Homework 2

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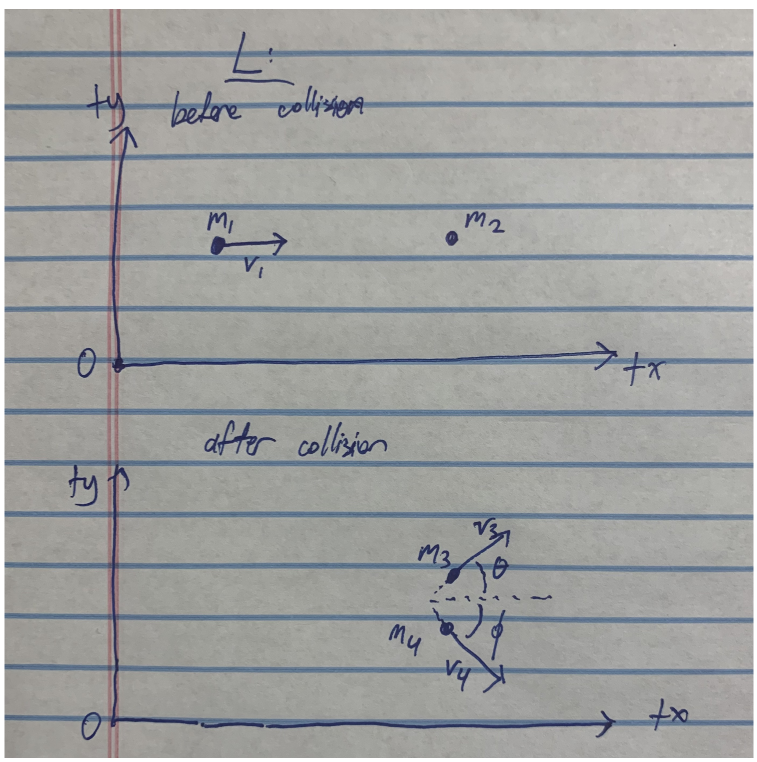
Question K&K 6.10. In the L system, a particle of mass m_1 with kinetic energy E_1 strikes a particle of mass m_2 initially at rest. A nuclear reaction occurs, with the release of a particle of mass m_3 at angle θ and energy E_3, and a particle of mass m_4 at angle ϕ and energy E_4. (Angles are measured in L from the incident line.) Neither ϕ nor E_4 are measured.

Find an expression for the energy Q released in the reaction in terms of the masses, energies E_1 and E_3, and the angle θ.



Solution. To find the energy Q released, let us use the conservation of mechanical energy to get the equation

$$E_1 = E_3 + E_4 + Q.$$

Thus, Q is rearranged as

$$Q = E_1 - E_3 - E_4 = \frac{1}{2}(m_1 v_1^2 - m_3 v_3^2 - m_4 v_4^2), \tag{1}$$

where v_1, v_3, and v_4 are the velocities of m_1, m_3, and m_4, respectively. In this system, momentum is also conserved. Since this is a two-dimensional collision, we are given two equations

$$m_1 v_{1,x} = m_1 v_1 = m_3 v_3 \cos\theta + m_4 v_4 \cos\phi, \tag{2}$$
$$m_1 v_{1,y} = 0 = m_3 v_3 \sin\theta - m_4 v_4 \sin\phi. \tag{3}$$

Next, let us find v_4, so that we can substitute it into (1). When analyzing v_4 by component by rearranging (2) and (3), we get

$$v_{4,x} = v_4 \cos\phi = \frac{m_1 v_1 - m_3 v_3 \cos\theta}{m_4},$$
$$v_{4,y} = v_4 \sin\phi = \frac{m_3 v_3 \sin\theta}{m_4}.$$

Then,

$$\begin{aligned} v_4^2 = v_{4,x}^2 + v_{4,y}^2 &= \left(\frac{m_1 v_1 - m_3 v_3 \cos\theta}{m_4}\right)^2 + \left(\frac{m_3 v_3 \sin\theta}{m_4}\right)^2 \\ &= \frac{(m_1 v_1)^2 - 2 m_1 m_3 v_1 v_3 \cos\theta + (m_3 v_3 \cos\theta)^2 + (m_3 v_3 \sin\theta)^2}{m_4^2} \\ v_4^2 &= \frac{(m_1 v_1)^2 - 2 m_1 m_3 v_1 v_3 \cos\theta + (m_3 v_3)^2}{m_4^2}. \end{aligned}$$

