A simplified yaw-attitude model for eclipsing GPS satellites

Article *in* GPS Solutions · January 2009

DOI: 10.1007/s10291-008-0092-1

1 author:

Jan Kouba
Natural Resources Canada

GPS Solut (2009) 13:1–12
DOI 10.1007/s10291-008-0092-1

ORIGINAL ARTICLE

A simplified yaw-attitude model for eclipsing GPS satellites

J. Kouba

Received: 2 January 2008 / Accepted: 5 March 2008 / Published online: 20 March 2008

Abstract A simplified yaw-attitude modeling, consistent with Bar-Sever (1996), has been implemented and tested in the NRCan PPP software. For Block IIR GPS satellite it is possible to model yaw-attitude control during eclipsing periods by using the constant hardware yaw rate of 0.20°/s. The Block IIR satellites maintain the nominal yaw attitude even during a shadow crossing (Y. E. Bar-Sever, private communication, 2007), except for the noon and shadow midnight turn maneuvers, both of which can be modeled and last up to 15 min. Thus, for Block IIR satellites it is possible to maintain continuous satellite clock estimation even during eclipsing periods. For the Block II/IIA satellites, it is possible to model satisfactorily the noon turns and also shadow crossing, thanks to the permanent positive yaw bias of 0.5°, implemented in November 1995. However, in order to model the Block II/IIA shadow crossings, satellite specific yaw rates should be used, either solved for or averaged yaw-rate solutions. These yaw rates as estimated by the Jet Propulsion Laboratory (JPL) can differ significantly from the nominal hardware values. The Block II/IIA post-shadow recovery periods, which last about 30 min, should be considered uncertain and cannot be properly modeled. Data from post-shadow recovery periods should, therefore, not be used in precise global GPS analyses (Bar-Sever 1996). For high-precision applications, it is essential that users implement a yaw-attitude model, which is consistent with the generation of the satellite clocks. Initial testing and analyses, based on the IGS and AC Final orbits and clocks have revealed that during eclipsing periods, significant inconsistencies in yaw-attitude modeling still exist amongst the IGS Analyses Centers, which contribute to the errors of the IGS Final clock combinations.

Keywords GPS · Yaw-attitude control · Precise point positioning · IGS clock solutions

J. Kouba (✉)
Geodetic Survey Division, Natural Resources Canada (NRCan),
615 Booth Street, Ottawa, Canada K1A 0E9
e-mail: kouba@geod.nrcan.gc.ca

Introduction

Attitude control of GPS satellites is dictated by two requirements or constraints, namely that the transmitting antenna always points toward the Earth and that the solar panel axis is perpendicular to the Sun direction to ensure that the panels are facing the Sun. These requirements necessitate that the satellites constantly rotate (yaw) along the antenna axis, which points toward the Earth. The *nominal yaw attitude* has the body-fixed Z-axis pointing to the Earth, the Y-axis is along the solar panels and perpendicular to the Sun direction, and the X-axis points either toward the Sun for Block II, IIA or away from it for Block IIR satellites, and it completes the right-handed coordinates system.

Knowing the satellite attitude, or orientation, is important for three reasons, firstly for the orbit determination in order to model the solar pressure effects correctly (e.g., Beutler et al. 1994). Secondly, to correct for the so called "phase wind-up" (Wu et al. 1993) since due to RHC (right-hand-circular) polarization, the received carrier phase depends on the mutual orientation of satellite and receiver antennas. Thirdly, to properly relate measurements to the center of mass of the Block II/IIA satellites, used e.g., by IGS orbit/clock products (Dow et al. 2005), since the Block II/IIA satellites have a significant offset of antenna phase center (of 27 cm in X). Of these three attitude-related errors, the orbital one is likely the smallest. The largest one, which can exceed 10 cm and affects both phase and pseudorange measurements, is caused by the Block II/IIA antenna phase center eccentricity. Note that this error does not exist for the new Block IIR satellites, since they have practically no antenna eccentricity in X and Y. The phase wind-up error portion of the yaw-attitude error affects only phase measurements and can also reach up to 10 cm. However, this error completely cancels out for double differenced phase observations, while for undifferenced (phase and pseudorange) solutions it is largely absorbed by satellite clock solutions.

The nominal yaw attitude has two singularities and brakes down when the (satellite–Earth) and (satellite–Sun) vectors are collinear, causing two 180° discontinuities. The collinear and nearly collinear vectors cause so called "noon" and "midnight" rapid turn maneuvers, which exceed maximum hardware yaw rates. The noon turn is at the closest point to the Sun and the midnight turn (for Block IIR only), when a satellite is in the Earth's shadow and is at the farthest point from the Sun. Furthermore, when entering the Earth's shadow, the solar sensor of Block II/IIA satellites can no longer control the yaw attitude, which causes those satellites to start yawing with maximum hardware yaw rates of about 0.10–0.13°/s. Both turn and shadow crossing maneuvers cause problems, since yaw attitude departs from the nominal one, though for the Block II/IIA satellites the noon turn is much milder and shorter than the shadow crossing. The noon and for the Block IIR satellites also midnight turn problems are due to insufficient hardware yaw rates, which cause the actual yaw angle to temporarily lag behind the nominal yaw attitude for up to 30 min and particularly so for the slow Block II/IIA satellites. The shadow crossings are much longer than the noon turns. Typically it takes up to 1 h to cross the Earth's shadow and for the Block II/IIA satellites additional 30 min to recover to the nominal yaw orientation after exiting a shadow. This means that for the Block II/IIA the actual yaw attitude may be unknown and quite different from the nominal one for up to almost 90 min!

The Jet Propulsion Laboratory (JPL) has pioneered a rigorous yaw-attitude modeling for eclipsing satellites (IGSMAIL-591; Bar-Sever 1996). They were also instrumental in persuading the GPS Control Command to bias all Block II/IIA satellites with the same yaw bias of +0.5° starting in November 1995, which makes the direction of the maximal yawing known during shadow crossing, making it possible to model yaw attitude. Bar-Sever (1996) developed rigorous deterministic models for both noon turn and shadow crossing maneuvers of the Block II/IIA satellites. The models require iterations as well as estimations of maximal yaw rates, since they were observed to change over time. However, according to Bar-Sever (1996), the variations of the yaw-rate solutions were caused by the changes in the sign of yaw bias, which were routinely done by the GPS Control Command prior November 1995. With a constant yaw bias for all the Block II/IIA satellites, the solved yaw rates became nearly constant so that yaw-rate solutions were even considered unnecessary (Bar-Sever 1996). For this deterministic model with yaw-rate solutions, it was still recommended to delete about 30 min of data, following the shadow exit, since the direction of maximal yawing for the recovery of the nominal yaw attitude is largely unpredictable (Bar-Sever 1996).

Knowledge of the satellite yaw attitude for the new Block IIR satellites is less critical since they do not have any eccentricity in X or Y, thus eliminating the most significant yaw-error contribution. Furthermore, their yaw rate is significantly higher (0.20°/s) than for the Block II/IIA one (up to 0.13°/s only), thus delaying and also shortening the noon and midnight turn maneuvers. Additionally, for β angles, the acute angle between the Sun direction and the orbit plane, with $|\beta| < 1.6°$, the Block IIR satellites can also include shadow yaw-attitude control, where the body X-axis points approximately in the satellite velocity direction. (IGSMAIL-1653). However, this special, Block IIR shadow yaw-attitude control is no longer used. Instead, the Block IIR satellites maintain the nominal yaw attitude even during a shadow crossing, as if they saw through the Earth (Y. E. Bar-Sever, private communication, 2007). This means that the eclipsing yaw regime of the Block IIR satellites is reduced only to the noon and midnight turn maneuvers which can last up to 15 min only.

