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50. $|\lambda I - A| = \begin{vmatrix} \lambda-2 & -6 \\ -3 & \lambda+1 \end{vmatrix} = (\lambda-2)(\lambda+1)-18 = \lambda^2-\lambda-20$

$= (\lambda-5)(\lambda+4) = 0 \quad \therefore \lambda = 5$ 또는 $\lambda = -4$

① $\lambda = 5$ 일 때

$\begin{pmatrix} 3 & -6 & 0 \\ -3 & 6 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 선도변수: x, 자유변수: $y = t$ $\quad x = 2t \;\; y = t$ 고유공간: $\{(2,1)t \mid t \in \mathbb{R}\}$ $\quad$ 고유벡터: $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

② $\lambda = -4$ 일 때

$\begin{pmatrix} -6 & -6 & 0 \\ -3 & -3 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 선도변수: x, 자유변수: $y = t$ $\quad x = -t \;\; y = t$ 고유공간: $\{(1,-1)t \mid t \in \mathbb{R}\}$ $\quad$ 고유벡터: $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$P = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \quad P^{-1} = \frac{1}{-2-1}\begin{pmatrix} -1 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{pmatrix} \quad D\begin{pmatrix} 5 & 0 \\ 0 & -4 \end{pmatrix}$

$A = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 5 & 0 \\ 0 & -4 \end{pmatrix}\begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{pmatrix}$

50.(2)

$$e^A = P e^D P^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} e^5 & 0 \\ 0 & e^{-4} \end{pmatrix} \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{pmatrix}$$

$$= \frac{1}{3} \begin{pmatrix} 2e^5 & e^{-4} \\ e^5 & -e^{-4} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2e^5+e^{-4} & 2e^5-2e^{-4} \\ e^5-e^{-4} & e^5+2e^{-4} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2e^5+e^{-4}}{3} & \frac{2e^5-2e^{-4}}{3} \\ \frac{e^5-e^{-4}}{3} & \frac{e^5+2e^{-4}}{3} \end{pmatrix}$$

(3)

$$A^{2023} = P P^{2023} P^{-1} = \frac{1}{3} \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 5^{2023} & 0 \\ 0 & (-4)^{2023} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2\cdot 5^{2023}+(-4)^{2023}}{3} & \frac{2\cdot 5^{2023}-2(-4)^{2023}}{3} \\ \frac{5^{2023}-(-4)^{2023}}{3} & \frac{5^{2023}+2\cdot(-4)^{2023}}{3} \end{pmatrix}$$

5. (1) $|\lambda I - A| = \begin{vmatrix} \lambda-2 & -1 \\ -3 & \lambda-4 \end{vmatrix} = \lambda^2-6\lambda+8-3 = \lambda^2-6\lambda+5 = (\lambda-5)(\lambda-1)=0$

$\therefore \lambda=1$ 또는 $\lambda=5$

① $\lambda=1$ 일 때

$\begin{pmatrix} -1 & -1 & 0 \\ -3 & -3 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 선도변수: x 자유변수: y 즉 $x=-y$

고유벡터: $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ 고유치: $\{(1,-1)t \mid t \in R\}$

② $\lambda=5$ 일 때

$\begin{pmatrix} 3 & -1 & 0 \\ -3 & 1 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 선도변수: x 자유변수: y 즉 $x=\frac{1}{3}y$

고유벡터: $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ 고유치: $\{(1,3)t \mid t \in R\}$

$D = \begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$ $P = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$ $P^{-1} = \frac{1}{4}\begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} \frac{3}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

51.(2) $e^A = P e^D P^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} e & 0 \\ 0 & e^5 \end{pmatrix} \begin{pmatrix} \frac{3}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

$= \left(\begin{array}{cc|c} e & e^5 & \\ -e & 3e^5 & \end{array}\right) \left(\begin{array}{cc|c} \frac{3}{4} & -\frac{1}{4} & \\ \frac{1}{4} & \frac{1}{4} & \end{array}\right) = \begin{pmatrix} \frac{3e+e^5}{4} & \frac{-e+e^5}{4} \\ \frac{-3e+3e^5}{4} & \frac{e+3e^5}{4} \end{pmatrix}$

(3) $A^{2023} = P D^{2023} P^{-1} = \left(\begin{array}{cc|c} 1 & 1 & \\ -1 & 3 & \end{array}\right) \left(\begin{array}{cc|c} 1 & 0 & \\ 0 & 5^{2023} & \end{array}\right) \begin{pmatrix} \frac{3}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$

$= \left(\begin{array}{cc|c} 1 & 5^{2023} & \\ -1 & 3\cdot 5^{2023} & \end{array}\right) \left(\begin{array}{cc|c} \frac{3}{4} & -\frac{1}{4} & \\ \frac{1}{4} & \frac{1}{4} & \end{array}\right) = \begin{pmatrix} \frac{3+5^{2023}}{4} & \frac{-1+5^{2023}}{4} \\ \frac{-3+3\cdot 5^{2023}}{4} & \frac{1+3\cdot 5^{2023}}{4} \end{pmatrix}$

52.(1)

