04_3_gradient_descent

2020년 11월 30일

In [1]:
```
import numpy as np
import matplotlib.pylab as plt
```

In [2]:
```
def function_2(x):
    if x.ndim == 1:
        return np.sum(x**2)
    else:
        return np.sum(x**2, axis=1) # x0^2 + x1^2
```

In [3]:
```
def numerical_gradient_single_point(f, x, verbose=False):
    h = 1e-4
    grad = np.zeros_like(x)
    if verbose:
        print('x.size={}'.format(x.size)) # (x0, x1) 을 입력으로 받음 --> 2

    for idx in range(x.size): #축별로 계산
        v_keep = x[idx]

        # f(x+h) 계산
        x[idx] = float(v_keep) + h #n차원 입력 중 해당 차원으로만 h를 더하고
        fxh1 = f(x)
        if verbose:
            print(x, '-->', fxh1)

        # f(x-h) 계산
        x[idx] = float(v_keep) - h #n차원 입력 중 해당 차원으로만 h를 빼서
        fxh2 = f(x)
```

```
            if verbose:
                print(x, '-->', fxh2)

            grad[idx] = (fxh1 - fxh2) / (2*h) #n차원 방향의 차분을 구함 !
            x[idx] = v_keep # 값 복원

            if verbose:
                print('grad[{}]={}'.format(idx, grad[idx]))
                print()
        return grad
```

```
In [4]: def numerical_gradient(f, X):
        if X.ndim == 1:
            return numerical_gradient_single_point(f, X) #한지점에서의 gradient를 구하는 함수
        else:
            grad = np.zeros_like(X)

            for idx, x in enumerate(X):
                grad[idx] = numerical_gradient_single_point(f, x)

            return grad
```

```
In [5]: def gradient_descent(f, init_x, lr =0.1, step_num= 100, return_history = False, verbos
        x = init_x
        vx = np.zeros((step_num+1, 2)) # x의 궤적의 변화를 기록하기 위한 list 초기화
        vx[0] = x

        for i in range(step_num):
            grad = numerical_gradient(f, x)

            x -= lr * grad
            if verbose:
                print(x, grad)

            vx[i+1] = x # keep x in the history list

        if return_history:
```

```
            return x, vx
        else:
            return x
```

return_history = False 중간값 뽑을 건지 구분하는 매개변수

```
In [6]: init_x = np.array([-3.0, 4.0])
```

초기값. -3, 4에서 출발함

```
In [7]: learning_rate = 0.1
```

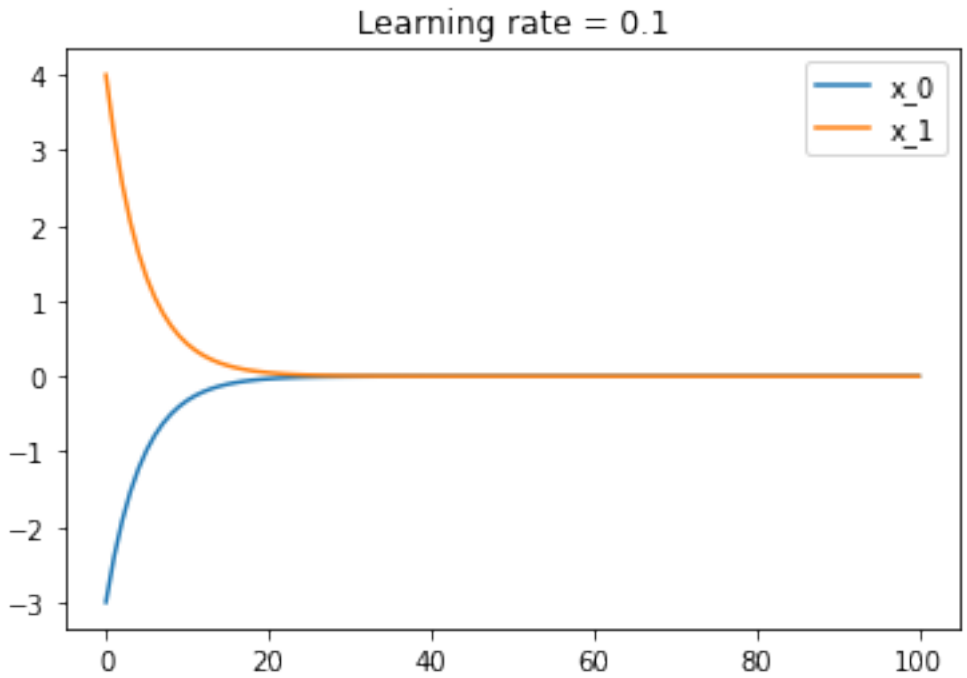
```
In [8]: x_final, x_hist = gradient_descent(function_2, init_x.copy(), lr = learning_rate, retu
```

```
In [9]: x_final
```

```
Out[9]: array([-6.11110793e-10,  8.14814391e-10])
```

```
In [10]: plt.plot(x_hist[:,0])
         plt.plot(x_hist[:,1])
         plt.legend(['x_0', 'x_1'])
         plt.title('Learning rate = {}'.format(learning_rate))
```

```
Out[10]: Text(0.5, 1.0, 'Learning rate = 0.1')
```



```
In [11]: init_x
```

```
Out[11]: array([-3.,  4.])
```