Putting this into (1) gives

$$\begin{aligned} Q &= \frac{1}{2}\left(m_1 v_1^2 - m_3 v_3^2 - m_4 v_4^2\right) \\ &= \frac{1}{2}\left(m_1 v_1^2 - m_3 v_3^2 - m_4 \cdot \frac{(m_1 v_1)^2 - 2 m_1 m_3 v_1 v_3 \cos\theta + (m_3 v_3)^2}{m_4^2}\right) \\ &= \frac{1}{2}\left(m_1 v_1^2 - m_3 v_3^2\right) - \frac{(m_1 v_1)^2 - 2 m_1 m_3 v_1 v_3 \cos\theta + (m_3 v_3)^2}{2 m_4} \\ &= E_1 - E_3 - \frac{1}{2 m_4}\left(m_1^2 v_1^2 - 2 m_1 m_3 v_1 v_3 \cos\theta + m_3^2 v_3^2\right). \end{aligned}$$

Now substitute

$$v_1^2 = \frac{2E_1}{m_1}, \qquad v_3^2 = \frac{2E_3}{m_3},$$

so that

$$m_1^2 v_1^2 = 2 m_1 E_1, \qquad m_3^2 v_3^2 = 2 m_3 E_3, \qquad v_1 v_3 = \sqrt{\frac{2E_1}{m_1}}\sqrt{\frac{2E_3}{m_3}}.$$

Therefore,

$$Q = E_1 \left(1 - \frac{m_1}{m_4}\right) - E_3 \left(1 + \frac{m_3}{m_4}\right) + \frac{2\sqrt{m_1 m_3 E_1 E_3}}{m_4} \cos\theta.$$

$\square$

Question K&K 12.7. *Note:* S refers to an inertial system x, y, z, t and S' refers to an inertial system x', y', z', t', moving along the x axis with speed v relative to S. The origins coincide at $t = t' = 0$. For numerical work, take $c = 3 \times 10^8\,\mathrm{m/s}$.

Assuming that $v = 0.6c$, find the coordinates in S' of the following events:

(a) $x = 4\,\mathrm{m},\ t = 0\,\mathrm{s}$.

(b) $x = 4\,\mathrm{m},\ t = 1\,\mathrm{s}$.

(c) $x = 1.8 \times 10^8\,\mathrm{m},\ t = 1\,\mathrm{s}$.

(d) $x = 10^9\,\mathrm{m},\ t = 2\,\mathrm{s}$.

Solution. Since the motion is along the x axis, $y = z = 0$, so $y' = y = 0$ and $z' = z = 0$ for all events. The Lorentz transformation (for S' moving at speed v along $+x$) is

$$\begin{cases} x' = \gamma(x - vt), \\ y' = y, \\ z' = z, \\ t' = \gamma\left(t - \dfrac{v}{c^2}x\right). \end{cases}$$

The Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - (0.6)^2}} = \frac{5}{4}.$$

Also note that

$$v = 0.6c = 0.6(3 \times 10^8) = 1.8 \times 10^8 \text{ m/s}, \qquad \frac{v}{c^2} = \frac{0.6}{c} = \frac{0.6}{3 \times 10^8} = 2 \times 10^{-9} \text{ s/m}.$$

(a) $x = 4\,\mathrm{m},\ t = 0\,\mathrm{s}$.

$$x' = \gamma(x - vt) = \frac{5}{4}(4 - 0) = 5 \text{ m},$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \frac{5}{4}\left(0 - (2 \times 10^{-9})(4)\right) = -1 \times 10^{-8} \text{ s}.$$

(b) $x = 4\,\mathrm{m},\ t = 1\,\mathrm{s}$.

