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ORIGINAL ARTICLE

Analysis of orbital configurations for geocenter determination with GPS and low-Earth orbiters

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Abstract We use a series of simulated scenarios to characterize the observability of geocenter location with GPS tracking data. We examine in particular the improvement realized when a GPS receiver in low Earth orbit (LEO) augments the ground network. Various orbital configurations for the LEO are considered and the observability of geocenter location based on GPS tracking is compared to that based on satellite laser ranging (SLR). The distance between a satellite and a ground tracking-site is the primary measurement, and Earth rotation plays important role in determining the geocenter location. Compared to SLR, which directly and unambiguously measures this distance, terrestrial GPS observations provide a weaker (relative) measurement for geocenter location determination. The estimation of GPS transmitter and receiver clock errors, which is equivalent to double differencing four simultaneous range measurements, removes much of this absolute distance information. We show that when ground GPS tracking data are augmented with precise measurements from a GPS receiver onboard a LEO satellite, the sensitivity of the data to geocenter location increases by more than a factor of two for Z-component. The geometric diversity underlying the varying baselines between the LEO and ground stations promotes improved global observability, and renders the GPS technique comparable to SLR in terms of information content for geocenter location determination. We assess a variety of LEO orbital configurations, including the proposed orbit for the geodetic reference antenna in space mission concept. The results suggest that a retrograde LEO with altitude near 3,000 km is favorable for geocenter determination.

Keywords Geocenter · GPS · LEO · Reference frame · Orbital configuration

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1 Introduction

The current conventional definition of geocenter motion can be stated as the movement of the center of mass (CM) of the Earth system—including the solid Earth, Oceans and Atmosphere—with respect to a reference system as realized by a globally distributed network of reference stations tied to the surface of the solid Earth (Dong et al. 2003; Lavallee et al. 2006). The CM is the point to which an Earth satellite orbit is connected through the equations of motion. The CM moves in the inertial space following the laws of physics under the influence of forces external to the Earth system. Redistribution of masses within the Earth system does not alter the path of CM in the inertial space. The surface of the solid Earth, however, moves with respect to CM as masses inside the Earth system redistribute.

From the perspective of an observer on the surface of the Earth, the location of the CM in the reference system changes continuously. Ideally one can define the origin of the reference system to coincide with the CM. In practice, however, this is not easy (if not impossible) to realize. The international terrestrial reference frame (ITRF) used to describe the locations of objects on and around the surface of the Earth is realized by defining the coordinates of a network of fiducial stations such that the origin of the coordinate system has zero translation and translation rate relative to the CM on the decadal time scales represented by the historical space-geodetic tracking record (Altamimi et al. 2011). On shorter time scales (hours to seasons), however, the location of the CM changes relative to the ITRF (Dong et al. 2003). Precise determination of the location of CM at a given time in a specific reference frame is one of the tasks of geodesy, and has profound impact on the geophysical interpretation of geodetic measurements (Wu et al. 2012).

Precise measurement of the distance between a satellite, the position of which is related to the Earth's CM through the equations of motion of the satellite, and a ground station, which is tied to the surface of the solid Earth, is the primary measurement for determining the geocenter location in a reference frame. When the CM is not at the origin of the reference frame that we use to describe the location of the ground station at the measurement time, the computed range (by assuming CM at the origin of the reference frame) between the satellite and the ground station is inconsistent with the directly measured range, leading to a residual error. Adjusting the origin of the reference frame by a translation, common to all ground stations in the tracking network, reduces the residual. Over a span of an orbital arc sufficiently long to capitalize on dynamical constraints, and with a sufficient number of ground stations, a constant vector of CM location in the reference frame can be estimated together with other orbital and ground-specific parameters by minimizing the overall residual errors. This "fiducial solution", sometimes called the "geometric approach" (Vigue et al. 1992; Cheng 1999; Pavlis 1999; Kang et al. 2009), determines the location of the CM with respect to the reference frame as represented by the fiducial network. Another approach, called "network shift approach", derives the geocenter location from a Helmert transformation between an adjusted network (with the positions for all stations estimated from satellite tracking data) and its corresponding polygonal figure in the reference frame (Heflin et al. 1992; Lavallee et al. 2006; Rebischung et al. 2014). The non-fiducial solution is ideally centered on the instantaneous CM, and thus provides a means of estimating geocenter location via transformation to the standard ITRF. Regardless of the approach, time series of the geocenter location estimates are then formed for successive solution arcs to depict the geocenter motion in the given reference frame.

Satellite laser ranging (SLR) provides absolute and unambiguous measurements of the distance between targeted satellites and ground stations. SLR is the conventional technique for monitoring geocenter motion (e.g., Watkins and Eanes 1997), and is presently the sole technique for aligning the long-term (secular) origin of the ITRF with CM (Altamimi et al. 2011). With the increasingly important role that GPS plays in geodesy, the prospects for measuring geocenter location from GPS tracking data have been widely investigated (Vigue et al. 1992; Kang et al. 2009; Haines et al. 2011). Geocenter location determination using DORIS tracking data has also been studied and improved in recent years (Gobinddass et al. 2009; Willis et al. 2010). The method of inferring the geocenter motion through the deformation of Earth's crust (Blewitt et al. 2001; Wu et al. 2012) is a different type of method that does not involve the CM directly and will not be discussed in this paper.

Despite the large amount of GPS tracking data as compared to SLR tracking, the quality of geocenter motion estimated from GPS ground tracking data has been widely perceived as inferior to SLR-based estimates, especially for the component aligned with the Earth's spin (Z) axis (e.g., Altamimi et al. 2007). Indeed, no GPS data are used in the ITRF origin realizations (Altamimi et al. 2011). A number of studies have attempted to explain the relative weakness of GPS-based estimation of the geocenter Z component. Rebischung et al. (2014) attribute that weakness to the need to estimate GPS receiver and transmitter clock corrections simultaneously with tropospheric parameters, and regard this as an intrinsic limit on the ability to sense geocenter location with GPS measurements. Meindl et al. (2013) investigated the absorption of GPS solar radiation pressure force model error into the Z component of geocenter estimation. On the other hand, efforts have been made over the last decade to improve the quality of GPS observed geocenter location with various new strategies (e.g., Haines et al. 2015). LEOs carrying precise GPS receivers have long been proposed as means for enhancing various GPS applications. For geodetic applications, the geodetic reference antenna in space (GRASP) mission concept has been proposed (Bar-Sever et al. 2012) to improve GPS contribution to ITRF. One of the tasks of GRASP is to enhance the geocenter location determination by GPS tracking and to combine it optimally with other geodetic techniques so that an improved and unified terrestrial reference frame can be cultivated to better serve the location services and geodetic applications in the international communities.