$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 0\ 2 & 0 \\ 0 & 1 & 3 & 5\ 0 & 0 \\ 1 & 3 & 2 & 5\ 2 & 0 \\ 1 & 0 & -7 & -8\ 0 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 2 & -1 & 0\ 2 & 0 \\ 0 & 1 & 3 & 5\ 0 & 0 \\ 0 & 1 & 3 & 5\ 0 & 0 \\ 0 & -2 & -6 & -8\ -2 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & -7 & 0\ 2 & 0 \\ 0 & 1 & 3 & 5\ 0 & 0 \\ 0 & 0 & 0 & 1\ -1 & 0 \\ 0 & 0 & 0 & 0\ 0 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & -7 & 0\ -8 & 0 \\ 0 & 1 & 3 & 0\ 5 & 0 \\ 0 & 0 & 0 & 1\ -1 & 0 \\ 0 & 0 & 0 & 0\ 0 & 0 \end{array}\right)$ (열: $x_1\ x_2\ x_3\ x_4\ x_5$)

선도변수: x_1, x_2, x_4 자유변수: $x_3 = s,\ x_5 = t$ $\Rightarrow (7s+8t, -3s-5t, s, t, t)$

$\therefore x_1 = 7s+8t,\ x_2 = -3s-5t,\ x_4 = t$

$\{(7,-3,1,0,0)s + (8,-5,0,1,1)t \mid s,t \in R\}$

영공간의 기저: $\{(7\ -3\ 1\ 0\ 0), (8\ -5\ 0\ 1\ 1)\}$

정답: 영공간의 기저: $\{(7\ -3\ 1\ 0\ 0), (8\ -5\ 0\ 1\ 1)\}$

52.(2)

$$A=\begin{pmatrix}1&2&-1&0&2\\0&1&3&5&0\\1&3&2&5&2\\1&0&-7&-8&0\end{pmatrix} \xrightarrow{\text{기본행연산}} R=\begin{pmatrix}1&0&-7&0&-8\\0&1&3&0&5\\0&0&0&1&-1\\0&0&0&0&0\end{pmatrix}$$

정답! 열공간의 기저: $\left\{\begin{pmatrix}1\\0\\1\\1\end{pmatrix},\begin{pmatrix}2\\1\\3\\0\end{pmatrix},\begin{pmatrix}0\\5\\5\\-8\end{pmatrix}\right\}$

(3)

(A의 영공간)$^{\perp}$ = A의 행공간 ⇒ 행공간의 기저: $\{(1\ 2\ -1\ 0\ 2),(0\ 1\ 3\ 5\ 0),(1\ 3\ 2\ 5\ 2)\}$

R의 행공간의 기저: $\{(1\ 0\ -7\ 0\ -8),(0\ 1\ 3\ 0\ 5),(0\ 0\ 0\ 1\ -1)\}$

①과 ②은 A의 행공간의 기저에 존재, ③은 $(0\ 1\ 3\ 0\ 5)+4(0\ 0\ 0\ 1\ -1)$ 성립

⑤은 $2(0\ 0\ 0\ 1\ -1)$로 성립

정답! ④

53.(1) $A=\begin{pmatrix}-3&-1&1&8&-3\\1&2&3&-1&1\\2&5&8&-1&2\end{pmatrix}\to\begin{pmatrix}1&2&3&-1&1\\-3&-1&1&8&-3\\2&5&8&-1&2\end{pmatrix}\to\begin{pmatrix}1&2&3&-1&1\\0&5&10&5&0\\0&1&2&1&0\end{pmatrix}$

$$\to\begin{pmatrix}1&2&3&-1&1\\0&1&2&1&0\\0&5&10&5&0\end{pmatrix}\to\begin{pmatrix}1&0&-1&-3&1\\0&1&2&1&0\\0&0&0&0&0\end{pmatrix}$$

정답! 해공간의 기저: $\{(1\ 0\ -1\ -3\ 1),(0\ 1\ 2\ 1\ 0)\}$

53.(2) $\begin{pmatrix} -3 & -1 & 1 & 8 & -3 \\ 1 & 2 & 3 & -1 & 1 \\ 2 & 5 & 8 & -1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & -1 & 1 \\ -3 & -1 & 1 & 8 & -3 \\ 2 & 5 & 8 & -1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & -1 & 1 \\ 0 & 5 & 10 & 5 & 0 \\ 0 & 1 & 2 & 1 & 0 \end{pmatrix}$

$\to \begin{pmatrix} 1 & 2 & 3 & -1 & 1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 5 & 10 & 5 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -1 & -3 & 1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

정답! 열공간의 기저: $\left\{ \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \right\}$

(3) $\begin{pmatrix} -3 & -1 & 1 & 8 & -3 \\ 1 & 2 & 3 & -1 & 1 \\ 2 & 5 & 8 & -1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & -1 & 1 \\ -3 & -1 & 1 & 8 & -3 \\ 2 & 5 & 8 & -1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & -1 & 1 \\ 0 & 5 & 10 & 5 & 0 \\ 0 & 1 & 2 & 1 & 0 \end{pmatrix}$

