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# A new model for GPS yaw attitude

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**Abstract.** modeling of the GPS satellite yaw attitude is a key element in high-precision geophysical applications. This fact is illustrated here as a new model for the GPS satellite yaw attitude is introduced. The model constitutes a significant improvement over the previously available model in terms of efficiency, flexibility and portability. The model is described in detail and implementation issues, including the proper estimation strategy, are addressed. The performance of the new model is analyzed and an error budget is presented. Finally, the implementation of the yaw bias on the GPS satellites is reviewed from its inception until it reached a steady state in November, 1995.

## Introduction

On June 6, 1994, the US Air Force implemented a yaw bias on most GPS satellites. By January 1995 the implementation was extended to all the satellites except SVN10. The yaw bias was implemented in order to fix a problem with the attitude control subsystem that prevented the yaw attitude of the satellite from being properly modeled during eclipse seasons (Bar-Sever et al. 1996). The yaw attitude of a biased GPS satellite during eclipse seasons is markedly different from the yaw attitude of a non-eclipsing satellite, or from that of an unbiased satellite. The yaw attitude of the GPS satellite has a profound effect on precise geophysical applications. Mismodeling the satellite attitude can cause decimeter-level range errors. It was necessary, therefore, to develop a special attitude model for biased GPS satellites.

The first attitude model written for the biased constellation was made freely available to the GPS community in the form of a collection of FORTRAN routines (Bar-Sever 1994). In addition to the yaw bias effects, the "noon maneuver" (see definition below) was modeled properly for the first time. For simplicity we will refer to this model throughout this paper as GYM94 (for "GPS Yaw attitude Model - 94"). GYM94 was implemented in the Jet Propulsion Laboratory's (JPL) GIPSY software (Zumberge et al. 1995) and, in various forms, in other high-precision geodetic software packages. The model was successfully used in JPL's routine processing of daily GPS orbits and ground stations coordinates for the International GPS Service (IGS), but it proved to be cumbersome to implement and very demanding in computer resources, namely, memory and CPU time.

In this paper we describe a new model for the GPS satellite yaw attitude, referred to as GYM95. It is considerably more efficient than GYM94 and its relative simplicity allows for easy implementation. We start by reviewing the basic problem of determining the yaw attitude of GPS satellites. We supply motivation by presenting new evidence for the importance of proper yaw modeling in some geophysical applications. We then describe the new model in detail, followed by an error analysis. Finally, the state of the GPS constellation with respect to the yaw bias is described and its implications on the usage of the yaw model are discussed.

## Background

The analysis that led to the implementation of the yaw bias on GPS satellites is described by Bar-Sever et al. (1996). A general description of the first yaw attitude model can also be found there. For completeness, we give here a brief summary.

The nominal yaw attitude of a GPS satellite is determined by satisfying two constraints. First, that the navigation antennae point toward the geocenter and second, that the normal to the solar array surface will be pointing at the Sun. To meet these two conditions the satellite has to yaw constantly. The yaw attitude algorithm described above is singular at two points - the intersections of the orbit with the Earth - Sun line. At these points the yaw attitude is not single-valued as any yaw angle allows optimal view of the Sun. In the vicinity of these singular points the yaw rate of the spacecraft, required to keep track of the Sun, is unbounded. This singularity problem was largely ignored prior to the release of GYM94. While this mismodeling problem could be easily fixed through the implementation of a finite limit on the spacecraft yaw rate, a bigger problem existed that could only be addressed by changing the Attitude Control Subsystem (ACS) on board the spacecraft. The ACS determines the yaw attitude of the satellite by using a pair of solar sensors mounted on the solar panels. As long as the Sun is visible, the signal from the solar sensors is a true representation of the yaw error. During shadow, in the absence of sunlight, the output from the sensors is essentially zero and the ACS is driven in an open loop mode by the noise in the system. It turns out that even a small amount of noise can be enough to trigger a yaw maneuver at maximum rate. To allow modeling of the yaw attitude of a GPS satellites, the ACS had to be biased by a small but fixed amount. Biasing the ACS means that the Sun sensor's signal is superposed with another signal (the bias) equivalent to an observed yaw error of 0.5° (the smallest bias allowed by the ACS). As a result, during periods when the Sun is observed, the satellite yaw attitude will be about 0.5° in error with respect to the nominal orientation . During shadow, this bias dominates the open loop noise and will yaw the satellite at full rate in the direction of the bias. Upon shadow exit, the yaw attitude of the satellite can be calculated and the Sun recovery maneuver can also be modeled. The yaw maneuvers executed by the spacecraft from shadow entry until nominal attitude is resumed are termed collectively “the midnight maneuver”. A somewhat similar maneuver is taking place near the other singularity of the nominal yaw attitude model, at the point where the satellite is closest to the Sun. This maneuver is termed “the noon maneuver”.

