case is useful in deciding against making share buybacks as it makes it possible to conclude that for any case of future retaining of earnings and investments that the company makes, the resulting value to eternal shareholders is smaller than the market-cap. This also follows from Eq. 3-25:

$$MarketCap > v \text{ and } ROE < d \Rightarrow MarketCap > v'$$

Eq. 3-36

This can also be written in terms of the earnings and value yield using Eq. 3-31 and Eq. 3-28:

$$Earnings\,Yield < d \text{ and } ROE < d \Rightarrow Value\,Yield < d$$

Eq. 3-37

3.8.8. Constant Retained Earnings

Let $Retain$ be the fraction of earnings retained in the company so the equity at the end of the year is:

$$Equity' = Equity + Retain \cdot Earnings$$

Eq. 3-38

The equity grows by the rate g:

$$1 + g = \frac{Equity'}{Equity} \Leftrightarrow g = ROE \cdot Retain$$

Eq. 3-39

Assume that earnings are retained each year for eternity and ROA, capital structure and hence ROE are constant, so earnings will grow each year by the rate $g = ROE \cdot Retain$. The remaining $Earnings \cdot (1 - Retain)$ and excess cash is paid out as dividends. Assume the risk of the increased earnings is identical to the risk of the existing earnings, so the same discount rate d is used in calculating their present value, and assume $d > g$. The value to eternal shareholders is then calculated similarly to Eq. 3-5:

$$\begin{aligned} v' &= Excess\ Cash + \sum_{t=1}^{\infty} Earnings \cdot (1 - Retain) \cdot \frac{(1 + ROE \cdot Retain)^{t-1}}{(1 + d)^t} \\ &= Excess\ Cash + \frac{Earnings \cdot (1 - Retain)}{d - ROE \cdot Retain} \end{aligned}$$

Eq. 3-40

This is only defined for $0 \leq Retain < 1$ because $Retain = 1$ would mean that all earnings are retained so there are no dividend payouts for eternity in which case $v' = Excess\ Cash$.

The value yield is the discount rate for which the value to eternal shareholders equals the market-cap, that is, the value yield is the rate of return a shareholder would get at the given market-price for the company's shares, when part of the earnings each year are retained to fund future earnings growth. It is calculated by setting $d = Value\ Yield$ in Eq. 3-40 and using the definition of earnings yield from Eq. 3-32:

$$\begin{aligned} & MarketCap = v'(Value\ Yield) \\ \Leftrightarrow\ & MarketCap = Excess\ Cash + \frac{Earnings \cdot (1 - Retain)}{Value\ Yield - ROE \cdot Retain} \\ \Leftrightarrow\ & Value\ Yield = \frac{Earnings \cdot (1 - Retain)}{MarketCap - Excess\ Cash} + ROE \cdot Retain \\ & = Earnings\ Yield \cdot (1 - Retain) + ROE \cdot Retain \end{aligned}$$

Eq. 3-41

Under these assumptions the value yield is therefore the weighted average of the earnings yield and ROE, where the weight is the fraction of earnings that are being retained each year.

3.8.9. Value Yield Bounds

Now assume ROE is constant forever but the amount of retained earnings may vary each year. At one extreme no future earnings are retained so the value yield equals the earnings yield, as shown by setting $Retain = 0$ in Eq. 3-41. At the other extreme all future earnings are retained so the value yield equals the ROE, as shown by setting $Retain = 1$ in Eq. 3-41, although it should be noted that this is a boundary case because the value yield is not well-defined when all earnings are retained for eternity. The value yield will be between these two bounds depending on how much of future earnings are retained. Whether the earnings yield or ROE is the upper or lower bound depends on which of the two is larger. The bounds for the value yield are therefore:

$$Min(Earnings\ Yield, ROE) \leq Value\ Yield \leq Max(Earnings\ Yield, ROE)$$

Eq. 3-42

It follows that if ROE equals the earnings yield then the value yield equals the earnings yield:

$$ROE = Earnings\ Yield \Rightarrow Value\ Yield = Earnings\ Yield$$

It also follows that retained earnings decrease the value yield if ROE is smaller than the earnings yield:

$$ROE \leq Earnings\ Yield \Rightarrow Value\ Yield \leq Earnings\ Yield$$

Eq. 3-43

This means that if the share price is sufficiently low so the earnings yield is greater than ROE, then the rate of return from buying shares at that price will decrease if any future earnings are retained rather than being paid out as dividends. However, this situation usually does not arise as companies with high ROE normally have a high market-cap and hence a low earnings yield, but if the situation does arise then a share buyback may be more appropriate than retaining and reinvesting earnings, as shown in section 4.7.

In certain cases it can be concluded that the value yield of one company is smaller than the value yield of another company, without having to forecast how much of their future earnings will be retained. If the yields and ROE of the two companies are subscripted 1 and 2 then it follows from Eq. 3-42 that:

$$Max(Earnings\ Yield_1, ROE_1) < Min(Earnings\ Yield_2, ROE_2) \Rightarrow Value\ Yield_1 < Value\ Yield_2$$

Eq. 3-44

3.9. Adjustments to Reported Earnings

Using the models above to calculate the value of a company to its eternal shareholders requires an estimate of the starting earnings. The earnings reported in the financial statements of a company are based on accrual-accounting and may not be an accurate measure of the cash earnings that could be paid out as dividends so adjustments may be required. For example, gains or losses may be reversed if they are either not expected to recur in the future or if they do not affect the company's cash holdings. Such gains or losses could arise from e.g. restructuring charges or the sale or impairments of assets. Although the assets were paid with cash in the past and a loss on sale or an impairment charge therefore represents a cash loss to the past shareholders, they do not affect the amount of cash available for future dividends, which is what determines the present value to eternal shareholders. There may also be tax implications of these gains or losses which would also have to be reversed.

There are other differences between accrual-based earnings and cash earnings. For example, capital expenditures are made in one accounting period but the cost is depreciated in the reported earnings over several accounting periods. The reported earnings can be adjusted for this by adding depreciation and subtracting capital expenditures, but it is generally not possible for the outsider to see how much of the capital expenditures are for maintenance of existing assets and how much is for purchase of new assets. This means the growth expected from an increase in assets requires an estimate of the rate of return on assets, which is provided by accrual-based accounting. So unless the company has significantly misstated the depreciation charges, for example if the company's industry is evolving so rapidly that assets become obsolete faster than expected, there is no need to make adjustments to the reported earnings as the formulas in section 3.8 can be used for valuation when earnings are retained and the depreciation charges are reasonably accurate.

Adjusting the reported earnings can be complicated and imprecise, and accounting rules may change over time. It is beyond the scope of this work to give details and the reader is instead referred to e.g. [**51**] for theories and techniques. The case studies in section 9 demonstrate simple earnings adjustments.

4. Valuation of Share Buybacks

It was argued in section 3.1 that share buybacks should be made for the sake of eternal shareholders, who rely solely on dividends as their source of income from owning the shares because the shares will never be sold. A share buyback uses cash that could otherwise have been paid out as dividends and decreases the number of shares outstanding hence increasing the potential for future dividends per share. The question is whether it is best for eternal shareholders to get a dividend payout now or a share buyback with its promise of increased future dividends per share.

4.1. Actual & Estimated Value

The formulas derived in this section calculate the actual value and equilibriums of share buybacks, provided the actual value without share buybacks is used as input. However, the actual value without share buybacks is typically not known in advance, as it depends on future earnings which are usually not known in advance. Hence, estimates of the value without share buybacks will normally have to be used as input in the formulas, which means the value of share buybacks and equilibriums are also only estimates.

4.2. Value without Share Buyback

First consider the value to eternal shareholders without share buybacks, that is, the present value of all future dividends which consist of the excess cash assumed to be paid out as dividends now and future earnings assumed to be paid out as dividends annually. The present value, pre-tax and not per share, is:

$$v = \ Excess\ Cash + \sum_{t=1}^{\infty} \frac{Earnings_t}{(1+d)^t}$$

Eq. 4-1

This is the basic valuation formula from Eq. 3-3.

Let $Shares$ be the current number of shares outstanding, assumed to have equal rights to dividends. Further assume the tax-rate on dividends remains the same for eternity for the individual shareholder but it can be different amongst shareholders. The post-tax value per share is then:

$$V = \frac{1 - TaxDividend}{Shares} \cdot v$$

Eq. 4-2

4.3. Value with Share Buyback

A share buyback reduces both the company's cash holdings and the number of shares outstanding. The cash reduction results in a decrease in the value to eternal shareholders v because the cash is no longer available for dividend payout. The reduction in the number of shares from the buyback is proportional to the share-price.

Let $Buyback$ be the amount used for the share buyback so the number of shares bought back is:

$$\frac{Buyback}{SharePrice} = Shares \cdot \frac{Buyback}{MarketCap}$$

Eq. 4-3

The number of outstanding shares after the buyback is then:

$$Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)$$

Eq. 4-4

The value to eternal shareholders with a share buyback, per share outstanding before the share buyback and after dividend tax, is then:

$$W = \frac{(v - Buyback) \cdot (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)}$$

Eq. 4-5

It is sometimes convenient to consider this without the dividend tax and number of shares:

$$w = \frac{v - Buyback}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 4-6

If a share buyback is made for excess cash then $v \geq Buyback \geq 0$ which means that $w \geq 0$. If instead a share buyback is made for borrowed money then it is possible to have $v < Buyback$ in which case $w < 0$. Valuation of share buybacks for borrowed money is studied in greater detail in section 4.9.

4.3.1. Alternative Definitions of v

The value to eternal shareholders without a share buyback is denoted v and is defined in Eq. 4-1 as the company's excess cash plus the present value of future earnings available for dividend payouts. This is deemed a sensible definition for use in valuing share buybacks, as previously discussed.

However, the value v can be defined differently yet result in the same mathematical properties for the share buyback equilibrium and relative value. This merely requires (1) that a share buyback decreases v with the buyback amount, and (2) that the number of shares is decreased as in Eq. 4-4, so the combined effect is that a share buyback transforms the value to eternal shareholders from v to w as in Eq. 4-6.

4.3.2. Relative Value

The value of share buybacks relative to the value with no share buybacks is Eq. 4-5 divided by Eq. 4-2:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 4-7

This formula is not defined for $v = 0$ and it is ill-defined for $v < 0$ where this formula suggests a share buyback increases value to eternal shareholders whereas the value is actually decreased. But these cases cannot arise because it is assumed that $v \geq Buyback \geq 0$ and hence $w \geq 0$, otherwise the share buyback would have to be made for borrowed money which requires different valuation formulas, see section 4.9.

The relative value is independent of the tax on dividends which means the tax can be different amongst shareholders without affecting the relative value and equilibrium of share buybacks, but the tax-rate is assumed to be constant for the individual shareholder for eternity. Future changes to the dividend tax-rate affect the value of current share buybacks as shown in section 4.4.

The relative value of a share buyback is usually written W/V to emphasize it is per share currently outstanding and after dividend tax. But the number of shares and the dividend tax both factor out in the calculation of W/V and hence are ignored, so Eq. 4-7 is equivalent to:

$$\frac{W}{V} = \frac{w}{v}$$

Eq. 4-8

4.3.3. Equilibrium

The equilibrium is where the value to eternal shareholders is unaffected by the share buyback so $W = V$. However, it is more useful to express the equilibrium in terms of an inequality so it is easy to see when the value to eternal shareholders is increased or decreased from a share buyback. The equilibrium is derived from Eq. 4-2 and Eq. 4-5, or equivalently from Eq. 4-7:

$$W > V \Leftrightarrow MarketCap < v$$

Eq. 4-9

Appendix 13.2 shows that the equilibrium can only be greater than v when future share buybacks are made at favourable share-prices. As future share-prices are unpredictable this may never be possible, so valuation of share buybacks must be done for the current year only.

Value Yield

If v is calculated using a constant discount rate d then it follows from Eq. 3-11 that the equilibrium in Eq. 4-9 is equivalent to:

$$W > V \Leftrightarrow d < Value\ Yield$$

Eq. 4-10

This allows for the assessment of whether a share buyback increases or decreases value to eternal shareholders by first calculating the value yield of v and then comparing it with the rate of return which could be obtained from alternative investments with similar risk characteristics.

4.3.4. Effect on Value

Figure 1 and Figure 2 demonstrate the above formulas for the relative value and equilibrium of a share buyback, which show that a share buyback magnifies the difference between $MarketCap$ and v. The value with a share buyback w is depicted for changing variables in Figure 3 and the relative value of a share buyback W/V is depicted in Figure 4. These are compared in Figure 5 where it can be seen that w changes linearly and W/V changes non-linearly with varying v, which is explored further in section 5 and used in section 6 to derive properties for share buyback valuation under stochastic uncertainty.

4.3.5. Effect on Price

The effect of a share buyback on the market price of the remaining shares is unclear. In case the share price is above the equilibrium then a share buyback would presumably have a negative effect on the future share price. In case the share price is below the equilibrium then a share buyback would presumably have a positive effect on the future share price. In order to accurately predict the effect of a share buyback on the future share price, we would therefore need to know the equilibrium value at the time of the share buyback, which depends on future earnings and is hence unknowable, and we would need to know how the shares would have been priced in the future without a share buyback and how the market participants value the effect of the share buyback, which is also unknowable.

It is argued in section 3.1 that share buybacks should be made for the sake of eternal shareholders who will never sell their shares, which circumvents the problem of predicting future share prices so the valuation can focus on predicting future earnings instead – which is difficult enough.

4.3.6. Effect on Equity per Share

A share buyback made from excess cash reduces the company's assets and equity by the buyback amount. The number of shares is reduced as in Eq. 4-4. This means the equity per share is reduced as a result of a share buyback whenever the equity is less than the market-cap:

$$\begin{aligned} &Equity\ per\ Share\ after\ Buyback < Equity\ per\ Share\ before\ Buyback \\ &\Leftrightarrow \frac{Equity - Buyback}{Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)} < \frac{Equity}{Shares} \Leftrightarrow Equity < MarketCap \end{aligned}$$

Eq. 4-11

The relative change in equity per share is:

$$\frac{Equity\ per\ Share\ after\ Buyback}{Equity\ per\ Share\ before\ Buyback} = \frac{\frac{Equity - Buyback}{Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)}}{\frac{Equity}{Shares}} = \frac{1 - \frac{Buyback}{Equity}}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 4-12

Note the similarity to the relative value of a share buyback in Eq. 4-7 only with v replaced by $Equity$.

4.3.7. Effect on P/Book

If the equity is less than the market-cap and the share price is unchanged following a share buyback then the P/Book ratio is increased. This follows from Eq. 4-11 because the equity per share is decreased. It can also be shown as follows.

The P/Book ratio before the share buyback is defined as:

$$P/Book = \frac{MarketCap}{Equity}$$

Eq. 4-13

A share buyback decreases the equity by the buyback amount. If the share-price is unchanged then the market-cap is also decreased by the buyback amount. The P/Book ratio after a share buyback is then:

$$(P/Book)' = \frac{MarketCap - Buyback}{Equity - Buyback}$$

Eq. 4-14

So a share buyback increases the P/Book ratio whenever the equity is less than the market-cap:

$$(P/Book)' > P/Book \Leftrightarrow \frac{MarketCap - Buyback}{Equity - Buyback} > \frac{MarketCap}{Equity} \Leftrightarrow Equity < MarketCap$$

Eq. 4-15

The relative change in P/Book resulting from a share buyback is found by dividing Eq. 4-14 with Eq. 4-13:

$$\frac{(P/Book)'}{P/Book} = \frac{1 - \frac{Buyback}{MarketCap}}{1 - \frac{Buyback}{Equity}}$$

Eq. 4-16

If the P/Book ratio is to remain unchanged after a share buyback then the share-price must be divided by this factor. If $Equity < MarketCap$ then the share-price must decrease and if $Equity > MarketCap$ then the share-price must increase in order for the P/Book ratio to remain unchanged after the share buyback.

4.3.8. Effect on Value Yield

The value yield is the discount rate that makes v equal to the market-cap, that is, the value yield is the rate of return an investor would get from buying shares at the given price and holding the shares for eternity. Using the definition of value yield from Eq. 3-12 and the definition of v from Eq. 4-1 gives:

$$MarketCap = Excess\ Cash + \sum_{t=1}^{\infty} \frac{Earnings_t}{(1 + Value\ Yield)^t}$$

Eq. 4-17

A share buyback decreases both the market-cap and v by the buyback amount. The value yield after the share buyback is denoted $Value\ Yield'$ and must satisfy the equation:

$$MarketCap - Buyback = Excess\ Cash - Buyback + \sum_{t=1}^{\infty} \frac{Earnings_t}{(1 + Value\ Yield')^t}$$

Eq. 4-18

The buyback amount can be removed from both sides of this equation and the result compared to Eq. 4-17. This shows that the value yield is unchanged from the share buyback:

$$MarketCap = Excess\ Cash + \sum_{t=1}^{\infty} \frac{Earnings_t}{(1 + Value\ Yield')^t} \Leftrightarrow Value\ Yield = Value\ Yield'$$

Eq. 4-19

So the share buyback does not change the rate of return an investor would get from buying shares at the given price and holding the shares for eternity. To assess whether a share buyback increases or decreases value to eternal shareholders, the value yield must therefore be compared to an appropriate rate of return, see section 4.3.3.

Note that the value yield is unchanged when the share buyback is made from excess cash, while recapitalizations can significantly change the value yield as demonstrated in section 9.2.

4.4. Changing Dividend Tax

If the dividend tax rate is constant for eternity then it is irrelevant to the relative value of a share buyback and hence the equilibrium, as shown in Eq. 4-7 and Eq. 4-9.

Consider now the case where the current year's dividend tax rate is $TaxDividend$ and for future years it is $TaxDividend'$. Assuming the excess cash is paid out as dividends immediately, the value without a share buyback is:

$$V = \frac{Excess\ Cash \cdot (1 - TaxDividend) + (1 - TaxDividend') \cdot \sum_{t=1}^{\infty} \frac{Earnings_t}{(1+d)^t}}{Shares}$$

Eq. 4-20

The value with a share buyback is:

$$W = \frac{(Excess\ Cash - Buyback) \cdot (1 - TaxDividend) + (1 - TaxDividend') \cdot \sum_{t=1}^{\infty} \frac{Earnings_t}{(1+d)^t}}{Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)}$$

Eq. 4-21

The relative value of a share buyback reduces to:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{Excess\ Cash + \frac{1 - TaxDividend'}{1 - TaxDividend} \cdot \sum_{t=1}^{\infty} \frac{Earnings_t}{(1+d)^t}}}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 4-22

The equilibrium below which a share buyback increases value to eternal shareholders is:

$$W > V \Leftrightarrow MarketCap < Excess\ Cash + \frac{1 - TaxDividend'}{1 - TaxDividend} \cdot \sum_{t=1}^{\infty} \frac{Earnings_t}{(1+d)^t}$$

Eq. 4-23

The dividend tax rate is constant for eternity when $TaxDividend' = TaxDividend$ so the equilibrium is v as in Eq. 4-9. When $TaxDividend' < TaxDividend$ the equilibrium is higher and it is lower when $TaxDividend' > TaxDividend$ because the equilibrium decreases as $TaxDividend'$ increases.

When deriving share buyback valuation formulas for changing dividend tax rates, it is important to match the tax rates with the dividend payouts rather than the earnings. For example, if cash earnings are retained for several years before being paid out as dividends then it is the tax rate at the time of the dividend payout rather than at the time of the earnings that should be used in the valuation formulas.

For simplicity the dividend tax rate is assumed to be constant for eternity in most of this treatise.

4.5. Multiple Share Buybacks

Multiple share buybacks can be made in succession without affecting the equilibrium share price because the decrease in market-cap after each share buyback is matched by the decrease in value v. This means that once you have determined the equilibrium share price below which a share buyback will increase value for eternal shareholders, this share price need not be recalculated after every share buyback unless there is a change in the value v apart from the amount used for share buybacks.

For example, if a company has excess cash holdings that are included in v and this cash is to be used for share buybacks then it will not affect the equilibrium share price that the cash is being spent during multiple rounds of share buybacks, as long as they occur within the same dividend and discounting period.

To see this, let the following be the result after the first share buyback:

$$MarketCap' = MarketCap - Buyback$$

$$v' = v - Buyback$$

$$Shares' = Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)$$

The equilibrium per share is then:

$$MarketCap < v \Leftrightarrow \frac{MarketCap}{Shares} < \frac{v}{Shares} \Leftrightarrow \frac{MarketCap'}{Shares'} < \frac{v'}{Shares'} \Leftrightarrow MarketCap' < v'$$

Eq. 4-24

4.6. Tax on Share Buybacks

Share buybacks were assumed in the above to be tax-free. A tax would decrease the number of shares bought back but it would not affect the decrease in the value v, so Eq. 4-5 becomes:

$$W = \frac{(v - Buyback) \cdot (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Buyback \cdot (1 - TaxBuyback)}{MarketCap}\right)}$$

Eq. 4-25

The relative value of a share buyback from Eq. 4-7 becomes:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback \cdot (1 - TaxBuyback)}{MarketCap}}$$

Eq. 4-26

The equilibrium from Eq. 4-9 becomes:

$$W > V \Leftrightarrow MarketCap < v \cdot (1 - TaxBuyback)$$

Eq. 4-27

4.7. Alternative Investment

Assume an investment is made for the funds that would otherwise have been used for a share buyback. This investment could, for example, be an acquisition of another company or the construction of a new production facility. The investment amount is denoted $Invest$ and the present value of the future return on that investment is denoted $Return$ which should be calculated using an appropriate discount rate that would be the same as the discount rate used for calculating v if the return has similar risk.

The value to eternal shareholders per share currently outstanding and after dividend tax is:

$$V_{Invest} = \frac{(v - Invest + Return) \cdot (1 - TaxDividend)}{Shares}$$

Eq. 4-28

The relative value of the investment is:

$$\frac{V_{Invest}}{V} = 1 + \frac{Return - Invest}{v}$$

Eq. 4-29

The value to eternal shareholders is increased whenever the present value of the return is greater than the investment made, that is:

$$V_{Invest} > V \Leftrightarrow Return > Invest$$

Eq. 4-30

The value of the investment relative to the value of a share buyback from Eq. 4-5 is (by assumption $Invest = Buyback$):

$$\frac{V_{Invest}}{W} = \left(1 + \frac{Return}{v - Invest}\right) \cdot \left(1 - \frac{Invest}{MarketCap}\right)$$

Eq. 4-31

The equilibrium where a share buyback starts to increase value relative to the investment is:

$$W > V_{Invest} \Leftrightarrow MarketCap < Invest \cdot \left(1 + \frac{v - Invest}{Return}\right)$$

Eq. 4-32

The equilibrium can also be formulated in terms of the return on the investment:

$$W > V_{Invest} \Leftrightarrow Return < \frac{Invest \cdot (v - Invest)}{MarketCap - Invest}$$

Eq. 4-33

These equilibriums are useful for deciding whether to make a share buyback or an alternative investment.

4.7.1. Return Factor

In case the present value of the return is a factor r of the value v, that is $Return = r \cdot v$, then the equilibrium in Eq. 4-32 which is formulated in terms of the market-cap becomes:

$$W > V_{Invest} \Leftrightarrow MarketCap < Invest \cdot \left(1 + \frac{1 - \frac{Invest}{v}}{r}\right)$$

Eq. 4-34

The equilibrium in Eq. 4-33 which is formulated in terms of the return becomes:

$$W > V_{Invest} \Leftrightarrow r < \frac{Invest \cdot \left(1 - \frac{Invest}{v}\right)}{MarketCap - Invest}$$

Eq. 4-35

4.7.2. Constant Rate of Return

Now assume the investment has a constant rate of return each year for eternity which is dubbed Return on Investment (ROI). The discount rate for calculating the present value of the return is denoted d' and should be chosen according to the risk of the return differing from that expected.

One use of these formulas is when a company contemplates either expanding its existing business by investing in more productive assets or making a share buyback. If a qualitative assessment deems the risk and expected return of the expansion to be similar to the existing business, then the discount rate would be $d' = d$ as used in the calculation of v, and ROI would be the expected ROE of the company.

The value to eternal shareholders with an investment, per share currently outstanding and after dividend tax, is found by inserting the present value of the return from Eq. 3-17 in Eq. 4-28:

$$V_{Invest} = \frac{\left(v + Invest \cdot \left(\frac{ROI}{d'} - 1\right)\right) \cdot (1 - TaxDividend)}{Shares}$$

Eq. 4-36

The relative value of the investment is:

$$\frac{V_{Invest}}{V} = 1 + \frac{Invest}{v} \cdot \left(\frac{ROI}{d'} - 1\right)$$

Eq. 4-37

The investment increases value to eternal shareholders when:

$$V_{Invest} > V \Leftrightarrow ROI > d'$$

Eq. 4-38

The value of the investment relative to a share buyback is found by inserting Eq. 3-17 in Eq. 4-31:

$$\frac{V_{Invest}}{W} = \left(1 + \frac{ROI}{d' \cdot \left(\frac{v}{Invest} - 1\right)}\right) \cdot \left(1 - \frac{Invest}{MarketCap}\right)$$

Eq. 4-39

The equilibrium where a share buyback starts to increase value relative to the investment is found by inserting Eq. 3-17 in Eq. 4-32 (by assumption $Invest = Buyback$):

$$W > V_{Invest} \Leftrightarrow MarketCap < Invest + (v - Invest) \cdot \frac{d'}{ROI}$$

Eq. 4-40

The equilibrium can also be formulated in terms of ROI:

$$W > V_{Invest} \Leftrightarrow ROI < d' \cdot \frac{v - Invest}{MarketCap - Invest}$$

Eq. 4-41

The equilibrium can also be formulated in terms of the investment amount (which equals the buyback amount by assumption):

$$W > V_{Invest} \Leftrightarrow Invest > \frac{MarketCap \cdot ROI - d' \cdot v}{ROI - d'}$$

Eq. 4-42

These formulas are not always well-defined and it is important to use them in the correct order. First use Eq. 4-38 to determine if the investment would increase value to eternal shareholders otherwise it should not be considered. Then use Eq. 4-9 to determine if a share buyback would increase value to eternal shareholders otherwise it should not be considered. If both the investment and a share buyback would increase value to eternal shareholders, then in order to determine which would increase value the most either use Eq. 4-39 to calculate their relative value or use one of the equilibriums above.

As shown by these equilibriums, the share buyback and alternative investment affect the value to eternal shareholders differently depending on the amount of the share buyback or investment. This is because the relative value to eternal shareholders is assumed to change linearly with the investment amount but it changes non-linearly with the share buyback amount, as demonstrated in Figure 6.

The optimal allocation between share buyback and investment is to make the one that is most valuable to eternal shareholders according to the equilibriums above. In case only a limited amount can be used for the optimal choice, that is, if the investment opportunities are limited or if only a limited number of shares are available for buyback, then the remainder of the amount should be used for the other choice which still increases value to eternal shareholders.

4.7.3. Interpretation of Present Value

The present value of an investment's return is a measure of the value relative to another investment or class of investments, whose future rate of return is the discount rate used in the present value calculation. The present value is used to gauge whether an investment has a return that is sufficient compared to other investments with similar risk characteristics. Proper choice of discount rate and proper interpretation of the present value is important when assessing the value of a share buyback relative to an alternative investment. See section 3.8.4 for a more detailed discussion.

4.8. Share Issuance to Fund Investment

A share issuance is the opposite of a share buyback in that new shares are issued to raise cash for the company. The increased number of shares causes dilution to existing shareholders. Share issuance may be done by a company to fund a new investment, as studied here, or it may be done to repay debt as studied in section 4.9.

Let $Issuance$ be the amount of the share issuance and let $SharePrice$ be the price at which the new shares are issued. The number of shares issued is:

$$\frac{Issuance}{SharePrice} = Shares \cdot \frac{Issuance}{MarketCap}$$

Eq. 4-43

The number of outstanding shares after the issuance is:

$$Shares \cdot \left(1 + \frac{Issuance}{MarketCap}\right)$$

Eq. 4-44

Note that this is the opposite of the reduction in shares from a share buyback in Eq. 4-4.

The cash raised by the company in the share issuance is assumed to be used entirely for the investment so that $Invest = Issuance$. Let $Return$ denote the present value of the return on the investment.

The value to eternal shareholders with the share issuance and investment, per share currently outstanding and after dividend tax, is then:

$$\begin{aligned} W_{Issuance} &= \frac{(v + Issuance - Invest + Return) \cdot (1 - TaxDividend)}{Shares \cdot \left(1 + \frac{Issuance}{MarketCap}\right)} \\ &= \frac{(v + Return) \cdot (1 - TaxDividend)}{Shares \cdot \left(1 + \frac{Issuance}{MarketCap}\right)} \end{aligned}$$

Eq. 4-45

The relative value of making the share issuance and investment is:

$$\frac{W_{Issuance}}{V} = \frac{1 + \frac{Return}{v}}{1 + \frac{Issuance}{MarketCap}}$$

Eq. 4-46

The relative value is non-linear in several variables. An example is plotted in Figure 18 and Figure 19.

The equilibrium where the share issuance and investment starts to increase value to eternal shareholders can be formulated in terms of different variables. The equilibrium formulated in terms of the market-cap is:

$$W_{Issuance} > V \Leftrightarrow MarketCap > v \cdot \frac{Issuance}{Return}$$

Eq. 4-47

The equilibrium can also be formulated in terms of the value v:

$$W_{Issuance} > V \Leftrightarrow v < MarketCap \cdot \frac{Return}{Issuance}$$

Eq. 4-48

The equilibrium can also be formulated in terms of the return on the investment:

$$W_{Issuance} > V \Leftrightarrow Return > Issuance \cdot \frac{v}{MarketCap}$$

Eq. 4-49

4.8.1. Partially Cash-Funded

Now consider the case where part of the investment is cash-funded. Let $CashInvest$ denote the amount of cash invested in addition to the share issuance. Let $Return'$ denote the present value of the gross return on the investment. Let $Return$ denote the return on the investment net of the cash investment:

$$Return = Return' - CashInvest \Leftrightarrow Return' = Return + CashInvest$$

Eq. 4-50

In calculating the value to eternal shareholders, the cash investment must first be subtracted from v because it is no longer available for dividend payouts, and the gross return on the investment is then added. Using the definition of $Return'$ from Eq. 4-50 this reduces to:

$$W_{Issuance} = \frac{(v - CashInvest + Return') \cdot (1 - TaxDividend)}{Shares \cdot \left(1 + \frac{Issuance}{MarketCap}\right)} = \frac{(v + Return) \cdot (1 - TaxDividend)}{Shares \cdot \left(1 + \frac{Issuance}{MarketCap}\right)}$$

Eq. 4-51

This is identical to Eq. 4-45 which means that the above formulas for the relative value and equilibriums can also be used for partially cash-funded investments by using $Return' - CashInvest$ instead of $Return$.

The relative value is:

$$\frac{W_{Issuance}}{V} = \frac{1 + \frac{Return}{v}}{1 + \frac{Issuance}{MarketCap}} = \frac{1 + \frac{Return' - CashInvest}{v}}{1 + \frac{Issuance}{MarketCap}}$$

Eq. 4-52

The equilibrium formulated in terms of market-cap is:

$$W_{Issuance} > V \Leftrightarrow MarketCap > v \cdot \frac{Issuance}{Return' - CashInvest}$$

Eq. 4-53

The equilibrium formulated in terms of v is:

$$W_{Issuance} > V \Leftrightarrow v < MarketCap \cdot \frac{Return' - CashInvest}{Issuance}$$

Eq. 4-54

The equilibrium formulated in terms of the gross return on the investment is:

$$W_{Issuance} > V \Leftrightarrow Return' > Issuance \cdot \frac{v}{MarketCap} + CashInvest$$

Eq. 4-55

4.9. Recapitalization

A company's capital structure is the combination of liabilities and equity that fund the company's assets. The capital structure can be changed through debt issuance and share buybacks (conversely, debt repayment and share issuance).

When a company's capital structure is changed it also changes the risk of default on the debt and hence the degree of uncertainty about the company's future earnings and dividends to shareholders. This requires a revaluation of the company's future earnings which must be taken into account when assessing whether the recapitalization increases value to eternal shareholders.

Two approaches for revaluation are considered here. First is revaluation by changing the discount rate used in the present value calculations. Second is adding a revaluation term to the present value of earnings.

4.9.1. Revaluation by Changing Discount Rate

Let d be the discount rate used for valuing the company without the debt. Assume all excess cash and future earnings are paid out as dividends so the value to eternal shareholders is:

$$v = v(d) = Excess\ Cash + \sum_{t=1}^{\infty} \frac{Earnings_t}{(1+d)^t}$$

The value per share currently outstanding and after dividend tax is then:

$$V = \frac{v \cdot (1 - TaxDividend)}{Shares}$$

Debt Increase & Share Buyback

Let $Debt$ be the amount of debt to be issued and assume the share buyback equals this amount. The debt is assumed to have a constant interest rate and no maturity so fixed interest payments are made starting a year from now and continuing for eternity. Further assuming the company is profitable and interest payments are tax-deductible, the interest payments provide a so-called tax-shield from the corporate tax.

Let d' be the discount rate used for revaluing the company with the debt to reflect the increased risk of defaulting on the debt. The value to eternal shareholders is then:

$$v' = Excess\ Cash + \sum_{t=1}^{\infty} \frac{Earnings_t - Debt \cdot InterestRate \cdot (1 - TaxCompany)}{(1+d')^t}$$
$$= Excess\ Cash + \sum_{t=1}^{\infty} \frac{Earnings_t}{(1+d')^t} - \frac{Debt \cdot InterestRate \cdot (1 - TaxCompany)}{d'}$$

The present value of the interest payments was calculated using Eq. 13-2 with zero growth.

Writing the value v calculated with the discount rate d' as $v(d')$, the value with debt v' equals:

$$v' = v(d') - \frac{Debt \cdot InterestRate \cdot (1 - TaxCompany)}{d'}$$

Eq. 4-56

The value per share currently outstanding and after dividend tax is then:

$$V' = \frac{v' \cdot (1 - TaxDividend)}{Shares}$$

Eq. 4-57

When the debt issuance is used entirely for a share buyback, the value per share after dividend tax is:

$$W_{DebtIncrease} = \frac{v' \cdot (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Debt}{MarketCap}\right)}$$

Eq. 4-58

The relative value of this recapitalization is:

$$\frac{W_{DebtIncrease}}{V} = \frac{v'}{v \cdot \left(1 - \frac{Debt}{MarketCap}\right)}$$

Eq. 4-59

The equilibrium where the recapitalization starts to increase value to eternal shareholders is:

$$W_{DebtIncrease} > V \Leftrightarrow MarketCap < \frac{Debt}{1 - \frac{v'}{v}}$$

Eq. 4-60

This equilibrium is less than the equilibrium for cash-funded share buybacks v, when the difference between the present value of the earnings that result from the change in discount rate, is greater than the difference between the debt amount and the present value of the interest payments:

$$\frac{Debt}{1 - \frac{v'}{v}} < v \Leftrightarrow v(d) - v(d') > Debt \cdot \left(1 - \frac{InterestRate \cdot (1 - TaxCompany)}{d'}\right)$$

Eq. 4-61

Debt Decrease & Share Issuance

Now consider a company that has debt outstanding which is repaid by issuing shares, that is, $Debt$ is the amount of debt to be repaid and the share issuance equals this amount. The company's value changes from being v' to v as the present value of the interest payments is added to v' and the remaining earnings are re-valued with the discount-rate d so as to reflect the decrease in risk of default. The number of shares increases accordingly, so the value per share currently outstanding is:

$$W_{DebtDecrease} = \frac{v \cdot (1 - TaxDividend)}{Shares \cdot \left(1 + \frac{Debt}{MarketCap}\right)}$$

Eq. 4-62

The relative value of the recapitalization is:

$$\frac{W_{DebtDecrease}}{V'} = \frac{v}{v' \cdot \left(1 + \frac{Debt}{MarketCap}\right)}$$

Eq. 4-63

The equilibrium where this recapitalization starts to increase value to eternal shareholders is:

$$W_{DebtDecrease} > V' \Leftrightarrow MarketCap > \frac{Debt}{\frac{v}{v'} - 1}$$

Eq. 4-64

This equilibrium is less than the equilibrium for cash-funded share buybacks v', when the difference between the present value of the earnings that result from the change in discount rate, is greater than the difference between the debt amount and the present value of the interest payments:

$$\frac{Debt}{\frac{v}{v'} - 1} < v' \Leftrightarrow v(d) - v(d') > Debt \cdot \left(1 - \frac{InterestRate \cdot (1 - TaxCompany)}{d'}\right)$$

Eq. 4-65

Reversibility

These recapitalizations are reversible if the share prices are identical. That is, if debt is first increased and shares bought back, and this is then reversed with debt repayment and share issuance at the same share price, then the value to eternal shareholders is unaffected. To see this consider the change in number of shares outstanding after the first recapitalization where debt is increased and shares bought back:

$$Shares' = Shares \cdot \left(1 - \frac{Debt}{MarketCap}\right)$$

The resulting market-cap is:

$$MarketCap' = MarketCap - Debt$$

Then making a share issuance and debt repayment merely reverses this change in shares and market-cap:

$$\begin{aligned} W_{DebtDecrease} &= \frac{v \cdot (1 - TaxDividend)}{Shares' \cdot \left(1 + \frac{Debt}{MarketCap'}\right)} \\ &= \frac{v \cdot (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Debt}{MarketCap}\right) \cdot \left(1 + \frac{Debt}{MarketCap - Debt}\right)} = V \end{aligned}$$

Eq. 4-66

Temporary Debt Increase

If a company has no excess cash but wants to make a share buyback, it can raise cash from a short-term loan for use in the share buyback and repay the loan from subsequent earnings.

Let $Debt$ be the amount of short-term debt raised and used for a share buyback. The interest is assumed to be tax-deductible. The debt and interest is assumed to be paid in full after one year so it should be discounted accordingly. Similar to the permanent recapitalizations above, the discount rate d is used to calculate the value to eternal shareholders without the short-term debt, i.e. $v = v(d)$, and the discount rate d' is used to calculate the value with short-term debt, i.e. $v' = v(d')$.

The value to eternal shareholders with a share buyback funded with short-term debt is:

$$W_{TempDebt} = \frac{\left(v' - \frac{Debt \cdot \left(1 + InterestRate \cdot (1 - TaxCompany)\right)}{1 + d'}\right) \cdot \ (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Debt}{MarketCap}\right)}$$

Eq. 4-67

The relative value of the share buyback is:

$$\frac{W_{TempDebt}}{V} = \frac{v' - \frac{Debt \cdot \left(1 + InterestRate \cdot (1 - TaxCompany)\right)}{1 + d'}}{v \cdot \left(1 - \frac{Debt}{MarketCap}\right)}$$

Eq. 4-68

The equilibrium where the share buyback starts to increase value to eternal shareholders is:

$$W_{TempDebt} > V \Leftrightarrow MarketCap < \frac{v}{\frac{v - v'}{Debt} + \frac{1 + InterestRate \cdot (1 - TaxCompany)}{1 + d'}}$$

Eq. 4-69

These formulas are very sensitive to a small increase in the discount rate d' from d which results in a large change in the relative value and equilibrium.

In case the temporary increase in debt is deemed negligible regarding the company's risk of default on any of its debt, the discount rate can remain unchanged, that is, $d' = d$ and hence $v' = v$, so the relative value of the share buyback is:

$$\frac{W_{TempDebt}}{V} = \frac{1 - \frac{Debt \cdot \left(1 + InterestRate \cdot (1 - TaxCompany)\right)}{v \cdot (1 + d)}}{1 - \frac{Debt}{MarketCap}}$$

Eq. 4-70

The equilibrium is:

$$W_{TempDebt} > V \Leftrightarrow MarketCap < v \cdot \frac{1 + d}{1 + InterestRate \cdot (1 - TaxCompany)}$$

Eq. 4-71

4.9.2. Adding a Revaluation Term

Revaluation can instead be done by adding a term to the present value of future earnings. The value to eternal shareholders before the recapitalization is denoted v as usual. The value after the recapitalization is denoted v' and is defined as v minus the amount of debt, plus the revaluation term. The debt is subtracted because it is assumed here to equal the present value of future interest payments. The revaluation term is positive if there is a valuation benefit from increasing the debt, while the revaluation term is negative if there is a valuation penalty from increasing the debt.

The recapitalized value is:

$$v' = v - Debt + Revaluation$$

Eq. 4-72

This can be rewritten as:

$$v = v' + Debt - Revaluation$$

Eq. 4-73

The values per share currently outstanding and after dividend tax are:

$$V = \frac{v \cdot (1 - TaxDividend)}{Shares}$$

Eq. 4-74

$$V' = \frac{v' \cdot (1 - TaxDividend)}{Shares}$$

Eq. 4-75

Debt Increase & Share Buyback

When debt is increased and used entirely for a share buyback, the per-share value to eternal shareholders becomes:

$$W_{DebtIncrease} = \frac{v' \cdot (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Debt}{MarketCap}\right)}$$

Eq. 4-76

The relative value of this recapitalization is:

$$\frac{W_{DebtIncrease}}{V} = \frac{1 - \frac{Debt - Revaluation}{v}}{1 - \frac{Debt}{MarketCap}}$$

Eq. 4-77

The equilibrium where the recapitalization starts to increase value to eternal shareholders is:

$$W_{DebtIncrease} > V \Leftrightarrow MarketCap < \frac{v}{1 - \frac{Revaluation}{Debt}}$$

Eq. 4-78

This equilibrium is less than the equilibrium for cash-funded share buybacks when the revaluation term is negative, that is, when there is a revaluation penalty for increasing the debt:

$$v > \frac{v}{1 - \frac{Revaluation}{Debt}} \Leftrightarrow Revaluation < 0$$

Eq. 4-79

Debt Decrease & Share Issuance

Now consider a company that has debt outstanding which is repaid by issuing shares, that is, $Debt$ is the amount of debt to be repaid and the share issuance equals this amount which increases the number of shares accordingly. The company's value changes from being v' to v as the present value of the interest payments (assumed to equal the debt amount) is added to v' and the revaluation term is also added.

After this recapitalization, the value per share becomes:

$$W_{DebtDecrease} = \frac{v \cdot (1 - TaxDividend)}{Shares \cdot \left(1 + \frac{Debt}{MarketCap}\right)}$$

Eq. 4-80

The relative value of the recapitalization is:

$$\frac{W_{DebtDecrease}}{V'} = \frac{1 + \frac{Debt - Revaluation}{v'}}{1 + \frac{Debt}{MarketCap}}$$

Eq. 4-81

The equilibrium where this recapitalization starts to increase value to eternal shareholders is:

$$W_{DebtDecrease} > V' \Leftrightarrow MarketCap > \frac{v'}{1 - \frac{Revaluation}{Debt}}$$

Eq. 4-82

This equilibrium is less than the equilibrium for a cash-funded share buyback when the revaluation term is negative, that is, when there is a revaluation penalty for increasing the debt – and hence a revaluation benefit for decreasing the debt:

$$v' > \frac{v'}{1 - \frac{Revaluation}{Debt}} \Leftrightarrow Revaluation < 0$$

Eq. 4-83

4.9.3. Discussion

In applying these formulas, a revaluation discount rate or revaluation term must be selected, which depends on the change in risk of defaulting on the debt and hence depends on the size of the interest payments relative to the company's future earnings. However, the mathematical relationship is unclear. Furthermore, the recapitalization equilibriums are very sensitive, so a small change in capital structure requiring a small revaluation may have a large impact on the equilibriums. This means it is difficult to quantify the value of a restructuring using these formulas and more theoretical research is necessary.

However, a rule of thumb is that a share buyback for borrowed money should only be made when the market-cap is significantly below the value to eternal shareholders. If the market-cap is near the value to eternal shareholders then the recapitalization has no potential for gain but a large risk of loss of shareholder value.

Conversely, a share issuance to repay debt should only be made when the market-cap is significantly above the value to existing shareholders, otherwise the dilution will decrease the value to existing shareholders. But a share issuance to repay debt is usually only done under distress where there is no choice. Therefore companies should avoid getting into unsustainable levels of debt as it may have to be repaid from a share issuance when the share price is low and hence causing a loss to existing shareholders.

4.10. Cash Repatriation

Some companies have operations in several countries and so-called repatriation taxes must be paid when foreign cash earnings are transferred to the home country of incorporation.

4.10.1. Repatriation Value without Share Buyback

Let $v_{Repatriate_t}$ denote the value of a company to its eternal shareholders when foreign held cash is repatriated after t years at which point it is taxed and the remainder is paid out as dividends to shareholders. The repatriation tax rate in year t is denoted $TaxRepatriate_t$. Until the time of repatriation, the cash is assumed to generate compounded interest income and the annual interest rate is denoted $InterestIncomeRate$ which should include the effect of foreign tax on interest income. The present value of the interest income is calculated with the discount rate d' which should be chosen as the interest rate anyone could earn plus a premium because the cash is not directly controlled by the shareholders. The present value of the company's future earnings are denoted $v_{Earnings}$ which may also be subject to repatriation tax as discussed in section 4.10.3 below.

The repatriated value to eternal shareholders is:

$$v_{Repatriate_t} = Excess\ Cash \cdot \left(\frac{1 + InterestIncomeRate}{1 + d'}\right)^t \cdot (1 - TaxRepatriate_t) + v_{Earnings}$$

Eq. 4-84

If the foreign held cash is repatriated and paid out as dividends now, i.e. $t = 0$, then the value is:

$$v_{Repatriate_0} = Excess\ Cash \cdot (1 - TaxRepatriate_0) + v_{Earnings}$$

Eq. 4-85

The value to eternal shareholders per share currently outstanding and after dividend tax is:

$$V_{Repatriate_t} = \frac{v_{Repatriate_t} \cdot (1 - TaxDividend)}{Shares}$$

Eq. 4-86

The value of repatriating the cash in year t relative to the value of repatriating the cash now is:

$$\frac{V_{Repatriate_t}}{V_{Repatriate_0}} = \frac{Excess\ Cash \cdot \left(\frac{1 + InterestIncomeRate}{1 + d'}\right)^t \cdot (1 - TaxRepatriate_t) + v_{Earnings}}{Excess\ Cash \cdot (1 - TaxRepatriate_0) + v_{Earnings}}$$

Eq. 4-87

Future repatriation is more valuable to shareholders when the future repatriation tax rate is lower than:

$$V_{Repatriate_t} > V_{Repatriate_0} \Leftrightarrow TaxRepatriate_t < 1 - \frac{1 - TaxRepatriate_0}{\left(\frac{1 + InterestIncomeRate}{1 + d'}\right)^t}$$

Eq. 4-88

The required future repatriation tax becomes negative if repatriation is done after t years:

$$t > \frac{\log(1 - TaxRepatriate_0)}{\log\left(\frac{1 + InterestIncomeRate}{1 + d'}\right)}$$

Eq. 4-89

The lowest possible repatriation tax is zero so this means that repatriation after t years will decrease the value to eternal shareholders regardless of the repatriation tax at that point in time.

4.10.2. Repatriation Value with Share Buyback

Assume a share buyback is made now for borrowed money and the interest on the loan compounds for t years. This is one example of a repayment schedule and formulas can be derived for others. The interest rate is denoted $InterestExpenseRate$ which should include the tax effect. The present value of the borrowed amount is calculated with the same discount rate d' as used for the foreign interest income so

the present values of the interest expense and income are comparable. The present value of the borrowed amount is:

$$Buyback \cdot \left(\frac{1 + InterestExpenseRate}{1 + d'}\right)^t$$

Eq. 4-90

Further assume that the foreign held cash is repatriated after t years and used to repay the debt. The value per share currently outstanding and after dividend tax is then:

$$W_{Repatriate_t} = \frac{\left(v_{Repatriate_t} - Buyback \cdot \left(\frac{1 + InterestExpenseRate}{1 + d'}\right)^t\right) \cdot (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)}$$

Eq. 4-91

If the foreign held cash is repatriated now and a part of it is used for share buyback and the rest for dividend payout, then this is the usual value with a share buyback from Eq. 4-5 with $v = v_{Repatriate_0}$:

$$W_{Repatriate_0} = \frac{\left(v_{Repatriate_0} - Buyback\right) \cdot (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)}$$

Eq. 4-92

The relative value of such a share buyback is the same as Eq. 4-7 with $v = v_{Repatriate_0}$:

$$\frac{W_{Repatriate_0}}{V_{Repatriate_0}} = \frac{1 - \frac{Buyback}{v_{Repatriate_0}}}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 4-93

The equilibrium is the same as Eq. 4-9 with $v = v_{Repatriate_0}$:

$$W_{Repatriate_0} > V_{Repatriate_0} \Leftrightarrow MarketCap < v_{Repatriate_0}$$

Eq. 4-94

The value of a share buyback made for borrowed cash relative to the value of a share buyback made for repatriated cash is:

$$\frac{W_{Repatriate_t}}{W_{Repatriate_0}} = \frac{v_{Repatriate_t} - Buyback \cdot \left(\frac{1 + InterestExpenseRate}{1 + d'}\right)^t}{v_{Repatriate_0} - Buyback}$$

Eq. 4-95

The equilibrium is:

$$W_{Repatriate_t} > W_{Repatriate_0}$$
$$\Leftrightarrow v_{Repatriate_t} - v_{Repatriate_0} > Buyback \cdot \left(\left(\frac{1 + InterestExpenseRate}{1 + d'}\right)^t - 1\right)$$

Eq. 4-96

Although interest income is earned on the foreign held cash it is considered a cost for shareholders because the company holding excess cash for extended periods of time is inferior to the shareholders having direct control over the cash. The interest expense from borrowing money is also a cost. So the future repatriation tax must be low enough to offset both these costs if shareholder value is to be increased.

Also note that if a share buyback from borrowed or repatriated cash is to be more valuable than a dividend payout, then the equilibrium from Eq. 4-94 must be satisfied as well.

4.10.3. Repatriation of Future Earnings

When valuing a company's future earnings in foreign countries, the valuation must take repatriation tax into account. If the future earnings are expected to be repatriated immediately then the repatriation tax should merely be included in the valuation. If instead the company is expected to withhold future earnings for several years before eventually repatriating them and paying them out as dividends, then the valuation should include both the expected repatriation tax as well as an increased discount rate because the earnings will be less valuable to shareholders when they are held as excess cash for years instead of immediately being paid out as dividends.

4.11. Stock Options

Assume a company has issued stock options in the past which are now to be exercised. The question is whether the company should issue new shares or buy back existing shares to offset the diluting impact of the exercised stock options.

Let $V_{Exercise}$ denote the post-tax present value per share of the company to its eternal shareholders, with the stock options exercised but without a share buyback to offset the diluting impact. Assume the cash received by the company from the exercised stock options is paid out as dividends to shareholders. If the cash is instead retained in the company then $V_{Exercise}$ changes according to the return on equity, see section 3.8. The value to eternal shareholders is:

$$V_{Exercise} = \frac{(v + Options \cdot ExercisePrice) \cdot (1 - TaxDividend)}{Shares + Options}$$

Eq. 4-97

Let $W_{Exercise}$ denote the post-tax present value per share of the company to its eternal shareholders, with the stock options exercised and shares bought back to offset the diluting impact. The value to eternal shareholders is:

$$W_{Exercise} = \frac{\big(v + Options \cdot (ExercisePrice - SharePrice)\big) \cdot (1 - TaxDividend)}{Shares}$$

Eq. 4-98

The relative value of buying back shares to offset the diluting impact of stock options is:

$$\frac{W_{Exercise}}{V_{Exercise}} = \frac{\big(v + Options \cdot (ExercisePrice - SharePrice)\big) \cdot \left(1 + \frac{Options}{Shares}\right)}{v + Options \cdot ExercisePrice}$$

Eq. 4-99

Equilibrium is where the share buyback for offsetting the diluting impact of the stock options does not change the value to eternal shareholders. As usual, the equilibrium is given as an inequality to clarify the condition under which the value to eternal shareholders is increased from making the share buyback:

$$W_{Exercise} > V_{Exercise} \Leftrightarrow MarketCap < \frac{v + Options \cdot ExercisePrice}{1 + \frac{Options}{Shares}}$$

Eq. 4-100

4.11.1. Tax

Taxes can also be included in these formulas. For example, if the tax is applied to the difference between the stock option's exercise-price and the market-price of shares at the time of exercise, the value to eternal shareholders with a share buyback is:

$$W_{Exercise} = (v + Options \cdot (ExercisePrice - SharePrice) \cdot (1 - TaxCompany)) \cdot \frac{(1 - TaxDividend)}{Shares}$$

Eq. 4-101

If shares are not bought back but there is a similar tax benefit from the stock options, the value is:

$$V_{Exercise} = \big(v + Options \cdot (ExercisePrice - (ExercisePrice - SharePrice) \cdot TaxCompany)\big) \cdot \frac{(1 - TaxDividend)}{Shares + Options}$$

Eq. 4-102

These formulas assume there are future earnings from which the expenses of stock options can be deducted so as to result in a tax benefit.

The relative value is:

$$\frac{W_{Exercise}}{V_{Exercise}} = \frac{(v + Options \cdot (ExercisePrice - SharePrice) \cdot (1 - TaxCompany)) \cdot \left(1 + \frac{Options}{Shares}\right)}{v + Options \cdot (ExercisePrice - (ExercisePrice - SharePrice) \cdot TaxCompany)}$$

Eq. 4-103

The equilibrium is:

$$W_{Exercise} > V_{Exercise} \Leftrightarrow MarketCap < \frac{v + Options \cdot ExercisePrice \cdot (1 - TaxCompany)}{1 + \frac{Options}{Shares} \cdot (1 - TaxCompany)}$$

Eq. 4-104

4.11.2. Issuance Value

When stock options are issued e.g. to employees as part of their compensation, a non-cash expense is entered in the financial statements of the company. However, if the actual expense of stock options is considered to be the difference between the exercise price and the market price of shares at the time the options are exercised, then the actual expense cannot be known in advance and an estimate of the expense is needed for the current financial statements.

Estimating the value of a stock option can be done using the Black-Scholes formula [**52**] [**53**] which assumes stock prices follow a random walk with constant volatility, that stocks and options are correctly priced so there is no possibility for increasing expected returns without also increasing the risk, that there is unlimited borrowing ability, no transaction costs, interest rates are constant and known in advance, etc. An alternative way of estimating the value of stock options uses the so-called binomial model by Cox et al. [**54**], which also assumes capital markets are perfect and stock prices follow a random walk. These assumptions are similar to EMH and hence disputed in this treatise.

It was shown above that shares should only be bought back to offset the diluting impact of stock options if the share price is favourable, which cannot be depended upon to occur. Hence, stock options should be assumed to have a dilutive impact upon exercising, which would mean the actual expense of stock options is the present value of the decrease in all future dividends to existing shareholders.

Dividend Protected Stock Options

First assume the stock options are so-called dividend protected, that is, dividend payouts accumulate after the options have been issued and before they have been exercised so the accumulated dividends are paid out when the options are exercised. For simplicity assume the accumulated dividends are held as cash in the company and do not earn any interest.

The value of the company to its eternal shareholders, per share currently outstanding and after dividend tax, is denoted $V_{Options}$ and is defined as the value without stock options v, plus the present value of the cash paid for exercising the stock options after k years which is assumed to be paid out as dividends (or retained in the company and hence subject to the valuation formulas of section 3.8), divided by the number of shares plus options as the dilution is effective on all future dividend payouts because of the dividend protection.

The value is:

$$V_{Options} = \frac{1 - TaxDividend}{Shares + Options} \cdot \left(v + \frac{Options \cdot ExercisePrice}{(1+d)^k} \right)$$

Eq. 4-105

The value prior to dividend taxation and for all shares currently outstanding is:

$$v_{Options} = \frac{Shares}{Shares + Options} \cdot \left(v + \frac{Options \cdot ExercisePrice}{(1+d)^k} \right)$$

Eq. 4-106

The expense of the stock options to the company's existing shareholders is:

$$v - v_{Options} = \frac{Options}{Shares + Options} \cdot \left(v - \frac{Shares \cdot ExercisePrice}{(1+d)^k} \right)$$

Eq. 4-107

The stock option expense increases as the number of years k increases and the limit is:

$$\lim_{k \to \infty} v - v_{Options} = \frac{Options}{Shares + Options} \cdot v$$

Eq. 4-108

This means the upper bound for the expense of dividend protected stock options is their diluting impact.

