# Homework 5

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**Question 1.** (Fleisch and Kinneman 1.4)

(a) How much does the phase of an electromagnetic wave with frequency of 100 kHz change in 1.5 μs at a fixed location?

(b) What is the difference in phase of a mechanical wave with period of 2 seconds and speed of 15 m/s at two locations separated by 4 meters at some instant?

*Solution.* For part (a), it is given that

$$\Delta\phi = \omega\Delta t,$$

where $\omega = 2\pi f$. Therefore,

$$\Delta\phi = 2\pi f\Delta t = 2\pi(10^5\text{ Hz})(1.5\times 10^{-6}\text{ s}) = \boxed{0.3\pi\text{ radians.}}$$

For part (b),

$$\Delta\phi = k\Delta x,$$

where $k = \frac{2\pi}{\lambda}$. Since $v = \lambda/T$, we can rearrange and substitute values such that

$$\Delta\phi = k\Delta x = \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{vT}(\Delta x) = \frac{2\pi}{(15\text{ m/s})(2\text{ s})}(4\text{ m}) = \boxed{\frac{4}{15}\pi\text{ radians.}}$$

□

**Question 2.** (King 5.1) A transverse wave traveling along a string is described by the function $y = 15\cos(0.25x + 75t)$, where $x$ and $y$ are in millimeters and $t$ is in seconds. Find the amplitude, wavelength, frequency and velocity of the wave. In what direction is the wave traveling?

*Solution.* Given that

$$y(x,t) = A\cos(kx + \omega t),$$

by basic substitution we get that $\boxed{A = 15}$, $k = 0.25$, and $\omega = 75$. Since $k = \frac{2\pi}{\lambda}$, by rearranging the equation we get that $\boxed{\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{0.25 \text{ rad mm}^{-1}} = 8\pi \text{ mm.}}$ Additionally, since $\omega = 2\pi f$,

$$\boxed{f = \frac{\omega}{2\pi} = \frac{75 \text{ rad / s}}{2\pi} = \frac{75}{2\pi} \text{ Hz.}}$$

Since we now know $\lambda$ and $f$, the velocity is hence

$$\boxed{v = \lambda f = (8\pi \text{ mm})\left(\frac{75}{2\pi} \text{ Hz}\right) = 300 \text{ mm/s.}}$$

Lastly since both the $kx$ and $\omega t$ term are positive, this implies that the wave is moving to the left (negative $x$-direction). $\square$

**Question 3.** Consider a piano string which plays middle C (262 Hz). The mass density of the string is 9 g/cm$^3$. Suppose the string has a diameter of 0.5 mm and its length is 100 cm. What is the tension in the string?

*Solution.* It is true that

$$v = \sqrt{\frac{F_T}{\mu}}, \tag{1}$$

where $\mu$ is the mass density. It is true that $\mu = \rho A$, where $A$ is the cross sectional area. The cross sectional area is a sphere with area $A$ equal to

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{0.5 \text{ mm}}{2}\right)^2 = 0.0625\pi \text{ mm}^2 \times \frac{(1 \text{ cm})^2}{(10 \text{ mm})^2} = 6.25\pi \times 10^{-4} \text{ cm}^2.$$

Therefore,

$$\mu = \rho A = (9 \text{ g/cm}^3)(6.25\pi \times 10^{-4}\text{cm}^2) = 5.625\pi \times 10^{-3} \text{ g/cm}.$$

Let us rearrange (1) in terms of $F_T$ as such:

$$F_T = \mu v^2.$$

Since the string is fixed at its two ends, the wavelength $\lambda$ is

$$\lambda = 2L = 2(100 \text{ cm}) = 200 \text{ cm}.$$

This implies that the velocity $v$ of the string is

$$v = \lambda f = (200 \text{ cm})(262 \text{ Hz}).$$

Therefore, substituting this all in gives us,

$$F_T = \mu v^2 = (5.625\pi \times 10^{-3} \text{ g/cm})[(200 \text{ cm})(262 \text{ Hz})]^2$$
$$\boxed{\approx 4.85 \times 10^7 \text{ g cm/s}^2 \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{1 \text{ m}}{100 \text{ cm}} = 485 \text{ N.}}$$

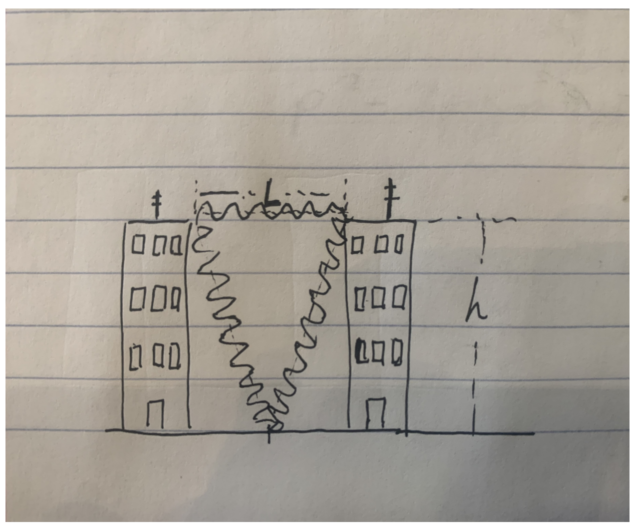
$\square$

**Question 4.** (After Ohanian Vol II, Chapter 39, Problem 39.23) Radio waves (98.0 MHz; $\lambda =$ 3.06 m) from a transmitter atop a tall building to a receiver atop of another equally tall ($h = 60$ m) building may follow either a direct path or else an indirect path involving a reflection on the ground. In this problem, you will explore configurations where this can lead to destructive interference of the two waves and a consequent fading of the radio signal.