$$x' = \frac{5}{4}\left(4 - (1.8 \times 10^8)(1)\right) \approx \frac{5}{4}\left(-1.8 \times 10^8\right) \approx -2.25 \times 10^8 \text{ m},$$

$$t' = \frac{5}{4}\left(1 - (2 \times 10^{-9})(4)\right) \approx \frac{5}{4}(1) \approx 1.25 \text{ s}.$$

(c) $x = 1.8 \times 10^8\,\mathrm{m},\ t = 1\,\mathrm{s}$.

$$x' = \frac{5}{4}\left(1.8 \times 10^8 - (1.8 \times 10^8)(1)\right) = 0 \text{ m},$$

$$t' = \frac{5}{4}\left(1 - (2 \times 10^{-9})(1.8 \times 10^8)\right) = \frac{5}{4}(1 - 0.36) = \frac{5}{4}(0.64) = 0.8 \text{ s}.$$

(d) $x = 10^9\,\text{m},\ t = 2\,\text{s}.$

$$x' = \frac{5}{4}\left(10^9 - (1.8 \times 10^8)(2)\right) = \frac{5}{4}(6.4 \times 10^8) = 8 \times 10^8 \text{ m},$$

$$t' = \frac{5}{4}\left(2 - (2 \times 10^{-9})(10^9)\right) = \frac{5}{4}(2 - 2) = 0 \text{ s}.$$

$\square$

Question K&K 12.8. Refer to the note and the sketch in Problem 12.7.

An event occurs in S at $x = 6 \times 10^8\,\mathrm{m}$, and in S' at $x' = 6 \times 10^8\,\mathrm{m}$, $t' = 4\,\mathrm{s}$. Find the relative velocity of the systems.

Solution. Since we know the primed coordinates, we use the inverse Lorentz transformation:

$$x = \gamma(x' + vt').$$

To make the problem easier to solve, divide both sides by c:

$$\begin{aligned}\frac{x}{c} &= \gamma\left(\frac{x'}{c} + \frac{v}{c}t'\right)\\ &= \gamma\left(\frac{x'}{c} + \beta t'\right),\end{aligned}$$

where $\beta = \frac{v}{c}$. Substituting $x = x' = 6 \times 10^8$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ gives

$$2 = \frac{2 + 4\beta}{\sqrt{1-\beta^2}}.$$

Squaring both sides and rearranging terms gives

$$1 - \beta^2 = (1 + 2\beta)^2 = 1 + 4\beta + 4\beta^2 \quad \Rightarrow \quad 5\beta^2 + 4\beta = 0.$$

Therefore, the solutions for β are

$$\beta = 0,\ -\frac{4}{5}.$$

However, substituting $\beta = -\frac{4}{5}$ does not satisfy the earlier equation since the right-hand side must be positive:

$$\frac{2 + 4\beta}{\sqrt{1-\beta^2}} = \frac{2 + 4\left(\frac{-4}{5}\right)}{\sqrt{1 - \left(\frac{4}{5}\right)^2}} = \frac{-6/5}{3/5} = -2 \neq 2.$$

Therefore this solution is extraneous. Hence, $\beta = 0$, implying $v = 0$. The two reference frames are not moving relative to each other.

$\square$

Question 4. Consider two events, A and B. Event A is at $(ct_A, x_A, 0, 0)$ and event B is at $(ct_B, x_B, 0, 0)$. Find the velocity of the system in which they occur simultaneously

Solution. Events at time t_A and t_B undergo time dilation when moving at certain speeds, and they occur at t'_A and t'_B such that

$$t'_A = \gamma\left(t_A - \frac{v}{c^2}x_A\right), \quad t'_B = \gamma\left(t_B - \frac{v}{c^2}x_B\right).$$

For the two events to occur simultaneously the difference between the times t'_A and t'_B must be zero as such

$$t'_A - t'_B = 0 = \gamma\left(t_A - \frac{v}{c^2}x_A\right) - \gamma\left(t_B - \frac{v}{c^2}x_B\right) = \gamma\left[t_A - t_B - \frac{v}{c^2}\left(x_A - x_B\right)\right].$$