100번까지 반복할 필요가 있을까?

30번 째에서 init_x가 0에 수렴하는 것을 볼 수 있다. -> 30번 정도만 반복해도 괜찮을 것.

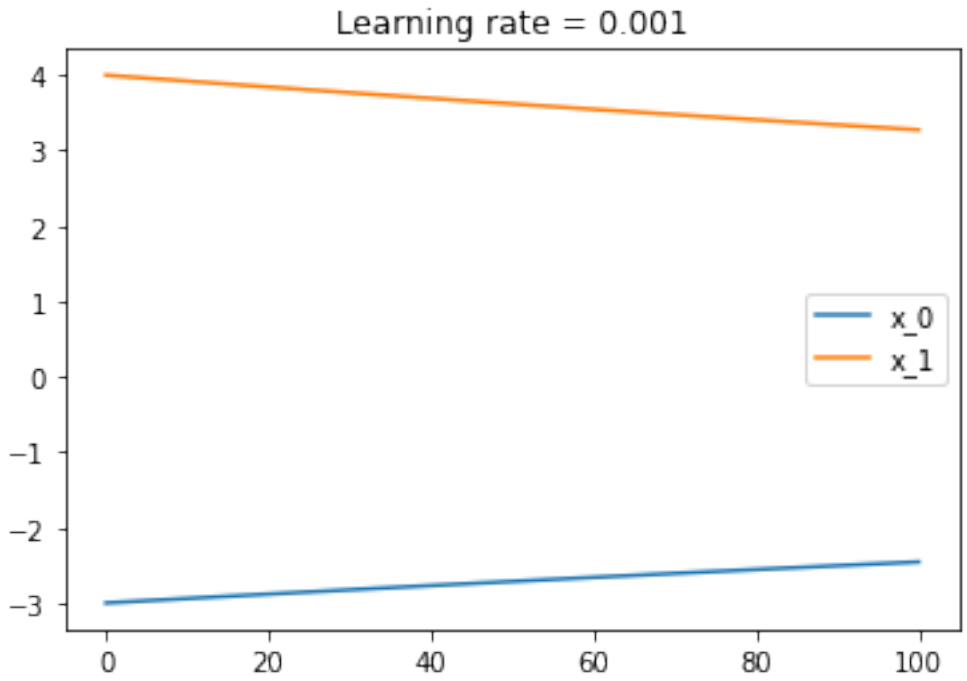
0.1 궤적 그리기

```
In [12]: def gradient3D(init_x):
             x0 = np.arange(-4, 4, 0.5 ) #-2부터 2까지 0.25의 간격으로 -16개
             x1 = np.arange(-4, 4, 0.5)
             X, Y = np.meshgrid(x0, x1)
             nrows = X.shape[0]
             ncols = X.shape[1]

             Xf = X.flatten()
             Yf = Y.flatten()
             arr = np.array([Xf, Yf])
             arr = arr.transpose()#axis = 1로했을 때 이것을 추가해야함

             Z = function_2(arr)
             Z = np.reshape(Z, (nrows,ncols))

             z_hist=function_2(init_x)
             z_hist.shape

             #graph
             fig = plt.figure()
             ax = plt.axes(projection="3d")

             ax.plot_wireframe(X, Y, Z)
             ax.plot(x_hist[:,0],x_hist[:,1],z_hist,'o-',c='orange')
```