The equilibrium where the stock options start to decrease the value of the company to its eternal shareholders is:

$$v_{Options} < v \Leftrightarrow ExercisePrice < \frac{v \cdot (1+d)^k}{Shares}$$

Eq. 4-109

An increase in the number of years k before the options are exercised causes an unbounded increase in the equilibrium exercise price.

The value of a stock option to its recipient is denoted $\tilde{V}$ and assumes the option is exercised after k years, after which the accumulated dividends are paid out and the resulting share is then assumed to be held for eternity. The value of the option to its recipient is the present value of all future dividends:

$$\tilde{V} = \frac{1 - TaxDividend}{Shares + Options} \cdot \left(\frac{1}{(1+d)^k} \cdot \sum_{t=0}^{k-1} Dividend_t + \sum_{t=k}^{\infty} \frac{Dividend_t}{(1+d)^t} \right)$$

Eq. 4-110

This can be used to decide whether it is more valuable to receive a cash bonus now or stock options to be exercised in k years, by considering whether the following holds:

$$Cash\ Bonus \cdot (1 - TaxIncome) > Options \cdot \left(\tilde{V} - \frac{ExercisePrice}{(1 + d')^k}\right)$$

Eq. 4-111

Where $TaxIncome$ is the income tax for the individual and d' is the discount rate used to calculate the present value of the exercise price for that individual, for example, d' could be the interest rate the individual could receive from low-risk government bonds.

Note that the expense of stock options to the company's existing shareholders is different from the value of the options to their recipient, because the dividends are withheld until year k before they are paid out upon exercising of the stock options, and this delay affects the value of the options to their recipient.

Not Dividend Protected

Now assume the stock options are not dividend protected, that is, dividends paid out before the options are exercised will only be paid out to existing shareholders and not be accumulated for the stock options.

Assume the options are exercised in year k just before dividends for that year are paid out. The value of the company to its eternal shareholders with the stock options is denoted $v_{Options}$ and is the present value of the dividends until year k, at which point the exercised stock options cause a diluting impact to all future dividends. The dilution factor is:

$$Dilution\ Factor = \frac{Shares}{Shares + Options}$$

Eq. 4-112

The cash received by the company from the exercising of the options is assumed to be paid out as dividends. The present value of the company to eternal shareholders is then:

$$v_{Options} = \sum_{t=0}^{k-1} \frac{Dividend_t}{(1 + d)^t} + \frac{Shares}{Shares + Options} \cdot \left(\sum_{t=k}^{\infty} \frac{Dividend_t}{(1 + d)^t} + \frac{Options \cdot ExercisePrice}{(1 + d)^k}\right)$$

Eq. 4-113

The value without options v can also be separated into two sums of dividends before and after year k:

$$v = \sum_{t=0}^{k-1} \frac{Dividend_t}{(1 + d)^t} + \sum_{t=k}^{\infty} \frac{Dividend_t}{(1 + d)^t}$$

Eq. 4-114

The expense of the stock options to the company's existing shareholders is then:

$$v - v_{Options} = \frac{Options}{Shares + Options} \cdot \left(\sum_{t=k}^{\infty} \frac{Dividend_t}{(1+d)^t} - \frac{Shares \cdot ExercisePrice}{(1+d)^k} \right)$$

Eq. 4-115

Assuming all dividends are non-negative, the upper bound of $v - v_{Options}$ is the diluting impact of the options because $\sum_{t=k}^{\infty} Dividend_t/(1+d)^t$ is upper bounded by v and decreases as k increases, and $Shares \cdot ExercisePrice/(1+d)^k$ decreases towards zero as k increases. So the upper bound of their difference is v, which is the same as for dividend protected stock options in Eq. 4-107.

The equilibrium where the stock options start to decrease the value of the company to its eternal shareholders is:

$$v_{Options} < v \Leftrightarrow ExercisePrice < \frac{(1+d)^k}{Shares} \cdot \sum_{t=k}^{\infty} \frac{Dividend_t}{(1+d)^t}$$

Eq. 4-116

The value of a stock option to its recipient assumes the option is exercised prior to the dividend payout for the k'th year and the resulting share is held for eternity. The present value of the option to its recipient is:

$$\tilde{V} = \frac{1 - TaxDividend}{Shares + Options} \cdot \sum_{t=k}^{\infty} \frac{Dividend_t}{(1+d)^t}$$

Eq. 4-117

This can be used with Eq. 4-111 to decide which of a cash bonus or stock option is more valuable.

Note that the value of an option to its recipient $\tilde{V}$ increases if dividends are withheld in the company as cash or reinvested at low rates of return and then paid out as dividends after the option has been exercised. If the cash is reinvested at high rates of return it will also cause an increase in the value of the option, but it is not necessary for the rate of return to be high in order for the value of the stock option to be increased. Section 3.8 showed that a low rate of return on retained earnings decreases value to eternal shareholders. This means a stock option that is not dividend protected misaligns the interests of shareholders and stock option recipients, by giving financial incentive for the option holders to behave contrary to the interests of shareholders in case only low returns on retained earnings are possible.

Dividends & Earnings

Dividends were used instead of earnings in the above formulas for valuation of stock options upon issuance, because the value and expense of an option depends on the actual dividends paid out after the option is exercised.

For options that are not dividend protected there can be a substantial difference in the expense and value of the options when using earnings instead of dividends in valuation, namely if earnings are retained and paid out as dividends after the stock options have been exercised.

For dividend protected options the difference between using dividends or earnings in valuation is the same as for valuation of the company's shares as described in section 3, namely that retained earnings will increase the value if the rate of return is greater than the discount rate.

Outstanding Stock Options

Stock options issued in the past and expected to be exercised in the future also affect the present value of a company to eternal shareholders, due to the expected cash payment for the options upon exercise and the dilution to the number of shares currently outstanding. The above valuation formulas for the issuance of stock options can also be used for valuing stock options that are already outstanding.

Future Issuance of Stock Options

If new stock options are expected to be issued in the future then it will affect the present value of the company to eternal shareholders, because that value depends in part on the exercise price of future stock options. The exercise price is usually the share price at the time the stock options are issued. If future share prices are fundamentally unpredictable, then the present value of a company expected to issue stock options in the future is fundamentally unpredictable as well. This means it is impossible to know if the current share price equals the present value of the company to eternal shareholders.

To illustrate this, assume new stock options are issued every year and exercised after a year just prior to dividend payouts. Let $Shares$ denote the number of shares currently outstanding and let $Options$ denote the number of options being issued now. Assume shares are not bought back so the number of shares increases by the number of options exercised each year. Further assume the ratio of options to shares is held constant so both the number of shares and options grow each year by the factor:

$$Annual\ Growth\ Factor\ of\ Options\ and\ Shares = 1 + \frac{Options}{Shares}$$

The dilution factor is the reciprocal of the growth factor:

$$Dilution\ Factor = \frac{1}{1 + \frac{Options}{Shares}} = \frac{Shares}{Shares + Options}$$

The number of shares and options, and hence the dilution, all grow exponentially over time.

Let $v_{Options}$ denote the value of the company to its eternal shareholders when options are issued and exercised in this manner for eternity. That is, added to the dividend payout each year is the cash received for the stock options which are assumed to be paid out as dividends immediately. The value is:

$$v_{Options} = \sum_{t=1}^{\infty} \left(\frac{Shares}{Shares + Options}\right)^{t} \cdot \frac{Dividend_t + Options \cdot \left(1 + \frac{Options}{Shares}\right)^{t-1} \cdot ExercisePrice_t}{(1+d)^t}$$

$$= \sum_{t=1}^{\infty} \frac{\left(\frac{Shares}{Shares + Options}\right)^{t} \cdot Dividend_t + \left(\frac{1}{Options} + \frac{1}{Shares}\right) \cdot ExercisePrice_t}{(1+d)^t}$$

Eq. 4-118

Note that this depends on the exercise prices of stock options issued in the future. The exercise price is usually just the share price at the time of issuance of the stock options. If the share price is fundamentally unpredictable then the value of the company to eternal shareholders is also fundamentally unpredictable.

As $v_{Options}$ cannot be determined accurately it would be useful to derive a lower bound that could be used to assess the value of the company regardless of how stock options are issued in the future. An obvious lower bound for $v_{Options}$ is when the exercise price of future stock options is zero, although this is an exaggeration of the expense of the stock options because they will only get exercised if the share price and hence exercise price increase over time. Unfortunately, a better bound for $v_{Options}$ is currently unknown.

A special case for the lower bound assumes $v_{Options}$ is as defined above, that the exercise price is zero and dividends grow by the rate g each year, so Eq. 13-2 gives:

$$v_{Options} = Dividend \cdot \sum_{t=1}^{\infty} \left(\frac{\left(\frac{Shares}{Shares + Options} \right) \cdot (1+g)}{1+d} \right)^t$$
$$= Dividend \cdot \left(\frac{1+g}{\frac{Options}{Shares} \cdot (1+d) + d - g} \right)$$

Eq. 4-119

The value without stock options is:

$$v = Dividend \cdot \frac{1+g}{d-g}$$

The relative value is:

$$\frac{v_{Options}}{v} = \frac{1}{1 + \frac{Options}{Shares} \cdot \frac{1+d}{d-g}}$$

Eq. 4-120

Discussion

When assessing whether share buybacks are likely to increase or decrease value to eternal shareholders, the value $v_{Options}$ which includes the impact of current and future stock options should be used instead of the value v without stock options. However, $v_{Options}$ is indeterminable as it relies on future share prices which are unpredictable. A lower bound for $v_{Options}$ could be used instead but the bound derived here is not of practical use as it is for a special case assuming the exercise price is zero, that a certain amount of options are issued annually, and dividends grow exponentially.

Therefore, when valuing share buybacks for a company, the expense of stock options as reported in the company's financial statements for prior years will merely be assumed to recur in the future. Note, however, that this too is an estimate with limited accuracy of the actual expense of the stock options.

Several important questions have been raised here concerning the valuation and effect of stock options. Stock options misalign the interests of their recipients and shareholders; the valuation of stock options is poorly understood and the accounting is inaccurate or even misleading; the value of stock options depends on the share price both at the point of issuance and exercise which may be unrelated to the work of the employee receiving the stock option. It is unclear why established companies use stock options rather than cash bonuses related to measurable performance goals for their individual employees. Stock options only seem appropriate for companies whose future is highly uncertain and who cannot afford to pay cash bonuses to their employees, but even then the value of stock options depends on the share price which may be too high or low and hence under- or over-compensate employees for their work.

4.12. Custom Valuation Formulas

This section derived formulas for valuation of standard share buyback scenarios. Formulas can be derived for special scenarios either by combining the basic formulas above or by deriving entirely new formulas. Derivation of share buyback valuation formulas is usually done by first defining the value per share without share buybacks V and the value with share buybacks W. The relative value of the share buyback is then calculated as the ratio W/V. The equilibrium where share buybacks start to increase value to eternal shareholders can be found by considering the inequality $W > V$. If W or V cannot be defined precisely it may be possible to consider boundary cases instead.

5. Non-Linearity of the Relative Value of Share Buybacks

The relative value of a share buyback changes non-linearly as demonstrated in Figure 4 and Figure 5. This can be formalized mathematically as follows.

5.1. Varying the Value to Eternal Shareholders

First consider the relative value of a share buyback W/V for varying values to eternal shareholders v. Assume the value is positive $v > 0$ so the relative value of a share buyback W/V in Eq. 4-7 is well-defined.

As v approaches positive zero, the limit of W/V is negative infinity:

$$\lim_{v \to 0^+} \frac{W}{V} = \lim_{v \to 0^+} \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = -\infty$$

Eq. 5-1

As v approaches positive infinity, the relative value of a share buyback converges to:

$$\lim_{v \to \infty} \frac{W}{V} = \lim_{v \to \infty} \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{MarketCap}{MarketCap - Buyback}$$

Eq. 5-2

The behaviour of W/V between these two extremes will now be studied.

5.1.1. First Derivative

Note that W/V is a continuous and monotonic function in the variable v when $v > 0$, which means it is also differentiable. The derivative of W/V with regards to the variable v is denoted $\partial_v W/V$ which is the rate of change of W/V around a given v when the other variables $MarketCap$ and $Buyback$ are held constant. The derivative is:

$$\partial_v \frac{W}{V} = \partial_v \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{MarketCap \cdot Buyback}{MarketCap - Buyback} \cdot v^{-2}$$

Eq. 5-3

Note that $\partial_v W/V$ is a continuous and monotonically decreasing function when $v > 0$.

As v approaches positive zero, the limit of $\partial_v W/V$ is infinity:

$$\lim_{v \to 0^+} \partial_v \frac{W}{V} = \lim_{v \to 0^+} \frac{MarketCap \cdot Buyback}{MarketCap - Buyback} \cdot v^{-2} = +\infty$$

Eq. 5-4

This means that near $v = 0$ the relative value of a share buyback W/V changes greatly when there is a small change in the value v and the other variables $MarketCap$ and $Buyback$ remain constant.

As v approaches infinity, the limit is zero:

$$\lim_{v\to\infty} \partial_v \frac{W}{V} = \lim_{v\to\infty} \frac{MarketCap \cdot Buyback}{MarketCap - Buyback} \cdot v^{-2} = 0^{+}$$

Eq. 5-5

Because $\partial_v\, W/V$ is continuous and monotonically decreasing it means that as v increases, the relative value of a share buyback W/V changes less.

5.1.2. Second Derivative

An alternative way of showing that the relative value W/V changes less as v increases is to consider the second derivative $\partial_v^2\, W/V$ which is the acceleration of W/V around a given v:

$$\partial_v^2 \frac{W}{V} = \partial_v \frac{MarketCap \cdot Buyback}{MarketCap - Buyback} \cdot v^{-2} = -2 \cdot \frac{MarketCap \cdot Buyback}{MarketCap - Buyback} \cdot v^{-3}$$

Eq. 5-6

This is negative for $v > 0$ which means the change in W/V is decelerating as v increases, that is, as the value to eternal shareholders v increases and $MarketCap$ and $Buyback$ remain constant, the relative value of a share buyback changes less.

5.1.3. Consequence

As a consequence of this non-linearity, a mispricing of $v = MarketCap - Error$ results in a decrease in the value to eternal shareholders that is relatively greater than the increase resulting from the opposite mispricing $v = MarketCap + Error$. This can also be seen from Figure 4 and Figure 5.

5.2. Varying the Buyback Amount

Now consider the relative value of a share buyback W/V for varying share buyback amounts which are denoted $Buyback$. The relative value is reprinted from Eq. 4-7 for easy reference:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}}$$

If $Buyback = 0$ then $W/V = 1$. If $Buyback = v$ and $MarketCap \neq v$ then $W/V = 0$.

As $Buyback$ approaches $MarketCap \neq v$, the limit of W/V is either positive or negative infinity:

$$\lim_{Buyback\to MarketCap} \frac{W}{V} = \lim_{Buyback\to MarketCap} \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \pm\infty$$

Eq. 5-7

The denominator $1 - Buyback/MarketCap$ approaches positive infinity as $Buyback$ approaches $MarketCap$ from below, so the numerator determines the sign of W/V. If $MarketCap < v$ then the numerator $1 - Buyback/v$ is positive as $Buyback$ approaches $MarketCap$ from below, so W/V

approaches positive infinity. If $MarketCap > v$ then the numerator is negative so W/V approaches negative infinity.

Note, however, that a negative relative value cannot actually occur because the share buyback is assumed to be made from excess cash so the buyback amount is upper bounded by v, that is, $Buyback \leq v$, and hence W/V is lower bounded by zero. Figure 3, Figure 4 and Figure 5 show W/V for $Buyback > v$ although this cannot actually occur without a recapitalization which requires different formulas for the relative value.

When $Buyback$ ranges between zero and $MarketCap$, the relative value of a share buyback W/V ranges between zero and infinity. So W/V maps a finite range onto an infinite range which means that W/V must be a non-linear function in the $Buyback$ variable.

5.2.1. Derivative

The derivative of W/V with regards to the $Buyback$ variable is denoted $\partial_{Buyback}\, W/V$ which is the rate of change of W/V around a given $Buyback$ when the other variables $MarketCap$ and v are held constant. The derivative is:

$$\partial_{Buyback}\frac{W}{V} = \partial_{Buyback}\frac{1-\frac{Buyback}{v}}{1-\frac{Buyback}{MarketCap}} = \frac{1-\frac{MarketCap}{v}}{\frac{Buyback^2}{MarketCap} - 2\cdot Buyback + MarketCap}$$

Eq. 5-8

The denominator is a 2nd degree polynomial in the variable $Buyback$ and it is positive when $Buyback$ ranges between zero and $MarketCap$ because it equals $MarketCap$ when $Buyback = 0$ and it equals zero when $Buyback = MarketCap$. The polynomial is continuous and monotonically decreasing when $Buyback$ goes from zero to $MarketCap$, so $\partial_{Buyback}\, W/V$ is monotonically increasing on that range and goes from $(v - MarketCap)/(v \cdot MarketCap)$ when $Buyback = 0$ to infinity when $Buyback = MarketCap$. The numerator $1 - MarketCap/v$ is independent of the variable $Buyback$ and is positive when $MarketCap < v$ and negative when $MarketCap > v$, so $\partial_{Buyback}\, W/V$ equals positive infinity when $Buyback = MarketCap$ and $MarketCap < v$ and it equals negative infinity when $MarketCap > v$.

5.2.2. Consequence

Altogether this means that the relative value of a share buyback W/V is a monotonic, non-linear function for varying share buyback amounts, and the effect of a share buyback on the value to eternal shareholders is greatly magnified when the share buyback amount is increased.

6. Valuation of Share Buybacks under Uncertainty

Assume the value to eternal shareholders is a stochastic variable which takes on different values according to some probability distribution. Let $\mathbb{V}$ denote the stochastic variable for the value to eternal shareholders without share buybacks.[4]

Let $\mathbb{W}$ denote the stochastic variable for the value to eternal shareholders with a share buyback, which is defined as a function of $\mathbb{V}$ using Eq. 4-6:

$$\mathbb{W} = \frac{\mathbb{V} - Buyback}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 6-1

Note that $\mathbb{V}$ and $\mathbb{W}$ are before dividend taxation and not per share, corresponding to v and w respectively.

Let $\mathbb{W}/\mathbb{V}$ denote the stochastic variable for the relative value of the share buyback which is defined as a function of $\mathbb{V}$ using Eq. 4-7:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{\mathbb{V}}}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 6-2

Similar to the non-stochastic case, the relative value is not defined for $\mathbb{V} = 0$ and it is ill-defined for $\mathbb{V} < 0$ where the formula suggests a share buyback increases value to eternal shareholders whereas the value is actually decreased. But these cases cannot arise because it is assumed the share buyback is made for excess cash so that $\mathbb{V} \geq Buyback \geq 0$ and hence $\mathbb{W} \geq 0$, otherwise the share buyback would have to be made for borrowed money which requires stochastic variants of the valuation formulas from section 4.9.

6.1. Simple Probability Distribution

Consider as a simple example a probability distribution where the value to eternal shareholders equals the market-cap plus or minus an error, with equal probability:

$$\Pr[\mathbb{V} = MarketCap - Error] = \Pr[\mathbb{V} = MarketCap + Error] = \frac{1}{2}$$

Eq. 6-3

Then the expected (or mean) value to eternal shareholders without a share buyback equals the market-cap:

$$E[\mathbb{V}] = \sum_{v} v \cdot \Pr[v] = (MarketCap - Error) \cdot \frac{1}{2} + (MarketCap + Error) \cdot \frac{1}{2} = MarketCap$$

Eq. 6-4

[4] It is customary to denote stochastic variables by capital letters but they are used for other purposes in this treatise.

The probability distribution for the value with a share buyback $\mathbb{W}$ is derived from the probability distribution for $\mathbb{V}$ and the definition of $\mathbb{W}$ in Eq. 6-1:

$$\Pr\left[\mathbb{W} = \frac{MarketCap - Error - Buyback}{1 - \frac{Buyback}{MarketCap}}\right] = \Pr\left[\mathbb{W} = \frac{MarketCap + Error - Buyback}{1 - \frac{Buyback}{MarketCap}}\right] = \frac{1}{2}$$

Eq. 6-5

The expected (or mean) value with a share buyback also equals the market-cap:

$$E[\mathbb{W}] = \sum_{w} w \cdot \Pr[w] = \frac{MarketCap - Error - Buyback}{1 - \frac{Buyback}{MarketCap}} \cdot \frac{1}{2} + \frac{MarketCap + Error - Buyback}{1 - \frac{Buyback}{MarketCap}} \cdot \frac{1}{2}$$
$$= MarketCap$$

Eq. 6-6

The probability distribution for the relative value of the share buyback $\mathbb{W}/\mathbb{V}$ is derived from the probability distribution for $\mathbb{V}$ and Eq. 6-2:

$$\Pr\left[\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{MarketCap - Error}}{1 - \frac{Buyback}{MarketCap}}\right] = \Pr\left[\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{MarketCap + Error}}{1 - \frac{Buyback}{MarketCap}}\right] = \frac{1}{2}$$

Eq. 6-7

The expected (or mean) relative value of the share buyback is:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = \sum_{w/v} \frac{w}{v} \cdot \Pr\left[\frac{w}{v}\right] = \frac{1 - \frac{Buyback}{MarketCap - Error}}{1 - \frac{Buyback}{MarketCap}} \cdot \frac{1}{2} + \frac{1 - \frac{Buyback}{MarketCap + Error}}{1 - \frac{Buyback}{MarketCap}} \cdot \frac{1}{2}$$

Eq. 6-8

By reduction it follows that the expected relative value of the share buyback is less than one whenever the squared error is positive:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] < 1 \Leftrightarrow Error^2 > 0$$

Eq. 6-9

So although the expected value with and without a share buyback are both equal to the market-cap, the expected relative value of a share buyback is less than one. The reason is that the relative value of a share buyback is non-linear, see Figure 5 and section 5. So if it turns out that the value of the company is actually $MarketCap - Error$ then a share buyback would have decreased the value to eternal shareholders relatively more than it would have been increased if the value turns out to be $MarketCap + Error$.

6.2. General Probability Distribution

Let $\mathbb{V}$ be a stochastic variable for the value to eternal shareholders without a share buyback, which is distributed according to the probability density function f satisfying:

$$\int_0^\infty f(v)\,dv = 1$$

Eq. 6-10

The expected value of $\mathbb{V}$ is:

$$E[\mathbb{V}] = \int_0^\infty v \cdot f(v)\,dv$$

Eq. 6-11

Assume the variance is positive $Var[\mathbb{V}] > 0$, otherwise the value to eternal shareholders would be known almost surely, which is contrary to the assumption that there is uncertainty about the value.

The expected value with a share buyback is calculated using the probability density function f and Eq. 6-1:

$$E[\mathbb{W}] = \int_0^\infty \frac{v - Buyback}{1 - \frac{Buyback}{MarketCap}} \cdot f(v)\,dv = \frac{\int_0^\infty v \cdot f(v)\,dv - Buyback \cdot \int_0^\infty f(v)\,dv}{1 - \frac{Buyback}{MarketCap}} = \frac{E[\mathbb{V}] - Buyback}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 6-12

This equals the market-cap whenever the expected value without a share buyback equals the market-cap:

$$E[\mathbb{V}] = MarketCap \Leftrightarrow E[\mathbb{W}] = MarketCap$$

Eq. 6-13

6.2.1. Relative Value

The expected (or mean) value of the reciprocal $1/\mathbb{V}$ is:

$$E\left[\frac{1}{\mathbb{V}}\right] = \int_0^\infty \frac{1}{v} \cdot f(v)\,dv$$

Eq. 6-14

The mean relative value of a share buyback is calculated using the probability density f and Eq. 6-2:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = \int_0^\infty \frac{1-\frac{Buyback}{v}}{1-\frac{Buyback}{MarketCap}} \cdot f(v)\,dv = \frac{\int_0^\infty f(v)\,dv - Buyback \cdot \int_0^\infty \frac{1}{v} \cdot f(v)\,dv}{1-\frac{Buyback}{MarketCap}}$$
$$= \frac{1 - Buyback \cdot E\left[\frac{1}{\mathbb{V}}\right]}{1-\frac{Buyback}{MarketCap}}$$

Eq. 6-15

6.2.2. Discrete Probability Distribution

These formulas also hold for a discrete probability distribution with expected value:

$$E[\mathbb{V}] = \sum_v v \cdot \Pr[v]$$

Eq. 6-16

The expected value of the reciprocal stochastic variable is then:

$$E\left[\frac{1}{\mathbb{V}}\right] = \sum_v \frac{1}{v} \cdot \Pr\left[\mathbb{V} = v\right]$$

Eq. 6-17

6.2.3. Equilibriums

In the non-stochastic case from section 4.3, the equilibrium market-cap below which a share buyback increases value to eternal shareholders, can be derived by directly comparing the value with and without a share buyback $W > V$ or equivalently by considering the inequality $W/V > 1$. But in the stochastic case these derivations result in different equilibriums.

Mean Equilibrium

The mean equilibrium is where the mean value with a share buyback from Eq. 6-12 equals the mean value without a share buyback from Eq. 6-11. As usual, the mean equilibrium is expressed as an inequality to clarify when the mean value is increased from a share buyback:

$$E[\mathbb{W}] > E[\mathbb{V}] \Leftrightarrow MarketCap < E[\mathbb{V}]$$

Eq. 6-18

Relative Equilibrium

The relative equilibrium is where the mean relative value of a share buyback equals one. It is calculated from Eq. 6-15 and is also known as the harmonic mean of $\mathbb{V}$. As usual, the relative equilibrium is expressed as an inequality to clarify when the mean relative value is greater than one:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow MarketCap < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]}$$

Eq. 6-19

Minimum Value

The mean and relative equilibriums are for mean outcomes of share buybacks. But a share buyback can be made at a market-cap below both the mean and relative equilibriums and still cause a loss to eternal shareholders if the actual value $\mathbb{V}$ turns out to be less than the market-cap.

If the value to eternal shareholders must be ensured to increase from a share buyback then the market-cap must be below the minimum possible value that $\mathbb{V}$ can take on:

$$MarketCap < Min[\mathbb{V}] \Rightarrow \mathbb{W} > \mathbb{V}$$

Eq. 6-20

This is not an equilibrium because equality and bi-implication do not hold.

Comparison of Equilibriums

The reciprocal function $1/\mathbb{V}$ is strictly convex because $\mathbb{V}$ is assumed to be positive. It is further assumed that $\mathbb{V}$ can take on multiple values because its variance is positive. This means the so-called Jensen inequality holds in its strict form:

$$E\left[\frac{1}{\mathbb{V}}\right] > \frac{1}{E[\mathbb{V}]}$$

Eq. 6-21

which is equivalent to:

$$\frac{1}{E\left[\frac{1}{\mathbb{V}}\right]} < E[\mathbb{V}]$$

Eq. 6-22

The left side of this inequality is the relative equilibrium from Eq. 6-19 and the right side is the mean equilibrium from Eq. 6-18, so the relative equilibrium is always less than the mean equilibrium.

By assumption $\mathbb{V} > 0$ and $Var[\mathbb{V}] > 0$ so the harmonic mean of $\mathbb{V}$ is greater than the minimum of $\mathbb{V}$:

$$Min[\mathbb{V}] < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]}$$

Eq. 6-23

The equilibriums are therefore ordered as follows:

Minimum Value < Relative Equilibrium < Mean Equilibrium

Eq. 6-24

When the value without a share buyback $\mathbb{V}$ is known with certainty so that $Var[\mathbb{V}] = 0$, then these equilibriums are all identical to the deterministic equilibrium in Eq. 4-9.

6.2.4. Increased Variance

A share buyback increases the variance of the value to eternal shareholders. The variance measures the spread of the possible values to eternal shareholders and the spread widens from a share buyback. This may be interpreted as increased uncertainty about the value to eternal shareholders as a result of a share buyback. It follows from the definition of $\mathbb{W}$ in Eq. 6-1 and the scaling property of the variance:

$$Var[\mathbb{W}] = \frac{Var[\mathbb{V}]}{\left(1 - \frac{Buyback}{MarketCap}\right)^2} > Var[\mathbb{V}]$$

Eq. 6-25

Similarly for the standard deviation:

$$Stdev[\mathbb{W}] = \frac{Stdev[\mathbb{V}]}{1 - \frac{Buyback}{MarketCap}} > Stdev[\mathbb{V}]$$

Eq. 6-26

The inequalities follow from the assumption that $MarketCap > Buyback > 0$ so that:

$$0 < \left(1 - \frac{Buyback}{MarketCap}\right)^2 < 1 - \frac{Buyback}{MarketCap} < 1$$

Eq. 6-27

6.2.5. Cumulative Distribution Function

The cumulative distribution function (CDF) for $\mathbb{V}$ is the probability that $\mathbb{V}$ is below some value v:

$$F(v) = \Pr\left[\mathbb{V} < v\right]$$

Eq. 6-28

The probability that a share buyback increases value to eternal shareholders is the probability that the market-cap is less than $\mathbb{V}$, which can be expressed in terms of the CDF:

$$\Pr[\mathbb{W} > \mathbb{V}] = \Pr[MarketCap < \mathbb{V}] = 1 - F(MarketCap)$$

Eq. 6-29

6.2.6. Comparison to Non-Stochastic Relative Value

Now compare the stochastic and non-stochastic relative value of a share buyback regardless of what the market-cap and buyback amount might be, using Eq. 6-15 and Eq. 4-7:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] < \frac{W}{V} \Leftrightarrow \frac{1 - Buyback \cdot E\left[\frac{1}{\mathbb{V}}\right]}{1 - \frac{Buyback}{MarketCap}} < \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} \Leftrightarrow E\left[\frac{1}{\mathbb{V}}\right] > \frac{1}{v}$$

Eq. 6-30

Assume $E[\mathbb{V}] = v$ and $Var[\mathbb{V}] > 0$, then the right side of this bi-implication holds because of Jensen's strict inequality in Eq. 6-21, and hence the left side holds. This means that uncertainty regarding the value to eternal shareholders causes the mean relative value of a share buyback to decrease. The difference is:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] - \frac{W}{V} = \frac{MarketCap \cdot Buyback}{MarketCap - Buyback} \cdot \left(\frac{1}{v} - E\left[\frac{1}{\mathbb{V}}\right]\right)$$

Eq. 6-31

The exact value of this difference depends on the probability distribution of $\mathbb{V}$.

6.3. Example

To demonstrate the application of the above formulas and some of their implications, consider a company whose value to eternal shareholders is a stochastic variable $\mathbb{V}$ which can take on the values 10 or 1000 with probabilities:

$$\Pr[\mathbb{V} = 10] = 0.9$$

$$\Pr[\mathbb{V} = 1000] = 0.1$$

Eq. 6-32

The expected value is calculated using Eq. 6-16:

$$E[\mathbb{V}] = \sum_{v} v \cdot \Pr[v] = 10 \cdot 0.9 + 1000 \cdot 0.1 = 109$$

6.3.1. Mean Equilibrium

The mean equilibrium from Eq. 6-18 is the mean value without a share buyback:

$$E[\mathbb{W}] > E[\mathbb{V}] \Leftrightarrow MarketCap < E[\mathbb{V}] = 109$$

So if $MarketCap = E[\mathbb{V}] = 109$ then the mean value with a share buyback is also $E[\mathbb{W}] = 109$.

6.3.2. Relative Value

The relative value of a share buyback is calculated using Eq. 6-2. For the case $\mathbb{V} = 10$ with $Buyback = 5$, the relative value is:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{\mathbb{V}}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{5}{10}}{1 - \frac{5}{109}} = \frac{109}{208} \simeq 52\%$$

For the case $\mathbb{V} = 1000$ the relative value is:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{5}{1000}}{1 - \frac{5}{109}} = \frac{21691}{20800} \simeq 104\%$$

So if $\mathbb{V} = 10$, which occurs with probability 0.9 (or 90%), then the share buyback results in a decrease in the value to eternal shareholders by about 48%. If instead $\mathbb{V} = 1000$, which occurs with probability 0.1 (or 10%), then the share buyback results in an increase in the value to eternal shareholders of about 4%.

The mean reciprocal value is calculated using Eq. 6-17:

$$E\left[\frac{1}{\mathbb{V}}\right] = \sum_{v} \frac{1}{v} \cdot \Pr[\mathbb{V} = v] = \frac{1}{10} \cdot 0.9 + \frac{1}{1000} \cdot 0.1 = \frac{901}{10000}$$

Eq. 6-33

This is used with Eq. 6-15 to calculate the mean relative value of the share buyback:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = \frac{1 - Buyback \cdot E\left[\frac{1}{\mathbb{V}}\right]}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - 5 \cdot \frac{901}{10000}}{1 - \frac{5}{109}} \simeq 58\%$$

This means the share buyback results in a large loss to eternal shareholders on average. The reason is that the relative value of a share buyback is non-linear, see Figure 5 and section 5. If it turns out the value of the company is actually $\mathbb{V} = 10$ which is less than the assumed $MarketCap = 109$, then a share buyback would have decreased the value to eternal shareholders by 48%. If instead it turns out the value is actually $\mathbb{V} = 1000$ then the value to eternal shareholders would only have increased by 4% as a result of the share buyback. The probability of the large loss is 0.9 (or 90%) while the probability of the small gain is 0.1 (or 10%) so the average outcome is a large loss to shareholders.

This shows that the mean equilibrium from Eq. 6-18 cannot be used by itself for assessing whether a share buyback is likely to increase value to eternal shareholders.

6.3.3. Relative Equilibrium

According to Eq. 6-21 the relative equilibrium is lower than the mean equilibrium and hence stronger. The relative equilibrium is calculated using Eq. 6-19 with the expected reciprocal value from Eq. 6-33:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow MarketCap < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]} = \frac{10000}{901} \simeq 11.1$$

So a share buyback made at a market-cap below 11.1 is expected to increase the relative value of the share buyback, meaning that potential losses to eternal shareholders will be relatively smaller than potential gains. But it does not mean that losses are avoided altogether.

For example, let $MarketCap = 10.5$ and $Buyback = 5$. The relative value of the share buyback is calculated using Eq. 6-2. For the case $\mathbb{V} = 10$ the relative value is:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{\mathbb{V}}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{5}{10}}{1 - \frac{5}{10.5}} = \frac{21}{22} \simeq 95\%$$

For the case $\mathbb{V} = 1000$ the relative value is:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{5}{1000}}{1 - \frac{5}{10.5}} = \frac{4179}{2200} \simeq 190\%$$

Weighting these by their respective probabilities from Eq. 6-32 gives the expected relative value:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = 95\% \cdot 0.9 + 190\% \cdot 0.1 \simeq 105\%$$

So on average the share buyback is expected to increase value to eternal shareholders. But the probability of this actually happening is only 0.1 (or 10%) in which case the value to eternal shareholders would have increased by 90% as a result of the share buyback. The probability of a loss, however, is 0.9 (or 90%) in which case the value to eternal shareholders would have decreased 5% as a result of the share buyback.

Although the relative equilibrium is stronger than the mean equilibrium, neither is sufficient for properly assessing the impact of a share buyback on the value to eternal shareholders.

6.3.4. Minimum Value

If a share buyback must be guaranteed to increase the value to eternal shareholders then the market-cap must be less than the minimum possible value that $\mathbb{V}$ can take on, see Eq. 6-20. In this case:

$$MarketCap < Min[\mathbb{V}] = 10 \;\Rightarrow\; \mathbb{W} > \mathbb{V}$$

For example, if $MarketCap = 9.5$ which is slightly below the minimum value of 10, then in case $\mathbb{V} = 10$ the relative value of the share buyback is:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{\mathbb{V}}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{5}{10}}{1 - \frac{5}{9.5}} = \frac{19}{18} \simeq 106\%$$

For the case $\mathbb{V} = 1000$ the relative value is:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{5}{1000}}{1 - \frac{5}{9.5}} = \frac{3781}{1800} \simeq 210\%$$

The expected relative value is:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = 106\% \cdot 0.9 + 210\% \cdot 0.1 \simeq 116\%$$

Both cases increase the value to eternal shareholders and on average the increase is 16%.

6.3.5. Increased Variance

Assume $MarketCap = E[\mathbb{V}] = 109$ and $Buyback = 5$. The value of a share buyback is calculated using Eq. 6-1. For the case $\mathbb{V} = 10$ it is:

$$\mathbb{W} = \frac{\mathbb{V} - Buyback}{1 - \frac{Buyback}{MarketCap}} = \frac{10 - 5}{1 - \frac{5}{109}} = \frac{545}{104} \simeq 5.24$$

For the case $\mathbb{V} = 1000$:

$$\mathbb{W} = \frac{1000 - 5}{1 - \frac{5}{109}} = \frac{108{,}455}{104} \simeq 1042.84$$

The mean value with a share buyback is calculated with the probabilities from Eq. 6-32:

$$E[\mathbb{W}] = \frac{545}{104} \cdot 0.9 + \frac{108{,}455}{104} \cdot 0.1 = 109$$

The mean squared value is:

$$E[\mathbb{W}^2] = \left(\frac{545}{104}\right)^2 \cdot 0.9 + \left(\frac{108{,}455}{104}\right)^2 \cdot 0.1 = \frac{1{,}176{,}516{,}025}{10{,}816} \simeq 108{,}776$$

The variance (or uncertainty) of the value with a share buyback is then:

$$Var[\mathbb{W}] = E[\mathbb{W}^2] - E[\mathbb{W}]^2 = \frac{1{,}176{,}516{,}025}{10{,}816} - 109^2 = 96{,}894$$

The variance of the value without a share buyback is:

$$Var[\mathbb{V}] = E[\mathbb{V}^2] - E[\mathbb{V}]^2 = 10^2 \cdot 0.9 + 1000^2 \cdot 0.1 - 109^2 = 88{,}209$$

Clearly $Var[\mathbb{W}] > Var[\mathbb{V}]$ so the uncertainty of the value to eternal shareholders increases as a result of the share buyback. The uncertainty can also be expressed in terms of the standard deviation which is the square root of the variance, $Stdev[\mathbb{V}] = 297$ and $Stdev[\mathbb{W}] \simeq 311$. Note that the uncertainty increases even though the share buyback was made at the mean equilibrium so that $MarketCap = E[\mathbb{V}] = E[\mathbb{W}]$. According to Eq. 6-25 the uncertainty increases regardless of the market-cap and buyback amount.

6.4. Implications

This proves that the dividend substitution hypothesis and the Modigliani-Miller dividend irrelevance hypothesis described in section 2.2, as well as the Efficient Market hypothesis described in section 2.1 are incorrect when there is uncertainty about the value of a company to its eternal shareholders, as there obviously is when that value depends on unknown future earnings. Because even if the stock-market had priced a stock to its expected value to eternal shareholders, a share buyback would increase the uncertainty of that value, and any potential losses from the share buyback would be relatively greater than any potential gains.

In practice, the probability distribution of the value to eternal shareholders should be considered so as to properly assess the effect of a share buyback. The greater the uncertainty about the value to eternal shareholders, the greater a margin of safety there should be between the market-cap and the average value, as suggested for investing in general by Graham and Dodd, see section 10.2.

6.5. Log-Normal Distribution

Let $\mathbb{V}_{Earnings} \sim \ln \mathcal{N}(\mu, \sigma^2)$ be a log-normal distributed stochastic variable for the present value of the company's future earnings. Then the value to eternal shareholders without a share buyback $\mathbb{V}$ consists of $\mathbb{V}_{Earnings}$ plus the excess cash, which is also known as a three-parameter log-normal variable:

$$\mathbb{V} = Excess\ Cash + \mathbb{V}_{Earnings}$$

Eq. 6-34

The excess cash is assumed to be used immediately for share buybacks or dividend payouts.

The expected (or mean) value is calculated using linearity of expectation and Eq. 13-9:

$$E[\mathbb{V}] = Excess\ Cash + E\left[\mathbb{V}_{Earnings}\right] = Excess\ Cash + e^{\mu + \frac{\sigma^2}{2}}$$

Eq. 6-35

The variance is unaffected by the excess cash and is the variance of the log-normal variable from Eq. 13-10:

$$Var[\mathbb{V}] = Var\left[\mathbb{V}_{Earnings}\right] = \left(e^{\sigma^2} - 1\right) \cdot e^{2\mu + \sigma^2}$$

Eq. 6-36

6.5.1. Mean Equilibrium

The mean equilibrium is found by inserting Eq. 6-35 in Eq. 6-18:

$$E[\mathbb{W}] > E[\mathbb{V}] \Leftrightarrow MarketCap < E[\mathbb{V}] = Excess\ Cash + e^{\mu+\frac{\sigma^2}{2}}$$

Eq. 6-37

6.5.2. Relative Equilibrium

The mean reciprocal $E[1/\mathbb{V}]$ for a three-parameter log-normal variable $\mathbb{V}$ has apparently not been derived in the literature but it can be estimated using statistical software and used with Eq. 6-19 to find the relative equilibrium. An example of this is given in section 9.8.5.

No Excess Cash

If the excess cash is zero then $\mathbb{V} = \mathbb{V}_{Earnings}$ is log-normal distributed with the two parameters μ and σ^2. The expected relative value of a share buyback can then be calculated by inserting Eq. 13-17 in Eq. 6-15:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = \frac{1 - Buyback \cdot E\left[\frac{1}{\mathbb{V}}\right]}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - Buyback \cdot \left(\frac{1}{E[\mathbb{V}]} + \frac{Var[\mathbb{V}]}{E[\mathbb{V}]^3}\right)}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 6-38

The relative equilibrium is found using Eq. 6-19 with Eq. 13-17:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow MarketCap < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]} = \frac{E[\mathbb{V}]^3}{E[\mathbb{V}]^2 + Var[\mathbb{V}]}$$

Eq. 6-39

The relative equilibrium can also be formulated in terms of the variance by rearranging the formula:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow Var[\mathbb{V}] < \frac{E[\mathbb{V}]^3}{MarketCap} - E[\mathbb{V}]^2$$

Eq. 6-40

The expected relative value and the relative equilibrium are depicted in Figure 7 for changing variables.

Comparison to Mean Equilibrium

Assume the excess cash is zero and $MarketCap = E[\mathbb{V}]$ which is the mean equilibrium from Eq. 6-37. Inserting this into Eq. 6-40 gives:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow Var[\mathbb{V}] < 0$$

But the right side never holds because the variance is always non-negative. So the inverse must hold:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] < 1 \Leftrightarrow Var[\mathbb{V}] > 0$$

Eq. 6-41

That is, when there is no excess cash and $\mathbb{V}$ is log-normal distributed with positive variance and the expected value equals the market-cap, then the expected relative value is less than 1, which means that any potential losses will be comparatively greater than any potential gains resulting from a share buyback. This can also be seen in Figure 7 and it was shown in section 6.2.3 to hold for any probability distribution.

6.5.3. Minimum Value

The minimum value that the three-parameter log-normal variable $\mathbb{V}$ can take on is the excess cash. A share buyback made at a market-cap below this value is ensured to increase the value to eternal shareholders, provided the remaining excess cash is paid out as dividends or reinvested at sufficiently high returns:

$$MarketCap < Min\,[\mathbb{V}] = Excess\ Cash\ \Rightarrow\ \mathbb{W}\ >\ \mathbb{V}$$

Eq. 6-42

6.5.4. Cumulative Distribution Function

The probability of a share buyback increasing value to eternal shareholders is the probability that the market-cap is less than $\mathbb{V}$, which can be expressed similarly to Eq. 6-29 but in terms of the cumulative distribution function F for $\mathbb{V}_{Earnings}$ defined in Eq. 13-13 and taking the excess cash into account:

$$\Pr[\mathbb{W}\ >\ \mathbb{V}] = \Pr\left[MarketCap < Excess\ Cash + \mathbb{V}_{Earnings}\right] = 1 - F(MarketCap - Excess\ Cash)$$

Eq. 6-43

6.5.5. Comparison to Non-Stochastic Relative Value

Now compare $E[\mathbb{W}/\mathbb{V}]$ to W/V. Assume $v = E[\mathbb{V}]$ and the excess cash is zero so $E[\mathbb{V}] = E\left[\mathbb{V}_{Earnings}\right]$. From Eq. 6-30 it follows that the expected relative value of a share buyback is decreased whenever the variance of $\mathbb{V}$ is positive. This can also be shown using Eq. 6-38:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] < \frac{W}{V} \Leftrightarrow \frac{1 - Buyback \cdot \left(\frac{1}{E[\mathbb{V}]} + \frac{Var[\mathbb{V}]}{E[\mathbb{V}]^3}\right)}{1 - \frac{Buyback}{MarketCap}} < \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} \Leftrightarrow Var[\mathbb{V}] > 0$$

Eq. 6-44

The difference is calculated from Eq. 6-31 and Eq. 13-17:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] - \frac{W}{V} = -\frac{MarketCap \cdot Buyback}{MarketCap - Buyback} \cdot \frac{Var[\mathbb{V}]}{v^3}$$

Eq. 6-45

Note how this difference is magnified as the variance and buyback amount increase, and as the value $v = E[\mathbb{V}]$ decreases. This is demonstrated in Figure 8.

6.6. Other Share Buyback Scenarios

Valuation of other share buyback scenarios is similar to the above when there is only one stochastic variable which is the value to eternal shareholders without a share buyback.

For example, the relative value and equilibrium of a share buyback to offset the diluting impact of stock options are calculated using Eq. 4-99 and Eq. 4-100 in which the stochastic variable $\mathbb{V}$ would be used instead of the deterministic variable v and the remaining variables $Options$, $Shares$, $ExercisePrice$ and $SharePrice$ are all known and hence the same as in the deterministic formulas. The resulting probability distribution for the relative value could then be used to determine the probability of a share buyback being more valuable than a share issuance for covering the exercised stock options.

6.7. Multiple Stochastic Variables

Share buyback valuation may have several sources of uncertainty with multiple stochastic variables. For example, valuing a share buyback relative to an alternative investment, as done for the deterministic case in section 4.7, may have an uncertain return on the investment as well as uncertainty about the company's other future earnings, which could be modelled as two separate stochastic variables.

Depending on the probability distributions and dependencies of the stochastic variables, it may not be possible to study this analytically, in which case Monte Carlo simulation may be useful for estimating the probability distribution and assessing the likelihood of e.g. a share buyback being more valuable to eternal shareholders than an alternative investment. Section 9.9 gives an example of Monte Carlo simulation with a single stochastic variable but the same principles apply to multiple stochastic variables.

7. Discount Rate

A discount rate is used in calculating the present value of a company to its eternal shareholders. There are several hypotheses on choosing the discount rate and the more popular ones are reviewed here. Recommendations are then made for choosing the discount rate used in valuation of share buybacks. As these discount rates have been derived from historical observations they may need updating eventually.

7.1. Review of Hypotheses

7.1.1. Williams

Originally, Williams [**1**] suggested the discount rate should be chosen by the individual shareholder doing the valuation because shareholders might have different views on the risks involved and the return demanded. Williams further suggested that valuation from the perspective of shareholders in general should use as discount rate the expected rate of return on various kinds of financial securities, such as long-term government bonds or an index of stocks. The method suggested here for selecting discount rates to be used in the valuation of share buybacks is an extension of Williams' ideas.

7.1.2. Modigliani & Miller

The hypothesis of Modigliani and Miller [**55**] assumes the market price of a stock equals its long-term value (similar to EMH described in section 2.1) and that stocks for different companies can be divided into classes of equivalent returns. From these and other strict assumptions it follows that the discount rate (which they call the cost of capital) and the capital structure of a company (the combination of liabilities and equity as means of funding the company's assets) are both irrelevant. However, the authors admit that capital markets do not behave according to their strict assumptions.

This hypothesis is ignored in this treatise, in part because it is based on EMH which is also ignored here, but also because it seems incorrect to consider different companies equivalent in terms of their potential returns. Although companies may be in the same industry, or even produce the same commodity, the companies will generally have operating differences that cause e.g. ROE to differ.

7.1.3. Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) by Sharpe [**56**] proposes that the cost of capital (discount rate) for an asset is the risk-free cost of capital plus a risk premium determined by multiplying the expected return of the overall market by the given asset's volatility in relation to the market. Sharpe's formulation of CAPM is vague and its application can be intricate with many estimates having to be made. For example, to determine the discount rate used for valuing a stock, one popular textbook by Koller et al. [**46**] suggests first using the interest rate of USA 10-year government bonds as the risk-free rate and then the historical risk premium of the stock market is claimed to be 4.5-5.5% which must be multiplied by a measure of volatility of the stock to the overall market, known as beta, whose estimation has many variables. Hence, CAPM is in fact an imprecise heuristic but may give the false impression of being a precise method for determining the discount rate. Another drawback of CAPM is that the risk premium of a stock is defined in terms of prices, that is, the historical price change of the stock market multiplied by the volatility of a stock's price relative to the overall market. Although many authors assert that financial risk equals price volatility to the extent where they use the two words interchangeably, it is incorrect to use that definition of risk when valuing share buybacks where shares are never to be sold again, and risk is therefore related

entirely to the degree of uncertainty about future earnings and dividends. See e.g. Buffett [**9**] and Klarman [**10**] for more extensive discussions of these issues and the absurdity of defining risk for long-term shareholders as share-price volatility.

7.1.4. Weighted Average Cost of Capital

The Weighted Average Cost of Capital (WACC) can be defined in several ways, see e.g. Nantell and Carlson [**57**]. The general idea is to summarize into one number the company's overall cost of capital (discount rate) taking into account the demanded return of all the different types of capital used for funding the company's assets and their proportion to the total capital. The intended purpose of WACC is for valuing new investments made by a company, so as to determine if the returns are sufficient to satisfy all stakeholders, not just equity holders. This means WACC is not applicable for valuing share buybacks because a single class of stock is being considered. Also note that as debt increases in relation to equity, the risk of default increases so a higher discount rate should be used in valuing the equity, but assuming the debt has lower interest rate than the demanded return on equity, the WACC decreases as debt increases in relation to equity, which again shows that using WACC as the discount rate for equity valuation is incorrect. WACC is merely mentioned here because it seems to be a common mistake to use it for equity valuation.

7.2. Relative Valuation

The *present* value of an investment can be interpreted as the *future* value of that investment relative to the *future* value of another investment. The discount rate used in calculating the present value of one investment is the rate of return on the other investment. This allows for the comparison of the expected return from investing in a company's stock relative to alternative investments such as government bonds, the overall stock market and venture companies. By comparing the expected return from investing in a specific stock to the returns of such well-known benchmarks, an assessment can be made whether the stock is mispriced concerning the risk involved and the availability of alternative investments with similar risk but different expected returns.

Risk is defined here as the degree of uncertainty concerning future returns in the form of earnings, dividends, bond yields, etc. differing from that expected or promised. Although this form of risk can be measured quantitatively by the volatility of past returns, e.g. the volatility of past earnings, the risk of future returns is more a qualitative assessment as will be seen from the case studies in section 9.

Sometimes the share buyback decision is clear almost regardless of the choice of discount rate used in valuation but in other cases the buyback decision is sensitive to the choice of discount rate, in which case the share buyback should perhaps not be made. For example, a share buyback may seem unlikely to increase value to eternal shareholders if the discount rate must be chosen to be a historically low yield on low-risk government bonds. Conversely, a share buyback for a mature and stable company may seem likely to increase value to eternal shareholders if the discount rate can be chosen as a rate of return normally associated with venture investments.

7.2.1. Tax

Discounting is used here to compare the return from investing in a company's stock relative to alternative investments such as government bonds, the overall stock market and venture companies. In general, interpreting the *present* value of an investment as the *future* value of that investment relative to the *future*

value of another investment, means that taxes must also be taken into consideration because taxes may be levied differently amongst investment types depending on a number of factors. For example, the tax-rate may depend on whether the investment is made in bonds or stocks, the duration of the holding period and the size of ownership interest. Some taxes are levied annually while others are instead levied after the full duration of the holding period. So taxes can have a significant impact on the present value of one investment relative to another. The individual investor should take this into account when calculating the present value of investments but it will now be shown to be irrelevant for share buybacks.

To better understand this, consider for example the present value of all future dividends from a stock relative to a government bond whose pre-tax yield is used as the discount rate d. If taxes on the stock dividends are levied upon payout of the dividends while taxes on the government bonds are levied annually, then the post-tax present value of the stock relative to the government bond is:

$$PostTax\ Present\ Value = \sum_{t=0}^{\infty} \frac{Dividend_t \cdot (1 - TaxDividend)}{(1 + d \cdot (1 - TaxOther))^t}$$

Eq. 7-1

If instead the tax on the government bonds is levied after the full holding period, then:

$$PostTax\ Present\ Value = \sum_{t=0}^{\infty} \frac{Dividend_t \cdot (1 - TaxDividend)}{(1 + d)^t \cdot (1 - TaxOther)} = \frac{1 - TaxDividend}{1 - TaxOther} \cdot \sum_{t=0}^{\infty} \frac{Dividend_t}{(1 + d)^t}$$

Eq. 7-2

Any taxation applied to the discount factor will lower the denominator in the present value calculation and hence increase the present value. The lower bound for the present value is therefore when there is no taxation on the discount factor.

In section 4.3 it was shown that a share buyback increases value to eternal shareholders when $MarketCap < v$, where v is the present value of all future earnings which was just shown to increase when tax is applied to the discount factor and the lower bound for v is when no tax is applied to the discount factor. So the expected rate of return from various alternative investments, such as government bonds, the overall stock market or venture companies, can be used as the discount rate in valuing a company relative to those alternative investments without adjusting for taxes or other fees levied on those alternative investments. The resulting equilibrium for share buybacks is valid for all shareholders regardless of their individual taxation on those alternative investments.

7.3. Asset Classes

There are numerous hypotheses on what determines the rate of return that can be obtained from different asset classes. The causal factors will not be discussed here but a brief overview of the relationship between different asset classes and their associated risks and returns is helpful in valuing share buybacks.

7.3.1. Return Rates

The basic tenets of return rates are perhaps easiest explained in terms of debt and interest rate. A lender allows a borrower the use of capital, such as cash, typically for a limited period of time. The lender expects

to receive the capital in return after the duration of the loan has expired. Additionally, the lender usually demands compensation in form of interest for providing the loan. If the interest is negative then the loan would be partly charity. There are several reasons why a positive interest rate is the norm. There is generally some risk that a loan may not be repaid in full or part and the lender should be compensated for taking this risk. The lender also forgoes other investment opportunities while the capital is committed to the loan, also known as opportunity cost, for which the lender should be compensated. Finally, the purchasing power of the principal amount of the loan may have decreased due to inflation by the time the loan is repaid, for which the lender should also be compensated.

Inflation is when the price of a product or service increases over time and deflation is when the price declines. Historically, most products and services in market economies have experienced inflation, although there are some exceptions, such as computers whose quality have improved exponentially but are being sold at the same or even lower price. General deflation in an economy may have negative effects such as consumers deferring their spending so economic activity decreases and eventually the price of larger assets including real estate and production plants may also decrease, which could lead to mass bankruptcies, unemployment, decrease in living standard and social unrest. Conversely, high inflation is also undesirable because it decreases the purchasing power of people's savings which in extreme cases may also lead to economic distress and social unrest. Another way of understanding inflation and deflation is to view money as a medium of storage and exchange of the value of people's labour, thus allowing for economic planning and easier exchange of goods and services in the future. When the purchase value of money changes too rapidly through inflation or deflation, the relation between labour and its resulting purchase power makes economic planning difficult, and the advantage of money as a uniform means of value storage and exchange erodes and becomes a disadvantage compared to bartering where goods and services are exchanged directly, which is very inefficient otherwise. So the aim of a nation is typically to have low inflation over time. As the actual rate of inflation is not known in advance, the interest on a loan should compensate the lender for the expected inflation during the period of the loan, otherwise the lender should expect to be able to consume less in the future than now for the amount of the loan.