GYM94 was used operationally in JPL's IGS processing from September 1994 to April 1995. It properly handled both the midnight maneuver and the noon maneuver by accounting for the yaw bias as well as the limit on the yaw rate. The model computed the satellite yaw angle through numerical integration of a control law, requiring a small step size for stability. Its output was a large file containing the yaw attitude history and, optionally, partial derivatives of the yaw attitude with respect to the yaw rate parameter. This file could later be interpolated to retrieve a yaw angle at the requested time. This process required relatively large amounts of computer memory and CPU time. In addition, the model's complex control law - a simulation of the on-board attitude determination algorithm, did not allow much physical insight into the problem. To overcome all these deficiencies the GYM95 model was created. GYM95 is an analytical model, in contrast to the numerical nature of GYM94. It is simple enough to be described by a small set of formulas, allowing easy implementation in different computing environments. It allows queries at arbitrary time points with great savings in computer resources. Finally, it allows more flexibility in tuning and adapting it to the changing conditions of the GPS constellation.

## The effects of GPS satellite yaw attitude on precise geophysical applications

Both the carrier phase and pseudorange are sensitive to GPS satellite yaw. Both are affected by changes in the position of the transmit antenna phase center. Unmodeled variations in phase center position due to yaw can cause up to 10 cm in range error in both the carrier-phase and pseudorange observables (Bar-Sever et al. 1996). In addition, errors in modeling the phase wind-up can lead to decimeter-level range errors (Wu et al. 1993). The wind-up error cancels out with double differencing but is present otherwise. The actual effect of this ranging error on geophysical applications depends on the quantity being estimated and on the estimation strategy. Since yaw modeling error is rather limited in time, its effects also depends on the number of eclipsing satellites and how often they are being observed during shadow crossing.

It was already established that using a proper yaw attitude model (either GYM94 or GYM95) can improve the accuracy of the GPS orbits and station positions when these are being solved simultaneously (Bar-Sever et al. 1996; Bar-Sever 1995). In this section we explore the effects of yaw attitude errors on the technique of precise point-positioning, which uses predetermined GPS orbits and clocks to solve for individual station position (Zumberge and Bertiger 1995). Since there is no double-differencing in this process, the wind-up error compounds with the phase center error. This technique in routinely used at JPL to estimate the position of stations in a globally-distributed sub-set of the IGS network. The estimation strategy uses GPS data over thirty hours at five-minute intervals to fix the station position. Simultaneously, The station clock is also solved as a white noise process and the total zenith troposphere delay is solved as a random walk process.

To demonstrate the possible errors that result from mismodeling GPS satellite yaw, the following experiment was performed. Twenty-eight globally-distributed stations were point positioned over the period June 10 - June 30, 1995, during which eight satellites were in eclipse. This experiment was repeated three times. First with the full yaw attitude model and the estimated yaw rates, then with the yaw attitude model but with nominal yaw rates and, finally, without the yaw attitude model. The nominal yaw rates were on average 10% in error, compared to the estimated yaw rates. For some satellites, though, they were 25% in error. These were the best a-priori values available at the time. Some stations never observed any satellite during its eclipse and, consequently, are unaffected by the choice of yaw model. Forced to be constant over the estimation period, station position is not very sensitive to the relatively small amount of data from midnight-maneuvering satellites. Consequently, with this estimation strategy, station position is usually affected little by the choice of the yaw model. Nevertheless, some stations can be affected at a significant level. This fact is demonstrated in Table 1 where we compare the quality of the position estimates for the three stations that are most affected by the choice of the yaw model.

Using nominal yaw rates was just as effective as using estimated yaw rates for all the stations except FORT. This is probably a consequence of the particular eclipsing satellites observed by FORT and the amount of error in their nominal yaw rates.

Naturally, estimated quantities that are allowed to vary in time are more sensitive to the yaw attitude of the satellites. One such quantity is the total zenith troposphere delay (TZD). This quantity has recently emerged as an important by-product of GPS positioning, with applications in climatology and weather prediction (Bevis et al. 1994). To be useful in these applications the TZD has to meet stringent accuracy requirements of better than 1 cm. Assuming now that the TZD estimates obtained with the full yaw model are "truth", and subtracting these estimates from estimates obtained without the yaw model, there are many cases where the differences significantly exceed 1 cm. Figure 1 depicts two examples: for FORT and BRMU. TZD values for BRMU (after subtracting the mean), estimated without the yaw model, are also presented in Figure 1 in order to demonstrate that the peaks in the error figure are indeed associated with anomalous features in the estimated value. Notice that although BRMU's position was hardly effected by the choice of the yaw model, its TZD estimates in the absence of a proper yaw model are unacceptably erroneous. All the peaks in Figure 1 correspond to epochs of observing an eclipsing satellite during its yaw maneuver.