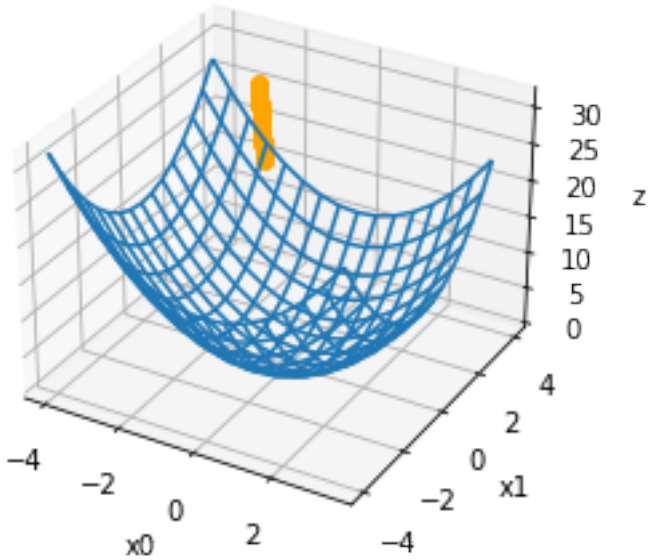
```
        ax.set_xlabel('x0')
        ax.set_ylabel('x1')
        ax.set_zlabel('z')
        plt.show()
```

Learning rate가 0.001일 때

```
In [13]: learning_rate = 0.001

In [14]: x_final, x_hist = gradient_descent(function_2, init_x.copy(), lr = learning_rate, retu

In [15]: print("final : ",x_final)

final :  [-2.45570041  3.27426722]


In [16]: plt.plot(x_hist[:,0])
         plt.plot(x_hist[:,1])
         plt.legend(['x_0', 'x_1'])
         plt.title('Learning rate = {}'.format(learning_rate))

Out[16]: Text(0.5, 1.0, 'Learning rate = 0.001')
```

