Quantum Computing with Qiskit and IBM Quantum systems

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IBM Quantum

- A bit

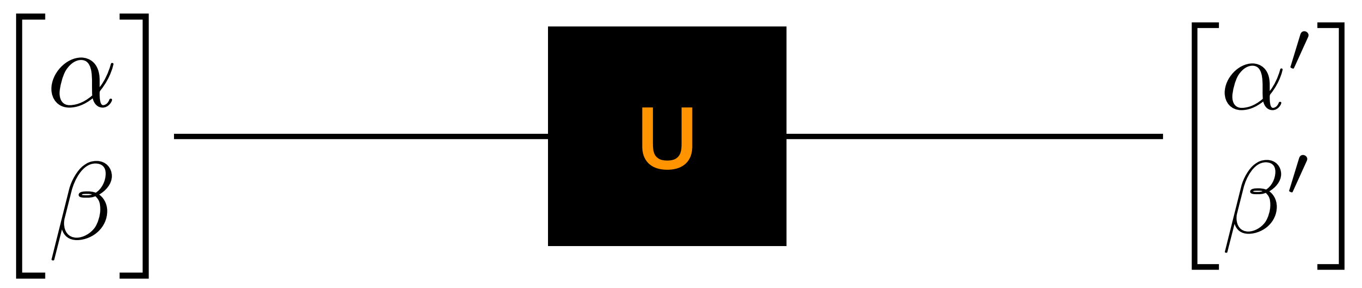
State of a bit is in a set, the set of possible states, {0, 1}

The state of a bit changes through logic operations / logic gates.

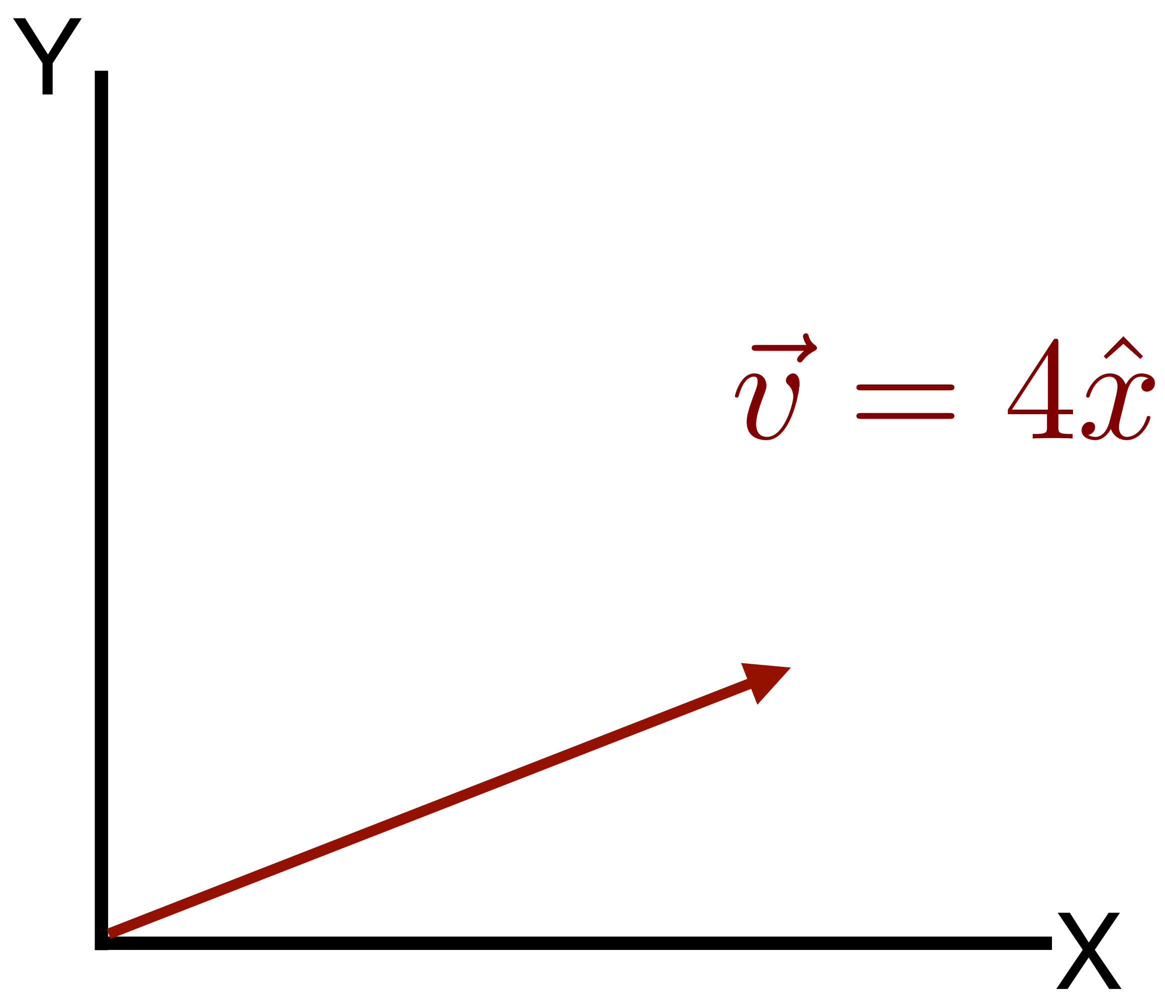
- **A qubit**

State of a qubit is a unit (normalized) vector in a two-dim complex vector space.

The state of a qubit changes through unitary operations / quantum gates.

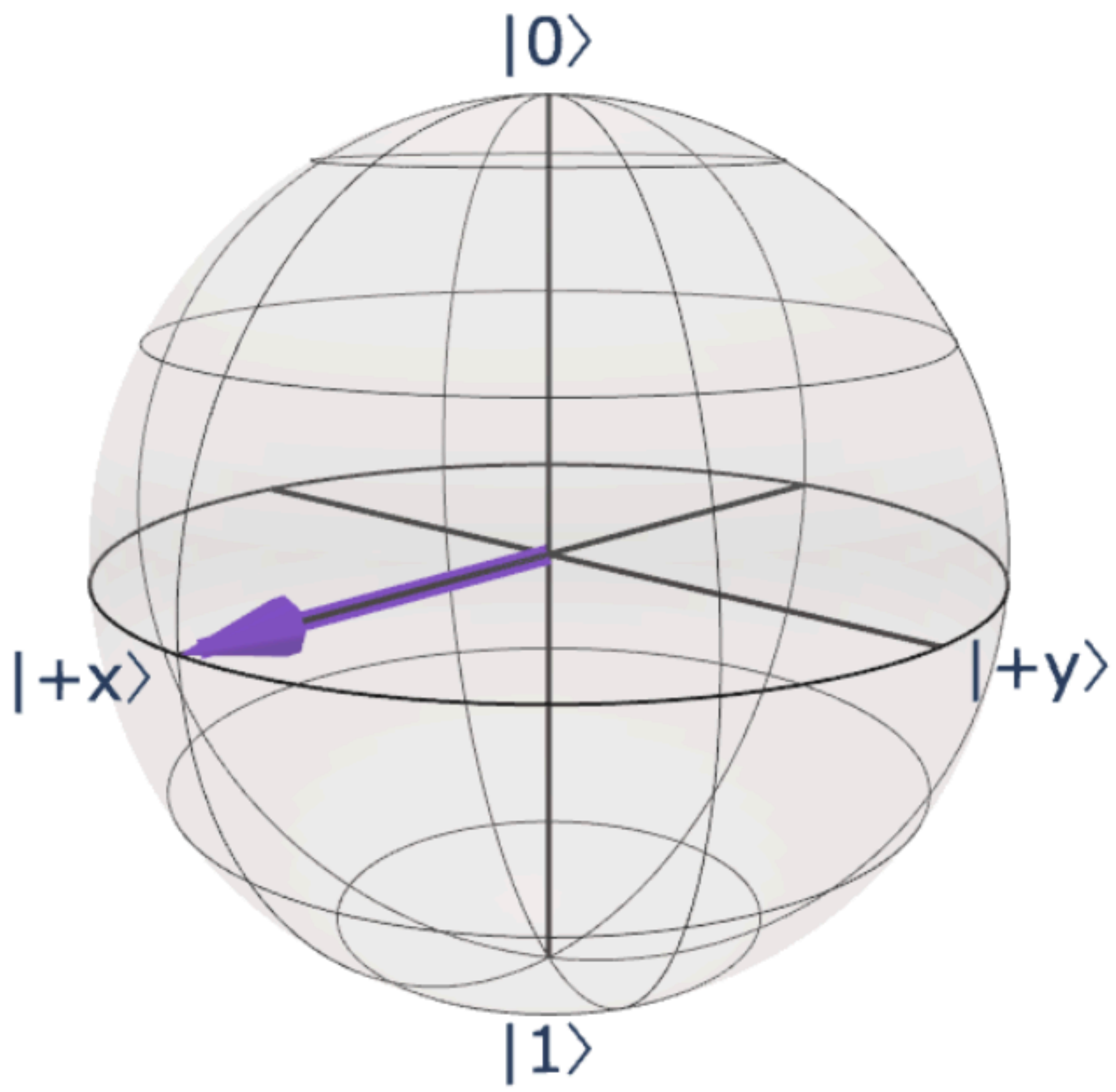


State of a qubit is a unit (normalized) vector in a two-dim complex vector space.



$$\vec{v} = 4\hat{x} + 2\hat{y} = 2(2\hat{x} + \hat{y}) = \begin{bmatrix} 4 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



- Computational basis states of a qubit

Two special quantum states corresponding to the 0 and 1 states of a classical bit

$$\langle\psi| = |\psi\rangle^\dagger$$

$$|0\rangle := \begin{bmatrix}1\\0\end{bmatrix} \qquad |1\rangle := \begin{bmatrix}0\\1\end{bmatrix}$$

$$\langle 0| := \begin{bmatrix}1 & 0\end{bmatrix} \qquad \langle 1| := \begin{bmatrix}0 & 1\end{bmatrix}$$

★ $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \neq 0$

- General states of a qubit (**superposition**)

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix}\alpha\\\beta\end{bmatrix} \qquad \alpha, \beta \in \mathbb{C} \qquad |\alpha|^2 + |\beta|^2 = 1$$

Superposition is one of the key attributes of the quantum computer.