In this paper, we study the geometric factors that limit the accuracy of geocenter location determination with GPS tracking data through various simulated tracking configurations. In particular, we study the determination of geocenter location by using GPS ground tracking and LEO tracking together and compare it with the scenario of using SLR LEO tracking. We focus on the observability aspect of measuring the geocenter location, defined in this paper as the location of the CM in a given reference frame as realized by a "fiducial solution". We investigate the achievable accuracy of the geocenter location determination in different scenarios based on the fundamentals of GPS measurement types, and provide recommendations on optimizing the measurement systems.

2 Method

In theory, geocenter determination using satellite tracking can be simplified into a geometric problem. As shown in

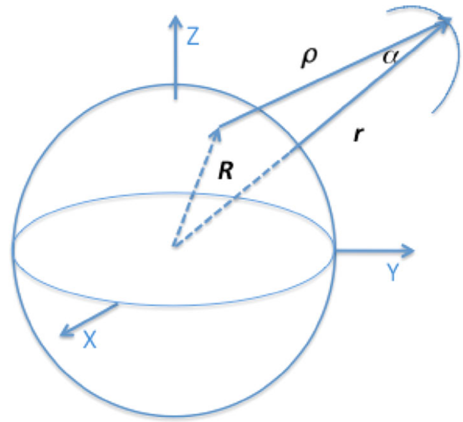


Fig. 1 Ground to satellite tracking geometry

Fig. 1, for example, the measurement of the distance ρ from the ground station at geocentric position $\boldsymbol{R}$ to the satellite at position $\boldsymbol{r}$ (both in Earth centered inertial coordinate system), is simply:

$$\rho = |\boldsymbol{r} - \boldsymbol{R}|, \quad \boldsymbol{R} = \boldsymbol{PNUXY}(\boldsymbol{R}_{\mathrm{f}} - \boldsymbol{R}_{\mathrm{g}}); \tag{1}$$

where $\boldsymbol{R}$ is converted from $\boldsymbol{R}_{\mathrm{f}}$, the ground station position in a terrestrial reference frame, through the coordinate transformations $\boldsymbol{P}$, $\boldsymbol{N}$, $\boldsymbol{U}$, $\boldsymbol{X}$ and $\boldsymbol{Y}$ for precession, nutation, Earth rotation and polar motion, respectively. $\boldsymbol{R}_{\mathrm{g}}$ is the geocenter location in the reference frame. A small difference, or error, between the measured and computed range, $\delta\rho$, can be caused by the error either in the modeled station position $\boldsymbol{R}$ or the modeled satellite position $\boldsymbol{r}$, or both. Since the satellite position $\boldsymbol{r}$ is computed by assuming that CM is at the origin of the frame, a displacement of the actual CM relative to the reference frame will give rise to systematic $\delta\rho$ errors for all tracking sites. Adjusting this displacement can minimize these systematic errors. The "fiducial method" fixes the $\boldsymbol{R}_{\mathrm{f}}$ in Eq. (1) for all fiducial stations to estimate $\boldsymbol{R}_{\mathrm{g}}$, while the "network shift" method zeroes out $\boldsymbol{R}_{\mathrm{g}}$ to estimate $\boldsymbol{R}_{\mathrm{f}}$ for every station and then derives $\boldsymbol{R}_{\mathrm{g}}$ through a Helmet transformation between the adjusted $\boldsymbol{R}_{\mathrm{f}}$'s and their nominal values in the reference frame. It should be noted that the critical role of $\boldsymbol{R}_{\mathrm{g}}$ in Eq. (1) is to translate the origin of the reference frame to coincide with the rotation axis of the Earth so that various rotation transformations can be performed correctly. Measuring $\boldsymbol{R}_{\mathrm{g}}$ in Eq. (1) means to measure the location of Earth's rotation axis in the XY-plane of the reference frame (X and Y component) and the distance between the geocenter and the XY-plane (Z component). This definition implicitly assumes that the CM is located on the Earth's rotation axis.

There are many factors that affect how well this geocenter location can be observed with range measurements. For example, the satellite altitude affects the maximum angle between the line of sight from a tracking site on the Earth and the satellite orbital plane been tracked, and how many tracking sites simultaneously observe the satellite. In addition to satellite altitude, other factors such as orbital inclination, rotation of the Earth, and distribution of the stations will also affect the observability of the geocenter location, making it difficult to perform analytical or semi-analytical analysis. Numerical simulation for a large variety of scenarios must therefore be performed to reveal the details of the effect of each factor.

While SLR provides a direct measurement of the distance between the satellite and the ground station, ground-based GPS tracking provides a comparatively weaker measurement type for determining the geocenter location. The estimation of GPS transmitter and receiver clock errors as white noise processes, which is equivalent to double differencing four simultaneous range measurements, removes the absolute distance information while retaining much of the baseline length information (Kuang et al. 1996). Estimation of carrier phase ambiguity biases and tropospheric delay of GPS signal further reduces the tracking data strength (Rebischung et al. 2014). However, the robust tracking configuration represented by a full GPS constellation and a tracking network with a large number of ground stations can significantly compensate for the weaknesses of the fundamental GPS observables. This tradeoff, as well as the details of the impact of each factor, can be adequately studied using numerical analysis with simulated measurements from a real tracking configuration.