Altogether this means a lender should demand interest from a loan, taking into account the expected inflation, the risk of the actual return of the loan differing from that promised, as well as the expected return on alternative investment opportunities available now or which can reasonably be expected to arise in the future. These principles of compensation for providing capital also hold for other types of investments.

When capital is committed for a finite amount of time it is typically called debt and the interest is usually determined in advance. When capital is committed indefinitely it is typically called equity and the return is a share in future earnings of the company. The financial security of debt is called a bond and the financial security of equity is called a share of stock. Bonds and shares typically also differ in their voting and payout rights, with bonds having first right to payouts and shares having the residual rights to earnings. This means bonds and stocks generally have distinct risk characteristics.

7.3.2. Benchmarks

This treatise uses three distinct classes of financial assets as benchmarks for the rate of return that can be expected from commonly available investments, which will then be used as the discount rate in valuing share buybacks relative to such alternative investments. The three classes of financial assets are:

- Government bonds of historically stable countries having had low risk of defaulting on the debt and interest payments, and low rates of return.
- Stock market index consisting of a diverse range of large and established companies which have historically had medium risk and medium rate of return.
- Venture index consisting of companies in their early stages of development which have historically had high risk and high rate of return.

The expected return and associated risk of different asset classes can be depicted as:

Venture Index > Stock Market Index > Low-Risk Government Bonds > Inflation

However, such relations are a statistical tendency, not a law of nature. Although it makes intuitive sense that venture investments are more uncertain than investment in a broad selection of established and profitable companies, the relation between the rate of return between government bonds and stocks of established companies is not always clear. For example, a company with international operations may thrive while its home country struggles to pay its debts.

Unusually profitable investment opportunities may arise if an asset is mispriced according to the expected risk and return. Qualitative insights are generally required to properly assess this. Assessing the relative risk of assets within the same overall risk class also requires qualitative insights, for example, how risky is the future of one company's earnings compared to the aggregate earnings of a broad stock market index, if the shares are owned for eternity? The credit ratings of companies could be used for this but they too rely on qualitative assessments, and credit ratings are not issued for all companies.

In practice, the valuation of share buybacks requires both quantitative and qualitative assessments. This treatise gives the formulas used in the quantitative assessment and describes the limits of their applicability which then requires qualitative assessment.

7.4. Government Bonds

Government bonds from stable countries are typically considered to have low risk of default on the promised return, but even they should not be considered entirely risk-free, see e.g. Reinhart and Rogoff [**58**] for a history of government defaults.

For use in valuation of share buybacks relative to government bonds, the discount rate would ideally be chosen as the current yield (annual rate of return) on a government bond with no maturity, that is, with an infinite life. However, government bonds usually have finite maturity periods that will have to be used instead, so discounting should model the repeated re-investment of the returns from government bonds in new government bonds. Hence, the current yield on government bonds of finite life is insufficient and future yields must be estimated somehow. One way of doing this is to extrapolate the past.

Figure 9 shows historical yields on long-term USA government bonds for the period April 1953 to January 2012. The statistics in Table 1 show the arithmetic mean annual yield on 10-year USA government bonds was 6.2%, geometric mean 5.7% and standard deviation 2.7%. Figure 10 shows the difference in yields of USA government bonds with 10, 20 and 30-year maturity periods and Table 1 shows that the 20 and 30-year maturity periods typically have yields of 0.2-0.3% (percentage points) more than the bonds with 10-year maturity. According to the data compiled by Homer and Sylla [**59**] for the longer period 1798-2005 as shown in Figure 11 (data for 2006-2012 is from the Federal Reserve), the annual yield on USA government bonds with various maturity periods (mostly long), terms and taxation had arithmetic mean about 4.6%, geometric mean 4.2% and standard deviation 2%. Considering all of this it seems reasonable to estimate the future yield on long-term USA government bonds to be 5% on average.

Different approaches are useful for valuing share buybacks relative to the expected return on government bonds. For example, the historical lowest interest rate on low-risk, long-term government bonds (2% for USA according to Table 1) can be used as the discount rate and is useful for demonstrating that a given market-cap of a company seems too high to merit share buybacks because it would mean the future of the company is more certain than low-risk government bonds at their historically lowest yield. Alternatively, the current yield on low-risk, long-term government bonds can be used as the discount rate for the first, say, 10 or 20 years of valuation, after which the expected yield on low-risk, long-term government bonds (5% for USA) can be used as the discount rate for eternity. This assumes not only that the future will resemble the past, but also that these are so-called zero coupon bonds, meaning they do not pay annual interest (called coupons) and the price of the bond is instead set lower than the face value which will be received at maturity. In reality, long-term government bonds typically pay interest semi-annually so a more accurate model would take into account the re-investing of these interest payments in new government bonds which would likely have an interest rate that would gradually approach the historical mean over time. However, the simple two-stage model is deemed sufficient for the purpose of valuing share buybacks so as to assess whether they are likely to increase or decrease value to eternal shareholders.

7.5. Stock Market

A stock market index is a subset of the market's stocks selected according to certain rules, for example, an index could contain stocks of companies in a certain industry, of a certain size, or in a certain country.

The present value of a company's shares relative to the return that could be obtained from investing in a broad stock market index is calculated using the expected rate of return for that stock market index as the discount rate.

7.5.1. S&P 500

The Standard & Poor's 500 (S&P 500) stock market index is used here and consists of 500 large companies traded on the stock market in USA and operating in a wide variety of industries including energy and utility, financial services, health care, information technology, heavy industry, consumer products, etc. The S&P 500 index may be used as a proxy for the entire stock market in USA as it covers about 75% of that market.[5]

[5] S&P 500 Fact Sheet, retrieved April 11, 2013: www.standardandpoors.com/indices/articles/en/us/?articleType=PDF&assetID=1221190434733

7.5.2. Extrapolation of Total Return

A simple approach to forecasting future returns of the stock market is to extrapolate its historical total return, that is, the annual rate of return from capital gains and reinvestment of dividends shown in Figure 12 for the period 1983-2011. But this approach is flawed because it ignores the current price level of the stock market relative to the future earnings of the underlying companies, hence implicitly assuming that the stock market is appropriately priced relative to its future earnings.

7.5.3. Equity Risk Premium

Another approach to forecasting future stock market returns is to use an Equity Risk Premium (ERP) derived from historical observations of stock market returns relative to low-risk government bonds. Then adding the historical average ERP to the current yield on government bonds gives the expected future rate of return of the stock market. This is an attractive technique for estimating future stock market returns because it is simple and uses two known numbers to estimate an unknown number. Several variations to ERP estimation have been proposed for taking into account macro-economic indicators, supply and demand, etc., see e.g. Hammond et al. [**60**] and Damodaran [**61**]. Unfortunately, the fundamental idea of ERP is flawed, firstly because it is incorrect to use averages in valuations with non-linear calculation and secondly because the ERP changes unpredictably over time, see Pedersen [**62**].

7.5.4. Value Yield

There is a better method for estimating the rate of return that can be expected from investing in a stock market index at its current market price and holding the shares for eternity. Consider the stock market index to be a single large conglomerate consisting of all the constituent companies of the index. Then estimate the value yield of this conglomerate given the current market price of its shares, that is, the index price. Recall from section 3.7 that the value yield is the annualized rate of return of all future dividend payouts when the shares are bought at the current market price and held for eternity.

The accumulation of equity capital and the resulting dividend growth is modelled and Monte Carlo simulated using historical data for the S&P 500 by Pedersen [**62**]. This means a wide range of changes in the macro economy, government bond yields, equity risk premium, etc. are taken into account through the use of historical data. Because the exact sequence of scenarios that will unfold in the future is unknown, the value yield of the S&P 500 index is a stochastic variable which turns out to be approximately normal distributed with mean and standard deviation depending on the current P/Book ratio of the S&P 500 index:

$$E[Value\ Yield] = \frac{9.1\%}{P/Book} + 4.4\%$$

Eq. 7-3

$$Stdev[Value\ Yield] = \frac{0.7\%}{(P/Book)^{1.34}} + 0.2\%$$

Eq. 7-4

For example, when the S&P 500 has $P/Book = 2.3$ the value yield mean and standard deviation is:

$$E[Value\ Yield] = \frac{9.1\%}{2.3} + 4.4\% \simeq 8.4\%$$

$$Stdev[Value\ Yield] = \frac{0.7\%}{2.3^{1.34}} + 0.2\% \simeq 0.4\%$$

That is, an investor who bought shares in the S&P 500 stock market index at a P/Book ratio of 2.3 could expect an annualized rate of return around 8.4% with standard deviation about 0.4%.

Table 2 shows the minimum and maximum $E[Value\ Yield]$ and $Stdev[Value\ Yield]$ for each year in the period 1983-2011, calculated using Eq. 7-3 and Eq. 7-4 with each year's minimum and maximum P/Book ratios from Figure 13. During the period 1983-2011 the average $E[Value\ Yield]$ was about 8.3% and the average $Stdev[Value\ Yield]$ was about 0.4%. Because the standard deviation is low, the mean value yield may be used as the discount rate in present value calculations thus giving a small error, see section 9.9.6.

Note that for a short period during 1984 the USA government bond yield reached 13.5% according to Figure 9 while the maximum mean value yield for the S&P 500 index was just above 12% according to Table 2, so at that point in time investors seemed to demand a higher rate of return from lending money to the government of USA than the return they might expect from a broad range of companies in USA. Also note that during year 2000 the USA government bond yield averaged 6% while the minimum mean value yield for the S&P 500 was 6.2% according to Table 2, so the annual rate of return that could be expected from holding the S&P 500 index for eternity was similar to the yield on USA government bonds.

7.5.5. Share Buybacks

In the estimation of the value yield for the S&P 500 index in Eq. 7-3 and Eq. 7-4 it is assumed that earnings are either used to pay dividends, or fund growth through retained earnings, or used for share buybacks at equilibrium share prices. However, share buybacks have historically been increased relative to dividends when the share-price was high, which means the value yield was decreased, see section 8.

7.5.6. Other Stock Market Indices

This section estimated the value yield for the broad S&P 500 stock market index but other stock market indices could be considered instead. For example, in valuing share buybacks for a company, the discount rate could be chosen as the value yield for an index of companies in the same industry.

7.6. Ventures

Ventures are companies at various stages of development before they have become profitable and can fund their development through retained earnings. The investor has high risk of not receiving future payouts from venture investments, either in the form of dividends or through the sale of the ownership stake, and there is high uncertainty of the size of any payout. Venture investors use different valuation methods and assessments of risk, see e.g. the study by Reid and Smith [**63**] who interviewed venture investors and found their discount rates ranged between 20-90% depending on the investor's preferences and the company's stage of development. This is the discount rate used in valuation so as to determine the size of the ownership stake an investment will buy. More relevant when valuing share buybacks relative to venture investments is the return that can be expected from diversified venture investments over time.

7.6.1. Dow Jones Venture Capital

The Dow Jones Venture Capital (DJVC) / Sand Hill index in Figure 16 covers approximately 18,000 venture companies for the period 1991-2010. Of those companies 11,000 have exited the index for the following reasons: 15% of the companies became publicly traded, 15% were acquired for more than had been invested, 20% were acquired for less than had been invested, and 50% became worthless. The small probability of success is the reason venture investors use high discount rates in their valuations so they get larger ownership stakes in the few venture companies that do eventually become profitable investments.

For the period 1991-2010 the DJVC had a geometric mean rate of return about 16%. This is the annual rate of return an investor would have made from investing in all venture companies in the index in proportion to their market-cap, and re-investing all payouts from public offerings and acquisitions into the remaining venture companies. This is before taxes and fees for the venture capital funds through which investments are typically made.

The total return of ordinary stock market indices was criticized in section 7.5 for not taking into account the price of the index relative to the expected earnings of the constituent companies of the index. The solution for stock market indices was to consider the value yield instead. But the value yield cannot be estimated for venture companies because they typically do not have any earnings and the financial data for their assets, equity, etc. are not publicly available for the DJVC index either. So the historical total return of the DJVC index must be used.

The duration of the DJVC index is 1991-2010 which is short and the returns are highly volatile with the monthly returns having arithmetic mean 1.69%, geometric mean 1.27% and a comparatively high standard deviation of 9%. The period also includes the so-called Dot-Com bubble around year 2000 where the market price increased rapidly and then collapsed for both venture and established public companies in certain high-tech industries. The period also includes the severe financial crisis of 2008-2009. This volatility of the DJVC index demonstrates the inherent difficulty of predicting future earnings of venture companies, especially because the transactions underlying the volatile DJVC index are likely made by investment professionals with experience valuing venture companies.

During the period 1991-2010 covered by this data for the DJVC index, the yield on long-term USA government bonds ranged between 2-8% yet prior to that in the first half of the 1980's it was 10-15%, see Figure 9. Obviously, in times when government bonds with comparatively lower risk have high returns, a venture investor will use a correspondingly higher discount rate when valuing venture companies, thus obtaining larger ownership stakes and higher expected rates of return. It would be desirable to estimate the future return of the DJVC index from the current yield on USA government bonds plus a risk premium based on the historical difference between the returns of the DJVC index and USA government bonds, but according to Pedersen [**62**] there is no predictable relation between these two asset classes.

It is not possible to make venture investments for the large amounts that can be used for share buybacks because the stock market for venture companies is too small, e.g. in 2010 there were USD 22b invested in

venture companies in USA[6] while the companies in the S&P 500 stock market index bought back shares for almost USD 200b net of share issuance.

Nevertheless, it is sometimes helpful to use historical rates of return for a venture index in the valuation of share buybacks to demonstrate the likelihood of a share buyback increasing value to eternal shareholders.

[6] National Venture Capital Association (NVCA), Thomson Reuters, Yearbook 2011, www.nvca.org

Case Studies

8. Share Buyback Trends

Before studying individual companies, consider Figure 14 which shows various financial ratios for the S&P 500 stock market index. First note that the ratio of equity to assets has steadily decreased from 29% in 1983 to 16% in 2008, which means the companies have increasingly funded their assets from liabilities rather than equity capital. Year 2008 was the time of a large financial crisis and stock market crash where the companies then made large share issuances to increase their equity capital to become 21% of assets in 2010. The figure also shows share buybacks and dividends in relation to earnings. During the period 1983-2011 dividends have decreased overall in relation to earnings while share buybacks net of issuance have increased in relation to earnings, although irregularly.

Figure 15 shows a scatter-plot of the P/Book ratios and share buybacks net of issuance relative to earnings. This suggests there is a tendency for companies to increase share buybacks as the P/Book increases. The P/Book ratio measures the market price of the shares relative to an estimate of their value to shareholders.

For example, Figure 14 shows the companies of the S&P 500 increased their share buybacks during 1994-1998 and according to Figure 13 the P/Book also increased during those years to reach its highest level in 1999 for the entire period 1983-2011. According to Table 2 the mean value yield of the S&P 500 was slightly above 6% during 1999-2000 and according to Figure 9 this was only slightly above the yield on USA government bonds. This implies that the future earnings of the S&P 500 were only slightly more risky than the yields on USA government bonds, or that the earnings of the S&P 500 would grow more than they had historically, or that the S&P 500 index was overpriced. In the following years the S&P 500 price decreased as did the yield on USA government bonds (which means the bond prices increased).

Another example is during 2003-2007 where the companies of the S&P 500 significantly increased their share buybacks while the P/Book ratio was somewhat above its average for the 28 year period, but in the stock market crash of 2008-2009 where the P/Book ratio greatly decreased, companies found it necessary to increase their equity capital by selling new shares rather than buying back shares.

As shown theoretically and empirically in this treatise, the strategy that managers should employ when either buying back or issuing shares is the opposite of what managers seem to have been doing in the recent past, if shareholder value is to be increased or even preserved.

9. Case Studies

The following are real-world case studies of share buyback valuations and analyses. The aim is to give the reader an understanding of how to apply the theory of share buyback valuation. Basic valuation models are used in order to focus on the aspects of share buyback valuation rather than the intricacies of accounting and earnings forecasting. Qualitative considerations have been omitted as they are subjective opinions and hence unsuitable for scientific treatment. Legal implications and the trading volume of stocks are ignored in most cases. With the exception of section 9.1, the names of the responsible corporate managers have been omitted as the purpose here is not to criticise or commend individuals.

Market-Cap Estimates

In these case studies the lowest and highest market-cap for a year are typically estimated by multiplying the weighted average basic number of shares outstanding with the lowest and highest share-price for that year. This is a convenient way of estimating the lowest and highest market-cap from readily available data, but in a year when a company issues or buys back shares it may cause the market-cap estimates to be imprecise, but the error is deemed acceptable because the market-cap estimates are typically only used to calculate various ratios where a low precision is required.

Share Issuance

In the financial data of these case studies the share issuance does not include any tax effects of exercised stock options. Although share issuance is often a result of stock option exercising, the accounting is complex and imprecise as described in section 4.11, and the share issuance data is typically only used here to give an indication of the approximate relation between share issuance and buyback.

9.1. Berkshire Hathaway

The company *Berkshire Hathaway* has a long history and was originally a textile producer. In 1964 Warren Buffett became the manager of the company after having accumulated a large share of the company's stock. Since then Buffett has used the company as his main investment vehicle and has built it into a large holding company with many subsidiaries in diverse industries. According to the company's financial report for 2011[7] the main segments by revenue were: Insurance was almost 26% of revenues, wholesale distribution and logistics were 23% of revenues, railroad almost 14% of revenues, and energy almost 8% of revenues. Within the insurance segment there is some diversification as different kinds of insurance are underwritten. The remaining 29% of revenues are highly diversified.

Buffett is the only manager referred to by name in this treatise for several reasons: Firstly because Buffett's philosophy on investing and share buybacks has greatly influenced the formal theory developed in this treatise and that must be acknowledged. Buffett is a well known investor and businessman of his time and knowing about his investment and business philosophy may also be helpful in understanding the underlying ideas of the treatise. Secondly because Buffett has set a good example that all managers should follow regarding the level of detail in public communications on share buyback policy.

9.1.1. Share Buyback Policy

On September 26, 2011 Berkshire Hathaway announced its policy for their first ever share buyback:[8]

> *"Our Board of Directors has authorized Berkshire Hathaway to repurchase Class A and Class B shares of Berkshire at prices no higher than a 10% premium over the then-current book value of the shares. In the opinion of our Board and management, the underlying businesses of Berkshire are worth considerably more than this amount, though any such estimate is necessarily imprecise. If we are correct in our opinion, repurchases will enhance the per-share intrinsic value of Berkshire shares, benefiting shareholders who retain their interest.*
>
> *Berkshire plans to use cash on hand to fund repurchases, and repurchases will not be made if they would reduce Berkshire's consolidated cash equivalent holdings below USD 20 billion. Financial strength and redundant liquidity will always be of paramount importance at Berkshire.*
>
> *Berkshire may repurchase shares in open market purchases or through privately negotiated transactions, at management's discretion. The repurchase program is expected to continue indefinitely and the amount of purchases will depend entirely upon the levels of cash available, the attractiveness of investment and business opportunities either at hand or on the horizon, and the degree of discount from management's estimate of intrinsic value. The repurchase program does not obligate Berkshire to repurchase any dollar amount or number of Class A or Class B shares."*

This press release informed shareholders about two things imperative to the assessment of a share buyback programme: 1) The price limit below which shares may be bought back, measured relatively to some simple accounting numbers available to everyone. The limit given here was relative to the book-value

[7] Form 10-K annual report for 2011 filed with US SEC:
www.sec.gov/Archives/edgar/data/1067983/000119312512079022/d280149d10k.htm
[8] Form 8-K filed with US SEC on September 26, 2011:
www.sec.gov/Archives/edgar/data/1067983/000119312511256263/d236452dex991.htm

of the shares, which will be analysed below, but it could have been any measure that is simple and clear for shareholders to evaluate. 2) How the share buybacks will be financed and what it means to the liquidity and solvency of the company.

The share buyback criteria are formulated in general terms so they will be applicable indefinitely unless the company changes fundamentally. Two exceptions should be noted, however. First, the required cash holdings are set to USD 20b but will presumably increase if the company grows, thus requiring a reformulation of the share buyback policy. Second, as shown below, the company has large holdings of financial securities, such as bonds and shares of stock of other companies, whose market prices fluctuate and hence affect the company's assets and equity. If, for example, the prices of those financial securities increase significantly beyond their likely long-term value, then the assets and equity of Berkshire Hathaway will also be significantly higher than their likely long-term value. This means the limit at which the company will make share buybacks, which is defined in terms of its market-cap in relation to its equity, should be adjusted so as to reflect the true long-term value of these financial securities and their impact on the company's equity.

Share Buyback Philosophy

Buffett has detailed his philosophy on share buybacks in several of his letters to shareholders, e.g. in 1999:[9]

> *"There is only one combination of facts that makes it advisable for a company to repurchase its shares: First, the company has available funds – cash plus sensible borrowing capacity – beyond the near-term needs of the business and, second, finds its stock selling in the market below its intrinsic value, conservatively-calculated. To this we add a caveat: Shareholders should have been supplied all the information they need for estimating that value. Otherwise, insiders could take advantage of their uninformed partners and buy out their interests at a fraction of true worth. (…) The business "needs" that I speak of are of two kinds: First, expenditures that a company must make to maintain its competitive position (…) and, second, optional outlays, aimed at business growth, that management expects will produce more than a dollar of value for each dollar spent (…). When available funds exceed needs of those kinds, a company with a growth-oriented shareholder population can buy new businesses or repurchase shares. If a company's stock is selling well below intrinsic value, repurchases usually make the most sense. (…) Now, repurchases are all the rage, but are all too often made for an unstated and, in our view, ignoble reason: to pump or support the stock price. The shareholder who chooses to sell today, of course, is benefitted by any buyer, whatever his origin or motives. But the continuing shareholder is penalized by repurchases above intrinsic value. Buying dollar bills for $1.10 is not good business for those who stick around. (…) Sometimes, too, companies say they are repurchasing shares to offset the shares issued when stock options granted at much lower prices are exercised. This "buy high, sell low" strategy is one many unfortunate investors have employed – but never intentionally! Managements, however, seem to follow this perverse activity very cheerfully. (…) Just because stock has been issued to satisfy options – or for any other reason – does not mean that stock should be repurchased at a price above intrinsic value."*

[9] Letter to shareholders of Berkshire Hathaway for 1999: www.berkshirehathaway.com/letters/1999htm.html

Buffet's philosophy is reminiscent of Graham's [**2**] [**3**], which is not a coincidence as Buffett was Graham's student. However, Graham expressed reservations against share buybacks particularly if the shareholders were not fully informed (as did Buffett above) or if the company discontinued dividends during times of financial crises to make share buybacks instead, thus forcing shareholders who needed the dividend income to sell their shares at too low prices for the benefit of the remaining shareholders.

9.1.2. Methods of Analysis

Two methods are available for assessing whether a share buyback is likely to increase or decrease value to eternal shareholders. First, according to Eq. 4-9 the present value of the company to its eternal shareholders, calculated with an appropriate discount rate, is the equilibrium below which a share buyback increases value to eternal shareholders. Second, according to Eq. 4-10 the value yield of the company can be compared directly to the same discount rate, because the value yield is the rate of return obtained from buying shares of the company at the given market price and holding the shares forever. The discount rate used in both cases is the rate of return on an alternative investment with similar risk characteristics. If the discount rate is constant then the two methods are equivalent.

The value yield is first considered because it results in simple comparisons for this particular company, although it cannot be used to calculate a relative value which is then done using present value calculations.

9.1.3. Value Yield Analysis

The company's share buyback policy quoted above, states that the price limit for making share buybacks is:

$$MarketCap \leq 110\% \cdot Equity$$

As the company has not announced an intention to liquidate and distribute the proceeds to shareholders, it must mean the company's value to shareholders derives from its future earnings, and as that value is formulated in terms of equity, management must believe future earnings are directly related to the equity.

It will be assumed in the following that the company's ROE is constant for eternity, although it has in fact been volatile historically and decreased substantially in recent years from its long-term average, as described below.

According to Eq. 3-42 the value yield of a company is between its earnings yield and ROE, depending on how much of future earnings are retained and invested in additional assets so as to increase future earnings of the company. The earnings yield for this company, at the price limit where share buybacks can be made, is derived using Eq. 3-33:

$$MarketCap \leq 110\% \cdot Equity \Rightarrow Earnings\ Yield \geq \frac{ROE}{110\%}$$

As the value yield ranges between the earnings yield and ROE when ROE is assumed to be constant, it means the value yield is at least ROE/110% when the market-cap is at or below 110% of the equity.

Excess Cash & Investments

The company has substantial holdings of cash and financial securities, some of which are needed to amply cover the expected liabilities of the company's large insurance operations, and some of which are likely in

excess of what is needed for such operations. Historically, the company has had large compounded returns on its investments in financial securities, so this valuation will not estimate and separate excess cash and financial securities but instead assume the company will continue to get returns from them.

Equity Change & ROE

The company's large holdings of financial securities still complicate the valuation though. For example, the market price for the company's holdings of financial securities in its insurance segment was almost USD 95b in 2008, while it was USD 118b in 2009. Compare this to total assets of USD 297b and equity of USD 131b in 2009.[10] A part of the annual change in the company's holdings of financial securities is due to changes in the market price of those financial securities. This change does not constitute actual profit or loss if unrealized, that is, if the financial securities are not sold the change in their market price is not reflected in the earnings statement of the company. However, the change is still reflected in the assets of the balance sheet, so a similar change is needed in the equity of the balance sheet, which is detailed in the financial statements as so-called other comprehensive income. For example, in 2007 the reported net income was USD 13.2b and other comprehensive income was USD (1.4)b, in 2008 net income was USD 5b and other comprehensive income was USD (17.3)b, and in 2009 net income was USD 8.1b and other comprehensive income was USD 13.7b. Although this was a particularly volatile period, the change in the company's equity has historically been significantly affected by changes in market price of its holdings of financial securities, changes which were not reflected in the earnings statement.

Because investing can be expected to remain a significant part of the company's future activities, valuation must use the comprehensive income instead of the reported net income so as to include the unrealized investment profit or loss. ROE is then defined as the comprehensive earnings for a year divided by the equity at the end of the previous year.

It was shown above that the company's value yield and hence share buyback equilibrium are related to its future ROE. When estimating the future ROE, the analysis should ideally involve the company's publicly disclosed holdings of financial securities, as these constitute a significant part of the company's assets and will hence affect ROA and ROE. Especially the holdings of stock in other companies may be mispriced and hence affect the share buyback equilibrium. However, such detailed analysis is left to the reader as the aim of this treatise is to demonstrate the general application of the share buyback theory and formulas rather than making detailed valuations with intricate accounting adjustments.

Historical ROE

In the period 1967-2010 the company has never made a dividend payout nor any share buybacks. In the period 1964-2010 the equity per share has grown at an annually compounded rate of 20.2% according to the letter to shareholders[11], but the growth-rate has decreased substantially in recent years. For the 15 years ending in fiscal year 2010 the annual growth rate in equity per share was 13.4%, for the 10 years ending in 2010 it was 9%, and for the 5 years ending in 2010 it was 10%. The company's management has candidly stated throughout the years that the equity growth rate would eventually decrease, meaning the

[10] Form 10-K annual report for 2009 filed with US SEC:
www.sec.gov/Archives/edgar/data/1067983/000119312510043450/d10k.htm
[11] Letter to shareholders of Berkshire Hathaway for 2010:
www.berkshirehathaway.com/letters/2010ltr.pdf

company would either invest retained earnings at a lower rate of return, or that its portfolio of existing investments would have a lower rate of return, or the company would start making dividend payouts and share buybacks such as those described in the policy quoted above.

A part of the reason that the company's comprehensive earnings and hence equity growth are volatile, is because of the company's large insurance segment and the volatile earnings associated with that industry. For example, in 2010 the reported net income from the insurance segment was 41% of the aggregate net income while it was only 29% in 2011. Furthermore, the company's large holdings of financial securities with volatile market prices cause volatile comprehensive earnings, as previously mentioned.

For this company, ROE is defined as the comprehensive earnings divided by the equity at the end of the previous year. As there were previously no dividend payouts or share buybacks, it means all comprehensive earnings were retained. Hence, the ROE is identical to the growth in equity per share for this past period.

Comparison to S&P 500

In assessing whether the company's share buybacks are likely to increase or decrease value to eternal shareholders, its value yield should be compared to the rate of return that can be expected from alternative investments of similar risk characteristics. Due to the diversified subsidiaries of the company, the obvious basis for comparison is with the diversified S&P 500 stock market index.

The value yield of the S&P 500 index is the rate of return that can be obtained from buying shares in that index and holding them for eternity. The value yield is not known in advance and must therefore be estimated. A stochastic model was used to estimate the probability distribution of the S&P 500 value yield which resulted in Table 2. For simplicity the mean value yield will be used here which ranged between 8.4-9.3% in 2011.

The value yield of the company Berkshire Hathaway was estimated above to be at least ROE/110% at the limit where the company would make share buybacks. If the risk of the actual value yield differing from this estimate is similar to the risk of the actual value yield of the S&P 500 index differing from its estimate, then the value yields of the two should be equal. This means Berkshire Hathaway's ROE should be 110% of the value yield estimates for the S&P 500 index, so the ROE should be at least 9.2-10.2%. In other words, for it to be likely that the share buybacks of Berkshire Hathaway will increase value to its eternal shareholders when the shares are bought back at a price less than 110% of its equity per share, the company must have a ROE of at least 9.2-10.2%. A range is given for the ROE because the value yield estimate of the S&P 500 changed during the year as the index price changed. Historically the ROE of Berkshire Hathaway has been around 20% although it has been considerably less in recent years averaging 9-10%. Whether the company's future ROE will be sufficient for the share buybacks to increase value to eternal shareholders is a qualitative assessment left to the reader.

Result of Share Buyback

On September 26, 2011 the company announced its policy and intention for making share buybacks. At this point in time the company's most recent quarterly financial report had been published on August 5, 2011[12].

[12] Form 10-Q filed with US SEC for 2nd quarter of 2011: www.sec.gov/Archives/edgar/data/1067983/000115752311004793/a6813983.htm

In this report the balance sheet date is June 30, 2011 where the shareholders' equity is listed at USD 163b. At this date there were 1.65m shares outstanding[13]. On September 23, 2011, the trading day prior to the company announcing its share buyback plan, the share price was about USD 100k which meant a market-cap about USD 165b. On September 26, 2011, after the share buyback plan had been announced, the share price increased to about USD 108k giving a market-cap about USD 178b, just below the limit market-cap of 110% of equity which was about USD 179b at that time. The share price was below the limit for only a few days and has steadily increased since then to become about USD 130k at the time of this writing in September 2012. According to the annual report for 2011 the company had bought back 633 shares for a total amount of USD 67m in these few days. As this is a tiny fraction of the 1.65m outstanding shares, the impact on the value of the company to its eternal shareholders is negligible. However, the exact magnitude of the impact on shareholder value cannot be calculated using value yield analysis because the value yield is unchanged from a share buyback, see section 4.3.8. Present value analysis is needed for this.

9.1.4. Present Value Analysis

The company's management believed the shares were worth considerably more than 110% of their book-value, which was the limit at which share buybacks could be made. Assume, for example, that the value to eternal shareholders was actually 120% of book-value so $v = USD\ 195.6b$ because the book-value was USD 163b on June 30, 2011.

The relative value of a share buyback is calculated using Eq. 4-7. For a share buyback of USD 67m at a market-cap of USD 178b, the relative value is:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 67m}{USD\ 195.6b}}{1 - \frac{USD\ 67m}{USD\ 178m}} \simeq 100.0034\%$$

That is, the share buyback would have increased value to eternal shareholders by about 0.0034% if the value to eternal shareholders was indeed $v = USD\ 195.6b$ as assumed here.

If the share buyback amount had been more substantial, say, USD 5b, then the relative value would be:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 5b}{USD\ 195.6b}}{1 - \frac{USD\ 5b}{USD\ 178m}} \simeq 100.26\%$$

That is, the share buyback would have increased the value to eternal shareholders by about 0.26%.

9.1.5. Conclusion

This section first used value yield analysis to assess whether a share buyback would increase or decrease value to eternal shareholders. But value yield analysis is limited because it cannot measure the magnitude of the impact on shareholder value, it can merely assess whether a share buyback increases or decreases value to eternal shareholders. Present value analysis is needed for measuring the magnitude of the impact of a share buyback on shareholder value and is therefore used in the remainder of this treatise.

[13] The company has two classes of shares. All these calculations use Class A equivalent share numbers.

9.2. Jackson Hewitt Tax Service

The company *Jackson Hewitt Tax Service* was founded in 1982 and does computerized preparation of tax returns through franchised and company-owned offices. In connection with the tax returns, customers can also obtain short-term, high-interest loans secured with the anticipated tax refund.

The company had a record fiscal year 2007 in terms of revenue and reported net income, see Table 3, but in its annual report's section on risk it is mentioned amongst other things that the tax authorities in USA are seeking to simplify the procedures for filing tax returns and shorten the processing period, which could drastically reduce the need for the company's services. [14] Despite their acknowledgement of significant exogenous risks and future uncertainty of their industry, the company restructured its capital by issuing debt and buying back shares, thus increasing the risk of default on the debt payments in case of future business adversity. In the following years revenue and earnings declined significantly, the cause of which was unclear to management so they hired a consultancy firm to assist in a turnaround. There were also several changes in top management from 2007 and onwards. The company filed for so-called chapter 11 bankruptcy on May 24, 2011 and the company expected its common stock to be cancelled and hence worthless after a restructuring. [15]

The company's shares started trading publicly on NYSE (New York Stock Exchange) in June 2004 under the ticker symbol JTX and the share price reached its peak in late 2006 (the company's fiscal year 2007) with a share price of USD 37.44. Less than five years later the share price was USD 0.19 before the stock was finally delisted from NYSE as the company entered bankruptcy proceedings. The shares were then traded on the OTC (Over the Counter) stock exchange for USD 0.05 even though the company had announced the existing equity would be worthless after the bankruptcy restructuring. As of this writing in September 2012 the company still exists but its previous shareholders have lost ownership. This section analyses the impact of the company's recapitalization on shareholder value and whether it could have been foretold and prevented.

9.2.1. Share Buyback Policy

The company's share buyback plan and policy was briefly mentioned in the annual report for 2005:[16]

> *"(…) our Board of Directors authorized a new USD 50.0 million share repurchase program (…) such repurchases may be made through open market purchases or privately negotiated transactions. Such repurchases depend on the prevailing market conditions, our liquidity requirements, contractual restrictions and other factors."*

[14] Form 10-K annual report for 2007 filed with US SEC:
www.sec.gov/Archives/edgar/data/1283552/000119312507147353/d10k.htm
[15] Form 8-K filed with US SEC on May 24, 2011:
www.sec.gov/Archives/edgar/data/1283552/000119312511149091/dex991.htm
[16] Form 10-K annual report for 2005 filed with US SEC:
www.sec.gov/Archives/edgar/data/1283552/000104746905020253/a2161020z10-k.htm

The company has since then renewed its share buyback plan. For example, on October 13, 2006, the board of directors announced in a press release the completion of a previous share buyback plan and presented a new plan for USD 200m with the Chairman and CEO (Chief Executive Officer) stating: [17]

> *"Prospects for long-term growth and continued cash flow generation, combined with increased flexibility under our credit facility, provide an opportunity for Jackson Hewitt to reinvest in the business as well as deliver value to shareholders through further share repurchases."*

The press release also declared that:

> *"Since its initial public offering in 2004, Jackson Hewitt has returned over USD 160 million in share repurchases and dividends, delivering to shareholders over 100% of the prior year net income in each of the past two years."*

Although the press release also announced its credit facility had increased to USD 450m and extended the maturity to 2011, it was not made clear either in that press release or the annual reports that the company was significantly recapitalizing by issuing debt and making share buybacks. Neither the press release nor the company's annual reports specified the criteria for making share buybacks, such as the share price being below a certain limit, so it was unknown to shareholders how the company would fund its share buybacks, how large they would be, and how they would likely affect the value of the company to shareholders.

9.2.2. Ex-Post Analysis

The value of a company to its eternal shareholders is defined as the present value of future dividends for eternity. Share buyback valuation usually deals with estimates of this value as the actual value can only be known after the company has seized to exist. The company in this example went bankrupt and its shares became worthless, so its actual value can be studied and compared to value estimates to determine how the actual value was affected by the recapitalization and if it could have been predicted.

Value without Share Buybacks

This part of the analysis will focus on the value to eternal shareholders in fiscal year 2006 when the company started making large share buybacks. In valuing share buybacks the excess cash and future earnings that can be paid out as dividends are used. The company did not have excess cash in 2006. The earnings that could be paid out as dividends are chosen here to be the Free Cash Flow (FCF), in this case defined as the operating cash flow minus investing cash flow, which is the amount of cash available each year after the company has made investments in an effort to maintain or grow future earnings. Using the financial data from Table 3 and a variable for the discount rate d, the present value of the FCF is:

$$v = \frac{FCF_{2006}}{1+d} + \cdots + \frac{FCF_{2011}}{(1+d)^5} = \frac{97}{1+d} + \frac{63}{(1+d)^2} + \frac{3}{(1+d)^3} + \frac{16}{(1+d)^4} + \frac{-28}{(1+d)^5} + \frac{-78}{(1+d)^5}$$

The negative FCF for 2010 and 2011 are merely subtracted instead of being modelled as a new share or debt issuance. Using as discount rate the yield on low-risk government bonds during 2006 which was around 5% according to Figure 9, results in the present value USD 85m. Using as discount rate the

[17] Form 8-K filed with US SEC on October 13, 2006: www.sec.gov/Archives/edgar/data/1283552/000119312506207271/dex991.htm

estimated mean value yield for the S&P 500 stock market index which was about 7.7% in 2006 according to Table 2 (average of the high and low mean value yields in that table), results in the present value USD 89m. The maximum present value is $v \simeq USD\ 97m$ which occurs when the discount rate is $d \simeq 21\%$. This value is less than the lowest market-cap of USD 647m for 2006, so the value yield is mathematically ill-defined as no discount rate can be chosen to make v equal that market-cap. Although it is not possible to mathematically express the compounded rate of loss from buying the company at the given market-cap, it is still clear that the present value of all future FCF is decidedly less than the market-cap in 2006. According to Eq. 4-9 this means the share buybacks of that year decreased value to eternal shareholders.

In this example, the negative FCF may be considered irrelevant to the shareholders because it merely meant that no dividend payment could be made but it did not require the shareholders to pay any cash to the company. The value yield is the discount rate that makes v equal the market-cap. The company's market-cap ranged between USD 647-1141m during fiscal year 2006. Using a computer spreadsheet with a numerical root-finding method gives a value yield between (48)% and (58)%, that is, a significant compounded loss, which is obviously much worse than the rate of return that could be obtained from low-risk government bonds which was around 5% during that year, or from the S&P 500 index whose mean value yield was estimated at around 7.7% for that year. According to Eq. 4-10 this means the share buyback decreased value to eternal shareholders.

Relative Value of Share Buyback

Consider now the value with share buybacks relative to the value without share buybacks. The value v is the value without share buybacks if all FCF were to be paid out as dividends. If the discount rate is chosen to be the estimated mean value yield of the S&P 500 stock market index, which was around 7.7% on average during 2006 according to Table 2, and the negative FCF is ignored, then v is USD 159m.

In fiscal year 2006 the company used USD 58m to buy back shares net of issuance. If the share buybacks were made at the lowest market-cap of the year, which was USD 647m, then according to Eq. 4-7 the relative value of the share buyback was:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 58m}{USD\ 159m}}{1 - \frac{USD\ 58m}{USD\ 647m}} \simeq 70\%$$

This means the share buyback in 2006 decreased the value to eternal shareholders by about 30% relative to the value if all positive FCF for that and future years had instead been paid out as dividends.

The present value of the remaining positive FCF was only USD 74m in 2007 when using the discount rate 7.7%, but the share buyback for that year was USD 139m and the market-cap was at least USD 875m. Using this with Eq. 4-7 gives the relative value of the share buyback -104%, which is negative because the amount used for share buyback was greater than the present value of the FCF. This was possible because a large part of the share buyback was funded by a debt issuance. The formula for calculating the relative value is not well-defined in this case and a recapitalization formula is needed. However, the recapitalization formulas of section 4.9 assume the company continues to exist indefinitely. Although formulas could be devised for companies with finite lives, it should be clear that either or both of shareholders and creditors will experience a loss when share buybacks are made above the equilibrium.

Value of Dividends

FCF was used above to measure the present value of what the company could have paid out as dividends. Most of the FCF was not used for dividends but for share buybacks. This affected the return to shareholders who bought shares and held them until they became worthless and therefore relied on dividends as the sole source of income and value from owning the shares.

To calculate the present value of the dividends, the decrease in the number of shares must be included in the calculation. Assuming for simplicity that dividends are paid out at the end of each fiscal year, that the shareholder bought shares at the beginning of the fiscal year, and shares are bought back during the year, the present value in 2006 of all future dividends was:

$$Present\ Value\ of\ Dividends = \frac{Dividend_{2006} \cdot \frac{Shares_{2005}}{Shares_{2006}}}{1+d} + \cdots + \frac{Dividend_{2011} \cdot \frac{Shares_{2005}}{Shares_{2011}}}{(1+d)^5}$$
$$= \frac{11 \cdot \frac{37.6}{36.1}}{1+d} + \frac{16 \cdot \frac{37.6}{33.3}}{(1+d)^2} + \frac{21 \cdot \frac{37.6}{29.6}}{(1+d)^3} + \frac{15 \cdot \frac{37.6}{28.5}}{(1+d)^4} + \frac{0 \cdot \frac{37.6}{28.6}}{(1+d)^5} + \frac{0 \cdot \frac{37.6}{28.8}}{(1+d)^5}$$

This equals the lowest market-cap of USD 647m during 2006 when the discount rate d is about (51)%, as found by a computer spreadsheet with a numerical root-finding method. That is, the shareholder who bought shares in 2006 and held them until the company went bankrupt and the shares became worthless, experienced a compounded annual loss of (51)%, when taxes are ignored.

Using as discount rate d the rate of return that could be expected from investing in the S&P 500 index in 2006 which is the estimated mean value yield of 7.7% according to Table 2, would give a present value of the dividends of USD 62m, which was far below the minimum market-cap of USD 647m during that year.

9.2.3. Ex-Ante Analysis

When the company started making large share buybacks in 2006, the company's management presumably did not know the company would go bankrupt a few years later, so the actual present value and the actual value yield as calculated above would have been unknown at the time and estimates would have had to be used in the share buyback valuation.

Estimating the value of a company without share buybacks involves estimating the excess cash the company could pay out as dividends now, as well as forecasting all future earnings that could be paid out as dividends, and forecasting ROE in case the company would retain some of the future earnings.

Earnings Yield

This analysis focuses on fiscal year 2007. In this year the company had no excess cash. In the period 2002-2005 the company's ROA had been stable around 7%. As the company was mostly equity-funded during this period the ROE was just slightly higher at 7-8%. In 2006 the ROA increased to 9% due to a decrease in assets from the cancellation of a large account receivable from the company's former parent company as a form of non-cash special dividend. The company also added significant debt during that year and used the proceeds to pay a special cash dividend to the former parent company. The effect was that both ROA and ROE increased in 2006. The company's FCF in 2005 and 2006 was significantly higher than the reported net income, but it was unlikely to recur indefinitely as it was due to e.g. working capital changes which could

easily be reversed and thus require cash in the future. It is therefore assumed that the reported net income is a good estimate of what the company could earn on its equity and pay out as dividends to shareholders in the long term.

If the company could indefinitely maintain its 15% ROE from 2006, then the expected earnings of 2007 would be USD 58m. The market-cap for 2007 ranged between USD 875-1246m which gives an earnings yield range of 4.7-6.6%. At the company's lowest market-cap the estimated earnings yield of 6.6% was below the estimated value yield of the S&P 500 stock market index, whose mean ranged between 7.4-7.8% during 2007, see Table 2. This implied that the company's future earnings were either less risky than those of the S&P 500 index, or the company could grow its future earnings with a sufficiently high ROE depending on how much of future earnings would be retained. At the company's highest market-cap the estimated earnings yield was only 4.7% which was approximately the average yield on low-risk government bonds during that year, see Figure 9. This implied that the company's future earnings either had similar risk as low-risk government bonds, or the company could grow its future earnings. Otherwise a share buyback would decrease value to eternal shareholders according to Eq. 4-10. As it turned out the company did grow its revenue and earnings in 2007 but it was not to last and revenue and earnings decreased significantly in the following years. While it is trivial to see this mistake in hindsight it would also have been preventable if the company had made this assessment and followed the principles of section 10.

Recapitalization

It was assumed above that the company used earnings to buy back shares, but in 2007 the company issued debt to buy back shares, which is called a recapitalization and must be valued differently.

The company can be valued relative to the S&P 500 stock market index by using the estimated value yield of the S&P 500 index as the discount rate, which was about 7.6% on average in 2007, see Table 2. Using the same assumptions as above, namely that the company can indefinitely maintain a ROE of 15%, it means the estimated earnings of the company for 2007 were USD 58m. If the company paid out all earnings as dividends and this level of earnings could be maintained indefinitely, then according to Eq. 3-5 the estimated value to eternal shareholders would be:

$$v = v(7.6\%) = \frac{USD\ 58m}{7.6\%} \simeq USD\ 763m$$

This is the value of the company to its eternal shareholders under these assumptions and relative to the rate of return that could be expected from investing in the S&P 500 index. In retrospect it is obvious that the company did not have similar risk characteristics as the S&P 500 index so a higher discount rate should have been used in valuation. It is usually a qualitative assessment to judge the risk of a particular company against e.g. the S&P 500 index and this is left to the reader. This example is still educational when using this discount rate.

If the company were to issue significant debt then its risk of financial distress would increase and this requires an increased discount rate in valuation. The choice of risk premium is again a qualitative assessment but as an example a risk premium of 1% (percentage points) is used here so the discount rate becomes $d' = 8.6\%$. The estimated value is then:

$$v(8.6\%) = \frac{USD\ 58m}{8.6\%} = USD\ 674m$$

According to Table 3 the company's share buyback net of share issuance was USD 139m in 2007 and debt increased USD 77m from the prior year. In this example the recapitalization for USD 77m will be valued and the remainder of the share buyback is ignored. According to the company's annual report it mostly had debt with variable interest rate which was 6.2% in 2007, but for simplicity it will be assumed to be a constant interest rate for eternity. The company's effective tax rate was 39.2% in that year which is also assumed to be constant for eternity. The value of the company with such interest payments for eternity is calculated using Eq. 4-56:

$$v' = v(d') - \frac{Debt \cdot InterestRate \cdot (1 - TaxCompany)}{d'}$$
$$= USD\ 674m - \frac{USD\ 77m \cdot 6.2\% \cdot (1 - 39.2\%)}{8.6\%} \simeq USD\ 640m$$

The value with share buybacks relative to the value without share buybacks is calculated using Eq. 4-59 with all this data and the lowest market-cap of USD 875m for that year:

$$\frac{W_{DebtIncrease}}{V} = \frac{v'}{v \cdot \left(1 - \frac{Debt}{MarketCap}\right)} = \frac{USD\ 640m}{USD\ 763m \cdot \left(1 - \frac{USD\ 77m}{USD\ 875m}\right)} \simeq 92\%$$

This means the recapitalization decreased the value to eternal shareholders by about 8% under these assumptions. Note that this was calculated using a discount rate valuing the unlevered company as being of similar risk as the S&P 500 index, which was obviously much too optimistic in hindsight, and a risk premium of one percentage point for valuing the levered company which was also much too optimistic. It was also assumed the share buyback was made at the lowest market-cap of the year. All this means the decrease in value to eternal shareholders would likely be greater as a result of such a recapitalization. If the recapitalization was to be merely value neutral to eternal shareholders then the company would have to grow its future earnings. This did occur but only temporarily and then the revenue and earnings decreased. Although this is easy to see in hindsight, the assessment did not use ex-post data and could therefore have been done before the company made the recapitalization.

Using the same assumptions as above with Eq. 4-60 gives the equilibrium where the recapitalization starts to increase value to eternal shareholders:

$$W_{DebtIncrease} > V \Leftrightarrow MarketCap < \frac{Debt}{1 - \frac{v'}{v}} \Leftrightarrow MarketCap < \frac{USD\ 77m}{1 - \frac{USD\ 640m}{USD\ 763m}} \simeq USD\ 478m$$

This equilibrium is much lower than the minimum market-cap of USD 875m for that year, which means the recapitalization could not have been made during that year, without risking a decrease in value to eternal shareholders if the company should fail to sufficiently grow its future earnings.

9.2.4. Conclusion

This analysis showed that the company's share buybacks and recapitalizations significantly decreased the value to shareholders who held their shares until they became worthless following the company's bankruptcy. These shareholders relied on dividends as the sole source of value. Had the company not paid any dividends at all then the value of the company would have been zero to these shareholders.

The company made share buybacks and recapitalizations in spite of large exogenous risks beyond the control of management, at a share price that would require future earnings growth with sufficiently high ROE if they were merely to be value neutral to eternal shareholders. Prior to making the share buybacks and recapitalizations, an assessment using the formulas of this treatise would have shown the recapitalizations were likely to decrease value to eternal shareholders.

The company's publicized share buyback policy and plans did not specify the criteria for making share buybacks in terms of a limit for the share price, nor did they specify that the share buybacks would be funded largely with debt and hence increase the risk of financial distress. It was therefore not possible for outsiders to properly assess the impact of the company's share buybacks and recapitalizations on the value to eternal shareholders.

This example disproves the substitution hypothesis from section 2.2 as it was shown that the value of a company to its eternal shareholders can change significantly as a result of share buybacks. Also disproved is the notion that the market price of a company's shares equals their value to eternal shareholders, as assumed in several of the hypotheses reviewed in section 2.

9.3. Microsoft

The company *Microsoft* was founded by two people in 1975 to produce computer software. It has since grown and in fiscal year 2012 it employed 94k people with annual revenues of almost USD 74b and reported net income of almost USD 17b (excluding a goodwill impairment it would be USD 23b).[18]

In fiscal year 2012 the company's revenue segments were: Software for business productivity including text editors and spreadsheets was almost 33% of revenue. Operating systems and related software for personal computers were 25% of revenue. Software for computer servers, databases, and development tools was 25% of revenue. Entertainment products were 13% of revenue. Internet services including search and advertisement were 4% of revenue. The largest segments are also the most profitable with operating profit margins (operating income divided by revenue) over 60%.

9.3.1. Share Buyback Policy

The company's share buyback policy is mentioned briefly in its annual report, e.g. in fiscal year 1999:[19]

> *(...) cash will be used to repurchase common stock to provide shares for employee stock option and purchase plans. The buyback program has not kept pace with employee stock option grants or exercises. (...) Microsoft enhances its repurchase program by selling put warrants. (...) Management believes existing cash and short-term investments together with funds generated from operations will be sufficient to meet operating requirements for the next 12 months. The Company's cash and short-term investments are available for strategic investments, mergers and acquisitions, other potential large-scale cash needs that may arise, and to fund an increased stock buyback program over historical levels to reduce the dilutive impact of the Company's employee stock option and purchase programs. (...)The Company repurchases its common stock in the open market to provide shares for issuing to employees under stock option and stock purchase plans. The Company's Board of Directors authorized continuation of this program in 2000.*

From this it would seem the company's primary and perhaps only motivation for share buybacks is to offset the dilutive impact of employee stock options and stock purchase plans. There is no mentioning of conditions for the share price to be below a certain limit before share buybacks are made.

In a press release from September 22, 2008 the company announced the completion of one share buyback programme and the beginning of another: [20]

> *(...) board of directors approved a new share repurchase program authorizing up to an additional USD 40 billion in share repurchases with an expiration of September 30, 2013. (...) Microsoft has returned over USD 115 billion to shareholders through a combination of share repurchases and dividends over the last five years. "These announcements illustrate our confidence in the long-term*

[18] Form 10-K annual report for 2012 filed with US SEC:
www.sec.gov/Archives/edgar/data/789019/000119312512316848/d347676d10k.htm
[19] Form 10-K annual report for 1999 filed with US SEC:
www.sec.gov/Archives/edgar/data/789019/0001032210-99-001375.txt
[20] Company's self-published press release for September 22, 2012:
www.microsoft.com/en-us/news/press/2008/sep08/09-22dividend.aspx
Form 8-K filed with US SEC on September 22, 2012 does not describe the share buyback programme:
www.sec.gov/Archives/edgar/data/789019/000119312508199030/d8k.htm

> *growth of the company and our commitment to returning capital to our shareholders," said [the] chief financial officer of Microsoft.*

This suggests the company believes in the hypothesis from section 2.2 that share buybacks can be used to 'return capital to shareholders' as a substitute for dividends, which is disproven both theoretically and empirically in this treatise. There is no mentioning of a limit share price below which share buybacks will be made and the annual report for 2011 does not detail this either.[21]

9.3.2. Cash Repatriation

The company has significant revenue and operating income from foreign countries which are taxed lower than domestic income and repatriation tax must be paid when cash from foreign income is transferred to the company's country of incorporation. In recent years the company has had about 2/3 of its pre-tax operating income in foreign countries. The company's effective tax rate has been 18-25% while its domestic tax rate is 35%. On June 30, 2012 the company had cash, equivalents and short-term investments totalling about USD 63b of which USD 54b were held by foreign subsidiaries and would be subject to repatriation tax if transferred to USA. Repatriation tax can have a significant impact on shareholder value which should be included in valuations. But for the sake of simplicity and to better demonstrate other aspects of share buyback valuation, the repatriation tax is ignored here and instead studied in detail in section 9.6.

9.3.3. Analysis of Fiscal Year 2000

During fiscal year 2000 the company's market-cap was at the historically highest, ranging between USD 313-622b, see Table 4. This market-cap was calculated using the weighted average basic number of shares outstanding for the year, multiplied by the highest and lowest share prices during the year. The diluted number of shares and hence the diluted market-cap were significantly higher. The company did not expense employee stock options and for fiscal year 1999 the value of the options as calculated using the Black-Scholes formula amounted to USD 1.1b while the reported net income was USD 7.8b.[22] This means the company's actual earnings attributable to shareholders were significantly lower than the reported net income if the stock options had been expensed, or the company's actual market-cap was significantly higher if the diluting impact of the stock options was included. This analysis uses the reported net income which means the actual value of the company is likely less, and the non-diluted market-cap is used which means the actual market-cap is likely higher. In combination this means the required earnings growth is likely higher if the share buybacks during fiscal year 2000 are to be value neutral to eternal shareholders.

Retained Earnings

At the end of fiscal year 1999, the company's cash, equivalents and short-term investments were USD 17.2b which was almost 45% of its assets. The company's ROA was 35% and ROE was 47% for the year. This suggests the company does not need much capital to operate and hence may not need to retain earnings in order to grow future earnings. It is therefore assumed in this analysis that all earnings could be paid out as dividends to shareholders, and future growth in revenue and earnings is due to an increase in customer demand rather than an increase in assets.

[21] Form 10-K annual report for 2011 filed with US SEC:
www.sec.gov/Archives/edgar/data/789019/000119312511200680/d10k.htm
[22] Company's self-published annual report for 1999:
download.microsoft.com/download/4/A/9/4A9500B6-2B69-4B40-B161-55504BC549D5/ar99.doc

Excess Cash

It is assumed that all the cash etc. at the end of fiscal year 1999 in the amount of USD 17.2b are in excess of the company's needs for operations. This is likely too high as some cash is usually needed to support operations, and there may also be taxes that would be incurred if e.g. cash earned and held in other countries were to be transferred to the USA and paid out as dividends. But this assumption of all cash being in excess provides a useful upper bound in valuation, as it means the actual value is likely less and hence the required earnings growth is likely greater than that derived here.

