학과: 게임학부
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61. (1) $\underbrace{\begin{pmatrix} 1 & -2 & -3 \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}}_{A}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$

$$\det(A) = \begin{vmatrix} 1 & -2 & -3 \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 7 & -2 & 0 \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = -\begin{vmatrix} 7 & -2 \\ 1 & -1 \end{vmatrix} = -(-7+2) = 5 \neq 0$$

∴ A의 행렬이 가역이므로 일대일이다.

정답: $\boxed{\text{일대일이다.}}$

(2) $C_{11} = +\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} = 1$ $\quad C_{21} = -\begin{vmatrix} -2 & -3 \\ -1 & 0 \end{vmatrix} = 3$ $\quad C_{31} = +\begin{vmatrix} -2 & -3 \\ 0 & 1 \end{vmatrix} = -2$

$C_{12} = -\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = -(0-1) = 1$ $\quad C_{22} = +\begin{vmatrix} 1 & -3 \\ 1 & 0 \end{vmatrix} = 3$ $\quad C_{32} = -\begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} = -7$

$C_{13} = +\begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} = (-2-0) = -2$ $\quad C_{23} = -\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} = -1$ $\quad C_{33} = +\begin{vmatrix} 1 & -2 \\ 2 & 0 \end{vmatrix} = 4$

$$A^{-1} = \frac{1}{5}\begin{pmatrix} 1 & 3 & -2 \\ 1 & 3 & -7 \\ -2 & -1 & 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & \frac{3}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{3}{5} & -\frac{7}{5} \\ -\frac{2}{5} & -\frac{1}{5} & \frac{4}{5} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{5} & \frac{3}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{3}{5} & -\frac{7}{5} \\ -\frac{2}{5} & -\frac{1}{5} & \frac{4}{5} \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{5}w_1 + \frac{3}{5}w_2 - \frac{2}{5}w_3 \\ \frac{1}{5}w_1 + \frac{3}{5}w_2 - \frac{7}{5}w_3 \\ -\frac{2}{5}w_1 - \frac{1}{5}w_2 + \frac{4}{5}w_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

정답: 표준행렬 $= \begin{pmatrix} \frac{1}{5} & \frac{3}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{3}{5} & -\frac{7}{5} \\ -\frac{2}{5} & -\frac{1}{5} & \frac{4}{5} \end{pmatrix}$, $\quad x_1 = \frac{1}{5}w_1 + \frac{3}{5}w_2 - \frac{2}{5}w_3$, $\quad x_2 = \frac{1}{5}w_1 + \frac{3}{5}w_2 - \frac{7}{5}w_3$, $\quad x_3 = -\frac{2}{5}w_1 - \frac{1}{5}w_2 + \frac{4}{5}w_3$

62. (1), (2), (4)

(3)은 공리가 아닙니다. 왜냐하면 양의 공리가 성립하지 않습니다. 근거로는 $\langle \vec{v},\vec{v}\rangle = 2(v_1)^2-3(v_2)^2$ 이되므로 $\langle \vec{v},\vec{v}\rangle \ge 0$가 무조건 성립 되지 않습니다.

(5) 양의 공리가 성립하지 않으므로 공리가 아닙니다. 근거로 $\langle \vec{v},\vec{v}\rangle = (v_1)^2+5v_1v_2+(v_2)^2 \ge 0$ 성립해야 하지만 $v_1=1$, $v_2=-\frac{1}{2}$ 이면 $\langle \vec{v},\vec{v}\rangle = -\frac{5}{4} \le 0$ 때문에 내적이 아니다.

63. (a), (c)

(b)는 $\langle p(x),p(x)\rangle = (a_0)^2+2a_0a_1+(a_1)^2 = (a_0+a_1)^2 \ge 0$ 는 만족하지만 $(a_0+a_1)^2=0$ 일때 $a_0=-a_1$ 일때가 존재하므로 양의 공리 법칙이 성립하지 않아 성립하지 않습니다.

(d)은 $\langle \vec{v},\vec{u}\rangle = u_1^2+4u_1u_2+3u_2^2=(u_1+3u_2)(u_1+u_2)$ 이므로 $u_1=2$, $u_2=-1$ 이면 $-1$ 이되므로 0보다 작아 양의 공리가 성립하지 않습니다.

64. (1) 정답! $\boxed{-7}$

$\langle p,q\rangle = (5\times -2)+(-1\times 0)+(1\times 3) = -10+3=-7$

(2) 정답! $\boxed{\sqrt{14}}$

$\|p\| = \sqrt{3\times 3+(-2)\times(-2)+(-1)\times(-1)} = \sqrt{9+4+1} = \sqrt{14}$

(3) 정답! $\boxed{\sqrt{38}}$

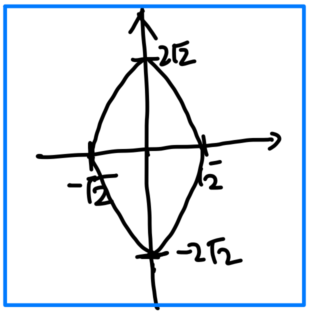
$d(p,q) = \|p-q\| = \|(5,-1,1)-(2,1,-4)\| = \|(3,-2,5)\| = \sqrt{9+4+25} = \sqrt{38}$