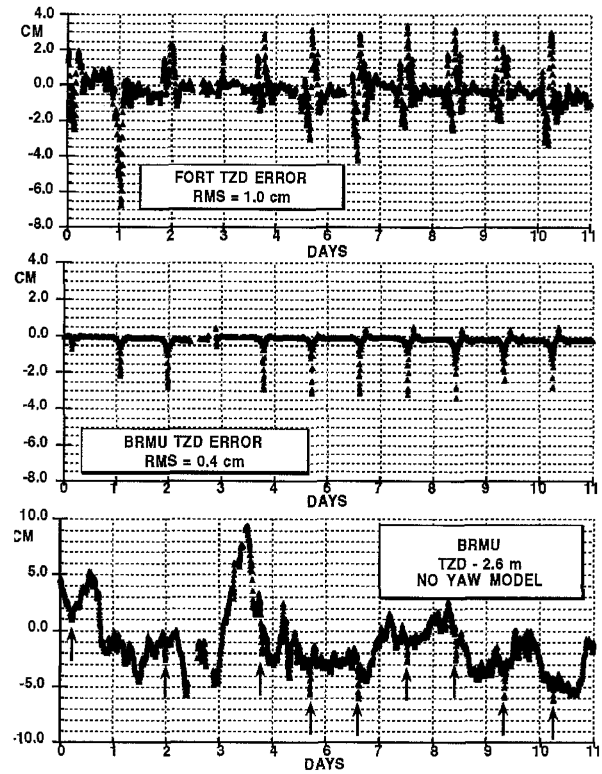


**Fig. 1.** Effects of omitting the GPS yaw model on estimates of total troposphere zenith delay (TZD) for FORT and BRMU. Estimates of TZD with the full yaw model (with estimated yaw rates) are considered truth. Top: TZD errors for FORT. Middle: TZD errors for BRMU. Bottom: estimated TZD for BRMU after a mean of 2.6 m was taken out and when GPS yaw model was not used. The arrows indicate the anomalous features of the estimates that correspond to the peaks in the middle figure.

**Table 1.** Repeatability ( in millimeters) in station position over eleven days as a function of the choice of GPS yaw model. The repeatability is measured as standard deviation in the radial, longitude and latitude components of the station position. Stations were point-positioned using the JPL IGS solutions for satellites and clocks. The formal error in the estimated 3D position is about 5 mm.

| | No Yaw Model | | | Yaw Model With Nominal Yaw Rates | | | Yaw Model With Estimates Yaw Rates | | |
|---|---|---|---|---|---|---|---|---|---|
| | rad | lon | lat | rad | lon | lat | rad | lon | lat |
| FORT | 18 | 16 | 4 | 17 | 13 | 5 | 14 | 13 | 6 |
| KOUR | 23 | 7 | 7 | 14 | 7 | 7 | 15 | 9 | 6 |
| HARV | 15 | 5 | 4 | 11 | 5 | 4 | 11 | 5 | 4 |

Using nominal yaw rates instead of estimated yaw rates in the yaw model resulted in TZD errors that were about a third as large as when no yaw model was used.

## The new yaw attitude model (GYM95)

### *Overview*

The yaw attitude of a GPS satellite can be divided into four regimes: nominal attitude, shadow crossing, post-shadow maneuver and noon maneuver. Most of the time (and for non-eclipsing satellites all the time) the satellite is in the nominal attitude regime. The post-shadow maneuver begins immediately after emerging from the Earth's shadow and lasts until the satellite has regained it nominal attitude. This phase can last from zero to 40 minutes. The noon maneuver does not occur until the beta angle goes below about 5° and can last between zero and 40 minutes. A typical orbit geometry during eclipse season is depicted in Figure 2.

We shall start by defining some important terms and then describe the yaw attitude during each of the four regimes including the governing formulas. Finally, we shall describe how to tie all the regimes together into one functional model and analyze any built-in errors.

### *Definitions*

orbit midnight: The point on the orbit furthest from the Sun.

orbit noon: The point on the orbit closest to the Sun.

orbit normal: Unit vector along the direction of the satellite's angular momentum, treating the satellite as a point-mass. ( = position x velocity where the order of the cross product is important)

sun vector: The unit vector in the direction from the Earth to the Sun.

beta angle: The acute angle between the Sun vector and the orbit plane. It is defined as positive if the Sun vector forms an acute angle with the orbit normal and negative otherwise.

orbit angle: The angle formed between the spacecraft position vector and orbit midnight, growing with the satellite's motion.

yaw origin: A unit vector that completes the spacecraft position vector to form an orthogonal basis for the orbit plane and is in the general direction of the spacecraft velocity vector.

spacecraft-fixed Z axis: The direction of the GPS navigation antennae.

nominal spacecraft-fixed X axis: A unit vector orthogonal to the Spacecraft-fixed Z axis such that it lies in the Earth-spacecraft-Sun plane and points in the general direction of the Sun. (Note: this definition is not single valued when the Earth, spacecraft and Sun are collinear.)

spacecraft-fixed X axis: A spacecraft-fixed vector, rotating with the spacecraft, such that far enough from orbit noon and orbit midnight it coincides with the nominal spacecraft-fixed X axis. Elsewhere it is a rotation of the nominal spacecraft-fixed X axis around the spacecraft-fixed Z axis.

nominal yaw angle: The angle between the nominal spacecraft-fixed X axis and the yaw-origin direction, restricted to be in [-180,180]. It is defined to have a sign opposite to that of the beta angle.

yaw angle: The angle between the spacecraft-fixed X axis and the yaw-origin direction, restricted to be in [-180,180]. Also termed "actual yaw angle".

yaw error: The difference between the actual yaw angle and the nominal yaw angle, restricted to be in [-180,180].

midnight maneuver: The yaw maneuver the spacecraft is conducting from shadow entry until it resumes nominal attitude sometime after shadow exit

noon maneuver: The yaw maneuver the spacecraft conducts in the vicinity of orbit noon when the nominal yaw rate would be higher than the yaw rate the spacecraft is able to maintain. It ends when the spacecraft resumes nominal attitude.

spin-up/down time: The time it takes for the spacecraft to spin up or down to its maximal yaw rate. The spacecraft is spinning down when it has to reverse its yaw rate.