Required Earnings Growth

Share buybacks can be valued relative to alternative investments such as the S&P 500 stock market index, by using the value yield estimate of the S&P 500 index as the discount rate d when valuing the company. Recall that the value yield estimate is the expected annual rate of return from buying shares at the given price and holding the shares for eternity. The mean value yield estimate of the S&P 500 index was 6.2% at its lowest during 2000, see Table 2.[23]

The earnings growth g required for the value yield of the company to be greater than the estimated value yield of the S&P 500 index, can be found using Eq. 3-9 which is used instead of Eq. 3-6 because the previous year's earnings are known and assumed to grow from the current year. It is assumed the reported net income is the earnings that could have been paid out as dividends. The formula for the required growth rate from Eq. 3-9 is reprinted here for easy reference:

$$MarketCap < v = Excess\ Cash + Earnings \cdot \frac{1+g}{d-g} \Leftrightarrow g > \frac{d - \frac{Earnings}{MarketCap - Excess\ Cash}}{1 + \frac{Earnings}{MarketCap - Excess\ Cash}}$$

According to Eq. 4-9 a share buyback increases value to eternal shareholders if earnings grow more than this rate g for eternity.

Using this formula with the estimated mean value yield of the S&P 500 index of 6.2% as the discount rate, the lowest market-cap of USD 313b during fiscal year 2000, the reported net income of USD 7.8b for fiscal year 1999 as the starting earnings, and assuming excess cash of USD 17.2b, means the earnings growth rate must satisfy:

$$MarketCap < v \Leftrightarrow g > \frac{6.2\% - \frac{USD\ 7.8b}{USD\ 313b - USD\ 17.2b}}{1 + \frac{USD\ 7.8b}{USD\ 313b - USD\ 17.2b}} \simeq 3.5\%$$

Using the highest market-cap during 2000 of USD 622b means the earnings growth rate must satisfy:

$$MarketCap < v \Leftrightarrow g > 4.8\%$$

So if shares had been bought back at the lowest market-cap for the year, and assuming the company had excess cash of USD 17.2b, then the company's earnings must grow at an annual rate of more than 3.5% for

[23] The company's fiscal year is not the calendar year but this value yield estimate is sufficiently precise for this study.

eternity, if the share buyback were to increase value to eternal shareholders, relative to the rate of return they could expect from investing in the S&P 500 index.

However, the estimated value yield of the S&P 500 index was historically low during year 2000 and about the same as the yield on USA long-term government bonds which was around 6% on average during that year. According to section 7.5.4 the historical average value yield estimate for the S&P 500 index was 8.3%, which can be used as the discount rate d when calculating the required earnings growth for the company. For the lowest market-cap during fiscal year 2000 of USD 313b and assuming the company had excess cash of USD 17.2b, means the earnings growth rate must then satisfy:

$$MarketCap < v \Leftrightarrow g > 5.5\%$$

Using the highest market-cap during 2000 of USD 622b means the earnings growth rate must satisfy:

$$MarketCap < v \Leftrightarrow g > 6.9\%$$

In summary, the lower bound on the required earnings growth for the share buyback to increase value to eternal shareholders was estimated to be 3.5%, provided shares were bought back at the lowest market-cap for the year and under various valuation assumptions. Buying back shares at a higher share price or changing the valuation assumptions significantly increased the required earnings growth.

Stock Options

During fiscal year 2000 a total of 396m stock options were exercised at a weighted average share price of USD 4.77 (numbers adjusted for a subsequent stock split)[24], so the total proceeds were almost USD 1.9b. Although the cash flow statement for that year lists share issuance of more than USD 2.2b, the cause of the difference is not important for this demonstration of the share buyback valuation formulas.

When stock options are exercised, management has a choice of issuing new shares or buying back shares. The equilibrium is where share buybacks start to increase value to eternal shareholders and for stock options exercising it is calculated using Eq. 4-100. The number of shares outstanding at the end of fiscal year 1999 was 10,218m and is used instead of the weighted average number of shares during the year, which includes the effect of the share buybacks being valued. The equilibrium is:

$$MarketCap < \frac{v + Options \cdot ExercisePrice}{1 + \frac{Options}{Shares}} = \frac{v + 396m \cdot USD\ 4.77}{1 + \frac{396m}{10{,}218m}} \simeq \frac{v + USD\ 1.9b}{1.0388}$$

Assuming the company's earnings grow by a constant rate for eternity starting this year, the value v is defined from Eq. 3-8:

$$v = Excess\ Cash + Earnings \cdot \frac{1+g}{d-g}$$

[24] Company's self-published annual report for 2000:
download.microsoft.com/download/D/A/4/DA46BC9A-6E9D-4481-9144-8E1652441850/msft_ar00.doc

The required growth rate is then:

$$MarketCap < \frac{v + USD\ 1.9b}{1.0388} \Leftrightarrow g > \frac{d - \frac{Earnings}{MarketCap \cdot 1.0388 - USD\ 1.9b - Excess\ Cash}}{1 + \frac{Earnings}{MarketCap \cdot 1.0388 - USD\ 1.9b - Excess\ Cash}}$$

Using the reported net income of USD 7.8b for fiscal year 1999 as the starting earnings and assuming the total cash holdings of USD 17.2b at the end of 1999 are in excess and could be paid out as dividends, and using the lowest market-cap of USD 313b for 2000, gives the required rate of earnings growth:

$$g > \frac{d - 2.5\%}{102.5\%}$$

Using the lowest value yield estimate for the S&P 500 index during 2000 of 6.2% would give:

$$g > \frac{6.2\% - 2.5\%}{102.5\%} \simeq 3.6\%$$

This earnings growth requirement assumed all cash holdings are in excess and the share buybacks are made at the lowest share price during the year, which means the actual required earnings growth is likely higher.

Free Cash Flow

The reported net income for fiscal year 1999 was USD 7.8b while the estimated Free Cash Flow (FCF) was USD 9.4b, according to Table 4. For fiscal year 2000 the reported net income was USD 9.4b (20.5% annual growth) and the estimated FCF was USD 10.5b (11.7% annual growth). As the FCF is considerably higher than the reported net income, it will now be used to calculate the share buyback equilibrium to see how it affects the share buyback value. Further assume the FCF for fiscal year 2000 of USD 10.5b was known in advance so it can be used as the starting earnings in valuation, which also means Eq. 3-6 is used instead of Eq. 3-9 to calculate the future earnings growth required. The cash holdings at the end of fiscal year 1999 of USD 17.2b are assumed to be in excess of what is needed to support operations and could therefore be paid out as dividends immediately. At the lowest market-cap of USD 313b during fiscal year 2000, the required earnings growth rate is:

$$MarketCap < \frac{v + USD\ 1.9b}{1.0388} \Leftrightarrow g > d - \frac{Earnings}{MarketCap \cdot 1.0388 - Excess\ Cash + USD\ 1.9b}$$
$$= d - \frac{USD\ 10.5b}{USD\ 313b - USD\ 17.2b + USD\ 1.9b} \simeq d - 3.5\%$$

Using the lowest value yield estimate for the S&P 500 index during 2000 of 6.2% would give:

$$g > 6.2\% - 3.5\% = 2.7\%$$

At the highest market-cap of USD 622b during that year the required earnings growth rate is:

$$g > d - 1.7\%$$

Using the value yield estimate for the S&P 500 index of 6.2% gives the required earnings growth rate:

$$g > 6.2\% - 1.7\% = 4.5\%$$

This was a historically low value yield estimate for the S&P 500 index because the stock prices were historically high in relation to estimated future earnings. According to section 7.5.4 the historical average value yield estimate for the S&P 500 index was 8.3% which can be used as the discount rate in calculating the required earnings growth rate. For the lowest market-cap of USD 313b it is:

$$g > 8.3\% - 3.5\% = 4.8\%$$

For the highest market-cap of USD 622b the required earnings growth rate is:

$$g > 8.3\% - 1.7\% = 6.6\%$$

So even though the comparatively higher FCF for fiscal year 2000 is used in valuation, earnings should still grow by 2.7% for eternity in order for the share buybacks to be value neutral to eternal shareholders. This was an optimistic valuation scenario assuming shares were bought back at their lowest market price during that year, and the valuation was done relative to the estimated value yield of the S&P 500 which was historically low during that year. Using less optimistic assumptions in the share buyback valuation means the required earnings growth would be several percentage points higher.

Actual Earnings Growth

In the period 1999 to 2012 the company's revenue grew 10.7% on average each year (geometric mean) while its reported net income available to common shareholders grew 6.2% on average (geometric mean). This includes growth from acquisitions which used cash from earnings and should be subtracted in valuation as it could not have been paid out as dividends when it was used for acquisitions. Still, the share buybacks from 2000 seem so far to have been at least value neutral and might have increased value to eternal shareholders, but earnings must continue to grow at a sufficient rate or the share buybacks from 2000 will ultimately turn out to have decreased value to eternal shareholders.

9.3.4. Analysis of Fiscal Year 2012

According to Table 4, at the end of fiscal year 2011 the company had assets of nearly USD 109b, of which cash, equivalents and short-term investments were almost USD 53b, and equity was USD 57b. ROE for 2011 was 50%. This suggests a substantial part of the company's cash holdings are in excess of its needs for operations and furthermore that the company does not require additional equity capital to grow its existing business, which means all its earnings are available for dividend payout.

Comparison to Alternative Investments

The company's market-cap in fiscal year 2012 ranged between USD 200-277b. Assuming the company can maintain its earnings from fiscal year 2011 of USD 23b, and the company had no excess cash, the earnings yield during fiscal year 2012 would range between 8.3-11.5% according to Eq. 3-14. If the company had, say, USD 30b in excess cash of its total USD 53b holdings of cash and equivalents at the end of fiscal year 2011, then the earnings yield during 2012 would be between 9.3-13.5%. Compare this to the yield on low-risk government bonds which were around 3-4% during calendar year 2011, see Figure 9, and the highest estimated mean value yield of 9.3% for the S&P 500 stock market index during calendar year 2011, see

Table 2. Also compare it to the historical rate of return of 16% for an index of venture companies, see section 7.6. This means either the company's future earnings were known to decrease, or the future earnings were almost as uncertain or risky as a diversified index of venture companies, or the company's shares were mispriced in relation to the uncertainty of the company's future earnings.

Required Earnings Growth

This analysis will value the company relative to the S&P 500 stock market index, whose mean value yield ranged between 8.4-9.3% during calendar year 2011, according to Table 2. Recall that the value yield is the rate of return a shareholder would get from buying shares at the given market price and holding them for eternity.

The earnings growth required in order for the company's market-cap to be less than its value can be calculated using Eq. 3-9, which assumes the company's earnings grow by the rate g in the first and each consecutive year for eternity, and that all earnings are available for dividend payout. Let the starting earnings be the reported net income from 2011 of USD 23b, let the discount rate d be the highest value yield for the S&P 500 index of 9.3%, so the required growth rate is:

$$MarketCap < v \Leftrightarrow g > \frac{9.3\% - \frac{USD\ 23b}{MarketCap - Excess\ Cash}}{1 + \frac{USD\ 23b}{MarketCap - Excess\ Cash}}$$

Using this formula with the highest market-cap during fiscal year 2012 of USD 250b and assuming the company has no excess cash, means the earnings growth rate must satisfy:

$$MarketCap < v \Leftrightarrow g > 0.1\%$$

Using the lowest market-cap during 2012 of USD 200b and assuming the company had excess cash of, say, USD 30b, means the earnings growth rate must satisfy:

$$MarketCap < v \Leftrightarrow g > -3.7\%$$

This means if shares had been bought back at the lowest market-cap of the year, and assuming the company had excess cash of USD 30b, then the company's earnings could have a compounded annual decrease of (3.7)% for eternity, and the company would still offer the same rate of return as could be expected from the S&P 500 stock market index. At the highest market-cap for the year and assuming the company had no excess cash, the earnings should only grow by 0.1% per year for eternity and the company's value yield would be the same as that expected for the S&P 500 index.

According to Eq. 4-9 a share buyback would increase value to eternal shareholders if earnings were to grow more than these rates. The reported net income for fiscal year 2012 was USD 17b, a decrease of 26%, although this included a non-cash goodwill impairment of USD 6b without which the reported net income would have been USD 23b, similar to that of 2011. Whether the earnings growth requirement can be satisfied in the long-term is a qualitative assessment left to the reader.

Dividends or Share Buybacks

Dividends in 2012 were USD 6.4b and share buybacks were USD 5b. If shares were bought back at the lowest market-cap during the year, and if the company had USD 30b of excess cash and the company's earnings would grow more than (3.7)% per year for eternity, then the value of the company to its eternal shareholders would have increased if the company had suspended the dividend and used the cash for share buybacks instead.

9.3.5. Conclusion

This analysis focused on fiscal years 2000 and 2012. In 2000 the company had net income of USD 9.4b, no dividend payouts and share buybacks of USD 4.9b which was 52% of net income. In 2012 the company had net income (excluding a goodwill impairment) of USD 23b, dividends of USD 6.4b and share buybacks of USD 5b which was 22% of net income. The analysis showed that in order for the share buybacks in year 2000 to be value neutral to eternal shareholders, the company's future earnings must grow at least 3.5% per year for eternity in the most optimistic valuation scenario considered, but the required earnings growth might actually be several percentage points higher. Conversely, the share buybacks in 2012 would increase value to eternal shareholders even if earnings should decrease each year for eternity. If the shares had been bought back at the lowest market-cap during 2012, the earnings decrease could be as much as (3.7)% compounded each year for eternity.

This means the company made large share buybacks relative to earnings in 2000 when the share price was high and would hence require future earnings growth for the share buybacks to increase value to eternal shareholders, while the company made small share buybacks relative to earnings in 2012 when the share price was low so future earnings could decrease substantially and the share buybacks would still increase value to eternal shareholders.

These analyses did not use ex-post data so the analyses could have been made by the company's management prior to the share buybacks. On September 23, 1999 during the company's fiscal year 2000, the company's president is quoted as having said: *"There is such an overvaluation of technology stocks that it is absurd. I would include our stock in that category. It is bad for the long-term worth of the economy."*[25] Yet ironically the company made its largest share buyback to date during that period.

Because the company's publicized share buyback policy does not mention the share price criteria for making share buybacks, it is impossible for outsiders to properly assess the valuation impact of the company's future share buybacks.

[25] Society of American Business Editors and Writers, Technology Conference, September 23, 1999: web.archive.org/web/20110108131206/http://sabew.org/1999/09/1999-news-sabews-technology-conference-moves-market/

9.4. McDonald's

The company *McDonald's* was started by two brothers in USA in 1948 as a single restaurant selling fast-food at low prices. At the end of 2011 the company had grown to employ 420k people and comprised 33.5k restaurants in 119 countries, of which 19% of the restaurants were operated by the company and the rest were franchised. In 2011 the company had USD 27b in total revenue consisting of the revenue from its own restaurants and revenue from the franchisees in the form of a percentage of their sales and rent of the restaurant sites which are owned or leased by the company.[26]

9.4.1. Share Buyback Policy

The company's share buyback policy is briefly mentioned in its annual reports. For the period 1998-2001 the wording is similar:[27]

> *"Given the Company's returns on equity and assets, management believes it is prudent to reinvest a significant portion of earnings back into the business and use free cash flow for share repurchase. Accordingly, the common stock dividend yield is modest. (...) The Company uses free cash flow and debt capacity to repurchase shares, as we believe this enhances shareholder value."*

In 2002 the company experienced almost a halving of its reported net income. The wording of the share buyback policy changed slightly to indicate that debt repayment would take priority over share buybacks:

> *"Given the Company's returns on equity and assets, management believes it is prudent to reinvest a significant portion of earnings back into the business and use excess cash flow* **for debt repayments** *and share repurchases. Accordingly, the common stock dividend yield is modest. (...) Average debt levels were higher in both years [2001-2002] because the Company used its available credit capacity to repurchase shares of its common stock."*

In the annual report for 2011 the wording was different but the meaning substantially the same:

> *"Given the Company's returns on equity, incremental invested capital and assets, management believes it is prudent to reinvest in the business in markets with acceptable returns and/or opportunity for long-term growth and use excess cash flow to return cash to shareholders through dividends, share repurchases or a combination of both. (...) We remain committed to returning all of our free cash flow (cash from operations less capital expenditures) to shareholders over the long-term via dividends and share repurchases."*

In none of these annual reports did the company's share buyback policy mention a criterion for share buybacks in terms of the share price being lower than some measure of value to eternal shareholders. The company seems to believe in the substitution hypothesis which claims that share buybacks can be used to return capital to shareholders as a substitute for dividends, which is shown in this treatise to be a fallacy. This means the company relies on the stock market to set the share price at or below the value to eternal shareholders, otherwise share buybacks will decrease value to them.

[26] Form 10-K annual report for 2011 filed with US SEC: www.sec.gov/Archives/edgar/data/63908/000119312512077317/d260574d10k.htm

[27] Form 10-K annual report for 1998 filed with US SEC: www.sec.gov/Archives/edgar/data/63908/0000950131-99-001938.txt

Additionally, the annual report from 1998 claims:

> *"The average option exercise price has consistently exceeded the average cost of treasury stock issued for option exercises because the Company prefunds the program through share repurchase. Thus, stock option exercises have generated additional capital, since cash received from employees has exceeded the Company's average acquisition cost of treasury stock. In addition, stock option exercises resulted in USD 319.6 million of tax benefits for the Company during the three years ended December 31, 1998."*

This apparently ignores the time value of money because the shares held in the treasury were likely bought back prior to the issuance of stock options and the price difference calculated on a first-in-first-out (FIFO) basis, otherwise the price could not consistently be less than the exercise price of the stock options. The claim that stock options are a source of income to existing shareholders is therefore misleading.

9.4.2. Analysis of Year 1999

The company had a peak market-cap during fiscal year 1999 which was not reached again until 2007, see Table 5. In 1999 share buybacks were USD 933m and dividends were USD 265m. The share buybacks will now be analysed both ex-ante and ex-post so as to assess whether they could reasonably be expected to increase or decrease value to eternal shareholders and what the actual effect was.

Ex-Ante Analysis

Assume the reported net income is a reasonable estimate of the earnings that could be paid out as dividends to shareholders if the company were to maintain that level of earnings for eternity. Although the company could perhaps increase its profit margin, it is assumed here that in order for the company to grow its revenue and earnings it must invest in more productive assets such as property and equipment, which means the company needs additional equity and liabilities. It is assumed that the company retains a certain fraction of earnings each year for eternity and that the Return on Equity (ROE) is also constant for eternity.

Table 5 shows the ROE starting in year 1998 where it was 18%, which will be assumed to be the future ROE for eternity. The reader may study the company's prior financial statements so as to better assess the historical and possible future ROE. The earnings for 1999 were not known in advance but assuming ROE is constant means the earnings for 1999 would equal ROE multiplied by the equity at the end of 1998, that is, 18% of USD 9.5b which is USD 1.7b, while the reported net income was actually USD 1.9b in 1999. According to the balance sheet of the company's annual report for 1998, the company had cash and equivalents of USD 299m while the long-term debt was USD 6.2b, which means the company had little or no excess cash. Using Eq. 3-32 with these assumptions and the lowest market-cap for 1999 of USD 49b gives the earnings yield:

$$Earnings\ Yield = \frac{Earnings}{MarketCap - Excess\ Cash} = \frac{USD\ 1.7b}{USD\ 49b - 0} \simeq 3.5\%$$

This earnings yield estimate of 3.5% was considerably lower than the yield that could be obtained from USA government bonds which was 5.9% on average in 1999, see Figure 9. Using the ex-post reported net income for 1999 of USD 1.95b gives an earnings yield of 4% which was also well below the yield of USA government bonds. This means the company's future earnings were either less risky than USA government

bonds, or the company would grow its future earnings at a sufficiently high ROE, or the company's shares were overpriced.

The value yield is the rate of return that a shareholder would obtain from buying the company's shares and holding them forever. Assuming ROE and the fraction of earnings being retained are constant for eternity, the value yield is calculated using Eq. 3-41 by weighting the earnings yield and ROE with the fraction of earnings that are retained each year for eternity:

$$Value\ Yield = Earnings\ Yield \cdot (1 - Retain) + ROE \cdot Retain \simeq 3.5\% \cdot (1 - Retain) + 18\% \cdot Retain$$

According to Eq. 4-10 a share buyback increases value to eternal shareholders relative to an alternative investment when the value yield is greater than the rate of return that could be obtained from the alternative investment. For this value yield to be greater than the average yield of USA government bonds of 5.9% would thus require:

$$Value\ Yield > 5.9\% \Leftrightarrow 3.5\% \cdot (1 - Retain) + 18\% \cdot Retain > 5.9\% \Leftrightarrow Retain > 17\%$$

That is, assuming ROE is 18% for eternity, the company would have to retain more than 17% of its earnings each year for eternity so as to grow future earnings sufficiently for the value yield to be greater than the yield of long-term USA government bonds which was 5.9% on average during that year. According to Eq. 3-39 this means the equity and hence earnings would grow each year for eternity at least by the rate:

$$g = ROE \cdot Retain > 18\% \cdot 17\% \simeq 3\%$$

Consider now the estimated value yield for the S&P 500 stock market index, which was historically low during 1999 with a mean ranging between 6.1-6.5% according to Table 2. This was slightly above the average yield of 5.9% on USA government bonds in 1999, meaning the required earnings growth rate was slightly higher than 3% in order for the value yield of the company to equal that of the S&P 500 index. But this was an abnormally low value yield for the S&P 500 index whose average value yield was 8.3% for the period 1983-2011, according to section 7.5.4. Using this average instead gives:

$$Value\ Yield > 8.3\% \Leftrightarrow 3.5\% \cdot (1 - Retain) + 18\% \cdot Retain > 8.3\% \Leftrightarrow Retain > 33\%$$

This means the earnings growth rate would be:

$$g = ROE \cdot Retain > 18\% \cdot 33\% \simeq 6\%$$

If instead the highest market-cap for 1999 of USD 67b was used in these calculations, then the earnings yield would be:

$$Earnings\ Yield = \frac{Earnings}{MarketCap} = \frac{USD\ 1.7b}{USD\ 67b} \simeq 2.5\%$$

Assuming both ROE and the fraction of earnings being retained are constant for eternity, this gives the value yield estimate:

$$Value\ Yield = Earnings\ Yield \cdot (1 - Retain) + ROE \cdot Retain \simeq 2.5\% \cdot (1 - Retain) + 18\% \cdot Retain$$

For this to be greater than the yield on USA government bonds which averaged 5.9% during 1999, it requires the following fraction of earnings to be retained for eternity:

$$Value\ Yield > 5.9\% \Leftrightarrow 2.5\% \cdot (1 - Retain) + 18\% \cdot Retain > 5.9\% \Leftrightarrow Retain > 22\%$$

The growth in equity and earnings for eternity would then be:

$$g = ROE \cdot Retain > 18\% \cdot 22\% \simeq 4\%$$

Instead using the average value yield for the S&P 500 index of 8.3% as the benchmark, gives the following requirement for earnings retaining each year for eternity:

$$Value\ Yield > 8.3\% \Leftrightarrow 2.5\% \cdot (1 - Retain) + 18\% \cdot Retain > 8.3\% \Leftrightarrow Retain > 37\%$$

The growth in equity and earnings for eternity would then be:

$$g = ROE \cdot Retain > 18\% \cdot 37\% \simeq 7\%$$

In summary, when the company's future ROE is assumed to be constant at 18% and if shares were bought back at the lowest market-cap during 1999, then in order for the value yield of the company to be greater than the yield that could be obtained from USA government bonds in that year, the company would have to retain at least 17% of its earnings each year so its earnings would grow 3% for eternity. At the highest market-cap for 1999 the company would have to retain 22% of its earnings each year so earnings would grow 4% for eternity. During 1999 the estimated value yield for the S&P 500 index was historically low because the index price was high relative to the earnings that could reasonably be expected from the constituent companies of the index. If instead the historical average value yield estimate for the S&P 500 index of 8.3% is compared to the estimated value yield of the company, then for those to be equal the company would have to retain between 33-37% of earnings each year in order to grow earnings 6-7% for eternity, depending on whether shares were bought back at the lowest or highest market-cap during 1999. If a risk premium would have been deemed appropriate for this company relative to the S&P 500 index, then the required earnings growth rate and earnings retaining fraction would have been higher still.

Ex-Post Analysis

The share buyback in 1999 created an obligation for the company to significantly increase its future earnings otherwise the share buyback would decrease value to eternal shareholders.

For the period 1999-2011 the company had geometric mean ROE of 21% with standard deviation 9%, geometric mean growth in revenue of 6% per year, and geometric mean growth in net income of 10% per year, according to the data in Table 5. So it appears that during this period the company has grown its earnings with a sufficiently high ROE to make the share buyback from 1999 be either value neutral or perhaps increase value to eternal shareholders, depending on what alternative investment would have been deemed appropriate as the benchmark for the risk and required return of the company.

The company's cash flows will now be studied in more detail. Free Cash Flow (FCF) is defined here as operating cash flow minus investing cash flow because the latter is primarily related to the main business. The company uses a different definition of FCF which merely subtracts capital expenditures from the operating cash flow, but this seems to result in an overstatement of FCF because it excludes e.g. the

purchase and sale of franchised restaurants which is a part of the business. Using our definition, FCF was USD 747m in 1999 which was much lower than the reported net income of USD 1.9b, see Table 5. The reason is that depreciation, which is an estimate used in accrual-based accounting of how much the company's assets are worn down during the year, was almost USD 1b and thus about half the capital expenditures of USD 1.9b, which is the actual amount of cash that was spent during the year for maintaining and extending the company's productive assets.[28] Provided the depreciation estimate is reasonably accurate then the comparatively larger capital expenditures means the company is investing in new productive assets presumably with the intention of growing its future revenue and earnings. Furthermore, the company's franchise programme in which restaurants are bought and sold, had a net cash usage of USD 99m in 1999, while the company also sold properties for USD 21m and used USD 316m for other unspecified investing purposes.

FCF is an estimate of what could have been paid out as dividends after the company has made investments, and there was no debt increase. So the value to eternal shareholders can be estimated as the sum of the present value of the FCF for a known period and a terminal value in which a mathematical model estimates the unknown FCF for eternity:

$$v = v_{FCF} + v_{Terminal}$$

The present value of the FCF for the period 1999-2011 is calculated using the data from Table 5:

$$\begin{aligned} v_{FCF} &= \frac{FCF_{1999}}{1+d} + \cdots + \frac{FCF_{2011}}{(1+d)^{13}} \\ &= \frac{USD\ 747m}{1+d} + \frac{USD\ 539m}{(1+d)^2} + \frac{USD\ 620m}{(1+d)^3} + \frac{USD\ 423m}{(1+d)^4} + \frac{USD\ 1{,}899m}{(1+d)^5} + \frac{USD\ 2{,}521m}{(1+d)^6} \\ &+ \frac{USD\ 2{,}519m}{(1+d)^7} + \frac{USD\ 3{,}068m}{(1+d)^8} + \frac{USD\ 3{,}726m}{(1+d)^9} + \frac{USD\ 4{,}292m}{(1+d)^{10}} + \frac{USD\ 4{,}096m}{(1+d)^{11}} \\ &+ \frac{USD\ 4{,}286m}{(1+d)^{12}} + \frac{USD\ 4{,}579m}{(1+d)^{13}} \end{aligned}$$

If the company is assumed to not grow after 2011 then the reported net income for 2011 is taken as an estimate of the earnings that could be paid out as dividends each year for eternity. The terminal value is then calculated using Eq. 13-1 with zero growth:

$$v_{Terminal} = \frac{USD\ 5{,}503m}{(1+d)^{14}} \cdot \frac{1+d}{d} = \frac{USD\ 5{,}503m}{d \cdot (1+d)^{13}}$$

Note that this is discounted by $(1+d)^{14}$ to give the present value in year 1999.

Using for discount rate d the historical average value yield estimate for the S&P 500 stock market index which was 8.3%, the value of the company was in 1999:

$$v = v_{FCF} + v_{Terminal} \simeq USD\ 16.6b + USD\ 23.5b \simeq USD\ 40.1b$$

[28] Form 10-K annual report for 1999 filed with US SEC: www.sec.gov/Archives/edgar/data/63908/0000950131-00-002087.txt

The amount used for share buyback was USD 933m in 1999. The relative value of the share buyback is calculated using Eq. 4-7 with the lowest market-cap of USD 48.7b for that year:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 933m}{USD\ 40.1b}}{1 - \frac{USD\ 933m}{USD\ 48.7b}} \simeq 99.6\%$$

That is, even though FCF grew about 23% per year in the period 1999-2011 the value to eternal shareholders still decreased by about 0.4% as a result of the share buyback in 1999.

This calculation used as discount rate the estimated value yield of the S&P 500 index of 8.3%. If a risk premium had been demanded so the discount rate would have been, say, $d = 10\%$, then the value to eternal shareholders would have been considerably less:

$$v \simeq USD\ 30.5b$$

The relative value of the share buyback at the lowest market-cap during 1999 would be:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 933m}{USD\ 30.5b}}{1 - \frac{USD\ 933m}{USD\ 48.7b}} \simeq 98.8\%$$

That is, the value to eternal shareholders would have decreased by about 1.2% as a result of the share buyback.

These calculations used the lowest market-cap of USD 48.7b during 1999. Instead using the highest market-cap of USD 67b gives:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 933m}{USD\ 30.5b}}{1 - \frac{USD\ 933m}{USD\ 67b}} \simeq 98.3\%$$

That is, the value to eternal shareholders would have decreased by about 1.7% as a result of the share buyback.

This means the share buyback of 1999 has decreased value to shareholders unless one of the following occurs. First, the earnings continue to grow beyond year 2011 at a sufficiently high ROE so the value v in 1999 becomes higher than the market-cap at which share buybacks were made in that year. Second, some of the share buybacks in year 2000 and beyond are made below the equilibrium value if the stock market should offer the opportunity; see the equilibrium theorem in appendix 13.2. Third, the company funds its investments in assets partially or wholly by an increase in debt, while this calculation of the value v assumed all investments were funded through earnings and hence understated the amount that could have been paid out as dividends.

The value of a share buyback is rarely known in advance because it depends on several unknown factors, but it is evident from this example that a share buyback made at a too high price creates a potentially long-

lasting obligation for future earnings growth if the share buyback is merely to be value neutral to eternal shareholders. Fortunately such an obligation can easily be avoided by instead using the cash to make a dividend payout, or repay debt, or retain the cash in anticipation of a lower share price in the future.

9.4.3. Analysis of Year 2003

In 1999 the company made share buybacks for more than 350% of the dividend payout at a time when the market-cap was high relative to earnings, thus creating an obligation for future earnings growth if the share buybacks were merely to be value neutral to eternal shareholders. Conversely, in 2003 dividends were USD 504m and share buybacks were USD 439m, or about 87% of dividends, see Table 5, while the lowest market-cap during 2003 was only about 22% of the highest market-cap of 1999. So the company decreased share buybacks relative to dividends although the market-cap had greatly decreased. This section gives both ex-ante and ex-post analysis of how the value to eternal shareholders would likely have been affected if the dividend payout in 2003 had instead been used for share buybacks.

Ex-Ante Analysis

The company had a large decrease in reported net income from USD 2b in 2000 to USD 0.9b in 2002, which included a USD 0.7b restructuring charge of which only USD 0.1b were after-tax cash charges. Excluding this the earnings for 2002 would have been USD 1.5b. The average ROE (both arithmetic and geometric mean) was 17% for the period 1998-2002 when calculated using the reported net income. The company had cash and equivalents of USD 330m and long-term debt of USD 9.7b at the end of fiscal year 2000, according to the balance sheet of the financial statements, so the company did not seem to have excess cash.[29]

Assuming ROE remains 17% for eternity then the earnings for 2003 would be 17% of the equity for 2002, which was USD 10.3b, that is, earnings would be USD 1.7b for 2003. Although the actual net income for 2003 was less than USD 1.5b, see Table 5, the earnings estimate of USD 1.7b will be used in this ex-ante analysis. Further assume the fraction of earnings being retained each year is constant for eternity then the value to eternal shareholders is calculated using Eq. 3-40, which is reprinted here for convenience:

$$v = Excess\ Cash + \frac{Earnings \cdot (1 - Retain)}{d - ROE \cdot Retain}$$

If the discount rate is chosen to be the average value yield estimate for the S&P 500 stock market index, which is 8.3% according to section 7.5.4, then the value to eternal shareholders is:

$$v = \frac{Earnings \cdot (1 - Retain)}{d - ROE \cdot Retain} = \frac{USD\ 1.7b \cdot (1 - Retain)}{8.3\% - 17\% \cdot Retain}$$

If no earnings are retained so $Retain = 0$ then the value is:

$$v = \frac{USD\ 1.7b}{8.3\%} \simeq USD\ 20.5b$$

Eq. 9-1

[29] Form 10-K annual report for 2002 filed with US SEC:
www.sec.gov/Archives/edgar/data/63908/000104746903008450/a2105107z10-k.htm

The lowest market-cap for 2003 was about USD 15.4b, see Table 5. If the dividends of USD 504m had instead been used for a share buyback at this lowest market-cap, then according to Eq. 4-7 the relative value of the share buyback would have been:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 504m}{USD\ 20.5b}}{1 - \frac{USD\ 504m}{USD\ 15.4b}} \simeq 100.8\%$$

That is, the value to eternal shareholders would have increased by about 0.8%.

Now assume 10% of earnings were retained each year for eternity and ROE was 17% for eternity, then the value to eternal shareholders would be:

$$v = \frac{Earnings \cdot (1 - Retain)}{d - ROE \cdot Retain} = \frac{USD\ 1.7b \cdot 90\%}{8.3\% - 17\% \cdot 10\%} = USD\ 23.2b$$

The relative value of making a share buyback instead of the dividend payout would then be:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 504m}{USD\ 23.2b}}{1 - \frac{USD\ 504m}{USD\ 15.4b}} \simeq 101.1\%$$

That is, the value to eternal shareholders would have increased by about 1.1%.

These calculations used a discount rate of 8.3% which was the average value yield estimate for the S&P 500 index for the period 1983-2011. If a risk premium would be deemed appropriate so the discount rate would instead be, say, 10%, then the value to eternal shareholders with no earnings growth would be:

$$v = \frac{Earnings}{d} = \frac{USD\ 1.7b}{10\%} \simeq USD\ 17b$$

Eq. 9-2

At the lowest market-cap of USD 15.4b during 2003, the relative value of the share buyback would be:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 504m}{USD\ 17b}}{1 - \frac{USD\ 504m}{USD\ 15.4b}} \simeq 100.3\%$$

So if earnings could be maintained at USD 1.7b for eternity, then using the cash designated for dividend payouts to make a share buyback instead, at the lowest market-cap during 2003, would likely have increased value to eternal shareholders somewhat. At the time, it would have been a qualitative assessment whether the assumptions of these calculations were reasonable.

Ex-Post Analysis

The value to eternal shareholders is now calculated for the year 2003 using the company's FCF for the period 2003-2011 from Table 5. FCF is used as an estimate of how much cash the company generated after funding its investments, and which could then be paid out as dividends to shareholders. The company's

share issuances were much lower than the share buybacks and dividends so the company's growth was not funded from the issuance of new equity capital. Debt changes would affect the amount of cash that would be available for dividend payouts but that is not taken into account here.

The value is calculated as the present value of FCF for the period 2003-2011 plus a terminal value which estimates the present value of the unknown earnings beyond year 2011:

$$v = v_{FCF} + v_{Terminal}$$

The present value of the FCF for the period 2003-2011 is:

$$\begin{aligned} v_{FCF} &= \frac{FCF_{2003}}{1+d} + \cdots + \frac{FCF_{2011}}{(1+d)^9} \\ &= \frac{USD\ 1{,}899m}{1+d} + \frac{USD\ 2{,}521m}{(1+d)^2} + \frac{USD\ 2{,}519m}{(1+d)^3} + \frac{USD\ 3{,}068m}{(1+d)^4} + \frac{USD\ 3{,}726m}{(1+d)^5} \\ &+ \frac{USD\ 4{,}292m}{(1+d)^6} + \frac{USD\ 4{,}096m}{(1+d)^7} + \frac{USD\ 4{,}286m}{(1+d)^8} + \frac{USD\ 4{,}579m}{(1+d)^9} \end{aligned}$$

Assuming the company stops growing after 2011 and the reported net income of USD 5.5b for 2011 is the earnings that could be paid out as dividends and it remains constant for eternity, means the terminal value is calculated using Eq. 13-1 with zero growth:

$$v_{Terminal} = \frac{USD\ 5{,}503m}{(1+d)^{10}} \cdot \frac{1+d}{d} = \frac{USD\ 5{,}503m}{d \cdot (1+d)^9}$$

The historical average value yield estimate for the S&P 500 stock market index is 8.3% according to section 7.5.4. Using this as the discount rate results in the following value of the company to eternal shareholders:

$$v = v_{FCF} + v_{Terminal} \simeq USD\ 20.1b + USD\ 32.3b \simeq USD\ 52.4b$$

Although this is dominated by the terminal value, the present value of the FCF for the period 2003-2011 was USD 20.1b which was still higher than the lowest market-cap of USD 15.4b in 2003. So according to Eq. 4-9 the terminal value should merely be greater than USD (4.7)b in order for a share buyback at the lowest market-cap of 2003 to increase value to eternal shareholders.

The relative value of share buybacks is calculated using Eq. 4-7, with the share buyback amount being USD 504m which were used for dividend payouts in 2003. At the lowest market-cap of USD 15.4b for the year, the relative value would be:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 504m}{USD\ 52.4b}}{1 - \frac{USD\ 504m}{USD\ 15.4b}} \simeq 102.4\%$$

That is, using the USD 504m for share buybacks instead of dividend payouts in 2003 would have increased the value to eternal shareholders by about 2.4%.

These calculations used the historical average rate of return that could be expected from the S&P 500 index as the discount rate in valuation. Using a risk premium so the discount rate is instead, say, 10%, the value decreases to:

$$v = v_{FCF} + v_{Terminal} \simeq USD\ 18.6b + USD\ 23.3b \simeq USD\ 41.9b$$

The relative value of a share buyback at the lowest market-cap of USD 15.4b during 2003 is then:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 504m}{USD\ 41.9b}}{1 - \frac{USD\ 504m}{USD\ 15.4b}} \simeq 102.1\%$$

That is, the value to eternal shareholders would have increased about 2.1% as a result of making a share buyback instead of a dividend payout.

Share Buyback or Alternative Investment, Ex-Ante Analysis

In 2003 the company made dividend payouts and share buybacks for a total of USD 943m, see Table 5. Suppose the company had instead invested in expanding its business rather than making dividend payouts and share buybacks. Which choice would have been more valuable to eternal shareholders?

Assume the annual return on the investment is constant for eternity so the formulas from section 4.7.2 can be used to compare the value of the investment to the value of a share buyback. In order to use those formulas both the investment and share buyback must increase value to eternal shareholders, which will therefore be assessed first.

According to the ex-ante analysis above, the value to eternal shareholders would have increased if share buybacks were made at the lowest market-cap of around USD 15b during 2003, and assuming all future earnings are either paid out as dividends or partly retained so as to increase future earnings at a constant ROE of 17% for eternity.

According to Eq. 4-38 the investment would increase value to eternal shareholders if the annual rate of return on the investment (ROI) is greater than the discount rate used to calculate the present value of the investment return. Various discount rates were used in the above valuations to show their effect on the share buyback decision. One discount rate was chosen to be the estimated value yield for the S&P 500 stock market index which was 8.3% on average during the period 1983-2011, see section 7.5.4. Another discount rate was chosen to be 10% so as to include a risk premium over the S&P 500 index if that would have been deemed appropriate according to a qualitative assessment. Assuming ROE is 17% for eternity, as was assumed in the valuation of the share buyback, ROI on the investment will also be 17% for eternity if the capital structure remains unchanged. This ROI is significantly higher than the discount rates used in this valuation which means the investment would increase value to eternal shareholders. Whether these assumptions are reasonable would have been a qualitative assessment at the time.

In calculating the relative value of a share buyback and investment, first assume earnings are USD 1.7b per year for eternity and no earnings are retained, and let the discount rate be the historical average value yield estimate of the S&P 500 index so that $d = d' = 8.3\%$.

The value to eternal shareholders when the company neither makes a share buyback or investment was then calculated in Eq. 9-1 to be $v \simeq USD\ 20.5b$. Assume ROI is 17% for eternity, and shares were bought at the lowest market-cap of USD 15.4b during 2003, and the amount to be used for either the share buyback or investment was USD 943m, which was the total amount the company used for share buybacks and dividends during 2003. The value of the investment relative to a share buyback is then calculated using Eq. 4-39:

$$\frac{V_{Invest}}{W} = \left(1 + \frac{ROI}{d' \cdot \left(\frac{v}{Invest} - 1\right)}\right) \cdot \left(1 - \frac{Invest}{MarketCap}\right)$$
$$= \left(1 + \frac{17\%}{8.3\% \cdot \left(\frac{USD\ 20.5b}{USD\ 943m} - 1\right)}\right) \cdot \left(1 - \frac{USD\ 943m}{USD\ 15.4b}\right) \simeq 103.1\%$$

This means the investment would be about 3.1% more valuable to eternal shareholders than a share buyback. This was at the lowest market-cap during 2003. At the highest market-cap of USD 34.3b during that year, the investment would be about 6.9% more valuable than a share buyback, partly because a share buyback at a market-cap of USD 34.3b would have decreased value to eternal shareholders if the value was only USD 20.5b, as assumed here.

The equilibrium is where a share buyback starts to increase value to eternal shareholders more than the investment and it is calculated using Eq. 4-40:

$$W > V_{Invest} \Leftrightarrow MarketCap < Invest + (v - Invest) \cdot \frac{d'}{ROI}$$
$$= USD\ 943m + (USD\ 20.5b - USD\ 943m) \cdot \frac{8.3\%}{17\%} \simeq USD\ 10.5b$$

The value to eternal shareholders is USD 20.5b and it follows from Eq. 4-9 that a share buyback increases value to eternal shareholders when the market-cap is below this value, but if an alternative investment is available that yields 17% per year for eternity, then the market-cap has to be less than USD 10.5b, which is about half the normal equilibrium of USD 20.5b, in order for the share buyback to be more valuable than the investment.

The equilibrium can also be formulated in terms of the ROI using Eq. 4-41. Assuming shares would be bought back at the lowest market-cap of USD 15.4b during 2003, the equilibrium is:

$$W > V_{Invest} \Leftrightarrow ROI < d' \cdot \frac{v - Invest}{MarketCap - Invest} = 8.3\% \cdot \frac{USD\ 20.5b - USD\ 943m}{USD\ 15.4b - USD\ 943m} \simeq 11.2\%$$

This means a share buyback would be more valuable than an investment if the annual rate of return on that investment is less than 11.2%.

The equilibrium can also be formulated in terms of the investment amount which by assumption equals the share buyback amount. According to Eq. 4-42 the equilibrium is:

$$W > V_{Invest} \Leftrightarrow Invest > \frac{MarketCap \cdot ROI - d' \cdot v}{ROI - d'} = \frac{USD\ 15.4b \cdot 17\% - 8.3\% \cdot USD\ 20.5b}{17\% - 8.3\%} \simeq USD\ 10.5b$$

This means that when ROI is 17% for eternity, the discount rate is 8.3%, the market-cap is USD 15.4b and the value to eternal shareholders is USD 20.5b without a share buyback or investment, then a share buyback is more valuable than an investment if the amount used for the share buyback or investment is greater than about USD 10.5b, which is far more than the company could spend on this in 2003 so the investment would be the optimal choice.

If the discount rate includes a risk premium so it is 10% instead of 8.3% then the value to eternal shareholders without a share buyback or investment is USD 17b according to Eq. 9-2. At the lowest market-cap of USD 15.4b during 2003, the value of the investment relative to that of a share buyback is:

$$\frac{V_{Invest}}{W} = \left(1 + \frac{ROI}{d' \cdot \left(\frac{v}{Invest} - 1\right)}\right) \cdot \left(1 - \frac{Invest}{MarketCap}\right) = \left(1 + \frac{17\%}{10\% \cdot \left(\frac{USD\ 17b}{USD\ 943m} - 1\right)}\right) \cdot \left(1 - \frac{USD\ 943m}{USD\ 15.4b}\right) \simeq 103.2\%$$

So the investment would increase value to eternal shareholders about 3.2% more than a share buyback.

The equilibrium formulated in terms of market-cap is:

$$W > V_{Invest} \Leftrightarrow MarketCap < Invest + (v - Invest) \cdot \frac{d'}{ROI} = USD\ 943m + (USD\ 17b - USD\ 943m) \cdot \frac{10\%}{17\%} \simeq USD\ 10.4b$$

The equilibrium formulated in terms of ROI is:

$$W > V_{Invest} \Leftrightarrow ROI < d' \cdot \frac{v - Invest}{MarketCap - Invest} = 10\% \cdot \frac{USD\ 17b - USD\ 943m}{USD\ 15.4b - USD\ 943m} \simeq 11.1\%$$

The equilibrium formulated in terms of the investment or share buyback amount is:

$$W > V_{Invest} \Leftrightarrow Invest > \frac{MarketCap \cdot ROI - d' \cdot v}{ROI - d'} = \frac{USD\ 15.4b \cdot 17\% - 10\% \cdot USD\ 17b}{17\% - 10\%} \simeq USD\ 13.1b$$

Note that when the discount rate is changed from 8.3% to 10%, the equilibriums formulated in terms of market-cap and ROI change little compared to the equilibrium formulated in terms of the investment or share buyback amount.

To demonstrate the effect that a change in investment and share buyback amount has on the relative value and equilibriums, consider the case from above where the discount rate is 10%. The equilibrium investment and share buyback amount is about USD 13.1b as calculated above. If $Invest = Buyback = USD\ 12b$ then the value of the share buyback is $W \simeq 133\% \cdot V$ as calculated from Eq. 4-7, and the value of the investment is $V_{Invest} \simeq 149\% \cdot V$ as calculated from Eq. 4-37. If instead $Invest = Buyback = USD\ 15b$ then the value of the share buyback is $W \simeq 453\% \cdot V$ and the value of the investment is $V_{Invest} = 162\% \cdot V$. Equilibrium is when $Invest = Buyback \simeq USD\ 13{,}114m$ so $W = V_{Invest} \simeq 154\% \cdot V$. The reason for share buybacks and investments affecting the value to eternal shareholders differently depending on the amount used for the share buyback or investment, is that the relative value to eternal shareholders changes linearly with the investment amount, while the relative value changes non-linearly with the share buyback amount and magnifies any mispricing between market-cap and v, as depicted in Figure 6.

9.4.4. Conclusion

The company made large share buybacks relative to dividends during 1999 when the market-cap was high relative to earnings, thus creating an obligation for future earnings growth in order for the share buybacks to merely be value neutral to eternal shareholders. Conversely, in 2003 the company's market-cap was low relative to its historical earnings but the company then decreased share buybacks relative to dividends. This is the opposite of what the company should have done. In 1999 the company should not have made any share buybacks but instead have made a dividend payout, debt repayment, or simply held the cash. In 2003 the company should have made a share buyback instead of a dividend payout so as to increase value to eternal shareholders. Although this is easy to see afterwards, the formulas of this treatise could have been used for ex-ante assessments that would have suggested the same.

The company's share buyback policy in year 2011 is substantially unchanged from the years studied here. In particular, the share buyback policy does not mention a price limit below which shares are believed to increase value to eternal shareholders, so similar share buyback mistakes may recur in the future.

Furthermore, it was shown that if the company could instead re-invest in its business for the funds that were used for dividend payouts and share buybacks in 2003, then an investment with an annual rate of return above 11.2% would have increased value to eternal shareholders more than a share buyback at the lowest market-cap of that year.

9.5. Hewlett-Packard

The company *Hewlett-Packard* (HP) was started by two people in USA in 1939 as a producer of electronic equipment. At the end of fiscal year 2011 the company had grown to employ almost 350k people worldwide with annual revenue of USD 127b and reported net income of USD 7.1b. Sale of computers accounted for 30% of total revenue, consulting and outsourcing services was 28% of revenue, sale of printers and scanners was 20% of revenue, sale of enterprise computer servers and equipment was 17% of revenue, and the remaining revenue was from software and financial services.[30]

9.5.1. Share Buyback Policy

The company's share buyback policy has remained largely unchanged in recent years. For example, the annual reports for fiscal years 2006 and 2011 both have this text with only the year being different:[31]

> *"HP repurchased shares in the fourth quarter of fiscal 2006 [2011] under an ongoing program to manage the dilution created by shares issued under employee stock plans as well as to repurchase shares opportunistically. This program, which does not have a specific expiration date, authorizes repurchases in the open market or in private transactions."*

The annual report for 2006 also has the following text which is absent in more recent years:

> *"The prices at which HP purchases shares under the PVSPP [prepaid variable share purchase program] are subject to a minimum and maximum that were determined in advance of any repurchases being completed, thereby effectively hedging HP's repurchase price. The exact number of shares to be repurchased is based upon the volume weighted average market price of HP's shares during each weekly settlement period, subject to the minimum and maximum price as well as regulatory limitations on the number of shares HP is permitted to repurchase. HP decreases its shares outstanding each settlement period as shares are physically received. HP will retire all shares repurchased under the PVSPP, and HP will no longer deem those shares outstanding."*

This suggests a share-price was first determined and a hedging mechanism then employed so as to guarantee shares would be bought at the predetermined price. But the share-price is not specified.

The annual report for 2011 describes the source of cash for share buybacks:

> *"We use cash generated by operations as our primary source of liquidity; we believe that internally generated cash flows are generally sufficient to support business operations, capital expenditures and the payment of stockholder dividends, in addition to discretionary investments and share repurchases."*

Although the source of cash for share buybacks is described, the company neglects to inform shareholders of the share-price below which shares may be bought back and how this relates to management's assessment of the company's value to eternal shareholders, which is the equilibrium above which share buybacks will decrease value to eternal shareholders. This information is crucial for outside shareholders to estimate the effect that share buybacks may have on the value to eternal shareholders. According to the

[30] Form 10-K annual report for 2011 filed with US SEC:
www.sec.gov/Archives/edgar/data/47217/000104746911010094/a2206500z10-k.htm
[31] Form 10-K annual report for 2006 filed with US SEC:
www.sec.gov/Archives/edgar/data/47217/000104746906015253/a2174889z10-k.htm

policy quoted above, the primary reason for the company to make share buybacks seems to be an effort to offset the diluting impact of employee stock options. Combined with the lack of valuation considerations this suggests the company's management misunderstands the effect share buybacks have on the value to eternal shareholders.

Terminology

A recent and typical earnings report of the company states: [32] *"(...) we returned USD 384 million to shareholders in the form of share repurchases and dividends."* According to the cash flow statement of that report USD 124m of these USD 384m were used for share buybacks. However, during that 3 month period the share-price decreased about 24% and the market-cap decreased about USD 8.6b. Since the company's management claims it returned USD 124m to shareholders via share buybacks, it implies they believe that in the absence of this share buyback the market-cap would instead have decreased by the USD 8.6b experienced and an additional USD 124m that were used for share buybacks. This is a common example of the absurd terminology used by both practitioners and scholars which hinders a proper understanding of share buyback valuation and hence the establishing of sound share buyback policies and practices.

9.5.2. Analysis of Share Buyback and Dividend Trends

The company's historical allocation between share buybacks and dividends is shown in the scatter-plots of Figure 17 which use the ex-post data from Table 6 in valuation. The plots show that as the company's market-cap increases relative to the present value of the company's future earnings (calculations are detailed below), the company has a tendency to increase its share buybacks. This is opposite of what the company should have done. Whether the share buybacks can be said to have decreased or increased value to eternal shareholders depends in part on the valuation assumptions, namely the choice of discount rate and terminal value, but even if all the share buybacks were accretive to shareholder value, the allocation between share buybacks and dividends should still have been opposite so that share buybacks would be increased as the market-cap became lower relative to the value to eternal shareholders.

The value to eternal shareholders is the equilibrium below which a share buyback increases value to eternal shareholders, see Eq. 4-9. This value can be estimated from the company's excess cash, plus the present value of the earnings which could have been paid out as dividends for a known period, plus a terminal value in which a mathematical model estimates the unknown earnings for eternity. The plots in Figure 17 use the Free Cash Flow (FCF) minus acquisitions as an estimate of the earnings that could be paid out as dividends during the period 1996-2012, and starting in year 2013 the annual earnings are assumed to be constant for eternity. The company appears to have had excess cash sometimes but for simplicity it will be assumed the company did not have excess cash during this period; the findings of this study are materially unaffected by this assumption. The discount rate used in this valuation is chosen to be the historical average value yield for the S&P 500 stock market index which is 8.3% according to section 7.5.4.

For example, the value to eternal shareholders at the beginning of fiscal year 2011 is calculated as:

$$v_{2011} = v_{FCF} + v_{Terminal}$$

[32] Form 8-K with unaudited 4th quarter and annual results for 2012 filed with US SEC: www.sec.gov/Archives/edgar/data/47217/000004721712000033/ex99-1_112012.htm

The present value of the FCF for the known period is calculated using the ex-post data from Table 6:

$$v_{FCF} = \frac{FCF_{2011} - Acquisitions_{2011}}{1+d} + \frac{FCF_{2012} - Acquisitions_{2012}}{(1+d)^2}$$
$$= \frac{USD\ 9{,}099m - USD\ 10{,}391m}{1+8.3\%} + \frac{USD\ 7{,}482m - USD\ 57m}{(1+8.3\%)^2} \simeq USD\ 5.1b$$

If the earnings that could be paid out as dividends to shareholders in 2013 are assumed to be, say, USD 6b, which is 5% of the revenue for 2012 corresponding to the company's average net profit margin for the period 1996-2012, and the earnings are assumed to remain constant for eternity, then their present value is calculated using Eq. 13-1 with zero growth:

$$v_{Terminal} = \frac{USD\ 6b}{(1+d)^3} \cdot \frac{1+d}{d} = \frac{USD\ 6b}{d \cdot (1+d)^2} = \frac{USD\ 6b}{8.3\% \cdot (1+8.3\%)^2} \simeq USD\ 61.6b$$

Note that this is discounted by $(1+d)^3$ to give the present value at the beginning of fiscal year 2011.

The estimated value to eternal shareholders is then:

$$v_{2011} = v_{FCF} + v_{Terminal} \simeq USD\ 5.1b + USD\ 61.6b \simeq USD\ 66.7b$$

The terminal value is the dominating factor and whether its estimate is reasonable is a qualitative assessment left to the reader.

Changing the assumptions in this valuation, e.g. increasing the discount rate to reflect that the future earnings of this company are more uncertain than the aggregate future earnings of the S&P 500 companies, or changing the terminal value, or using reported net income instead of FCF, all result in the scatter-plots of Figure 17 having similar trend-lines but the horizontal axis and hence the share buyback equilibrium is shifted. Furthermore, the company's large acquisition in 2002 was paid with stock which complicates the calculation of FCF and diluting impact. A simple solution is to remove year 2002 from the scatter-plots in Figure 17 which has a negligible impact on the trend-lines so the findings of this study are unaffected.

Year 2010

Fiscal year 2010 is an example of the company making a large share buyback while the market-cap was high in relation to the estimated value of the company. The ex-post data from Table 6 with calculations similar to above are used to estimate the value to eternal shareholders at the beginning of 2010:

$$v_{2010} \simeq USD\ 61b$$

The share buyback for that year was USD 11b. The lowest market-cap for the year was about USD 87b. If the share buyback had been made at this market-cap, then according to Eq. 4-7 the relative value of the share buyback would be:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 11b}{USD\ 61b}}{1 - \frac{USD\ 11b}{USD\ 87b}} \simeq 94\%$$

That is, the share buyback would have decreased value to eternal shareholders by about 6%.

If instead the share buyback had been made at the highest market-cap of USD 127b during that year, the relative value of the share buyback would be:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 11b}{USD\ 61b}}{1 - \frac{USD\ 11b}{USD\ 127b}} \simeq 90\%$$

That is, the share buyback would have decreased value to eternal shareholders by about 10%.

