

Introduction to Share Buyback Valuation

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This paper is a short introduction to the treatise which is available at:

www.hvass-labs.org/people/magnus/publications/pedersen2012share-buyback.pdf

"The future depends on what you do now" – Gandhi

¹ If you find any parts difficult to understand, or if you encounter any errors, whether logical, mathematical, spelling, grammatical or otherwise, please mark the document and e-mail it to: [Magnus \(at\) Hvass-Labs \(dot\) Org](mailto:Magnus(at)Hvass-Labs(dot)Org)

² See last page for revision history.

Executive Summary

- Share buybacks have an unpredictable effect on the share price.
- Share buybacks are generally not a reliable way of returning capital to shareholders.
- Share buybacks magnify any mispricing of the shares compared to the value to long-term shareholders, which is the company's excess cash plus the present value of future earnings that can be paid out as dividends.
- Buybacks of overpriced shares cause much greater losses to long-term shareholders compared to the gains from buying back underpriced shares.
- Share buybacks can be valued relative to alternative investments, acquisitions, restructurings, etc. to assess which one is the most valuable to long-term shareholders.
- Share buybacks for offsetting the diluting impact of stock options should only be made when the shares are underpriced.
- Dividend tax is irrelevant to share buybacks if the future tax rate is constant or unknown.
- The greater the uncertainty about the value to long-term shareholders, the greater a margin of safety is required between the share price and the average value to long-term shareholders.
- Share buybacks increase the uncertainty of the value to long-term shareholders.
- The probability distribution of possible outcomes of a share buyback should be considered so as to properly assess the likely effect on shareholder value.
- Managers must inform shareholders about the share price limit below which shares may be bought back, and the source of cash. This information is needed for outsiders to value the share buybacks.

1. Introduction

This paper presents the main aspects of the formal theory for share buyback valuation from [1], which is referred to here as *the treatise* and gives more detailed mathematical proofs, valuation and equilibrium formulas for more share buyback scenarios, more case studies, discussions, etc.

The prevalent hypothesis amongst finance scholars and practitioners is that share buybacks can substitute for dividend payouts as a way for companies to transfer capital to their shareholders, possibly with a tax advantage to the shareholders depending on their individual tax-rates for capital gains and dividends. For a survey of this and other share buyback hypotheses see Baker et al. [2]. The substitution hypothesis is disproven here both theoretically and empirically, which is important because share buybacks have increased greatly in recent years, possibly due to the common belief that the substitution hypothesis was valid. During the year 2012, the companies in the S&P 500 stock market index had aggregate net income of about USD 780b, made dividend payouts of about USD 280b and share buybacks of almost USD 400b.³ It is shown in the treatise that the companies in the S&P 500 index have had a historical tendency to increase share buybacks when the share price was high relative to the estimated value to long-term shareholders, and conversely decrease share buybacks when the share price was relatively low, which is the opposite of what should have been done.

Share buybacks are shown to be value neutral to long-term shareholders only if the market-cap (market capitalization, that is, the market value of all the company's shares) equals the excess cash plus the present value of future earnings that can be paid out as dividends, and share buybacks magnify any mispricing as demonstrated in Figure 1 and Figure 2. When there is uncertainty about the future earnings, as there usually is, then the probability distribution should be considered so as to properly assess the likely impact of a share buyback on the value to long-term shareholders. If the future dividend tax rate is constant or unknown then it is irrelevant to the value of share buybacks.

³ S&P Dow Jones Indices, Press Release, June 19, 2013:
<http://eu.spindices.com/documents/index-news-and-announcements/20130619-sp-500-buyback.pdf>

2. Value without Share Buyback

In order to calculate the value of a share buyback, the value without a share buyback must first be defined. Share buybacks should be made when they increase value to the remaining shareholders rather than the selling shareholders, which ultimately means that share buybacks should increase value to so-called eternal shareholders who never sell their shares and thus derive value from the shares only through the receipt of dividends. In Williams' theory of investment value [3] the value of a company to its eternal shareholders is defined as the present value of all future dividends.

The present value of the dividend for future year t is the amount that would have to be invested today with an annual rate of return d , also called the discount rate, so as to compound into becoming $Dividend_t$ after t years:

$$Present\ Value\ of\ Dividend_t \cdot (1 + d)^t = Dividend_t \Leftrightarrow Present\ Value\ of\ Dividend_t = \frac{Dividend_t}{(1 + d)^t}$$

Eq. 2-1

Let v denote the present value of all future dividends to eternal shareholders, prior to dividend tax and not per share. Assume the discount rate d is constant forever. The present value v is then:

$$v = \sum_{t=0}^{\infty} Present\ Value\ of\ Dividend_t = \sum_{t=0}^{\infty} \frac{Dividend_t}{(1 + d)^t}$$

Eq. 2-2

Many companies only payout a fraction of their earnings as dividends so the value is instead expressed in terms of the company's excess cash that can be paid out as dividends now and the present value of all future earnings that can potentially be paid out as dividends. The earnings for the current year are denoted $Earnings_1$ which are available for dividend payout a year from now and should therefore be discounted by $1 + d$, and so forth. The value to eternal shareholders is then:

$$v = Excess\ Cash + \sum_{t=1}^{\infty} \frac{Earnings_t}{(1 + d)^t}$$

Eq. 2-3

As the excess cash is separated in this valuation formula, any future interest income from that cash must be deducted from $Earnings_t$.

Let $TaxDividend$ be the dividend tax rate so the post-tax value per share is:

$$V = \frac{v \cdot (1 - TaxDividend)}{Shares}$$

Eq. 2-4

3. Value with Share Buyback

The formulas derived in this section calculate the actual value and equilibriums of a share buyback, provided the actual value without a share buyback is used as input. However, the actual value without a share buyback is typically not known in advance, as it depends on future earnings which are usually not known in advance. Hence, estimates of the value without a share buyback will normally have to be used as input, which means the equilibriums and share buyback values are also only estimates.

A share buyback reduces both the company's cash holdings and the number of shares outstanding. The cash reduction results in a decrease in the value to eternal shareholders v because the cash is no longer available for dividend payout. The reduction in the number of shares from the buyback is proportional to the share-price.

The market capitalization, that is, the total market-price for all the outstanding shares is:

$$MarketCap = Shares \cdot SharePrice$$

Eq. 3-1

Let *Buyback* be the amount used for the share buyback so the number of shares bought back is:

$$\frac{Buyback}{SharePrice} = Shares \cdot \frac{Buyback}{MarketCap}$$

Eq. 3-2

The number of outstanding shares after the buyback is then:

$$Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)$$

Eq. 3-3

The value to eternal shareholders with a share buyback, per share outstanding before the share buyback and after dividend tax, is then:

$$W = \frac{(v - Buyback) \cdot (1 - TaxDividend)}{Shares \cdot \left(1 - \frac{Buyback}{MarketCap}\right)}$$

Eq. 3-4

It is sometimes convenient to consider this without the dividend tax and number of shares:

$$w = \frac{v - Buyback}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 3-5

3.1. Relative Value

The relative value of a share buyback is the value with a share buyback from Eq. 3-4 divided by the value without a share buyback from Eq. 2-4:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 3-6

Because the number of shares and dividend tax both factor out in the division, this is equivalent to:

$$\frac{W}{V} = \frac{w}{v}$$

Eq. 3-7

The relative value in Eq. 3-6 is only well-defined for $v > 0$. Also note that the dividend tax is irrelevant to the relative value of a share buyback as long as it remains constant for eternity.