In principle, the observability of a quantity in an observation system is reflected by its corresponding variance, or formal error, in the full covariance matrix obtained through the associated estimation process. For a linearized observation system:

$$\boldsymbol{y} = H\boldsymbol{x} + \boldsymbol{\varepsilon}; \quad E(\boldsymbol{\varepsilon}) = 0,\, E\left[\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T\right] = C \tag{2}$$

where $\boldsymbol{y}$ is an $m \times 1$ observation vector, $\boldsymbol{x}$ is an $n \times 1$ parameter vector, H is the so-called $m \times n$ design matrix that maps the parameter vector into the predicted observation, and $\boldsymbol{\varepsilon}$ is an $m \times 1$ random observational error vector that has zero mean value and covariance matrix C. If $\mathrm{rank}(H) = n$, then the weighted least square estimation of $\boldsymbol{x}$ from Eq. (2) is:

$$\tilde{\boldsymbol{x}} = (H^T C^{-1} H)^{-1} H^T C^{-1} \boldsymbol{y};$$
$$E(\tilde{\boldsymbol{x}}) = \boldsymbol{x},\, E[(\tilde{\boldsymbol{x}} - \boldsymbol{x})(\tilde{\boldsymbol{x}} - \boldsymbol{x})^T] = (H^T C^{-1} H)^{-1} \tag{3}$$

which is unbiased and has minimum variance (e.g., Tapley et al. 2004). The covariance matrix, $(H^T C^{-1} H)^{-1}$, reflects the effect of the random measurement errors on the estimated parameter vector $\boldsymbol{x}$ through the observation system, including the observability of the parameters. A weakly observed quantity, or a parameter determined poorly by the observation system, tends to have large formal error (defined as the square root of the corresponding diagonal element in the covariance matrix). For an observation system with deficiency, $\mathrm{rank}(H) < n$, certain components of the parameter vector are unobservable and the solution of Eq. (3) does not exist. A priori information or constraint conditions have to be applied to obtain reasonable solutions from the observation Eq. (2). In such cases, the resulting formal error depends on the uncertainty assigned to the a priori information or implied in the constraint conditions (Vanicek and Krakiwsky 1982). In particular, the unobservable components corresponding to the zero eigenvalues of $H^T C^{-1} H$ are totally determined from the a priori information. For those components, the formal error does not reflect the observability by the observation system and can be confusing or even misleading. In this study, we focus on the feature of the sensitivity of the tracking system and defer the rank deficient problem by holding all ground station positions fixed in the predefined reference frame.

In addition to the random error ε, if there is a systematic contribution, $B\eta$, from (uncompensated) measurement model and/or dynamic model errors, then the estimated parameter vector in Eq. (3) is not unbiased but has the error:

$$\Delta\tilde{\boldsymbol{x}} = (H^T C^{-1} H)^{-1} H^T C^{-1} B\boldsymbol{\eta} \tag{4}$$

This perturbation by the systematic error depends not only on the design matrix H, but also the systematic-error mapping matrix B. Although the absorption of the model error into the parameter estimation depends on the "correlation" between H and B matrices, and may look like an observability issue when one uses an empirical function to compensate the model errors, we do not mix the two issues. We do not regard the modeling errors as intrinsic weakness in observability of the measurement system because we believe that models are improvable. Modeling errors can be reduced through model improvement without sacrificing observability. One example is the solar radiation pressure model for GPS satellites. Using the GSPM model developed at JPL (Bar-Sever and Kuang 2004, 2005), one needs to estimate only one scale factor for each satellite, while with the so-called DYB model developed at CODE (Beutler et al. 1994) 9 parameters need to be estimated for each satellite. There is apparently an observability difference due to the number of estimated parameters, but there is no evidence showing that the one-parameter GSPM model is less accurate than the nine-parameter model (Sibthorpe et al. 2011). In this study, we focus on the fundamental observability of the measurement system, and leave the perturbation by systematic model errors to separate study.

We use 40 ground stations as our standard case in our simulated tracking configurations, based on an actual global distribution used in GPS orbit determination practice (Fig. 2). Although it is less realistic on account of cost and logistical constraints, these same stations are used in SLR tracking simulation, implying that differences between the effects of the tracking data types can be directly compared. A real constellation of 30 GPS satellites is used in the simulation. In keeping with the real strategy used in the JPL precision GPS

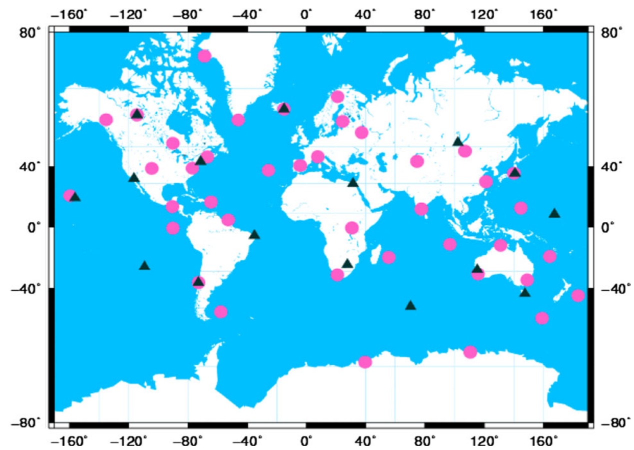


Fig. 2 Distribution of the ground stations used in the simulation. The *red circles* are stations in the 40-station plan and *black triangles* the SLR stations in the 16-station plan

Table 1 Tracking data types and their stochastic properties in the simulation

Data type	Noise level (cm)	Minimum elevation angle (°)
Ground PC	50	7.5
Ground LC	0.65	7.5
LEO PC	50	0
LEO LC	0.65	0
Ground SLR (two-way)	1.3	20

orbit and clock production, we chose 30-h orbit arcs and 7.5° elevation angle cut-off for our study (Desai et al. 2011). The ionosphere-free combinations of pseudorange (PC) and carrier phase (LC) measurements are simulated for GPS tracking, and two-way range measurements are simulated for SLR tracking, all at a 5-min data rate. Corresponding data noise levels and values of the elevation angle cut-off are listed in Table 1. Note that the SLR data noise level is equivalent to 6.5 mm one-way, the same level as GPS LC data noise level being used.

The GIPSY-OASIS software developed by JPL is used to carry out the simulation. Over the last two decades, this software has been used for generating JPL's precision GPS orbit and clock products as well as precise orbit determination products for various LEO satellites with onboard GPS tracking. The filter implementation in GIPSY allows all parameters to be estimated as either constant (bias) parameters or stochastic processes. The estimated parameters and their stochastic properties for this simulation study are listed in Table 2. We use the GSPM (Bar-Sever and Kuang 2004, 2005) to model the solar radiation pressure for GPS satellites, with one scale factor estimated for each satellite. To compensate possible short term modeling errors in GSPM, we also estimate first order Gauss-Markov process noise for the solar

Table 2 Model parameters and their stochastic properties in the simulation

Parameters	Numbers	Estimation and a priori sigma
Orbital initial states	6 per satellite	Constant (1 km, 1 m/s)
Solar radiation pressure scale	1 per satellite	Constant (100 %)
Y-bias	1 per GPS satellite	Constant (1 micron/s^2)
Drag coefficient	1 per LEO satellite	Constant (1,000 %)
Component GPS SRP scale	3 per GPS satellite per hour	First order Gauss-Markov (4-h, 1 %)
Transmitter clock bias	1 per satellite per data epoch	White noise (300 m)
Receiver clock bias	1 per receiver per data epoch	White noise (300 m)
Wet troposphere zenith delay	1 per GPS station per data epoch	Random walk (0.05 mm/$s^{1/2}$)
GPS Carrier phase bias	1 per phase data pass	White noise (300 m)
Geocenter components	3 per 30-h orbit arc	Constant (1 m)

radiation pressure scale factor in each component in the GPS spacecraft-fixed frame, comprising three parameters per GPS satellite. For LEO satellite, we only estimate one solar radiation pressure scale factor for the spacecraft components (box and wings) model.