(4) 정답! $\boxed{-\frac{\sqrt{7}}{7}}$

$\cos\theta = \frac{5\times 0+1\times(-3)+(-2)\times 1}{\sqrt{1+4}\cdot\sqrt{25+9+1}} = \frac{-5}{\sqrt{5}\times\sqrt{35}} = -\frac{1}{\sqrt{7}} = -\frac{\sqrt{7}}{7}$

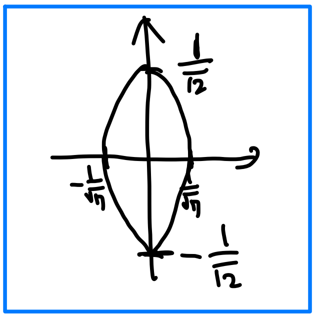
65. (1) $\vec{u}=(x,y)$라고 하면 $\|\vec{u}\| = \sqrt{\langle \vec{u},\vec{u}\rangle} = \sqrt{\frac{1}{2}x^2+\frac{1}{8}y^2}=1 \Rightarrow \frac{x^2}{2}+\frac{y^2}{8}=1$

정답!




(2) $\vec{u}=(x,y)$라고 하면 $\|\vec{u}\| = \sqrt{\langle \vec{u},\vec{u}\rangle} = \sqrt{7x^2+2y^2}=1 \Rightarrow 7x^2+2y^2=1$

정답!



66.

$$\|(1,-2)\| = \sqrt{3\times1\times1+5\times(-2)\times(-2)} = \sqrt{3+20} = \sqrt{23}$$

정답 : $\boxed{\sqrt{23}}$

67.

$$\langle \vec{u},\vec{v}\rangle = (u_1\ u_2)\begin{pmatrix}5 & -2\\ -2 & 3\end{pmatrix}\begin{pmatrix}v_1\\ v_2\end{pmatrix} = (5u_1-2u_2\quad -2u_1+3u_2)\begin{pmatrix}v_1\\ v_2\end{pmatrix}$$

$$= 5u_1v_1-2u_2v_1-2u_1v_2+3u_2v_2$$

$$\|(1,-2)\| = \sqrt{5\times1\times1-2\times(-2)\times1-2\times1\times(-2)+3\times(-2)\times(-2)} = \sqrt{5+4+4+12}$$

$$= \sqrt{25} = 5$$

정답 : $\boxed{5}$

68.

$$\vec{u} = \begin{pmatrix}u_1\\ u_2\end{pmatrix},\quad \vec{v} = \begin{pmatrix}v_1\\ v_2\end{pmatrix}$$

$$\langle \vec{u},\vec{v}\rangle = (u_1\ u_2)\begin{pmatrix}3 & -1\\ -1 & 2\end{pmatrix}\begin{pmatrix}v_1\\ v_2\end{pmatrix} = (3u_1-u_2\quad -u_1+2u_2)\begin{pmatrix}v_1\\ v_2\end{pmatrix}$$

$$= 3u_1v_1-u_2v_1-u_1v_2+2u_2v_2$$

$$\cos\theta = \frac{-3-1+2+4}{\sqrt{3+2+2}\cdot\sqrt{3-4+8}} = \frac{2}{\sqrt{7}\cdot\sqrt{7}} = \frac{2}{7}$$

정답 : $\boxed{\frac{2}{7}}$

69. (1)

$$\|p\| = \sqrt{\int_{-1}^{1} (2-3x)^2 dx} = \sqrt{\int_{-1}^{1} 9x^2-6x+4\, dx} = \sqrt{\left[3x^3-3x^2+4x\right]_{-1}^{1}}$$

$$= \sqrt{4-(-3-3-4)} = \sqrt{4+10} = \sqrt{14}$$

정답 : $\boxed{\sqrt{14}}$

(2)

$$\cos\theta = \frac{\langle p, q\rangle}{\|p\|\ \|q\|} = \frac{\int_{-1}^{1} -8+6x\, dx}{\sqrt{\int_{-1}^{1} 4\, dx}\cdot\sqrt{\int_{-1}^{1} 9x^2-24x+16}}$$

$$= \frac{\left[3x^2-8x\right]_{-1}^{1}}{\sqrt{\left[4x\right]_{-1}^{1}}\cdot\sqrt{\left[3x^3-12x^2+16x\right]_{-1}^{1}}} = 0$$

$\neq 0$

$\cos\theta = 0$ 이므로 $\theta = 90°$ 정답 : $\boxed{90°}$

70. (1)

정답 : $\boxed{x=3t,\ y=-4t,\ z=-5t}$

(2)

정답 : $\boxed{2x-4y+3z=0}$

(3) $\left(\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 2 & -1 & 4 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & -5 & 10 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right)$

선도변수 : $x, y$ 자유변수 : $z=t$ $x=t,\ y=2t,\ z=t$

정답 : $\boxed{x=-t,\ y=2t,\ z=t}$