3.2. Equilibrium

The equilibrium is where the value to eternal shareholders is unaffected by the share buyback so $W = V$. But it is more useful to express the equilibrium in terms of an inequality so it is easy to see when the value to eternal shareholders is increased or decreased from a share buyback. The equilibrium is found by comparing the value with a share buyback from Eq. 3-4 to the value without a share buyback from Eq. 2-4:

$$W > V \Leftrightarrow MarketCap < v$$

Eq. 3-8

So a share buyback increases value to eternal shareholders when the market-cap is less than the value without a share buyback v . In the treatise it is shown that the only way for the equilibrium to be greater than v is when a future share buyback is made below the future equilibrium. As future share-prices are unpredictable this may never be possible, so valuation of a share buyback is done for the current year only.

3.3. Recapitalization

When a share buyback is funded with debt rather than excess cash, it effectively means the company is making a recapitalization, which requires more complicated share buyback valuation and equilibrium formulas that are provided in the treatise.

3.4. Alternative Investment

Assume an investment is made for the funds that would otherwise have been used for a share buyback. This investment could, for example, be an acquisition of another company or the construction of a new production facility. The investment amount is denoted $Invest$ and the present value of the total future return on that investment is denoted $Return$ which should be calculated using an appropriate discount rate that would be the same as the discount rate used for calculating v if the return has similar risk.

The value to eternal shareholders per share currently outstanding and after dividend tax is then:

$$V_{Invest} = \frac{(v - Invest + Return) \cdot (1 - TaxDividend)}{Shares}$$

Eq. 3-9

The value to eternal shareholders is increased whenever the present value of the return is greater than the investment amount, that is:

$$V_{Invest} > V \Leftrightarrow Return > Invest$$

Eq. 3-10

The value of the investment relative to the value of a share buyback is found by dividing Eq. 3-9 with Eq. 3-4 (by assumption $Invest = Buyback$):

$$\frac{V_{Invest}}{W} = \left(1 + \frac{Return}{v - Invest}\right) \cdot \left(1 - \frac{Invest}{MarketCap}\right)$$

Eq. 3-11

The equilibrium where a share buyback starts to increase value relative to the investment is:

$$W > V_{Invest} \Leftrightarrow MarketCap < Invest \cdot \left(1 + \frac{v - Invest}{Return}\right)$$

Eq. 3-12

The equilibrium can also be formulated in terms of the return on the investment:

$$W > V_{Invest} \Leftrightarrow Return < \frac{Invest \cdot (v - Invest)}{MarketCap - Invest}$$

Eq. 3-13

3.5. Stock Options

Assume a company has issued stock options in the past which are now to be exercised. The question is whether the company should issue new shares or buy back existing shares to offset the diluting impact of the exercised stock options.

Let $V_{Exercise}$ denote the post-tax present value per share to eternal shareholders, with the stock options exercised but without a share buyback to offset the diluting impact. Assume the cash received by the company from the exercised stock options is paid out as dividends to shareholders. The value to eternal shareholders is:

$$V_{Exercise} = \frac{(v + Options \cdot ExercisePrice) \cdot (1 - TaxDividend)}{Shares + Options}$$

Eq. 3-14

Let $W_{Exercise}$ denote the post-tax present value per share, with the stock options exercised and shares bought back to offset the diluting impact. The value to eternal shareholders is:

$$W_{Exercise} = \frac{(v + Options \cdot (ExercisePrice - SharePrice)) \cdot (1 - TaxDividend)}{Shares}$$

Eq. 3-15

The relative value of buying back shares to offset the diluting impact of stock options is:

$$\frac{W_{Exercise}}{V_{Exercise}} = \frac{(v + Options \cdot (ExercisePrice - SharePrice)) \cdot \left(1 + \frac{Options}{Shares}\right)}{v + Options \cdot ExercisePrice}$$

Eq. 3-16

Equilibrium is where the share buyback does not affect the value to eternal shareholders. As usual, the equilibrium is given as an inequality to clarify the condition under which the value to eternal shareholders is increased from making the share buyback:

$$W_{Exercise} > V_{Exercise} \Leftrightarrow MarketCap < \frac{v + Options \cdot ExercisePrice}{1 + \frac{Options}{Shares}}$$

Eq. 3-17

4. Share Buyback Valuation under Uncertainty

Now let the value to eternal shareholders be a stochastic variable which takes on different values according to some probability distribution. Let \mathbb{V} denote the stochastic variable⁴ for the value to eternal shareholders without share buybacks and assume its variance is positive $Var[\mathbb{V}] > 0$, otherwise the value to eternal shareholders would be known almost surely, which is contrary to the assumed uncertainty about the value.

Let \mathbb{W} denote the stochastic variable for the value to eternal shareholders with a share buyback, which is defined as a function of \mathbb{V} using Eq. 3-5:

$$\mathbb{W} = \frac{\mathbb{V} - Buyback}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 4-1

The expected (or mean) value with a share buyback is:

$$E[\mathbb{W}] = \frac{E[\mathbb{V}] - Buyback}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 4-2

Let \mathbb{W}/\mathbb{V} denote the stochastic variable for the relative value of a share buyback, which is defined as a function of \mathbb{V} using Eq. 3-6:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{Buyback}{\mathbb{V}}}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 4-3

The mean relative value of a share buyback is:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = \frac{1 - Buyback \cdot E\left[\frac{1}{\mathbb{V}}\right]}{1 - \frac{Buyback}{MarketCap}}$$

Eq. 4-4

4.1. Equilibriums

There are two equilibriums for mean outcomes of share buybacks with stochastic values. But a share buyback can be made at a market-cap below both these equilibriums and still cause a loss to eternal shareholders if the actual value turns out to be lower. Only share buybacks at a market-cap below the minimum possible value ensure that the value to eternal shareholders is increased.

⁴ It is customary to denote stochastic variables by capital letters but they are used for other purposes in this paper.