For a given tracking strategy, we vary the orbital height, inclination and ascending node of the LEO in order to generate a variety of simulated tracking scenarios. Formal errors of the estimated geocenter location parameters under each simulated scenario are collected to evaluate their variation as the geometric factors change. In particular, we compare the formal errors of the estimated geocenter location parameters using GPS tracking against those obtained using SLR tracking data under the same scenario, to study the effect of different tracking data types.

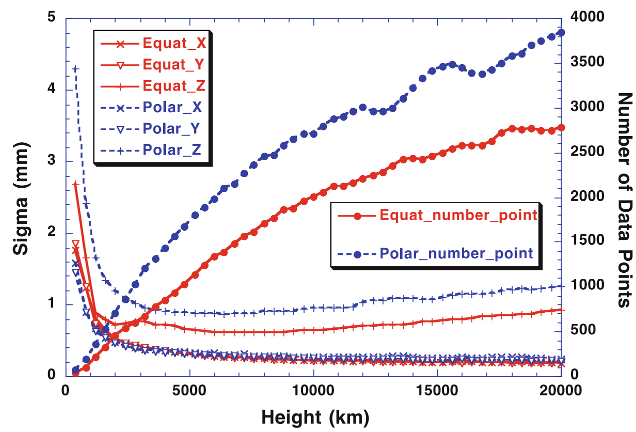


Fig. 3 Formal error for SLR tacking determined geocenter location, equatorial and polar orbits, as function of orbital heights

3 Results

3.1 Ground SLR tracking to one satellite

We first study the scenario of 40 ground stations tracking one satellite using SLR measurements. The satellite orbit (6 initial states plus solar radiation pressure scale factor and drag coefficient) and three components of the geocenter location are estimated using a 30-h orbital arc. Figure 3 shows the formal errors of the estimated geocenter location components for an equatorial orbit and a polar orbit, as function of orbital height. For both orbits, the formal errors for the estimated geocenter location in the XY-plane decrease monotonically as the orbital height increases, while the formal error for Z component reaches minimum near the orbital height of 6,000 km (LAGEOS orbit height is 5,900 km).

In the case of the polar orbit, the component of the estimated geocenter location along the spin (Z) axis has larger formal error than the equatorial orbit does, even though more data points are observed for the former (Fig. 3). This demonstrates the important role of the orbital plane in geocenter location determination. As we pointed out earlier, the Z component of the geocenter location as expressed in Eq. (1) is the distance between CM and the XY-plane of the reference frame. An equatorial orbit plane serves as a realization of the plane passing CM and parallel to the XY-plane, thus the out-of-plane information in the tracking data contributes fully to the determination of the Z component of the geocenter location. In contrast, for a polar orbit the out-of-plane information in the tracking data makes no contribution to Z direction, resulting in poorer observability of the Z component of the geocenter location.

This feature is more clearly illustrated in Fig. 4, which shows the effect of the satellite orbit inclination for satellites at two different orbital heights. Figure 4a shows the formal error of estimated geocenter location for a LEO at the height of 1,400 km (one of the orbital heights proposed for GRASP). The Z component is least well observed for a polar orbit, while the X and Y component are slightly better observed at the inclination of 90°.

Another important factor for the geocenter determination is the Earth rotation. Since the X and Y components of the geocenter location in Eq. (1) represent the location of the

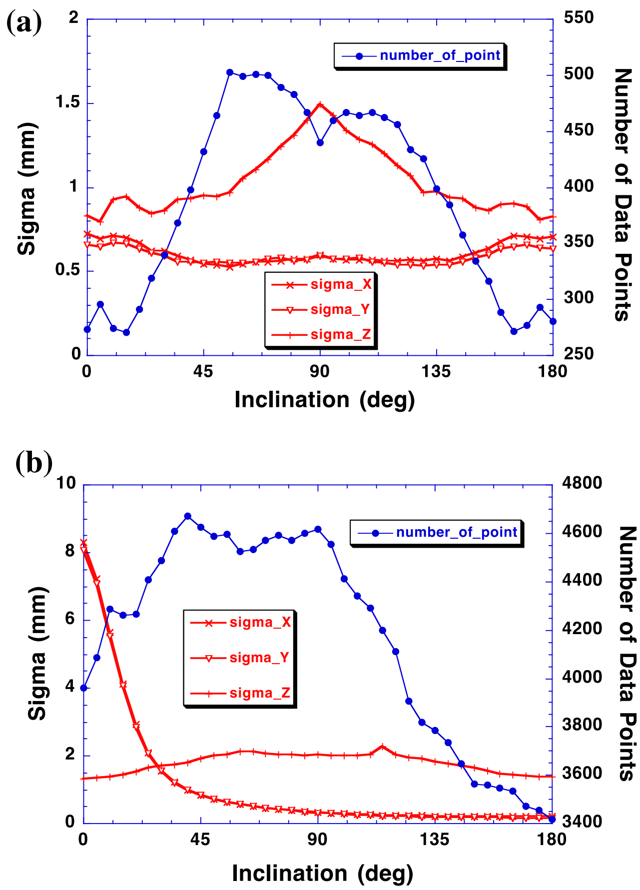


Fig. 4 Formal errors for geocenter location estimated from SLR tracking for LEO and GEO orbits as function of orbital inclination angles. **a** 1,400 km LEO orbits. **b** 36,000 km GEO orbits