- Bloch Sphere

- General states of a qubit in a bloch sphere representation

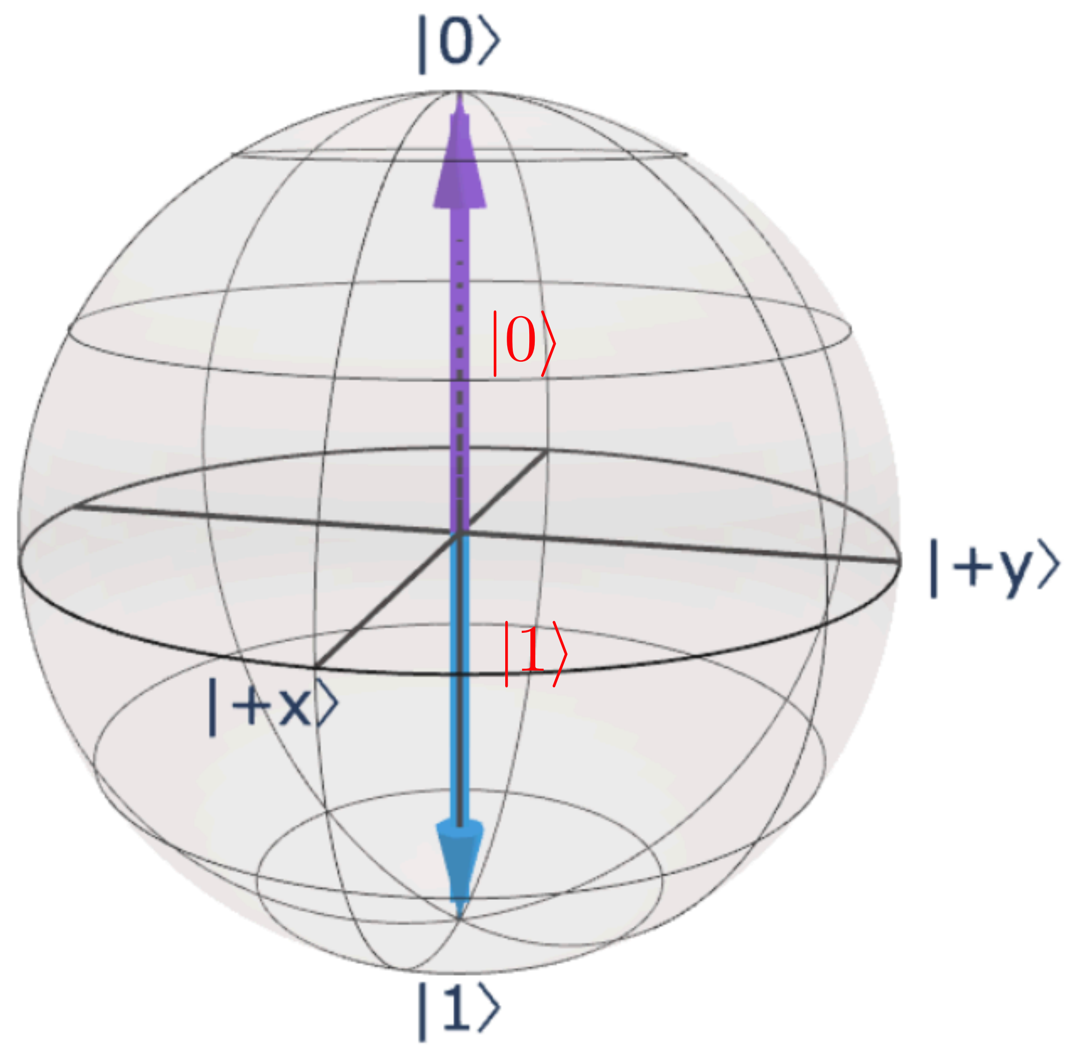
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle \qquad 0 \le \theta \le \pi \quad 0 \le \phi \le 2\pi$$

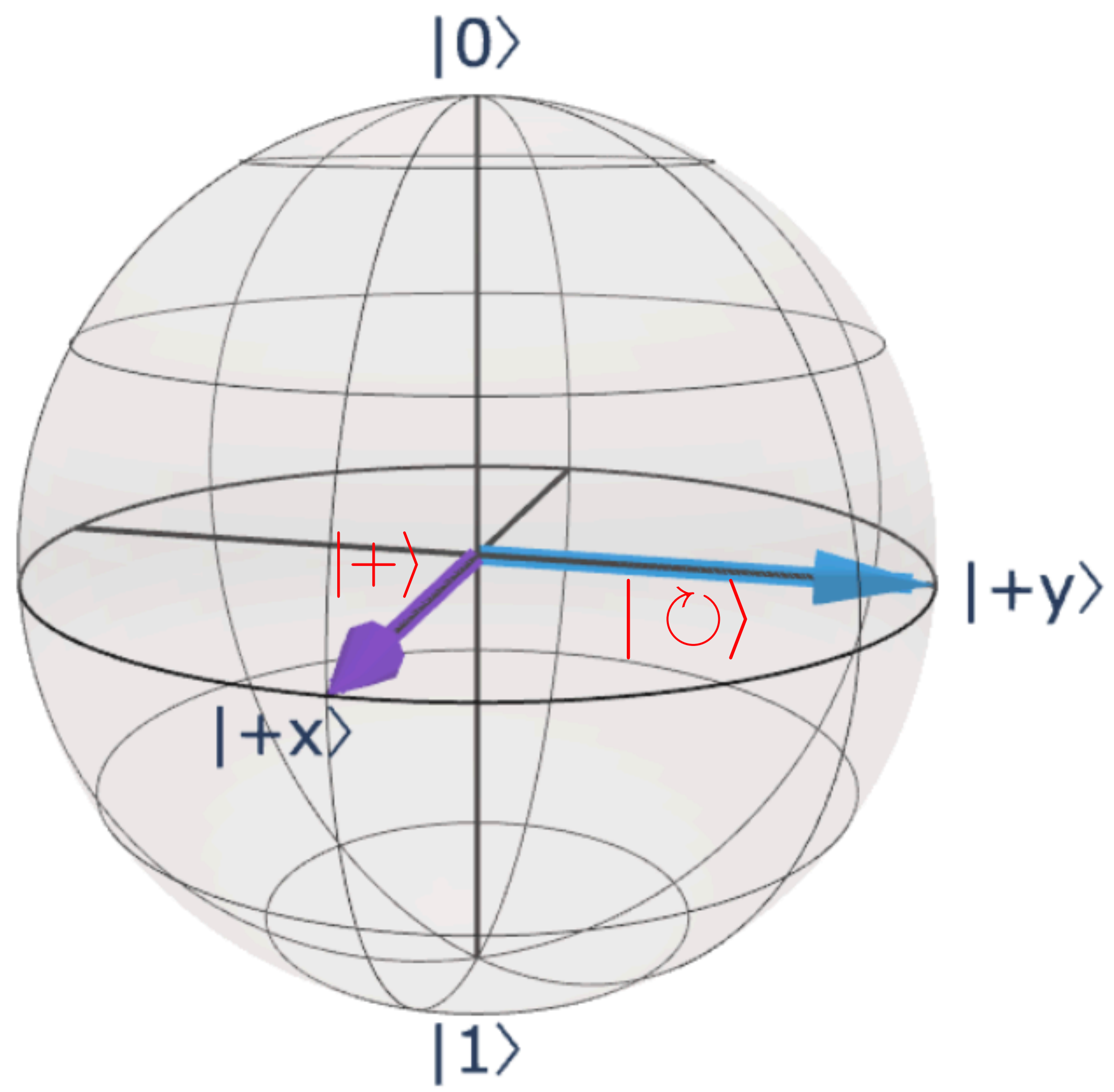
- Examples

$$|0\rangle \to (\theta = \phi = 0) \qquad |1\rangle \to (\theta = \pi, \phi = 0)$$

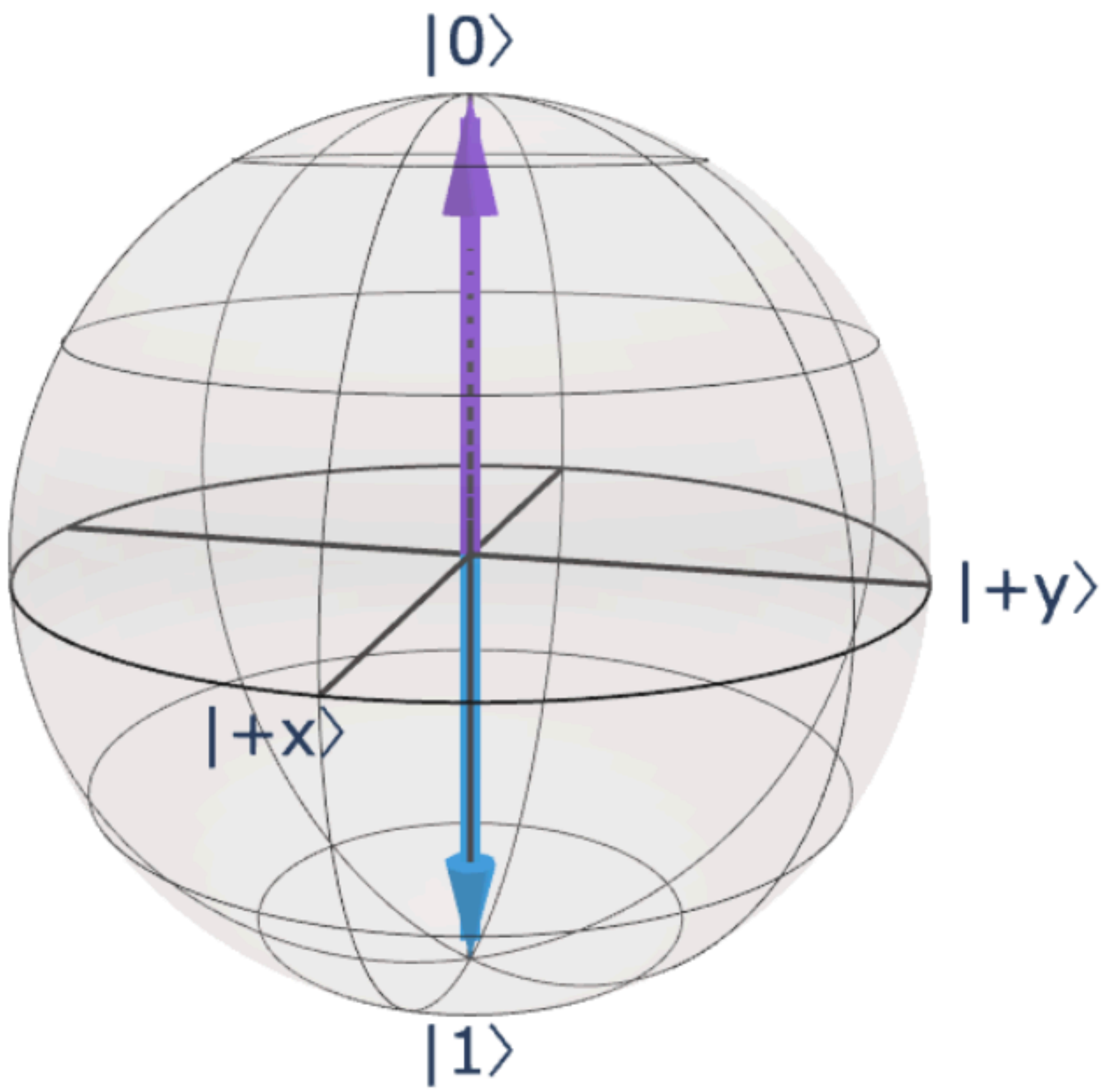
$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \to (\theta = \pi/2, \phi = 0)$$

$$|\circlearrowright\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) \to (\theta = \pi/2, \phi = \pi/2)$$