Mean Equilibrium

The mean equilibrium is where the mean value with a share buyback $E[\mathbb{W}]$ from Eq. 4-2 equals the mean value without a share buyback $E[\mathbb{V}]$. Writing it as an inequality clarifies when the mean value is increased:

$$E[\mathbb{W}] > E[\mathbb{V}] \Leftrightarrow MarketCap < E[\mathbb{V}]$$

Eq. 4-5

Relative Equilibrium

The relative equilibrium is where the mean relative value of a share buyback equals one, which is calculated from Eq. 4-4 and is also known as the harmonic mean of \mathbb{V} . The relative equilibrium is expressed as an inequality to clarify when the mean relative value is greater than one:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow MarketCap < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]}$$

Eq. 4-6

Minimum Value

Only share buybacks made at a market-cap below the minimum possible value of \mathbb{V} ensure that the value to eternal shareholders is increased:

$$MarketCap < Min[\mathbb{V}] \Rightarrow \mathbb{W} > \mathbb{V}$$

Eq. 4-7

This is not an equilibrium because equality and bi-implication do not hold.

Comparison of Equilibriums

The treatise shows that the equilibriums are ordered as follows:

$$Minimum\ Value < Relative\ Equilibrium < Mean\ Equilibrium$$

Eq. 4-8

At the mean equilibrium, the mean value with and without a share buyback both equal the market-cap:

$$E[\mathbb{V}] = MarketCap \Leftrightarrow E[\mathbb{W}] = MarketCap$$

Eq. 4-9

But when the mean equilibrium is satisfied, the mean relative value of a share buyback is less than one, as shown in the treatise using a well-known mathematical theorem called Jensen's inequality:

$$E[\mathbb{V}] = MarketCap \Rightarrow E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] < 1$$

Eq. 4-10

This is because the relative value of a share buyback is non-linear, see Figure 3 and Figure 4. If it turns out the value of the company is actually less than the market-cap, then a share buyback would have decreased

the value to eternal shareholders relatively more than it would have been increased if the value turns out to be higher than the market-cap. On average the potential losses are greater than the potential gains.

4.2. Increased Uncertainty

A share buyback increases the standard deviation of the value to eternal shareholders, which measures the uncertainty or spread of possible values. This follows from the definition of \mathbb{W} in Eq. 4-1, the scaling property of the standard deviation and the assumption that $MarketCap > Buyback > 0$, so that:

$$Stdev[\mathbb{W}] = \frac{Stdev[\mathbb{V}]}{1 - \frac{Buyback}{MarketCap}} > Stdev[\mathbb{V}]$$

Eq. 4-11

4.3. Implications

This proves that the dividend substitution hypothesis, as well as the Efficient Market hypothesis [4] and the Modigliani-Miller dividend irrelevance hypothesis [5] are incorrect when there is uncertainty about the value of a company to its eternal shareholders, as there obviously is when that value depends on unknown future earnings. Because even if the stock-market had priced a stock to its expected (or mean) value to eternal shareholders, a share buyback would increase the uncertainty (standard deviation) of that value, and any potential losses from the share buyback would be relatively greater than any potential gains.

In practice, the probability distribution of the value to eternal shareholders should be considered so as to properly assess the effect of a share buyback. The greater the uncertainty is about the value to eternal shareholders, the greater a margin of safety there should be between the market-cap and the mean value, as suggested for investing in general by Graham and Dodd [6].

5. Discount Rate

There are many proposals in the literature for selecting discount rates for use in valuation. The treatise suggests three distinct classes of financial securities as benchmarks for the rate of return that can be expected from commonly available investments, which are then used as the discount rate in valuing share buybacks relative to such alternative investments. The three classes of financial securities are:

- Government bonds of historically stable countries having had low risk of defaulting on the debt and interest payments, and low rates of return. For long-term USA government bonds the average annual rate of return has been about 5% for the period 1798-2012.
- Stock market index consisting of diverse, large and established companies which have historically had medium risk and medium rate of return. The rate of return from buying a stock market index at its current market price and holding the shares for eternity is called the value yield and is estimated using Monte Carlo simulation in [7]. For the S&P 500 index the estimated mean value yield is $E[\text{Value Yield}] = 9.1\% / (P/Book) + 4.4\%$ which is calculated from the current P/Book of the S&P 500 index. For the period 1984-2011 the mean value yield was about 8.3% on average.
- Venture index consisting of companies in their early stages of development which have historically had high risk and high rate of return in the long term. For the Dow Jones Venture Capital (DJVC) / Sand Hill index the average annual rate of return was about 16% during the period 1991-2010.

6. Case Study: Hewlett-Packard

Consider as an example the company *Hewlett-Packard* (HP) which was started by two people in USA in 1939 as a manufacturer of electronic equipment. At the end of fiscal year 2011 the company employed almost 350k people worldwide with annual revenue of USD 127b and reported net income of USD 7.1b.⁵

The focus here is on demonstrating the share buyback valuation formulas, not the intricacies of accounting and earnings forecasting. Qualitative assessments are required for many of the assumptions and are left to the reader who is encouraged to experiment with different assumptions in the formulas. The treatise gives a more detailed and slightly different analysis of this company, as well as case studies of other companies demonstrating more aspects of the share buyback valuation theory.

6.1. Share Buyback Policy

The company's share buyback policy has remained largely unchanged in recent years. For example, the annual reports for fiscal years 2006 and 2011 both have this text with only the year being different:⁶

"HP repurchased shares in the fourth quarter of fiscal 2006 [2011] under an ongoing program to manage the dilution created by shares issued under employee stock plans as well as to repurchase shares opportunistically. This program, which does not have a specific expiration date, authorizes repurchases in the open market or in private transactions."

The company neglects to inform shareholders of the share-price below which shares may be bought back and how this relates to management's assessment of the company's value to eternal shareholders, which is crucial information for outsiders to estimate the effect of the share buybacks on the value to eternal shareholders. The primary reason for the company's share buybacks seems to be the intention of offsetting the diluting impact of employee stock options. Combined with the lack of valuation considerations this suggests the company's management misunderstands the effect that share buybacks have on the value to eternal shareholders.

6.2. Terminology

A recent and typical earnings report of the company states:⁷ *"(...) we returned USD 384 million to shareholders in the form of share repurchases and dividends."* According to the cash flow statement of that report, USD 124m of these USD 384m were used for share buybacks. However, during that 3 month period the share-price decreased about 24% and the market-cap decreased about USD 8.6b. Since the company's management claims it returned USD 124m to shareholders via share buybacks, management must believe that in the absence of this share buyback the market-cap would instead have decreased by the USD 8.6b experienced and an additional USD 124m that were used for share buybacks.

This is a common example of the absurd terminology used by both practitioners and scholars which hinders a proper understanding of share buyback valuation and hence the establishing of sound share buyback policies and practices.