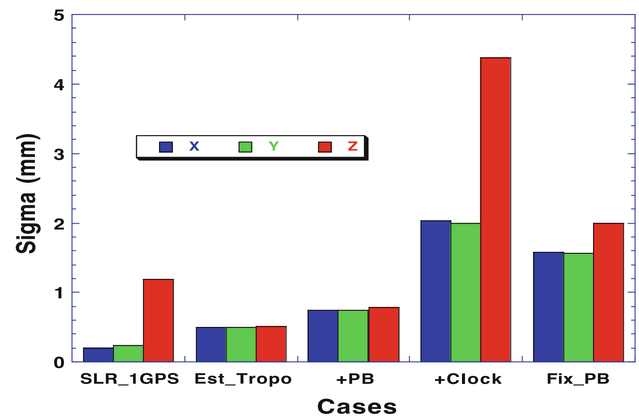


Fig. 5 Formal errors of geocenter location estimation using ground-only GPS tracking compared with SLR tracking

axis of Earth rotation in the XY plane of the reference frame, relative motion between the satellite and ground stations due to Earth rotation has to be observed to determine the X and Y components accurately. Without this relative motion in the measurement, the geocenter location is poorly determined by definition. Figure 4b illustrates this effect in a special way. At the height of 36,000 km, the orbit with 0° inclination is geo-synchronous. The X and Y components of the geocenter location are poorly determined under this tracking scenario, because there is no relative motion between the tracking stations and the satellite. As the orbit inclination increases, the relative motion becomes faster and the observability of X and Y components improves rapidly. On the contrary, the Z component estimation does not benefit from this relative motion.

3.2 Ground GPS tracking to full GPS constellation

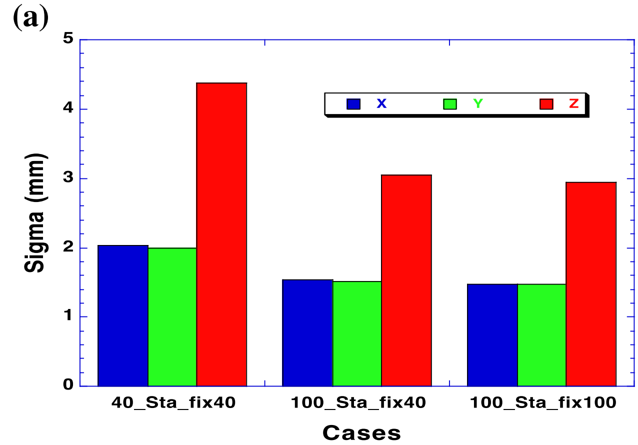
We use the same 40 ground stations to study the tracking configuration for 30 GPS satellites with both pseudorange and carrier phase measurements. In addition to the orbital parameters for all GPS satellites and the geocenter location parameters over the orbit arc, we also need to solve for other parameters due to the one-way radiometric nature of the GPS observations. These parameters include the random-walk zenith wet troposphere delays at each ground station, real-valued phase ambiguity biases for each continuous tracking pass, and clock biases at each measurement time for all transmitters and receivers (except one reference clock). These parameters are listed in Table 2. Figure 5 shows the formal errors of the geocenter location determined using ground GPS tracking data to all 30 GPS satellites, with different measurement model parameters. For comparison we also plot the formal error of the geocenter location determined from SLR tracking to one GPS satellite from the same 40 ground stations. Because of the estimation of troposphere zenith delay for all the ground stations, the formal errors of the GPS determined geocenter location are even larger than that determined from SLR tracking to only one satellite, except for Z component. With 30 times more tracking data points, the formal error of the Z component determined from the GPS tracking is smaller than that from the SLR tracking only by a factor of 2.4 instead of $\sqrt{30}$. With estimation of the carrier phase biases, the formal errors for the estimated geocenter location increase further. Estimating clock biases as white noise degrades the observability most, increasing the formal errors by more than a factor of four for Z component. Apparently the reduction of the observability for geocenter location from ground GPS tracking is significant due to the estimation of the clock, phase biases, and troposphere parameters.

In Fig. 5 we also plot the formal errors for the case where clocks and tropospheric zenith delay parameters are estimated, but not phase biases. This gives an optimistic idea of how much the geocenter location determination can be improved if all the phase bias ambiguities can be successfully resolved. In reality, not all phase bias ambiguities can be successfully resolved, and only the double-differenced phase bias ambiguities instead of one-way phase bias ambiguities can be resolved. We also tested adding Earth Orientation Parameters (bias and rate) to the estimate list; they have very little effect on the determination of geocenter location. Estimating the gradient of tropospheric zenith delay does increase the formal errors of geocenter location moderately. Even with all these parameters included in the estimation, the geocenter location remains recoverable, and we find little to support the conclusion by Rebischung et al. (2014) that the geocenter motion is practically unobservable with GPS tracking.

3.3 Ground and LEO GPS tracking

Kang et al. (2009) pioneered a new method for monitoring geocenter motion by using GRACE GPS tracking data. They used double-differenced GPS measurements between GRACE satellites and ground stations, with all ground stations held fixed to a defined frame, and limited correction applied to IGS GPS orbit products (Dow et al. 2009). With this method they observed annual and semiannual geocenter variations in good agreement with SLR solution, except the Z component. Following Kang et al. (2009) we added one LEO satellite equipped with an onboard GPS receiver to the ground GPS tracking configuration simulation described in previous Sect. (3.2). The onboard GPS receiver is simulated with one up-looking antenna tracking all visible GPS satellites with both pseudorange and carrier phase measurements. Only the main lobe of the transmitted signal (with half beam angle of 22°) is tracked. This limits the number of "visible" GPS satellites and total number of data points above the LEO height of 3,500 km. All the parameters mentioned in previous two sections are solved for, including the orbits of the LEO and all GPS satellites.

Figure 6 shows the formal errors of the geocenter location determined from such tracking configuration, with the LEO orbit at 1,400 km altitude and with 100° inclination angle. For comparison, the ground-only GPS tracking scenario in Fig. 5 is re-plotted in Fig. 6. In addition, a 100-station ground-only tracking scenario is also simulated to test the effect of additional ground stations. Two estimation strategies are tested, one by fixing only the 40 core stations, as done in previous section but estimating the other 60 stations, the other by fixing all 100 stations. Both cases improve the formal errors of the estimated geocenter location by almost the same amount, as shown in Fig. 6a. The amount of reduction in the formal error of Z component is close to what one would expect from the ratio of numbers of stations ($1-\sqrt{(40/100)}$, or 37 %). In contrast, with the addition of one LEO tracking GPS the geocenter location estimation improves moderately if the LEO orbit is determined in near-kinematic mode (by estimating white noise accelerations with sigma of 15,000 nm/s^2 in each of the radial, along-track and cross-track directions). Through dynamic determination of the LEO orbit, however, the formal error of the Z component of the estimated geocenter location is reduced by more than half as shown in Fig. 6b.