An improper yaw-attitude modeling can cause small errors of a few cm in position, tropospheric, and clock solutions (Bar-Sever 1996). A possible remedy is to delete the data of eclipsing satellites for intervals during the noon turns, shadow crossings, and the subsequent recovery periods. This works well for static precise point positioning (PPP), since it can almost instantly recover from data outages of a single satellite. For converged solutions, assuming that all the parameters are already known, the new ambiguity can be determined from a single-phase measurement. However, when satellite clocks are also estimated in a global phase solution, data deletions are undesirable. They can weaken user satellite clock solutions due to the ambiguity initializations and subsequent slow convergence, which can last for many minutes. Retaining the eclipsing data segments and associated yaw-attitude errors, on the other hand, can cause errors, which may significantly affect the solutions even for non-eclipsing satellite clocks (Bar-Sever 1996).

The main purpose of this paper is to review eclipsing yaw-attitude modeling of IGS solutions and to investigate the possibility of a simplified yaw-attitude modeling for eclipsing satellites in order to maintain continuous data processing and consequently to improve solutions.

Eclipsing yaw-attitude control in IGS solutions

According to analysis descriptions (ftp://igscb.jpl.nasa.gov/igscb/center/analysis/), several IGS Analysis Centers (ACs) ignore eclipsing yaw-attitude control; however, the IGS analysis descriptions may be out of date. Only JPL, GFZ (Geoforschungszentrum, Germany), and EMR (NRCan) ACs appear to include the JPL yaw-eclipsing models and also solve for satellite-dependent yaw rates for the Block II/IIA satellites. These ACs, in their global processing, also delete the 30-min data interval after a Block II/IIA satellite exits from an eclipsing shadow. JPL made their yaw-rate solutions freely available (see Bar-Sever 1996; ftp://sideshow.jpl.nasa.gov/pub/jpligsac/). No yaw-rate solutions or data deletions are usually applied for the new Block IIR satellites. An initial inspection of IGS and AC Final clock solutions during the GPS Weeks 1438 and 1439 (July 29–August 11, 2007) has revealed that GFZ deleted the 30 min of the post-shadow recovery periods for all the satellites, even for the Block IIR ones. EMR included the Block II/IIA post-shadow 30-min periods in their post-processed 30-s clock submissions (this has been corrected in October 2007). Recently (October 2007), JPL has started to submit their estimated (5-min) clocks, rather than the post-processed 30-s clocks, submitted previously.

The above discussions and Fig. 1 demonstrate an unsatisfactory and inconsistent treatment of eclipsing satellites by some of the IGS Analysis ACs. Figure 1 shows AC Final clock differences with respect to the IGS Final combined clocks for JPL, MIT (Massachusetts Institute of Technology, USA), EMR, CODE (Center for Orbit Determination in Europe), ESA (European Space Agency), and GFZ ACs. It also reveals a problem with experimental IGS 30-s clock combinations, which attempts to combine 30-s and 5-min AC clock solutions. Since the GPS week 1406 (December 17, 2006), IGS in addition to the regular 5-min clock combinations also combines experimentally several of 30-s AC clocks. Note that the IGS clocks even included the 30-min intervals, following the shadow exits of the Block II/IIA satellites, which are excluded in some of the AC clock solutions! Though currently (since October 2007) both IGS clock combinations also exclude the 30-min II/IIA post-shadow recovery periods and since January 2008, the IGS 30-s clock problem has also been largely corrected (G. Gendt, personnal communication).

Figure 2 examines the recent (GPS Weeks 1438 and 1439) shadow yaw-rate solutions of JPL for two Block IIA satellites, PRN 5 and 30. For both satellites, one can see a nearly constant yaw rate of about 0.1°/s, which is quite different from the nominal hardware values of 0.122 and 0.119°/s. Though, at the start of eclipsing season, when shadow crossings are short, the PRN30 solutions differed from the subsequent eclipsing periods with longer shadows. The typical formal sigma for yaw-rate shadow solutions is 0.002°/s and for the noon solutions they can be up to an order of magnitude larger (Bar-Sever 1996). Since the Block IIR yaw rates are applicable only to noon or midnight turns, which are up to 2 times shorter than the Block II/IIA turn maneuvers, JPL in most cases did not solve for the Block IIR yaw rates and used the nominal hardware value of 0.20°/s.

To prevent a possible confusion with the changing yaw rates of the nominal yaw attitude, the constant nominal maximum hardware yaw rates, here and after, are referred to as "hardware yaw rates". Table 1 compares one-year averages of the JPL shadow crossing yaw-rate solutions with the hardware values (see ftp://sideshow.jpl.nasa.gov/pub/GPS_yaw_attitude/nominal_yaw_rates) for all the Block II/IIA satellites, which were operational during the year of August 2006–August 2007. From Table 1 one can see that for most satellites the solution averages are significantly different from the hardware values and for three satellites (PRN 5, 15, and 29), the differences even exceed 0.02°/s. Such large yaw-rate differences can cause yaw-attitude errors of more than 60°. Note that the statistics of Table 1 excluded a few apparent outliers, which were usually occurring for short shadows at the beginning or end of eclipsing seasons. The corresponding JPL noon yaw-rate solutions (not shown here) have different averages with much larger variations, and furthermore, they are frequently equal to the hardware values, implying no yaw-rate estimations. That is why they were not considered here.

New yaw-attitude model

The nominal yaw-attitude orientation is specified by the unit vector (in ITRF) of the satellite body X-axis, which is obtained by the vector products of the ITRF unit vectors of the satellite ($\bar{x}_s$) and Sun ($\bar{x}_{sun}$):

$$\bar{X} = -[(\bar{x}_{sun} \times \bar{x}_s) \times \bar{x}_s]. \quad (1)$$

Since the Block IIR body X-axis points in the opposite direction than the Block II/IIA one, the sign of the above formula is reversed for the Block IIR satellites. The $\bar{X}$-unit vector is subsequently used for the X-antenna eccentricity of the Block II/IIA satellites and for all satellites when computing the phase-wind up corrections. For a different yaw-attitude control, it is thus necessary only to rotate appropriately the $\bar{X}$-unit vector around the body Z-axis.

Models developed by Bar-Sever (1996) for eclipsing yaw-attitude control, after some simplifications and approximations, are also followed here. More specifically, the Block II/IIA yaw bias of + 0.5° is neglected and it is only used for the sign of maximum yawing during shadows. This approximation can introduce a yaw bias of up to

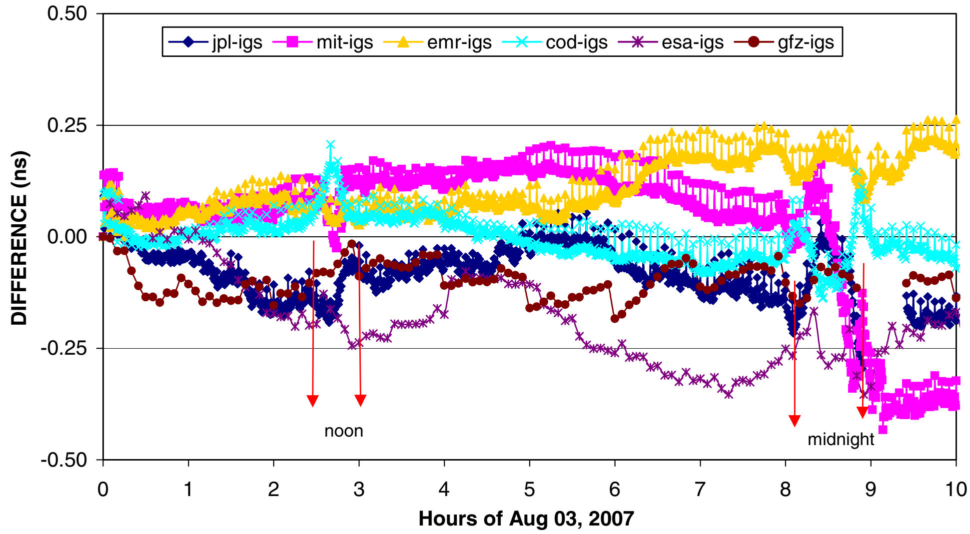

Fig. 1 AC PRN 5 clock differences during the noon and midnight turns ($|\beta| < 1.6°$) on August 3, 2007. The problem of combination of 30-s and 5-min AC clocks caused the regular, 5-min spikes in the IGS 30-s clock combinations (this problem has been largely corrected in January 2008 (G. Gendt, personal communication))