⁵ Form 10-K annual report for 2011 filed with US SEC:

www.sec.gov/Archives/edgar/data/47217/000104746911010094/a2206500z10-k.htm

⁶ Form 10-K annual report for 2006 filed with US SEC:

www.sec.gov/Archives/edgar/data/47217/000104746906015253/a2174889z10-k.htm

⁷ Form 8-K with unaudited 4th quarter and annual results for 2012 filed with US SEC:

www.sec.gov/Archives/edgar/data/47217/000004721712000033/ex99-1_112012.htm

6.3. Year 2010

Calculating the effect of a share buyback requires the value to eternal shareholders without a share buyback v from Eq. 2-3. Assuming the annual earnings that could be paid out as dividends are constant for eternity means the value without a share buyback is calculated from a so-called geometric series with zero growth, which is known to be:

$$v = \text{Excess Cash} + \sum_{t=1}^{\infty} \frac{\text{Earnings}}{(1+d)^t} = \text{Excess Cash} + \frac{\text{Earnings}}{d}$$

Eq. 6-1

For simplicity assume the company did not have any excess cash, which means a share buyback would be made for borrowed cash and hence be a recapitalization which requires more complicated valuation formulas that are given in the treatise, but the standard valuation formulas will be used in this example.

According to Table 1, in fiscal year 2010 the company had reported net income of about USD 8.8b and Free Cash Flow (FCF) is estimated at about USD 8.4b. Assume this FCF estimate is the earnings that could be paid out as dividends and that it will be constant for eternity.

The annual rate of return one would get from buying shares in the S&P 500 index and holding the shares for eternity is called the value yield of the S&P 500. During 2010 it was about 8.8% on average according to the treatise. Using this as the discount rate estimates the return of the company relative to the return that could be expected from investing in the S&P 500 index. The present value of the future earnings is:

$$v = \frac{\text{Earnings}}{d} = \frac{\text{USD } 8.4b}{8.8\%} \simeq \text{USD } 95b$$

The share buyback amount in 2010 was about USD 11b, see Table 1. The relative value of a share buyback is calculated using Eq. 3-6. At the lowest market-cap of about USD 86.5b during 2010, the relative value is:

$$\frac{W}{V} = \frac{1 - \frac{\text{Buyback}}{v}}{1 - \frac{\text{Buyback}}{\text{MarketCap}}} = \frac{1 - \frac{\text{USD } 11b}{\text{USD } 95b}}{1 - \frac{\text{USD } 11b}{\text{USD } 86.5b}} \simeq 101.3\%$$

That is, the share buyback would have increased the value to eternal shareholders by about 1.3%. At the highest market-cap of about USD 127b the relative value of the share buyback is about 96.8% which means a loss of about 3.2%. This used the assumption that future earnings would be USD 8.4b for eternity and the risk of the future earnings is similar to the risk of the future aggregate earnings of the companies in the S&P 500 index. If a risk premium had been deemed appropriate so the discount rate was instead, say, $d = 10\%$, then the value without a share buyback would be $v \simeq \text{USD } 84b$ and the relative value would be between 95-99.6%, depending on the market-cap at the time of share buyback during that year.

When the relative value is less than one it means the share buyback creates an obligation for future earnings to grow if the share buyback is to be merely value neutral to eternal shareholders, otherwise the share buyback results in a loss to eternal shareholders.

6.4. Acquisition of Autonomy

In fiscal year 2011 the company HP acquired the company *Autonomy* for USD 10.2b in cash.⁸ Autonomy was founded by two people in 1996 and at the end of 2010 it employed about 1.9k people, having annual revenue of USD 870m and reported net income of USD 217m.

HP could choose between acquiring Autonomy or use the cash for making a share buyback instead. The question is which would have been more valuable to the eternal shareholders of HP. To answer this we first need to know the value of HP without either the acquisition or share buyback. The following is an ex-ante analysis so as to simulate the valuation with the information available at the time.

According to Table 1, HP had reported net income of almost USD 7.1b and FCF of about USD 9.1b in fiscal year 2011. If the FCF is assumed to be the earnings available for dividend payouts and it is constant for eternity, then using Eq. 6-1 with a discount rate of, say, $d = 10\%$ gives the value to eternal shareholders:

$$v = \text{Excess Cash} + \frac{\text{Earnings}}{d} = \frac{\text{USD } 9.1b}{10\%} = \text{USD } 91b$$

First consider the case where Autonomy was acquired for its fair value, that is, the acquisition price equals the present value of the return: $\text{Invest} = \text{Return} = \text{USD } 10.2b$. According to Table 1 the lowest market-cap of HP was about USD 45b during 2011. The value to HP's eternal shareholders of the Autonomy acquisition relative to a share buyback for the same amount is calculated using Eq. 3-11:

$$\frac{V_{\text{Invest}}}{W} = \left(1 + \frac{\text{Return}}{v - \text{Invest}}\right) \cdot \left(1 - \frac{\text{Invest}}{\text{MarketCap}}\right) = \left(1 + \frac{\text{USD } 10.2b}{\text{USD } 91b - \text{USD } 10.2b}\right) \cdot \left(1 - \frac{\text{USD } 10.2b}{\text{USD } 45b}\right) \approx 87\%$$

That is, the acquisition of Autonomy is worth 13% less than a share buyback under these assumptions. Note that because $V_{\text{Invest}} = V$ in this case then $V_{\text{Invest}}/W = V/W$ which is just the reciprocal of Eq. 3-6.

In case the return on the Autonomy acquisition is only half its acquisition price then the value of the acquisition relative to a share buyback at the lowest market-cap of USD 45b would be $V_{\text{Invest}}/W \approx 82\%$. In the opposite case where the return on the Autonomy acquisition is twice the acquisition price, the value of the acquisition relative to a share buyback would be $V_{\text{Invest}}/W \approx 97\%$.

The equilibrium market-cap below which a share buyback is more valuable than the acquisition is calculated using Eq. 3-12 and depends on the present value of the return on the acquisition. Assuming the return is twice the acquisition price, the equilibrium is:

$$W > V_{\text{Invest}} \Leftrightarrow \text{MarketCap} < \text{Invest} \cdot \left(1 + \frac{v - \text{Invest}}{\text{Return}}\right) = \text{USD } 10.2b \cdot \left(1 + \frac{\text{USD } 91b - \text{USD } 10.2b}{\text{USD } 20.4b}\right) \approx \text{USD } 50.6b$$

⁸ In February 2011 the company Autonomy had 242.7m shares outstanding according to its annual report for 2010, which is currently unavailable from the company's website (www.autonomy.com). The acquisition offer was USD 42.11 per share giving a total amount of USD 10.2b, which was apparently accepted by the shareholders of Autonomy. Hewlett-Packard's Form 8-K for August 18, 2011 and annual report for 2011, both filed with US SEC: www.sec.gov/Archives/edgar/data/47217/000004721711000039/ex99-1_082011.htm www.sec.gov/Archives/edgar/data/47217/000104746911010094/a2206500z10-k.htm

The equilibrium can also be formulated in terms of the required return on the acquisition using Eq. 3-13. At the lowest market-cap of about USD 45b during the year, the equilibrium is:

$$W > V_{Invest} \Leftrightarrow Return < \frac{Invest \cdot (v - Invest)}{MarketCap - Invest} = \frac{USD\ 10.2b \cdot (USD\ 91b - USD\ 10.2b)}{USD\ 45b - USD\ 10.2b} \simeq USD\ 24b$$

That is, under these valuation assumptions and at the lowest market-cap of USD 45b during that year, the present value of the return on the Autonomy acquisition would have to be more than USD 24b in order for the acquisition to be more valuable than a share buyback to the eternal shareholders of HP.

