

$$1. a(1, 3, 2) + b(6, 0, -12) + c(1, -1, 2) = (0, \frac{1}{3}, 2)$$

$$(a, 3a, 2a) + (6b, 0, -12b) + (c, -c, 2c) = (0, \frac{1}{3}, 2)$$

$$\begin{cases} a + 6b + c = 0 \\ 3a - c = \frac{1}{3} \\ 2a - 12b + 2c = 2 \end{cases} = \begin{cases} a + 6b + c = 0 \\ 9a - 3c = 1 \\ a - 6b + c = 1 \end{cases} \rightarrow \begin{cases} 2a + 2c = 1 \\ 12b = -1 \end{cases}$$

$$9a - 3c = 2a + 2c \Rightarrow 7a = 5c \Rightarrow a = \frac{5}{7}c$$

$$12b = -1 \Rightarrow b = -\frac{1}{12}$$

$$a + 6b + c = 0 \Rightarrow a - \frac{1}{2} + c = 0 \Rightarrow a + c = \frac{1}{2}$$

$$\frac{5}{7}c + c = \frac{1}{2} \Rightarrow \frac{12}{7}c = \frac{1}{2} \Rightarrow c = \frac{7}{24}$$

$$a + \frac{7}{24} = \frac{12}{24} \Rightarrow a = \frac{5}{24}$$

정답!

$$\vec{u} = \frac{5}{24} \vec{v}_1 - \frac{1}{12} \vec{v}_2 + \frac{7}{24} \vec{v}_3$$

$$2. |\vec{u}| = \sqrt{4+16+1+9} = \sqrt{30}$$

\vec{u} 와 반대방향으로 크기가 3인 벡터: $-\frac{3}{\sqrt{30}}(-2, 4, -1, 3)$

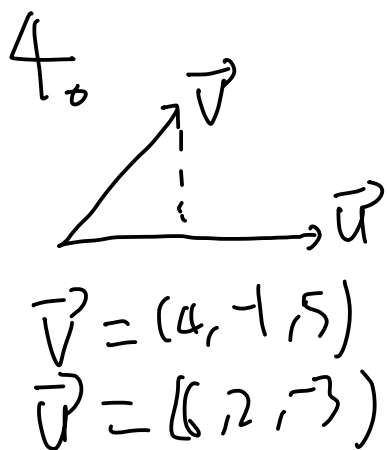
정답: $\sqrt{30}, \left(\frac{6}{\sqrt{30}}, -\frac{12}{\sqrt{30}}, \frac{3}{\sqrt{30}}, -\frac{9}{\sqrt{30}}\right)$

$$3. \cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| \cdot |\vec{v}|} = \frac{2-6+1}{\sqrt{4+9+1} \cdot \sqrt{1+4+1}} \quad 2 \cdot 4$$

$$= \frac{-3}{\sqrt{14} \cdot \sqrt{6}} = \frac{-3}{2\sqrt{21}} = -\frac{3\sqrt{21}}{84}$$

$$\theta = \cos^{-1}\left(-\frac{\sqrt{21}}{28}\right)$$

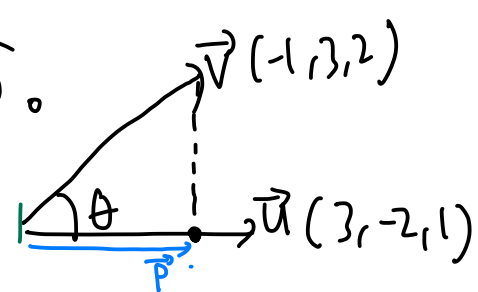
정답: $\cos^{-1}\left(-\frac{\sqrt{21}}{28}\right)$

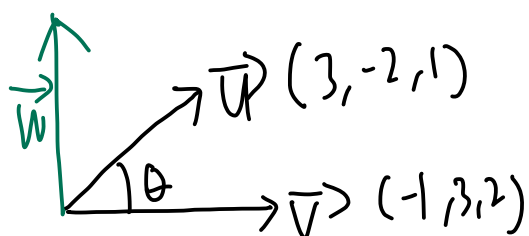


$$\text{proj}_{\vec{u}} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|^2} \cdot \vec{u} = \frac{24-2-15}{36+4+9} (6, 2, -3)$$

$$= \frac{7}{49} (6, 2, -3) = \left(\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\right)$$