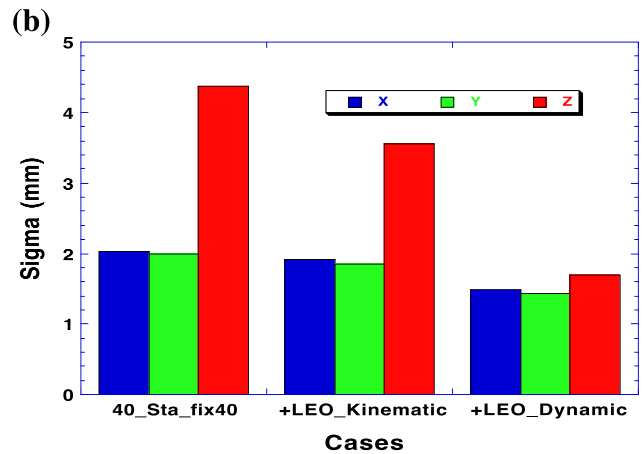


Fig. 6 Formal error of geocenter location estimation using ground-only and LEO-augmented GPS tracking. **a** Improvement by adding ground stations. **b** Improvement by adding LEO satellite

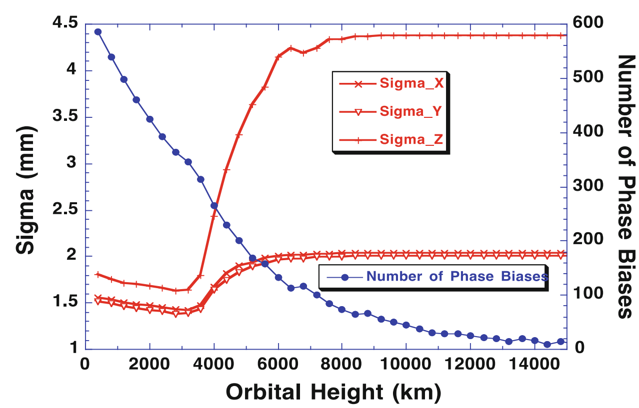


Fig. 7 Formal error of geocenter location estimation as function of the LEO satellite height

Figure 7 shows the formal errors of the estimated geocenter location as a function of the orbital height, for a LEO with inclination angle of 100° (the proposed inclination for GRASP). Instead of decreasing monotonically as the orbital height increases (like what is shown in Fig. 3), the formal errors increase rapidly after the orbital height reaches 3,500 km, due to the reduced number of observation limited by the beam angle of the transmitted signal. Above 3,500 km, a receiver may be outside the main lobe of a transmitted signal even if the transmitter is still "visible" by line of sight. At higher altitude, the receiver sees fewer transmitters, collecting no data when it reaches the GPS orbit height. Therefore, above certain height the formal sigma values stay at the level determined by ground tracking only.

Figure 8 shows the formal errors of the estimated geocenter location as a function of the orbit inclination, for a LEO with orbital altitude of 1,400 km. Comparing it with Fig. 4a, we see a very different pattern of formal error for

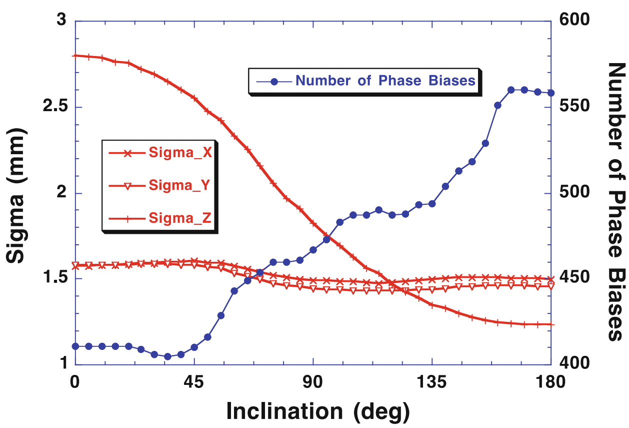


Fig. 8 Formal error of geocenter location estimation as function of the inclination angle of a 1,400-km orbit

Z component. Instead of having a maximum at polar orbit, the formal error of the Z component estimated from LEO GPS tracking decreases monotonically as the orbit inclination increases. A retrograde LEO shows better observability than a prograde one does. The reason for this may lie in the number of phase biases as shown in Fig. 8. This total number of satellite passes (continuous LEO-GPS tracking without phase break) from the onboard receiver over the 30 hours increases as the LEO orbit inclination angle increases. Since the GPS orbits are prograde orbits, a retrograde orbiting receiver has faster relative motion with respect to the GPS satellites, resulting in faster change of tracking geometry. This effect is better illustrated in Fig. 9 by the sky view of the onboard receiver antenna. For the retrograde and prograde equatorial obits shown in Fig. 9, the total numbers of observations is about the same. However, the sky view for the retrograde orbit shows more data passes, because the retrograde orbiter receiver observes a faster turnover of GPS satellites. For the purpose of measuring the length of the moving baselines, tracking more GPS satellites is apparently preferable to tracking fewer satellites over longer periods of time.

3.4 Additional tracking scenarios

In reality, the 40-station SLR tracking scenario is not likely to happen. A high-quality SLR tracking station is expensive to build. Because a SLR station cannot track multiple satellites without proper scheduling and coordinating among SLR stations, the scheduling can be increasingly complicated as the number of SLR satellites increases. In contrast, it is relatively