6.5. Restructuring

In 2012 the company HP announced that its workforce of almost 350k employees was to be reduced by 29k employees over the following years in an effort to decrease the company's expenses, increase research and development, and improve profitability.⁹ The effect on the value to eternal shareholders is now studied.

There are generally three components in the valuation of a corporate restructuring so as to determine its effect on the value to eternal shareholders. First there is the present value of the company to its eternal shareholders without the restructuring which is the value v as usual. Then the present value of the restructuring costs must be subtracted and the present value of the future cost savings must be added. This is similar to the valuation of an investment in Eq. 3-9 where the investment amount is the present value of the restructuring costs, i.e. $Invest = Restructuring\ Costs$, and the return is the present value of future savings resulting from the restructuring, i.e. $Return = Savings$, so that:

$$V_{Restructuring} = \frac{(v - Restructuring\ Costs + Savings) \cdot (1 - TaxDividend)}{Shares}$$

According to Table 1 the company's FCF in 2012 was about USD 7.5b. Assuming this is the amount that could be paid out as dividends and it is constant for eternity, and using a discount rate of, say, $d = 10\%$, then the value to eternal shareholders without a restructuring is calculated using Eq. 6-1:

$$v = \frac{USD\ 7.5b}{10\%} \simeq USD\ 75b$$

The company expects the total restructuring charges to be USD 3.7b over a few years. For simplicity, these restructuring costs are assumed to consist entirely of cash which are paid immediately rather than over multiple years, so that $Restructuring\ Costs = USD\ 3.7b$.

The company does not detail its expected savings from the restructuring so an estimate must be made. At the end of fiscal year 2011 the company had almost 350k employees so a reduction of 29k employees corresponds to about 8.3%. If the earnings are assumed to increase in direct proportion to the reduction in employees, and the FCF of USD 7.5b for 2012 is used as the basis earnings available for dividend payout without a restructuring, then the present value of the restructuring savings is:

$$Savings = \frac{8.3\% \cdot USD\ 7.5b}{10\%} \simeq USD\ 6.2b$$

⁹ Form 10-Q filed with US SEC for the quarter ending July 31, 2012:
www.sec.gov/Archives/edgar/data/47217/000104746912008732/a2210845z10-q.htm

Using the definitions of $V_{Restructuring}$ and V , the relative value of the restructuring can be reduced to:

$$\frac{V_{Restructuring}}{V} = 1 + \frac{Savings - Restructuring Costs}{v} \simeq 1 + \frac{USD 6.2b - USD 3.7b}{USD 75b} \simeq 103.3\%$$

That is, the restructuring increases value to eternal shareholders by about 3.3%.

If instead the restructuring cost of USD 3.7b is used for a share buyback at the market-cap of about USD 27.5b at the time of this writing in late December 2012, then the relative value of a share buyback is calculated using Eq. 3-6:

$$\frac{W}{V} = \frac{1 - \frac{Buyback}{v}}{1 - \frac{Buyback}{MarketCap}} \simeq \frac{1 - \frac{USD 3.7b}{USD 75b}}{1 - \frac{USD 3.7b}{USD 27.5b}} \simeq 109.8\%$$

The value of a share buyback relative to the restructuring is then:

$$\frac{W}{V_{Restructuring}} = \frac{W/V}{V_{Restructuring}/V} \simeq \frac{109.8\%}{103.3\%} \simeq 106.3\%$$

That is, the share buyback is worth about 6% more than the restructuring to the eternal shareholders of HP.

For a share buyback to be more valuable to eternal shareholders than the restructuring, the present value of the savings from the restructuring must be less than about USD 11b, as calculated using Eq. 3-13:

$$\begin{aligned} W > V_{Restructuring} &\Leftrightarrow Savings < \frac{Restructuring Costs \cdot (v - Restructuring Costs)}{MarketCap - Restructuring Costs} \\ &= \frac{USD 3.7b \cdot (USD 75b - USD 3.7b)}{USD 27.5b - USD 3.7b} \simeq USD 11b \end{aligned}$$

6.6. Uncertainty

To demonstrate the effect of uncertainty on share buyback valuation, let the value of HP to its eternal shareholders without a share buyback be a stochastic variable \mathbb{V} that can take on either of two values with equal probability: USD 50b or 150b. The mean value is:

$$E[\mathbb{V}] = USD 50b \cdot \frac{1}{2} + USD 150b \cdot \frac{1}{2} = USD 100b$$

If $MarketCap = E[\mathbb{V}] = USD 100b$ then $E[W] = MarketCap = USD 100b$ according to Eq. 4-9, so the mean value with and without a share buyback both equal the market-cap.

The relative value of a share buyback is calculated using Eq. 4-3. For $\mathbb{V} = USD 50b$, $Buyback = USD 5b$ and $MarketCap = E[\mathbb{V}] = USD 100b$ the relative value is:

$$\frac{W}{\mathbb{V}} = \frac{1 - \frac{Buyback}{\mathbb{V}}}{1 - \frac{Buyback}{MarketCap}} = \frac{1 - \frac{USD 5b}{USD 50b}}{1 - \frac{USD 5b}{USD 100b}} \simeq 94.7\%$$

For $\mathbb{V} = USD\ 150b$ the relative value is:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{USD\ 5b}{USD\ 150b}}{1 - \frac{USD\ 5b}{USD\ 100b}} \approx 101.8\%$$

The mean relative value is:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = 94.7\% \cdot \frac{1}{2} + 101.8\% \cdot \frac{1}{2} \approx 98.3\%$$

So even though the market-cap equals the mean value with and without a share buyback, the mean relative value of the share buyback is only about 98%, because the potential loss outweighs the potential gain. If a share buyback is made at a market-cap of USD 100b and it turns out the value of the company is only $\mathbb{V} = USD\ 50b$ then the eternal shareholders would lose about 5.3% of their value, which is much larger than the gain of only about 1.8% if instead the value turns out to be $\mathbb{V} = USD\ 150b$.

The relative equilibrium ensures the potential losses and gains are balanced. First we need to calculate:

$$E\left[\frac{1}{\mathbb{V}}\right] = \frac{1}{USD\ 50b} \cdot \frac{1}{2} + \frac{1}{USD\ 150b} \cdot \frac{1}{2} = \frac{1}{USD\ 75b}$$

The relative equilibrium is then calculated using Eq. 4-6:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow MarketCap < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]} = USD\ 75b$$

This is considerably less than the mean equilibrium which is $E[\mathbb{V}] = USD\ 100b$.