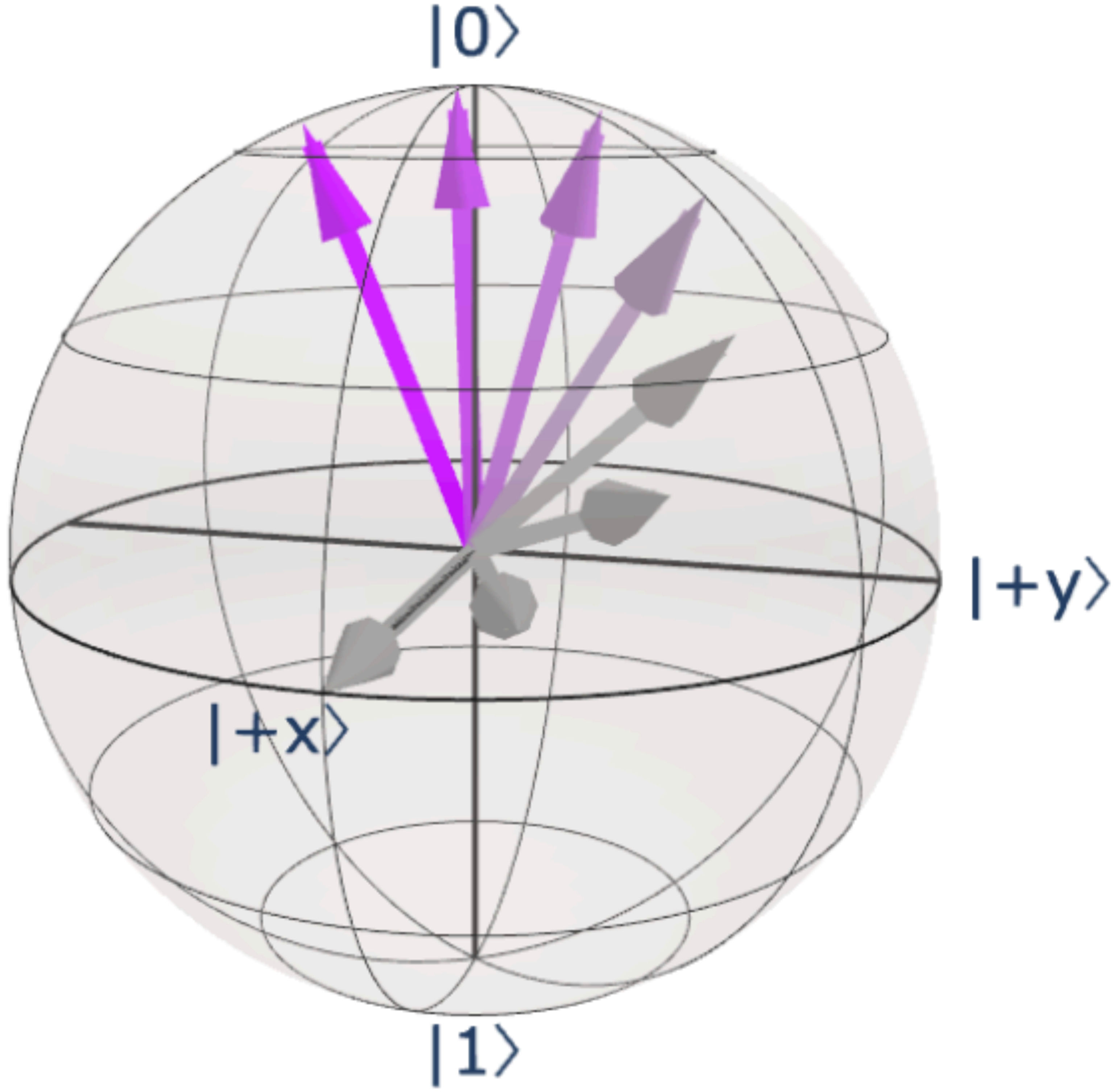




- A bit



- A qubit



- A bit

Encode 1 on a bit:

- **A qubit**

Encode $|\psi\rangle (= \alpha|0\rangle + \beta|1\rangle)$ on a qubit :

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad\qquad = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

- Unitary Operation / Quantum Gate

Generalization of real rotations in two-dim complex vector space

$$U^\dagger = U^{-1}, \quad UU^\dagger = U^\dagger U = I$$

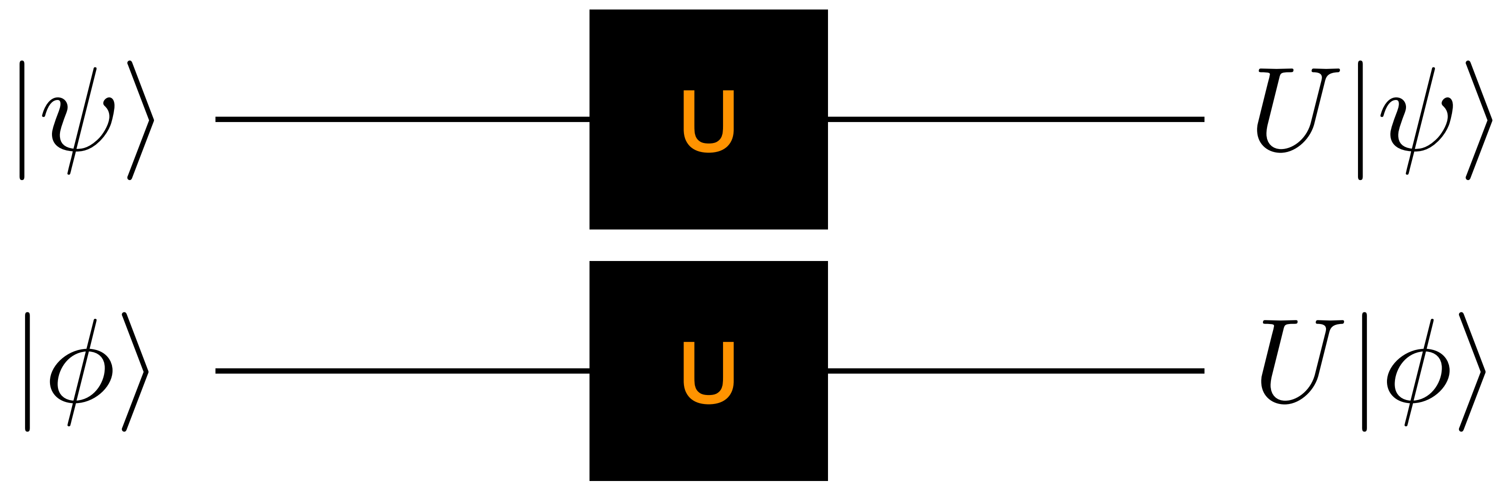
Linear

Reversible

Preserves the logical relation between the states

e.g. length preserving

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |\phi\rangle = \gamma|0\rangle + \delta|1\rangle \qquad \langle\phi|\psi\rangle = \begin{bmatrix}\gamma^* & \delta^*\end{bmatrix}\begin{bmatrix}\alpha \\ \beta\end{bmatrix}$$



$$\langle\phi|U^\dagger U|\psi\rangle = \langle\phi|\psi\rangle$$

- Unitary Operation / Quantum Gate

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \;\text{———}\boxed{X}\text{———}\; |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

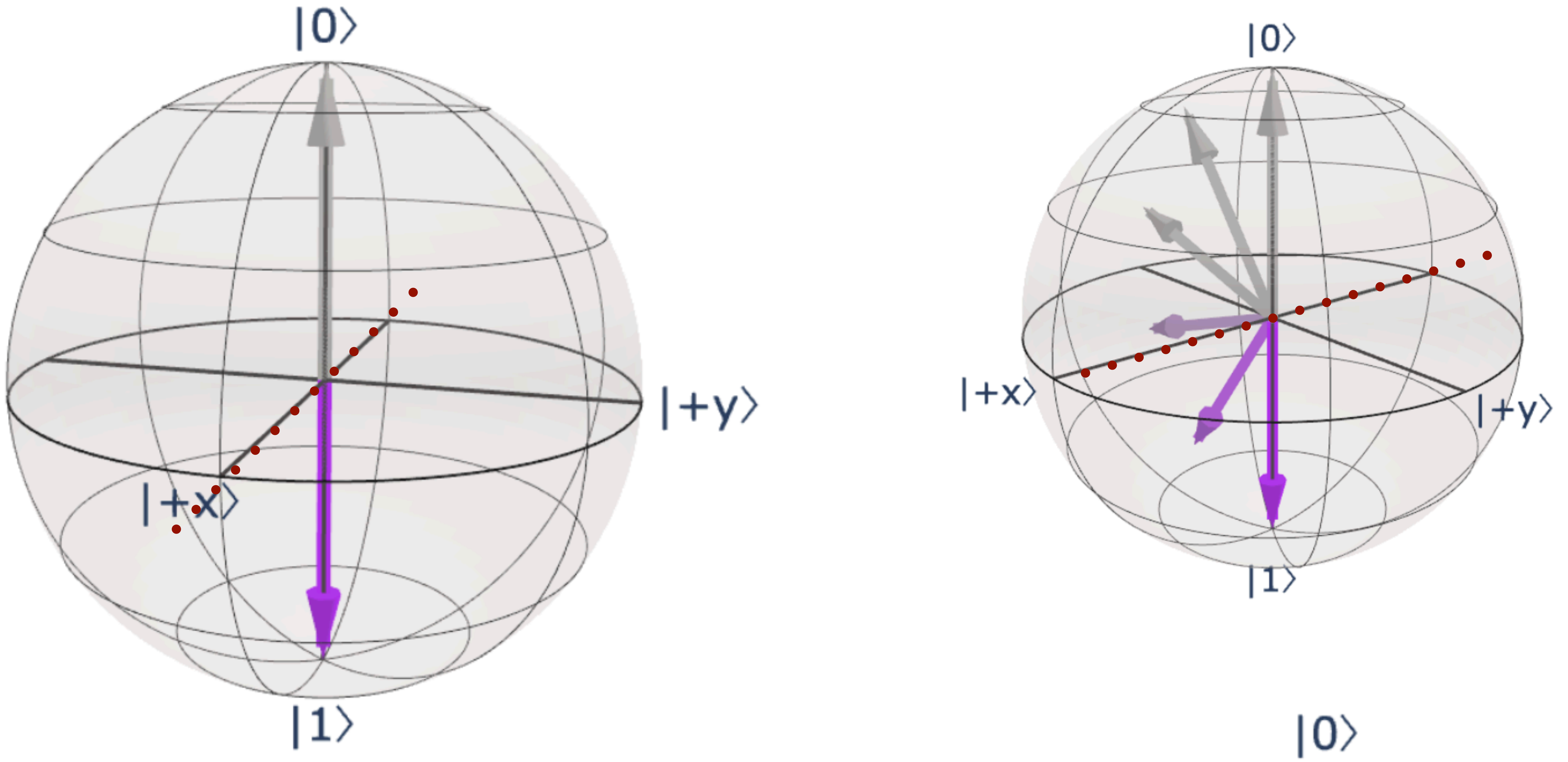
$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \;\text{———}\boxed{X}\text{———}\; |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \;\text{———}\boxed{H}\text{———}\; |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