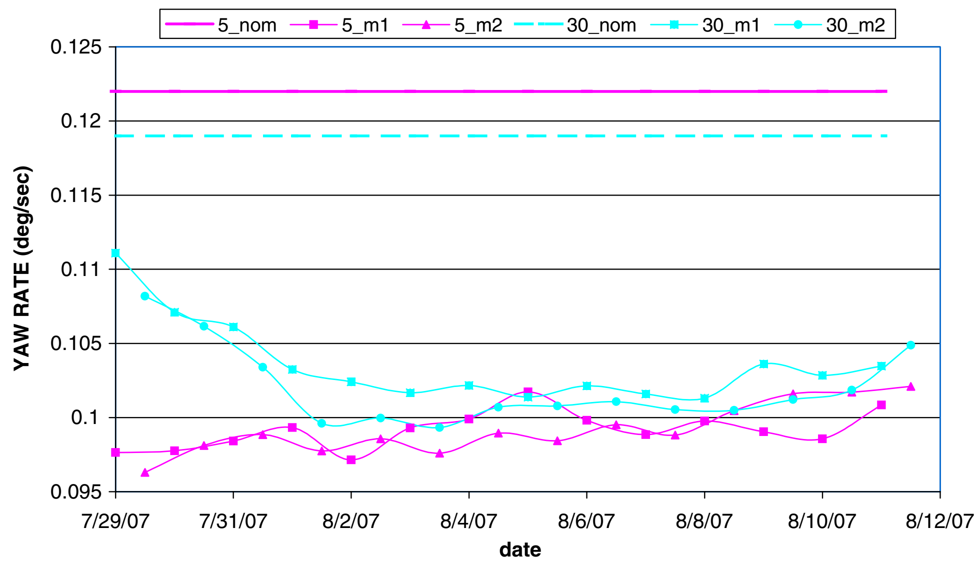

Fig. 2 JPL yaw-rate solutions for Block IIA PRN 5 and 7, during the GPS Weeks of 1438 and 1439. The nominal hardware values (*_nom*) as well as two shadow-crossing solutions (*_m1* and *_m2*) are shown each day for both satellites

6° at the beginning of a noon turn and about 2° at a shadow entry (Bar-Sever 1996). Furthermore, for all satellites, including the Block IIR ones, approximations of the order of the orbit eccentricity, or about 1° of yaw angle have also been introduced. Most of the above approximations and errors are commensurate with the uncertainties of entering shadows and/or noon or midnight turn maneuvers which can reach up to 15° (Bar-Sever 1996).

The eclipsing yaw-attitude control consists of three distinct regimes, (1), the noon (for all satellites) and midnight (for the Block IIR satellites only) turns; (2), the maximum yawing during a shadow crossing for the Block II/IIA satellites only; and (3), the post-shadow recovery to the nominal yaw attitude for the Block II/IIA satellites only. Since the Block II/IIA post-shadow recovery, lasting up to 30 min, is largely uncertain, it is not considered here, and as suggested by Bar-Sever (1996) the data from these periods should be excluded in all precise GPS analyses.

Noon and midnight turn maneuvers

The modeling of the noon and midnight turns is conceptually the same. It consists in finding the time before a midnight or noon turn, when the satellite can no longer keep up with the rapidly changing nominal yaw angle. Usually this happens when the nominal yaw-attitude rate exceeds the hardware one. So, the start of a turn maneuver depends on the maximum hardware yaw rate (or on the hardware yaw rate rates for $|\beta| < 0.3°$, see below). The nominal yaw angle ψ_n, which is the angle between $\bar{X}$ and the satellite velocity unit vector $\bar{v}$, can be computed from the Sun angle β and the orbit angle μ (the geocentric angle between the satellite and the orbit midnight, growing with the satellite motion). β can be simply computed from the unit vectors $\bar{v}, \bar{x}_s$, and $\bar{x}_{sun}$ as

$$\beta + \pi = \cos^{-1}[(\bar{v} \times \bar{x}_s) \cdot \bar{x}_{sun}], \tag{2}$$

where $\bar{v}$ is the inertial geocentric satellite velocity unit vector that can be approximated from the ITRF satellite velocity $\dot{\bar{x}}_s$ and the Earth's rotational velocity ω by

$$v_1 = \dot{x}_{s1} - \omega \cdot x_{s2}; \quad v_2 = \dot{x}_{s2} + \omega \cdot x_{s1}; \quad v_3 = \dot{x}_{s3}. \tag{3}$$

According to Bar-Sever (1996) for a Block II/IIA satellite ψ_n is:

$$\psi_n = \mathrm{ATAN2}(-\tan\beta, \sin\mu), \tag{4}$$

where ATAN2 is the usual FORTRAN function of $\tan^{-1}$, giving signed angles between $(-\pi,\pi)$. For the Block II/IIA satellites, the sign of ψ_n is always opposite to that of β. For the Block IIR satellites, due to the 180° reversal of $\bar{X}$, ψ_n is

$$\psi_n = \mathrm{ATAN2}(\tan\beta, -\sin\mu) \tag{5}$$

and the nominal yaw rate is

$$\dot{\psi}_n = \dot{\mu}\tan\beta\cos\mu/(\sin^2\mu + \tan^2\beta), \tag{6}$$

where $\dot{\mu} = 0.00836^{\circ}/\mathrm{s}$ is the average orbital angular velocity.

Given the hardware (maximum) yaw rate R and by substituting $\mu = 0$ into Eq. 6, one can approximate the maximum β angle limit

$$\beta_0 = \tan^{-1}(\dot{\mu}/R). \tag{7}$$

so that for all $|\beta| < \beta_0$ the yaw rate $\dot{\psi}_n$ of the nominal yaw-attitude turns will exceed the satellite maximum (hardware) yaw rate R.

Substituting the hardware yaw rates (Table 1) into the above Eq. 7 yields the β_0 limits of 3.6°–4.9° for the Block II/IIA and the 0.2°/s yaw rate yields 2.4° for the Block IIR satellites. This means that currently one need not be concerned about Block II/IIA noon turn maneuvers for all $|\beta| > 4.9°$ and about Block IIR noon and midnight turns for $|\beta| > 2.4°$.

For noon turns and $|\beta| < \beta_0$, the orbit angle μ, corresponding to the start of the turn maneuvers, can be conveniently approximated from Eq. 6 by setting $|\dot{\psi}_n| = R$, and by using Eq. 7 along with the small angle approximations of $\sin\alpha \approx \tan\alpha \approx \alpha$ and $\cos\alpha \approx 1$, which are valid for the small angles β_0, β, and μ. These approximations yield

$$\mu(t_s) = \pi - \sqrt{\beta_0|\beta| - \beta^2} \tag{8}$$

and for the midnight turns

$$\mu(t_s) = -\sqrt{\beta_0|\beta| - \beta^2}. \tag{9}$$

From the above two equations and Fig. 3 one can then compute the starting times of a turn maneuver t_s

$$t_s = t_m - \sqrt{\beta_0|\beta| - \beta^2}/\dot{\mu}. \tag{10}$$

From Eq. 10 one can see that for very small β, the time t_s, when the nominal yaw rate of a rapid, nearly 180° turn will start to exceed the hardware rate, will be very near and before the middle time t_m.