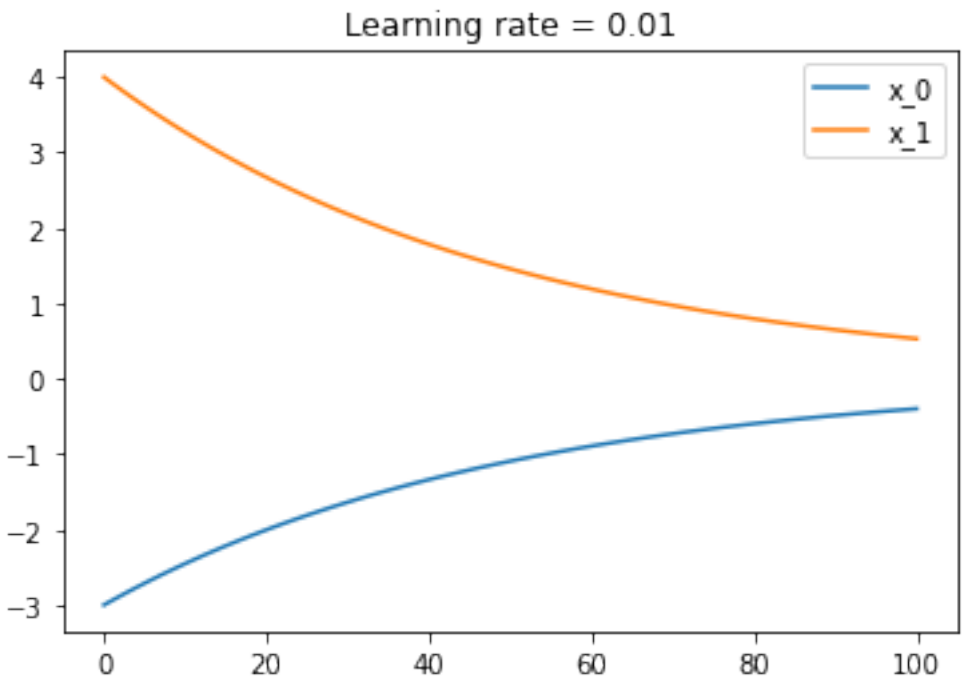


```
In [17]: gradient3D(x_hist)
```

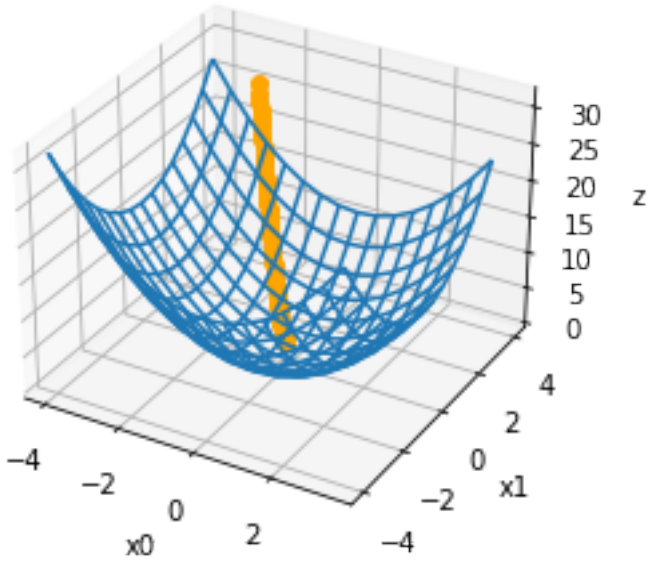


Learning rate가 0.01일 때

```
In [18]: learning_rate = 0.01

In [19]: x_final, x_hist = gradient_descent(function_2, init_x.copy(), lr = learning_rate, retu

In [20]: print("final : ",x_final)

final :  [-0.39785867  0.53047822]


In [21]: plt.plot(x_hist[:,0])
         plt.plot(x_hist[:,1])
         plt.legend(['x_0', 'x_1'])
         plt.title('Learning rate = {}'.format(learning_rate))

Out[21]: Text(0.5, 1.0, 'Learning rate = 0.01')
```