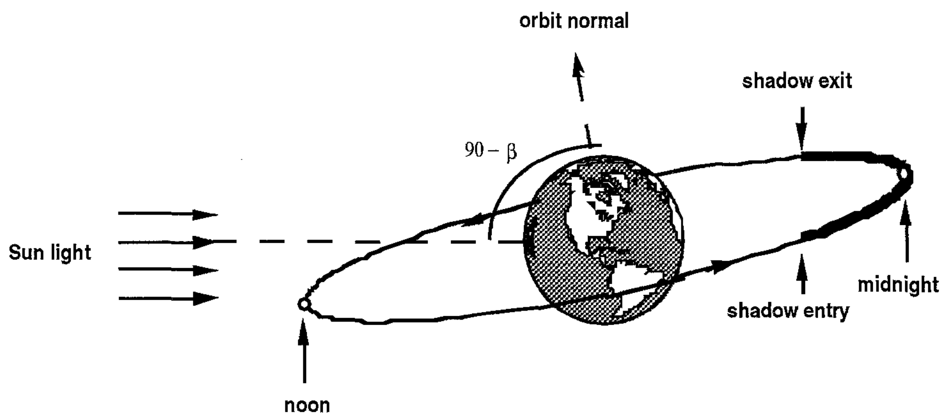


**Fig. 2.** Geometry of an eclipsing orbit

*Notations*

| | |
|---|---|
| $\mu$ | - The orbit angle |
| $\beta$ | - The beta angle |
| E | - The Earth-Spacecraft-Sun angle |
| b | - The yaw bias inserted in the satellite ACS |
| B | - The actual yaw angle induced by b |
| $\Psi$ | - The actual yaw angle |
| $\Psi_n$ | - The nominal yaw angle |
| t | - Current time, in seconds |
| $t_i$ | - Time of shadow entry |
| $t_e$ | - Time of shadow exit |
| $t_n$ | - Start time of the noon maneuver |
| $t_1$ | - The spin-up/down time |
| $\Psi_i$ | - Yaw angle upon shadow entry |
| $\Psi_e$ | - Yaw angle upon shadow exit |
| R | - The maximal yaw rate of the satellite |
| RR | - The maximal yaw rate rate (yaw acceleration) of the satellite |

Angle units, i.e., radians or degrees, will be implied by context. Radians will be usually used in formulas and degrees will be usually used in the text.

FORTRAN function names are used whenever possible with the implied FORTRAN functionality, e.g., ATAN2(a,b) is used to denote arc-tangent(a/b) with the usual FORTRAN sign convention.

*The nominal attitude regime*

The realization of the two requirements for the satellite orientation mentioned above, yields the following formula for the nominal yaw angle:

$$\Psi = \mathrm{ATAN2}(-\tan\beta, \sin\mu) + \mathrm{B}(\mathrm{b}, \beta, \mu) \qquad (1)$$

where $\beta$ is the beta angle, $\mu$ is the orbit angle, measured from orbit midnight in the direction of motion and B is the yaw bias (see below). It follows from this formula that the sign of the yaw angle is always opposite that of the beta angle.

Ignoring the time variation of the slow-changing beta angle leads to the following formula for the yaw rate (there are simpler formulas but they contain removable singularities which are undesirable for computer codes):

$$\dot{\Psi}_n = \dot{\mu} \tan\beta \cos\mu/(\sin^2\mu + \tan^2\beta) + \dot{\mathrm{B}}(\mathrm{b}, \beta, \mu) \qquad (2)$$

where $\mu$ varies little in time and can safely be replaced by 0.0083 degrees/second. Notice that the sign of the nominal yaw rate is the same as the sign of the beta angle in the vicinity of orbit midnight ($\mu$ = 0).

The singularity of these two formulas when $\beta$ = 0 and $\mu$ = 0, 180 is genuine and cannot be removed.

*The yaw bias*

Like any medicine, the yaw bias has its side effects. Outside the shadow it introduces yaw "errors" that are actually larger that 0.5°. To fully understand this we have to describe the ACS hardware, which is beyond the scope of this paper. The underlying reason is that the output of the solar sensor is proportional not to the yaw error but to its sine, and it is also proportional to the sine of the Earth-Spacecraft-Sun angle, E. It turns out that in order to offset a bias of b degrees inserted in the ACS, the satellite has to actually yaw B degrees where B is given by:

$$\mathrm{B}(\mathrm{b}, \beta, \mu) = \mathrm{B}(\mathrm{b,E}) = \sin^{-1}(0.0175\ \mathrm{b}/\sin \mathrm{E}) \qquad (3)$$

where 0.0175 is a hardware-dependent proportionality factor. The Earth-Spacecraft-Sun angle, E, the beta angle, $\beta$ and the orbit angle, $\mu$, satisfy the following approximate relationship:

$$\cos \mathrm{E} = \cos\beta \cos\mu \qquad (4)$$

where E is restricted to [0°,180°]. Formula (3) becomes singular for E less than 0.5013°. This has no effect on the actual yaw because a small value of E implies that the spacecraft is in the middle of a midnight or noon maneuver and is already yawing at full rate. The actual yaw bias, B, becomes significant only for moderately low values of the Earth-Spacecraft-Sun angle, E. For example, for E = 5° which is the typical value at the noon maneuver entry, the actual yaw bias is B ≈ 6°. The bias will, therefore, affect the yaw attitude during the noon maneuver, but it will have little effect on the midnight maneuver which begins at E angles of around 13°, for which B ≈ 2°.