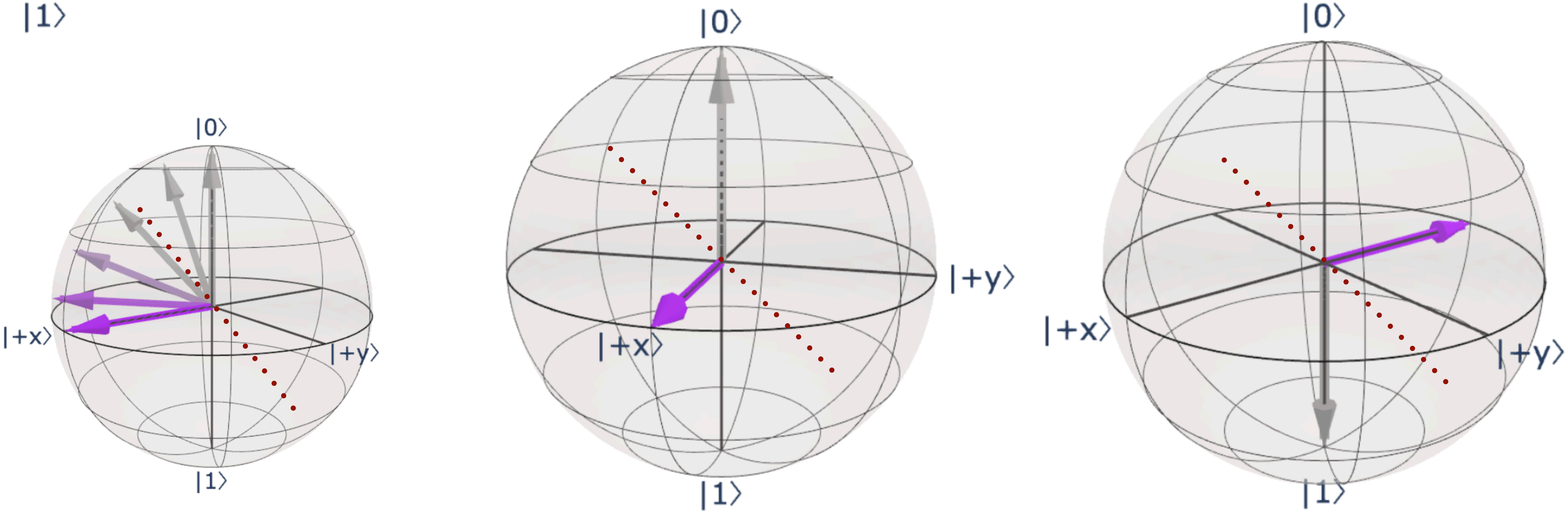
$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \;\text{———}\boxed{H}\text{———}\; |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

- X gate



- H gate



- Unitary Operation / Quantum Gate

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \xrightarrow{\quad X \quad} |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \xrightarrow{\quad X \quad} |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \xrightarrow{\quad X \quad} |\psi'\rangle = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

$$= \alpha|0\rangle + \beta|1\rangle \qquad\qquad = \alpha|1\rangle + \beta|0\rangle$$

$$= \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad\qquad = \alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- Phase

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$$

$$= |\alpha|e^{i\theta_0}|0\rangle + |\beta|e^{i\theta_1}|1\rangle$$

$$|\psi\rangle = \underbrace{e^{i\theta_0}}_{\text{Global phase}}\left(\ |\alpha||0\rangle + |\beta|\underbrace{e^{i\phi}}_{\text{Relative phase}}|1\rangle\ \right),\ \ \phi = \theta_1 - \theta_0$$

Global phase difference : Not detectable

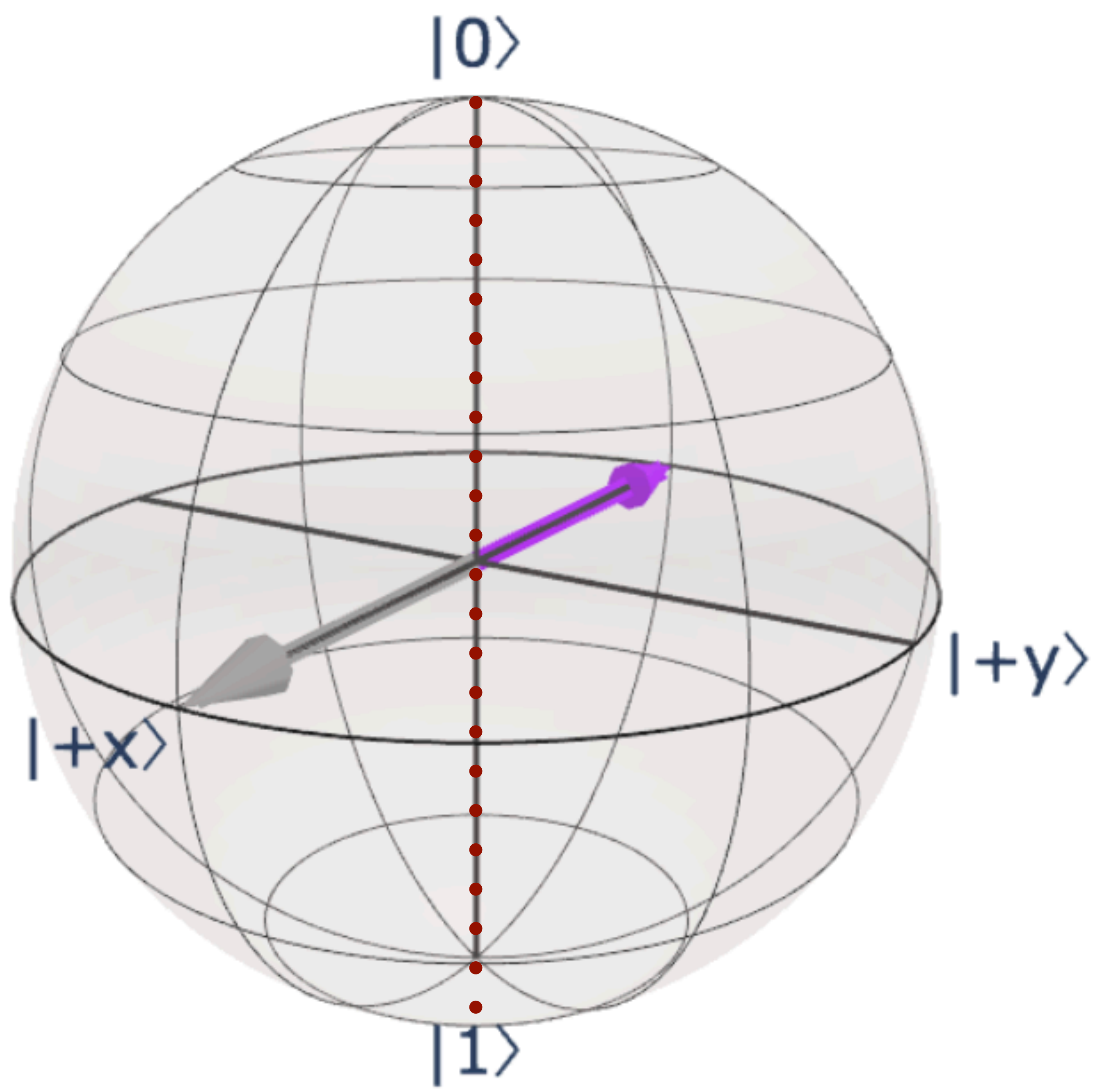
Relative phase difference : Detectable

- Phase gate Z

: π rotation around z axis

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \;—\boxed{Z}—\; e^{i0}|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$$

$$|1\rangle = \begin{bmatrix}0\\1\end{bmatrix} \;—\boxed{Z}—\; e^{i\pi}|1\rangle = \begin{bmatrix}0\\-1\end{bmatrix}$$