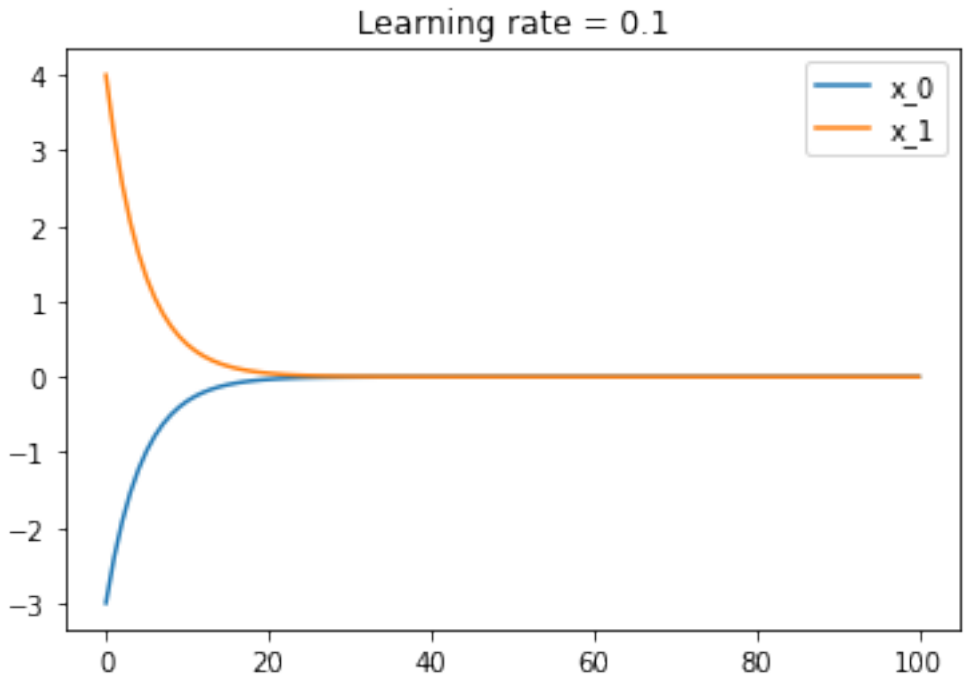


In [22]: `gradient3D(x_hist)`

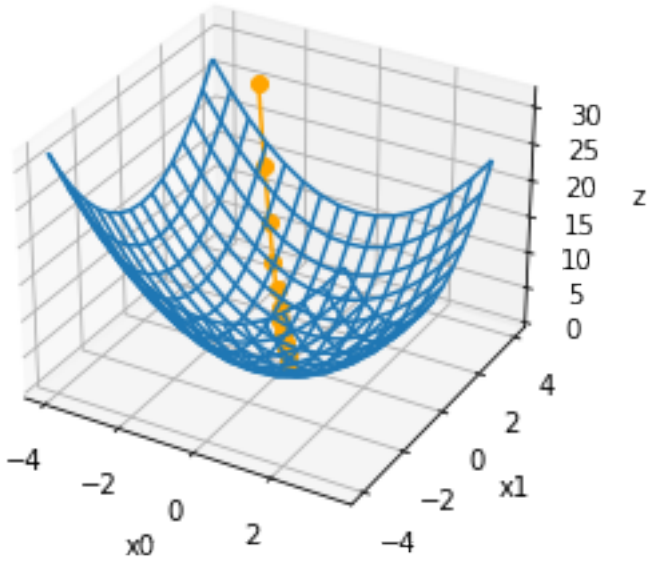


Learning rate가 0.1일 때

```
In [23]: learning_rate = 0.1

In [24]: x_final, x_hist = gradient_descent(function_2, init_x.copy(), lr = learning_rate, retu

In [25]: print("final : ",x_final)

final :  [-6.11110793e-10  8.14814391e-10]


In [26]: plt.plot(x_hist[:,0])
         plt.plot(x_hist[:,1])
         plt.legend(['x_0', 'x_1'])
         plt.title('Learning rate = {}'.format(learning_rate))

Out[26]: Text(0.5, 1.0, 'Learning rate = 0.1')
```




```
In [27]: gradient3D(x_hist)
```