Year 2012

Fiscal year 2012 is an example of the company significantly decreasing share buybacks while the market-cap was low relative to the estimated value to eternal shareholders. The ex-post data from Table 6 is used with calculations similar to above, still assuming earnings are USD 6b for eternity starting in 2013 and the discount rate is 8.3%. The estimated value to eternal shareholders at the beginning of 2012 is:

$$v_{2012} \simeq USD\ 73.7b$$

Assume that all FCF for the year 2012 in the amount of USD 7.5b were used for a share buyback. At the highest market-cap of USD 59b during that year, the relative value of a share buyback would be:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 7.5b}{USD\ 73.7b}}{1 - \frac{USD\ 7.5b}{USD\ 59b}} \simeq 103\%$$

At the lowest market-cap of USD 27b during that year, the relative value of a share buyback would be:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 7.5b}{USD\ 73.7b}}{1 - \frac{USD\ 7.5b}{USD\ 27b}} \simeq 124\%$$

That is, at the highest market-cap during 2012 the share buyback would have increased value to eternal shareholders by about 3%. At the lowest market-cap during that year, the share buyback would have increased value to eternal shareholders by about 24%.

In late November 2012, at the time of this writing, the company's market-cap is about USD 24b, which means a share buyback of USD 7.5b would increase value to eternal shareholders by about 31% under these valuation assumptions.

It is a qualitative assessment left to the reader whether the company's future earnings can be at least USD 6b for eternity and whether a 8.3% discount rate is adequate, as assumed in all these valuations.

9.5.3. Analysis of Autonomy Acquisition

In fiscal year 2011 the company HP acquired the company *Autonomy* for USD 10.2b in cash.[33] Autonomy was founded by two people in 1996 and at the end of 2010 it employed about 1.9k people, having annual revenue of USD 870m and reported net income of USD 217m. Autonomy provides information technology for understanding and searching unstructured data.

Required Earnings Growth

Assuming there are no synergy effects between the companies HP and Autonomy and that there are no tax implications when Autonomy becomes a subsidiary of HP, Autonomy can be valued as a separate entity in the usual manner.

At the end of 2010 the company had more than USD 1b in cash and equivalents, some of which may have been in excess and would be separated in normal valuation. However, the acquisition price of USD 10.2b was net of Autonomy's cash holdings which will therefore be ignored in this valuation.

Financial data for Autonomy is shown in Table 7, according to which the company's reported net income and FCF were similar during the period 2005-2010, so the reported net income will be taken here as a reasonable estimate of the starting earnings the company could pay out as dividends. In 2010 those earnings were USD 217m and as the acquisition price of the company was USD 10.2b the earnings yield was about 2.1%. During that year the yield on long-term USA government bonds was about 3.4% on average, see Figure 9. This either means Autonomy's future earnings were deemed more certain than the yield on long-term USA government bonds, or future earnings were expected to grow, or both.

Assuming the company's earnings grow at a constant rate for eternity starting in 2011, Eq. 3-9 can be used to determine the minimum required annual growth rate g, if the rate of return to eternal shareholders is to be greater than what could be obtained from an alternative investment, in this case the average yield on USA government bonds of 3.4% during 2011 which is used as the discount rate d:

$$g > \frac{d - \frac{Earnings}{MarketCap - Excess\ Cash}}{1 + \frac{Earnings}{MarketCap - Excess\ Cash}} = \frac{3.4\% - \frac{USD\ 217m}{USD\ 10.2b}}{1 + \frac{USD\ 217m}{USD\ 10.2b}} \simeq 1.2\%$$

That is, in order for the company Autonomy to have a rate of return greater than what could be obtained from long-term USA government bonds in 2011, the company would have to grow its future earnings by at least 1.2% each year for eternity.

[33] In February 2011 the company Autonomy had 242.7m shares outstanding according to its annual report for 2010, which is currently unavailable from the company's website (www.autonomy.com). The acquisition offer was USD 42.11 per share giving a total amount of USD 10.2b, which was apparently accepted by the Autonomy shareholders. Hewlett-Packard's Form 8-K for August 18, 2011 and annual report for 2011, both filed with US SEC:
www.sec.gov/Archives/edgar/data/47217/000004721711000039/ex99-1_082011.htm
www.sec.gov/Archives/edgar/data/47217/000104746911010094/a2206500z10-k.htm

However, during 2011 the yield on USA government bonds was considerably lower than its historical average of 5%, see section 7.4. Using this historical average as the discount rate instead, gives the required earnings growth rate:

$$g > \frac{5\% - \frac{USD\ 217m}{USD\ 10.2b}}{1 + \frac{USD\ 217m}{USD\ 10.2b}} \simeq 2.8\%$$

Using as discount rate the historical average value yield for the S&P 500 stock market index of 8.3%, see section 7.5.4, which was slightly below the lowest mean value yield estimate in 2011, see Table 2, gives the required earnings growth rate:

$$g > \frac{8.3\% - \frac{USD\ 217m}{USD\ 10.2b}}{1 + \frac{USD\ 217m}{USD\ 10.2b}} \simeq 6\%$$

Eq. 9-3

If a risk premium is deemed necessary because Autonomy's future earnings are deemed more uncertain than the aggregate earnings of the S&P 500 companies, so the discount rate is instead chosen to be, say, $d = 10\%$, then the required growth rate is:

$$g > \frac{10\% - \frac{USD\ 217m}{USD\ 10.2b}}{1 + \frac{USD\ 217m}{USD\ 10.2b}} \simeq 7.7\%$$

In summary, the acquisition price of Autonomy implies HP believes that Autonomy's future earnings are either more certain than the promised yield on long-term USA government bonds, or the company is expected to grow its earnings by at least 1.2% for eternity, or both. If instead Autonomy's future earnings are deemed equally uncertain as the aggregate future earnings of the companies in the S&P 500 index, then the required earnings growth for Autonomy would be 6% each year for eternity in order for the acquisition price to equal the present value of Autonomy's future earnings. If a risk premium is deemed appropriate because Autonomy's earnings are perceived to be more uncertain than the aggregate future earnings of the S&P 500 companies, then Autonomy must grow its earnings further still. Furthermore, it was assumed that the earnings growth would not be funded from retained earnings or issuance of new equity capital otherwise the growth rates would have to be greater still.

In the period 2005-2010 the compounded annual growth rate for Autonomy's revenue was 55% while it was 89% for its reported net income. However, Autonomy made large acquisitions during this period which may have significantly contributed to this growth. Whether the company's risk characteristics and future earnings growth possibilities justify the acquisition price is a qualitative assessment left to the reader.

Relative Value of Acquisition and Share Buyback

In 2011 HP could choose between acquiring Autonomy or use the cash for making a share buyback instead. The question is which would have been more valuable to the eternal shareholders of HP. To answer this we

first need to know the value of HP without either the acquisition or share buyback. The following is an ex-ante analysis so as to simulate the valuation with the information available at the time.

In 2010 HP's reported net income was USD 8.8b and the FCF estimate was USD 8.4b, see Table 6. If HP's future earnings that could be paid out as dividends are assumed to be, say, USD 8.5b each year for eternity, then Eq. 13-2 with zero growth gives the value to eternal shareholders. The discount rate can be chosen as 8.3% which was the historical average value yield for the S&P 500 stock market index, provided the future earnings of HP were deemed equally uncertain as the aggregate future earnings of the companies in the S&P 500 index. The value of HP to its eternal shareholders would then be:

$$v = \frac{Earnings}{d} = \frac{USD\ 8.5b}{8.3\%} \simeq USD\ 102b$$

However, the reported net incomes decreased substantially in 2011 and 2012 and although FCF increased in 2011 it too decreased substantially in 2012, see Table 6. So it can be argued that the discount rate used in this valuation should include a risk premium to reflect the greater uncertainty of HP's future earnings relative to those of the S&P 500 index, in which case the value v would decrease, e.g. if $d = 10\%$ then $v = USD\ 85b$. A compromise between these two estimates is used in the following so that:

$$v = USD\ 90b$$

At the end of fiscal year 2010, HP had USD 10.9b in cash and equivalents, while its total current assets were USD 54b compared to total current liabilities of USD 49b and long-term debt of USD 15b.[34] During 2011 the company made a recapitalization by increasing debt while making share buybacks and acquisitions. This requires variations of the valuation formulas in section 4.7 and 4.9 but for simplicity the company is merely assumed to have had no excess cash and the debt increase is ignored.

Different scenarios will now compare the value of the Autonomy acquisition to that of a share buyback. In the first scenario the price HP paid to acquire Autonomy is assumed to equal its present value to HP's eternal shareholders, that is, $Invest = Return = USD\ 10.2b$. This means there is no change in the value to eternal shareholders from making the acquisition which is then said to be value neutral. Since $V_{Invest} = V$ the usual share buyback equilibrium from Eq. 4-9 holds:

$$W > V_{Invest} \Leftrightarrow W > V \Leftrightarrow MarketCap < v \Leftrightarrow MarketCap < USD\ 90b$$

According to Table 6, HP's highest market-cap during fiscal year 2011 was about USD 103b, but for a large part of that year the market-cap was slightly lower, perhaps USD 95b on average. If the value estimate of USD 90b was correct then a share buyback at these market-cap levels would decrease the value to eternal shareholders. So in this scenario the share buyback would decrease the value to eternal shareholders while the acquisition of Autonomy would be value neutral to HP's eternal shareholders.

[34] Form 10-K annual report for 2010 filed with US SEC:
www.sec.gov/Archives/edgar/data/47217/000104746910010444/a2201180z10-k.htm

In 2012 HP made an impairment of the goodwill and intangible assets that arose from the Autonomy acquisition of the prior year in the amount of USD 8.8b.[35] This was accompanied by an accusation of accounting fraud against Autonomy prior to its acquisition, which HP claimed had inflated the acquisition price. This will be discussed in more detail below. To estimate the value of the acquisition relative to a share buyback with the realization that the acquisition price was too high, Eq. 4-31 is used with $Invest = Buyback = USD\ 10.2b$ and $Return = USD\ 10.2b - USD\ 8.8b = USD\ 1.4b$ which is the acquisition price minus the impairment charge in the year following the acquisition of Autonomy (the time value of money is ignored). The value of the acquisition relative to a share buyback is calculated using Eq. 4-31, and at the highest market-cap of USD 103b during 2011 it was:

$$\frac{V_{Invest}}{W} = \left(1 + \frac{Return}{v - Invest}\right) \cdot \left(1 - \frac{Invest}{MarketCap}\right) = \left(1 + \frac{USD\ 1.4b}{USD\ 90b - USD\ 10.2b}\right) \cdot \left(1 - \frac{USD\ 10.2b}{USD\ 103b}\right) \simeq 91.7\%$$

That is, even if the share buyback had been made at a market-cap of USD 103b which was higher than the equilibrium of USD 90b and thereby causing a loss to HP's eternal shareholders, the acquisition of Autonomy was still an 8.3% greater loss in comparison to the share buyback.

During fiscal year 2011 the lowest market-cap for HP was about USD 45b. This occurred after the acquisition of Autonomy was announced so there may be a wholly or partial causal effect between the two events. However, if it is assumed that HP could have chosen between the Autonomy acquisition or making a share buyback when the market-cap was USD 45b, then the relative value would be:

$$\frac{V_{Invest}}{W} = \left(1 + \frac{Return}{v - Invest}\right) \cdot \left(1 - \frac{Invest}{MarketCap}\right) = \left(1 + \frac{USD\ 1.4b}{USD\ 90b - USD\ 10.2b}\right) \cdot \left(1 - \frac{USD\ 10.2b}{USD\ 45b}\right) \simeq 78.7\%$$

That is, under these valuation assumptions, the value to HP's eternal shareholders from making the acquisition of Autonomy was about 21.3% less than the value of a share buyback.

If the acquisition of Autonomy had instead been value neutral to the eternal shareholders of HP, then a share buyback for USD 10.2b at the market-cap of USD 45b would have resulted in the value to eternal shareholders increasing about 15% according to Eq. 4-7:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 10.2b}{USD\ 90b}}{1 - \frac{USD\ 10.2b}{USD\ 45b}} \simeq 115\%$$

[35] Form 8-K with unaudited 4th quarter and annual results for 2012 filed with US SEC: www.sec.gov/Archives/edgar/data/47217/000004721712000033/ex99-1_112012.htm

Now consider the value of the Autonomy acquisition to be, say, twice the price paid, that is, $Invest = USD\ 10.2b$ and $Return = 2 \cdot Invest = USD\ 20.4b$, then at the lowest market-cap of USD 45b during 2011, the value of the acquisition relative to a share buyback would be:

$$\frac{V_{Invest}}{W} = \left(1 + \frac{Return}{v - Invest}\right) \cdot \left(1 - \frac{Invest}{MarketCap}\right) = \left(1 + \frac{USD\ 20.4b}{USD\ 90b - USD\ 10.2b}\right) \cdot \left(1 - \frac{USD\ 10.2b}{USD\ 45b}\right) \simeq 97\%$$

That is, even if the present value of Autonomy had actually been twice its acquisition price, a share buyback at the lowest market-cap of USD 45b would still have been about 3% more valuable to the eternal shareholders of HP.

The equilibrium where the share buyback starts to become more valuable than the acquisition can be formulated in terms of the present value of the return on the acquisition using Eq. 4-33:

$$W > V_{Invest} \Leftrightarrow Return < \frac{Invest \cdot (v - Invest)}{MarketCap - Invest} \Leftrightarrow Return < \frac{USD\ 10.2b \cdot (USD\ 90b - USD\ 10.2b)}{USD\ 45b - USD\ 10.2b} \simeq USD\ 23.4b$$

That is, as long as the present value of the return on the acquisition is less than about USD 23.4b, then a share buyback at a market-cap of USD 45b would be more valuable to eternal shareholders of HP.

In order for the present value of the Autonomy acquisition to be higher than USD 23.4b, the required earnings growth rate for Autonomy can be calculated using Eq. 3-9 as also done above. Using as discount rate 8.3% which is the historical average value yield for the S&P 500 index, gives a required growth rate:[36]

$$g > \frac{8.3\% - \frac{USD\ 217m}{USD\ 23.4b}}{1 + \frac{USD\ 217m}{USD\ 23.4b}} \simeq 7.3\%$$

That is, Autonomy's earnings would have to grow at least 7.3% for eternity if the present value should be at least USD 23.4b. If a discount rate of, say, $d = 10\%$ was used instead, then Autonomy's required earnings growth rate would be $g > 9\%$ for eternity. Otherwise a share buyback for HP at a market-cap of USD 45b would be more valuable to the eternal shareholders of HP. This assumed Autonomy's earnings could grow without retaining earnings or issuing new equity capital otherwise the required growth rates would be greater still.

Whether these valuation assumptions were reasonable at the time of the acquisition would have been a qualitative assessment which is left to the reader.

Accusation of Fraud

In November 2012 HP wrote down the goodwill and intangible assets that arose from the acquisition of Autonomy in the prior year. The acquisition amount was USD 10.2b which was written down by USD 8.8b. HP also accused Autonomy of having falsified its financial statements prior to the acquisition in an effort to

[36] Although Eq. 3-9 is formulated in terms of the market-cap it may be used with any number.

increase the apparent revenue and profitability of Autonomy and hence increase the acquisition price. HP did not disclose what it believed the true revenue and earnings were.

If Autonomy's financial statements were correct so its earnings for 2010 were USD 217m and this was assumed to grow at a constant annual rate for eternity, then in order for the acquisition of Autonomy to offer the same rate of return to the eternal shareholders of HP as could be expected from investing in the S&P 500 stock market index, it was shown in Eq. 9-3 that Autonomy's earnings would have to grow by 6% for eternity.

If the true earnings for Autonomy were actually USD 100m instead of the reported USD 217m in 2010, then the required earnings growth rate would have been:

$$g > \frac{d - \frac{Earnings}{MarketCap - Excess\ Cash}}{1 + \frac{Earnings}{MarketCap - Excess\ Cash}} = \frac{8.3\% - \frac{USD\ 100m}{USD\ 10.2b}}{1 + \frac{USD\ 100m}{USD\ 10.2b}} \simeq 7.2\%$$

If the true earnings were USD 50m then the required earnings growth rate would have been:

$$g > \frac{8.3\% - \frac{USD\ 50m}{USD\ 10.2b}}{1 + \frac{USD\ 50m}{USD\ 10.2b}} \simeq 7.8\%$$

That is, for the Autonomy acquisition to offer the same rate of return that could be expected from the S&P 500 index, the future earnings of Autonomy would have to grow 6% each year for eternity if the start earnings for 2010 were USD 217m as reported in the published financial statements. If instead the start earnings were really only USD 100m because of fraud then the earnings growth rate would have to be 7.2% for eternity. If the start earnings were only USD 50m then the earnings growth rate would have to be 7.8% for eternity. If a risk premium would have been deemed appropriate so the discount rate would be greater than the expected return on the S&P 500 index, then the required earnings growth rates would have increased as well. If the earnings growth should be funded through retained earnings or issuance of new equity capital then the required growth rate would be higher still. Whether any of this would have made a difference in the decision for HP to acquire Autonomy is a qualitative assessment left to the reader.

9.5.4. Analysis of Restructuring

In 2012 the company HP announced that their workforce of almost 350k employees was to be reduced by 29k employees over the following years in an effort to decrease the company's expenses, increase research and development, and improve profitability.[37] The effect of this on the value to eternal shareholders will now be studied.

There are generally three components in the valuation of a corporate restructuring so as to determine its effect on the value to eternal shareholders. First there is the present value of the company to its eternal shareholders without the restructuring which is the value v as usual. From this the present value of the restructuring costs must be subtracted and the present value of the future cost savings must be added. This

[37] Form 10-Q filed with US SEC for the quarter ending July 31, 2012: www.sec.gov/Archives/edgar/data/47217/000104746912008732/a2210845z10-q.htm

is similar to the valuation of an investment in Eq. 4-28 where the investment amount is the present value of the restructuring costs, i.e. $Invest = Restructuring\ Costs$, and the return is the present value of future savings resulting from the restructuring, i.e. $Return = Savings$, so that:

$$V_{Restructuring} = \frac{(v - Restructuring\ Costs + Savings) \cdot (1 - TaxDividend)}{Shares}$$

According to Table 6 the company's FCF in 2012 was about USD 7.5b. If this is assumed to be the amount that could be paid out as dividends and it is constant for eternity, then the value to eternal shareholders without a restructuring is calculated using Eq. 13-2 with zero growth. If these future earnings are deemed to have the same uncertainty or risk as the aggregate earnings of the companies in the S&P 500 stock market index, then the discount rate can be chosen to be the historical average value yield estimate of 8.3% for the S&P 500 index, see section 7.5.4. The value of the company without a restructuring is then:

$$v = \frac{USD\ 7.5b}{8.3\%} \simeq USD\ 90b$$

The company expects aggregate restructuring charges to be USD 3.7b over a few years. For simplicity, it is assumed that these restructuring costs consist entirely of cash which are paid immediately rather than over multiple years, so that $Restructuring\ Costs = USD\ 3.7b$ in the formula above.

The company does not detail its expected savings from the restructuring so an estimate must be made. At the end of fiscal year 2011 the company had almost 350k employees so a reduction of 29k employees corresponds to about 8.3%. If the earnings are assumed to increase in direct proportion to the reduction in employees, and the FCF of USD 7.5b for 2012 is used as the basis earnings available for dividend payout without a restructuring, then the present value of the restructuring savings is:

$$Savings = \frac{8.3\% \cdot USD\ 7.5b}{8.3\%} \simeq USD\ 7.5b$$

Using the definitions of $V_{Restructuring}$ and V the relative value of the restructuring can be reduced to:

$$\frac{V_{Restructuring}}{V} = 1 + \frac{Savings - Restructuring\ Costs}{v} \simeq 1 + \frac{USD\ 7.5b - USD\ 3.7b}{USD\ 90b} \simeq 104\%$$

If instead the restructuring cost of USD 3.7b is used for a share buyback at the market-cap of about USD 24b at the time of this writing in late November 2012, then the relative value of a share buyback is:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} \simeq \frac{1 - \frac{USD\ 3.7b}{USD\ 90b}}{1 - \frac{USD\ 3.7b}{USD\ 24b}} \simeq 113\%$$

The value of a share buyback relative to the restructuring is then:

$$\frac{W}{V_{Restructuring}} = \frac{W/V}{V_{Restructuring}/V} \simeq \frac{113\%}{104\%} \simeq 109\%$$

That is, the share buyback is worth about 9% more than the restructuring to the eternal shareholders of HP.

A share buyback for USD 3.7b is more valuable to eternal shareholders than a restructuring costing the same amount, if the present value of the savings from the restructuring is less than about USD 16b, as calculated using Eq. 4-33:

$$W > V_{Restructuring} \Leftrightarrow Savings < \frac{Restructuring\ Costs \cdot (v - Restructuring\ Costs)}{MarketCap - Restructuring\ Costs}$$
$$= \frac{USD\ 3.7b \cdot (USD\ 90b - USD\ 3.7b)}{USD\ 24b - USD\ 3.7b} \simeq USD\ 16b$$

If instead the entire FCF of USD 7.5b for 2012 was used for a share buyback at the market-cap of USD 24b, then the relative value of the share buyback would be:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} \simeq \frac{1 - \frac{USD\ 7.5b}{90b}}{1 - \frac{USD\ 7.5b}{USD\ 24b}} \simeq 133\%$$

This share buyback is worth about 28% more than the restructuring to the eternal shareholders of HP:

$$\frac{W}{V_{Restructuring}} = \frac{W/V}{V_{Restructuring}/V} \simeq \frac{133\%}{104\%} \simeq 128\%$$

Return Factor

If the share buyback amount equals the restructuring costs, and the present value of the future savings from the restructuring is assumed to be a factor r of the value v, then Eq. 4-34 gives the equilibrium below which a share buyback is more valuable to eternal shareholders than a restructuring. Note that Eq. 4-34 is formulated in terms of the return on an investment, so the restructuring costs are considered to be the investment amount and the return is the present value of the future savings. The equilibrium is:

$$W > V_{Restructuring} \Leftrightarrow MarketCap < Restructuring\ Costs \cdot \left(1 + \frac{1 - \frac{Restructuring\ Costs}{v}}{r}\right)$$
$$\simeq USD\ 3.7b \cdot \left(1 + \frac{1 - \frac{USD\ 3.7b}{USD\ 90b}}{8.3\%}\right) \simeq USD\ 46b$$

That is, assuming the value to eternal shareholders is $v = USD\ 90b$ and it increases by $r = 8.3\%$ from a restructuring which costs USD 3.7b, then a share buyback for the same amount as the restructuring is more valuable to eternal shareholders if the market-cap is less than USD 46b.

The equilibrium can also be formulated in terms of the return factor r using Eq. 4-35. Assuming $MarketCap = USD\ 24b$ as it is at the time of this writing in late November 2012, the equilibrium is:

$$W > V_{Restructuring} \Leftrightarrow r < \frac{Restructuring\ Costs \cdot \left(1 - \frac{Restructuring\ Costs}{v}\right)}{MarketCap - Restructuring\ Costs}$$
$$\simeq \frac{USD\ 3.7b \cdot \left(1 - \frac{USD\ 3.7b}{USD\ 90b}\right)}{USD\ 24b - USD\ 3.7b} \simeq 17\%$$

That is, under these assumptions of the value v, restructuring costs and market-cap, the restructuring is less valuable to eternal shareholders than a share buyback, if the restructuring results in an increase in the value to eternal shareholders that is less than about 17%.

These valuations made a number of assumptions about the future earnings of the company, restructuring costs and savings, and the choice of discount rate. Whether these assumptions are reasonable is a qualitative assessment left to the reader who is encouraged to experiment with different assumptions.

9.5.5. Conclusion

It was shown that the company has had a tendency to increase share buybacks as the market-cap increased relative to the estimated value to eternal shareholders, which is the opposite of what should have been done. This may be expected to continue in the future because the company's share buyback policy currently does not mention any criteria for buying back shares when the market-cap is below some estimate of the value to eternal shareholders.

It was also shown that the company made a large acquisition at a price which requires substantial future earnings growth of the acquired company, if the rate of return is to equal that expected from the S&P 500 stock market index. Furthermore, the acquisition was made at a time when the company's own market-cap was low relative to various estimates of the value to eternal shareholders, which means a share buyback might have significantly increased the value to eternal shareholders relative to what the acquisition seems to have accomplished.

A company that makes investments, acquisitions and share buybacks at too high prices decreases value to eternal shareholders, which may then be followed by a so-called restructuring in an effort to increase future earnings e.g. by decreasing expenses through a reduction in the number of employees. This company is currently undergoing such a restructuring in which the work-force is to be reduced by more than 8%. However, the company's market-cap is currently low, provided a historical average profitability can be maintained in the future, which means a share buyback would be significantly more accretive to the value to eternal shareholders than a restructuring. But the company's share buybacks are currently only a small fraction of what they were a few years ago when the market-cap was high.

In general, misallocation of capital negatively affects both shareholders and employees. It occurs when investments, acquisitions and share buybacks are made at too high prices compared to the return that can reasonably be expected, or conversely when opportunities for investments, acquisitions or share buybacks are available at low prices relative to the expected return but are being ignored.

9.6. Apple

The company *Apple* was founded in USA by two people in 1976 to produce computers. It has since grown to employ 76k people in fiscal year 2012 with annual revenue of almost USD 156b and reported net income of almost USD 42b.[38]

In fiscal year 2012 the company's revenue segments were: Smart-phones and related services were 51% of revenue. Tablet computers were 21% of revenue. Desktop and portable computers were 15% of revenue. Music related products and services were 9% of revenue. Other products and services were 4% of revenue.

9.6.1. Share Buyback Policy

The company's share buyback policy is described in its annual report for 2012:

> *"(...) repurchase up to USD 10 billion of the Company's common stock beginning in 2013. The repurchase program is expected to be executed over a three-year period with the primary objective of neutralizing the impact of dilution from future employee equity grants and employee stock purchase programs. The repurchase program does not obligate the Company to acquire any specific number of shares. The Company anticipates that it will utilize approximately USD 45 billion of domestic cash to pay dividends, repurchase shares, and to remit withheld taxes related to net share settlement of restricted stock units in the first three years of the dividend and stock repurchase programs. The Company anticipates the cash used for future dividends and the repurchase program will come primarily from current domestic cash and from on-going U.S. operating activities and the cash generated from such activities."*

This states that the primary reason for the company's share buybacks is to offset the dilutive impact of employee stock grants and options. By itself this is not a valid reason for making share buybacks as shown in section 4.11, because share buybacks decrease value to eternal shareholders if the share price is too high. A price limit for the share buybacks must therefore be determined and publicized if shareholders are to properly assess the impact of share buybacks on the value to eternal shareholders. The company does, however, describe the source of cash for the share buybacks which is also necessary in valuation.

Following a public debate in which shareholders disapproved of the company amassing excess cash in foreign countries, the company greatly increased the share buyback authorization and also indicated that future dividends and share buybacks may be funded from borrowings:

> *"In April 2013, the Company's Board of Directors increased the share repurchase program authorization from USD 10 billion to USD 60 billion. The Company's share repurchase program does not obligate it to acquire any specific number of shares. (…)The Company anticipates the cash used for future dividends and the share repurchase program will come primarily from its current domestic cash, cash generated from on-going U.S. operating activities and from borrowings."*

It is possible that the company's share buyback policy has changed in other ways following the shareholder debate, but the revised policy is not described further in the company's most recent quarterly report.

[38] Form 10-K annual report for fiscal year 2012 filed with US SEC: www.sec.gov/Archives/edgar/data/320193/000119312512444068/d411355d10k.htm

9.6.2. Analysis of Repatriation Tax

Although the corporate tax rate is 35% in the company's country of incorporation, the company's effective tax rate is only about 25% because its foreign earnings are taxed less and not repatriated.

Repatriated Cash

The repatriation tax is not detailed in the company's financial statements so assumptions will be made in the following in order to demonstrate the valuation formulas.

Having the excess cash earn interest for t years after which it is repatriated will increase value to shareholders if the future repatriation tax is lower than the equilibrium from Eq. 4-88. If the future repatriation tax rate is not lower than this, then the future repatriation will decrease shareholder value:

$$TaxRepatriate_t < 1 - \frac{1 - TaxRepatriate_0}{\left(\frac{1 + InterestIncomeRate}{1 + d'}\right)^t}$$

During the period 2010-2012 the company earned 0.75% to 1% annual interest on the investments of its excess cash. In calculating the present value of the interest income, the discount rate should be somewhat higher than the interest rate because the cash is not directly controlled by the shareholders. Assuming the future interest rate is 1%, the discount rate is $d' = 2\%$, the holding period is $t = 5$ years, and the current repatriation tax rate is 15%, then the future repatriation tax rate must be less than about 11%:

$$TaxRepatriate_5 < 1 - \frac{1 - 15\%}{\left(\frac{1.01}{1.02}\right)^5} \simeq 11\%$$

The number of years t after which the repatriation will decrease value to shareholders even if the repatriation tax becomes zero, is calculated using Eq. 4-89:

$$t > \frac{\log(1 - TaxRepatriate_0)}{\log\left(\frac{1 + InterestIncomeRate}{1 + d'}\right)} = \frac{\log(1 - 15\%)}{\log\left(\frac{1.01}{1.02}\right)} \simeq 16$$

So under these assumptions, repatriation will decrease value to shareholders after 16 years regardless of the future repatriation tax.

Future Earnings

In fiscal year 2012 only 37% of the company's revenue and operating income originated from within USA and the remaining operating income is subject to repatriation tax which should be taken into account when calculating the present value of future earnings. If the cash from future earnings are withheld for several years before being repatriated, then the present value of the interest income should also be added.

Assume for simplicity that the future earnings are repatriated immediately and there are tax treaties with the foreign countries so the total tax rate equals the tax rate of the home country. The repatriated earnings are then calculated by dividing the earnings with the effective tax rate and multiplying with the domestic tax rate. According to Table 8, the company's earnings and Free Cash Flow (FCF) for fiscal year 2012 were almost equal and around USD 41.6b on average.

According to the annual report the effective tax rate was about 25% and the tax rate for the home country USA was 35%, so the repatriated earnings would be:

$$Repatriated\ Earnings = Earnings \cdot \frac{1 - TaxDomestic}{1 - TaxEffective} = USD\ 41.5b \cdot \frac{65\%}{75\%} \simeq USD\ 36b$$

If the company's future earnings are deemed to be equally uncertain as the future earnings of the companies in the S&P 500 stock market index, then the value yield for the S&P 500 can be used as the discount rate in calculating the present value of the company's future earnings. At the time of this writing in July 2013, the P/Book ratio for the S&P 500 is estimated at about 2.4 so the mean value yield is about 8.2% according to Eq. 7-3. If the company's future earnings are assumed to be constant for eternity then Eq. 13-2 with zero growth gives the present value:

$$v_{Earnings} = \frac{Repatriated\ Earnings}{d} = \frac{USD\ 36b}{8.2\%} \simeq USD\ 439b$$

Eq. 9-4

Value without Share Buyback

On March 30, 2013 the company had almost USD 145b in cash and various low-risk investments of which USD 102b were held by foreign subsidiaries and generally subject to repatriation tax if transferred to the parent company in USA.[39]

The value to eternal shareholders is a variation of Eq. 4-84 because not all cash must be repatriated. It is assumed that the domestic cash is paid out as dividends immediately and the foreign cash is held for t years and earns an interest after which it is repatriated and paid out as dividends, so the value is:

$$v_{Repatriate_t} = Excess\ Cash\ Domestic + Excess\ Cash\ Foreign \cdot (1 - TaxRepatriate_t) \cdot \left(\frac{1 + InterestIncomeRate}{1 + d'}\right)^t + v_{Earnings}$$

Using the $v_{Earnings}$ calculated in Eq. 9-4 and assuming all cash are in excess of what is required for future operations, the value is:

$$v_{Repatriate_t} \simeq USD\ 43b + USD\ 102b \cdot (1 - TaxRepatriate_t) \cdot \left(\frac{1.01}{1.02}\right)^t + USD\ 439b$$

Eq. 9-5

If the foreign held cash is repatriated now so that $t = 0$ then the value is:

$$v_{Repatriate_0} \simeq USD\ 102b \cdot (1 - TaxRepatriate_0) + USD\ 482b$$

[39] Form 10-Q filed with US SEC for 2nd quarter of 2013: www.sec.gov/Archives/edgar/data/320193/000119312513168288/d501596d10q.htm

It is unclear from the company's financial statements what the current repatriation tax rate is on its foreign cash holdings. If it is 15% then the value is:

$$v_{Repatriate_0} \simeq USD\ 569b$$

Eq. 9-6

Value with Share Buyback from Domestic Cash

The company has domestic cash holdings of about USD 43b so share buybacks up to that amount do not require repatriation of foreign cash or domestic borrowings. The equilibrium market-cap below which share buybacks increase value to eternal shareholders is $v_{Repatriate_t}$ from Eq. 9-5. Assuming the foreign held cash is repatriated now at a tax rate of 15% then $v_{Repatriate_0} = \text{USD } 569\text{b}$ as calculated in Eq. 9-6. At the time of this writing in July 2013 the company's market-cap is about USD 372b which is significantly less than this equilibrium so a share buyback would increase the value to eternal shareholders.

The relative value is calculated similarly to Eq. 4-93 or Eq. 4-7. If the entire domestic cash holdings of USD 43b are used for the share buyback, then the relative value is:

$$\frac{W_{Repatriate_0}}{V_{Repatriate_0}} = \frac{1 - \frac{Buyback}{v_{Repatriate_0}}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD\ 43b}{USD\ 569b}}{1 - \frac{USD\ 43b}{USD\ 372b}} \simeq 104.5\%$$

That is, the share buyback would increase value to eternal shareholders by about 4.5%, provided these valuation assumptions are reasonable, which is a qualitative assessment left to the reader.

Value with Share Buyback from Borrowed Money

Now assume the company's domestic cash of USD 43b is paid out as dividends immediately and borrowings of USD 17b are made to fund a share buyback. The company did not have any debt outstanding in fiscal year 2012 so its interest rate is unknown. Assume the company would have to pay interest of 4% net of tax benefits. The present value of the interest payments is calculated with the same discount rate $d' = 2\%$ as used for calculating the present value of the interest income from the foreign held cash so the difference between interest income and expense is comparable. The present value of the interest payments is calculated using Eq. 4-90 assuming the interest accumulates for t years:

$$Buyback \cdot \left(\frac{1 + InterestExpenseRate}{1 + d'}\right)^t = USD\ 17b \cdot \left(\frac{1.04}{1.02}\right)^t$$

Borrowing cash to make a share buyback and repaying the loan from repatriated cash in t years, is more valuable to eternal shareholders than repatriating the cash immediately when Eq. 4-96 is satisfied, which is calculated by inserting the above variables in the formula:

$$\begin{aligned}
&W_{Repatriate_t} > W_{Repatriate_0} \\
&\Leftrightarrow v_{Repatriate_t} - v_{Repatriate_0} > Buyback \cdot \left(\left(\frac{1 + InterestExpenseRate}{1 + d'}\right)^t - 1\right) \\
&\Leftrightarrow USD\ 102b \cdot \left((1 - TaxRepatriate_t) \cdot \left(\frac{1.01}{1.02}\right)^t - (1 - TaxRepatriate_0)\right) \\
&\qquad > USD\ 17b \cdot \left(\left(\frac{1.04}{1.02}\right)^t - 1\right)
\end{aligned}$$

For example, if $t = 5$ and $TaxRepatriate_0 = 15\%$ then the future repatriation tax rate must be less than:

$$W_{Repatriate_5} > W_{Repatriate_0} \Leftrightarrow TaxRepatriate_5 < 8.9\%$$

So under these assumptions, it is more valuable to shareholders if a share buyback is made for money that is borrowed for 5 years and then repaid with repatriated cash, rather than making the share buyback for repatriated cash now, if the current repatriation tax is 15% and the future repatriation tax is less than 8.9%.

The relative value is calculated using Eq. 4-95. For example, if $t = 5$ and $TaxRepatriate_5 = 5\%$ then:

$$\begin{aligned}
\frac{W_{Repatriate_t}}{W_{Repatriate_0}} &= \frac{v_{Repatriate_t} - Buyback \cdot \left(\frac{1 + InterestExpenseRate}{1 + d'}\right)^t}{v_{Repatriate_0} - Buyback} \\
&= \frac{USD\ 43b + USD\ 102b \cdot (1 - 0.05) \cdot \left(\frac{1.01}{1.02}\right)^5 + USD\ 439b - USD\ 17b \cdot \left(\frac{1.04}{1.02}\right)^5}{USD\ 43b + USD\ 102b \cdot (1 - 0.15) + USD\ 439b - USD\ 17b} \\
&\simeq 101.6\%
\end{aligned}$$

So under these assumptions it is 1.6% more valuable to eternal shareholders to make a share buyback for borrowed money which is repaid with repatriated cash in 5 years, rather than repatriating the cash now.

9.6.3. Conclusion

This section demonstrated the valuation formulas for repatriating cash. It was shown that if the value to shareholders is to be maintained or increased from waiting to repatriate cash, then the future repatriation tax rate must decrease sufficiently to offset the cost of having excess cash earn low interest income.

The formulas were also used to evaluate whether share buybacks made for borrowed or repatriated cash is most valuable to eternal shareholders, which again depends on the future repatriation tax rate.

9.7. Facebook

The company *Facebook* was founded in USA in 2004 by five people for providing internet-based social networking. At the end of 2013 the company had grown to employ more than 6,000 people. In 2013 the company had USD 7.9b in revenue of which 89% came from advertising and the rest from fees.[40]

This case study values a share issuance used to fund an acquisition rather than share buybacks.

9.7.1. Value of Facebook

The company has two classes of common stock, class A and B. The classes have the same liquidation and dividend rights but class B shares have 10 votes per share while class A shares only have 1 vote per share. Only the dividend and liquidation rights are relevant in this valuation so the number of class A and B shares are merely added when calculating the market-cap of the company.

On January 28, 2014 the company had a total of about 2.55b shares outstanding.[40] On February 19, 2014 the company's share-price was around USD 68.50, which gives a market-cap of nearly USD 175b.

Discount Rate

A discount rate is needed to calculate the present value of a company's future earnings. If the company's future earnings are deemed to have risk characteristic similar to the future earnings of the S&P 500 stock market index, then the value yield of the S&P 500 can be used as the discount rate. Recall that the value yield is the annualized rate of return an investor would get from buying shares and holding them for eternity.

On September 30, 2013 the book value for the S&P 500 index was USD 693.22 per share.[41] However, the book-value on February 19, 2014 is required here and must therefore be estimated. There are 142 days between these two dates. According to [**62**] the mean book-value for the S&P 500 on February 19, 2014 can be estimated from the known book-value on September 30, 2013 using this formula:

$$E\left[BookValue_{Feb.19,2014}\right] \simeq BookValue_{Sep.30,2013} \cdot 1.04462^{142/365} \simeq USD\ 693.22 \cdot 1.017 \simeq USD\ 705$$

That is, the book-value for the S&P 500 index is estimated to be around USD 705 on February 19, 2014. On that day the price of the S&P 500 was around USD 1840. This gives a P/Book ratio of $1840/705 \simeq 2.61$.

The mean value yield for the S&P 500 is then estimated using the P/Book ratio with Eq. 7-3:

$$E[Value\ Yield] = \frac{9.1\%}{P/Book} + 4.4\% \simeq \frac{9.1\%}{2.61} + 4.4\% \simeq 7.9\%$$

That is, an investor who bought shares in the S&P 500 stock-market index on February 19, 2014 and would hold the shares for eternity could expect an average annualized rate of return of 7.9% according to that model. This estimate can be used as the discount rate in present value calculations so as to value the future earnings of a company relative to an investment in the S&P 500 index.

[40] Form 10-K annual report for 2013 filed with US SEC:
www.sec.gov/Archives/edgar/data/1326801/000132680114000007/fb-12312013x10k.htm
[41] Financial Data for S&P 500:
www.spindices.com/documents/additional-material/sp-500-eps-est.xlsx

Constant Earnings

According to Table 10, the reported net income of Facebook was about USD 1.5b for year 2013. Assuming these are the earnings that could be paid out as dividends and they remain constant for eternity means that the value to eternal shareholders can be calculated using Eq. 3-8 with zero growth. Further assume that, say, USD 10b of the company's cash holdings are in excess and can also be paid out as dividends. The estimate of 7.9% for the S&P 500 value yield is used as the discount rate as described above. The value to eternal shareholders is then:

$$v_{Net\ Income} = Excess\ Cash + \frac{Net\ Income}{d} = USD\ 10b + \frac{USD\ 1.5b}{7.9\%} \simeq USD\ 29b$$

If instead the company's Free Cash Flow (FCF) for 2013 of about USD 2.8b is used as an estimate for the earnings that can be paid out as dividends and this too is assumed to remain constant forever, then the value to eternal shareholders is:

$$v_{FCF} = Excess\ Cash + \frac{Free\ Cash\ Flow}{d} = USD\ 10b + \frac{USD\ 2.8b}{7.9\%} \simeq USD\ 45b$$

Required Earnings Growth

The company's market-cap was about USD 175b on February 19, 2014 which is much higher than the estimates of the value to eternal shareholders of USD 29b and USD 45b, that were calculated under the assumption of constant future earnings. This means future earnings growth is required if the company is to have a rate of return that is similar to that expected for the S&P 500 index.

The earnings growth g required for the value of the company v to be greater than its market-cap can be calculated using Eq. 3-9 if the grow rate is assumed to be constant each year for eternity. The formula is reprinted here for easy reference:

$$MarketCap < v = Excess\ Cash + Earnings \cdot \frac{1+g}{d-g} \Leftrightarrow g > \frac{d - \frac{Earnings}{MarketCap - Excess\ Cash}}{1 + \frac{Earnings}{MarketCap - Excess\ Cash}}$$

The mean value yield for the S&P 500 index was estimated above to be about 7.9% and is used as the discount rate d, the company's market-cap was about USD 175b on February 19, 2014, the reported net income of about USD 1.5b for year 2013 is the starting earnings, and USD 10b of the company's cash holdings are assumed to be in excess and available for dividends, so the earnings growth rate must satisfy:

$$MarketCap < v \Leftrightarrow g > \frac{7.9\% - \frac{USD\ 1.5b}{USD\ 175b - USD\ 10b}}{1 + \frac{USD\ 1.5b}{USD\ 175b - USD\ 10b}} \simeq 6.9\%$$

That is, in order for the company's market-cap to be smaller than its value to eternal shareholders, would require earnings growth of at least 6.9% each year for eternity.

If the company's Free Cash Flow (FCF) of about USD 2.8b is used instead of the reported net income as the earnings that can be paid out as dividends, then the required growth rate is about 6.1%. If a risk premium is

deemed appropriate because the company's future earnings are more uncertain than those of the S&P 500 index, then the required earnings growth rate would be higher still.

Whether these earnings growth rates are feasible is a qualitative assessment left to the reader.

9.7.2. Acquisition of WhatsApp

On February 19, 2014, Facebook announced its acquisition of the company *WhatsApp* for USD 4b cash and about 184m shares and 46m restricted stock units (RSUs).[42] The terms of the RSUs are not described so in the following the RSUs will be treated as if they were ordinary shares so a total of 230m shares were issued by Facebook to pay for the acquisition of WhatsApp. On that day Facebook's share-price was around USD 68.50 so the shares and RSUs would have a total market price around USD 15.8b.

This acquisition is mathematically equivalent to a company making an investment from the proceeds of a new share issuance which can be valued using the formulas in section 4.8.

Value of WhatsApp

There is no publicly available financial data for WhatsApp so it is impossible for outsiders to value the impact of the acquisition on the value to the eternal shareholders of Facebook. Different scenarios are therefore considered here so as to demonstrate the usage of the valuation formulas.

Equilibrium

The equilibrium where Facebook's share issuance and acquisition of WhatsApp starts to increase value to the eternal shareholders of Facebook is given in Eq. 4-47 and depends on several variables. For example, if Facebook's value to eternal shareholders without the acquisition of WhatsApp is $v = USD\ 140b$, the share issuance and acquisition amount is USD 15.8b, and the present value of the return on the WhatsApp acquisition is USD 20b net of the cash investment of USD 4b, then the equilibrium is:

$$W_{Issuance} > V \Leftrightarrow MarketCap > v \cdot \frac{Issuance}{Return} = USD\ 140b \cdot \frac{USD\ 15.8b}{USD\ 20b} = USD\ 110.6b$$

That is, under these assumptions the market-cap of Facebook must be greater than USD 110.6b in order for the acquisition and share issuance to increase value to the eternal shareholders of Facebook.

The equilibrium can also be written in terms of the required value v for Facebook using Eq. 4-48 with the same assumptions as above and the market-cap USD 175b from February 19, 2014:

$$W_{Issuance} > V \Leftrightarrow v < MarketCap \cdot \frac{Return}{Issuance} = USD\ 175b \cdot \frac{USD\ 20b}{USD\ 15.8b} \simeq USD\ 221.5b$$

That is, under these assumptions the value to eternal shareholders of Facebook without the share issuance and acquisition of WhatsApp must be less than USD 221.5b in order for the share issuance and acquisition to increase the value to eternal shareholders of Facebook.

[42] Form 8-K filed with US SEC on February 19, 2014:
www.sec.gov/Archives/edgar/data/1326801/000132680114000010/exhibit991_pressrelease219.htm

The equilibrium can also be written in terms of the return from the WhatsApp acquisition required to increase the value to eternal shareholders of Facebook. Using Eq. 4-49 with the same assumptions as above gives:

$$W_{Issuance} > V \Leftrightarrow Return > Issuance \cdot \frac{v}{MarketCap} = USD\ 15.8b \cdot \frac{USD\ 140b}{USD\ 175b} = USD\ 12.64b$$

This return is net of the cash payment of USD 4b which must be added so as to obtain the gross return of USD 16.64b. That is, under these assumptions the present value of the gross return on the WhatsApp acquisition must be greater than USD 16.64b in order for the share issuance and acquisition to increase the value to eternal shareholders of Facebook.

Figure 18 and Figure 19 show the equilibrium for varying choices of v and $Return$.

Relative Value

The relative value is calculated using Eq. 4-46 and measures how much the acquisition and share issuance of WhatsApp is worth to the eternal shareholders of Facebook. The relative value depends on the mispricing of both Facebook and WhatsApp.

When the shares of Facebook are overpriced and the shares of WhatsApp are underpriced then they both contribute to an increase in the value to Facebook's eternal shareholders. For example, if the present value of the return on the WhatsApp acquisition is $Return = USD\ 20b$ which is higher than the acquisition price of USD 15.8b (note that both of these amounts are net of the cash payment of USD 4b), and the value to eternal shareholders of Facebook without the acquisition and share issuance is $v = USD\ 140b$ which is lower than its market-cap of USD 175b, then the relative value is:

$$\frac{W_{Issuance}}{V} = \frac{1 + \frac{Return}{v}}{1 + \frac{Issuance}{MarketCap}} = \frac{1 + \frac{USD\ 20b}{USD\ 140b}}{1 + \frac{USD\ 15.8b}{USD\ 175b}} \simeq 104.8\%$$

That is, the share issuance to pay for the acquisition of WhatsApp would increase the value to eternal shareholders of Facebook by about 4.8% under these assumptions.

When the shares of Facebook are underpriced and the shares of WhatsApp are overpriced then they both contribute to a decrease in the value to Facebook's shareholders. For example, if Facebook's value is $v = USD\ 250b$ which is higher than its market-cap of USD 175b, and the WhatsApp acquisition has $Return = USD\ 10b$ which is lower than its acquisition price, then the relative value is:

$$\frac{W_{Issuance}}{V} = \frac{1 + \frac{USD\ 10b}{USD\ 250b}}{1 + \frac{USD\ 15.8b}{USD\ 175b}} \simeq 95.4\%$$

That is, the share issuance to pay for the acquisition of WhatsApp would decrease the value to eternal shareholders of Facebook by about 4.6% under these assumptions.

Figure 18 and Figure 19 show the relative value for varying choices of v and $Return$.

9.7.3. Conclusion

It was demonstrated that the equilibrium and relative value of an acquisition paid with a share issuance depend on the mispricing of the shares of both the acquiring and acquired companies.

If the shares of the acquiring company (in this case Facebook) are underpriced and the shares of the acquired company (in this case WhatsApp) are overpriced, then they both contribute to a decrease in the value to the eternal shareholders of the acquiring company (Facebook).

Conversely, if the shares of the acquiring company (Facebook) are overpriced and the shares of the acquired company (WhatsApp) are underpriced, then they both contribute to an increase in the value to eternal shareholders of the acquiring company (Facebook).

9.8. The9

The company *The9* was incorporated in the Cayman Islands in 1999 for licensing, operating and developing computer games and related entertainment services. Historically, the company has had its management, headquarters and the majority of its operations in China. The company made an Initial Public Offering (IPO) of its stock on the NASDAQ stock exchange in USA in late 2004, in which one American Depository Share (ADS) is equivalent to one ordinary share. Because the company is operating in China, incorporated in the Cayman Islands, has at least one important subsidiary in USA, and its shares are traded in USA, there are a number of complications when assessing its value to shareholders, the least of which is currency conversion. For example, China has special laws for distribution of dividends, special laws requiring national ownership of companies doing internet related business so a licensing agreement is required with an intermediate operating company owned by Chinese residents, there are limitations on shareholder rights, differences in auditing requirements, etc.[43] For the sake of simplicity and because the main focus here is on demonstrating various aspects of share buyback valuation, all of these complex issues are ignored.

9.8.1. Share Buyback Policy

During four months of 2007 the company's share-price decreased almost 60% which was followed by the company issuing a statement in which the co-founder, CEO and chairman of the board proclaimed:[44]

> *"Our business and competitive position in China continue to be strong and therefore we are confident of the company's future performance. We think that the current shares price level do not reflect the company's value and potential. Mirroring this confidence, our board of directors has authorized the company to repurchase up to USD 50 million of its own stock. We believe that this is a good investment opportunity for the company to repurchase its own shares at the prevailing price level."*

The company increased revenue in the following year by almost 34%, see Table 9, after which revenue, net income and Free Cash Flow (FCF) decreased significantly so the revenue of 2011 was about 8% of the revenue in 2007 when the above statement was issued.

The company issued a new statement regarding share buybacks in June 2011:[45]

> *"(...) Board of Directors has approved a share repurchase program to purchase up to USD 25 million of its American Depositary Shares over the next 12 months."*

In December 2012, shortly before the writing of this analysis, a similarly worded statement was issued:[46]

> *"(...) Board of Directors has authorized a share repurchase program under which the Company may purchase up to USD 10 million of its American Depositary Shares over the next 12 months."*

[43] Form 424B4 prospectus filed with US SEC:
www.sec.gov/Archives/edgar/data/1296774/000119312504213576/d424b4.htm
[44] Form 6-K filed with US SEC on November 20, 2007:
www.sec.gov/Archives/edgar/data/1296774/000119312507252471/d6k.htm
[45] Form 6-K filed with US SEC on June 13, 2011:
www.sec.gov/Archives/edgar/data/1296774/000095012311058822/c18746e6vk.htm
[46] Form 6-K filed with US SEC on December 6, 2012:
www.sec.gov/Archives/edgar/data/1296774/000119312512492760/d449734d6k.htm

These statements, as well as the company's annual reports, do not detail the share price below which shares will be bought back, nor the source of cash, which is crucial information for assessing the impact of share buybacks on the value of the company.

9.8.2. Analysis of Year 2007

The share buybacks of 2007 are now studied from ex-ante and ex-post perspectives.

Ex-Ante Analysis

In the company's financial statements published the day before the share buyback programme for 2007 was announced, the balance sheet which is dated a few months before reports cash and equivalents in the amount of CNY 2.1b and total liabilities in the amount of CNY 0.4b.[47] For the sake of this ex-ante valuation, assume the company's excess cash were $CNY\ 2.1b - CNY\ 0.4b = CNY\ 1.7b$ and would be paid out as dividends immediately. In reality, the company invested most of its cash which will be analysed ex-post below.

The company had recently issued and sold a large number of shares so there were about 29.4m shares outstanding at the time, which means the 27.4m weighted average number of shares for 2007 listed in Table 9 are significantly distorted and the actual number of shares outstanding will be used to calculate the market-cap. The lowest share-price during the end of that year was about USD 21.50 and using the year's average exchange rate of USDCNY 7.58, see Table 9, means the lowest market-cap during the final months of 2007 was about CNY 4.8b.

Assume the company's annual earnings are constant for eternity and are all paid out as dividends. The value to eternal shareholders is then calculated using Eq. 13-2 with zero growth. If the company's future earnings were deemed equally uncertain as those of the aggregate companies of the S&P 500 stock-market index, then the value yield estimate of the S&P 500 index could have been used as the discount rate in valuation. According to Table 2, the lowest mean value yield estimate for the S&P 500 index was 7.4% during 2007. The estimated value to eternal shareholders is then:

$$v = Excess\ Cash + \frac{Earnings}{d} \simeq CNY\ 1.7b + \frac{Earnings}{7.4\%}$$

The objective is now to determine the earnings that would be required for the share buyback of 2007 to increase value to eternal shareholders.

According to Eq. 4-9, a share buyback funded with excess cash increases value to eternal shareholders when:

$$MarketCap < v \simeq CNY\ 1.7b + \frac{Earnings}{7.4\%}$$

Using the lowest market-cap of CNY 4.8b and rearranging the inequality gives:

$$Earnings > (CNY\ 4.8b - CNY\ 1.7b) \cdot 7.4\% \simeq CNY\ 229m$$

[47] Form 6-K filed with US SEC for 3rd quarter of 2007:
www.sec.gov/Archives/edgar/data/1296774/000119312507249825/d6k.htm

That is, in order for the share buyback of 2007 to increase value to eternal shareholders under these assumptions of excess cash, constant earnings, lowest market-cap and the risk of future earnings being similar to that of the S&P 500 index, the annual earnings would have to be at least CNY 229m for eternity. Whether this would have seemed reasonable at the time is a qualitative assessment left to the reader.

Ex-Post Analysis

According to Table 9, the reported net income for 2007 was CNY 241m while the FCF was estimated at CNY 127m. In 2008 the reported net income decreased significantly while the FCF increased significantly. Then in the period 2009-2011 both the reported net income and FCF decreased substantially and became negative. The share buyback in 2007 will now be studied using this ex-post data.

The company's main source of revenue, a licensing agreement to operate a popular computer game in China, was unexpectedly terminated in 2009 and in the following years the company sought to acquire and develop new products and services, hence using its cash reserves for such investments. On June 30, 2012, the company reported cash and equivalents of about CNY 793m, and in the prior three quarters the average operating losses were about CNY 126m per quarter.[48]

Assume the company will use all its cash reserves to finish development of new products, which will start to generate earnings in year 2013 that are constant for eternity. Using the same discount rate $d = 7.4\%$ as above, the value of the company in 2007 is calculated using Eq. 13-2 with zero growth, and additional discounting so as to calculate the present value in 2007 of the earnings starting in 2013:

$$v = \frac{1}{(1+d)^5} \cdot \frac{Earnings}{d} \simeq \frac{Earnings}{10.6\%}$$

The requirement for a share buyback in 2007 to increase value to eternal shareholders would then be:

$$MarketCap < v = \frac{Earnings}{10.6\%} \Leftrightarrow Earnings > 10.6\% \cdot MarketCap$$

At the lowest market-cap of CNY 4.8b this would mean the required earnings would be:

$$Earnings > 10.6\% \cdot MarketCap \simeq CNY\ 509m$$

That is, starting in 2013 the company would need to have annual earnings of at least CNY 509m, if the share buyback in 2007 was to have increased value to eternal shareholders under these assumptions.

A discount rate of $d = 7.4\%$ was used above, which implies the company's future earnings are equally uncertain as those of the companies in the S&P 500 index, but in reality the company's earnings turned out to be significantly more uncertain. Also considering that the company is developing new products upon which future earnings rely, it might be more suitable to use a discount rate such as $d = 16\%$ which is the historical average annual return from an index of venture investments, see section 7.6. Using this discount rate means the required annual earnings from 2013 and onwards would be at least CNY 1.6b, if the share buyback in 2007 was to have increased value to eternal shareholders.