$$Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$$

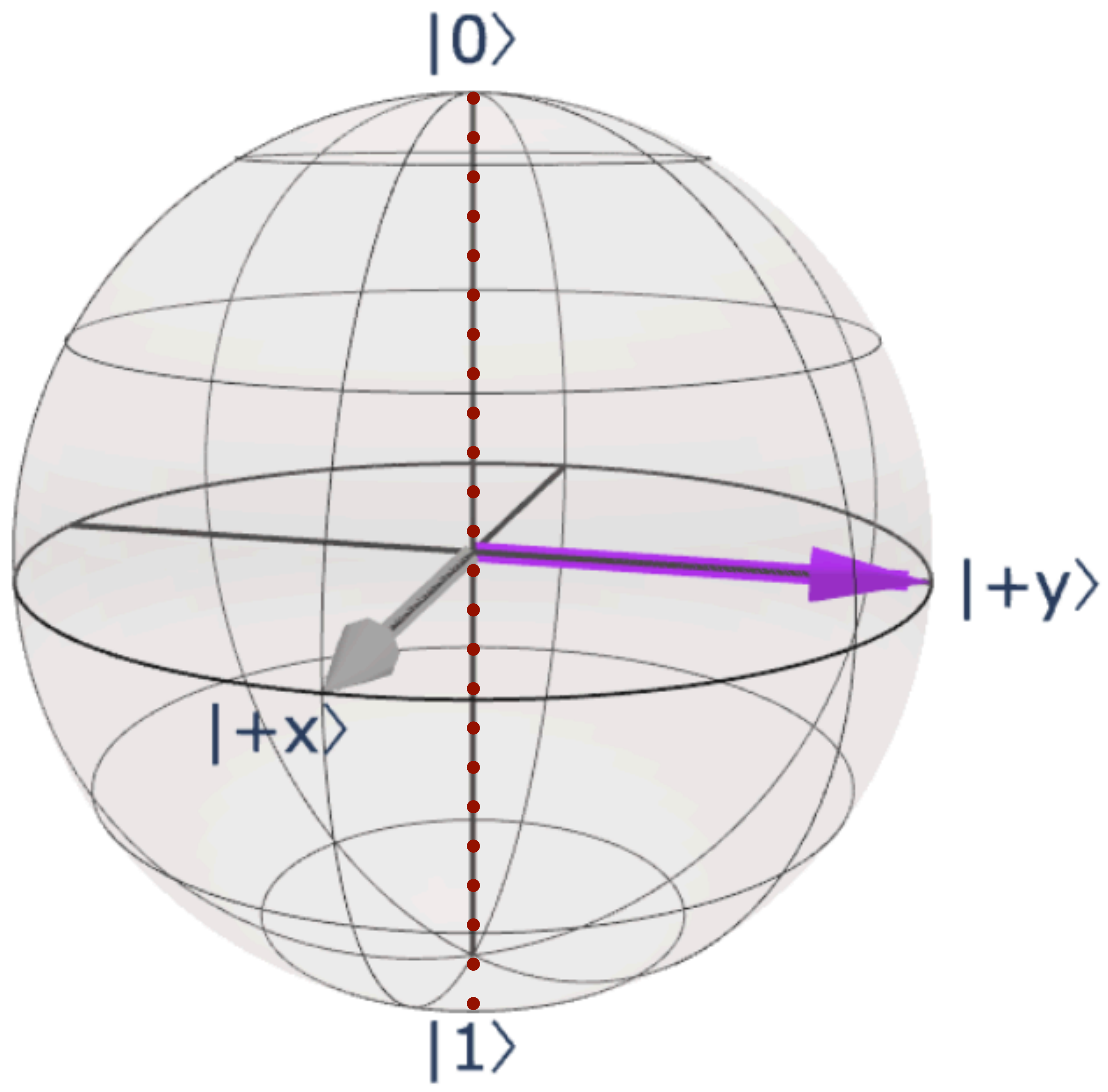
$$\begin{aligned} \underline{Z|+\rangle} &= \frac{1}{\sqrt{2}}Z(|0\rangle + |1\rangle) \\ &= \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi}|1\rangle) \\ &= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \underline{|-\rangle = \begin{bmatrix}1\\-1\end{bmatrix}} \end{aligned}$$

- Phase gate S

: $\frac{\pi}{2}$ rotation around z axis

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \xrightarrow{\;S\;} e^{i0}|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$$

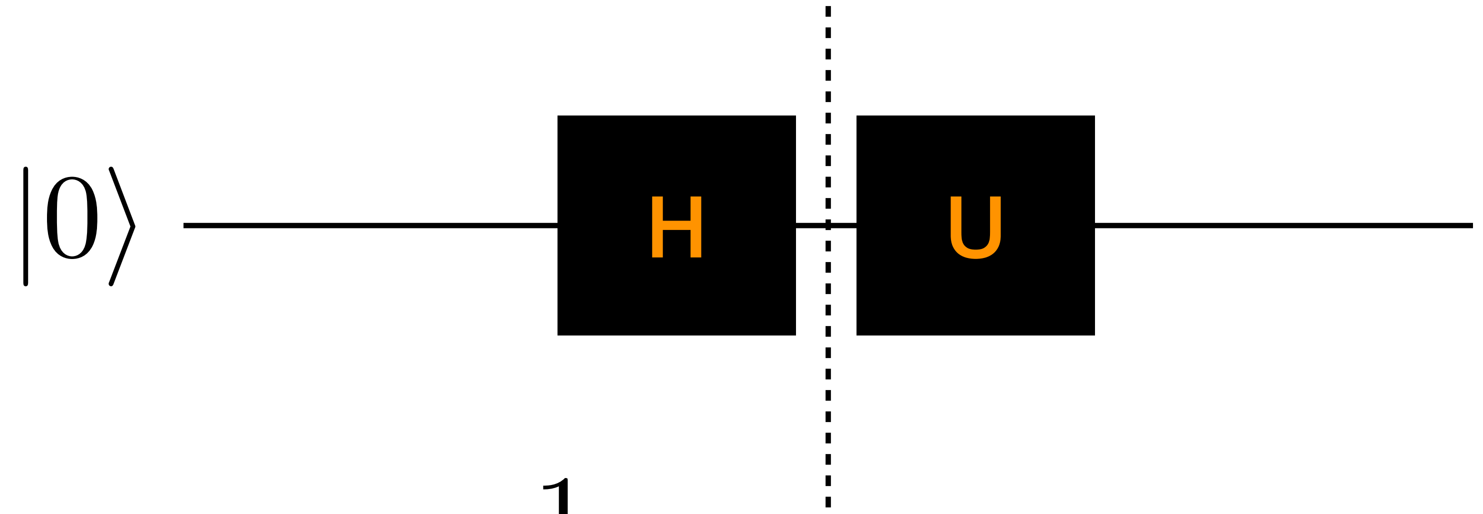
$$|1\rangle = \begin{bmatrix}0\\1\end{bmatrix} \xrightarrow{\;S\;} e^{i\frac{\pi}{2}}|1\rangle = \begin{bmatrix}0\\i\end{bmatrix}$$



$$S = \begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$$

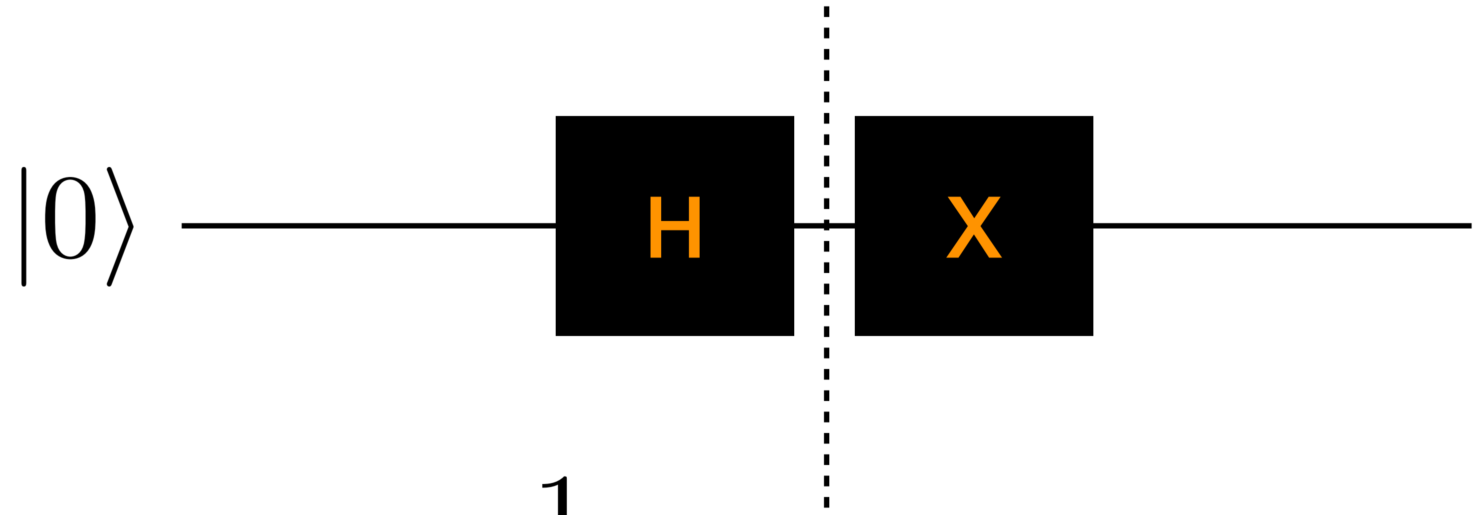
$$\begin{aligned} S|+\rangle &= \frac{1}{\sqrt{2}}S(|0\rangle + |1\rangle) \\ &= \frac{1}{\sqrt{2}}(|0\rangle + e^{i\pi/2}|1\rangle) \\ &= \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle) = \begin{bmatrix}1\\i\end{bmatrix} \end{aligned}$$

-Superposition and Quantum Parallelism



$$\frac{1}{\sqrt{2}}(U|0\rangle + U|1\rangle)$$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$



$$\frac{1}{\sqrt{2}}(X|0\rangle + X|1\rangle) = \frac{1}{\sqrt{2}}(|1\rangle + |0\rangle)$$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

- **Measurements** (non - unitary)

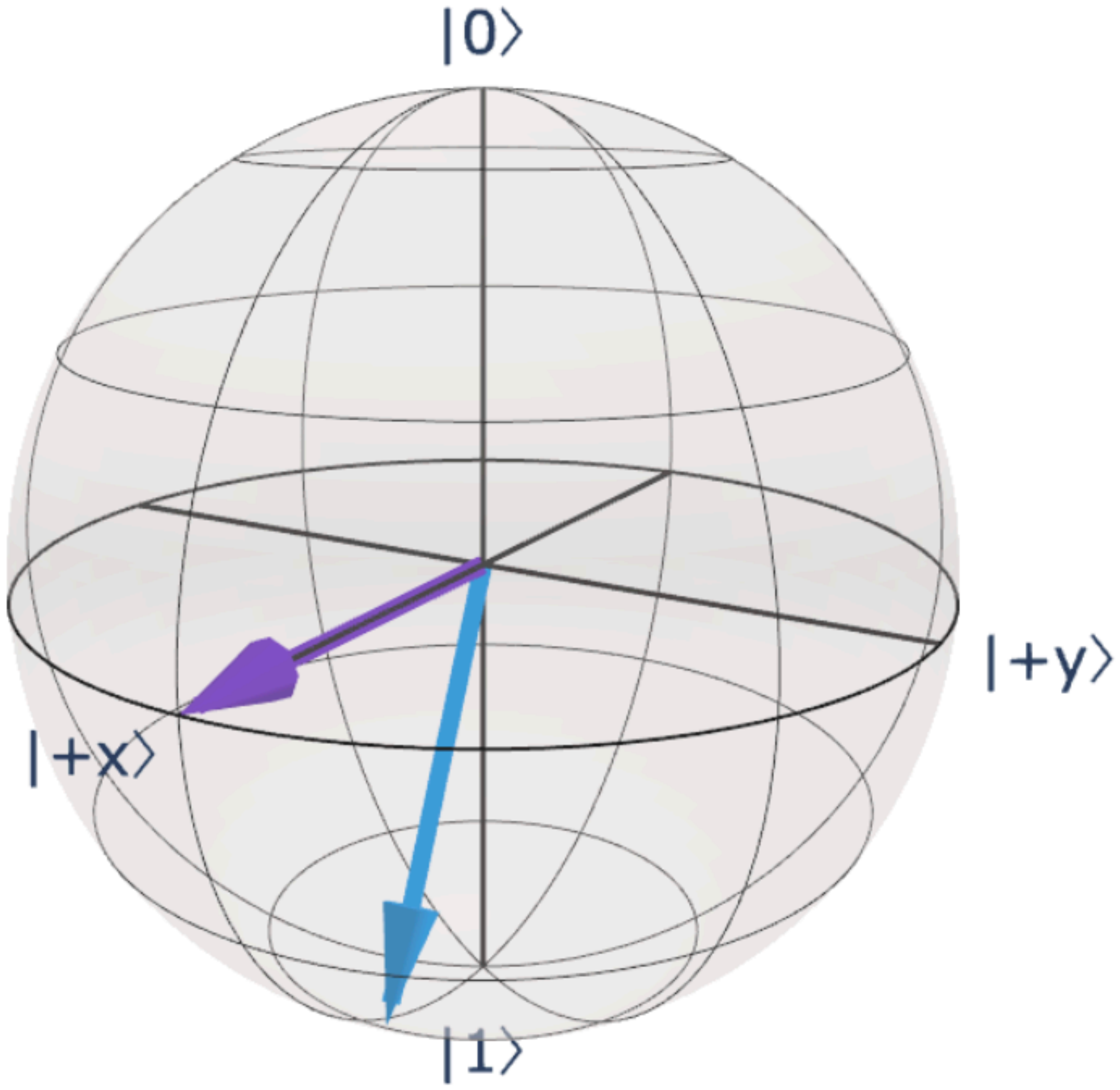
State of a qubit is not directly observable.