The middle time t_m of the nominal yaw-attitude turn is obtained from the considerations and schematics of Fig. 3, using the satellite–Earth–Sun angle E. First, the angle E is obtained by the dot product of unit vectors $\bar{x}_s$ and $\bar{x}_{sun}$ for the noon turns

Table 1 The Block II/IIA satellite yaw rates; the nominal hardware yaw rates (*Nom.*) and the mean values (*mean*) of JPL shadow crossing yaw-rate solutions during August 2006–August 2007

PRN	SVN Blk	Nom.	Mean	σ	Difference
1	32 IIA	0.1230	0.1046	0.0030	0.0184
3	33 IIA	0.1230	0.1255	0.0006	−0.0025
4	34 IIA	0.1230	0.1249	0.0005	−0.0019
5	35 IIA	0.1220	0.1003	0.0013	0.0217
6	36 IIA	0.1270	0.1230	0.0009	0.0040
7	37 IIA	0.1280	0.1136	0.0025	0.0144
8	38 IIA	0.1030	0.1169	0.0018	−0.0139
9	39 IIA	0.1280	0.1253	0.0016	0.0027
10	40 IIA	0.0980	0.0999	0.0013	−0.0019
15	15 II	0.1340	0.1092	0.0013	0.0248
24	24 IIA	0.1120	0.0960	0.0016	0.0160
25	25 IIA	0.1010	0.0838	0.0028	0.0172
26	26 IIA	0.1230	0.1284	0.0020	−0.0054
27	27 IIA	0.1200	0.1183	0.0028	0.0017
29	29 IIA	0.1270	0.1024	0.0004	0.0246
30	30 IIA	0.1190	0.1042	0.0016	0.0148
32	23 IIA	0.1140	0.1100	0.0022	0.0040

Also shown are the standard deviations (σ) of the mean values and the (Nom. − mean) differences (units: °/s)

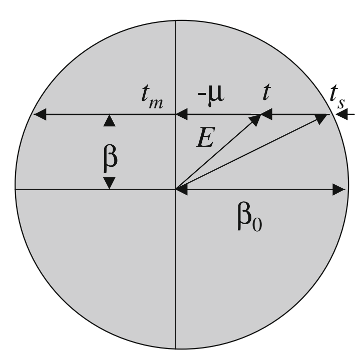

Fig. 3 Schematic view of midnight (shadow) turn, as seen from the Earth; showing relations amongst the Sun angles β_0, β, the orbit angle μ, and the satellite–Earth–Sun angle E. Also shown are the start, current, and middle satellite epochs (t_s, t, and t_m), respectively

$$E = \cos^{-1}(\bar{x}_s \cdot \bar{x}_{sun}) \tag{11}$$

and for the midnight turns

$$E = \cos^{-1}(\bar{x}_s \cdot \bar{x}_{sun}) - \pi. \tag{12}$$

Then, from the diagram of Fig. 3, one obtains for all the epochs t before the middle of a turn

$$t_m = t - \mu/\dot{\mu} = t + \sqrt{E^2 - \beta^2}/\dot{\mu} \tag{13}$$

and for all t after the middle of the turn,

$$t_m = t - \sqrt{E^2 - \beta^2}/\dot{\mu}. \tag{14}$$

Once the start time t_s of the turn maneuver is known, the turn yaw-attitude modeling, applicable to the Block II/IIA noon turns, is simple. For any epoch $t > t_s$ it consists of rotating the body $\bar{X}$ unit vector to the yaw angle

$$\psi(t) = \mathrm{ATAN2}[-\tan\beta, \sin\mu(t_s)] + \mathrm{SIGN}[R, \dot{\psi}_n(t_s)] \cdot (t - t_s), \tag{15}$$

where the SIGN is the usual FORTRAN sign function, yielding the yaw rate R of the same sign as the nominal yaw rate $\dot{\psi}_n(t_s)$ at the start of the turn. Both the noon and midnight turns of Block IIR are then modeled in the same fashion, except for the 180° reversal of $\bar{X}$;

$$\psi(t) = \mathrm{ATAN2}[\tan\beta, -\sin\mu(t_s)] + \mathrm{SIGN}[R, \dot{\psi}_n(t_s)] \cdot (t - t_s). \tag{16}$$

The eclipsing yaw attitude of Eq. 15 or 16 is terminated when the lagging angle $\psi(t)$ catches up with the nominal yaw attitude (the nominal yaw angle $\psi_n(t)$ of Eq. 4 or 5), at which point the current epoch is saved for back-substitution (if needed) as the end epoch t_e. It can take up to 30 min for the Block II/IIA and up to 15 min for the Block IIR satellites before the above turns are completed. Since usually both the nominal yaw rate $\dot{\psi}_n(t)$ and the hardware one R are very close (or even identical) at the start of a turn maneuver, there is no spin-up period required for most turns. However, for a very sharp turn ($|\beta| < 0.3°$), due to an insufficient hardware yaw-rate rates (accelerations) RR (see below), the actual $\psi(t)$ can actually start to lag behind $\psi_n(t)$ even sooner than the turn start time of Eq. 10, possibly resulting in a short wind-up period of <1 min. However, this is also neglected here.

Yaw-attitude control model for shadow crossing of Block II/IIA satellites

Since the Block IIR satellites maintain the nominal yaw attitude even during shadows, there is no need for any special yaw-attitude modeling, except for a short midnight turn, which is completely analogous to the noon turns already covered in the preceding subsection. However, the Block II/IIA satellites do require a special yaw-attitude modeling during a shadow crossing.

Assuming a point light source, a satellite would eclipse behind the Earth's shadow with an angle E_{sh} of

$$E_{sh} = R_E/r_s, \tag{17}$$

where R_E is the Earth's mean radius and r_s is the satellite radius vector. Substituting 6,371 km for R_E and 26,561 km as an average of the satellite radius r_s yields $E_{sh} = 13.74°$. Since the Sun is not a point light source, it produces a penumbra of about 0.5°(Bar-Sever 1996); so E_{sh} of 13.5° is likely a good choice for the Earth's shadow limit, since this puts it in the middle of the penumbra. So, all satellites with $|\beta| < 13.5°$ will experience an eclipsing period.

The start and middle epochs (t_s, t_m) of a shadow crossing are approximated from Eq. 13 or 14 and Fig. 3, while using $E_{sh} = 13.5°$ in place of β_0, i.e.,

$$t_s = t_m - \sqrt{E_{sh}^2 - \beta^2} \Big/ \dot{\mu} \tag{18}$$

and the shadow exit time t_e then is

$$t_e = t_m + \sqrt{E_{sh}^2 - \beta^2} \Big/ \dot{\mu}. \tag{19}$$

The spin-up or spin-down time t_1, i.e., the time to reach the maximal yaw rate R, depends on the nominal yaw rates at the shadow entry $(\dot{\psi}_n(t_s))$ and it is equal to

$$t_1 = [\mathrm{SIGN}(R, b) - \dot{\psi}_n(t_s)]/\mathrm{SIGN}(RR, b), \tag{20}$$

where $b = +0.5°$ is the permanent yaw bias of the Block II/IIA satellites and RR is the yaw rate rate (acceleration) of about 0.00165 and 0.0018°/s^2 for the Block IIA and II, respectively (Bar-Sever 1996). For $t_s < t \leq (t_s + t_1)$ the Block II/IIA shadow yaw angle then is

$$\begin{aligned}\psi(t) = {} & \mathrm{ATAN2}[-\tan\beta, \sin\mu(t_s)] + [(\dot{\psi}_n(t_s) \\ & + \mathrm{SIGN}(RR, b) \cdot (t - t_s)/2](t - t_s),\end{aligned} \tag{21}$$

and for $(t_s + t_1) < t < t_e$

$$\begin{aligned}\psi(t) = {} & \mathrm{ATAN2}[-\tan\beta, \sin\mu(t_s)] + \dot{\psi}_n(t_s) \cdot t_1 \\ & + \mathrm{SIGN}(RR, b) \cdot t_1^2/2 + \mathrm{SIGN}(R, b)(t - t_s - t_1).\end{aligned} \tag{22}$$

For the 30-min post-shadow recovery of the Block II/IIA satellites (when $t_e < t < (t_e + 30$ min)), the yaw attitude is largely uncertain and the corresponding data should not be used in precise global analyses (Bar-Sever 1996). After (t_e + 30 min) the nominal yaw attitude is resumed.

Testing and evaluation

The approximate models outlined in the previous section were implemented in the NRCan PPP software (GPS Pace) (Heroux and Kouba 2001), except for the spin-up effect (20) that was neglected. The new PPP software version was initially tested, using data of August 3, 2007 for the IGS station AMC2 (Colorado Springs, Co., USA). In order to avoid IGS clock combination problems and inconstancies of AC clock solutions, seen in Fig. 1, only the IGS orbits/clocks and EMR orbits/clocks were used. Since the satellite clocks are fixed in PPP solutions, the phase residuals of back-substituted static PPP solutions are the best approximations of the actual slant errors caused by yaw-attitude errors and were, therefore, used for testing of the new yaw-attitude model here.