Learning rate가 1.0일 때

```
In [28]: learning_rate =  1.0
```

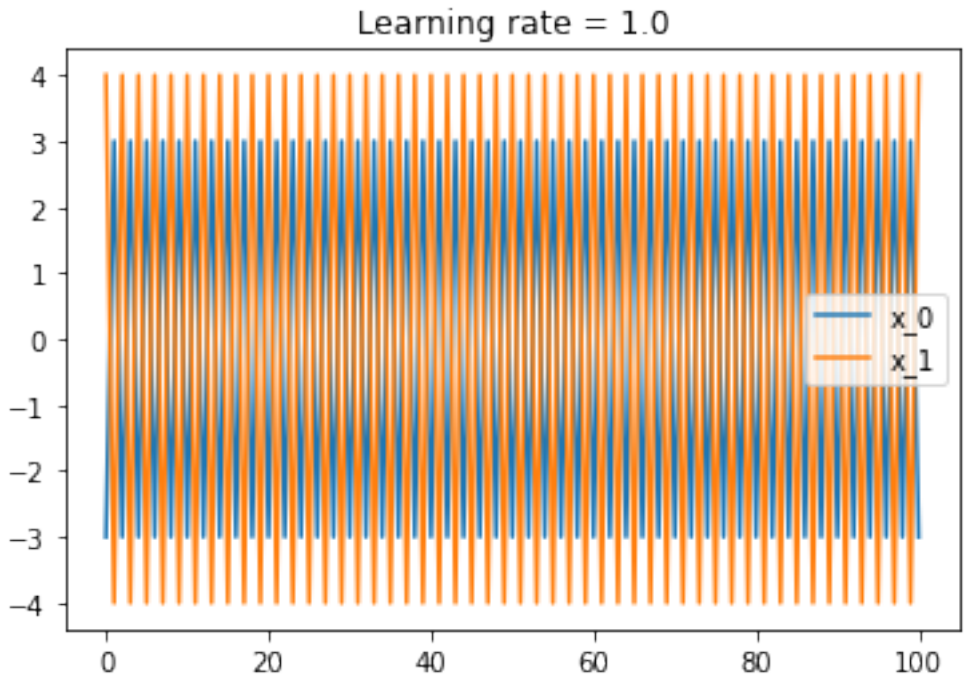
```
In [29]: x_final, x_hist = gradient_descent(function_2, init_x.copy(), lr = learning_rate, retu
```

```
In [30]: print("final : ",x_final)
```

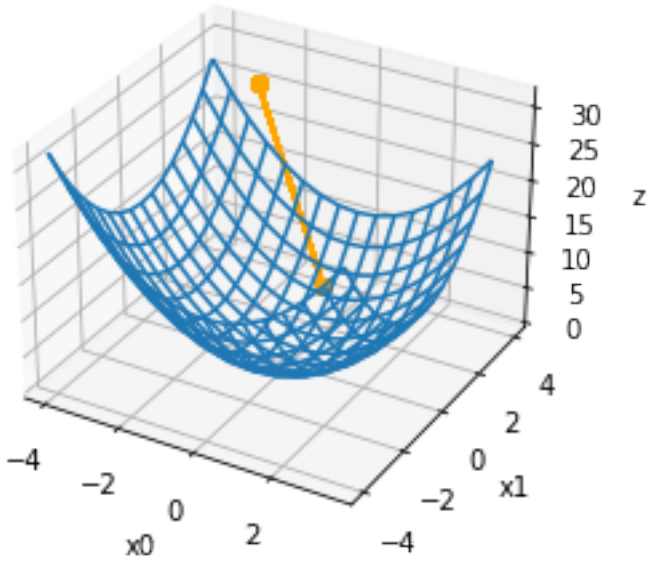
```
final :  [-3.  4.]
```

```
In [31]: plt.plot(x_hist[:,0])
         plt.plot(x_hist[:,1])
         plt.legend(['x_0', 'x_1'])
         plt.title('Learning rate = {}'.format(learning_rate))
```

```
Out[31]: Text(0.5, 1.0, 'Learning rate = 1.0')
```