The bias rate, $\dot{\mathrm{B}}$, is given by:

$$\dot{\mathrm{B}}(\mathrm{b}, \beta, \mu) = -0.0175\ \mathrm{b} \cos \mathrm{E} \cos\beta \sin\mu\ \dot{\mu}/(\cos \mathrm{B} \sin^3 \mathrm{E}) \qquad (5)$$

The ACS bias, b, can be ±0.5° or 0°. Starting November 1995 the bias is set to +0.5° on all satellites. Before that, with few exceptions to be discussed below, the bias was set to b = -SIGN(0.5, $\beta$) since this selection was found to expedite the Sun recovery time after shadow exit.

*The shadow crossing regime*

As soon as the Sun disappears from view, the yaw bias alone is steering the satellite. On some satellites the yaw bias has a sign opposite to that of the beta angle. To "correct" for the bias-induced error such a satellite has to reverse its yaw rate upon shadow entry. For those satellites with bias of equal sign to that of the beta angle there is no yaw reversal. The bias is large enough to cause the satellite to yaw at full rate until shadow exit when, finally, the bias can be compensated. The yaw angle during shadow crossing depends, therefore, on three parameters: The yaw angle upon shadow entry, $\Psi_i$, the yaw rate upon shadow entry, $\dot{\Psi}_i$, and the maximal yaw rate, R. Let $t_i$ be the time of shadow entry, $t_e$ be the time of shadow exit, and let t be the current time and define:

$$t_1 = (\mathrm{SIGN}(R,b) - \dot{\Psi}_i)/\mathrm{SIGN}(RR,b) \quad (6)$$

to be the spin-up/down time. Then the yaw angle during shadow crossing, $\Psi$, is given by:

$$\Psi = \Psi_i + \dot{\Psi}_i\,(t - t_i) + 0.5\ \mathrm{SIGN}(RR,b)\,(t - t_i)^2 \quad (7a)$$

for $t < t_i + t_1$ and

$$\Psi = \Psi_i + \dot{\Psi}_i\, t_1 + 0.5\ \mathrm{SIGN}(RR,b)\, t_1^2 + \mathrm{SIGN}(R,b)\,(t - t_i - t_1) \quad (7b)$$

for $t_e > t > t_i + t_1$.

Using these formulas, the singularity problem of the nominal attitude at midnight is avoided.

*The post-shadow maneuver*

The post-shadow maneuver is the most delicate part of the yaw attitude model. The post-shadow maneuver depends critically upon the yaw angle at shadow exit. The ACS is designed to reacquire the Sun in the fastest way possible. Upon shadow exit the ACS has two options: one is to continue yawing at the same rate until the nominal attitude is resumed, or second, to reverse the yaw rate and yaw at full rate until the nominal attitude is resumed. In this model we assume that the decision is based on the difference between the actual yaw angle and the nominal yaw angle upon shadow exit and we denote this difference by D. If $t_e$ is the shadow exit time then:

$$D = \Psi_n(t_e) - \Psi(t_e) - \mathrm{NINT}((\Psi_n(t_e) - \Psi(t_e))/360)\ 360 \quad (8)$$

and the yaw rate during the post-shadow maneuver will be SIGN(R,D).

Given the yaw angle upon shadow exit, the yaw rate upon shadow exit, SIGN(R,b), and the yaw rate during the post-shadow maneuver, we can compute the actual yaw angle during the post-shadow maneuver by using formula (7) with the appropriate substitutions. This yields:

$$t_1 = (\mathrm{SIGN}(R,D) - \mathrm{SIGN}(R,b))/\mathrm{SIGN}(RR,D) \quad (9)$$

$$\Psi = \Psi(t_e) + \mathrm{SIGN}(R,b)\,(t - t_e) + 0.5\ \mathrm{SIGN}(RR,D)\,(t - t_e)^2 \quad (10a)$$

for $t < t_e + t_1$ and

$$\Psi = \Psi(t_e) + \mathrm{SIGN}(R,b)\, t_1 + 0.5\ \mathrm{SIGN}(RR,D)\, t_1^2 + \mathrm{SIGN}(R,D)\,(t - t_e - t_1) \quad (10b)$$

for $t_n > t > t_e + t_1$.

$t_n$ is the end time of the post-shadow maneuver. The post-shadow maneuver ends when the actual yaw attitude, derived from formula 10, becomes equal to the nominal yaw attitude. In GYM95 $t_n$ is determined by an iterative process that brackets the root of the equation $\Psi(t) = \Psi_n(t)$, where the time dependence of $\Psi_n(t)$ is introduced by substituting $\mu = \mu_e + 0.0083*(t\text{-}t_e)$ in formula 1. This equation can be solved as soon as the satellite emerges from shadow. Once the time of resuming nominal yaw is reached we switch back to that regime.