However, a share buyback may still cause a loss to eternal shareholders even though the market-cap is less than the mean equilibrium of USD 100b and less than the relative equilibrium of USD 75b. This is because the mean and relative equilibriums are for average outcomes. If a share buyback must be guaranteed to increase the value to eternal shareholders then the market-cap must be below the minimum possible value that \mathbb{V} can take on, see Eq. 4-7:

$$MarketCap < Min[\mathbb{V}] = USD\ 50b \Rightarrow \mathbb{W} > \mathbb{V}$$

Share buybacks also increase the uncertainty about the value to eternal shareholders. The standard deviation of the value without a share buyback is $Stdev[\mathbb{V}] = USD\ 50b$ but it is $Stdev[\mathbb{W}] \approx USD\ 53b$ when a share buyback is made for USD 5b and the market-cap is USD 100b which is the mean equilibrium.

6.6.1. Extreme Example

This example further demonstrates the need to use probability distributions rather than averages in share buyback valuation.

Let the value without a share buyback be a stochastic variable \mathbb{V} that can take on either of two values: USD 10b with probability 0.9 (or 90%) or USD 1000b with probability 0.1 (or 10%). The mean value is:

$$E[\mathbb{V}] = USD\ 10b \cdot 0.9 + USD\ 1000b \cdot 0.1 = USD\ 109b$$

The relative value of a share buyback is calculated using Eq. 4-3. For $\mathbb{V} = \text{USD } 10b$, $\text{Buyback} = \text{USD } 5b$ and $\text{MarketCap} = E[\mathbb{V}] = \text{USD } 109b$ the relative value is:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{\text{Buyback}}{\mathbb{V}}}{1 - \frac{\text{Buyback}}{\text{MarketCap}}} = \frac{1 - \frac{\text{USD } 5b}{\text{USD } 10b}}{1 - \frac{\text{USD } 5b}{\text{USD } 109b}} \simeq 52\%$$

For $\mathbb{V} = \text{USD } 1000b$ the relative value is:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{\text{USD } 5b}{\text{USD } 1000b}}{1 - \frac{\text{USD } 5b}{\text{USD } 109b}} \simeq 104\%$$

So if $\mathbb{V} = \text{USD } 10b$, which occurs with probability 0.9 (or 90%), then the share buyback results in a decrease in the value to eternal shareholders by about 48%. If instead $\mathbb{V} = \text{USD } 1000b$, which occurs with probability 0.1 (or 10%), then the share buyback results in an increase in the value to eternal shareholders by about 4%. On average the potential loss far outweighs the potential gain:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = 52\% \cdot 0.9 + 104\% \cdot 0.1 \simeq 57\%$$

The relative equilibrium ensures that the potential losses and gains are in balance:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] > 1 \Leftrightarrow \text{MarketCap} < \frac{1}{E\left[\frac{1}{\mathbb{V}}\right]} \simeq \text{USD } 11.1b$$

So a share buyback made at a market-cap below USD 11.1b is expected to increase the relative value to eternal shareholders. But it does not guarantee that losses are avoided altogether. For example, if $\text{MarketCap} = \text{USD } 10.5b$ and $\text{Buyback} = \text{USD } 5b$ then for $\mathbb{V} = \text{USD } 10b$ the relative value is:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{\text{Buyback}}{\mathbb{V}}}{1 - \frac{\text{Buyback}}{\text{MarketCap}}} = \frac{1 - \frac{\text{USD } 5b}{\text{USD } 10b}}{1 - \frac{\text{USD } 5b}{\text{USD } 10.5b}} \simeq 95\%$$

For $\mathbb{V} = \text{USD } 1000b$ the relative value is:

$$\frac{\mathbb{W}}{\mathbb{V}} = \frac{1 - \frac{\text{USD } 5b}{\text{USD } 1000b}}{1 - \frac{\text{USD } 5b}{\text{USD } 10.5b}} \simeq 190\%$$

Weighting these by their probabilities gives the mean relative value:

$$E\left[\frac{\mathbb{W}}{\mathbb{V}}\right] = 95\% \cdot 0.9 + 190\% \cdot 0.1 \simeq 105\%$$

So on average the share buyback is expected to increase value to eternal shareholders. But the probability of this actually happening is only 0.1 (or 10%) in which case the value to eternal shareholders would have

increased by 90% as a result of the share buyback. The probability of a loss, however, is 0.9 (or 90%) in which case the value to eternal shareholders would have decreased 5% as a result of the share buyback.

A share buyback is only guaranteed to increase the value to eternal shareholders if the market-cap is below the minimum possible value that V can take on, see Eq. 4-7:

$$MarketCap < Min[V] = USD\ 10b \Rightarrow W > V$$

For example, if $MarketCap = USD\ 9b$ and $Buyback = USD\ 5b$ then for $V = USD\ 10b$ the relative value of the share buyback is 113% while it is 224% for $V = USD\ 1000b$, so both possible outcomes increase the value to eternal shareholders.

The uncertainty is also increased as a result of a share buyback. For example, the standard deviation of the value without a share buyback is $Stdev[V] \simeq USD\ 297b$ but it is $Stdev[W] \simeq USD\ 311b$ when a share buyback is made for USD 5b and the market-cap is USD 109b which is the mean equilibrium.

6.7. Summary

According to Table 1, the company made share buybacks for about USD 1.6b in 2012, while the average share buybacks were USD 8.3b per year in the period 2005-2011 during which the market-cap was typically 3-4 times higher than it was in late 2012. If the share buybacks in previous years were to have been merely value neutral to eternal shareholders then the company's future earnings would have to increase, while constant future earnings would be sufficient in order for a share buyback at the low market-cap in late 2012 to be much more valuable to eternal shareholders than prior share buybacks, the acquisition of Autonomy, and possibly also the company's restructuring in which 29k employees will lose their jobs.

7. Conclusion

Share buybacks magnify any mispricing between the share price and the value to eternal shareholders, which is the excess cash and present value of future earnings available for dividend payouts. Buybacks of overpriced shares cause losses to eternal shareholders that are disproportionately greater than the gains from buybacks of underpriced shares. Managers should therefore use caution and skepticism about the future prospects of their company when making share buybacks, so as to ensure they do not make share buybacks that will later prove to have actually decreased value to eternal shareholders.

When there is uncertainty about the future earnings, as there usually is, then the mean present value is misleading in share buyback valuation and the probability distribution should be considered so as to properly assess the likely impact of a share buyback on the value to eternal shareholders.

Misallocation of capital negatively affects both shareholders and employees. It occurs when investments, acquisitions and share buybacks are made at too high prices compared to the return that can reasonably be expected; or conversely when opportunities for investments, acquisitions or share buybacks are available at low prices but are being ignored.