Obtain the information encoded in qubits by measurement in the computational basis.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \qquad \alpha, \beta \in \mathbb{C} \qquad |\alpha|^2 + |\beta|^2 = 1$$

$P_{|0\rangle} = |\alpha|^2$ If get 0 $\Rightarrow$ $|\psi\rangle = |0\rangle$

$P_{|1\rangle} = |\beta|^2$ If get 1 $\Rightarrow$ $|\psi\rangle = |1\rangle$



$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

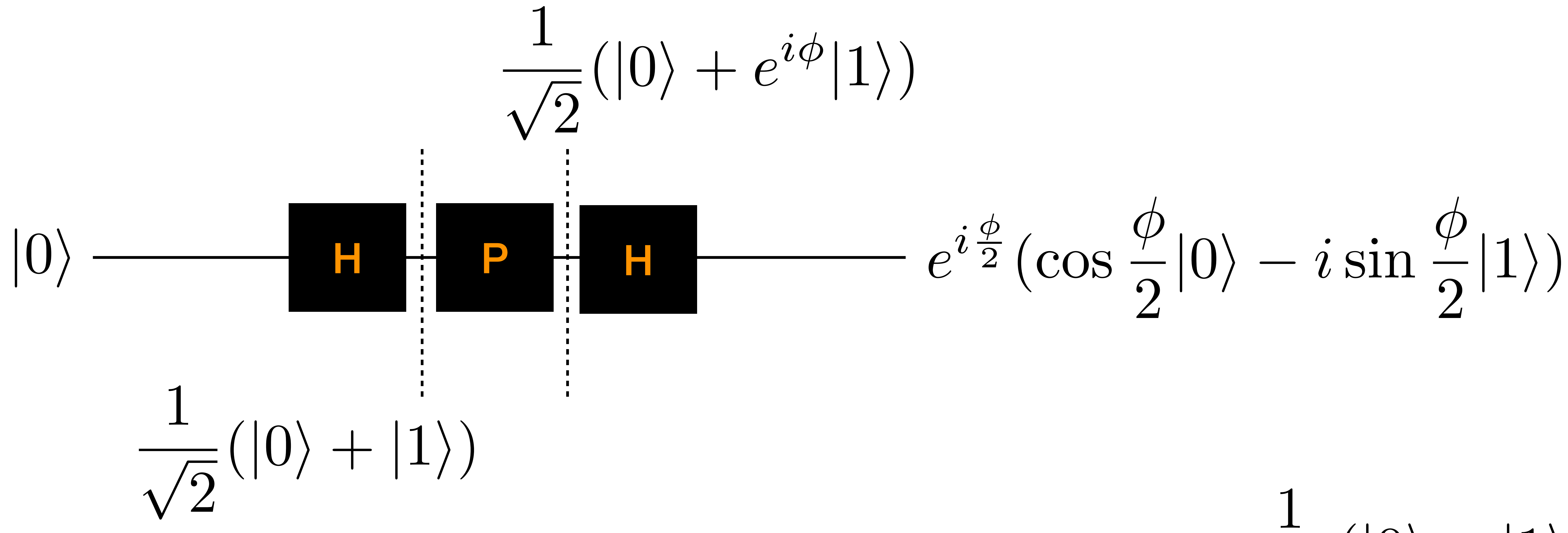
$$P_{|0\rangle} = P_{|1\rangle} = \frac{1}{2}$$

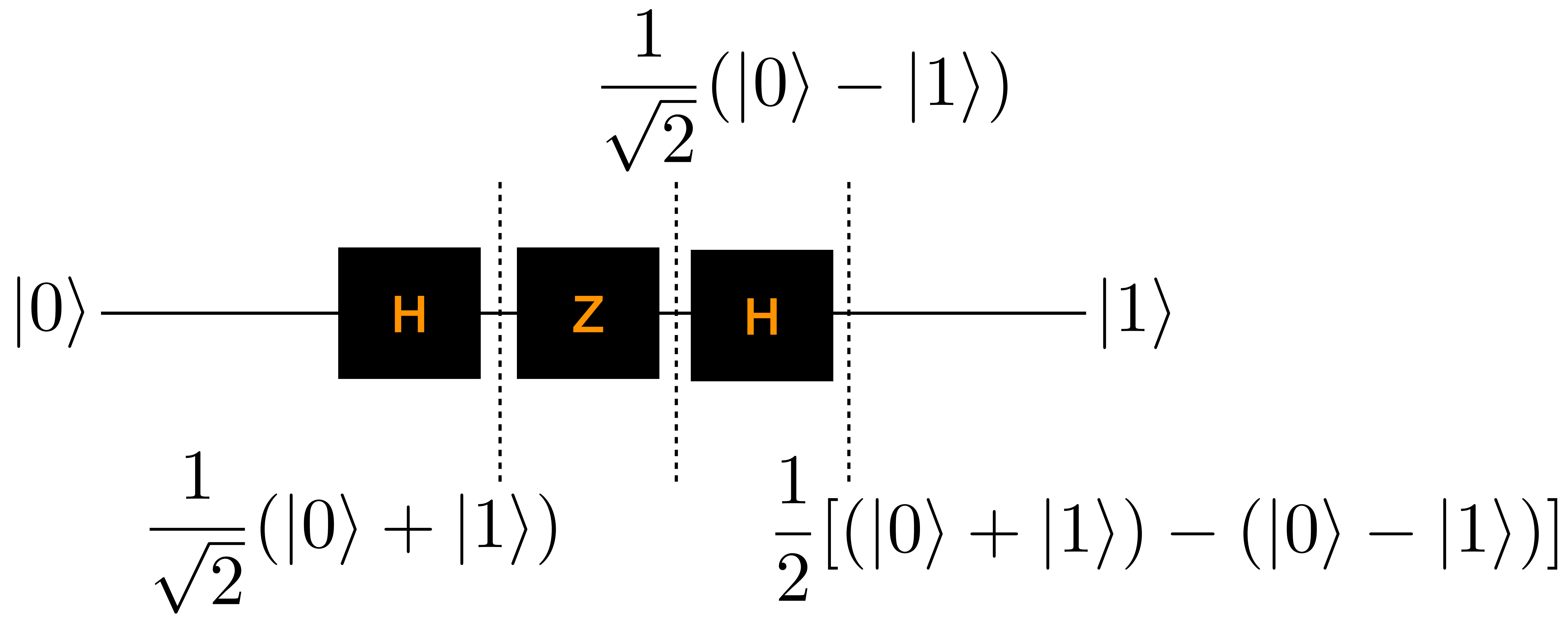
$$|\psi\rangle = (0.16 + i0.19)|0\rangle + (0.54 + i0.8)|1\rangle$$

$$= \begin{bmatrix}0.16 + i0.19\\0.54 + i0.8\end{bmatrix}$$

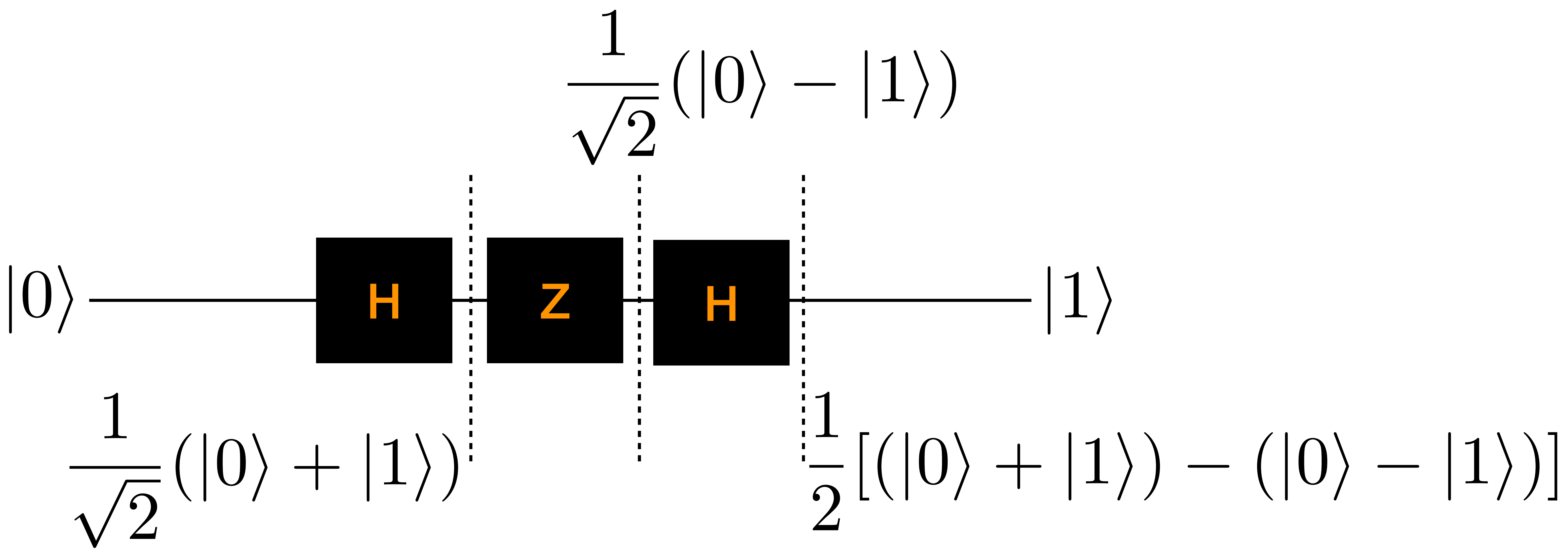
$$P_{|0\rangle} = 0.06 \qquad P_{|1\rangle} = 0.9$$