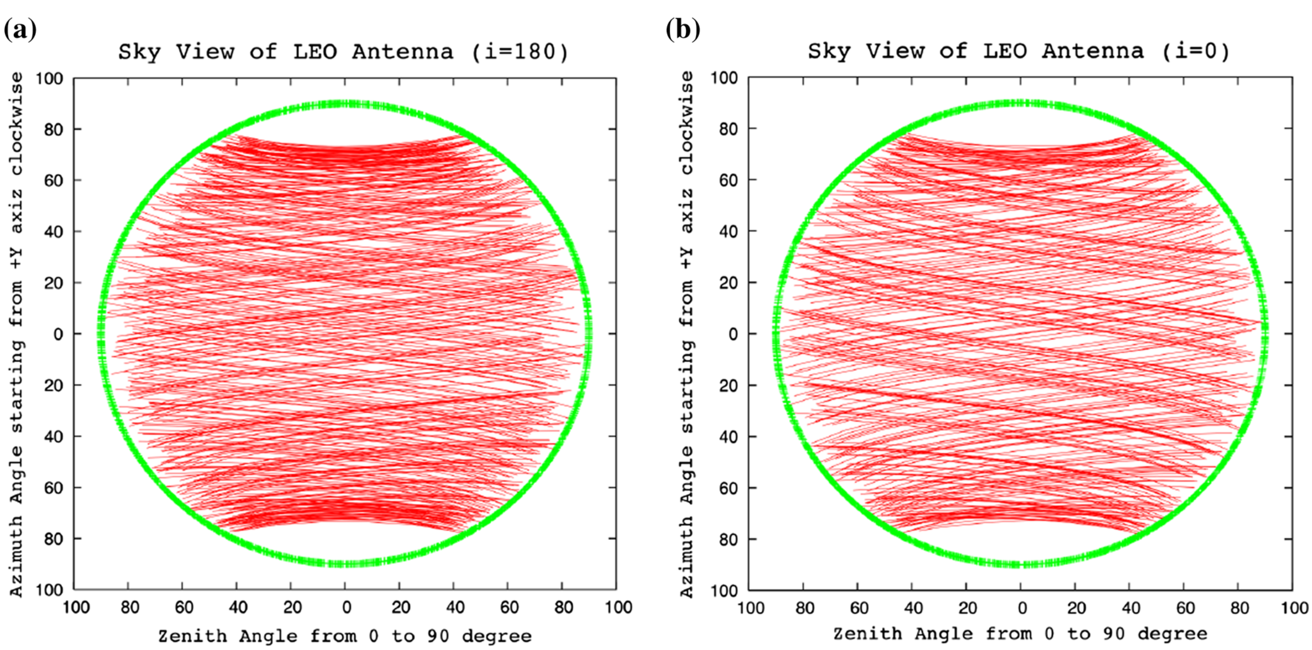


Fig. 9 Sky view of the onboard GPS receiver antenna, for retrograde and prograde orbits. **a** Retrograde equatorial orbit. **b** Prograde equatorial orbit

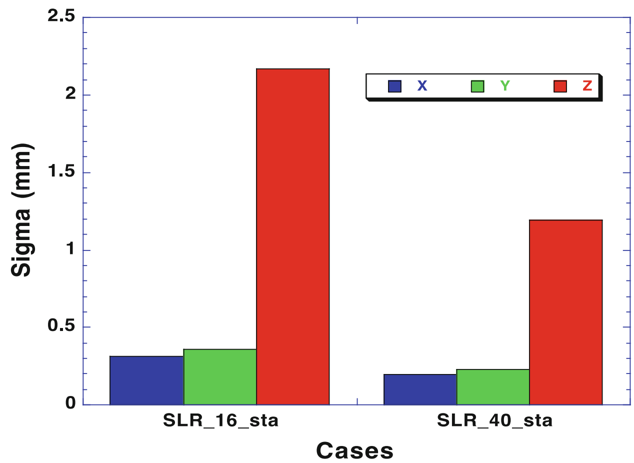


Fig. 10 Formal errors of estimated geocenter location due to different number of SLR tracking stations

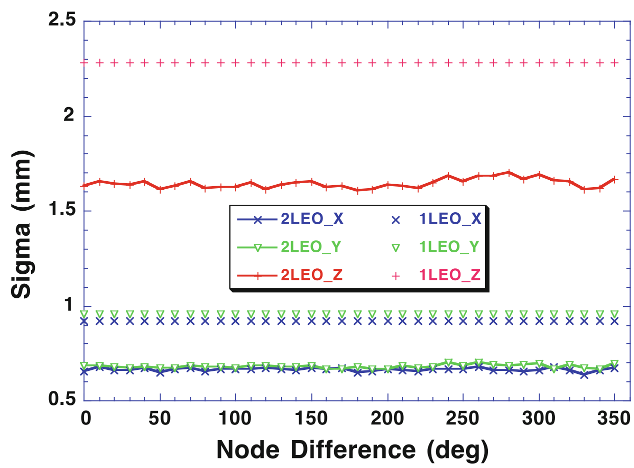


Fig. 11 Formal errors of geocenter location estimated from SLR tracking to two satellites

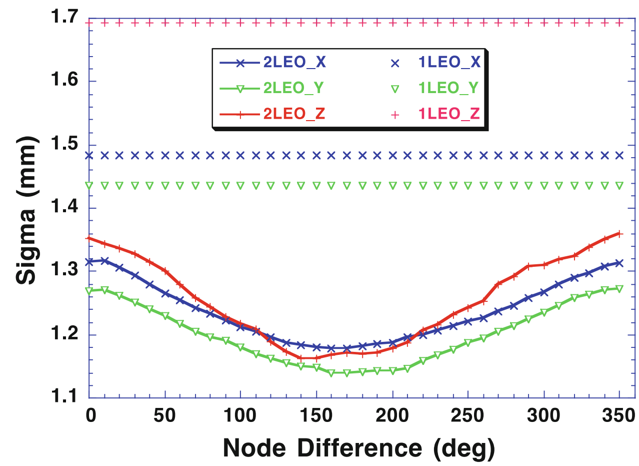


Fig. 12 Formal Errors of geocenter location estimated from GPS tracking to two LEOs

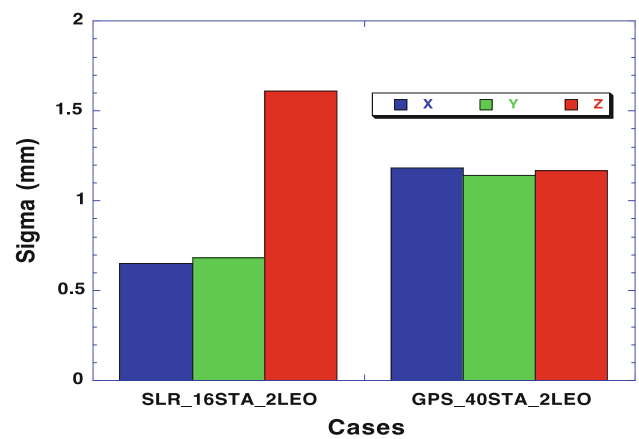


Fig. 13 Optimum formal errors from two-LEO SLR and GPS tracking

easy to extend the GPS tracking to a large ground network, due to the easy deployment of tracking stations. The LEO GPS tracking is completely independent of ground tracking, implying there is no additional burden on the ground stations as LEOs are added to the tracking configuration.