(a) If the distance between the buildings is $L$, what are the ray path lengths for the direct and for the indirect path (assume flat, bare ground between the buildings)? Make a sketch of the situation.

(b) Formulate an equation which expresses a condition for destructive interference at the receiver. Note: Because reflected waves suffer a phase shift, you will have to add $\lambda/2$ to the length of the indirect path (i.e. a phase shift of $\pi$ for the indirect path due to the electrodynamics of reflection) for your criterion to be correct.

(c) Solve for $L$ under the assumption that $L >> 100$ m. What is the maximum finite distance $L$ in meters between the buildings that will lead to destructive interference?




*Solution.* For part (a), let $h = 60$ m be the height of both buildings and $L$ be the horizontal distance between them. There are two primary paths for the radio waves. In the direct path, both the transmitter and receiver are at the same height $h$. Therefore, the direct path $d_1$ is simply the horizontal distance between the buildings $d_1 = L$.

The indirect path involves a reflection off the ground. Due to the symmetry of the equal heights ($h$), the reflection point occurs exactly at the midpoint $L/2$. This path forms two congruent right-angled triangles with base $L/2$ and height $h$. Using the Pythagorean theorem, the direct path $d_2$ is

$$d_2 = 2\sqrt{\left(\frac{L}{2}\right)^2 + h^2} = \sqrt{L^2 + 4h^2}.$$

For part (b), destructive interference occurs when the total phase difference between the two waves at the receiver is an odd multiple of $\pi$. Hence, the condition for destructive interference is that the effective path difference must be an odd multiple of half-wavelengths:

$$(d_2 - d_1) + \frac{\lambda}{2} = \left(m + \frac{1}{2}\right)\lambda \quad \text{for } m = 0, 1, 2, \ldots$$

Substituting the expressions for $d_1$ and $d_2$:

$$\sqrt{L^2 + 4h^2} - L + \frac{\lambda}{2} = m\lambda + \frac{\lambda}{2} \Rightarrow \boxed{\sqrt{L^2 + 4h^2} - L = m\lambda.}$$

For part (c), let us use a Taylor approximation $(1 + x)^n \approx 1 + nx$:

$$\sqrt{L^2 + 4h^2} = L\sqrt{1 + \frac{4h^2}{L^2}} \approx L + \frac{2h^2}{L}.$$

Substituting this approximation back into our interference condition from part (b):

$$\left(L + \frac{2h^2}{L}\right) - L \approx m\lambda \tag{2}$$

$$\frac{2h^2}{L} \approx m\lambda \implies L \approx \frac{2h^2}{m\lambda} \tag{3}$$

To find the maximum finite distance $L$, we choose the smallest non-zero integer for $m$ (since $m = 0$ would imply $L \to \infty$). Setting $m = 1$:

$$L_{\text{max}} = \frac{2h^2}{\lambda} \tag{4}$$

Using the values $h = 60$ m and $\lambda = 3.06$ m:

$$L_{\text{max}} = \frac{2(60)^2}{3.06} = \frac{7200}{3.06} \approx 2352.94 \text{ m} \tag{5}$$

The maximum finite distance $L$ that will lead to destructive interference is approximately $\boxed{\text{2353 meters.}}$ □

**Question 5.** (After Bernstein problem 4.2) Chemical processes typically involve energies on the order of 1 eV. What, then, is the typical wavelength of light emitted in the course of chemical reactions? What part of the electromagnetic spectrum does this fall into? Nuclear processes involve energies on the order of 1 MeV. Where in the spectrum of electromagnetic radiation are the photons that may be emitted in a nuclear reaction?

*Solution.* Using the relation $E = \frac{hc}{\lambda}$, where

$$hc = (6.63 \times 10^{-34}\ \text{J s})(3 \times 10^{8}\ \text{m/s}) \times \frac{10^{9}\ \text{nm}}{1\text{m}} \times \frac{1\ \text{eV}}{1.6 \times 10^{-19}\ \text{J}} \approx 1240\ \text{eV nm},$$

$$\boxed{\lambda = \frac{1240\ \text{eV} \cdot \text{nm}}{1\ \text{eV}} = 1240\ \text{nm}.}$$

The wavelength 1240 nm lies in the *infared* region of the electromagnetic spectrum. In nuclear processes where $E = 1\ \text{MeV} = 1 \times 10^{6}\ \text{eV}$,

$$\boxed{\lambda = \frac{1240\ \text{eV} \cdot \text{nm}}{10^{6}\ \text{eV}} = 1.24 \times 10^{-3}\ \text{nm} = 1.24 \times 10^{-12}\ \text{m} = 1.24\ \text{pm}.}$$

The wavelength 1.24 pm lies in the *gamma* region of the electromagnetic spectrum. □