*The noon maneuver regime*

The noon maneuver regime starts in the vicinity of orbit noon, when the nominal yaw rate reaches its maximal allowed value and ends when the actual yaw attitude catches up with the nominal regime. First we have to identify the starting point and this can be done by finding the root, $t_n$, of the equation $\dot{\Psi}_n(t) = -\mathrm{SIGN}(R, \beta)$, where $\dot{\Psi}_n(t)$ is the nominal yaw rate from formula 2. After the start of the noon maneuver the yaw angle is governed by formula (7), again, with the proper substitutions. This yields:

$$\Psi = \Psi_n(t_n) - \mathrm{SIGN}(R, \beta)\,(t - t_n) \quad (11)$$

The end time is found by the same procedure that is used to find the end time of the post-shadow maneuver.

*The complete model*

Satellite position and velocity, as well as the timing of shadow crossings are required inputs to GYM95. The model is able to bootstrap, though, if these input values are unavailable far enough into the past. For example, if the satellite is potentially in the post-shadow regime upon first query, there is a need to know the shadow entry time so that all the inputs to formulas (9) and (10) are known. If this shadow entry time is missing from the input, the model can compute it approximately as well as the shadow exit time. Once all the timing information is available, yaw angle queries can be made at arbitrary time points. The model will determine the relevant yaw regime and compute the yaw angle using the correct formula. Given the above formulas it is an easy matter to compute the partial derivatives of the yaw angle with respect to any parameter of the problem, the most important of which is the maximal yaw rate, R.

*Model fidelity*

The fidelity of the model is a measure of how accurately it describes the true behavior of the satellite. This is hard to measure because there is no high quality telemetry from the satellite and because the estimated value of the main model parameter, namely, the yaw rate, depends on many other factors besides the attitude model itself: data, estimation strategy and other models for the orbit and the radiometric measurements. Nevertheless, some conservative evaluation of the accuracy of GYM95 is possible, based on experience accumulated with the use of this model and its predecessor, GYM94.

The nominal attitude regime is believed to be very accurate. The only source of error is mispointing of the satellite which is poorly understood and relatively small (of the order of 1° around the pitch, yaw and roll axes). Compensations for the dynamic effect of this error source were discussed by Kuang et al. (1995) and Beutler et al. (1994) where it was treated, properly, within the context of the solar pressure model.

Modeling the midnight maneuver accurately is difficult. Inherent uncertainties such as the exact shadow entry and exit time are persistent error sources. Inaccuracies in shadow entry time are more important that inaccuracies in shadow exit time because errors in the former are propagated by the model throughout the midnight maneuver. In contrast, error in the shadow exit time will affect the post-shadow maneuver only. Either way, the inaccuracy will be manifested through a constant error in the yaw angle, which can be partially compensated through the estimation of the yaw rate. The length of the penumbra region is usually about 60 seconds. Sometime during this period the yaw bias begins to dominate the signal from the solar sensor. GYM95 puts that time midway into the penumbra. The maximum timing error is, therefore, less than 30 seconds. A worst-case scenario, ignoring the short spin-up/down period and using a yaw rate of 0.13 degrees/second, will give rise to a constant yaw error of $30*0.13 \approx 4$ degrees throughout the midnight maneuver. A more realistic estimate is 3°, even before applying yaw rate compensation, after which the RMS error will remain the same but the mean is expected to vanish. Another error source is the uncertainty in the value of the maximal yaw rate rate, RR. This parameter is weakly observable and therefore difficult to estimate. The nominal value for RR that is used in GYM95 is 0.00165 degrees/sec$^2$ for Block IIA satellites and 0.0018 for Block II. The uncertainty in those values should to be less than 30%. The long-term effects of a yaw rate rate error can be computed from the second part of Formula (7) to be:

$$\Psi(RR) = [\dot{\Psi}_i \, SIGN(R,b) - 0.5\, \dot{\Psi}_i^2 - SIGN(R,b)^2]/SIGN(RR,b) \qquad 0.5$$

A worst-case scenario assuming $\dot{\Psi}_i = -SIGN(R,b) = 0.13$ and 30% error in the yaw rate rate would give rise to a yaw error of about 5°. These assumptions also imply a very short shadow duration, guaranteeing that the error will not be long-lasting. For long shadow events $\dot{\Psi}_i \approx 0$ and the resulting yaw error is about 1°. Again, this error can be partially offset by estimating the yaw rate.

The main error source for the noon maneuver is the timing uncertainty of the onset of the maneuver. This uncertainty is not expected to be larger than two minutes. A timing error of two minutes will cause a constant yaw error of about 15°, assuming a yaw rate of 0.13 deg/sec. The relatively short duration of the noon maneuver diminishes somewhat the effects of such a large error. Estimating the yaw rate will decrease the error further.

The yaw rate, R, is the key parameter in the model. Since it is time-integrated, a small error in R will cause a yaw error which is growing in time. For example, an error of 0.01°/sec, which is typically less than 10% of the value of R, will give rise to a 30° error in yaw at the end of a 50-minute shadow event. Therefore, great care should be exercised in choosing values for the yaw rates or, alternatively, they should be estimated. Estimated yaw rates available from JPL (see below) are believed to be accurate to better than 0.002°/sec (1 $\sigma$) based on their formal errors.