[48] Form 6-K filed with US SEC for 2nd quarter of 2012:
www.sec.gov/Archives/edgar/data/1296774/000119312512367366/d402544d6k.htm

Whether these valuation assumptions and resulting earnings requirements are reasonable, is a qualitative assessment left to the reader.

9.8.3. Analysis of Year 2011

Two valuation scenarios are now considered. First the liquidation value to eternal shareholders without a share buyback and then the liquidation value with a prior share buyback.

On June 30, 2011, the company had cash and equivalents of CNY 1,345m and total liabilities of CNY 294m, according to its most recent financial statements published at the time.[49] Assuming there were no costs of liquidation and the company's other assets were worthless, the proceeds from liquidation would be the company's cash and equivalents minus total liabilities, that is, CNY 1,051m. The company was spending considerable amounts of cash during this period and at the end of the year its cash minus total liabilities was CNY 706m, see Table 9. The share buyback date considered here is approximately half-way between these two dates, so the liquidation value at that date is estimated to be about:

$$v \simeq CNY\ 880m$$

The lowest share-price of 2011 occurred during the last quarter of the year and was USD 2.90 which is about CNY 18.71 using the average exchange rate for the year of USDCNY 6.45, see Table 9. There were about 25m shares outstanding so the market-cap was about:

$$MarketCap \simeq CNY\ 468m$$

Although only a small number of shares were traded at this low market-cap, it will be used to demonstrate the large impact share buybacks can have to shareholder value under certain conditions.

According to the company's previously announced share buyback policy quoted above, the company allowed for share buybacks up to $USD\ 25m \simeq CNY\ 161m$, which is therefore used in this valuation. As the share buyback would be funded with excess cash, Eq. 4-7 gives the relative value of the share buyback:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} \simeq \frac{1 - \frac{CNY\ 161m}{CNY\ 880m}}{1 - \frac{CNY\ 161m}{CNY\ 468m}} \simeq 125\%$$

That is, if a share buyback in the amount of CNY 161m was made when the market-cap was CNY 468m and the company was liquidated immediately afterwards, the share buyback would increase the value per share by about 25%.

[49] Form 6-K filed with US SEC for 2nd quarter of 2011:
www.sec.gov/Archives/edgar/data/1296774/000119312511224811/d6k.htm

Per-Share Numbers

The per-share numbers will now be studied in detail. There were about 25m shares outstanding, so the liquidation value per share without a share buyback is calculated using Eq. 4-2. The dividend tax (or liquidation tax) is a variable here because it may be different amongst shareholders:

$$V = \frac{v \cdot (1 - TaxDividend)}{Shares} \simeq \frac{CNY\ 880m}{25m} \cdot (1 - TaxDividend) \simeq CNY\ 35.20 \cdot (1 - TaxDividend)$$

Assuming the market-cap was CNY 468m and the share buyback amount was CNY 161m, then according to Eq. 4-4 the number of shares outstanding would be reduced to:

$$Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right) = 25m \cdot \left(1 - \frac{CNY\ 161m}{CNY\ 468m}\right) \simeq 16.4m$$

According to Eq. 4-5, the value per share after a share buyback would be:

$$W = \frac{(v - Buyback) \cdot (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)} \simeq \frac{CNY\ 880m - CNY\ 161m}{16.4m} \cdot (1 - TaxDividend)$$
$$\simeq CNY\ 43.84 \cdot (1 - TaxDividend)$$

So a share bought at the lowest price of USD 2.90, i.e. CNY 18.71, would receive CNY 35.20 before taxes if the company liquidated immediately. But if instead the company made a share buyback prior to the liquidation, then the liquidation proceeds would be CNY 43.84 per share before taxes, which is about 25% more than the liquidation proceeds without the prior share buyback.

There are several likely reasons why the company did not make a share buyback and then liquidated. As mentioned above, only few shares were traded at the lowest price so the impact of a share buyback on shareholder value would have been considerably smaller. Even so, the company's market-cap was lower than its apparent liquidation value so the question remains why they did not liquidate. The answer is that only the small number of shareholders who bought at the lowest share-price would experience a profit from the liquidation. The majority of the shareholders had paid more for their shares in the past and/or been invested for several years, so they would presumably prefer that the company pursued its turnaround strategy rather than liquidate for the benefit of only a small group of new shareholders.

This example demonstrated that the anti-takeover hypothesis described in section 2.5 is a mathematical fallacy. The reason is that share buybacks should generally only be made when they are beneficial to eternal shareholders, that is, when the market-cap is less than the value to eternal shareholders. But if the share-price remains unchanged following a share buyback, the difference between market-cap and value to eternal shareholders is magnified from a share buyback, which means the company becomes a more attractive takeover target as a result of the share buyback.

9.8.4. Discrete Stochastic Variable

The above analysis for year 2011 is now extended so the value to eternal shareholders is considered to be a stochastic variable $\mathbb{V}$ which can take on two values with equal probability:

$$\Pr[\mathbb{V} = CNY\ 440m] = \Pr[\mathbb{V} = CNY\ 1{,}320m] = \frac{1}{2}$$

This means there is uncertainty about the value to eternal shareholders which may be either CNY 440m or CNY 1,320m with equal probability. This could be two scenarios if the company did not liquidate and instead invested its cash with a potential loss or profit. These two values are merely chosen as an example.

Mean Equilibrium

The expected (or mean) value without a share buyback is calculated using Eq. 6-16:

$$E[\mathbb{V}] = \sum_{v} v \cdot \Pr[v] = CNY\ 440m \cdot \frac{1}{2} + CNY\ 1{,}320m \cdot \frac{1}{2} = CNY\ 880m$$

The stochastic variable for the value with a share buyback is denoted $\mathbb{W}$ and is defined in Eq. 6-1:

$$\mathbb{W} = \frac{\mathbb{V} - Buyback}{1 - \frac{Buyback}{MarketCap}}$$

The probability distribution of $\mathbb{W}$ is derived from the probability distribution of $\mathbb{V}$:

$$\Pr\left[\mathbb{W} = \frac{CNY\ 440m - Buyback}{1 - \frac{Buyback}{MarketCap}}\right] = \Pr\left[\mathbb{W} = \frac{CNY\ 1{,}320m - Buyback}{1 - \frac{Buyback}{MarketCap}}\right] = \frac{1}{2}$$

Assuming $MarketCap = E[\mathbb{V}] = CNY\ 880m$ and a share buyback is made for CNY 110m, gives:

$$\Pr\left[\mathbb{W} = \frac{CNY\ 440m - CNY\ 110m}{1 - \frac{CNY\ 110m}{CNY\ 880m}}\right] = \Pr\left[\mathbb{W} = \frac{CNY\ 1{,}320m - CNY\ 110m}{1 - \frac{CNY\ 110m}{CNY\ 880m}}\right] = \frac{1}{2}$$

This reduces to:

$$\Pr\left[\mathbb{W} = \frac{8}{7} \cdot CNY\ 330m\right] = \Pr\left[\mathbb{W} = \frac{8}{7} \cdot CNY\ 1{,}210m\right] = \frac{1}{2}$$

The expected (or mean) value with a share buyback is:

$$E[\mathbb{W}] = \sum_{w} w \cdot \Pr[w] = \frac{8}{7} \cdot CNY\ 330m \cdot \frac{1}{2} + \frac{8}{7} \cdot CNY\ 1{,}210m \cdot \frac{1}{2} = CNY\ 880m$$

So the expected value with and without a share buyback both equal the market-cap. This is because $MarketCap = E[\mathbb{V}]$ is the mean equilibrium from Eq. 6-18 which causes $E[\mathbb{W}] = E[\mathbb{V}] = MarketCap$.

Relative Value

The stochastic variable for the relative value of a share buyback is denoted $\mathbb{W}/\mathbb{V}$ and is defined in Eq. 6-2:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{\mathbb{V}}}{1 - \frac{Buyback}{MarketCap}}$$

Its probability distribution is derived from that of $\mathbb{V}$:

$$\Pr\left[\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{CNY\ 440m}}{1 - \frac{Buyback}{MarketCap}}\right] = \Pr\left[\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{CNY\ 1{,}320m}}{1 - \frac{Buyback}{MarketCap}}\right] = \frac{1}{2}$$

Using the same assumptions as above that $MarketCap = CNY\ 880m$ and $Buyback = CNY\ 110m$, gives:

$$\Pr\left[\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{CNY\ 110m}{CNY\ 440m}}{1 - \frac{CNY\ 110m}{CNY\ 880m}}\right] = \Pr\left[\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{CNY\ 110m}{CNY\ 1{,}320m}}{1 - \frac{CNY\ 110m}{CNY\ 880m}}\right] = \frac{1}{2}$$

This reduces to:

$$\Pr\left[\frac{\mathbb{W}}{\mathbb{V}} = \frac{6}{7}\right] = \Pr\left[\frac{\mathbb{W}}{\mathbb{V}} = \frac{22}{21}\right] = \frac{1}{2}$$

The expected relative value of the share buyback is:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = \sum_{w/v} \frac{w}{v} \cdot \Pr\left[\frac{w}{v}\right] = \frac{6}{7} \cdot \frac{1}{2} + \frac{22}{21} \cdot \frac{1}{2} = \frac{20}{21} \simeq 95\%$$

So on average the share buyback is expected to decrease the value to eternal shareholders by 5% even though the expected value with and without the share buyback both equal the market-cap. The reason is that the relative value is non-linear. If it turns out that the actual value is $\mathbb{V} = CNY\ 440m$ then the value to eternal shareholders would have decreased by 1/7 (about 14%). Conversely, if the actual value turns out to be $\mathbb{V} = CNY\ 1{,}320m$ then the value to eternal shareholders would have increased by 1/21 (almost 5%). These scenarios have equal probability of occurring so on average the loss will outweigh the gain.

Relative Equilibrium

The relative equilibrium ensures that the mean relative value of a share buyback is greater than one. Its calculation requires the reciprocal stochastic variable which is calculated using Eq. 6-17:

$$E\left[\frac{1}{\mathbb{V}}\right] = \sum_{v} \frac{1}{v} \cdot \Pr[\mathbb{V} = v] = \frac{1}{CNY\ 440m} \cdot \frac{1}{2} + \frac{1}{CNY\ 1{,}320m} \cdot \frac{1}{2} = \frac{1}{CNY\ 660m}$$

The relative equilibrium is then calculated using Eq. 6-19:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow MarketCap < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]} = CNY\ 660m$$

This is significantly lower than the mean equilibrium which is $E[\mathbb{V}] = CNY\ 880m$.

But even if the market-cap is less than the relative equilibrium, say, $MarketCap = CNY\ 600m$ then there is still a probability of 0.5 (or 50%) that the actual value of the company will turn out to be $\mathbb{V} = CNY\ 440m$ so there will be a loss to eternal shareholders as a result of a share buyback.

Minimum Value

If a share buyback must be guaranteed to increase value to eternal shareholders then it must be made at a market-cap below the minimum possible value that $\mathbb{V}$ can take on, which is CNY 440m here, see Eq. 6-20:

$$MarketCap < Min[\mathbb{V}] = CNY\ 440m\ \Rightarrow\ \mathbb{W} > \mathbb{V}$$

Increased Variance

To calculate the variance of the value without a share buyback, first calculate: [50]

$$E[\mathbb{V}^2] = \sum_v v^2 \cdot \Pr[v] = (CNY\ 440m)^2 \cdot \frac{1}{2} + (CNY\ 1{,}320m)^2 \cdot \frac{1}{2} = CNY^2\ 968{,}000m^2$$

This is used with a well-known identity of the variance:

$$Var[\mathbb{V}] = E[\mathbb{V}^2] - E[\mathbb{V}]^2 = CNY^2\ 968{,}000m^2 - CNY^2\ 774{,}400m^2 = CNY^2\ 193{,}600m^2$$

The variance of the value with a share buyback is calculated using Eq. 6-25 and the same assumptions as above, namely that $MarketCap = E[\mathbb{V}] = CNY\ 880m$ and $Buyback = CNY\ 110m$:

$$Var[\mathbb{W}] = \frac{Var[\mathbb{V}]}{\left(1 - \frac{Buyback}{MarketCap}\right)^2} = \frac{CNY^2\ 193{,}600m^2}{\left(1 - \frac{CNY\ 110m}{CNY\ 880m}\right)^2} \simeq CNY^2\ 252{,}865m^2$$

So $Var[\mathbb{W}] > Var[\mathbb{V}]$ which means the uncertainty of the value to eternal shareholders increases as a result of the share buyback. The uncertainty can also be expressed in terms of the standard deviation which is the square root of the variance, $Stdev[\mathbb{V}] = \text{CNY } 440\text{m}$ and $Stdev[\mathbb{W}] \simeq \text{CNY } 503\text{m}$.

It is shown in Eq. 6-25 that the variance increases regardless of the market-cap and buyback amount.

[50] The units of the variance are squared, which means the currency CNY and the factor m (million) are both squared.

9.8.5. Log-Normal Stochastic Variable

Assume the value to eternal shareholders is a stochastic variable $\mathbb{V}$ which consists of excess cash that can be paid out as dividends now plus a log-normal distributed stochastic variable $\mathbb{V}_{Earnings} \sim \ln \mathcal{N}(\mu, \sigma^2)$ for the present value of future earnings, see Eq. 6-34:

$$\mathbb{V} = Excess\ Cash + \mathbb{V}_{Earnings}$$

Assumptions

In late 2011, the company had cash in excess of liabilities estimated at CNY 880m. The share buyback policy allowed for a share buyback of about CNY 161m. So assume the company will invest all its cash reserves in its turnaround except for CNY 161m which will either be used for a share buyback or dividend payout.

The mean value without a share buyback is assumed to be the same as above:

$$E[\mathbb{V}] = CNY\ 880m$$

The mean of $\mathbb{V}_{Earnings}$ is then:

$$E\left[\mathbb{V}_{Earnings}\right] = E[\mathbb{V}] - Excess\ Cash = CNY\ 880m - CNY\ 161m = CNY\ 719m$$

The standard deviation of $\mathbb{V}$ is unaffected by the excess cash. Assume it is the same as above:

$$Stdev[\mathbb{V}] = Stdev\left[\mathbb{V}_{Earnings}\right] = CNY\ 440m$$

The variance is the squared standard deviation:

$$Var[\mathbb{V}] = Var\left[\mathbb{V}_{Earnings}\right] = \left(Stdev\left[\mathbb{V}_{Earnings}\right]\right)^2 = CNY\ 193{,}600m$$

The log-normal distribution parameters for $\mathbb{V}_{Earnings}$ are calculated using Eq. 13-11 and Eq. 13-12:

$$\mu = \ln\left(E\left[\mathbb{V}_{Earnings}\right]\right) - \frac{1}{2}\ln\left(1 + \frac{Var\left[\mathbb{V}_{Earnings}\right]}{E\left[\mathbb{V}_{Earnings}\right]^2}\right) = \ln(719) - \frac{1}{2}\ln\left(1 + \frac{193{,}600}{516{,}961}\right) \simeq 6.42$$

$$\sigma^2 = \ln\left(1 + \frac{Var\left[\mathbb{V}_{Earnings}\right]}{E\left[\mathbb{V}_{Earnings}\right]^2}\right) = \ln\left(1 + \frac{193{,}600}{516{,}961}\right) \simeq 0.32$$

The mean reciprocal $E[1/\mathbb{V}]$ is needed for calculating the mean relative value and the relative equilibrium, but it is apparently not derived analytically in the literature for a three-parameter log-normal variable such as $\mathbb{V}$. It can be estimated using statistical software (e.g. the freely available R package) by drawing e.g. 100k samples from a log-normal distribution for $\mathbb{V}_{Earnings}$ with the above parameters μ and σ. The result is:

$$E\left[\frac{1}{\mathbb{V}}\right] = E\left[\frac{1}{Excess\ Cash + \mathbb{V}_{Earnings}}\right] \simeq (CNY\ 727m)^{-1}$$

Eq. 9-7

Mean Equilibrium

Share buybacks below the mean equilibrium ensure that the mean value to eternal shareholders increases. Using the above assumptions with the definition of the mean equilibrium from Eq. 6-18 gives:

$$E[\mathbb{W}] > E[\mathbb{V}] \Leftrightarrow MarketCap < E[\mathbb{V}] = CNY\ 880m$$

The mean value with a share buyback $E[\mathbb{W}]$ is calculated from Eq. 6-12. For example, if the market-cap is CNY 870m and the buyback amount is CNY 161m then $E[\mathbb{W}]$ is slightly greater than $E[\mathbb{V}]$:

$$E[\mathbb{W}] = \frac{E[\mathbb{V}] - Buyback}{1 - \frac{Buyback}{MarketCap}} = \frac{CNY\ 880m - CNY\ 161m}{1 - \frac{CNY\ 161m}{CNY\ 870m}} \simeq CNY\ 883m$$

Relative Value

The mean relative value is calculated using Eq. 6-15 and Eq. 9-7. If a share buyback is made for CNY 161m at the mean equilibrium $MarketCap = E[\mathbb{V}] = CNY\ 880m$ then the mean relative value is:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = \frac{1 - Buyback \cdot E\left[\frac{1}{\mathbb{V}}\right]}{1 - \frac{Buyback}{MarketCap}} \simeq \frac{1 - \frac{CNY\ 161m}{CNY\ 727m}}{1 - \frac{CNY\ 161m}{CNY\ 880m}} \simeq 95\%$$

So on average the share buyback decreases the value to eternal shareholders by about 5% even though the expected value with and without the share buyback both equal the market-cap. As in the above example for a discrete probability distribution, the reason is that the relative value is non-linear so the potential losses outweigh the potential gains.

Relative Equilibrium

The relative equilibrium ensures the mean relative value is greater than one. It is calculated using Eq. 6-19 and Eq. 9-7:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow MarketCap < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]} \simeq CNY\ 727m$$

For example, if a share buyback was made at the lowest market-cap of CNY 468m during 2011, see Table 9, then the mean relative value is:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = \frac{1 - Buyback \cdot E\left[\frac{1}{\mathbb{V}}\right]}{1 - \frac{Buyback}{MarketCap}} \simeq \frac{1 - \frac{CNY\ 161m}{CNY\ 727m}}{1 - \frac{CNY\ 161m}{CNY\ 468m}} \simeq 119\%$$

That is, the share buyback would increase value to eternal shareholders by 19% on average.

Probability

Both the mean and relative equilibriums are insufficient for properly assessing the likely impact of a share buyback on the value to eternal shareholders. The probability distribution must be considered as well.

The probability of a share buyback increasing value to eternal shareholders is the probability that $\mathbb{V}$ is greater than the market-cap, which is calculated using Eq. 6-43 with the function F being the log-normal cumulative distribution function from Eq. 13-13 which is calculated using statistical software. Assuming the market-cap is CNY 727m (the relative equilibrium) and the excess cash is CNY 161m, gives:

$$\Pr[\mathbb{W} > \mathbb{V}] = \Pr\left[MarketCap < Excess\ Cash + \mathbb{V}_{Earnings}\right] = 1 - F(MarketCap - Excess\ Cash)$$
$$= 1 - F(727 - 161) \simeq 0.56$$

So there is a probability of about 0.56 (or 56%) that a share buyback made at a market-cap of CNY 727m (the relative equilibrium) would increase the value to eternal shareholders, under these assumptions of the mean and variance of the log-normal distribution. At the lowest market-cap of CNY 468m during 2011 the probability is 0.89 (or 89%) that the value to eternal shareholders is increased from the share buyback.

Minimum Value

A share buyback is guaranteed to increase the value to eternal shareholders when the market-cap is less than the minimum possible value that $\mathbb{V}$ can take on, see Eq. 6-42. In this case the minimum possible value is the excess cash which is assumed to be CNY 161m:

$$MarketCap < Min[\mathbb{V}] = Excess\ Cash = CNY\ 161m\ \Rightarrow\ \mathbb{W} > \mathbb{V}$$

Increased Variance

Share buybacks increase the uncertainty of the value to eternal shareholders as measured by the variance or standard deviation. The standard deviation of the value without a share buyback was assumed to be $Stdev[\mathbb{V}] = CNY\ 440m$. If a share buyback is made for CNY 161m with $MarketCap = E[\mathbb{V}] = CNY\ 880m$ then the standard deviation of the value with a share buyback is $Stdev[\mathbb{W}] \simeq CNY\ 540m$ and for a market-cap of CNY 468m it is $Stdev[\mathbb{W}] \simeq CNY\ 673m$, which are greater than $Stdev[\mathbb{V}] = CNY\ 440m$.

9.8.6. Conclusion

The company made a share buyback in 2007 because management believed the company would do well in the future and the company's shares were under-priced. In the following years the company's main source of revenue was unexpectedly terminated and the company started investing its large cash holdings in the development of new products and services.

It was shown here that in order for the share buyback in 2007 to increase value to eternal shareholders, the earnings from the company's new products would have to be significantly greater than the company's previous earnings, because there was uncertainty about the success of the new products and there would be a delay of several years before the new products would start to generate earnings.

In 2011 the company announced a new share buyback programme. It was shown here that the company's shares were trading below their liquidation value at the time, and a share buyback prior to liquidation of the company would have resulted in increased liquidation proceeds to the remaining shareholders. The company did not liquidate but continued its turnaround strategy of developing new products. This meant there was uncertainty about the value to eternal shareholders which was modelled here as a stochastic variable whose valuation was demonstrated with different assumptions.

9.9. Coca-Cola

The company *Coca-Cola* (abbreviated as Coke) was incorporated in 1919 in USA but its origin is significantly older. The company produces beverages that are sold worldwide. In 2012 the company employed 150k people and had revenue of about USD 48b with net income of about USD 9b.[51]

9.9.1. Share Buyback Policy

The company's share buyback policy is mentioned briefly in its annual report, e.g. in fiscal year 2012:

> *Since the inception of our initial share repurchase program in 1984 through our current program as of December 31, 2012, we have purchased approximately 3.0 billion shares of our Company's common stock at an average price per share of USD 12.75. (...) We currently expect to repurchase an additional USD 3.0 billion to USD 3.5 billion of our stock during 2013, net of proceeds from the issuance of stock due to the exercise of employee stock options.*

The same annual report suggests the company may make short-term borrowings to fund share buybacks:

> *(...) the Company believes its current liquidity position is strong, and we will continue to meet all of our financial commitments for the foreseeable future. These commitments include, but are not limited to, regular quarterly dividends, debt maturities, capital expenditures, share repurchases and other obligations (...) Our debt management policies, in conjunction with our share repurchase programs and investment activity, can result in current liabilities exceeding current assets.*

The motivation and purpose of the company's share buyback programme is not mentioned, neither is a share price limit below which share buybacks will be made which is necessary for outsiders to properly assess the effect on the value to eternal shareholders.

9.9.2. Monte Carlo Simulation

Monte Carlo simulation is the use of computers to simulate numerous outcomes of a mathematical model so as to estimate the probability distribution. This is useful when the model cannot be studied analytically.

Equity Growth Model

A simple equity growth model is used by Pedersen [**62**] to simulate the future equity, earnings and payouts of Coke, based on historical data for its return on equity and fraction of earnings being retained. This is a reasonable model for companies whose earnings are related to their equity capital. Whether this is a reasonable model for this particular company is a qualitative assessment left to the reader.

The earnings for year t are found by multiplying the equity at the end of the previous year with the Return on Equity (ROE):

$$Earnings_t = ROE_t \cdot Equity_{t-1}$$

Eq. 9-8

[51] Form 10-K annual report for 2012 filed with US SEC:
www.sec.gov/Archives/edgar/data/21344/000002134413000007/a2012123110-k.htm

The earnings retained in the company are the earnings minus dividends and share buybacks, plus share issuance:

$$Retain_t = Earnings_t - Dividends_t - Buyback_t + Issuance_t$$

Eq. 9-9

The payout is defined as the earnings that are not being retained in the company. Using the term payout for share buybacks is misleading as noted in section 2.2, but it is used here as convenient notation and the actual meaning should be kept in mind. Using the definitions in Eq. 9-8 and Eq. 9-9 gives:

$$Payout_t = Earnings_t - Retain_t = ROE_t \cdot Equity_{t-1} \cdot \left(1 - \frac{Retain_t}{Earnings_t}\right)$$

Eq. 9-10

The equity grows from the retained earnings:

$$Equity_t = Equity_{t-1} + Retain_t$$

Eq. 9-11

The present value of all future payouts is the value to eternal shareholders:

$$v = \sum_{t=1}^{\infty} \frac{Payout_t}{(1+d)^t} = \sum_{t=1}^{\infty} \frac{Earnings_t - Retain_t}{(1+d)^t}$$

Eq. 9-12

Monte Carlo simulation of the equity growth model consists of first sampling the historical ROE and $Retain/Earnings$ data and then using the samples with the above formulas to calculate the future equity and payouts. The historical data used here is for the period 1993-2012 and Table 11 shows the subset for 1997-2012.

Decreasing ROE & Retain Ratios

It is shown in [**62**] that if the historical ROE and $Retain/Earnings$ for Coke are repeated randomly in the future, and the same is done for the S&P 500 stock market index using its historical data, then Coke's equity will outgrow the equity of the S&P 500 in 65 years. It seems unrealistic that a beverage producer will grow to dominate the stock markets in USA, so the growth assumptions must be wrong.

A simple solution is to decrease Coke's ROE and $Retain/Earnings$ over time by multiplying the samples with scales that decrease over time. The scale on $(Retain/Earnings)_t$ is set to:

$$ScaleRetain_t = 0.5 \cdot 0.9^t + 0.5$$

The scale on ROE_t is set to:

$$ScaleROE_t = 0.33 \cdot 0.95^t + 0.67$$

The Retain scale is 0.95 in year $t = 1$, in year 10 it is about 0.67, in year 50 it has almost converged to 0.5. The ROE scale is 0.9835 in year 1, in year 10 it is about 0.87, in year 50 it is about 0.70, and it converges to 0.67 as t increases. These scales were chosen so their combined effect is a scale that converges to 0.335 which gives the mean equity growth rate for Coke:

$$ScaleROE \cdot ScaleRetain \cdot E\left[ROE \cdot \frac{Retain}{Earnings}\right] = 0.335 \cdot 13.2\% \simeq 4.42\%$$

This is slightly less than the mean growth rate of 4.46% for the equity of the S&P 500 index, see [**62**]. The rates of decrease for these scales were chosen somewhat arbitrarily and the reader is encouraged to redo the experiments with different assumptions, using the source-code and data files provided below.

Source-Code and Data

The source-code and data files used in this Monte Carlo simulation are available at:

www.hvass-labs.org/people/magnus/publications/pedersen2013monte-carlo.zip

9.9.3. Analysis of Coca-Cola's Value Relative to S&P 500

Coke's future payouts resulting from the Monte Carlo simulation can be discounted with the future rate of return that can be obtained from the S&P 500 stock market index. This gives the value of Coke's future payouts relative to an investment in the S&P 500 index.

The future return on the S&P 500 is itself uncertain and its probability distribution can be estimated using Monte Carlo simulation with the same equity growth model as used for Coke but with data for the S&P 500 instead. This is done in [**62**] and results in Eq. 7-3 for the mean and Eq. 7-4 for the standard deviation of the annualized rate of return that an investor would get from buying shares in the S&P 500 index and holding the shares for eternity, which is called the value yield. Samples from the value yield distribution of the S&P 500 are then used as the discount rate in calculating the present value of Coke's future payouts.

The P/Book ratio of the S&P 500 is about 2.4 at the time of this writing in early July 2013. A log-normal distribution is used here for the S&P 500 value yield with the mean calculated from Eq. 7-3 and the standard deviation from Eq. 7-4, that is, $E[Value\ Yield] = 8.2\%$ and $Stdev[Value\ Yield] = 0.4\%$. The results of the present value calculations would have been similar if a normal distribution had been used.

A risk premium of 2% (percentage points) is added to the sampled value yield before using it as a discount rate. Adding a risk premium is normally done if a greater return is demanded from investing in a specific company over the S&P 500, but in this case it is because the present values would otherwise all be greater than Coke's market-cap so the share buyback valuation formulas could not be demonstrated properly.

Earnings Yield & Growth

The company's reported net income for fiscal year 2012 was USD 9b, see Table 11, and its market-cap is about USD 180b at the time of this writing in early July 2013. This gives an earnings yield of 5% which is much less than the mean value yield of 8.2% for the S&P 500 even without a risk premium. This means the company must grow its future earnings if a share buyback at this market-cap is to be as valuable to eternal shareholders as an investment in the S&P 500 is expected to be.

The mean of Coke's Monte Carlo simulated earnings for the first year is about 7.6% of the current market-cap, or USD 13.7b which seems too large considering last year's reported net income was only USD 9b and free cash flow was only USD 8b, see Table 11. The reason is that the first year's simulated earnings are calculated as the historical ROE, which is 42.6% on average for the period 1993-2012, multiplied by the current equity which is about USD 32.5. For the period 1993-1997 the average ROE was 60% while it was only 33% for the recent period 2008-2012. The Monte Carlo simulation samples the historical ROE at random so the higher ROE from the early years are just as likely to occur in the simulation as the lower ROE from recent years. The simulation could be redone with different assumptions.

In this simulation, after 10 years the mean earnings is about 16% of the initial market-cap, after 50 years the mean earnings is about 104% of the initial market-cap, and after 100 years the mean earnings is about 899% of the initial market-cap.

Share Buyback Valuation

Figure 20 shows the distribution of Coke's value to eternal shareholders without a share buyback $\mathbb{V}$ that results from 10k Monte Carlo simulations, with mean $E[\mathbb{V}] \simeq USD\ 263b$ and standard deviation $Stdev[\mathbb{V}] \simeq USD\ 63b$.

The mean equilibrium from Eq. 6-18 ensures that the mean value to eternal shareholders increases from a share buyback:

$$E[\mathbb{W}] > E[\mathbb{V}] \Leftrightarrow MarketCap < E[\mathbb{V}] \simeq USD\ 263b$$

For example, if Coke's market-cap is USD 180b as it is at the time of this writing in July 2013, and a share buyback is made for USD 3b, then the mean value to eternal shareholders becomes $E[\mathbb{W}] \simeq USD\ 264b$ which is calculated using Eq. 6-1 with the distribution of $\mathbb{V}$ from Figure 20. This is slightly higher than the mean value without the share buyback $E[\mathbb{V}] \simeq USD\ 263b$.

The mean equilibrium does not ensure that the expected relative value of a share buyback is greater than one, which means that relative losses are expected to be greater than relative gains. For example, if a share buyback is made for USD 3b at a market-cap of USD 255b then the mean relative value of the share buyback is $E[\mathbb{W}/\mathbb{V}] \simeq 0.9997$, as calculated using Eq. 6-15.

The relative equilibrium from Eq. 6-19 ensures the relative value of a share buyback is greater than one:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow MarketCap < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]} \simeq USD\ 249b$$

This is close to the mean equilibrium of USD 263b because the standard deviation of $\mathbb{V}$ is small at USD 63b.

For example, if a share buyback is made for USD 3b at a market-cap of USD 180b, then $E[\mathbb{W}/\mathbb{V}] \simeq 100.5\%$ which means such a share buyback increases the value to eternal shareholders by about 0.5% on average. But neither the mean nor the relative equilibrium guarantees that a share buyback increases the value to eternal shareholders.

Only a share buyback made at a market-cap below the minimum possible value that $\mathbb{V}$ can take on guarantees that the value to eternal shareholders is increased, see Eq. 6-20:

$$MarketCap < Min[\mathbb{V}] \simeq USD\ 113b\ \Rightarrow\ \mathbb{W}\ >\ \mathbb{V}$$

However, none of this is sufficient for properly assessing the impact of a share buyback on the value to eternal shareholders. The probability distribution must be considered to assess the likelihood of different outcomes. The probability that a share buyback increases value to eternal shareholders equals the probability that the market-cap is less than the value without a share buyback:

$$\Pr[\mathbb{W}\ >\ \mathbb{V}] = \Pr\left[MarketCap < \mathbb{V}\right]$$

This probability can be determined directly from the cumulative distribution function (CDF) of $\mathbb{V}$, see Eq. 6-29. The CDF in Figure 20 shows that this probability is about 0.94 (or 94%). This can also be seen from Figure 21 which shows the CDF of the relative value of a share buyback $\mathbb{W}/\mathbb{V}$:

$$\Pr[\mathbb{W}\ >\ \mathbb{V}] = \Pr[\mathbb{W}/\mathbb{V} > 1] \simeq 0.94$$

The probability distributions can also be used to answer other questions. For example, the probability is about 0.5 (or 50%) that the relative value of the share buyback is greater than 1.005, which means the probability is about 0.5 that a share buyback increases value to eternal shareholders by 0.5% or more.

9.9.4. Analysis of Coca-Cola's Value Relative to DJVC

Coke's future payouts resulting from the Monte Carlo simulation described above can be discounted with the rate of return that can be obtained from investing in the DJVC venture capital index described in section 7.6.1. This gives the value of Coke's future payouts relative to an investment in the DJVC index.

The future return of the DJVC index is itself uncertain. Venture companies cannot be modelled using the equity growth model used for the S&P 500 stock market index because venture companies typically do not have any earnings. The future return of the DJVC index is therefore simulated by sampling and compounding its historical monthly returns for the period 1991-2010, which are very volatile with mean 1.65% and standard deviation 9%, see [**62**] for details.

The simulated compounded returns of the DJVC index are then used as the discount rates in calculating the present value of Coke's future payouts. No risk premium is added.

There is no theoretical argument as to why the historical monthly returns of the DJVC index should merely be repeated randomly in the future, so the following results should be interpreted with skepticism. They are mainly intended to demonstrate various aspects of stochastic share buyback valuation.

Share Buyback Valuation

Figure 22 shows the distribution of Coke's value to eternal shareholders without a share buyback $\mathbb{V}$ that results from 10k Monte Carlo simulations, with $E[\mathbb{V}] \simeq USD\ 279b$ and $Stdev[\mathbb{V}] \simeq USD\ 786b$.

The mean equilibrium from Eq. 6-18 ensures that the mean value to eternal shareholders increases as a result of a share buyback:

$$E[\mathbb{W}] > E[\mathbb{V}] \Leftrightarrow MarketCap < E[\mathbb{V}] \simeq USD\ 279b$$

For example, if Coke's market-cap is USD 180b as it is at the time of this writing in July 2013, and a share buyback is made for USD 3b, then the mean value to eternal shareholders becomes $E[\mathbb{W}] \simeq USD\ 281b$ which is calculated using Eq. 6-1 with the distribution of $\mathbb{V}$ from Figure 22. This is slightly higher than the mean value without the share buyback $E[\mathbb{V}] \simeq USD\ 279b$.

The relative equilibrium from Eq. 6-19 ensures the relative value of a share buyback is greater than one:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow MarketCap < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]} \simeq USD\ 110b$$

This is far below the mean equilibrium USD 279b because the standard deviation of $\mathbb{V}$ is high at USD 786b.

For example, if a share buyback is made for USD 3b at a market-cap of USD 180b which is above the relative equilibrium of USD 110b, then the mean relative value of the share buyback is $E[\mathbb{W}/\mathbb{V}] \simeq 0.989$ as calculated using Eq. 6-15, which is less than one so the relative losses are on average greater than the relative gains resulting from the share buyback.

If instead a share buyback is made for USD 3b at a market-cap of USD 100b, then $E[\mathbb{W}/\mathbb{V}] \simeq 100.3\%$ which means the share buyback increases the value to eternal shareholders by about 0.3% on average.

Neither the mean nor the relative equilibrium guarantees that a share buyback increases value to eternal shareholders, which requires for the market-cap to be below the minimum possible value that $\mathbb{V}$ can take on, see Eq. 6-20:

$$MarketCap < Min[\mathbb{V}] \simeq USD\ 14b \Rightarrow \mathbb{W} > \mathbb{V}$$

This is much lower than both the mean and relative equilibriums of USD 279b and USD 110b respectively.

The probability that a share buyback increases value to eternal shareholders can be determined directly from the CDF in Figure 22:

$$\Pr[\mathbb{W} > \mathbb{V}] = \Pr[MarketCap < \mathbb{V}] \simeq 0.39$$

This can also be determined from the CDF of the relative value of a share buyback in Figure 23.

9.9.5. Comparison

As shown above, the present value of Coke's future payouts varies greatly depending on the discount rate. Using as discount rate the value yield of the S&P 500 results in Coke's present value ranging between USD 115b and 603b with mean USD 263b and standard deviation USD 63b. Using instead the simulated and compounded return of the DJVC index as the discount rate results in Coke's present value ranging between USD 14b and 41,000b with mean USD 279b and standard deviation USD 786b.

Although the mean annualized return of DJVC is about 16% while it is about 10.2% for the S&P 500 (including a 2% risk premium), the mean present value is greatest when using DJVC as discount rate. This is because the simulated DJVC returns are highly volatile so very low compounded returns do occur which result in very high present values that increase the average.

This shows that the variance of the discount rate can greatly affect the variance of the present value.

9.9.6. Jensen's Inequality

Jensen's inequality states that using the mean of a stochastic variable in non-linear calculations gives inaccurate results. Because the discounting in present value calculations is non-linear, using the mean discount rate under-estimates the present value. This is similar to Eq. 6-21 and detailed in [**62**].

If the mean value yield of the S&P 500 is used as the discount rate, then the resulting mean present value of Coke is USD 260b which is slightly less than the USD 263b when samples of the S&P 500 value yield were used as discount rates. If the mean annualized return of the DJVC is used as the discount rate, then the mean present value is USD 120b, which is significantly less than the USD 279b when samples of the compounded DJVC return were used as discount rates. The inaccuracy from using the mean discount rate increases as the variance of the discount rate increases.

Because the variance of the S&P 500 value yield is small compared to its mean, the other case studies using the mean value yield are deemed sufficiently accurate for demonstration purposes.

9.9.7. Conclusion

This case study demonstrated the stochastic share buyback equilibriums, how they relate to each other and the necessity for considering the probability distribution in addition to the equilibriums so as to properly assess the impact of a share buyback on the value to eternal shareholders. The effect of Jensen's inequality when using mean discount rates for present value calculations was also demonstrated.

More specifically, an equity growth model was used with historical data for the company's ROE and retained earnings to estimate the probability distribution of its value to eternal shareholders. This was done by Monte Carlo simulating the model thousands of times. Under the assumptions of this model, it was shown that there is a very high probability that share buybacks will increase the company's value to eternal shareholders, even though the company's current earnings yield is low, because the model shows there is a very high probability of sufficient earnings growth. Whether the assumptions of the model are reasonable is a qualitative assessment left to the reader who may redo the experiment with other assumptions.

Conclusion

10. The Share Buyback Decision for Managers

When a company has cash holdings or generates cash from operations, managers must first decide if the company has too much debt to ensure long-term solvency of the company, in which case the debt should be repaid until it has reached sustainable levels. Having sustainable debt is of primary importance because other capital allocation decisions are almost irrelevant to the shareholders if the company goes bankrupt and the shareholders then risk losing the entire value of their shares. The level of debt a company can sustain depends on its future operating earnings from which interest payments are to be made. Forecasting future earnings is usually a qualitative assessment.

If the company's debt is deemed sustainable then the company must decide whether to make investments in its existing business, e.g. by upgrading its production plants to maintain or increase competitiveness, or make acquisitions of other companies, or retain cash for investments or acquisitions that can reasonably be expected in the future. Share buybacks should also be considered at this stage as an alternative to investments and acquisitions, because a sufficiently low share price may cause a share buyback to increase shareholder value more than the investments or acquisitions, see section 4.7.

If the company has sustainable debt and it has made all investments and acquisitions that are available with satisfactory expected returns, and the company still has cash in excess of what it will likely need in the future, then the choice is between dividend payout or share buyback. Share buybacks should only be made when the share price is sufficiently low compared to the value to eternal shareholders, see sections 4 and 6, otherwise a dividend payout should be made. Because a low share price in relation to value may rarely occur, share buybacks should be considered the exception rather than the rule. When such low share prices do occur, the value to eternal shareholders can be increased by suspending dividends and using the proceeds for share buybacks instead.

Share buybacks for borrowed money should only be considered when the market-cap is significantly below the value to eternal shareholders. Because increasing a company's debt also increases the risk of default and bankruptcy. So if the market-cap is near the value to eternal shareholders then the share buyback for borrowed money would have no potential for gain but would increase the risk of loss to shareholders.

10.1. Similarity to Investing

Share buyback valuation is in many ways similar to valuation of long-term investments, but it is significantly easier to implement in practice for several reasons. Firstly because only one stock is being considered where as allocation of an investment portfolio may consider a universe of many thousand financial securities. Secondly because a share buyback is optional and can easily be avoided if the company's management is uncertain about its effect on shareholder value. Thirdly because the holding period for shares bought back by a company is eternity so there is no speculation about a future selling price.

10.2. Margin of Safety

The calculation of a company's value to eternal shareholders involves many assumptions about the future. Neither managers nor shareholders can accurately predict the future earnings of a company, so the calculation of a company's value to eternal shareholders is inherently imprecise. Graham's solution to this difficult problem was to instead have a margin of safety between the share price paid and the estimated

long-term value of the share, summarized thus [**3**]: *"the function of the margin of safety is, in essence, that of rendering unnecessary an accurate estimate of the future"*.

Without a margin of safety, a share buyback creates an obligation for the company to sustain or grow future earnings if the share buyback is merely to be value neutral to eternal shareholders. This obligation can easily be avoided by making a dividend payout instead of a share buyback, which should be the norm when managers are in doubt.

10.3. Timing Share Buybacks

As the share price becomes lower relative to the value to eternal shareholders, share buybacks become more accretive to shareholder value. Ideally, management should wait and time share buybacks so as to achieve the lowest purchase price the stock market will offer. But if future share prices are fundamentally unpredictable then by trying to time the share buyback, management would effectively be engaging in an act of stock market speculation which is not the duty of corporate management. It is instead recommended that managers make share buybacks when the share price becomes sufficiently low relative to the share buyback equilibrium and an appropriate margin of safety, and perhaps spread the share buybacks evenly over time or increase the amount of share buybacks if the share price decreases further.

10.4. Shareholder Information & Voting

This treatise shows that share buybacks can have a large impact on the value of a company to its eternal shareholders. It is therefore imperative that management gives accurate and complete information to shareholders regarding the share buyback policy and progress.

The share buyback policy communicated to shareholders must detail the share price limit below which share buybacks will be made, as well as the source of funds e.g. from excess cash, operating earnings or debt issuance, otherwise shareholders cannot determine the likely impact to shareholder value. The company must report at least quarterly the number and price of shares that have been bought back.

The shareholders should ideally be asked to vote on the share buyback policy because the capital ultimately belongs to them and they ought to determine how it should be allocated. A suggestion would be for management to first set a share price limit defined in terms of accounting numbers from the company's financial statements, e.g. a certain multiple of the book-value per share, or a multiple of the average earnings per share for the past few years, or whatever management believes is a good approximation for the company's value to eternal shareholders. Shareholders are then asked to vote on two issues:

1) At what percentage of the given share price limit would you want share buybacks to be made, provided management does not have other more profitable investment opportunities at hand?
2) How big a percentage of dividends would you want to be temporarily suspended and instead used for share buybacks, provided the share price is lower than the given limit?

The averages of the shareholders' votes can then be publicized and used as the price limit for share buybacks and the percentage of dividends that may be suspended and used for share buybacks.

11. Analysing Share Buybacks for Investors

A company's management has access to more detailed financial information than outsiders, so it may seem that management's estimate of the company's value would be the most accurate. But section 9 gave several examples of companies where both the management and stock market were incorrect. So the thorough shareholder should trust neither the value estimates of the company's management nor the stock market's price, and instead do an independent valuation of the company to assess whether its share buybacks are likely to increase or decrease value to eternal shareholders.

However, this currently poses a problem because most companies neither detail the share price limit nor the source of funds for making share buybacks, which are essential in estimating the likely value impact from share buybacks. It is suggested that shareholders do the following:

1) Adjust their estimates of the company's value by using the share buyback valuation formulas from section 4 similar to the case studies in section 9 with various assumptions for share buyback amounts and share prices. The greater the uncertainty, the wider the margin of safety there should be between the estimated value and the share price one should be willing to pay.
2) Contact the company's management or department for investor relations and demand that the share buyback policy is made sufficiently detailed and possibly also put to a shareholder vote. If enough shareholders contact management it may eventually have effect.

12. Conclusion

This treatise presented a formal theory for valuation of share buybacks in different scenarios, including share buybacks in connection with recapitalizations, alternative investments and stock option exercising.

It was shown that share buybacks affect the value to eternal shareholders depending on the share price relative to the excess cash and present value of future earnings available for dividend payouts, and a share buyback magnifies any mispricing. Buying back overpriced shares causes a much greater loss to eternal shareholders than the gains from buying back underpriced shares. If the future dividend tax rate is constant or unknown then it does not affect the value of share buybacks compared to dividend payouts.

A distinction must be made between the estimated and actual value of a share buyback. The actual value of a share buyback is typically not known in advance because it depends on the future earnings available for dividend payouts which are usually not known in advance. Therefore an estimate of the future earnings must be made so as to assess whether a share buyback is likely to increase or decrease value to eternal shareholders. In practice, using a range of valuation assumptions or a probability distribution helps in understanding the likely impact a share buyback will have on shareholder value.

It was shown both theoretically and empirically that the so-called substitution hypothesis is a fallacy. This hypothesis claims that share buybacks can substitute for dividend payouts as a way for companies to return capital to shareholders, possibly with a tax advantage depending on the individual shareholder's tax rates for capital gains and dividends. The substitution hypothesis is particularly important because it forms the basis of most academic hypotheses on share buybacks and may also be the cause of companies now making share buybacks that are substantially larger than dividend payouts. The companies in the well-known S&P 500 stock market index were shown to have had a historical tendency to increase share buybacks as the share price increases relative to an estimate of the value to eternal shareholders, which is the opposite of what the companies should have done. This was also shown for individual companies.

Corporate managers must properly inform their shareholders about the share buyback policy, especially regarding the share-price below which shares may be bought back and the source of cash, which is necessary in assessing the impact on shareholder value. Since share buybacks made at too high share prices cause great losses to eternal shareholders, it is recommended that managers use caution and skepticism about the future prospects of their company when making share buybacks, to ensure they do not make share buybacks that will later prove to have actually decreased value to eternal shareholders.

Appendix

13. Appendix

13.1. Geometric Series

The following is a well-known property of geometric series:

$$\sum_{t=0}^{\infty} x^t = \frac{1}{1-x} \text{ for } |x| < 1$$

Starting the summation at $t = 1$ gives:

$$\sum_{t=1}^{\infty} x^t = \sum_{t=0}^{\infty} x^t - 1 = \frac{1}{1-x} - 1 = \frac{x}{1-x} \text{ for } |x| < 1$$

So for $g < d$:

$$\sum_{t=0}^{\infty} \left(\frac{1+g}{1+d}\right)^t = \frac{1+d}{d-g}$$

Eq. 13-1

$$\sum_{t=1}^{\infty} \left(\frac{1+g}{1+d}\right)^t = \frac{1+d}{d-g} - 1 = \frac{1+g}{d-g}$$

Eq. 13-2

13.2. Future Share Buybacks

This appendix shows that future share buybacks should be ignored in the equilibrium formula.

13.2.1. Value without Share Buybacks

The value to eternal shareholders without share buybacks is the present value of all future dividends which consists of the excess cash assumed to be paid out as dividends now and future earnings assumed to be paid out as dividends annually. The present value, pre-tax and not per share, is:

$$v = \ Excess\ Cash + v_1$$

Eq. 13-3

where:

$$v_1 = \sum_{t=1}^{\infty} \frac{Earnings_t}{(1+d)^t}$$

This is based on the basic valuation formulas from section 3.

Let $Shares$ be the current number of shares outstanding, assumed to have equal rights to dividends. Further assume the tax-rate on dividends remains the same for eternity for the individual shareholder but it can be different amongst shareholders. The post-tax value per share is then:

$$V = \frac{1 - TaxDividend}{Shares} \cdot v$$

Eq. 13-4

13.2.2. Value with Share Buybacks

Consider now the value to eternal shareholders with share buybacks. Let the time-index $t = 0$ be now and assume the current dividends are: $Dividend_0 = Excess\ Cash - Buyback_0$, and for future years: $Dividend_t = Earnings_t - Buyback_t$. Let $Shares$ be the current number of shares outstanding and let $Shares_t$ be the number of shares outstanding after the share buyback but before the dividend payout of year t. Assume $MarketCap_t > 0$ and $MarketCap_t \geq Buyback_t \geq 0$ for all t. Ignore transaction costs for the share buybacks. The number of shares outstanding after a share buyback occurring now is:

$$Shares_0 = Shares \cdot \left(1 - \frac{Buyback_0}{MarketCap_0}\right)$$

Eq. 13-5

The number of shares outstanding in year t can be defined recursively:

$$Shares_t = Shares_{t-1} \cdot \left(1 - \frac{Buyback_t}{MarketCap_t}\right)$$

which reduces to:

$$Shares_t = Shares \cdot \prod_{k=0}^{t} \left(1 - \frac{Buyback_k}{MarketCap_k}\right)$$

This gives the following present value to eternal shareholders for arbitrary share buybacks now and in the future, post-tax and per share currently outstanding:

$$W = \left(\frac{Excess\ Cash - Buyback_0}{Shares_0} + \sum_{t=1}^{\infty} \frac{Earnings_t - Buyback_t}{Shares_t \cdot (1+d)^t}\right) \cdot (1 - TaxDividend)$$

Eq. 13-6

which can also be written as:

$$W = \frac{1 - TaxDividend}{Shares} \cdot w$$

where the total, not per share, pre-tax value of the company to its eternal shareholders is:

$$w = \frac{Excess\ Cash - Buyback_0 + w_1}{1 - \frac{Buyback_0}{MarketCap_0}}$$

and w_1 is defined as:

$$w_1 = \sum_{t=1}^{\infty} \frac{Earnings_t - Buyback_t}{(1+d)^t \cdot \prod_{k=1}^{t}\left(1 - \frac{Buyback_k}{MarketCap_k}\right)}$$

In this mathematical model the value to eternal shareholders is zero if $Buyback_t = Earnings_t$ for all t as that would mean $Dividend_t = 0$ for eternity and dividends are assumed to be the only source of value to eternal shareholders. In reality, however, the dividend/buyback policy can be changed so that dividends will eventually be paid out, at least when majority ownership has been obtained. This will be ignored in the following for the sake of simplicity.

13.2.3. Relative Value

The relative value of share buybacks is calculated from Eq. 13-6 divided by Eq. 13-4:

$$\frac{W}{V} = \frac{\frac{Excess\ Cash - Buyback_0 + w_1}{1 - \frac{Buyback_0}{MarketCap_0}}}{v}$$

Eq. 13-7

13.2.4. Equilibrium

The equilibrium is derived from Eq. 13-7 with $Buyback_0 > 0$:

$$W > V \Leftrightarrow MarketCap_0 < \frac{Buyback_0}{1 - \frac{Excess\ Cash - Buyback_0 + w_1}{v}}$$

Because w_1 is included in this formula it means the equilibrium for a share buyback being made now also depends on future share buybacks. This makes it necessary to show a lower bound for the equilibrium under reasonable assumptions of future share buybacks. Using the definition of v from Eq. 13-3 the equilibrium is at least v when:

$$Equilibrium \geq v \Leftrightarrow \frac{Buyback_0}{1 - \frac{Excess\ Cash - Buyback_0 + w_1}{v}} \geq v \Leftrightarrow w_1 \geq v_1$$

Recall that w_1 is the value to eternal shareholders with future share buybacks and v_1 is the value without future share buybacks. So this equation states that the equilibrium for a present-day share buyback is at least v whenever future share buybacks do not decrease the value. As the company can choose not to make share buybacks in the future it is always possible to have:

$$w_1 = v_1 \Leftrightarrow Equilibrium = v$$

If the present-day equilibrium is to be greater than v it requires for the future share buybacks to increase value to eternal shareholders:

$$Equilibrium > v \Leftrightarrow w_1 > v_1$$

So to avoid a decrease in value to eternal shareholders if shares are bought back today at a market-cap greater than v, it is necessary to buy back shares in the future at favourable market prices so that $w_1 > v_1$. Assuming future share prices are unpredictable then such an obligation for future share buybacks should be avoided because it may be impossible to fulfil. This means that share buybacks must be valued as if they are onetime occurrences.

13.3. Properties of Log-Normal Distribution

Let $\mathbb{V} \sim \ln \mathcal{N}(\mu, \sigma^2)$ be a log-normal distributed stochastic variable. The probability density function f is:

$$f(v) = \frac{1}{v \cdot \sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(\ln v - \mu)^2}{2\sigma^2}}$$

Eq. 13-8

The expected (or mean) value is known to be:

$$E[\mathbb{V}] = e^{\mu + \frac{\sigma^2}{2}}$$

Eq. 13-9

The variance is known to be:

$$Var[\mathbb{V}] = \left(e^{\sigma^2} - 1\right) \cdot e^{2\mu + \sigma^2}$$

Eq. 13-10

If the mean and variance are given, then the parameters μ and σ^2 are known to be:

$$\mu = \ln(E[\mathbb{V}]) - \frac{1}{2}\ln\left(1 + \frac{Var[\mathbb{V}]}{E[\mathbb{V}]^2}\right)$$

Eq. 13-11

$$\sigma^2 = \ln\left(1 + \frac{Var[\mathbb{V}]}{E[\mathbb{V}]^2}\right)$$

Eq. 13-12

The cumulative distribution function (CDF) is the probability that $\mathbb{V}$ is below some value. It is known to be:

$$F(v) = \Pr[\mathbb{V} < v] = \frac{1}{2} + \frac{1}{2} \cdot \operatorname{erf}\left[\frac{ln\, v - \mu}{\sqrt{2\sigma^2}}\right]$$

Eq. 13-13

The erf function is the so-called error function which is defined as an integral that is omitted here because it cannot be reduced to an elementary function and hence requires an approximation which is easiest done in mathematical computer software.

13.3.1. Reciprocal

The reciprocal $1/\mathbb{V}$ is also log-normal distributed but with negative parameter μ:

$$\frac{1}{\mathbb{V}} \sim \ln \mathcal{N}(-\mu, \sigma^2)$$

Eq. 13-14

The mean is found using Eq. 13-9:

$$E\left[\frac{1}{\mathbb{V}}\right] = e^{-\mu+\frac{\sigma^2}{2}}$$

Eq. 13-15

The mean can also be written as:

$$E\left[\frac{1}{\mathbb{V}}\right] = e^{-\mu+\frac{\sigma^2}{2}} = e^{(\mu-\mu)-\mu+\frac{\sigma^2}{2}} = e^{\mu+\frac{\sigma^2}{2}-2\cdot\mu} = E[\mathbb{V}] \cdot e^{-2\mu}$$

Eq. 13-16

Using Eq. 13-11 this is identical to:

$$E\left[\frac{1}{\mathbb{V}}\right] = E[\mathbb{V}] \cdot e^{-2\mu} = E[\mathbb{V}] \cdot e^{-2\left(\ln(E[\mathbb{V}])-\frac{1}{2}\ln\left(1+\frac{Var[\mathbb{V}]}{E[\mathbb{V}]^2}\right)\right)} = \frac{1}{E[\mathbb{V}]} + \frac{Var[\mathbb{V}]}{E[\mathbb{V}]^3}$$

Eq. 13-17

13.4. S&P 500 Data

Data for the S&P 500 stock market index has been collected by the staff at the Customized Research Department of Compustat. [52] The data was then used to calculate the various financial ratios.

The S&P 500 stock market index consists of 500 companies that are weighted according to certain changes and events that affect their capitalization. The weights are proprietary and could not be obtained. Instead, the data for the individual companies in the S&P 500 index has merely been aggregated (summed).