Figure 4 shows the PPP phase residuals obtained at station AMC2 with the nominal attitude and with the new yaw-attitude model during a shadow eclipsing for the Block IIA PRN 5, when using IGS and EMR orbits/clocks. Here one can see that when the nominal yaw attitude was used during the shadow, the errors were almost 10 cm. With the new yaw-attitude model, the errors were considerably smaller, though could still reach up to 5 cm near the end of the eclipsing interval. This is caused by an error of the PRN 5-hardware yaw rate, used in the new yaw-attitude model. When the average yaw rate of 0.1003°/s was used instead of the hardware rate of 0.122°/s (Table 1), the EMR phase residuals become small and nearly zero, except for the last few data points, which are caused by observations taken at low elevation angles (<10°). Note that for this testing 5° elevation data cut-off was used, as well as the original GPS Pace version, which normally excludes all eclipsing periods (including noon turns), but modified for this testing to retain the eclipsing data, while maintaining the nominal yaw attitude. It is encouraging to see that during this shadow eclipsing, in spite of the problems of the new IGS (30-s) clock combination, seen in Fig. 1, the PPP residuals with IGS clocks (Fig. 4) had similar behavior as PPP with EMR clocks, which employed a proper eclipsing yaw-attitude orientation.

Given the several and significant changes in AC clock submissions and in IGS combinations since the initial testing of August 2007, new tests with recent data were considered necessary. Specifically, EMR and IGS no longer include the 30-min post-shadow recovery period of the Block IIA satellites (there are no Block II satellites left!), GFZ no longer deletes any Block IIR periods and JPL has started to contribute only directly estimated 5-min clocks. This is why a new testing, with recent data was, considered necessary. The GPS Week 1446, and in particular its Day 5, September 28, 2007, has experienced numerous eclipses. Ten satellites were eclipsing during that day, some with very low β angle of nearly 0°. Consequently, September 28, 2007 was chosen for the evaluation and testing. The IGS station GODE (Goddard Space Flight Center, Greenbelt, Md., USA) was selected for this purpose, since it had a very good visibility of the eclipsing periods, including all ten eclipsing satellites (5 Block IIR's and 5 IIA's), during 12 eclipsing periods (6 shadow crossings and 6 noon-turn maneuvers).

Figure 5 shows GODE PRN 7 phase residuals of September 28, 2007 for static PPP with the new yaw-attitude model and using JPL, EMR, and CODE orbits/clocks. For comparisons also shown are the phase residuals of the IGS 30-s combined clocks with the new yaw-attitude model and solutions with a special version of GPS Pace, where the nominal yaw attitude was used all the time, including during all the eclipsing periods. These nominal yaw-attitude solutions have labels appended with ("*no*") in Fig. 5.

As one can see from Fig. 5, for the Block IIA PRN 7, only JPL and EMR clocks produced small, nearly zero residuals with the new yaw-attitude model, and particularly so for the estimated JPL clocks. On the other hand, the EMR and JPL residuals with the nominal yaw attitude gave vastly different and large residuals, exceeding 10 cm. The situation is reversed for CODE clocks, which during the eclipsing period have the smallest residuals with the nominal yaw attitude and significantly different and large residuals with the new yaw-attitude modeling. This indicates that CODE likely employed only the nominal yaw attitude, even during the eclipsing periods. Note that for the new yaw-attitude model, the hardware yaw rate of 0.1280°/s was used, which differs by 0.014°/s from the mean JPL solution of Table 1. The IGS 30-s clock residuals with the new yaw-attitude model, as expected, fall in between CODE and zeros. The IGS residuals also show the problem of combining 30-s and 5-min AC clocks; this has already been noticed in Fig. 1, but for a different date and a different satellite.

Figure 6 shows the AC and IGS PRN 7 30-s clocks, corresponding to Fig. 5, each corrected for an offset and daily slope. For completeness, all the available AC clocks, including MIT and GFZ ones, are also shown here. Note that on this day and several others of the GPS Week 1446, there were no ESA clocks available. Like in Fig. 5, CODE and to a smaller extent the IGS combined clocks show a different behavior (a drift) during the shadow crossing. Apart from the 30-s IGS combination problems also noticeable here, there are large clock resets of MIT clocks, in particular at the end of the shadow period. On the other hand, the shape of MIT clocks tends to indicate that MIT likely employed a proper yaw model during this Block IIA eclipsing. Figs. 5 and 6 also demonstrate that even for Block II/IIA, satellite clocks also largely absorbed the errors caused by wrong yaw-attitude orientation of the satellite antenna eccentricity. Or conversely, Fig. 5

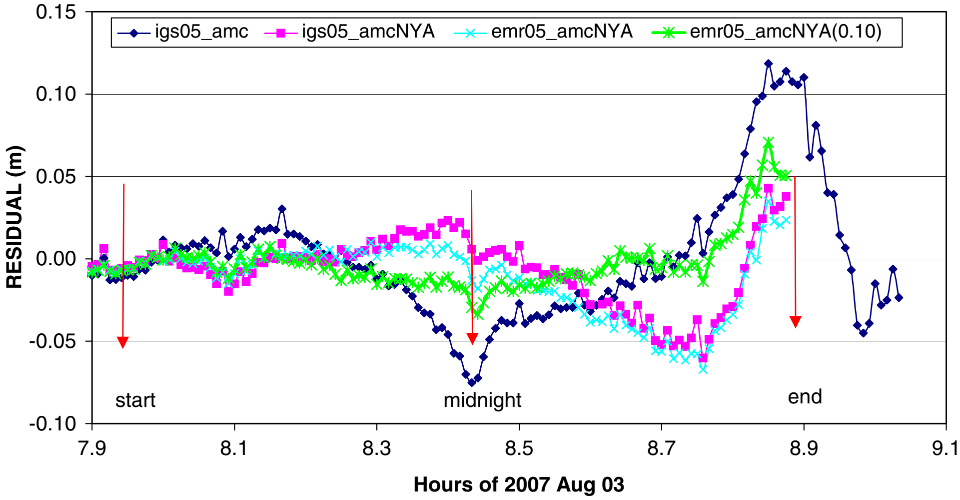

Fig. 4 AMC2 PPP phase residuals during a deep shadow eclipsing (β = 0.6°) of the Block II satellite (PRN 5) on August 03, 2007; using IGS Final orbits/clocks with the nominal yaw attitude (*igs05_amc*) and with the new yaw-attitude model (*igs05_amcNYA*). Also shown are the phase residuals of the new yaw-attitude model for EMR 30-s clocks (*emr05_amcNYA*). For the new yaw-attitude model, the hardware yaw-rate of 0.122°/s was used. The residuals *emr05_amc-NYA(0.10)* were generated with the average of JPL yaw-rate solutions of 0.1003°/s, see Table 1

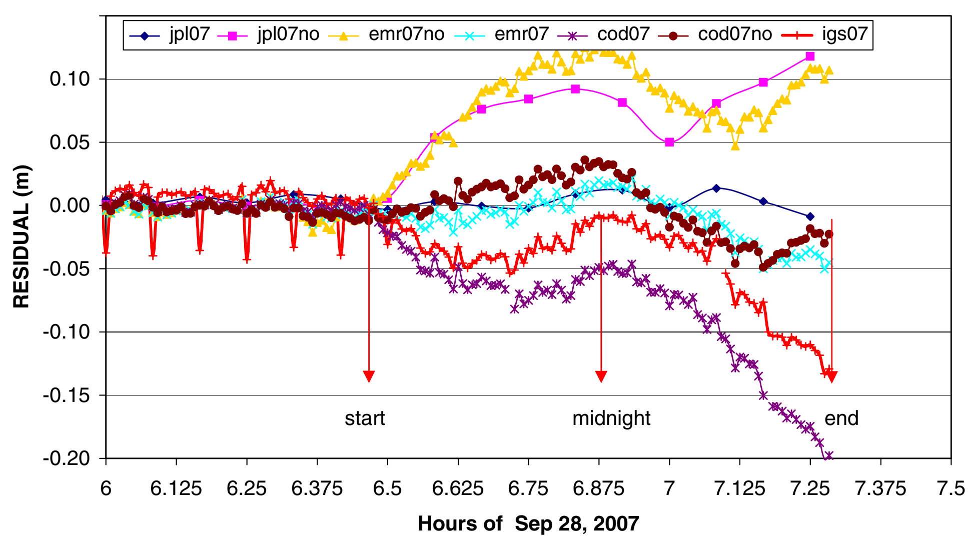

Fig. 5 AC PPP (GODE) phase residuals of the Block IIA PRN 7 during a shadow crossing on September 28, 2007; with the new (Block IIA) shadow yaw-attitude model (yawing at the maximum rate due to the permanent yaw bias) and the nominal one (**no*). CODE clocks appear to be consistent with the nominal yaw attitude, while the IGS residuals fall in between. Note the combination problem of the new IGS 30-s clocks. PPP phase residual noise is <1 cm

indicates that acceptable PPP solutions can likely be obtained even with a "wrong" yaw-attitude control, provided that PPP solutions use the same ("wrong") yaw-attitude model used for the satellite clock solutions.