$\to \begin{pmatrix} 1 & 2 & 3 & -1 & 1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 5 & 10 & 5 & 0 \end{pmatrix} \to \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ \begin{pmatrix} 1 & 0 & -1 & -3 & 1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$

선두변수: x_1, x_2

자유변수: x_3, x_4, x_5

nullity(A)=3 정답! nullity(A)=3

(4) (A의 영공간)$^{\perp}$ = A의 행공간의 기저: $\{(-3\ -1\ 1\ 8\ -3), (1\ 2\ 3\ -1\ 1)\}$

A의 행사다리꼴을 R ⇒ R의 행공간의 기저: $\{(1\ 0\ -1\ -3\ 1), (0\ 1\ 2\ 1\ 0)\}$

①은 $-(-3\ -1\ 1\ 8\ -3)$로 성립, ②는 $2(1\ 2\ 3\ -1\ 1)+(0\ 1\ 2\ 1\ 0)$로 성립, ③은 R의 행공간의 기저에 존재, ④는 $(1\ 2\ 3\ -1\ 1)+(0\ 1\ 2\ 1\ 0)$로 성립

정답! ⑤

54.(1) 4개, 이유? 선도변수개수 ≤ min(4,7)=4

(2) 6개, 이유? 계약 행사다리꼴이 성립해야 하므로 선도변수적어도 1개가 필요하다. 그러므로 rank(A)=1이다.

rank(A)+nullity(A)=7 ⇒ 1+nullity(A)=7 ⇒ nullity(A)=6

(3) 4개, 이유? 선도변수개수 ≤ min(7,4) = 4

(4) 3개, 이유? rank(A)+nullity(A)=4 이 성립되므로 rank(A)가 최소 개수가 되어야 한다. 하지만 rank(A)는 0은 성립할 수 없다. 왜냐하면 계약 행사다리꼴이 성립해야 하므로 선도변수 적어도 1개가 필요하다.

1+nullity(A)=4 ⇒ nullity(A)=3

55. $\begin{pmatrix} \cos(-60^\circ) & -\sin(-60^\circ) \\ \sin(-60^\circ) & \cos(-60^\circ) \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right)$

$= \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$ 정답!

$\boxed{\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}}$

56. $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}\begin{pmatrix} \cos(120^\circ) & -\sin(120^\circ) \\ \sin(120^\circ) & \cos(120^\circ) \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & 0 \\ \frac{\sqrt{3}}{2} & 0 \end{pmatrix}$

$= \begin{pmatrix} -\frac{1}{6} & 0 \\ \frac{\sqrt{3}}{6} & 0 \end{pmatrix}$ 정답!

$\boxed{\begin{pmatrix} -\frac{1}{6} & 0 \\ \frac{\sqrt{3}}{6} & 0 \end{pmatrix}}$

57. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \cos(-135^\circ) & -\sin(-135^\circ) \\ \sin(-135^\circ) & \cos(-135^\circ) \end{pmatrix}$

$= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \cos(-135^\circ) & -\sin(-135^\circ) \\ \sin(-135^\circ) & \cos(-135^\circ) \end{pmatrix} = \begin{pmatrix} \sin(-135^\circ) & \cos(-135^\circ) \\ 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 0 \end{pmatrix}$ 정답! $\boxed{\begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 0 \end{pmatrix}}$

58. $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} \sqrt{3} & 1 \\ 1 & -\sqrt{3} \end{pmatrix}$ 정답: $\boxed{\begin{pmatrix} \sqrt{3} & 1 \\ 1 & -\sqrt{3} \end{pmatrix}}$

59. $\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}\cos 150^\circ & -\sin 150^\circ\\ \sin 150^\circ & \cos 150^\circ\end{pmatrix}\begin{pmatrix}2\\4\end{pmatrix}$

$=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}-\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & -\frac{\sqrt{3}}{2}\end{pmatrix}\begin{pmatrix}2\\4\end{pmatrix}=\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -\frac{1}{2}\end{pmatrix}\begin{pmatrix}2\\4\end{pmatrix}=\begin{pmatrix}1-2\sqrt{3}\\ -\sqrt{3}-2\end{pmatrix}$

정답! $\boxed{\begin{pmatrix}-2\sqrt{3}+1\\ -\sqrt{3}-2\end{pmatrix}}$

60. 직선 l을 나타내는 행렬을 F라고 정의하면

$F=\begin{pmatrix}-1&0\\0&1\end{pmatrix}\begin{pmatrix}\cos 45^\circ & -\sin 45^\circ\\ \sin 45^\circ & \cos 45^\circ\end{pmatrix}=\begin{pmatrix}-1&0\\0&1\end{pmatrix}\begin{pmatrix}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}=\begin{pmatrix}-\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}$

$FA=\begin{pmatrix}-\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}0\\\sqrt{2}\end{pmatrix}$ $\quad FB=\begin{pmatrix}-\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2}\end{pmatrix}$

$FC=\begin{pmatrix}-\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}\begin{pmatrix}0\\-1\end{pmatrix}=\begin{pmatrix}-\frac{\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{2}\end{pmatrix}$

정답! $\boxed{A=\left(0,\sqrt{2}\right),B=\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right),C=\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)}$