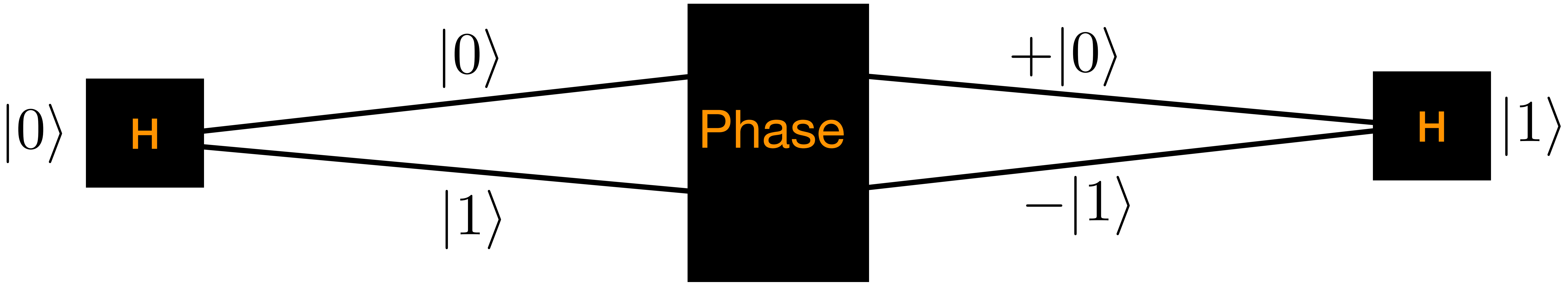
- Interference

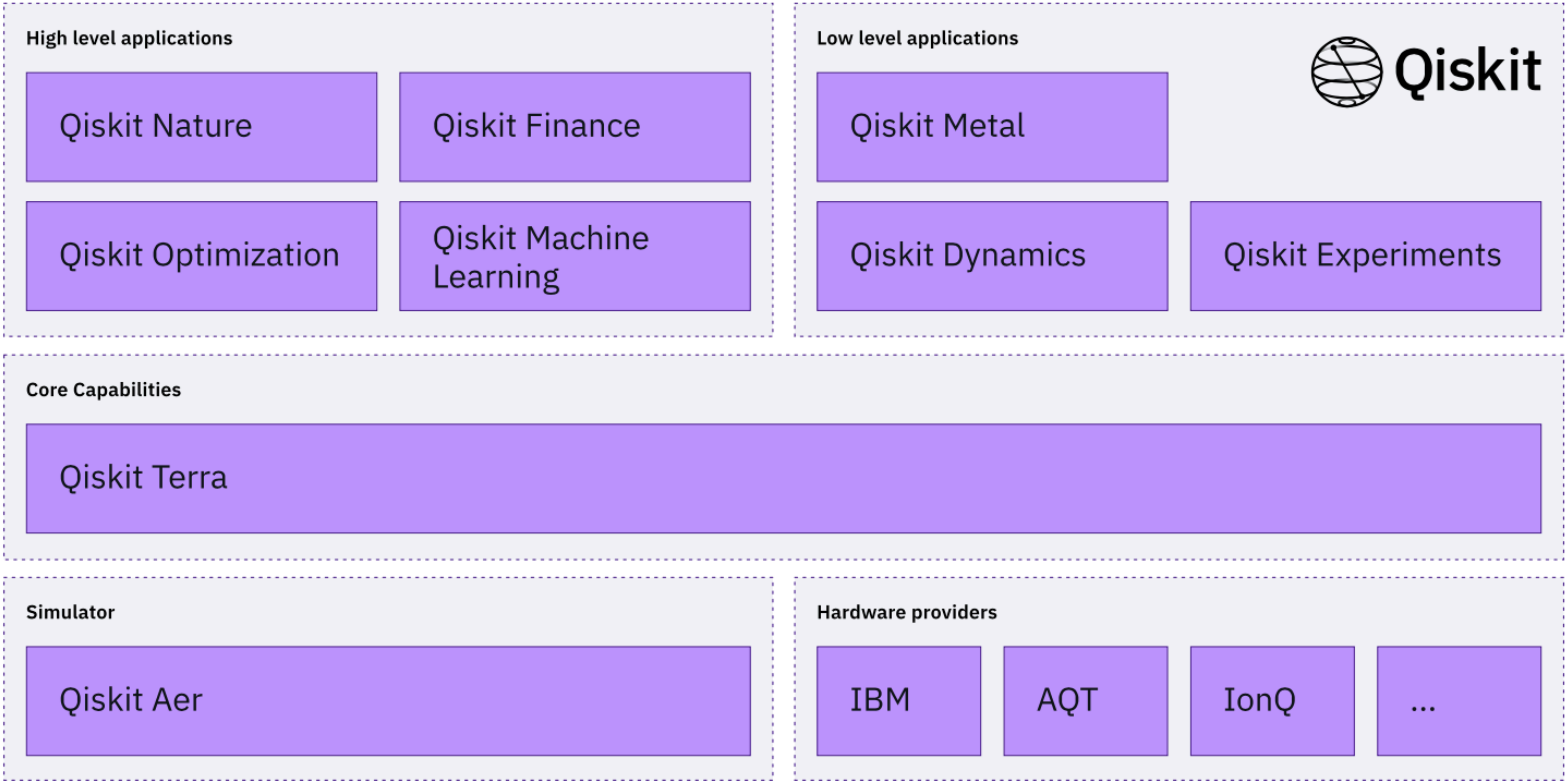




- Interference






High level applications
Qiskit Nature
Qiskit Finance
Qiskit Optimization
Qiskit Machine Learning
Low level applications
Qiskit
Qiskit Metal
Qiskit Dynamics
Qiskit Experiments
Core Capabilities
Qiskit Terra
Simulator
Qiskit Aer
Hardware providers
IBM
AQT
IonQ
...

- Simulating Quantum Circuits

Can simulate quantum systems using classical computers.

Limited to ~50 qubits

Qubits	State	Memory
2	$\alpha\|00\rangle + \beta\|01\rangle + \gamma\|10\rangle + \delta\|11\rangle$	64 bytes
4	16 terms	256 bytes
8	256 terms	4096 bytes (4 KB)
16	65356 terms	1048576 bytes (1MB)
32	4.3 billion	68719476736 bytes (64 GB)
50	1.2 quadrillion	18014398509481984 bytes (16 EB)

- IBM Quantum website

IBM Quantum website provides a cloud-based platform for using quantum computers.

You can program quantum computers using qiskit in a Jupyter Notebook environment or explore and execute circuits graphically using the circuit Composer.

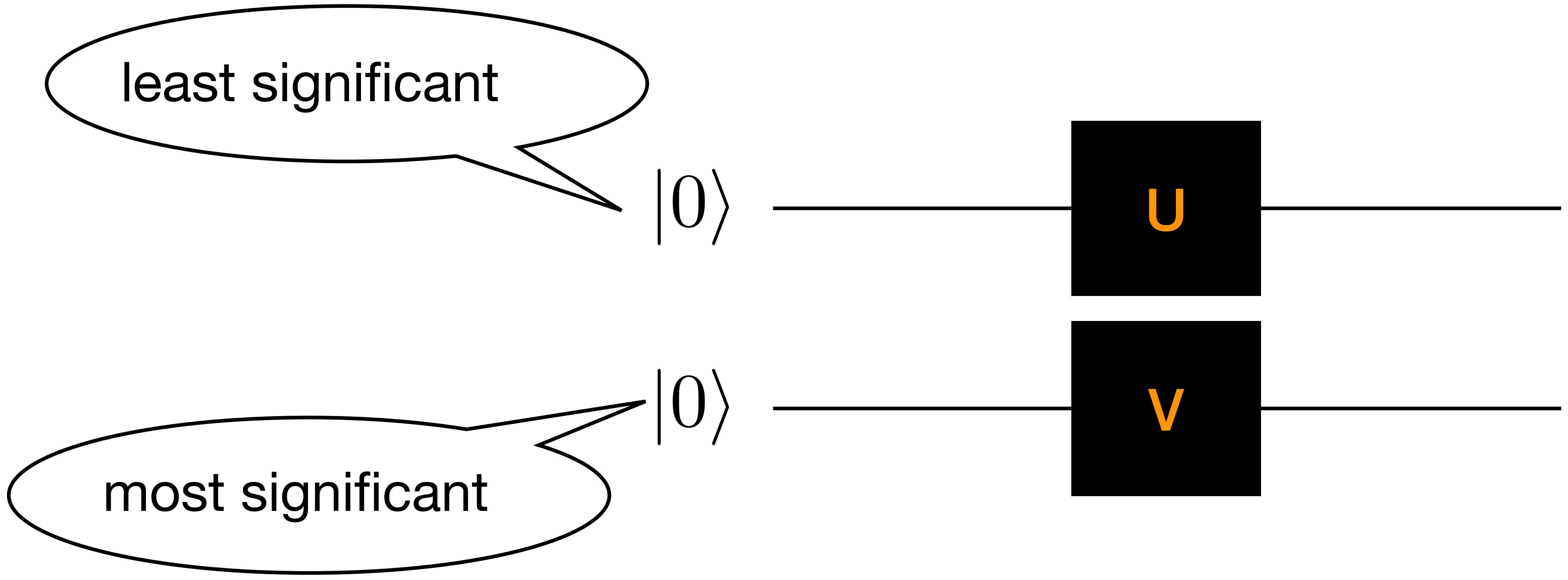
- Composite System : 2 qubit case

State of 2 qubit system is a unit (normalized) vector in a 2^2dim complex vector space.

The state of 2 qubits changes through unitary operations / quantum gates.