Before Share Buyback	
$MarketCap$	
$v = 95\% \cdot MarketCap$	
Share Buyback is The Portion in Dark Grey	
$Buyback = 20\% \cdot MarketCap$	
$Buyback \simeq 21\% \cdot v$	
After Share Buyback	
$MarketCap$	
$v = 93.75\% \cdot MarketCap$	

Figure 1: Demonstration of the effect of a share buyback when the value to eternal shareholders is $v = 95\% \cdot MarketCap$ and the share buyback is 20% of the $MarketCap$. If the share-price remains unchanged then the mispricing is magnified slightly as the value to eternal shareholders decreases to become 93.75% of $MarketCap$, down from 95% prior to the share buyback. According to Eq. 3-6 the value of the share buyback relative to a dividend payout is about 98.7%, that is, the value to eternal shareholders decreases about 1.3% as a result of the share buyback. If instead $MarketCap = 95\% \cdot v$ then the value to eternal shareholders would have increased by about 1.3% as a result of the share buyback. The equilibrium is when $MarketCap = v$ so the share buyback has equal effect on $MarketCap$ and v .

Before Share Buyback	
$MarketCap$	
$v = 25\% \cdot MarketCap$	
Share Buyback is The Portion in Dark Grey	
$Buyback = 20\% \cdot MarketCap$	
$Buyback = 80\% \cdot v$	
After Share Buyback	
$MarketCap$	
$v = 6.25\% \cdot MarketCap$	

Figure 2: Demonstration of the effect of a share buyback when the value to eternal shareholders is $v = 25\% \cdot MarketCap$ and the share buyback is 20% of the $MarketCap$. If the share-price remains unchanged then the mispricing is magnified greatly as the value to eternal shareholders decreases to become 6.25% of $MarketCap$, down from 25% prior to the share buyback. According to Eq. 3-6 the value of the share buyback relative to a dividend payout is 25%, that is, the value to eternal shareholders decreases 75% as a result of the share buyback. If instead $MarketCap = 25\% \cdot v$ then the value to eternal shareholders would have increased by 300% as a result of the share buyback. The equilibrium is when $MarketCap = v$ so the share buyback has equal effect on $MarketCap$ and v .

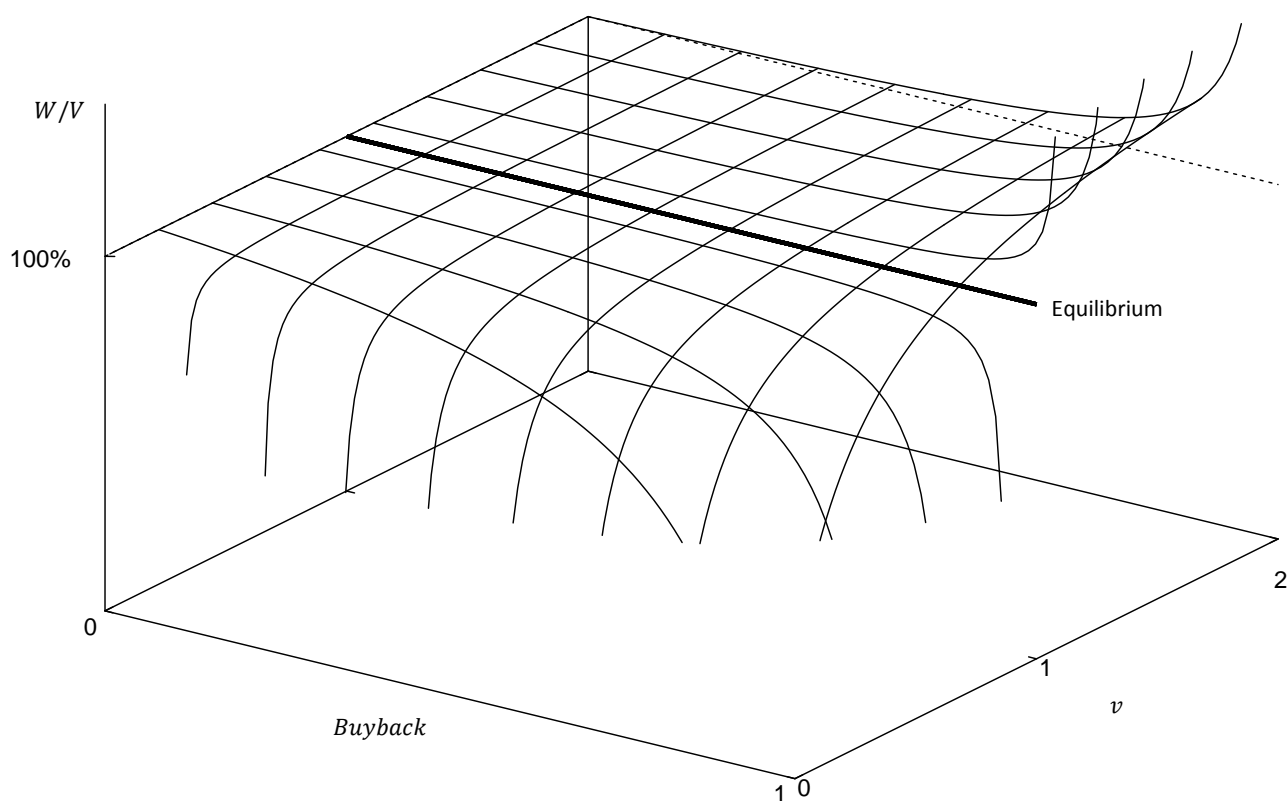


Figure 3: The relative value of a share buyback W/V (equivalently w/v) plotted by changing the variables in Eq. 3-6. The variables have been normalized so $MarketCap = 1$, $Buyback$ varies between 0 and 1, and the value to eternal shareholders v varies between 0 and 2. The plot shows that the value with a share buyback W is greater than the value without a share buyback V when $v > MarketCap = 1$, and vice versa. Equilibrium is where the value to eternal shareholders is unaffected by the share buyback so that $W = V$ which occurs when $v = MarketCap = 1$. Note the non-linearity of W/V as a function of v and compare this to the linearity of the absolute value of a share buyback as shown in Figure 4.