Although unlikely, errors from different sources can augment. In that case the maximal error for each regime is as follows: 2° for the nominal yaw regime, 9° for the midnight maneuver regime and 15° for the noon maneuver regime. Typical errors are expected to be less than half these values.

## Operational aspects

Continuous changes in the implementation of the yaw bias in the ACS of GPS satellites and occasional hardware problems require appropriate adjustments to the yaw attitude model. This section reviews the changes in the GPS constellation affecting the yaw attitude since the initial implementation of the yaw bias on June 6, 1994.

Initially, the yaw bias was inserted into all GPS satellites except those with a reaction wheel failure (SVNs 14, 18 and 20 at the time). SVN 10 does not allow for a yaw bias. On January 9, 1995, a reaction wheel failure on SVN 16 forced the GPS operators to switch off its yaw bias. Then, on January 31 the Air Force agreed to extend the implementation of the yaw bias to the satellites with a reaction wheel failure and the implementation was carried out a week later. Currently, all 24 operational satellites are yaw biased. The four satellites with reaction wheel failure cannot yaw at the same rate as a healthy satellite and their yaw rate is about 23% smaller.

The yaw bias can be set positive or negative. It can be shown that if the sign of the yaw bias is opposite that of the beta angle, the Sun reacquisition time after shadow exit is minimized. For this reason the US Air Force had routinely switched the sign of the yaw bias in a satellite whenever the beta angle crossed zero, such that b = - SIGN(0.5, $\beta$). Due to operational constraints it was impossible to carry out this switch exactly when $\beta$ = 0 and it was actually carried out within 24 hours of the beta angle sign change. The actual time of each bias change can be obtained from the file "pub/GPS_yaw_attitude/yaw_bias_table" on node 128.149.70.41. Unexplained anomalies in the estimated yaw rates (see below) that were correlated with the bias sign switch led us to request that the Air Force stop switching the bias sign. The request was granted on an experimental basis and, gradually, from September 1995 to November 1995 the yaw bias on all GPS satellites was set to +0.5° and it remained unchanged ever since. As a result, the above-mentioned anomalies disappeared, making the yaw rates much more predictable and, essentially, removing the need to estimate them. The precise history of the yaw bias for every satellite can be found in the file "pub/GPS_yaw_attitude/yaw_bias_table".

**The estimated yaw rates**

As part of the implementation of the GYM models at JPL the yaw rates of all eclipsing satellites are estimated for every midnight maneuver and every noon maneuver. In JPL's GIPSY software this is done by treating the yaw rate as a piecewise constant parameter for each satellite. The parameter value is allowed to change twice per revolution, mid-way between noon and midnight. JPL routinely publishes the final estimates for the yaw rates. They are available as daily text files via anonymous ftp to 128.149.70.41, directory "pub/jpligsac". Unfortunately, due to a software bug, the archived yaw rates for dates prior to February 16, 1995, were in error. Figure 3 depicts the estimated yaw rates for each eclipsing satellite, for each midnight maneuver and for each noon maneuver, from February 16 to April 26, 1995. The accuracy of the estimates depends on the amount of data available during each maneuver and this, in turn, is proportional to the duration of the maneuver. The longer the maneuver the better the estimate. The effect of a reduced estimation accuracy during short maneuvers is mitigated by the fact that the resulting yaw error is also proportional to the duration of the maneuver. For long maneuvers, e.g., midnight maneuver at the middle of eclipse season, the estimates are accurate to 0.002°/sec, which leads to a maximal yaw error of about 6°. A similar error level is expected for short maneuvers. Noon maneuvers occur only during the middle part of the eclipse season. In Figure 3 they can be distinguished from midnight maneuver rates by the larger formal errors associated with them, since they are typically short events of 15 to 30 minutes duration. As a result, the scatter of the noon maneuver rates is larger than that of the midnight maneuver rates. Toward the edges of the eclipse season the quality of the yaw rate estimates drops, again because of the short duration of the shadow events.

The most striking feature in Figure 3 is the discontinuity of the estimated yaw rates in the middle of eclipse season, corresponding to the beta angle crossing zero. No plausible explanation is currently available for this jump. SVN 29 is the only satellite that does not have a jump discontinuity; this is also the only satellite that does not undergo a bias switch in the middle of eclipse season. SVN 31 is the only satellite with a jump from high yaw rates to low yaw rates as the beta angle transitions from positive the negative. The ratio of the high yaw rate values to the low yaw rate values is about 1.3 for all satellites. Within each half of the eclipse season the midnight yaw rates are fairly constant, varying by 10% or less. This behavior was found to be 100% correlated with the event of the yaw bias sign switch, taking place around the time the beta angle crosses zero. After November 1995, when the yaw bias was set permanently to +0.5°, no discontinuities were observed.

A table containing a-priori yaw rate values for each satellite is available on internet node 128.149.70.41, directory "pub/GPS_yaw_attitude/nominal_yaw_rates". For midnight maneuvers taking place after November 1995, these rates are believed to be accurate to 0.005°/sec (1 $\sigma$). This should be accurate enough for most users, eliminating the need to estimate the yaw rates. Estimated yaw rates for each satellite and for each midnight maneuver and noon maneuver are also freely available from JPL with about a week delay. They can be FTPed from the relevant sub-directories of "pub/jpligsac". These yaw rates are believed to be accurate to 0.002°/sec (1 $\sigma$).