정답: $\left(\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\right)$

5. 
$$\vec{P} = \frac{-3-6+2}{9+4+1} (3, -2, 1)$$
$$= \frac{-7}{14} (3, -2, 1)$$
$$= \left(-\frac{3}{2}, 1, -\frac{1}{2}\right)$$



\vec{W} : \vec{V} 의 \vec{U} 에 직교하는 벡터 성분

$$\vec{W} = \vec{V} - \text{proj}_{\vec{U}} \vec{V}$$
$$= (-1, 3, 2) - \frac{1}{2} (-1, 3, 2)$$
$$= (-1, 3, 2) + \left(\frac{1}{2}, -\frac{3}{2}, -1\right)$$
$$= \left(-\frac{1}{2}, \frac{3}{2}, 1\right)$$

정답! $\left(-\frac{3}{2}, 1, -\frac{1}{2}\right), \left(-\frac{1}{2}, \frac{3}{2}, 1\right)$

6. $\vec{PQ} = (-4, 4, 6)$

(a) $\frac{x-3}{-4} = \frac{y+1}{4} = \frac{z+2}{6}$ 이 방향 벡터는 $(-4, 4, 6)$ ①

(b) \vec{PQ} 방향을 법선 벡터로 하면 $-4x+4y+6z=0$ 이거나 점 $(1, 1, 4)$ 대입하면 $-4+4+24=24 \neq 0$ 이므로 틀린 말이다. ②

(c) 방향 벡터는 $(2, -2, -3)$ 이므로 벡터 $-\frac{1}{2}\vec{PQ}$ 와 같다. ③

(d) 법선 벡터 $(1, -5, 4)$ 를 \vec{n} 이라고 하면 $\vec{n} \perp \vec{PQ}$ 이므로 $\vec{n} \cdot \vec{PQ} = 0$.

$(1, -5, 4) \cdot (-4, 4, 6) = -4 - 20 + 24 = 0$ 즉 맞다. ④

(e) $\vec{AB} = (-2, 2, -3)$ 이고 $\vec{BA} = (2, -2, 3)$ 이므로 $\frac{1}{2}\vec{PQ} = (-2, 2, 3)$ 이랑 다르므로 평행은 아니다. ⑤

정답! $(a), (c), (d)$

7. $\vec{PQ} = (-1, 2, -2)$

① $(2, 3, -1) + t(-1, 2, -2) = (-t+2, 2t+3, -2t-1)$

② $x = -t+2, y = 2t+3, z = -2t-1$

③ $\frac{x-2}{-1} = \frac{y-3}{2} = \frac{z+1}{-2}$

8. 법선 벡터 : $(5, -1, 3)$

스칼라 방정식: $5(x-3) - 1(y-2) + 3(z-1) = 5x - y + 3z - 15 + 2 - 3$
 $= 5x - y + 3z - 16 = 0$

정답! $5x - y + 3z = 16$

9. 평면의 법선 방정식: $(2+t_1-t_2, -3-2t_1+5t_2, -2+4t_1+t_2)$

평면의 매개변수 방정식: $x = 2+t_1-t_2, y = -3-2t_1+5t_2, z = -2+4t_1+t_2$

법선 벡터를 $\vec{n} = (a, b, c)$ 로 둔다. $\vec{n} \cdot \vec{u} = \vec{n} \cdot \vec{v} = 0$

$(a, b, c) \cdot (1, -2, 4) = a - 2b + 4c = 0$

$(a, b, c) \cdot (-1, 5, 1) = -a + 5b + c = 0 \xrightarrow{+} a - 5b - c = 0$

$a - 2b + 4c = a - 5b - c \rightarrow 3b = -5c \rightarrow b = -\frac{5}{3}c$

$a - 5b - c = 0 \rightarrow a + \frac{25}{3}c - c = 0 \rightarrow a = -\frac{22}{3}c$

$(a, b, c) = (-\frac{22}{3}c, -\frac{5}{3}c, c) = (-22, -5, 3) = \vec{n}$

스칼라 방정식: $22(x-2) - 5(y+3) + 3(z+2) = 22x - 5y - 3z - 35 = 0$

정답!
 매개변수 방정식: $x = 2+t_1+t_2, y = -3-2t_1+5t_2, z = -2+4t_1+t_2$
 스칼라 방정식: $22x - 5y - 3z - 35 = 0$