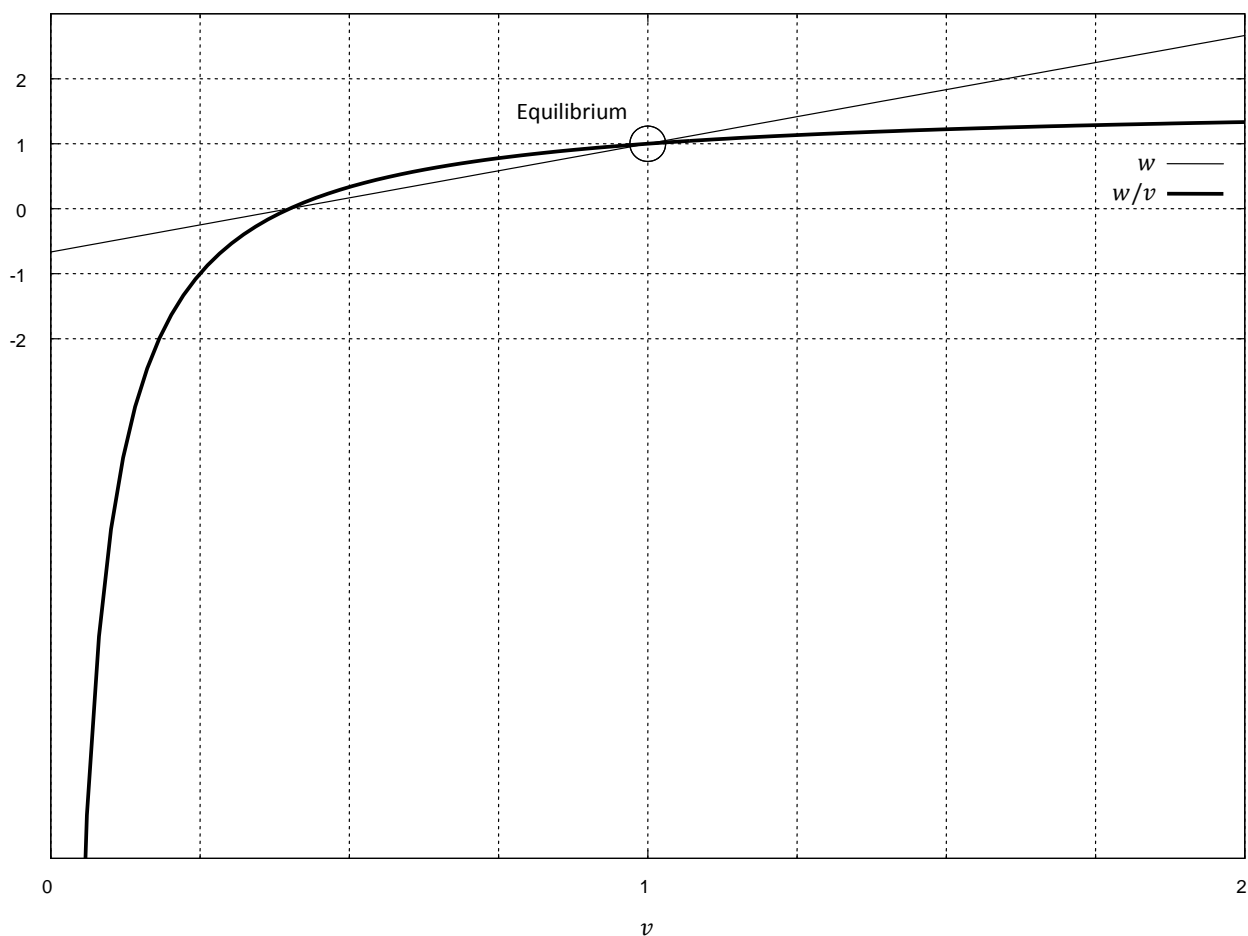


Figure 4: The relative value of a share buyback w/v (equivalently W/V) calculated from Eq. 3-6, compared to the absolute value of a share buyback w calculated from Eq. 3-5. The plot shows the absolute value of a share buyback w changes linearly as a function of v while the relative value w/v changes non-linearly as a function of v . The variables have been normalized so $MarketCap = 1$, $Buyback$ is set to 0.4 as an example, and the value to eternal shareholders v varies between 0 and 2. The share buyback equilibrium is where the value to eternal shareholders is unaffected by the share buyback so that $w = v$ (equivalently $W = V$) which is marked here as a circle and occurs when $v = MarketCap = 1$. This plot is an excerpt of the 3-dimensional plot in Figure 3 obtained by setting constant one axis at $Buyback = 0.4$.

USD Millions	2005	2006	2007	2008	2009	2010	2011	2012
Revenue	86,696	91,658	104,286	118,364	114,552	126,033	127,245	120,357
Net Income	2,398	6,198	7,264	8,329	7,660	8,761	7,074	(12,650)
Free Cash Flow (FCF)	6,575	9,373	7,143	12,026	10,179	8,391	9,099	7,482
Acquisitions	(641)	(855)	(6,793)	(11,248)	(391)	(7,977)	(10,391)	(54)
Dividends	(926)	(894)	(846)	(796)	(766)	(771)	(844)	(1,015)
Share Issuance	1,161	2,538	3,103	1,810	1,837	2,617	896	716
Share Buyback	(3,514)	(7,779)	(10,887)	(9,620)	(5,140)	(11,042)	(10,117)	(1,619)
Assets	77,317	81,981	88,699	113,331	114,799	124,503	129,517	108,768
Equity	37,176	38,144	38,526	38,942	40,517	40,781	39,004	22,833
ROA	3%	8%	9%	9%	7%	8%	6%	-10%
ROE	6%	17%	19%	22%	20%	22%	17%	-32%
Shares, Basic	2,879	2,782	2,630	2,483	2,388	2,319	2,094	1,974
Shares, Diluted	2,909	2,852	2,716	2,567	2,437	2,372	2,128	1,974
Market-Cap, Low	54,010	78,230	100,519	74,564	60,631	86,545	45,021	27,370
Market-Cap, High	84,067	110,918	139,048	132,791	117,490	126,965	103,423	58,993

Table 1: Selected financial data for the company *Hewlett-Packard*. Free Cash Flow is defined as Operating Cash Flow minus Capital Expenditures plus proceeds from sale of Property, Plant and Equipment. Acquisitions are net of cash required, and net of cash proceeds from divested business units. ROA and ROE are calculated as Net Income for a year divided by the Assets and Equity for the previous year, respectively. Basic and diluted shares outstanding are weighted averages for the year. Market-Cap is approximated as the number of basic shares multiplied by the minimum and maximum share-price for the year. Share buyback for 2006 includes USD 1.7b in prepayment for future share buybacks, the meaning of which is unclear.¹⁰

¹⁰ Form 10-K annual reports (2005-2011), Form 8-K unaudited 4th quarter report (2012), all filed with US SEC: www.sec.gov/cgi-bin/browse-edgar?action=getcompany&CIK=0000047217&type=10-k
www.sec.gov/Archives/edgar/data/47217/000004721712000033/ex99-1_112012.htm

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9. Revision History

- 2014, August 13: Corrected the FCF for year 2012 in Table 1 and updated section 6.5. (27 pages)
- 2013, October 1: Rewrote section 4.2. Minor text revisions. Minor layout changes. (27 pages)
- 2013, August 3: Renamed stochastic equilibriums. Minor text revisions. (24 pages)
- 2013, July 24: Rewrote several parts. Added executive summary. (24 pages)
- 2013, January 4: Minor text revisions. (20 pages)
- 2013, January 1: First edition. (19 pages)