In the period 1983-2011 the S&P 500 index consisted mostly of companies reporting their financial statements in USD currency. Of the 500 companies in the index an average of 497 companies used USD currency each year. To avoid distorting effects of non-USD currencies those would either have to be converted into USD or removed from the data. For simplicity and because so few companies reported in non-USD currency they were removed from the data.

[52] The Compustat database was accessed through the facilities of the Collaborative Research Center 649 on Economic Risk at the Humboldt University of Berlin, Germany.

The S&P 500 index is being studied here as if it was one big conglomerate. This means the financial data such as assets, equity, earnings and dividends are aggregated. This differs from the accounting used in actual conglomerates where the consolidated financial statements would adjust for inter-company, intra-conglomerate dependencies such as revenue and liabilities. Making such consolidated financial statements is a complex process requiring access to financial details of the companies in question which is only available to those companies and their auditors. The sum of financial data is deemed sufficiently accurate for this study.

Compustat provides the data items PRSTKC and SSTK for the amount of share buyback and issuance, respectively, where the preferred and common stocks are combined. However, the other data items being considered in this study are for the common stock alone, which means comparisons and calculations made using these numbers for share buyback and issuance contain an error as the preferred stock is included. But the error is negligible because the preferred equity is only 4% of the common equity on average for the period 1983-2011.

Cash flows associated with tax benefits of stock options have been ignored as they are negligible.

Figures & Tables

Before Share Buyback	
$MarketCap$	
$v = 95\% \cdot MarketCap$	
Share Buyback is The Portion in Dark Grey	
$Buyback = 20\% \cdot MarketCap$	
$Buyback \simeq 21\% \cdot v$	
After Share Buyback	
$MarketCap$	
$v = 93.75\% \cdot MarketCap$	

Figure 1: Demonstration of the effect of a share buyback when the value to eternal shareholders is $v = 95\% \cdot \boldsymbol{MarketCap}$ and the share buyback is 20% of the $\boldsymbol{MarketCap}$. If the share-price remains unchanged then the mispricing is magnified slightly as the value to eternal shareholders decreases to become 93.75% of $\boldsymbol{MarketCap}$, down from 95% prior to the share buyback. According to Eq. 4-7 the value of the share buyback relative to a dividend payout is about 98.7%, that is, the value to eternal shareholders decreases about 1.3% as a result of the share buyback. If instead $\boldsymbol{MarketCap} = 95\% \cdot \boldsymbol{v}$ then the value to eternal shareholders would have increased by about 1.3% as a result of the share buyback. The equilibrium is when $\boldsymbol{MarketCap} = \boldsymbol{v}$ so the share buyback has equal effect on $\boldsymbol{MarketCap}$ and $\boldsymbol{v}$.

Before Share Buyback	
$MarketCap$	
$v = 25\% \cdot MarketCap$	
Share Buyback is The Portion in Dark Grey	
$Buyback = 20\% \cdot MarketCap$	
$Buyback = 80\% \cdot v$	
After Share Buyback	
$MarketCap$	
$v = 6.25\% \cdot MarketCap$	

Figure 2: Demonstration of the effect of a share buyback when the value to eternal shareholders is $v = 25\% \cdot \boldsymbol{MarketCap}$ and a share buyback is 20% of the $\boldsymbol{MarketCap}$. If the share-price remains unchanged then the mispricing is magnified greatly as the value to eternal shareholders decreases to become 6.25% of $\boldsymbol{MarketCap}$, down from 25% prior to the share buyback. According to Eq. 4-7 the value of the share buyback relative to a dividend payout is 25%, that is, the value to eternal shareholders decreases 75% as a result of the share buyback. If instead $\boldsymbol{MarketCap} = 25\% \cdot \boldsymbol{v}$ then the value to eternal shareholders would have increased by 300% as a result of the share buyback. The equilibrium is when $\boldsymbol{MarketCap} = \boldsymbol{v}$ so the share buyback has equal effect on $\boldsymbol{MarketCap}$ and $\boldsymbol{v}$.

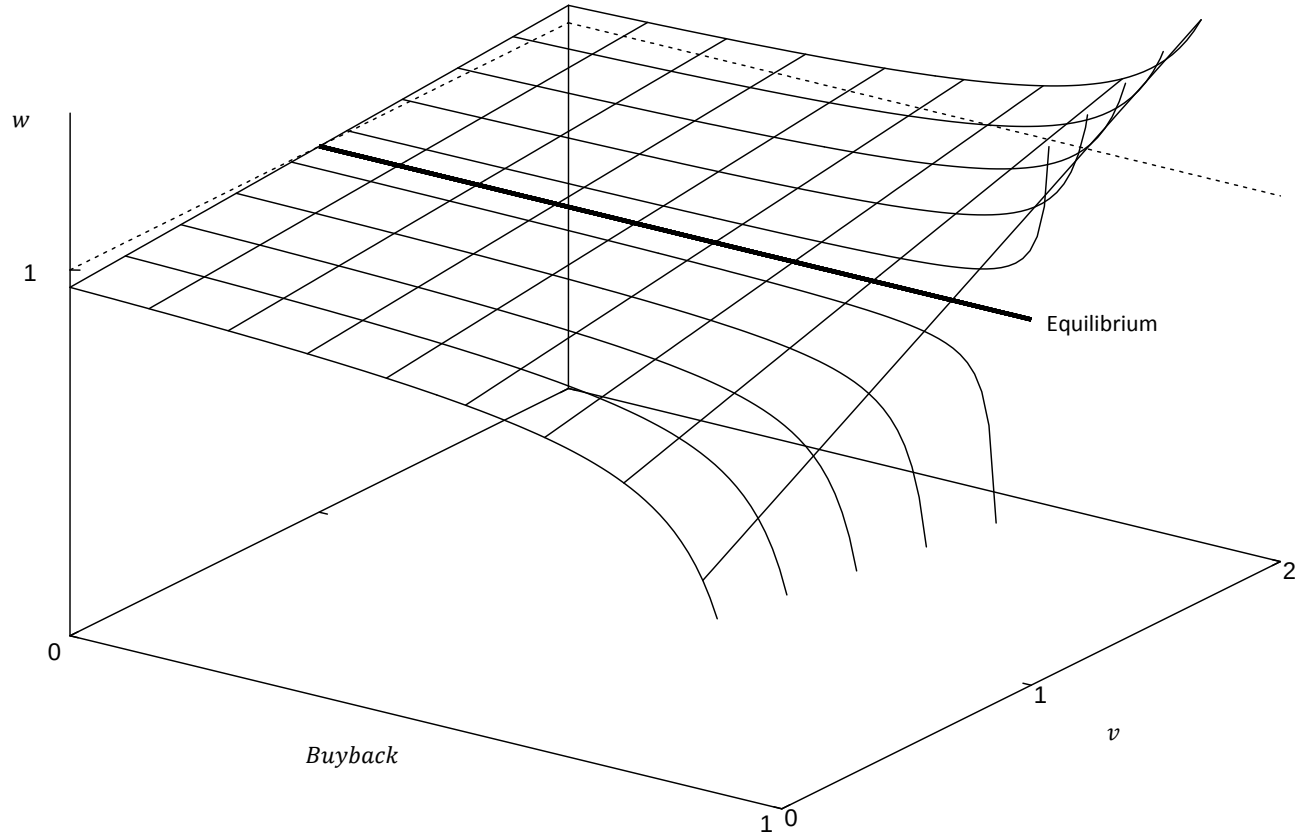


Figure 3: The value with a share buyback w (also called the absolute value of a share buyback) plotted by changing the variables in Eq. 4-6. The variables have been normalized so $MarketCap = 1$, $Buyback$ varies between 0 and 1, and the value to eternal shareholders v varies between 0 and 2. The plot shows that the value with a share buyback w is greater than the value without a share buyback v when $v > MarketCap = 1$, and vice versa. Equilibrium is where the value to eternal shareholders is unaffected by the share buyback so that $w = v$ (equivalently $W = V$) which occurs when $v = MarketCap = 1$. Note the linearity of w as a function of v and compare this to the non-linearity of the relative value of a share buyback W/V (equivalently w/v) as shown in Figure 4 and Figure 5.

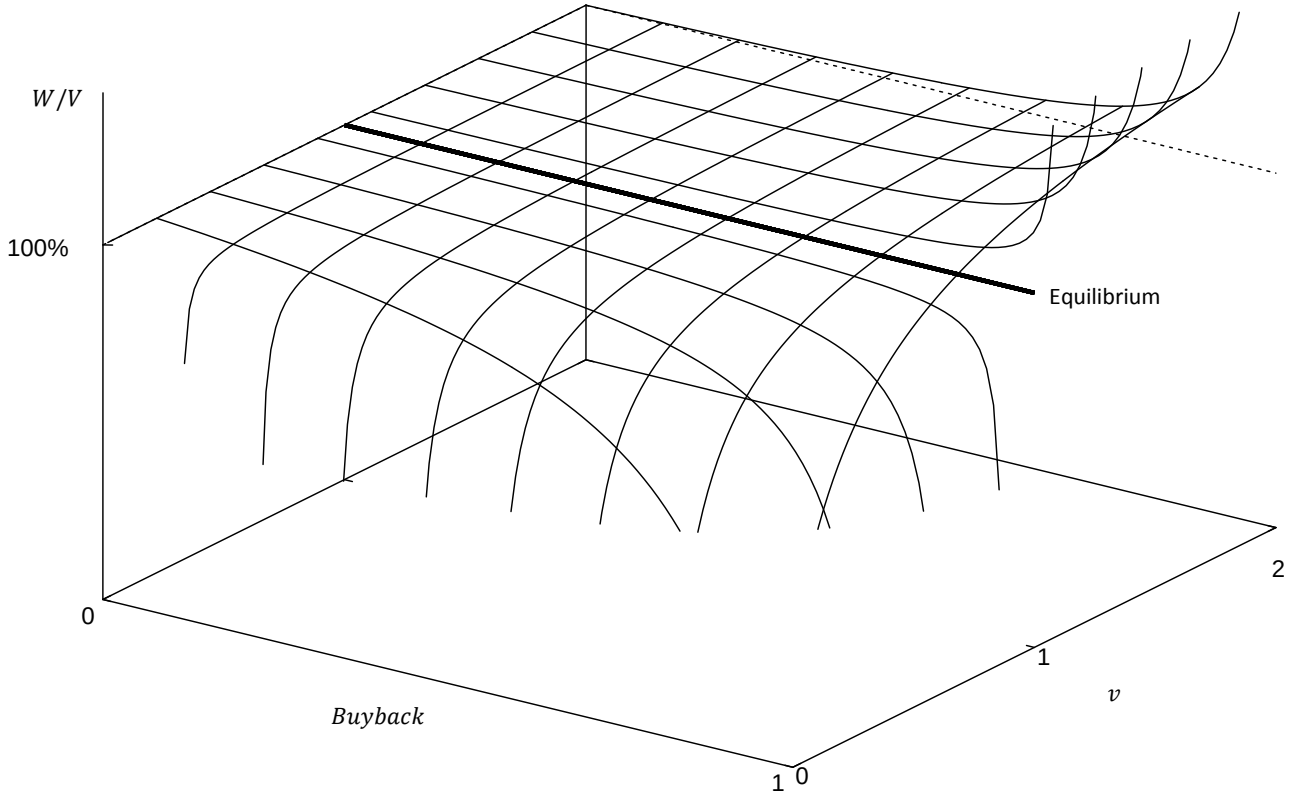


Figure 4: The relative value of a share buyback W/V (equivalently w/v) plotted by changing the variables in Eq. 4-7. The variables have been normalized so $MarketCap = 1$, $Buyback$ varies between 0 and 1, and the value to eternal shareholders v varies between 0 and 2. The plot shows that the value with a share buyback W is greater than the value without a share buyback V when $v > MarketCap = 1$, and vice versa. Equilibrium is where the value to eternal shareholders is unaffected by the share buyback so that $W = V$ which occurs when $v = MarketCap = 1$. Note the non-linearity of W/V as a function of v and compare this to the linearity of the absolute value of a share buyback as shown in Figure 3 and Figure 5.

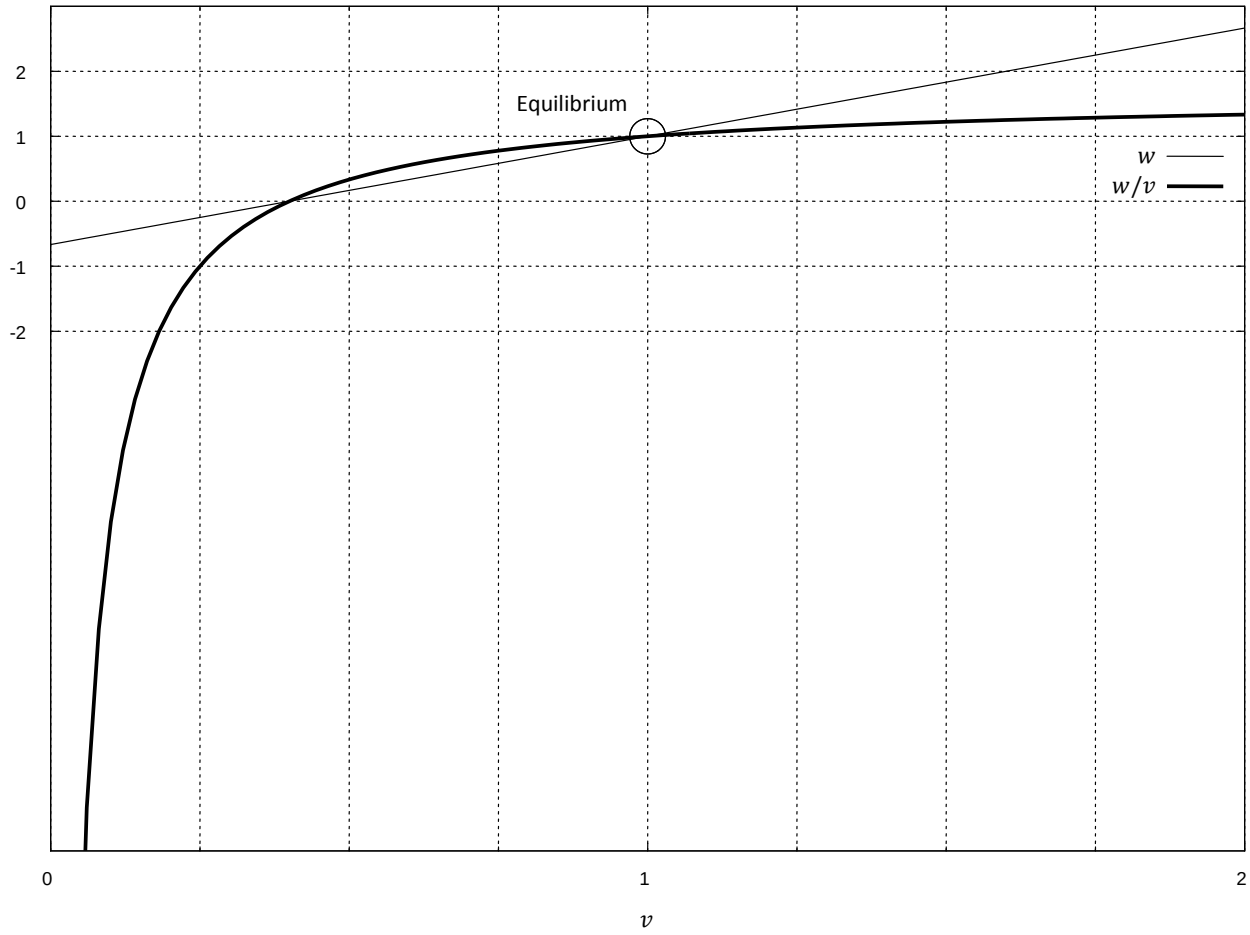


Figure 5: The relative value of a share buyback w/v (equivalently W/V) calculated from Eq. 4-7, compared to the absolute value of a share buyback w calculated from Eq. 4-6. The plot shows that the absolute value of a share buyback w changes linearly as a function of v while the relative value w/v changes non-linearly as a function of v. The variables have been normalized so $MarketCap = 1$, $Buyback$ is set to 0.4 as an example, and the value to eternal shareholders v varies between 0 and 2. The share buyback equilibrium is where the value to eternal shareholders is unaffected by the share buyback so that $w = v$ (equivalently $W = V$) which is marked here as a circle and occurs when $v = MarketCap = 1$. This plot is an excerpt of the 3-dimensional plots in Figure 3 and Figure 4 obtained by setting constant one axis at $Buyback = 0.4$.

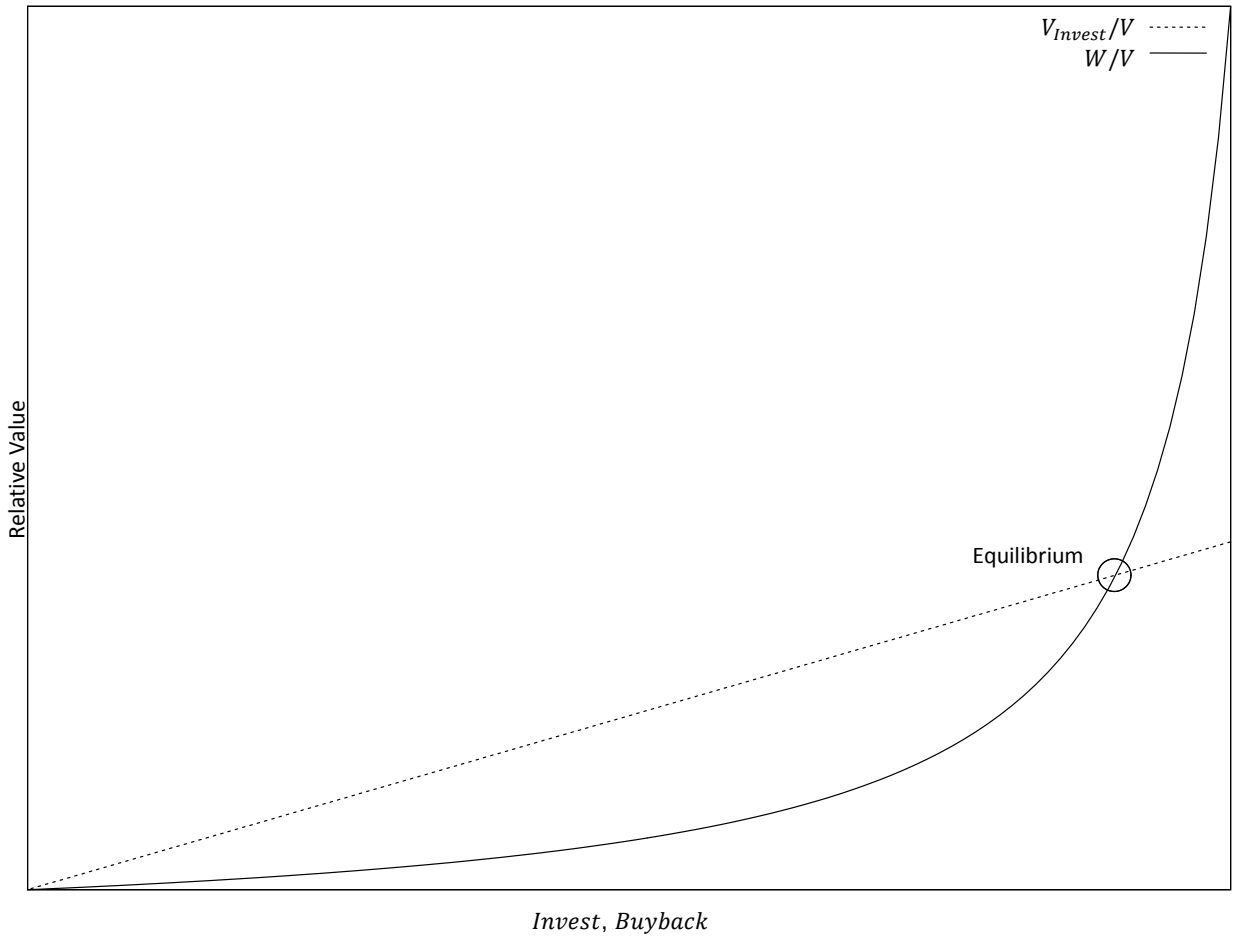


Figure 6: The relative value of a share buyback W/V calculated from Eq. 4-7, compared to the relative value of an investment with constant rate of return V_{Invest}/V calculated from Eq. 4-37. The plot shows that the relative value to eternal shareholders changes linearly with the investment amount but changes non-linearly with the share buyback amount. The equilibrium where the share buyback and alternative investment have equal effect on the value to eternal shareholders is marked by a circle and can be calculated from Eq. 4-42. This plot uses the example from section 9.4.3 with the annual rate of investment return $ROI = 17\%$, discount rate $d' = 10\%$, value to eternal shareholders $v = USD\ 17b$, and $MarketCap = USD\ 15.4b$.

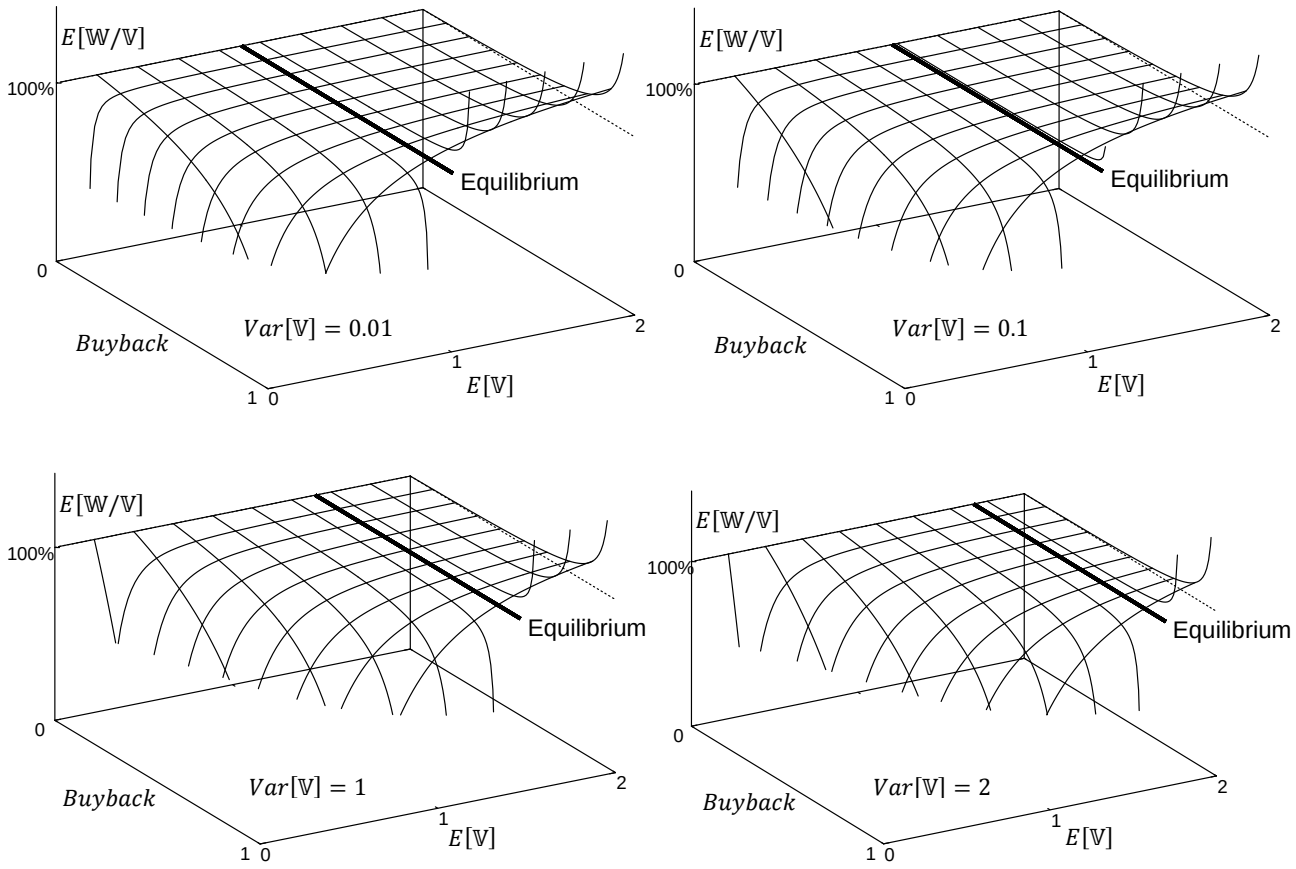


Figure 7: The mean relative value of a share buyback $E[\mathbb{W}/\mathbb{V}]$ when the value without a share buyback is a log-normal distributed stochastic variable $\mathbb{V}$. The plots are made by changing the variables in Eq. 6-38. The variables have been normalized so $MarketCap = 1$, $Buyback$ varies between 0 and 1, and the mean value to eternal shareholders $E[\mathbb{V}]$ varies between 0 and 2. The relative equilibrium is where $E[\mathbb{W}/\mathbb{V}]$=1 which is calculated using Eq. 6-39 and shown as thick lines in these plots, which shows that the relative equilibrium is significantly affected as the variance $Var[\mathbb{V}]$ increases. Compare this to Figure 4 for the non-stochastic case where the equilibrium is fixed.

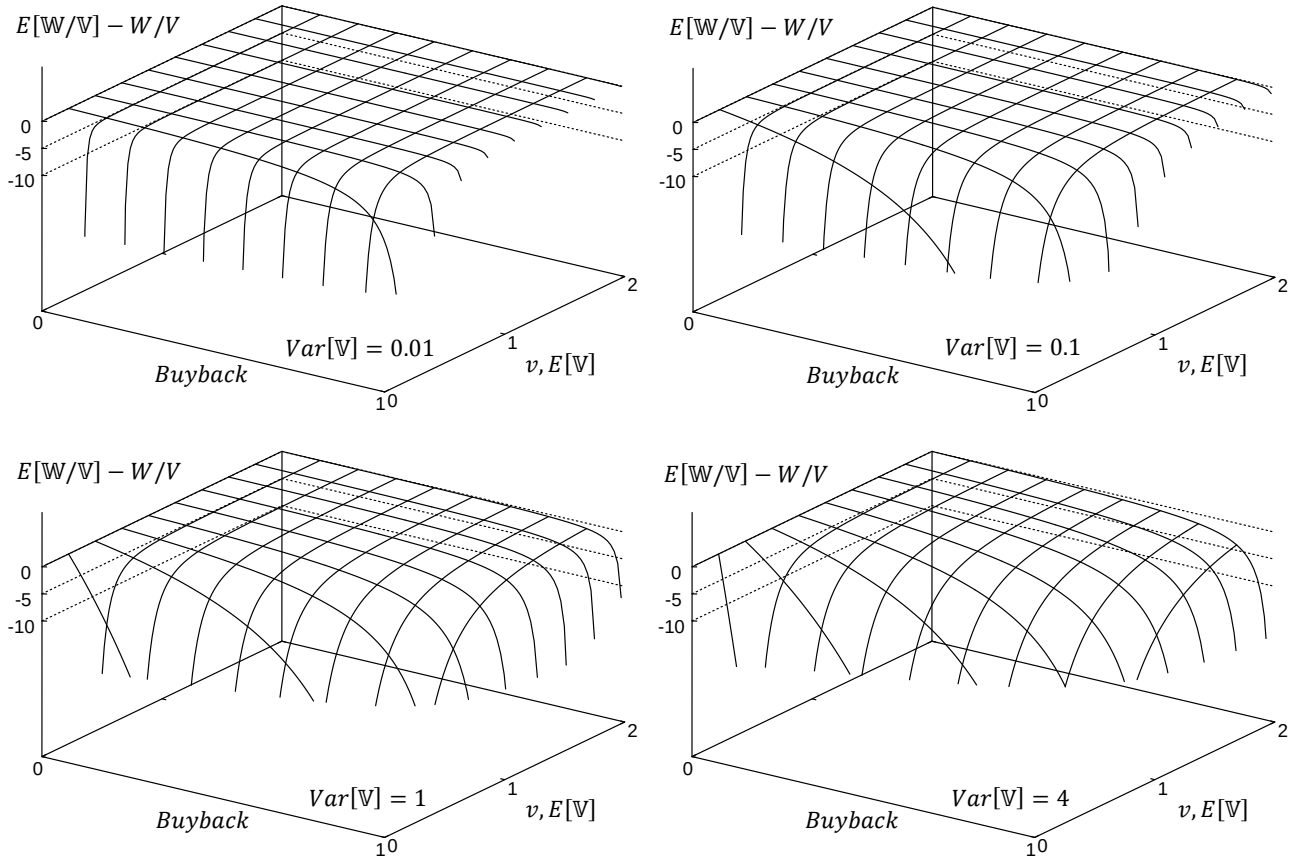


Figure 8: Difference between the stochastic and non-stochastic relative value of a share buyback $E[\mathbb{W}/\mathbb{V}] - W/V$, when $\mathbb{V}$ is a log-normal distributed stochastic variable. Plots are made by changing the variables in Eq. 6-45. The variables have been normalized so $MarketCap = 1$, $Buyback$ varies between 0 and 1, and the value to eternal shareholders $v = E[\mathbb{V}]$ varies between 0 and 2. The plots show that the difference $E[\mathbb{W}/\mathbb{V}] - W/V$ is magnified as the value $v = E[\mathbb{V}]$ decreases and the variance $Var[\mathbb{V}]$ and buyback amount increase.

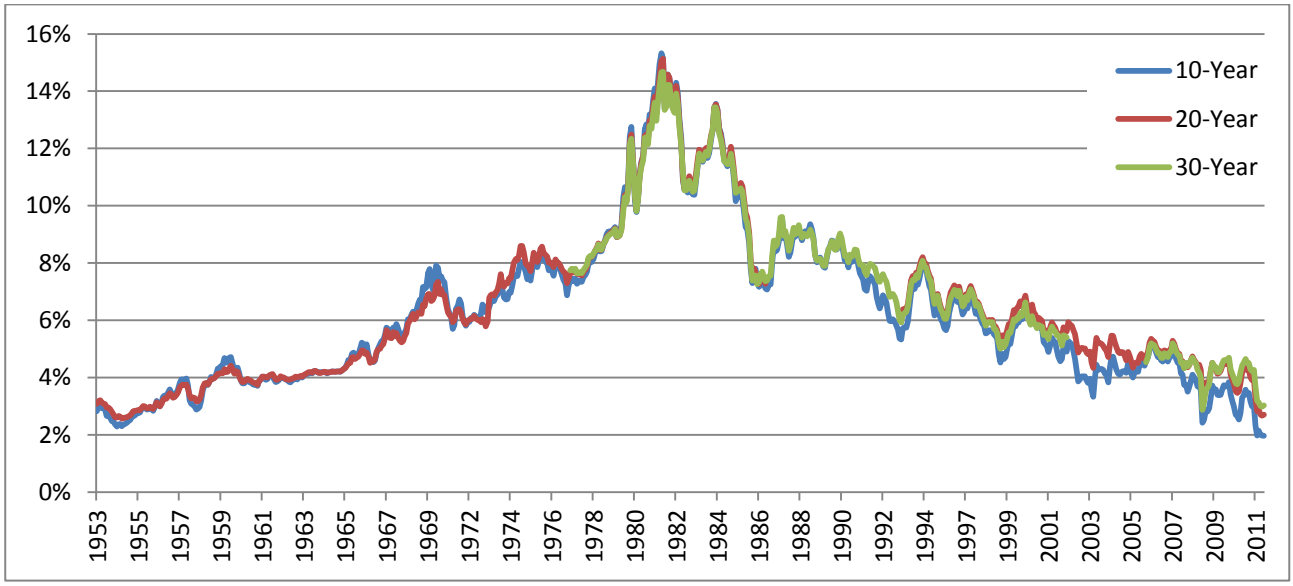


Figure 9: USA government bond yields (US Treasury, Constant Maturity Rate) for 10, 20 and 30 year maturity periods. Observations are monthly averages. For bonds with 10-year maturity the observation range is 1953-04 to 2012-01. For bonds with 20-year maturity the observation range is the same but the bond series was discontinued in the period 1987-01 to 1993-09. For bonds with 30-year maturity the observation range is 1977-02 to 2012-01 but the bond series was discontinued in the period 2002-03 to 2006-01.[53]

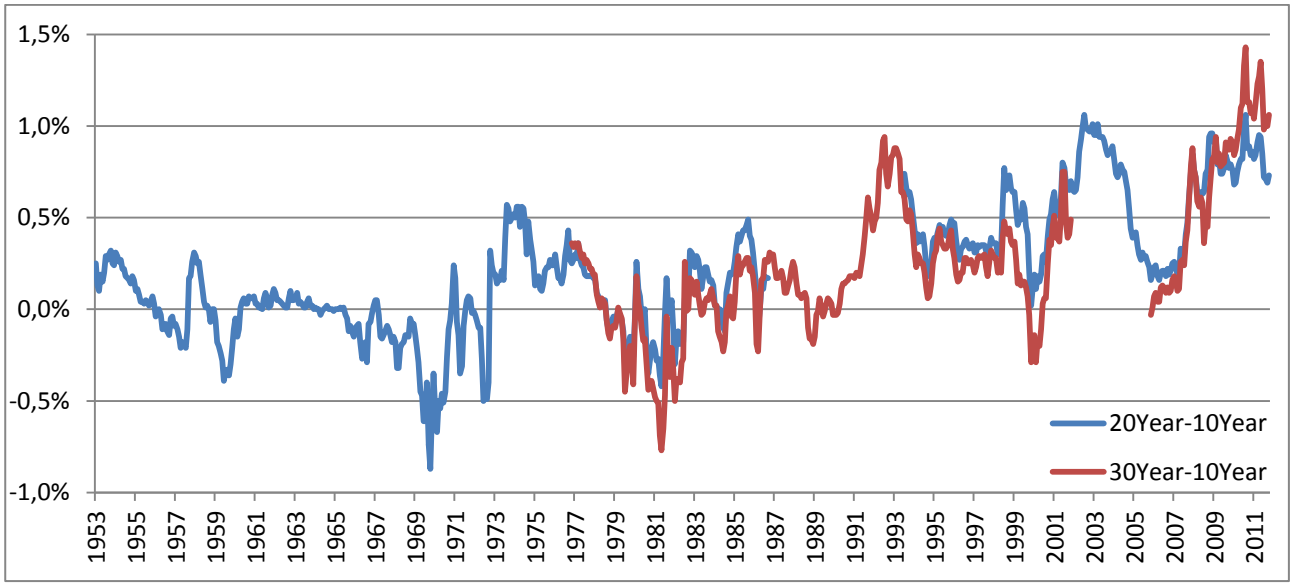


Figure 10: Difference in percentage points between the series in Figure 9.

Maturity	Min	Max	Arith. Mean	Geo. Mean	Std. Dev.
10-Year	1.97%	15.32%	6.24%	5.69%	2.72%
20-Year	2.57%	15.13%	6.22%	-	2.70%
30-Year	2.87%	14.68%	7.69%	-	2.66%
20Year-10Year	-0.9%	1.0%	0.2%	-	0.4%
30Year-10Year	-0.8%	1.4%	0.3%	-	0.4%

Table 1: Statistics for the yield on USA government bonds from Figure 9 and Figure 10.

[53] The GS10, GS20 and GS30 time series from research.stlouisfed.org/fred2/

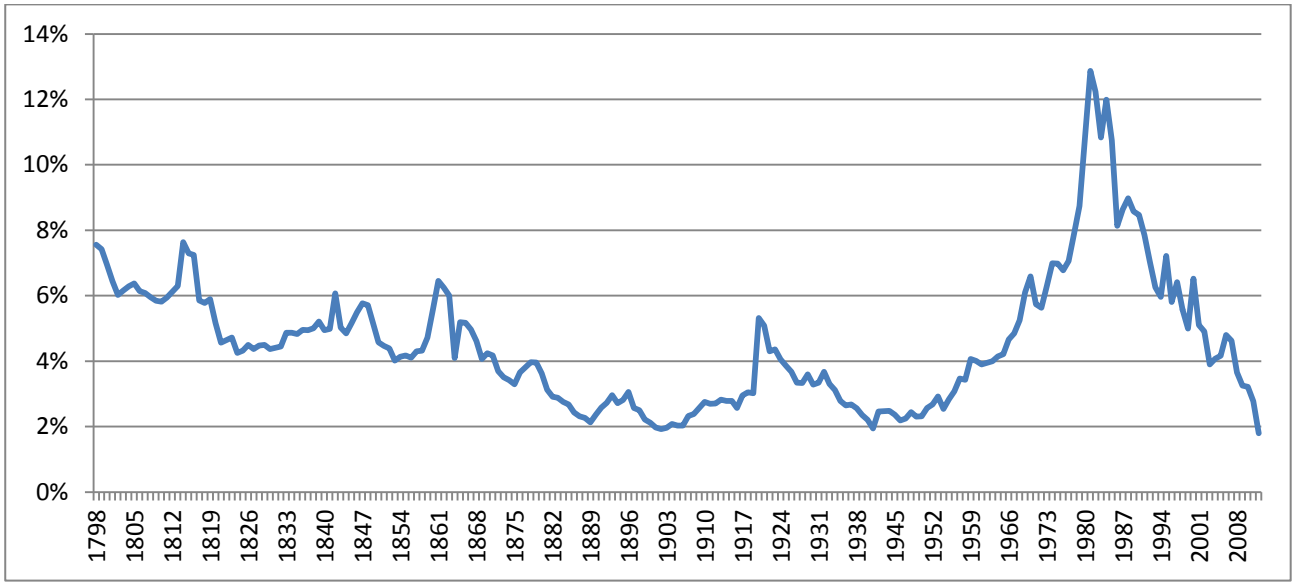


Figure 11: USA government bond yields for the period 1798-2012. Bonds have varying maturity, terms and taxation during this period. These are annual averages so the extreme bond yields shown in Figure 9 are smoothened somewhat.[54]

[54] Data for the period 1798-2005 is from Homer & Sylla [**59**] tables 38, 46, 48, 51, 87, and data for the period 2006-2012 is from the Federal Reserve:
www.federalreserve.gov/releases/h15/data.htm

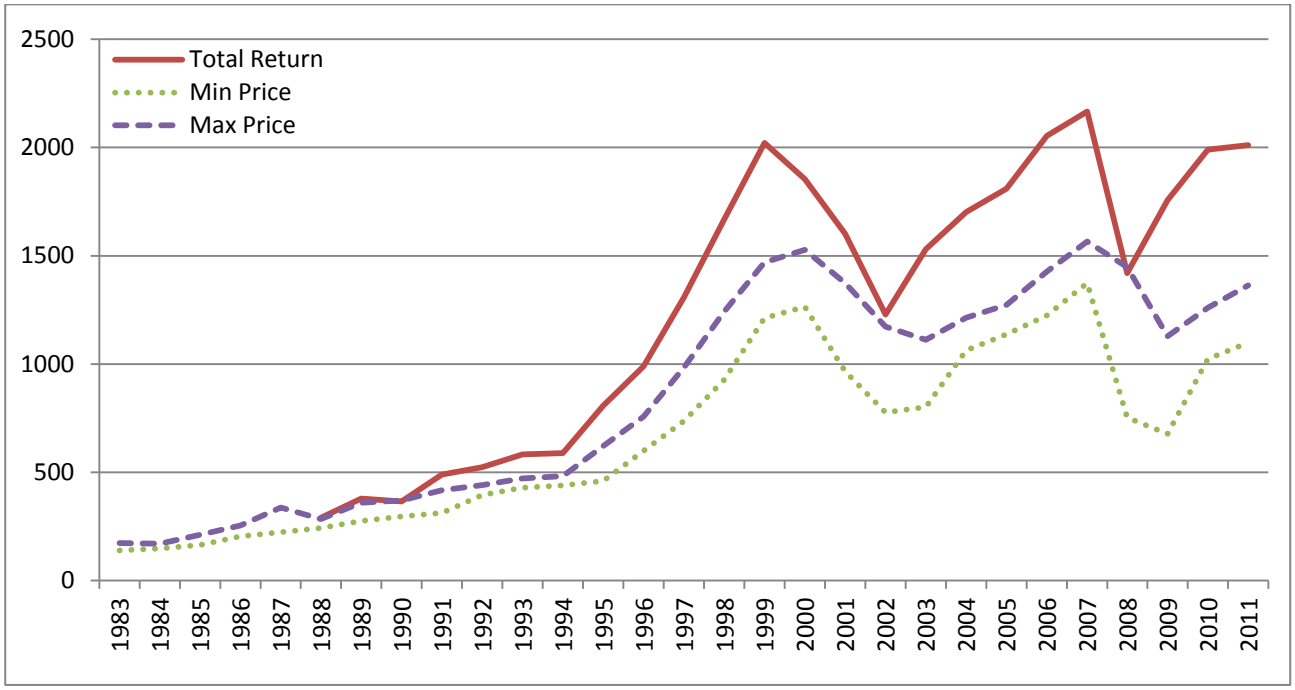


Figure 12: S&P 500 price range during each year and the total return at year-end starting in 1988.[55]

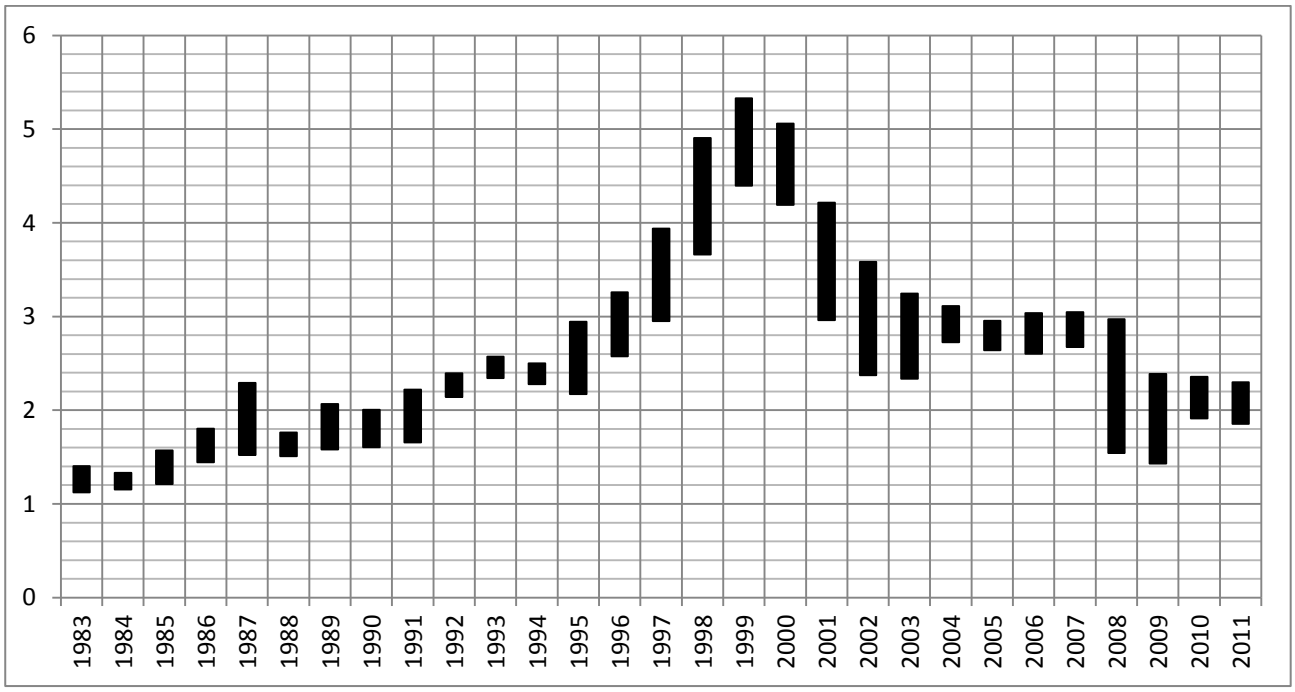


Figure 13: Range of P/Book ratios for the S&P 500 stock market index.

[55] www.compustat.com

		Year	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992
Value Yield	Mean	Min	10.9%	11.3%	10.2%	9.5%	8.4%	9.6%	8.8%	9.0%	8.5%	8.2%
		Max	12.5%	12.3%	11.9%	10.7%	10.4%	10.5%	10.2%	10.1%	9.9%	8.7%
	Stdev	Min	0.6%	0.7%	0.6%	0.5%	0.4%	0.5%	0.5%	0.5%	0.4%	0.4%
		Max	0.8%	0.8%	0.7%	0.6%	0.6%	0.6%	0.6%	0.6%	0.6%	0.5%

		Year	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
Value Yield	Mean	Min	7.9%	8.0%	7.5%	7.2%	6.7%	6.3%	6.1%	6.2%	6.6%	6.9%
		Max	8.3%	8.4%	8.6%	7.9%	7.5%	6.9%	6.5%	6.6%	7.5%	8.2%
	Stdev	Min	0.4%	0.4%	0.4%	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%	0.3%
		Max	0.4%	0.4%	0.4%	0.4%	0.4%	0.3%	0.3%	0.3%	0.4%	0.4%

		Year	2003	2004	2005	2006	2007	2008	2009	2010	2011
Value Yield	Mean	Min	7.2%	7.3%	7.5%	7.4%	7.4%	7.5%	8.2%	8.3%	8.4%
		Max	8.3%	7.7%	7.9%	7.9%	7.8%	10.3%	10.8%	9.2%	9.3%
	Stdev	Min	0.3%	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%	0.4%
		Max	0.4%	0.4%	0.4%	0.4%	0.4%	0.6%	0.6%	0.5%	0.5%

Table 2: Value yield mean and standard deviation for the S&P 500 stock market, calculated using Eq. 7-3 and Eq. 7-4 with each year's minimum and maximum P/Book ratios from Figure 13.

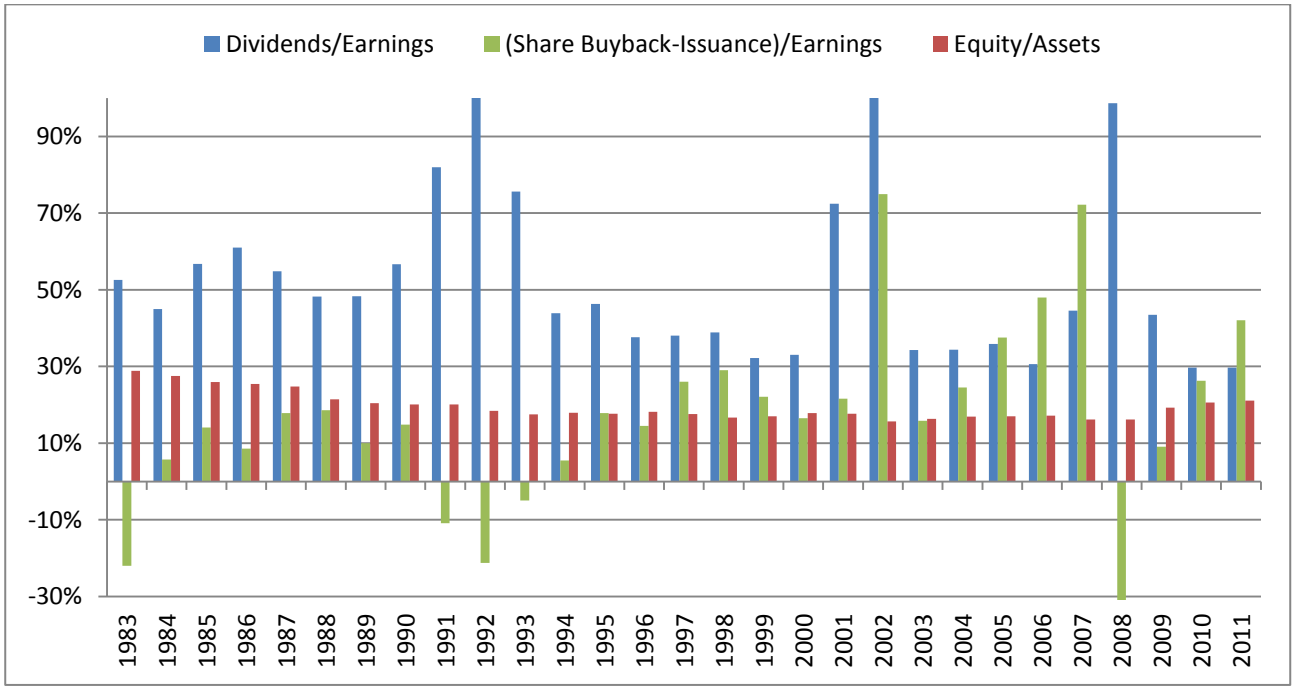


Figure 14: Ratios for the S&P 500 stock market index. The dividends to earnings ratio exceeds the plot area in 1992 (171%), 2002 (166%) and 2008 (99%) because the earnings were very low in those years but dividends were not decreased correspondingly. The net share buybacks to earnings ratio exceeds the plot area in 2008 (-41%) because large share issuances were made.[56]

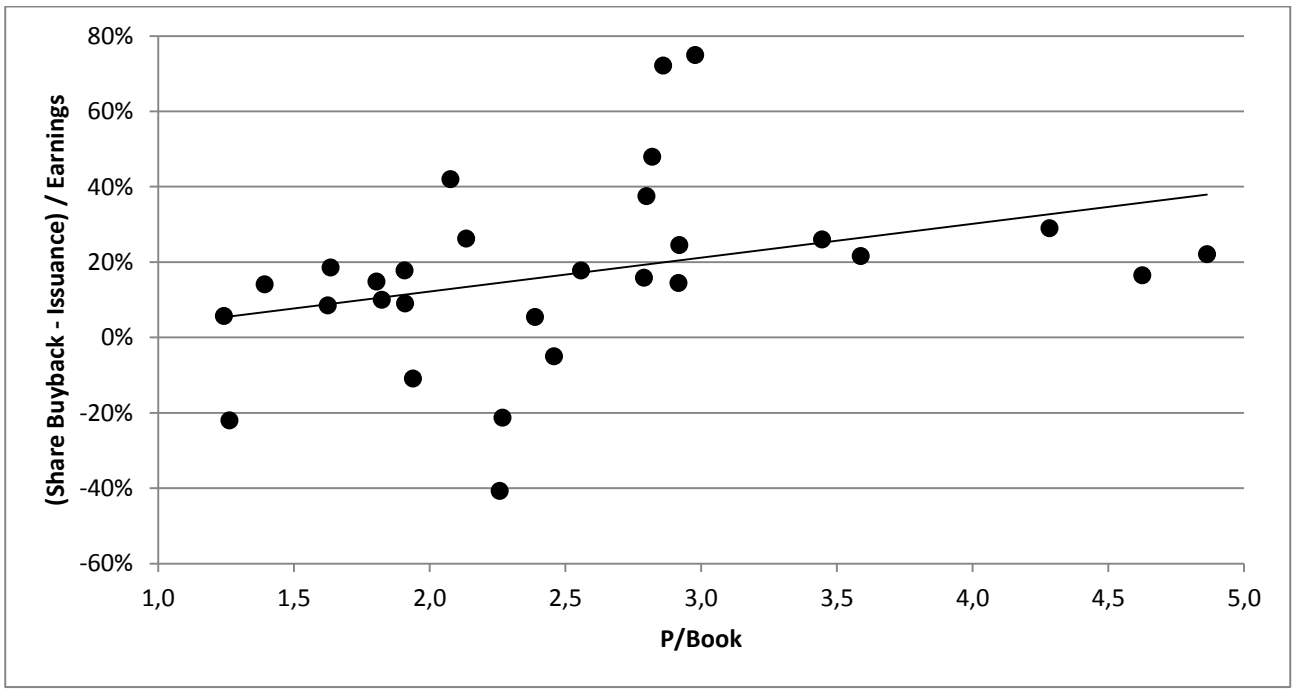


Figure 15: Scatter plot of the average P/Book ratio from Figure 13 and the net share buyback ratio from Figure 14 for the S&P 500 stock market index in the period 1983-2011. The trend-line results from linear regression of the data points and suggests there is a tendency for companies to increase share buybacks as the P/Book increases.

[56] Data is documented in appendix 13.4.