Figures 7 and 8 show residuals and the clocks of the Block IIR PRN23 satellite for a very sharp noon turn ($\beta \sim 0.05°$). This extreme turn maneuver lasted about 14 min and the first (30-s) data epoch after the maneuver start had a yaw orientation more than 100° away from the nominal one! Furthermore, this particular noon turn required a spin-up period (i.e., accelerating from nearly zero up to the maximum hardware yaw rate of 0.200°/s) of about 2 min. As discussed before, the spin-up period has been neglected here and can cause yaw errors up to 8°. Except for the CODE clocks, which likely use the nominal yaw attitude even during eclipsing as already seen above, and the first three EMR epochs after the start of the noon-maneuver, the new yaw-attitude model gave a very good agreement with nearly zero residuals. However, the rather abrupt and suspicious three-epoch spike of EMR at the turn start was not expected and it may be due to a late start of the EMR yaw-attitude model, since there is no such spike in the corresponding MIT residuals. Furthermore, in Fig. 8 the EMR clocks also clearly show the corresponding three-epoch anomaly, which is not present in MIT and the other AC clocks, and which also indicate that an inadequate turn modeling, likely due to a late start, was absorbed into the EMR clock solutions. That these three anomalistic epochs

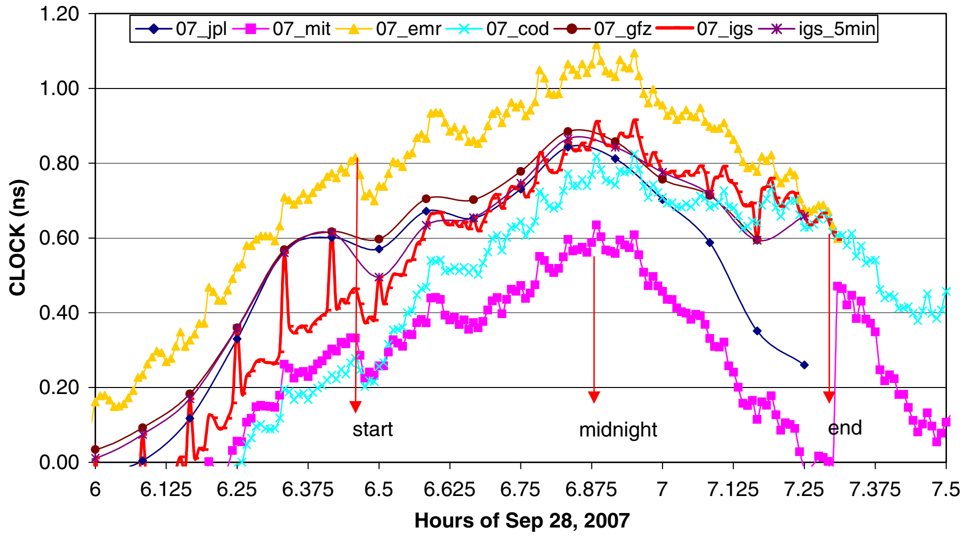

Fig. 6 AC clocks (corrected for daily drifts) of the Block IIA PRN 7 during the shadow crossing of Fig. 5. CODE clocks behave differently from the rest of ACs, while IGS clocks fall in between. IGS as well as JPL, EMR, GFZ currently also exclude a half-hour after the shadow exit. The new IGS 30-s combination has a tooth-like, 5-min problem (see Fig. 9). Note the large reset of MIT clocks at the end of the shadow crossing. (There were no ESA clocks on this day.)

are consistent with the nominal yaw attitude is also indicated by the EMR residuals (*emr07no*), which were generated with the nominal yaw attitude and which show no spike for the three problematic epochs.

Figure 9, which shows AC PRN 7 clock differences with respect to the new IGS 30-s combined clocks, again demonstrates the strange nature and severity of the new IGS combination problem (corrected in January 2008). Note that usually this problem of regular 5-min spikes nearly does not exist during the eclipsing period. Also note that IGS no longer provides clocks for the 30-min post-shadow recovery of the Block IIA satellites; hence, there are no differences during this period in Fig. 9.

Probably the best test of the validity and benefits of the new eclipsing yaw-attitude model are kinematic PPP solutions. During a heavy eclipsing, such as was the case on September 28, 2007, deleting the eclipsing data, as it was done in the original version of the NRCan PPP software, could result in insufficient data. On the other hand, when mismodeled yaw attitude is retained during eclipsing, it is absorbed into epoch position solutions, which are independent from adjacent epochs and consequently free to adjust. Figures 10–12 compare kinematic PPP position solution repeatability (latitude, longitude, and height) for station GODE on September 28, 2007 using IGS, EMR, and JPL orbits/clocks, obtained with the original software version that deletes the eclipsing data and with the new version, employing the new yaw-attitude model, while retaining all data, except for the 30-min Block IIA recovery periods. Also shown are all the 12 eclipsing periods (six shadows and six noon turns). Around 04:30, when two satellites were in the shadow (see Figs. 10–12), the original version solution experienced severe problems since only the minimum four satellites remained for the PPP position solutions. The corresponding formal standard deviations (not shown here) were equally

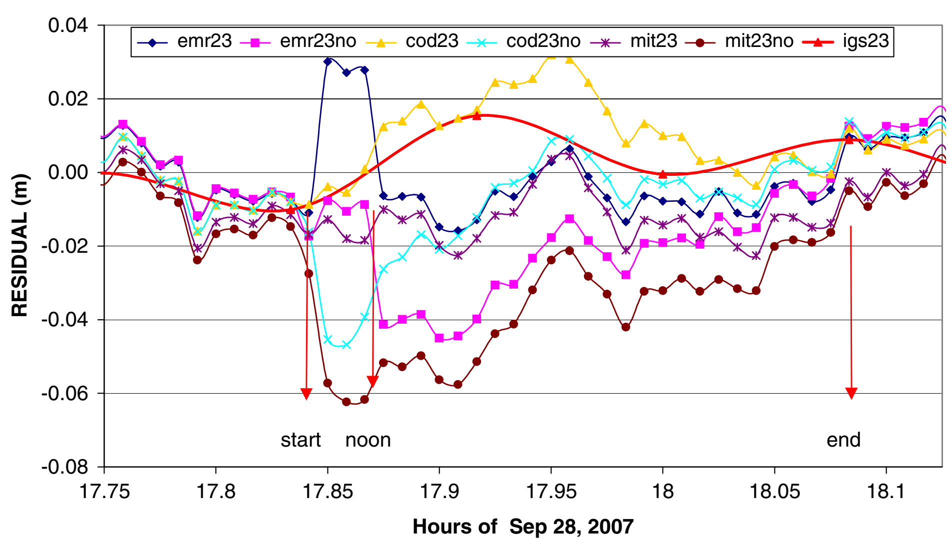

Fig. 7 AC PPP (GODE) phase residuals of the Block IIR PRN23 during a deep noon turn ($\beta \sim 0.05°$) on September 28, 2007; with the new yaw-attitude model (noon turn) and the nominal one (no turn maneuver) (**no*). The turn maneuver starts about 1 min before the noon and lasts about 14 min. Unlike the rest of ACs, CODE appears to agree with the nominal yaw attitude (no turn maneuver). EMR turn maneuver appears to start 1 min too late (see also Fig. 8). The PPP phase residual noise is <1 cm