The noon maneuver yaw rates seem to be more variable than the midnight maneuver rates. This is not only a consequence of the weak observability but also of the fact that the spacecraft is subject to a varying level of external torque during the noon maneuver as the eclipse seasons progresses. Nevertheless, JPL will continue to estimate the yaw rates for every midnight maneuver and for every shadow maneuver in order to maintain the highest accuracy and in order to monitor the system.

The modeling of the post-shadow maneuver is a problem for which a satisfactory solution has not yet been found. The source of the problem is the presence of the post-shadow regime which makes the estimation of the yaw rate into a nonlinear problem. There is always a critical value of the yaw rate such that for higher values the spacecraft will reverse its yaw upon shadow exit and for lower values the spacecraft will retain its yaw rate until the end of the midnight maneuver. If this critical value falls in the range of feasible yaw rates - which it often does - it becomes very difficult to determine the kind of maneuver

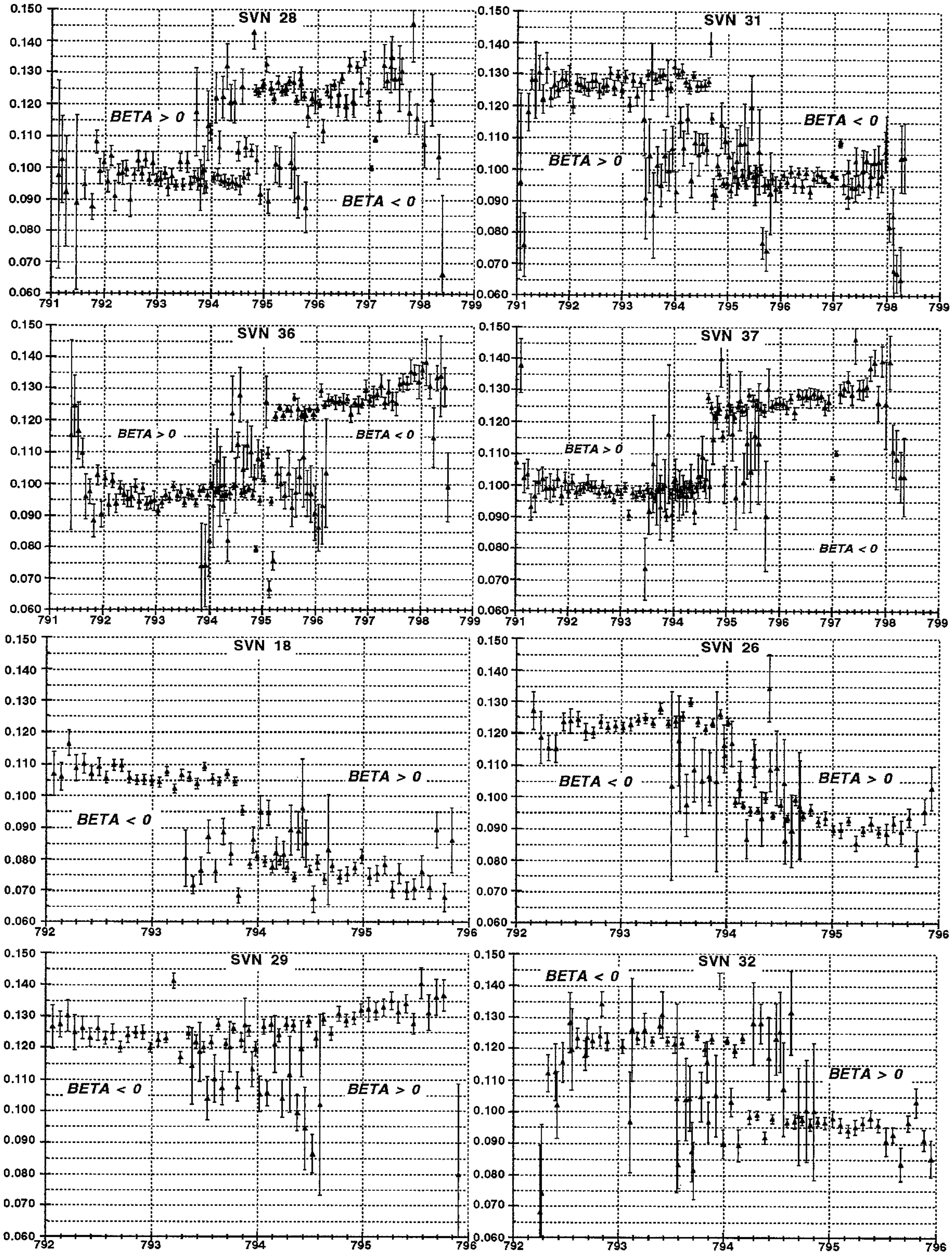


**Fig. 3.** Estimated yaw rates (deg/sec) with their formal errors (σ) vs. GPS week. SVNs 28, 31, 36, 37 are coplanar (C-plane). So are SVNs 18, 26, 29 and 32 (F-plane).

taking place upon shadow exit. To avoid this post-shadow ambiguity we have been rejecting measurement data from shadow exit until about 30 minutes thereafter.

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