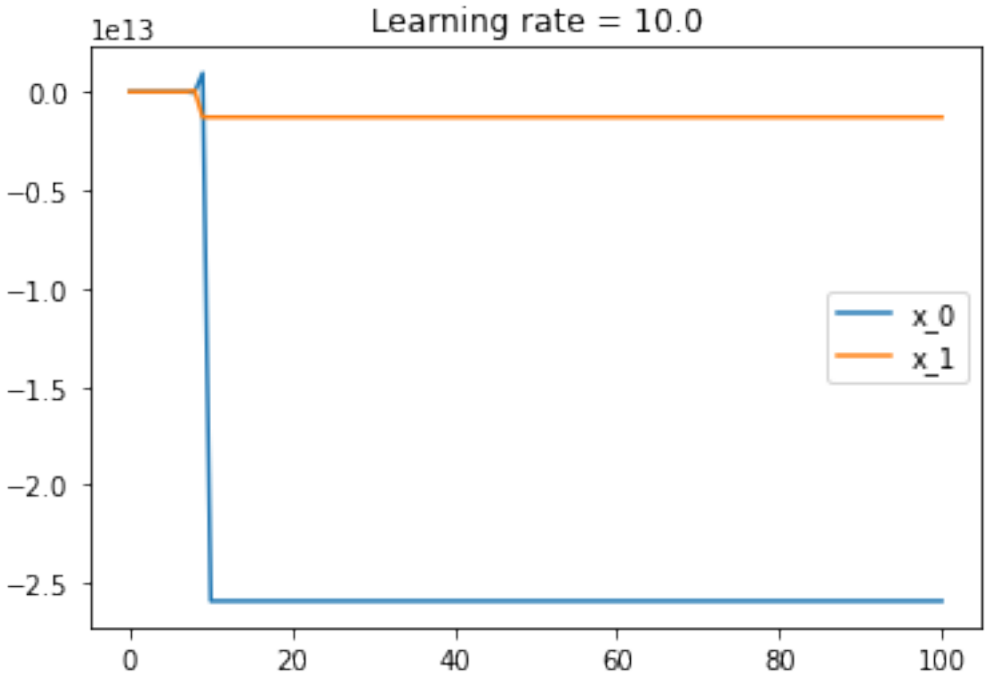


In [32]: `gradient3D(x_hist)`

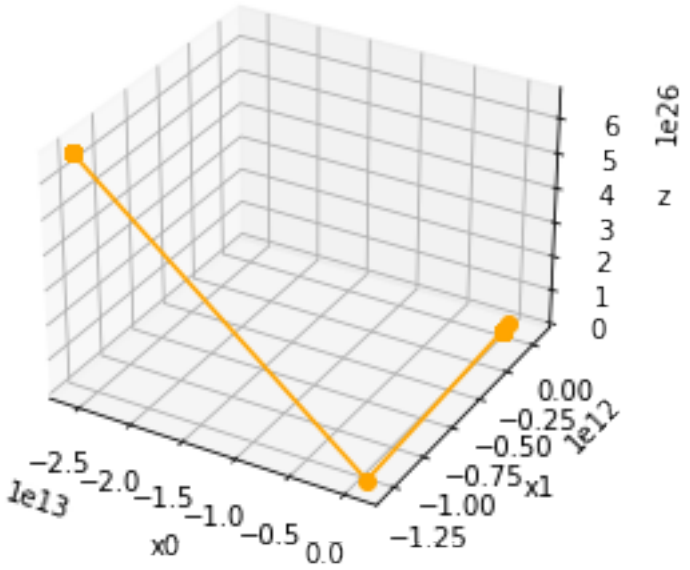


Learning rate가 10.0일 때

```
In [33]: learning_rate =  10.0

In [34]: x_final, x_hist = gradient_descent(function_2, init_x.copy(), lr = learning_rate, retu

In [35]: print("final : ",x_final)

final :  [-2.58983747e+13 -1.29524862e+12]


In [36]: plt.plot(x_hist[:,0])
         plt.plot(x_hist[:,1])
         plt.legend(['x_0', 'x_1'])
         plt.title('Learning rate = {}'.format(learning_rate))

Out[36]: Text(0.5, 1.0, 'Learning rate = 10.0')
```




```
In [37]: gradient3D(x_hist)
```


6
5
4
3
2
1
0
1e26
z
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
1e12
x1
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
1e13
x0