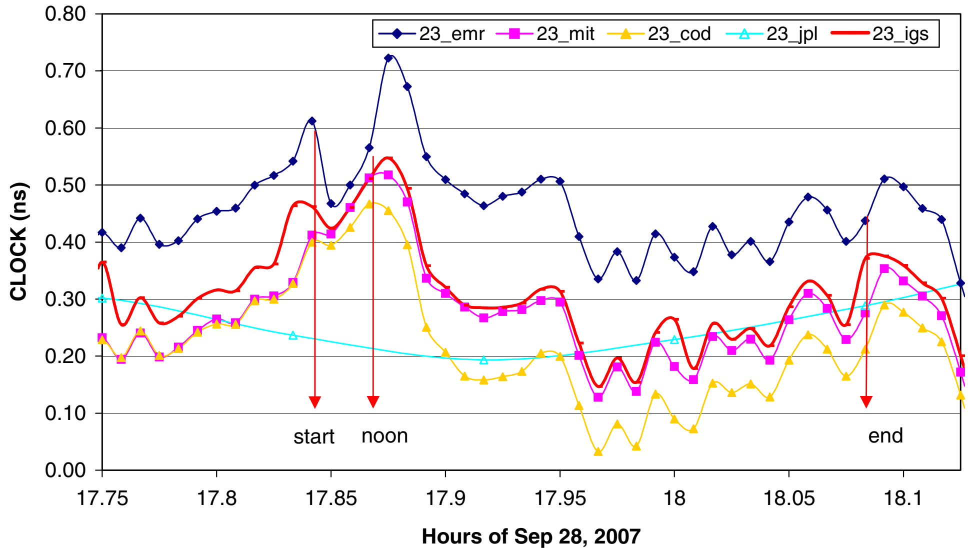

Fig. 8 AC clocks (corrected for daily drifts) of the Block IIR PRN23 during the deep noon turn of Fig. 7. CODE clocks have a slope after the start of the turn. EMR clocks have three outliers, the first three epochs after the start of the noon maneuver; this is likely due to a delayed start (~2 min) of EMR noon turn maneuver model. Note that the 5-min sampling misses this possible 2-min delay of the turn maneuver

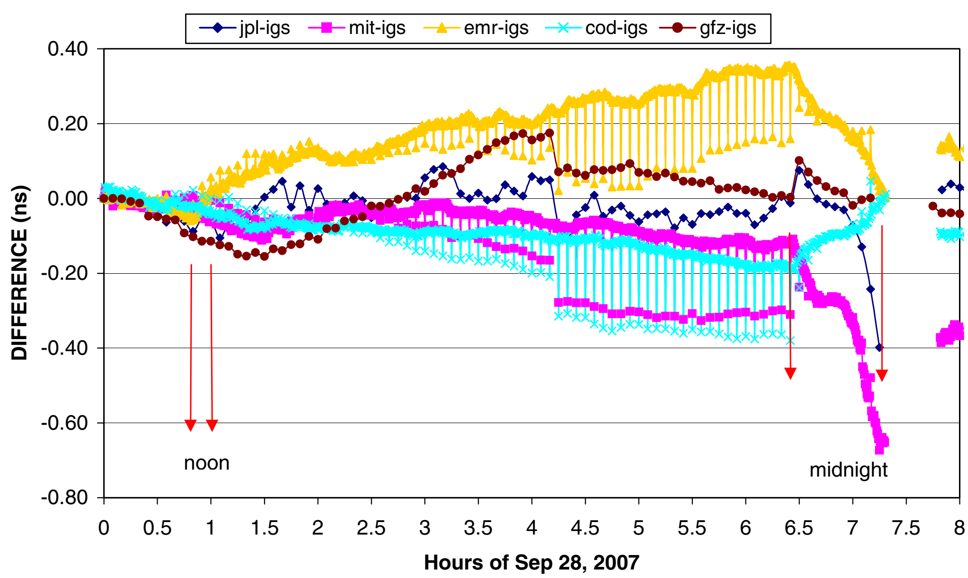

Fig. 9 AC clock differences of the Block IIA PRN 7 on September 28, 2007; with respect to the new IGS 30-s clock combination, during two eclipsing periods ($\beta \sim -5°$), corresponding to Figs. 5 and 6. CODE clocks behave differently, in particularly during the shadow. The IGS 30-s, tooth-like, combination problem (corrected in January 2008), apparent in all the AC 30-s clock differences, and does not exist for the AC 5-min clock residuals. Note that IGS no longer provides the 30-min intervals after the Block II/IIA shadow exits

large (~1 m!) during this period. However, the solutions with the new yaw-attitude model (using the hardware yaw rates) did not have any problems and appeared to be of similar quality in or out of the eclipsing periods. The impact of noon turns is considerably less significant; nevertheless at times it is still noticeable, in particularly for the latitude and height solutions.

Discussions and conclusions

Incorrect yaw attitude during eclipsing of Block II/IIA satellites can cause range and clock errors of up to 15 cm. The Block II/IIA noon and shadow-crossing intervals can be accurately modeled provided that yaw rate is solved for or satellite specific average values are used. For some Block IIA satellites the nominal hardware yaw rates differed from averaged solutions by more than 20% (0.02°/s), which can cause yaw-attitude errors of more than 60°. However, the 30-min post-shadow recovery period of the Block II/IIA satellites cannot be accurately modeled and should be excluded from precise global GPS analyses.

Block IIR satellites (currently form about a half of all the GPS satellites) have significantly smaller errors, caused by an incorrect yaw-attitude control, since they have no antenna phase center eccentricity in the body *X*- or *Y*-axis. Furthermore, the duration of the Block IIR noon and midnight turns are significantly shorter than for the Block II/IIA and they have no post-shadow recovery. Since during shadow crossings, the Block IIR satellites maintain the

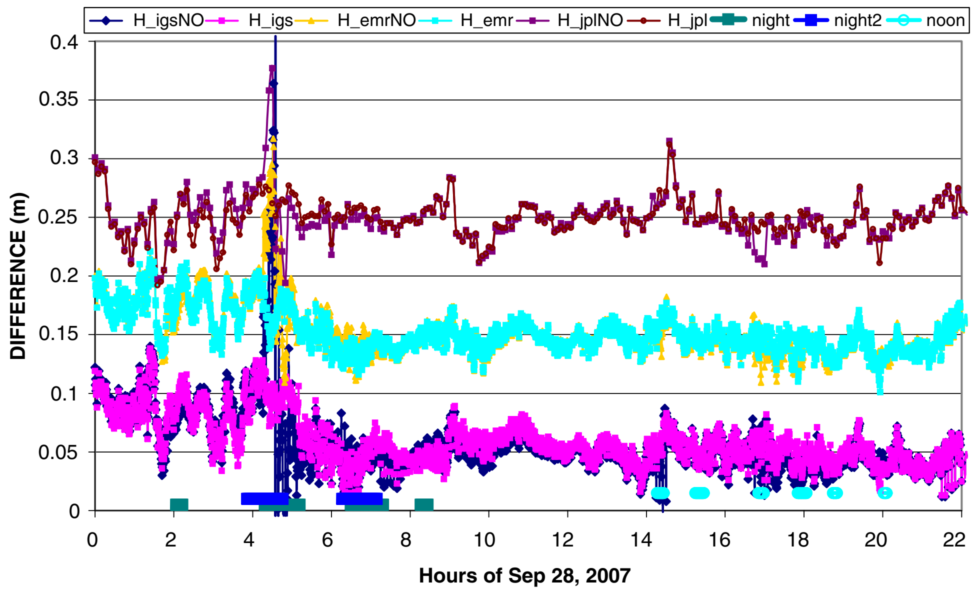

Fig. 10 Kinematic PPP latitude position solution differences for station GODE during eclipsing of 10 satellites on September 28, 2007; with the original PPP version (*NO)* that deletes all the eclipsing (shadow/noon) periods and the new PPP version with new yaw-attitude modeling, during shadows (*night* for Block IIA and *night2* for Block IIR satellites), and all the noon turns (*noon*), with the new IGS, EMR, and JPL orbits/clocks. For display purposes, EMR and JPL PPP solutions are offset by 10 and 20 cm, respectively