Ex) n = 2 : 4-dim $\quad |00\rangle, \quad |01\rangle, \quad |10\rangle, \quad |11\rangle$

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$



$$a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

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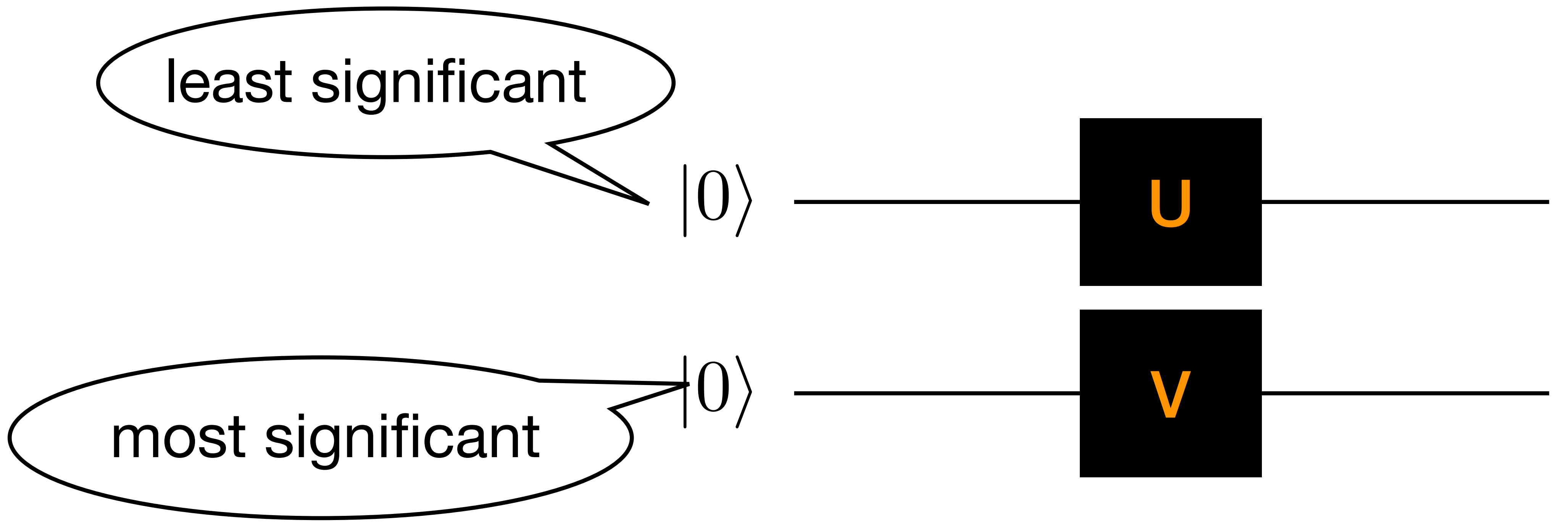
Ex) n = 2 : 4-dim $\quad |00\rangle, \quad |01\rangle, \quad |10\rangle, \quad |11\rangle \qquad \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \quad \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$



$$\alpha|0\rangle + \beta|1\rangle$$

$$\gamma|0\rangle + \delta|1\rangle$$

$$= a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

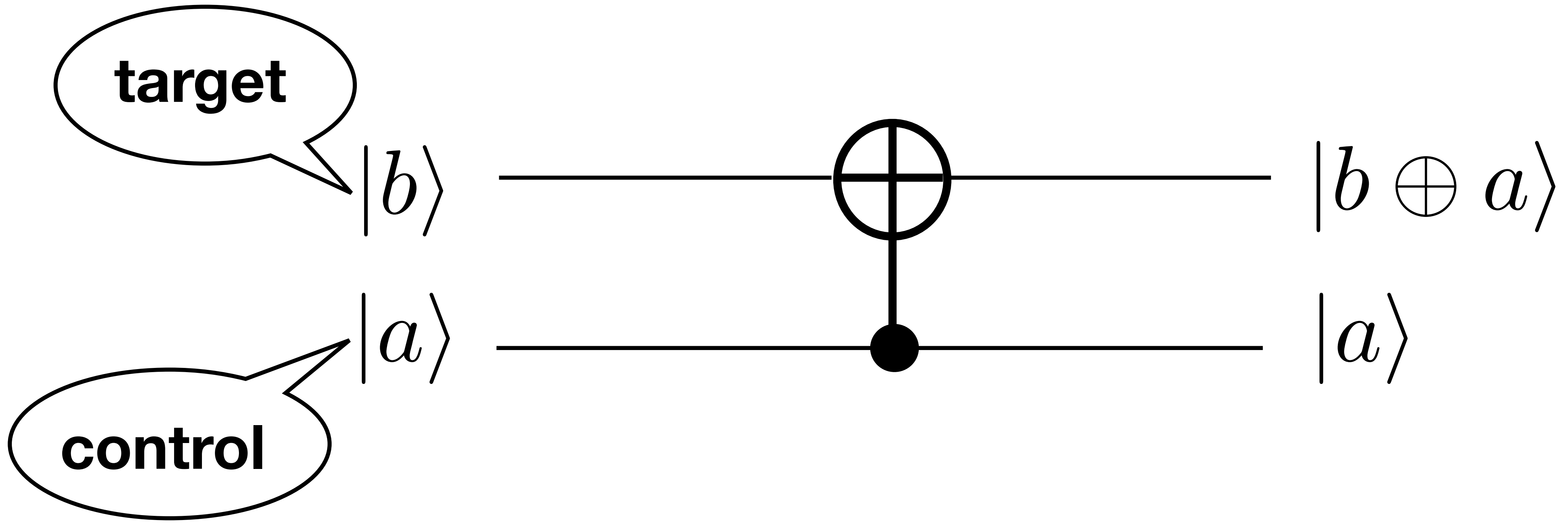
$$(\gamma|0\rangle + \delta|1\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle) = \gamma\alpha|00\rangle + \gamma\beta|01\rangle + \delta\alpha|10\rangle + \delta\beta|11\rangle$$

$$= a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

: Product state, each sub system can be prepared independently

- Two-Qubit Gate, CNOT

: **controlled-not**, or controlled-x gate



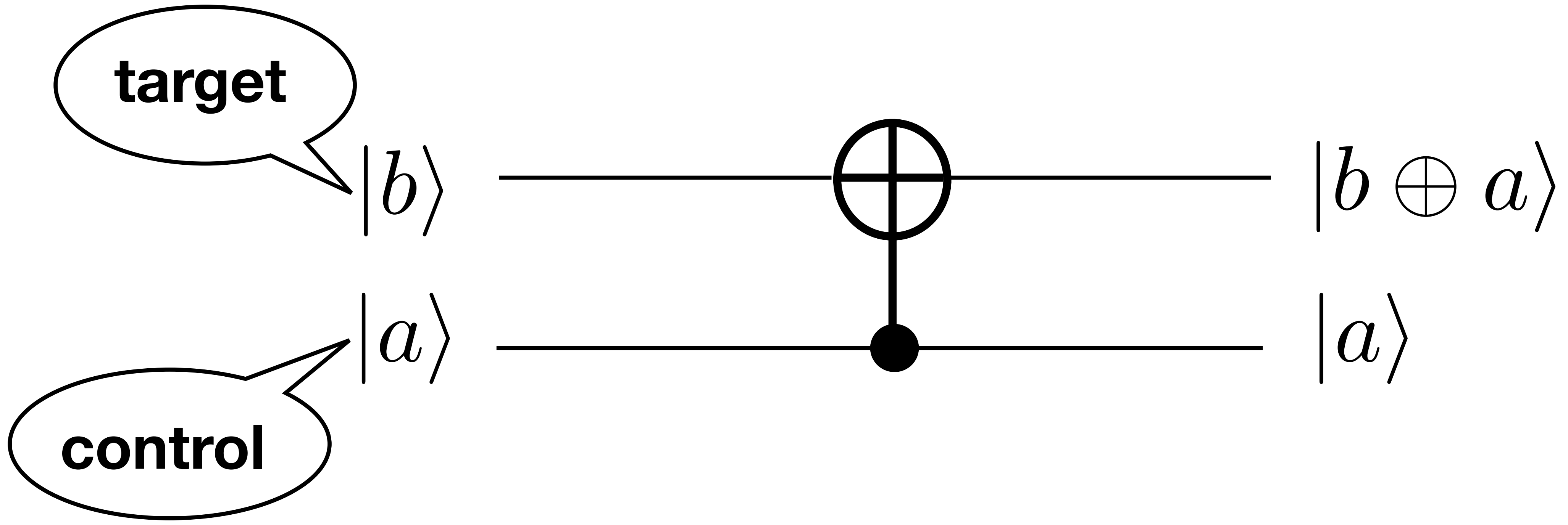
$$U_{CN}|ab\rangle = |a\ b \oplus a\rangle$$

$$|00\rangle \rightarrow |00\rangle; \quad |01\rangle \rightarrow |01\rangle; \quad |10\rangle \rightarrow |11\rangle; \quad |11\rangle \rightarrow |10\rangle$$

What happens if $|a\rangle = \dfrac{|0\rangle + |1\rangle}{\sqrt{2}}$ and $|b\rangle = |0\rangle$?

- Two-Qubit Gate, CNOT

: **controlled-not**, or controlled-x gate



$$U_{CN}|ab\rangle = |a \; b \oplus a\rangle$$

$$|00\rangle \to |00\rangle; \quad |01\rangle \to |01\rangle; \quad |10\rangle \to |11\rangle; \quad |11\rangle \to |10\rangle$$

What happens if $|a\rangle = \dfrac{|0\rangle + |1\rangle}{\sqrt{2}}$ and $|b\rangle = |0\rangle$?

$$\begin{aligned} U_{CN}|ab\rangle &= \frac{1}{\sqrt{2}} U_{CN}(|0\rangle + |1\rangle)|0\rangle \\ &= \frac{1}{\sqrt{2}}(U_{CN}|00\rangle + U_{CN}|10\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \end{aligned}$$

- Entanglement

Non-classical correlation between quantum systems

Like interference, **entanglement** is a key quantum phenomena for quantum computation.

Entangled states can be utilized as a resource to perform computational tasks that are impossible for classical systems : teleportation, superdense coding

ex) $$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

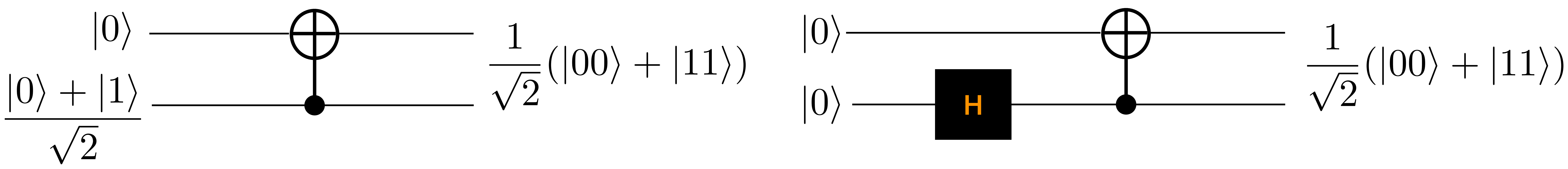
Bell State

: maximally entangled state, cannot be written as a product state.

describe the composite system completely

nothing is known about the individual subsystems

Quantum Circuit to construct bell state



Quantum teleportation