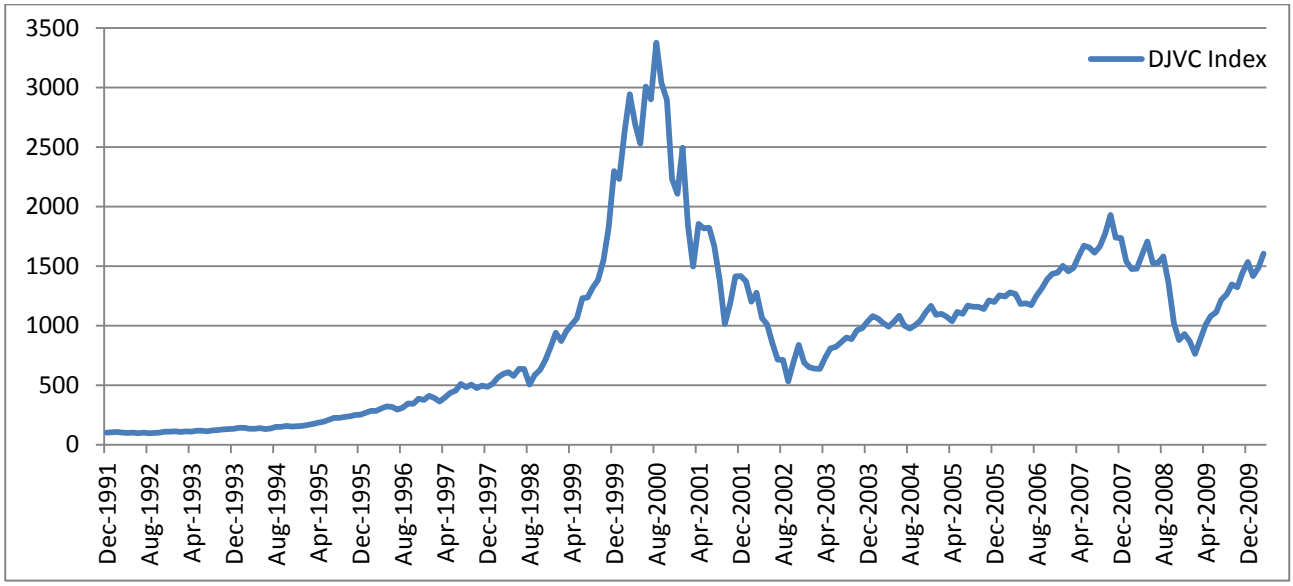


Figure 16: Dow Jones Venture Capital (DJVC) / Sand Hill index. During this period the index covered approximately 18,000 venture companies with 60,000 valuation events (rounds of funding, public offerings, acquisitions and shutdowns). The observation range is December 1991 to March 2010. The peak in year 2000 is known as the Dot-Com bubble and the valley in 2008-2009 is a severe financial crisis.[57]

[57] www.sandhillecon.com
www.djindexes.com/venturecapital/

USD Millions	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Revenue	62	76	157	172	206	232	275	293	279	248	214	90
Net Income	(2)	11	43	41	43	50	58	65	32	19	(272)	(33)
Free Cash Flow (FCF)	-	(13)	(19)	51	54	106	97	63	3	16	(28)	(78)
Dividends	-	-	-	-	0	(8)	(11)	(16)	(21)	(15)	0	0
Share Issuance	-	-	-	-	-	0.4	3	4	12	0.01	0	0
Share Buyback	-	-	-	-	-	0	(61)	(142)	(99)	0	0	0
Assets	563	575	618	662	726	675	588	574	600	608	346	389
Equity	518	528	571	612	655	396	388	303	237	244	(25)	(56)
Debt	-	-	-	-	0	175	50	127	231	232	274	362
Interest Income, Net	2.9	1.8	1.4	0.8	0.3	(5.2)	(6.4)	(8.1)	(12.6)	(12.9)	(19.6)	(32)
ROA	-	2%	7%	7%	6%	7%	9%	11%	6%	3%	-	-
ROE	-	2%	8%	7%	7%	8%	15%	17%	11%	8%	-	-
Shares, Basic	-	-	-	-	37.5	37.6	36.1	33.3	29.6	28.5	28.6	28.8
Shares, Diluted	-	-	-	-	37.5	37.8	36.4	33.8	29.9	28.5	28.6	28.8
Market-Cap, Low	-	-	-	-	-	623	647	875	323	80	42	0.2
Market-Cap, High	-	-	-	-	-	958	1141	1246	1021	508	217	66

Table 3: Selected financial data for the company *Jackson Hewitt Tax Service*. For 2011 the data is only for the first 3 quarters of the fiscal year before the company went bankrupt. In fiscal 2005 the company's shares became publicly traded. A special cash dividend of USD 175m was paid in 2005 to its former parent company and is not listed here. A receivable account of USD 132m from the parent company was cancelled in 2005 as a non-cash dividend payout. Net Loss for 2010 includes a non-cash USD 274m goodwill impairment. Free Cash Flow is defined here as Operating Cash Flow minus Investing Cash Flow. ROA and ROE are calculated as Net Income for a year divided by the Assets and Equity for the previous year, respectively. Basic and diluted shares outstanding are weighted averages for the year. Market-Cap is calculated as the number of basic shares multiplied by the minimum and maximum share-price for the year; which should ideally have been calculated using the quarterly numbers of shares outstanding but is sufficiently accurate for this analysis.[58]

[58] Form S-1 prospectus (years 2000-2003), Form 10-K annual reports (years 2004-2010), Form 10-Q quarterly report for the quarter ending January 31, 2001 (year 2011), all filed with US SEC:
www.sec.gov/Archives/edgar/data/1283552/000119312504041498/ds1.htm
www.sec.gov/cgi-bin/browse-edgar?action=getcompany&CIK=0001283552&type=10-k
www.sec.gov/Archives/edgar/data/1283552/000119312511061597/d10q.htm

USD Millions	1997	1998	1999	2000	2001	2002	2003	2004
Revenue	11,936	15,262	19,747	22,956	25,296	28,365	32,187	36,835
Net Income	3,454	4,490	7,785	9,421	7,346	5,355	7,531	8,168
Free Cash Flow (FCF)	4,190	6,224	9,447	10,547	12,319	13,739	14,906	13,517
Acquisitions	-	(190)	79	-	-	-	(1,063)	(4)
Dividends	-	-	-	-	-	-	(857)	(1,729)
Share Issuance	744	959	1,350	2,245	1,620	1,497	2,120	2,748
Share Buyback	(3,101)	(2,468)	(2,950)	(4,896)	(6,074)	(6,069)	(6,486)	(3,383)
Assets	14,387	22,357	38,321	51,694	58,830	69,910	81,732	94,368
Cash & Equivalents	8,966	13,927	17,236	23,798	31,600	38,652	49,048	60,592
Equity	10,777	16,627	28,438	41,368	47,289	54,842	64,912	74,825
ROA	-	31%	35%	25%	14%	9%	11%	10%
ROE	-	42%	47%	33%	18%	11%	14%	13%
Shares, Basic	9,564	9,728	10,056	10,378	10,683	10,811	10,723	10,803
Shares, Diluted	10,488	10,724	10,964	11,072	11,148	11,106	10,882	10,894
Market-Cap, Low	64,270	143,488	220,629	313,312	221,672	262,815	229,686	260,892
Market-Cap, High	161,345	264,018	480,828	622,369	438,003	392,277	312,254	323,658

USD Millions	2005	2006	2007	2008	2009	2010	2011	2012
Revenue	39,788	44,282	51,122	60,420	58,437	62,484	69,943	73,723
Net Income	12,254	12,599	14,065	17,681	14,569	18,760	23,150	16,978
Free Cash Flow (FCF)	15,793	12,826	15,532	18,430	15,918	22,096	24,639	29,321
Acquisitions	(207)	(649)	(1,150)	(8,053)	(868)	(245)	(71)	(10,112)
Dividends	(36,112)	(3,545)	(3,805)	(4,015)	(4,468)	(4,578)	(5,180)	(6,385)
Share Issuance	3,109	2,101	6,782	3,494	579	2,311	2,422	1,913
Share Buyback	(8,057)	(19,207)	(27,575)	(12,533)	(9,353)	(11,269)	(11,555)	(5,029)
Assets	70,815	69,597	63,171	72,793	77,888	86,113	108,704	121,271
Cash & Equivalents	37,751	34,161	23,411	23,662	31,447	36,788	52,772	63,040
Equity	48,115	40,104	31,097	36,286	39,558	46,175	57,083	66,363
ROA	13%	18%	20%	28%	20%	24%	27%	16%
ROE	16%	26%	35%	57%	40%	47%	50%	30%
Shares, Basic	10,839	10,438	9,742	9,328	8,945	8,813	8,490	8,396
Shares, Diluted	10,906	10,531	9,886	9,470	8,996	8,927	8,593	8,506
Market-Cap, Low	259,269	223,999	216,564	250,643	133,012	193,886	192,978	199,740
Market-Cap, High	324,953	296,230	306,678	349,800	254,933	278,314	250,115	276,648

Table 4: Selected financial data for the company *Microsoft*. Free Cash Flow is defined as Operating Cash Flow minus Capital Expenditures (there were no proceeds from sale of property, plant and equipment listed during this period, which would have been added otherwise). Acquisitions are of companies and do not include acquired minority interests, patents and technology. Cash & Equivalents also includes short-term investments. ROA and ROE are calculated as Net Income for a year divided by the Assets and Equity for the previous year, respectively. Basic and diluted shares outstanding are weighted averages for the year. Market-Cap is calculated as the number of basic shares multiplied by the minimum and maximum share-price for the year. The share numbers and prices are adjusted for stock-splits. Years 1997-2000 had a small issue of convertible preferred stock.[59]

[59] Company's self-published annual reports (1997-2000), Form 10-K annual reports filed with US SEC (2001-2012): www.microsoft.com/investor/AnnualReports/default.aspx
www.sec.gov/cgi-bin/browse-edgar?action=getcompany&CIK=0000789019&type=10-K

USD Millions	1997	1998	1999	2000	2001	2002	2003	2004
Revenue	11,409	12,421	13,259	14,243	14,870	15,406	17,140	19,065
Net Income	1,642	1,550	1,948	1,977	1,637	893	1,471	2,279
Free Cash Flow (FCF)	225	818	747	539	620	423	1,899	2,521
Dividends	(221)	(239)	(265)	(281)	(288)	(297)	(504)	(695)
Share Issuance	-	-	-	-	-	-	171	581
Share Buyback	(765)	(1,162)	(933)	(2,002)	(1,090)	(687)	(439)	(605)
Assets	18,242	19,784	20,983	21,684	22,535	24,194	25,838	27,838
Equity	8,852	9,465	9,639	9,204	9,488	10,281	11,982	14,201
ROA	-	8%	10%	9%	8%	4%	6%	9%
ROE	-	18%	21%	21%	18%	9%	14%	19%
Shares, Basic	1,379	1,365	1,355	1,323	1,290	1,273	1,270	1,260
Shares, Diluted	1,410	1,406	1,404	1,357	1,309	1,282	1,277	1,274
Market-Cap, Low	29,045	30,457	48,699	34,901	31,928	19,311	15,392	30,920
Market-Cap, High	37,836	54,259	67,154	57,722	45,227	39,107	34,290	41,530

USD Millions	2005	2006	2007	2008	2009	2010	2011
Revenue	20,460	20,895	22,787	23,522	22,745	24,075	27,006
Net Income	2,602	3,544	2,395	4,313	4,551	4,946	5,503
Free Cash Flow (FCF)	2,519	3,068	3,726	4,292	4,096	4,286	4,579
Dividends	(842)	(1,217)	(1,766)	(1,823)	(2,235)	(2,408)	(2,610)
Share Issuance	768	976	1,138	548	332	463	334
Share Buyback	(1,228)	(2,959)	(3,943)	(3,919)	(2,797)	(2,699)	(3,363)
Assets	29,989	28,974	29,392	28,462	30,225	31,975	32,990
Equity	15,146	15,458	15,280	13,383	14,034	14,634	14,390
ROA	9%	12%	8%	15%	16%	16%	17%
ROE	18%	23%	15%	28%	34%	35%	38%
Shares, Basic	1,260	1,234	1,188	1,127	1,092	1,066	1,032
Shares, Diluted	1,274	1,252	1,212	1,146	1,107	1,080	1,045
Market-Cap, Low	34,474	39,155	50,264	51,605	55,080	65,090	74,448
Market-Cap, High	44,969	55,135	75,664	75,509	70,707	86,282	104,232

Table 5: Selected financial data for the company *McDonald's*. Free Cash Flow is defined as Operating Cash Flow minus Investing Cash Flow because the investments are primarily related to the company's main business. There were no acquisitions of other companies during this period. ROA and ROE are calculated as Net Income for a year divided by the Assets and Equity for the previous year, respectively. Basic and diluted shares outstanding are weighted averages for the year. Market-Cap is calculated as the number of basic shares multiplied by the minimum and maximum share-price for the year. Share issuances are proceeds from stock option exercising, which is not specified separately in the cash flow statements prior to 2003.[60]

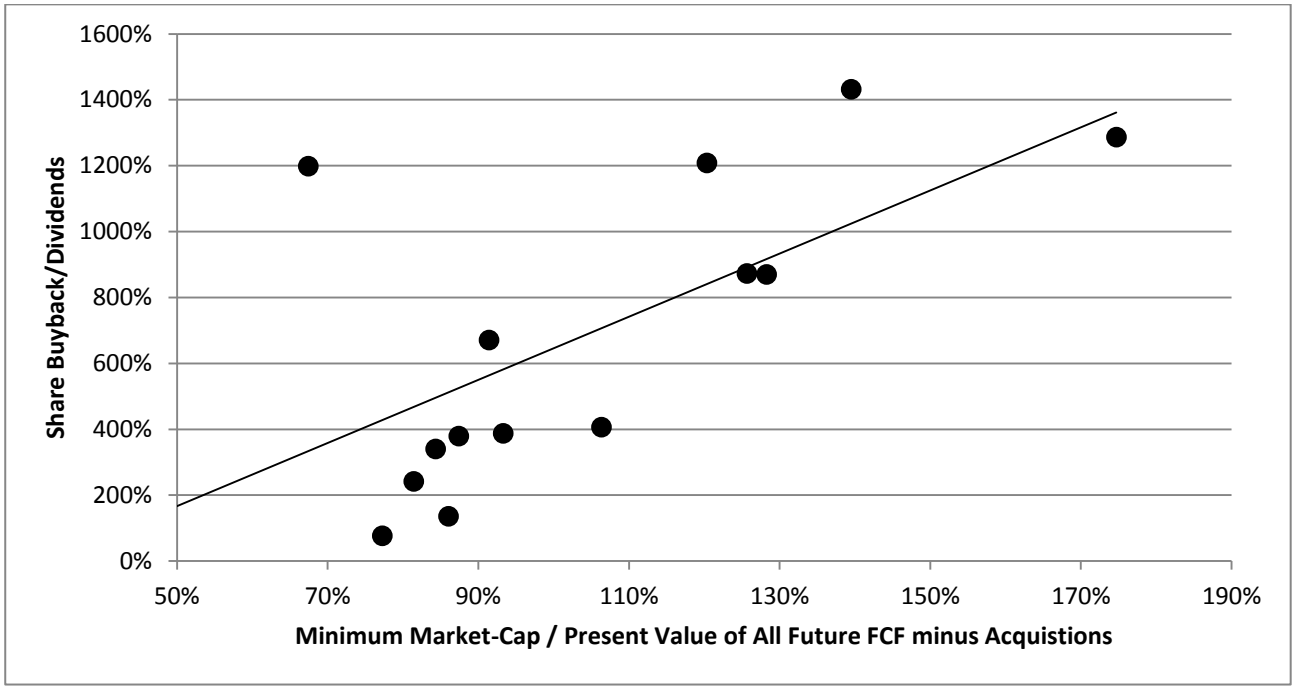
[60] Form 10-K annual reports filed with US SEC (1997-2011):
www.sec.gov/cgi-bin/browse-edgar?action=getcompany&CIK=0000063908&type=10-K

USD Millions	1996	1997	1998	1999	2000	2001	2002	2003	2004
Revenue	31,613	35,358	39,330	42,371	48,870	45,226	56,588	73,061	79,905
Net Income	2,586	3,119	2,945	3,491	3,697	408	(903)	2,539	3,497
Free Cash Flow (FCF)	1,571	1,872	3,436	2,504	2,388	1,481	4,096	4,415	3,409
Acquisitions	0	0	0	0	448	223	4,436	(149)	(1,124)
Dividends	(450)	(532)	(625)	(650)	(638)	(621)	(801)	(977)	(972)
Share Issuance	363	419	467	660	748	354	377	482	570
Share Buyback	(1,089)	(724)	(2,424)	(2,643)	(5,570)	(1,240)	(671)	(751)	(3,309)
Assets	27,699	29,852	31,708	35,297	34,009	32,584	70,710	74,716	76,138
Equity	13,438	16,155	16,919	18,295	14,209	13,953	36,262	37,746	37,564
ROA	11%	10%	11%	10%	1%	-3%	4%	5%	11%
ROE	23%	18%	21%	20%	3%	-6%	7%	9%	23%
Shares, Basic	2,104	2,052	2,068	2,018	1,979	1,936	2,499	3,047	3,024
Shares, Diluted	-	2,114	2,144	2,104	2,077	1,974	2,499	3,063	3,055
Market-Cap, Low	39,845	44,246	50,214	58,333	71,501	28,072	27,889	45,248	49,896
Market-Cap, High	59,833	73,423	84,400	117,296	152,383	91,844	58,801	71,665	78,987

USD Millions	2005	2006	2007	2008	2009	2010	2011	2012
Revenue	86,696	91,658	104,286	118,364	114,552	126,033	127,245	120,357
Net Income	2,398	6,198	7,264	8,329	7,660	8,761	7,074	(12,650)
Free Cash Flow (FCF)	6,575	9,373	7,143	12,026	10,179	8,391	9,099	7,482
Acquisitions	(641)	(855)	(6,793)	(11,248)	(391)	(7,977)	(10,391)	(54)
Dividends	(926)	(894)	(846)	(796)	(766)	(771)	(844)	(1,015)
Share Issuance	1,161	2,538	3,103	1,810	1,837	2,617	896	716
Share Buyback	(3,514)	(7,779)	(10,887)	(9,620)	(5,140)	(11,042)	(10,117)	(1,619)
Assets	77,317	81,981	88,699	113,331	114,799	124,503	129,517	108,768
Equity	37,176	38,144	38,526	38,942	40,517	40,781	39,004	22,833
ROA	3%	8%	9%	9%	7%	8%	6%	-10%
ROE	6%	17%	19%	22%	20%	22%	17%	-32%
Shares, Basic	2,879	2,782	2,630	2,483	2,388	2,319	2,094	1,974
Shares, Diluted	2,909	2,852	2,716	2,567	2,437	2,372	2,128	1,974
Market-Cap, Low	54,010	78,230	100,519	74,564	60,631	86,545	45,021	27,370
Market-Cap, High	84,067	110,918	139,048	132,791	117,490	126,965	103,423	58,993

Table 6: Selected financial data for the company *Hewlett-Packard*. Free Cash Flow is defined as Operating Cash Flow minus Capital Expenditures plus proceeds from sale of Property, Plant and Equipment. Acquisitions are net of cash required, and net of cash proceeds from divested business units. In 2002 a company was acquired for stock so new shares were issued and the acquired company's cash appear here as a positive cash inflow. ROA and ROE are calculated as Net Income for a year divided by the Assets and Equity for the previous year, respectively. Basic and diluted shares outstanding are weighted averages for the year. Market-Cap is calculated as the number of basic shares multiplied by the minimum and maximum share-price for the year, and is adjusted for stock-splits. Share buyback for 2006 includes USD 1.7b in prepayment for future share buybacks, the meaning of which is unclear.[61]

[61] Form 10-K annual reports (1996-2011), Form 8-K unaudited 4th quarter report (2012), all filed with US SEC:
www.sec.gov/cgi-bin/browse-edgar?action=getcompany&CIK=0000047217&type=10-k
www.sec.gov/Archives/edgar/data/47217/000004721712000033/ex99-1_112012.htm



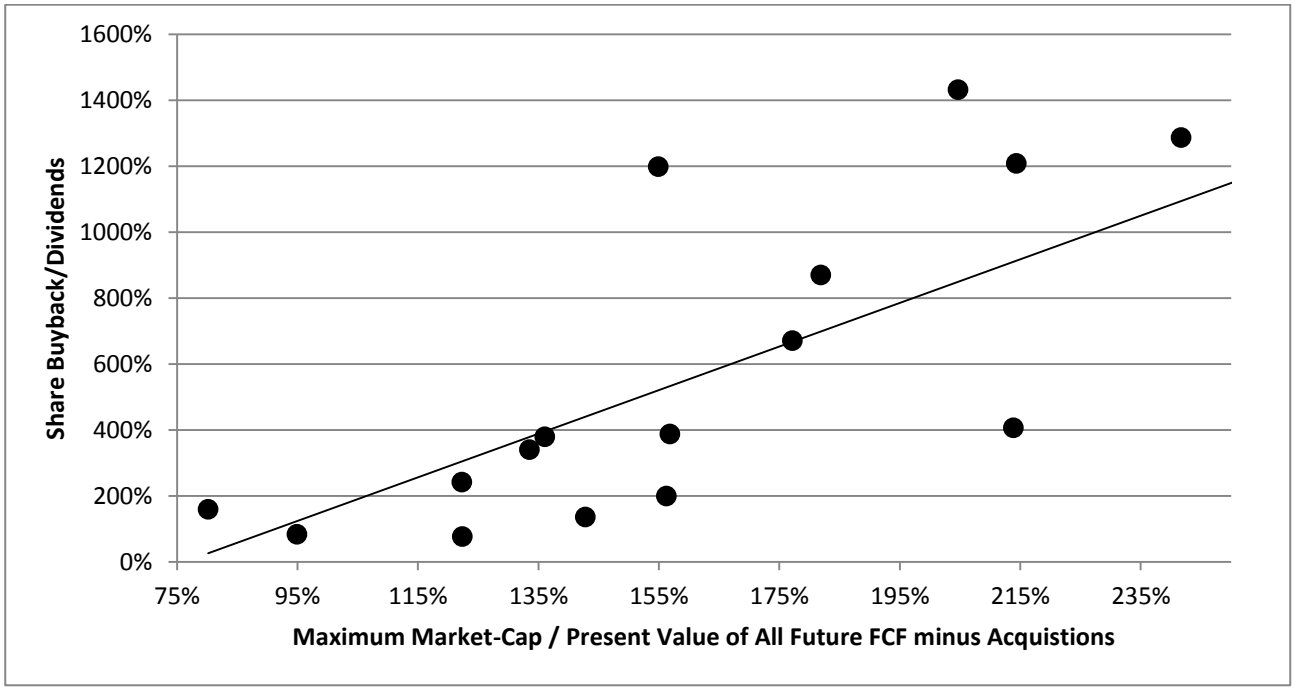


Figure 17: For the company *Hewlett-Packard*, the ratio of share buybacks to dividends relative to the ratio of the minimum and maximum market-cap for a given year relative to the present value of all future Free Cash Flow (FCF) minus acquisitions. The share buyback equilibrium is when the market-cap equals the value to eternal shareholders, that is, the present value of all future earnings that could be paid out as dividends. The ex-post financial data from Table 6 is used in these valuations which are detailed in section 9.5.2. The trend-lines result from linear regression of the data points and suggest there is a tendency for the company to increase share buybacks relative to dividends when the market-cap increases relative to the value of the company, which is the opposite of what the company should have done. Using Net Income instead of FCF, or changing the assumptions of the present value calculations such as increasing the discount rate from the 8.3% used here and/or changing the terminal value which is estimated from an assumed constant annual earnings of USD 6b for eternity starting in fiscal year 2013, the trend-lines remain similar but the horizontal axis is shifted, which means the share buybacks move in relation to the equilibrium where they are value neutral to eternal shareholders, but the tendency remains of increasing share buybacks with increasing market-cap.

USD Millions	2005	2006	2007	2008	2009	2010
Revenue	96	251	343	503	740	870
Net Income	9	39	62	132	192	217
Free Cash Flow (FCF)	15	35	65	121	204	191
Acquisitions	(353)	(23)	(381)	(6)	(630)	(79)
Assets	614	685	1,216	1,320	2,280	3,281
Cash & Equivalents	69	121	93	199	243	1,061
Equity	459	572	998	1,120	1,702	2,067
ROA		6%	9%	11%	15%	10%
ROE		8%	11%	13%	17%	13%

Table 7: Selected financial data for the company *Autonomy*. Free Cash Flow is defined as Operating Cash Flow minus Capital Expenditures minus Product Development Expenses plus proceeds from sale of Property, Plant and Equipment. Minor joint venture and equity investments were also made which are not shown here. ROA and ROE are calculated as Net Income for a year divided by the Assets and Equity for the previous year, respectively.[62]

[62] Company's self-published annual reports (2006, 2008, 2010), currently unavailable from the company's website: www.autonomy.com

USD Millions	1996	1997	1998	1999	2000	2001	2002	2003	2004
Revenue	9,833	7,081	5,941	6,134	7,983	5,363	5,742	6,207	8,279
Net Income	(816)	(1,045)	309	601	786	(25)	65	57	266
Free Cash Flow (FCF)	452	148	818	774	726	(47)	(85)	125	758
Acquisitions	-	(384)	(10)	-	-	-	(52)	-	-
Dividends	(14)	-	-	-	-	-	-	-	-
Share Issuance	39	34	41	86	85	42	105	53	427
Share Buyback	-	-	-	(75)	(116)	-	-	(26)	-
Assets	5,364	4,233	4,289	5,161	6,803	6,021	6,298	6,817	8,039
Cash & Equivalents	1,745	1,459	2,300	3,226	4,027	4,336	4,337	4,566	5,464
Equity	2,058	1,200	1,642	3,104	4,107	3,920	4,095	4,223	5,063
ROA	-	(19%)	7%	14%	15%	(0%)	1%	1%	4%
ROE	-	(51%)	26%	37%	25%	(0%)	2%	1%	6%
Shares, Basic	495	504	528	573	649	691	710	721	743
Shares, Diluted	495	504	672	697	721	691	724	723	775
Market-Cap, Low	1,980	1,607	1,683	4,083	8,236	4,709	4,966	4,730	7,319
Market-Cap, High	5,259	3,749	5,775	11,479	24,399	9,370	9,269	8,328	14,117

USD Millions	2005	2006	2007	2008	2009	2010	2011	2012
Revenue	13,931	19,315	24,006	37,491	42,905	65,225	108,249	156,508
Net Income	1,328	1,989	3,496	6,119	8,235	14,013	25,922	41,733
Free Cash Flow (FCF)	2,275	1,563	4,484	8,397	8,946	16,474	30,077	41,454
Acquisitions	-	40	-	(220)	-	(638)	(244)	(350)
Dividends	-	-	-	-	-	-	-	(2,488)
Share Issuance	543	318	365	483	475	912	831	665
Share Buyback	-	(355)	(3)	(124)	(82)	-	-	-
Assets	11,516	17,205	25,347	36,171	47,501	75,183	116,371	176,064
Cash & Equivalents	8,261	10,110	15,386	24,490	33,992	51,011	81,570	121,251
Equity	7,428	9,984	14,532	22,297	31,640	47,791	76,615	118,210
ROA	17%	17%	20%	24%	23%	30%	34%	36%
ROE	26%	27%	35%	42%	37%	44%	54%	55%
Shares, Basic	808	844	865	882	893	909	924	935
Shares, Diluted	857	878	889	902	907	925	936	945
Market-Cap, Low	15,069	40,402	62,799	101,818	69,833	164,256	254,100	331,150
Market-Cap, High	42,986	72,922	134,075	179,011	168,688	266,819	390,723	659,112

Table 8: Selected financial data for the company *Apple*. Free Cash Flow is defined as Operating Cash Flow minus purchase of Property, Plant, Equipment and Intangible Assets, plus proceeds from sale of such assets. Acquisitions are of companies and do not include acquired cash. Cash & Equivalents also includes short- and long-term investments. ROA and ROE are calculated as Net Income for a year divided by the Assets and Equity for the previous year, respectively. Basic and diluted shares outstanding are weighted averages for the year. Market-Cap is calculated as the number of basic shares multiplied by the minimum and maximum share-price for the year. The share numbers and prices are adjusted for stock-splits. Years 1997-2000 had a small issue of convertible preferred stock.[63]

[63] Form 10-K annual reports filed with US SEC:
www.sec.gov/cgi-bin/browse-edgar?action=getcompany&CIK=0000320193&type=10-K

CNY Millions	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Revenue	3	16	35	465	986	1,280	1,711	761	103	106
Net Income	(29)	32	12	72	312	241	97	(405)	(500)	(284)
Free Cash Flow (FCF)	2	56	(19)	(70)	474	127	571	(204)	(430)	(378)
Acquisitions & Investments	(4)	-	(34)	(230)	(80)	29	(299)	(29)	(47)	38
Dividends	-	-	-	-	-	-	-	(201)	-	-
Share Issuance	-	-	813	4	58	1,282	9	1	0.09	0.7
Share Buyback	-	-	-	-	-	(107)	(245)	(157)	-	(21)
Assets	20	156	1,027	1,214	1,625	3,246	3,263	2,325	1,857	1,629
Cash & Equivalents	6	63	793	488	938	2,215	2,153	1,675	1,416	1,072
Equity	(58)	(13)	865	942	1,336	2,086	2,719	2,011	1,531	1,263
Shares, Basic	9.9	9.9	10.3	24.2	24.5	27.4	27.7	25.4	25.1	25.0
Shares, Diluted	9.9	17.2	14.5	24.8	24.6	27.6	27.7	25.4	25.1	25.0
Market-Cap, Low	-	-	4,208	2,710	3,023	4,062	1,911	1,089	628	468
Market-Cap, High	-	-	4,829	5,644	6,410	10,891	5,463	2,887	1,469	1,369
USDCNY Avg. Exch. Rate	8.28	8.28	8.28	8.18	7.96	7.58	6.92	6.83	6.76	6.45

Table 9: Selected financial data for the company *The9*. Free Cash Flow is defined as Operating Cash Flow minus Capital Expenditures minus purchase of Intangible Assets plus proceeds from sale of Property, Plant and Equipment. Acquisitions are net of cash required, include non-controlling equity investments, and are net of cash proceeds from sale of such investments. It is unclear if these investment amounts include long-term financial securities held as an alternative to cash. Basic and diluted shares outstanding are weighted averages for the year. Market-Cap is calculated as the number of basic shares multiplied by the minimum and maximum share-price for the year, and multiplied by the average exchange rate for the year because the share-price is in USD but the financial data is in CNY. Market-Cap for 2004 is calculated as the number of common shares outstanding at the end of 2004 multiplied by the low and high share prices for that year, because the IPO was in late 2004 which distorts the weighted average basic number of shares outstanding. Net Income and Equity are net of minority interests. Prior to the IPO in 2004 the company had a more complex capital structure with preferred shares and convertible debt which is ignored here.[64]

[64] Form 20-F annual reports (data for years 2002-2003 are from 2004 report) filed with US SEC: www.sec.gov/cgi-bin/browse-edgar?action=getcompany&CIK=0001296774&type=20-f

USD Millions	2007	2008	2009	2010	2011	2012	2013
Revenue	153	272	777	1,974	3,711	5,089	7,872
Net Income	(138)	(56)	229	372	668	32	1,491
Free Cash Flow (FCF)	(55)	(88)	66	188	470	37	2849
Acquisitions & Investments	-	-	3	(22)	(27)	(913)	(369)
Dividends	0	0	0	0	0	0	0
Share Issuance	-	-	9	506	1,026	6,777	1,504
Share Buyback	-	-	0	0	0	0	0
Assets	-	-	1,109	2,990	6,331	15,103	17,895
Cash & Equivalents	-	-	633	1,785	3,908	9,626	11,449
Equity	-	-	868	2,162	4,899	11,755	15,470
Shares, Basic	-	-	-	-	1,294	2,006	2,420
Shares, Diluted	-	-	-	-	1,508	2,116	2,517
Market-Cap, Low	-	-	-	-	-	35,205	54,861
Market-Cap, High	-	-	-	-	-	90,270	141,764

Table 10: Selected financial data for the company *Facebook*. Cash and equivalents also include marketable securities. Free Cash Flow is defined as Operating Cash Flow minus purchase and lease of Property and Equipment. Acquisitions are net of cash required. Basic and diluted shares outstanding are weighted averages for the year. Market-Cap is calculated as the number of basic shares multiplied by the minimum and maximum share-price for the year, which is imprecise because the number of shares changed greatly during these years. Net Income and Equity are net of minority interests. Prior to the IPO in 2012 the company had a more complex capital structure with convertible preferred shares which is ignored here. Share issuances are net proceeds from issuance of new shares plus proceeds from exercise of stock options; various tax effects are excluded here although they are substantial.[65]

[65] Form 10-K annual reports, Form S-1 prospectus, all filed with US SEC:
www.sec.gov/cgi-bin/browse-edgar?action=getcompany&CIK=0001326801&type=10-k
www.sec.gov/Archives/edgar/data/1326801/000119312512034517/d287954ds1.htm

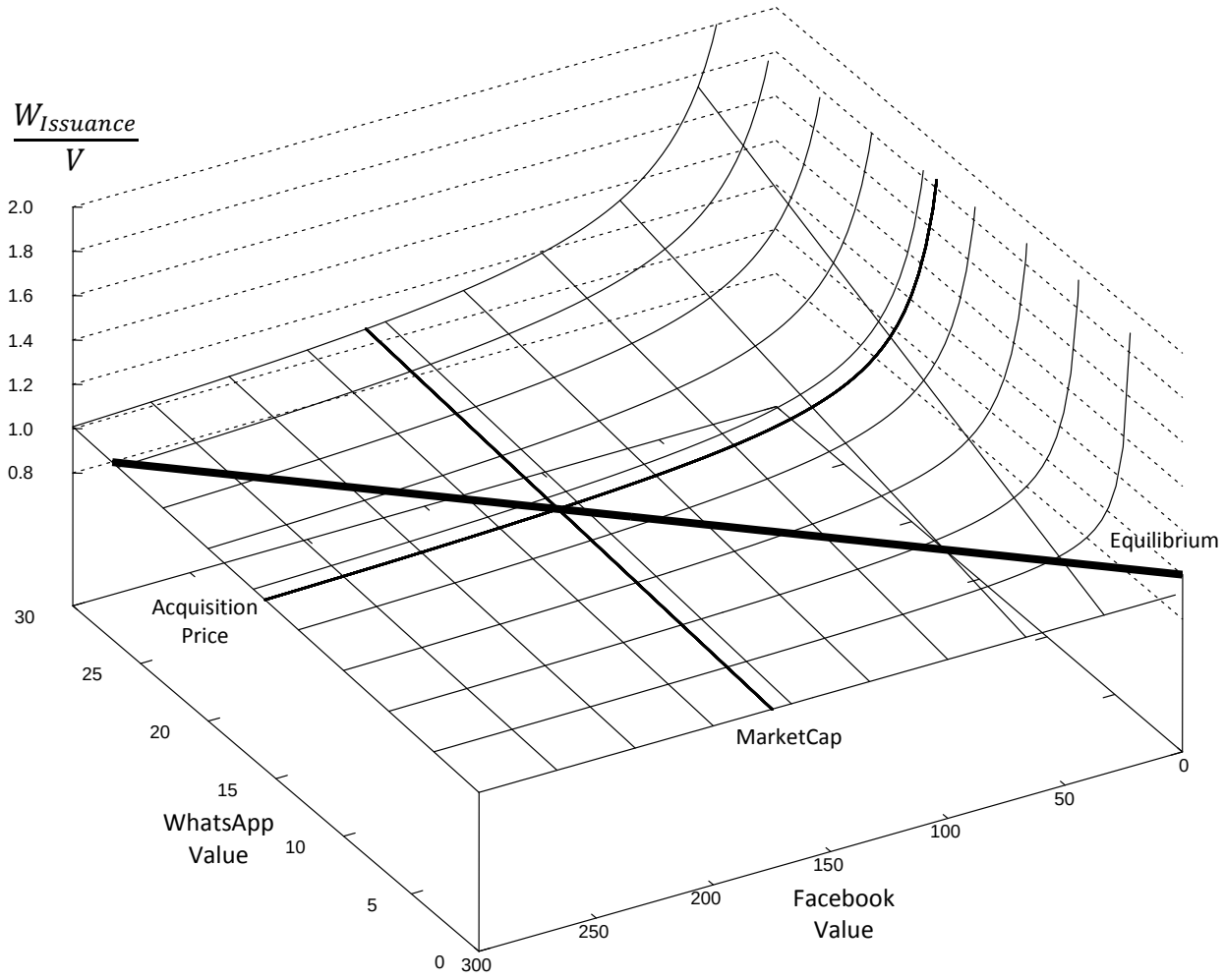


Figure 18: The relative value of Facebook's acquisition of WhatsApp paid with a share issuance, as described in section 9.7. The relative value $W_{Issuance}/V$ is shown on the vertical axis. Relative values larger than one mean that the acquisition increases value to the eternal shareholders of Facebook and vice versa for relative values less than one. The horizontal axes show Facebook's value to eternal shareholders v and WhatsApp's value to eternal shareholders, denoted $Return$ in Eq. 4-46, both in USD billions. Plot is made by changing these variables in Eq. 4-46. The bold lines show Facebook's market-cap of USD 175b and WhatsApp's acquisition price of USD 15.8b which was paid in shares of Facebook. The extra bold line shows the equilibrium where the acquisition of WhatsApp does not affect the value to the eternal shareholders of Facebook. The value of WhatsApp is shown net of the USD 4b that was paid in cash and must be added so as to obtain the gross value of the return on WhatsApp.

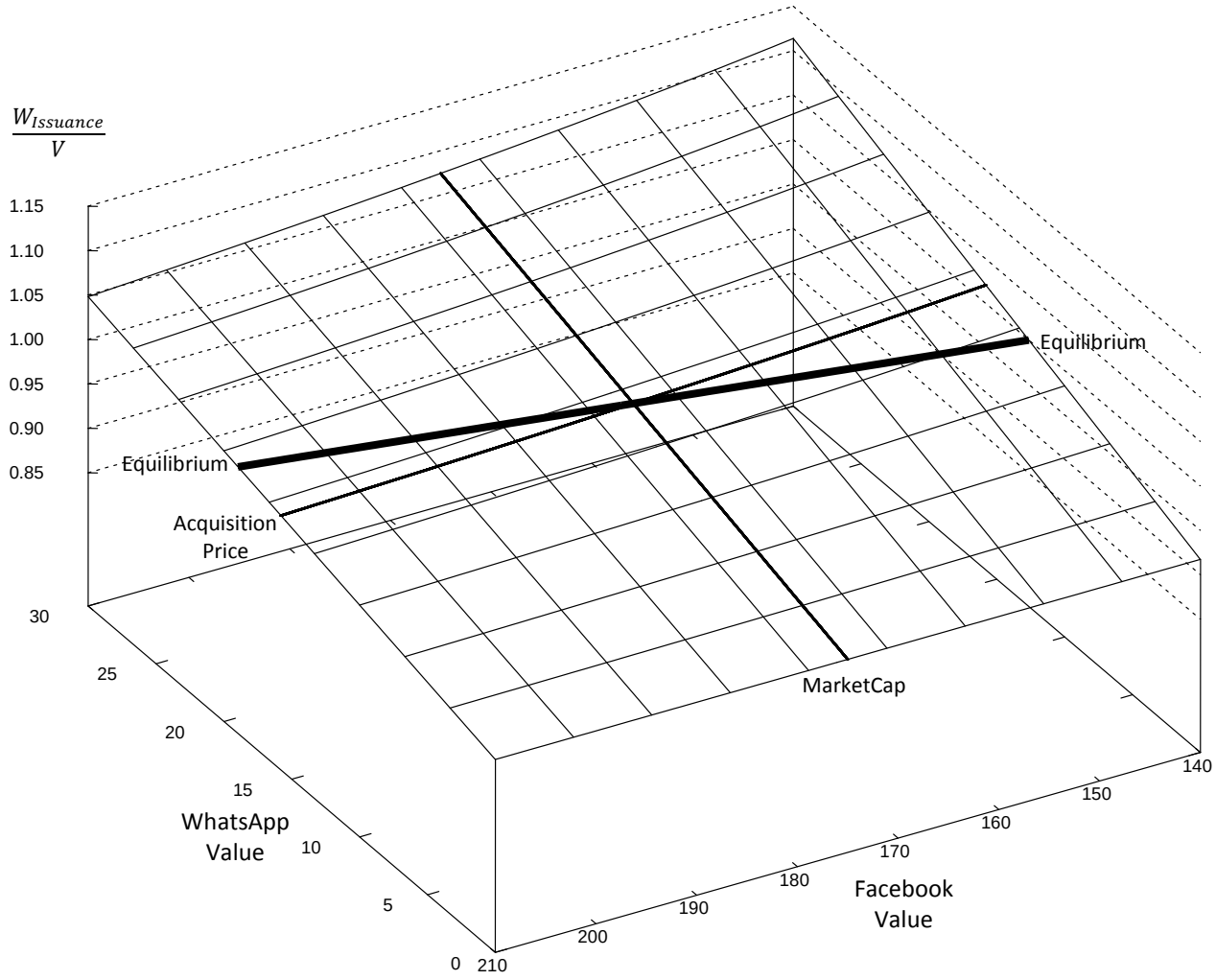


Figure 19: Excerpt of Figure 18 for a more limited range of Facebook values.

USD Millions	1997	1998	1999	2000	2001	2002	2003	2004
Revenue	16,611	16,301	16,767	17,354	17,545	19,564	21,044	21,962
Net Income	4,129	3,533	2,431	2,177	3,969	3,050	7,762	7,638
Free Cash Flow (FCF)	3,011	2,624	2,859	2,897	3,432	3,960	4,731	5,554
Acquisitions	(1,100)	(1,428)	(1,876)	(397)	(651)	(544)	(359)	(267)
Dividends	(1,387)	(1,480)	(1,580)	(1,685)	(1,791)	(1,987)	(2,166)	(2,429)
Share Issuance	150	302	168	331	164	107	98	193
Share Buyback	(1,262)	(1,563)	(15)	(133)	(277)	(691)	(1,440)	(1,739)
Assets	16,881	19,145	21,623	20,834	22,417	24,501	27,410	31,441
Equity	7,274	8,403	9,513	9,316	11,366	11,800	14,090	15,935
ROA	39%	21%	13%	10%	19%	14%	32%	28%
ROE	67%	49%	29%	23%	43%	27%	66%	54%
Retain/Earnings	39%	22%	41%	32%	52%	16%	55%	48%
Shares, Basic	4,954	4,934	4,938	4,954	4,974	4,956	4,918	4,852
Shares, Diluted	5,030	4,992	4,974	4,974	4,974	4,966	4,924	4,858
Market-Cap, Low	126,649	132,305	116,808	106,214	105,374	106,306	91,008	92,916
Market-Cap, High	179,905	219,415	161,720	165,662	154,667	143,501	125,163	129,791

USD Millions	2005	2006	2007	2008	2009	2010	2011	2012
Revenue	23,104	24,088	28,857	31,944	30,990	35,119	46,542	48,017
Net Income	4,872	5,080	5,981	5,807	6,824	11,809	8,854	9,019
Free Cash Flow (FCF)	5,612	4,662	5,741	5,732	6,297	7,451	6,655	8,008
Acquisitions	(637)	(901)	(5,653)	(759)	(300)	(2,511)	(977)	(1,535)
Dividends	(2,678)	(2,911)	(3,149)	(3,521)	(3,800)	(4,068)	(4,300)	(4,595)
Share Issuance	230	148	1,619	586	662	1,666	1,569	1,489
Share Buyback	(2,055)	(2,416)	(1,838)	(1,079)	(1,518)	(2,961)	(4,513)	(4,559)
Assets	29,427	29,963	43,269	40,519	48,671	72,921	79,974	86,174
Equity	16,355	16,920	21,744	20,472	24,799	31,003	31,635	32,790
ROA	15%	17%	20%	13%	17%	24%	12%	11%
ROE	31%	31%	35%	27%	33%	48%	28%	29%
Retain/Earnings	8%	(2%)	44%	31%	32%	55%	16%	15%
Shares, Basic	4,784	4,696	4,626	4,630	4,628	4,616	4,568	4,504
Shares, Diluted	4,786	4,700	4,662	4,672	4,658	4,667	4,646	4,584
Market-Cap, Low	96,422	92,417	105,380	93,271	86,636	114,177	140,009	149,938
Market-Cap, High	108,262	115,874	148,726	151,841	137,567	152,051	163,946	183,133

Table 11: Selected financial data for the company *Coca-Cola*. Free Cash Flow is defined as Operating Cash Flow minus purchase of Property, Plant and Equipment, plus proceeds from sale of such assets. Acquisitions are mainly of bottlers and trademarks and gross of proceeds from sale of such assets because they are not reported separately in the cash flow statements. ROA and ROE are calculated as Net Income for a year divided by the Assets and Equity for the previous year, respectively. Retain is calculated using Eq. 9-9. Basic and diluted shares outstanding are weighted averages for the year. Market-Cap is calculated as the number of basic shares multiplied by the minimum and maximum share-price for the year. The share numbers and prices are adjusted for stock-splits.[66]

[66] Form 10-K annual reports filed with US SEC:
www.sec.gov/cgi-bin/browse-edgar?action=getcompany&CIK=0000021344&type=10-K

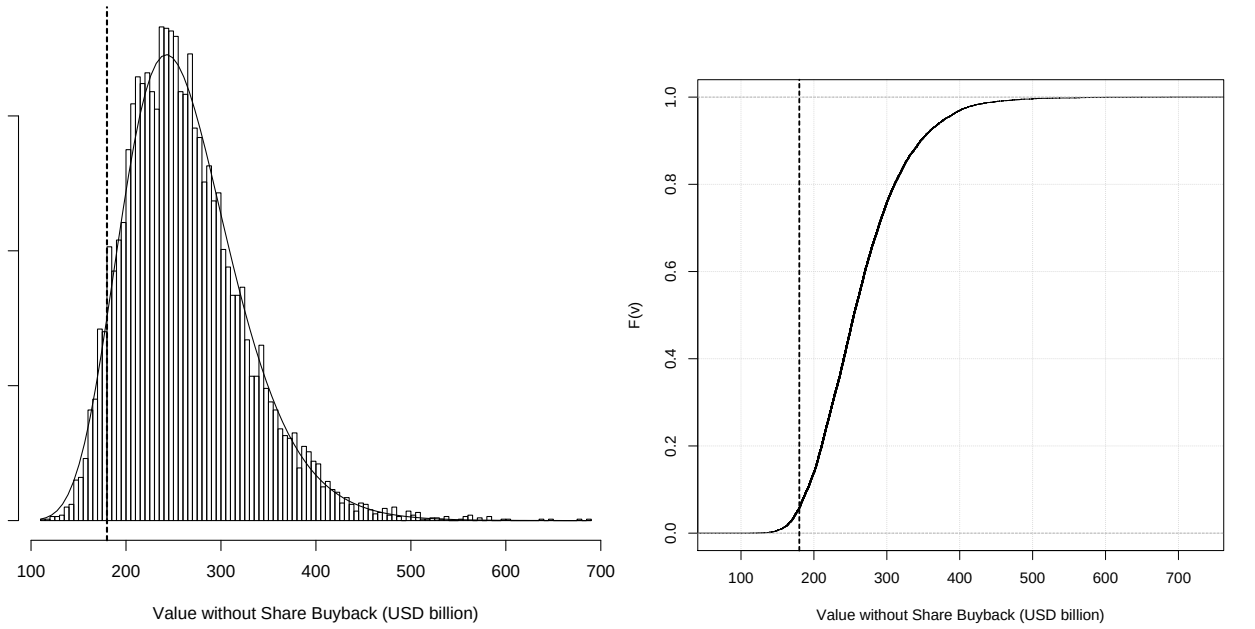


Figure 20: Histogram (left) and cumulative distribution function (CDF, right) for Coca-Cola's value to eternal shareholders without a share buyback $\mathbb{V}$, resulting from Monte Carlo simulated payouts which are discounted with sampled value yields of the S&P 500 stock market index, as described in section 9.9.3. The histogram has a log-normal curve fitted. The dotted line is Coca-Cola's current market-cap of USD 180b. Share buybacks increase value to eternal shareholders when the market-cap is less than the value without a share buyback, that is, for values to the right of the dotted lines.

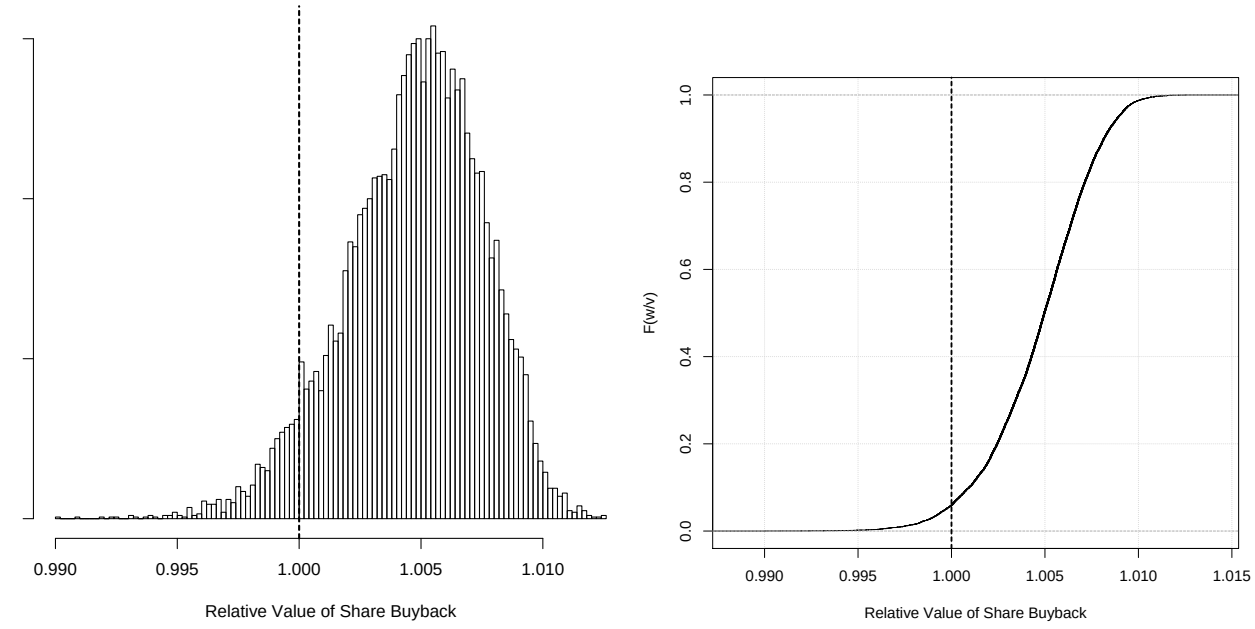


Figure 21: Histogram (left) and cumulative distribution function (CDF, right) for Coca-Cola's relative value of a share buyback $\mathbb{W}/\mathbb{V}$, calculated using Eq. 6-2 with the distribution for $\mathbb{V}$ in Figure 20, a share buyback of USD 3b and market-cap of USD 180b, as described in section 9.9.3. The dotted line shows a relative value of one where share buybacks do not affect the value to eternal shareholders. Share buybacks increase value to eternal shareholders when the relative value is greater than one, that is, for relative values to the right of the dotted lines.

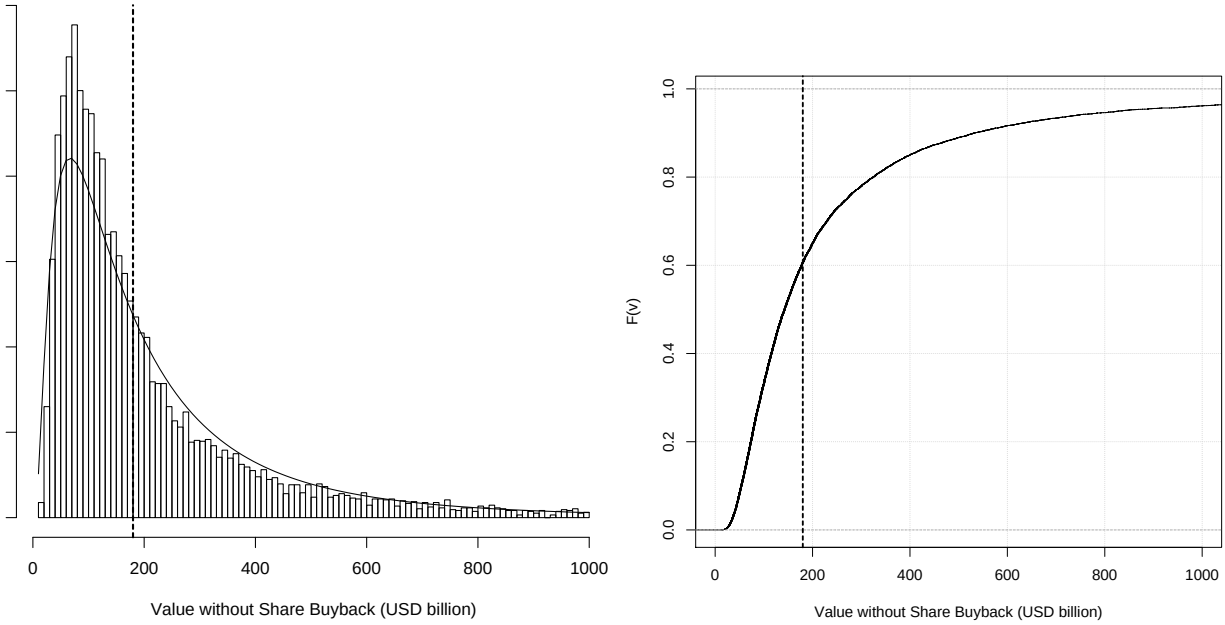


Figure 22: Histogram (left) and cumulative distribution function (CDF, right) for Coca-Cola's value to eternal shareholders without a share buyback $\mathbb{V}$, resulting from Monte Carlo simulated payouts which are discounted with simulated compounded returns of the DJVC venture capital index, as described in section 9.9.4. The range of values is USD 14b to 41,000b but the plots have been limited to USD 1,000b for clarity. The histogram has a log-normal curve fitted. The dotted line is Coca-Cola's current market-cap of USD 180b. Share buybacks increase value to eternal shareholders when the market-cap is less than the value without a share buyback, that is, for values to the right of the dotted lines.

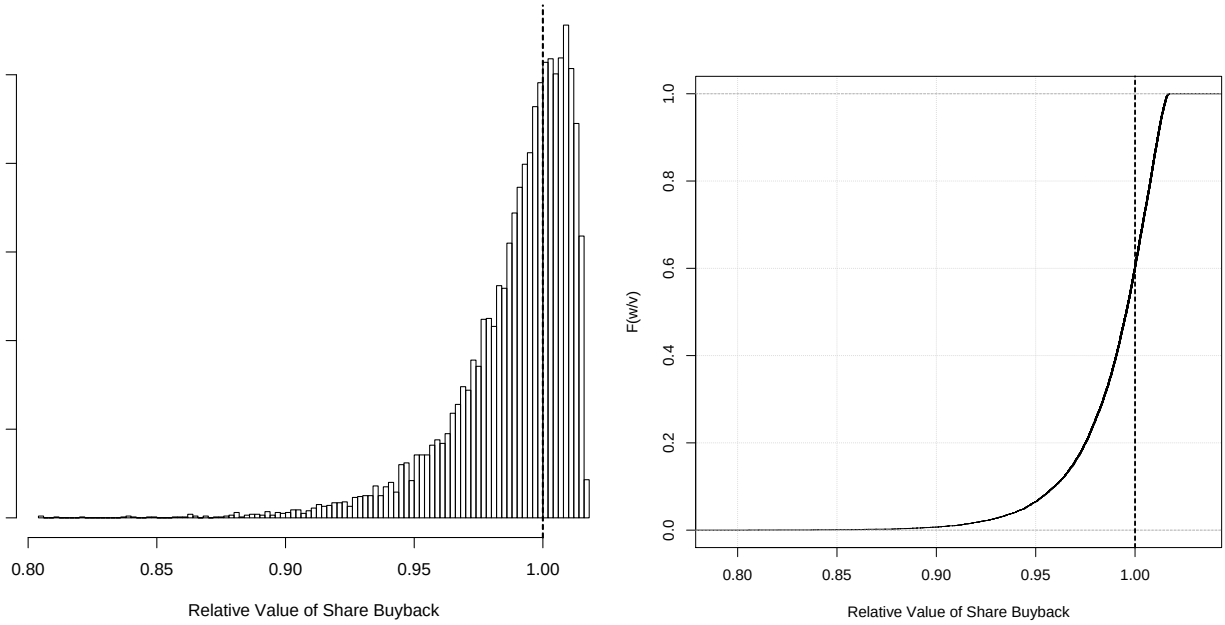


Figure 23: Histogram (left) and cumulative distribution function (CDF, right) for Coca-Cola's relative value of a share buyback $\mathbb{W}/\mathbb{V}$, calculated using Eq. 6-2 with the distribution for $\mathbb{V}$ in Figure 22, a share buyback of USD 3b and market-cap of USD 180b, as described in section 9.9.4. The dotted line shows a relative value of one where share buybacks do not affect the value to eternal shareholders. Share buybacks increase value to eternal shareholders when the relative value is greater than one, that is, for relative values to the right of the dotted lines.

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15. Revision History

- 2014, August 13: Added section 2.9. Corrected the FCF for year 2012 in Table 6 and updated section 9.5. Minor text revisions. (210 pages)
- 2014, March 28: Added section 4.8.1. Minor text revisions in section 9.7.2 and Figure 18. Minor changes to Table 3 and Table 10. (210 pages)
- 2014, March 13: Added case study for Facebook (section 9.7). Added section 4.8. Extended section 4.9. Added Figure 18, Figure 19. Minor text revisions. (209 pages)
- 2014, January 15: Added sections 4.3.1, 4.3.8. Extended section 9.1. Minor text revisions. (197 pages)
- 2013, September 30: Added sections 4.3.6, 4.3.7, 5.2. Rewrote section 6.2.4. Extended section 9.5.4. Minor text revisions. (196 pages)
- 2013, August 5: Added case studies for Apple (section 9.6) and Coca-Cola (section 9.9). Added executive summary. Added sections 3.8.4, 4.3.4, 4.3.5, 4.10, 9.3.2. Moved part of section 4 to appendix 13.2. Rewrote sections 3.8.1, 4.7, 6, 8 and 9.8. Rewrote sections 7 and 9 to use [**62**]. Removed some figures. Removed section 'Main Contributions'. Changed ordering of some sections. Added Figure 11, Figure 13, Figure 14, Figure 15, Table 2. Replaced Figure 8, Figure 12, Figure 17. Added reference numbers for many formulas. Minor text revisions. (193 pages)
- 2013, January 4: Corrected error in section 9.5.3 where $Return = USD\ (8.8)b$ but should have been $USD\ 1.4b$. Added sections 4.4, 6.2.4. Extended section 9.8.3. Added Figure 7. Minor text revisions. (151 pages)
- 2012, December 19: Added case study for The9 (section 9.8). Added section 2.7. Separated and extended section 5. Extended and clarified section 6. Added Figure 3, Figure 5, Figure 8. Corrected fonts and other issues with some figures. Minor text revisions. (148 pages)
- 2012, November 29: Added case study for Hewlett-Packard (section 9.5). Added sections 4.7.2, 6. Extended sections 4.7, 9.4.3. Rearranged section 3.7. Added Figure 4 and Figure 6. Minor revisions of the abstract, introduction and conclusion. Minor text revisions. (130 pages)
- 2012, October 27: Added case study for McDonald's (section 9.4). Added sections 4.1, 0. Added Figure 1 and Figure 2, and a figure since removed. Changed ordering of some sections. Minor clarifications and text revisions. (101 pages)
- 2012, October 12: First edition. (87 pages)