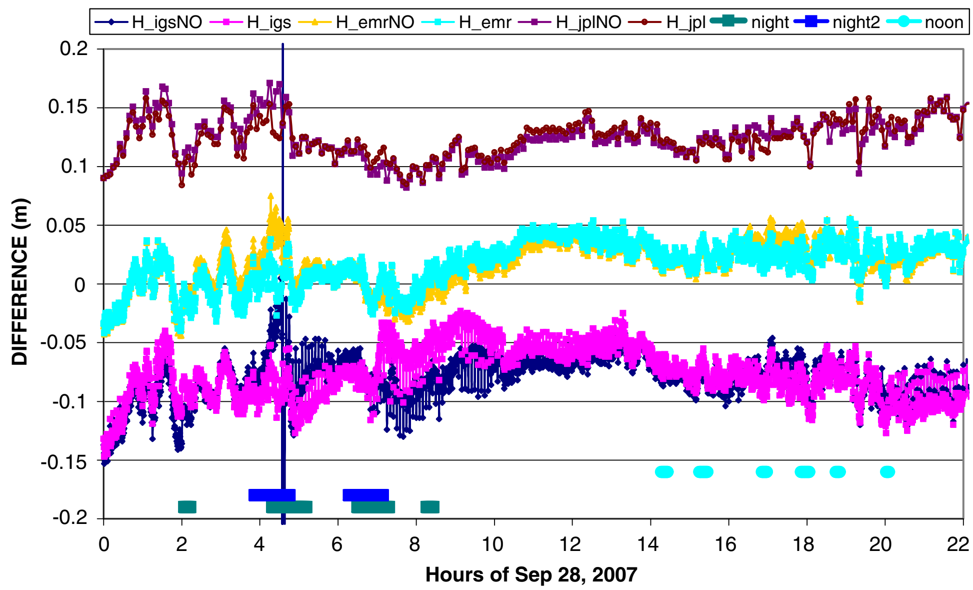

Fig. 11 The same as Fig. 10, but for longitudes. (Note here the IGS 30-s combination problem!)

nominal yaw attitude as if they saw through the Earth, Block IIR shadow yaw-attitude control reduces down to a midnight turn that typically lasts <15 min.

For phase-based GPS solutions the phase wind-up portion of a yaw-attitude error is completely absorbed by the satellite clock solutions (or eliminated by double differencing). For combined phase/pseudorange solutions, the yaw-attitude errors are nearly absorbed by the clock solutions, provided that the relative weighting of pseudorange observations is sufficiently weak and the sampling interval is sufficiently long. So, for Block IIR, phase-only positioning with tropospheric estimation, or phase/pseudorange positioning with sufficiently weak pseudorange weighting and 30-s data sampling, any reasonable yaw-attitude orientation can be used provided that PPP users utilize the same yaw-attitude model used for the satellite clock generation, even an incorrect one (!). The satellite clock solutions also absorbed most of the Block II/IIA yaw-attitude errors, caused by the significant antenna phase center offset in the body *X*-axis, as indicated by Figs. 5 and 7. Consequently, for the sake of completeness and benefits of PPP users, even the problematic 30-min intervals of the Block II/IIA post-shadow recovery could be included in post-processed (e.g., back-substituted) and IGS combined satellite clocks. They could be based, for example, on the nominal yaw attitude. However, such clock solutions of the Block II/IIA post-recovery periods can be used only for PPP position estimation and cannot be used in timing analyses of satellite clocks, since they are subject to significant errors. Also, the Block II/IIA post-recovery data

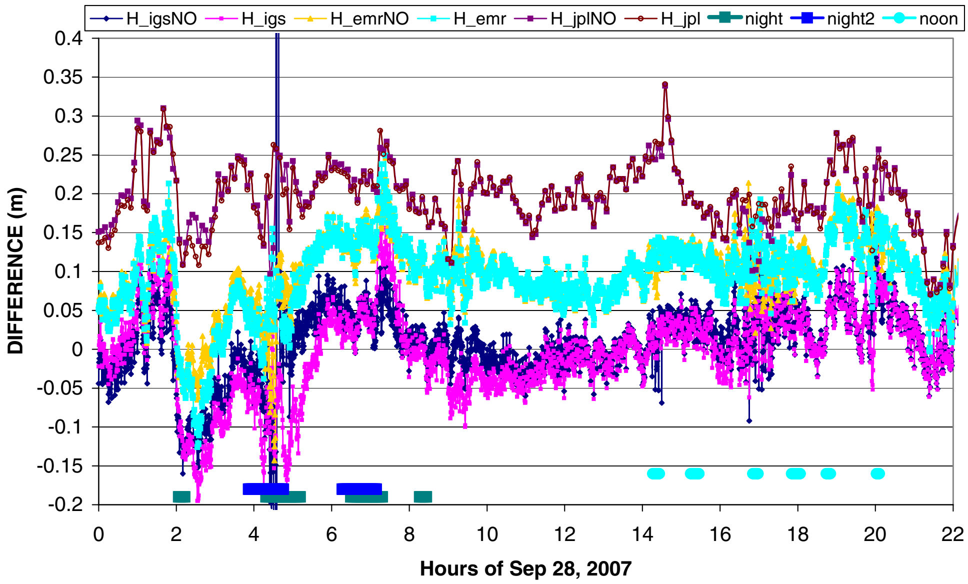

Fig. 12 The same as Fig. 10, but for heights

should not be used in global GPS analyses since it may decrease the accuracy of solutions for other satellites and station solution parameters (Bar-Sever 1996)

The above testing, as well as additional ones not shown here, provides a strong indication that the new eclipsing yaw attitude models and the approximations, described in Section "New yaw-attitude model", are sound and properly implemented in the NRCan PPP software. The new yaw-attitude model employs numerous simplifications and approximations, all of which, except for possible errors of the hardware yaw-rate values (Table 1) should not exceed 10° in the yaw-attitude. The most dramatic improvements were seen for the Block II/IIA shadow-crossing intervals.

The new IGS combinations of 30-s clocks had problems (till January 2008), caused by the combination of 30-s and 5-min sampling of AC clocks. This unfortunately may have compromised and even diminished the value of the IGS 30-s clock combinations for precise PPP solutions. For IGS clock combination it is also essential that the eclipsing data be treated consistently by all ACs. Consistent yaw-attitude models should be used along with consistent Block II/IIA yaw rates, either solved for or averaged ones.

Acknowledgements IGS data and Final solution products (Dow et al. 2005) were analyzed and used here. *Pierre Tetreault* of NRCan has kindly agreed to read this paper and provided the author with useful suggestions and comments.

References

Bar-Sever YE (1996) A new model for GPS yaw-attitude. JoG 70:714–723

Beutler G, Brockmann E, Gurtner W, Hugentobler U, Mervart L, Rothacher M, Verdun A (1994) Extended orbit modeling technique at CODE processing center of the international GPS service for geodynamics (IGS): theory and initial results. Manuscr Geod 19:367–386

Dow JM, Neilan RE, Gendt G (2005) The International GPS Service (IGS): celebrating the 10th Anniversary and looking to the next decade. Adv Space Res 36(3):320–326. doi:10.1016/j.asr.2005.05.125

Héroux P, Kouba J (2001) GPS precise point positioning with IGS orbit products. Phys Chem Earth A 26:573–578

IGS Mail # 591: http://igscb.jpl.nasa.gov/mail/igsmail/1994/msg00166.html

IGS Mail # 1653: http://igscb.jpl.nasa.gov/mail/igsmail/1997/msg00151.html

Wu JT, Wu SC, Hajj GA, Bertiger WI, Lichten SM (1993) Effects of antenna orientation on GPS carrier phase. Manuscr Geod 18:91–98