Figure 10 shows the formal error of the estimated geocenter location for a more realistic16-station SLR tracking scenario, compared to the 40-station scenarios. The formal error in the Z component is significantly larger due to the reduced number of SLR stations, and reaches a level similar to that of the dynamic solution obtained from LEO GPS tracking (Fig. 6b).

Figure 11 shows the formal error of the estimated geocenter location by tracking two satellites with the 16 SLR stations. The two satellites orbit at same altitude of 1,400 km and inclination of 100° but with different ascending nodes. The formal error improves by a factor of about $\sqrt{2}$ for all components, compared to the one-satellite tracking scenario, regardless of how much the two orbital planes separate.

Adding another LEO to the GPS tracking scenario improves the estimation of geocenter location too, but the amount of improvement varies as the separation between the two orbital planes changes, as shown in Fig. 12. The improvement is most significant when the two orbital planes separate by 180°, where the formal error for the estimated Z component improves by a factor of about$\sqrt{2}$. For the same two LEO satellites at altitude of 1,400 km and inclination of 100°, the observability of geocenter location from GPS tracking is comparable to that from SLR tracking, as shown in Fig. 13.

4 Summary

We use numerical simulations to study the observability of geocenter location with different satellite tracking configurations. The purpose of this is not to develop a complete error budget for a particular mission. Instead, we focus on formal errors to assess the relative strength of GPS tracking scenarios for determining geocenter location. In particular, we study the effects of adding LEO GPS tracking to ground GPS tracking on overcoming the intrinsic weakness in GPS measurements for geocenter location determination, as compared to SLR measurements. The formal errors, which do not include the effect of dynamic model errors, are not intended to capture the absolute accuracy of geocenter location determination in this study. Unlike the dynamic model errors, the relative change of formal errors in this study is not very sensitive to orbital arc length thus the results in this study apply to longer orbit arc and we expect to see percentage-wise similar improvement of observability over orbit arcs longer than 30 h as well.

The formal errors obtained through the fiducial solution in this study may be a little optimistic because we hold all station positions fixed. In real applications one needs to estimate some or even all ground stations' positions to account for real deformation of the network. This complicates the formal error analysis due to the lack of explicit definition of reference datum. However, this datum definition issue is common to both SLR tracking and GPS tracking. It does not affect the comparison of the information content in the two tracking systems. The datum definition can be solved in various ways. For example, one can always make a fiducial-free solution first to account for the deformation of the network, and use the "network shift" method to transform the freely determined network to a reference frame. In this way we can define a reference frame of the epoch. The analysis in this paper can be performed then by holding all the network station positions fixed to such defined coordinates. This definition is similar to imposing the "no-net rotation" constraint in a fiducial-free solution and should not affect the analysis of the formal errors of the geocenter location estimated under various tracking configurations and the geometric factors impacting geocenter location determination.

Our simulations show that a dynamically defined orbital plane plays an important role in geocenter location determination by serving as the non-rotating reference frame in inertial space. The X and Y components of the geocenter location are determined mainly through the relative motion between the orbit and ground stations due to the Earth rotation. Earth rotation does not help the determination of Z component. Using either SLR or GPS tracking, the Z component is less well determined than the other two components. The Z component of the geocenter location is mainly determined from the measured distance between ground stations and the orbital plane; thus an equatorial orbit is better for Z component determination. Compared to SLR tracking, ground GPS tracking is weaker for locating geocenter due to the estimation of transmitter and receiver clock biases, tropospheric delay and carrier phase biases in the measurements. However, with a large number of ground stations, ground GPS tracking data can still be used to determine the geocenter location reasonably well.

Adding a LEO to the GPS tracking configuration improves the observability of geocenter location to the level comparable to SLR tracking, especially for Z component. The varying baselines between the LEO and GPS ground stations significantly enhance the observability of Z component of the geocenter, surpassing the performance of SLR when tracking two LEOs. Results from investigations using real GPS measurements have already shown improvement in geocenter location determination by adding LEO GPS tracking data to ground GPS tracking data. Haines et al. (2015) added GRACE GPS tracking data to a ground network of up to 46 stations, and determined the geocenter location in a series of 3-day arc solutions over an 11-years period. They see significant improvement in the repeatability of their solutions by adding GRACE tracking data. The bias and trend of the estimated Z component both agree better with ITRF2008/IGb08, although the differences are within the estimated errors. More dramatically, the RMS of the residuals after these signals are removed is improved from 11.1 to 4.7 mm, representing an 80 % reduction in variance. This is a strong evidence of the improved observability of geocenter location. Since dynamic model errors at GRACE orbit height is typically larger than that at GPS orbit height, there must be significantly large amount of new information that is brought in by the added GRACE GPS tracking data to improve the precision of the geocenter location estimates.

The GPS+LEO configuration, as has been proposed for the GRASP mission, offers additional practical advantages relative to SLR tracking. In particular, there is no requirement for line-of-sight visibility between the LEO and ground stations. Tracking is continuous, and there is no need to schedule a ground station for LEO tracking; thus, the network can be easily expanded and LEOs can be independently added. A retrograde LEO near 3,000 km altitude is most appealing because the fast changing tracking geometry provides better measurements of the moving baselines. Adding multiple LEOs in different orbit planes can further improve the observability of geocenter location and is a promising direction for future mission concepts.

A big challenge in LEO-augmented GPS tracking as compared to LEO SLR tracking is the dynamic modeling of the LEO orbit. Satellites carrying GPS receivers usually have large sizes and complex shapes, partially due to the power consumption and data communication requirements. It is more difficult to precisely model the dynamics of these satellites than that of the geodetic satellites specifically designed for SLR tracking, such as LAGEOS. The effect of these dynamic model errors, such as atmospheric drag model and solar radiation pressure model errors, on geocenter location determination should be studied by perturbing these models with different types of errors (e.g., as exercised by Bar-Sever et al. 2012; Meindl et al. 2013). Indeed, the GRASP mission was designed to minimize the modeling errors and to support optimal estimation of geocenter location and other reference frame parameters. However, the force model errors do not represent an intrinsic weakness in observability. While the impact of force model errors is a separate issue from the observability, improved dynamic modeling of GPS-bearing geodetic satellites certainly warrants additional research. Advances in force modeling, however, are only meaningful for systems in which the geocenter location is observable in the first place. Force models are improvable. In particular, if a high-quality accelerometer is carried onboard the LEO, then the effect of dynamical model errors can be minimized and the accuracy of the geocenter location determination can be close to what we analyzed here.

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