The Lorentz factor γ can never be zero. Thus,

$$t_A - t_B - \frac{v}{c^2}\left(x_A - x_B\right) = 0$$

Rearranging the equation in terms of v we get the velocity of the system in order for the events A and B to occur simultaneously:

$$v = \frac{c^2(t_A - t_B)}{x_A - x_B}.$$

□

Question K&K 12.6. The pole-vaulter has a pole of length l_0, and the farmer has a barn $\frac{3}{4}l_0$ long. The farmer bets that he can shut the front and rear doors of the barn with the pole completely inside. The bet being made, the farmer asks the pole-vaulter to run into the barn with a speed $v = c\sqrt{3}/2$. In his case the farmer observes the pole to be Lorentz contracted to $l = l_0/2$, and the pole fits into the barn with ease. The farmer slams the door when the pole is inside, and claims the bet. The pole-vaulter disagrees: he sees the barn contracted by a factor of 2, so the pole can't possibly fit inside. Let the farmer and barn be in system S and the pole-vaulter in system S'. Call the leading end of the pole A, and the trailing end B.

(a) The farmer in S sees A reach the rear door at $t_A = 0$, and closes the front door at the same time $t_A = t_B = 0$. What is the length of the pole as seen in S'?

(b) The pole-vaulter in S' sees A reach the rear door at t'_A. Where does he see B at this instant?

(c) Show that in S', A and B do not lie inside the barn at the same instant.

Solution. Let us solve each part:

(a) In the S' reference frame, the pole-vaulter (and the pole) are at rest. Hence, the pole has its proper length in S', so the length is l_0.

(b) In the S frame, the barn is at rest and has length $L = \frac{3}{4}l_0$. The pole-vaulter runs with

$$v = \frac{\sqrt{3}}{2}c, \qquad \beta = \frac{v}{c} = \frac{\sqrt{3}}{2}, \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = 2.$$

In S, event A (leading end reaches the rear door) occurs at

$$(t_A, x_A) = (0, L) = \left(0, \frac{3}{4}l_0\right).$$

Transform this event to S' using

$$x' = \gamma(x - vt), \qquad t' = \gamma\left(t - \frac{vx}{c^2}\right).$$

Then

$$x'_A = \gamma(x_A - vt_A) = 2\left(\frac{3}{4}l_0 - 0\right) = \frac{3}{2}l_0,$$

and

$$t'_A = \gamma\left(0 - \frac{v}{c^2}\cdot\frac{3}{4}l_0\right) = -\gamma\frac{v}{c^2}\left(\frac{3}{4}l_0\right).$$

At the instant $t' = t'_A$, the pole is at rest in S' and has length l_0, so the trailing end B is located at

$$x'_B = x'_A - l_0 = \frac{3}{2}l_0 - l_0 = \frac{l_0}{2}.$$

To determine whether B is inside the barn at this instant, we find the S'-position of the front door at the same time $t' = t'_A$. The front door is the worldline $x = 0$ in S. For $x = 0$, the time transform gives

$$t' = \gamma t \quad \Longrightarrow \quad t = \frac{t'}{\gamma}.$$

Therefore at $t' = t'_A$, we have $t = t'_A/\gamma$, and the S' position of the front door is

$$x'_{\text{front}} = \gamma(0 - vt) = -\gamma v \left(\frac{t'_A}{\gamma}\right) = -vt'_A.$$

Substitute the value of t'_A:

$$x'_{\text{front}} = -v\left(-\gamma \frac{v}{c^2} \cdot \frac{3}{4} l_0\right) = \gamma \frac{v^2}{c^2} \cdot \frac{3}{4} l_0 = \gamma \beta^2 \cdot \frac{3}{4} l_0.$$

With $\gamma = 2$ and $\beta^2 = 3/4$,

$$x'_{\text{front}} = 2 \cdot \frac{3}{4} \cdot \frac{3}{4} l_0 = \frac{9}{8} l_0.$$

Since $x'_B = \frac{1}{2} l_0 < \frac{9}{8} l_0 = x'_{\text{front}}$, the pole-vaulter sees B outside of the front door.

(c) We already proved that when A reaches the back door, then B is outside the front door. Both ends cannot lie inside the barn at one instant. Hence, in S', A and B are never inside the barn simultaneously.

$\square$