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71.

$$(\vec{u})_S = \frac{(2,-1,1)\cdot(1,1,-1)}{1+1+1}(1,1,-1) + \frac{(2,-1,1)\cdot(1,0,1)}{1+0+1}(1,0,1) + \frac{(2,-1,1)(-1,2,1)}{1+4+1}(-1,2,1)$$

$$= \frac{2-1-1}{3}(1,1,-1) + \frac{2+1}{2}(1,0,1) + \frac{-2-2+1}{6}(-1,2,1)$$

$$= 0(1,1,-1) + \frac{3}{2}(1,0,1) + \frac{-\cancel{3}^{\,1}}{\cancel{6}_{\,2}}(-1,2,1)$$

$$= \left(\frac{3}{2},0,\frac{3}{2}\right) + \left(\frac{1}{2},-1,-\frac{1}{2}\right) = (2,-1,1)$$

정답: $\boxed{(\vec{u})_S = (2,-1,1)}$

72. $\vec{u}_3 = (a,b,c) = \left(\frac{1}{\sqrt6}, -\frac{2}{\sqrt6}, \frac{1}{\sqrt6}\right)$

$$\vec{u}_1\cdot\vec{u}_3 = \frac{a}{\sqrt3} + \frac{b}{\sqrt3} - \frac{c}{\sqrt3} = 0 \Rightarrow a+b-c=0$$

$$\vec{u}_2\cdot\vec{u}_3 = \frac{a}{\sqrt2} + \frac{c}{\sqrt2} = 0 \Rightarrow a+c=0$$

$$\Downarrow$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right)$$

선도변수: a, b $a=-t$ $b=2t$ $c=t$
자유변수: $c=t$ $\vec{u}_3 = (1,-2,1) = \left(\frac{1}{\sqrt6}, -\frac{2}{\sqrt6}, \frac{1}{\sqrt6}\right)$

$$U = (\vec{u}_1\cdot\vec{v}, \vec{u}_2\cdot\vec{v}, \vec{u}_3\cdot\vec{v}) = \left(\frac{2}{\sqrt3}+\frac{2}{\sqrt3}+\frac{3}{\sqrt3}, \frac{2}{\sqrt2}-\frac{3}{\sqrt2}, \frac{2}{\sqrt6}-\frac{4}{\sqrt6}+\frac{3}{\sqrt6}\right)$$

$$= \left(\frac{7}{\sqrt3}, -\frac{1}{\sqrt2}, \frac{1}{\sqrt6}\right)$$

정답: $\boxed{U = \left(\frac{7}{\sqrt3}, -\frac{1}{\sqrt2}, \frac{1}{\sqrt6}\right)}$

73.

$\langle (1,0),(0,1)\rangle = 3\cdot 1\cdot 0 + 5\cdot 0\cdot 1 = 0$ 이므로 직교한다.

$\|(1,0)\| = \sqrt{3\cdot 1\cdot 1+5\cdot 0\cdot 0} = \sqrt{3}$, $\|(0,1)\| = \sqrt{3\cdot 0\cdot 0+5\cdot 1\cdot 1} = \sqrt{5}$

정규 직교집합 $= \left\{\left(\frac{1}{\sqrt{3}}, 0\right), \left(0, \frac{1}{\sqrt{5}}\right)\right\}$

74. $A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ -2 & -1 \end{pmatrix} \to u_1 = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}, u_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \Rightarrow V_1 = u_1 = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$

$V_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} - \left(\left(\frac{2}{3}\ -\frac{1}{3}\ -\frac{2}{3}\right)\cdot\left(1\ 0\ -1\right)\right)\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{2}{3} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} - \frac{4}{3}\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{2}{3} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} -\frac{8}{9} \\ \frac{4}{9} \\ \frac{8}{9} \end{pmatrix} = \begin{pmatrix} \frac{1}{9} \\ \frac{4}{9} \\ -\frac{1}{9} \end{pmatrix}$

$q_1 = \begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{2}{3} \end{pmatrix}, q_2 = \begin{pmatrix} \frac{1}{3\sqrt{2}} \\ \frac{4}{3\sqrt{2}} \\ -\frac{1}{3\sqrt{2}} \end{pmatrix} \Rightarrow Q = \begin{pmatrix} \frac{2}{3} & \frac{1}{3\sqrt{2}} \\ -\frac{1}{3} & \frac{4}{3\sqrt{2}} \\ -\frac{2}{3} & -\frac{1}{3\sqrt{2}} \end{pmatrix}$ $R = \begin{pmatrix} \langle u_1, q_1\rangle & \langle u_2, q_1\rangle \\ 0 & \langle u_2, q_2\rangle \end{pmatrix} = \begin{pmatrix} 3 & \frac{4}{3} \\ 0 & \frac{\sqrt{2}}{3} \end{pmatrix}$

정답!

$$\boxed{\begin{pmatrix} 2 & 1 \\ -1 & 0 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3\sqrt{2}} \\ -\frac{1}{3} & \frac{4}{3\sqrt{2}} \\ -\frac{2}{3} & -\frac{1}{3\sqrt{2}} \end{pmatrix}\begin{pmatrix} 3 & \frac{4}{3} \\ 0 & \frac{\sqrt{2}}{3} \end{pmatrix}}$$

75. $A=\begin{pmatrix}2&-1\\1&0\\-4&1\end{pmatrix} \to u_1=\begin{pmatrix}2\\1\\-4\end{pmatrix},\ u_2=\begin{pmatrix}-1\\0\\1\end{pmatrix} \Rightarrow v_1=u_1=\begin{pmatrix}2\\1\\-4\end{pmatrix}$

$$v_2=\begin{pmatrix}-1\\0\\1\end{pmatrix}-\left(\frac{-6}{21}\right)\begin{pmatrix}2\\1\\-4\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix}-\begin{pmatrix}-\frac{4}{7}\\-\frac{2}{7}\\\frac{8}{7}\end{pmatrix}=\begin{pmatrix}-\frac{7}{7}+\frac{4}{7}\\0+\frac{2}{7}\\\frac{7}{7}-\frac{8}{7}\end{pmatrix}=\begin{pmatrix}-\frac{3}{7}\\\frac{2}{7}\\-\frac{1}{7}\end{pmatrix}$$

$$q_1=\begin{pmatrix}\frac{2}{\sqrt{21}}\\\frac{1}{\sqrt{21}}\\\frac{-4}{\sqrt{21}}\end{pmatrix}\quad q_2=\begin{pmatrix}-\frac{3}{\sqrt{14}}\\\frac{2}{\sqrt{14}}\\-\frac{1}{\sqrt{14}}\end{pmatrix}\to Q=\begin{pmatrix}\frac{2}{\sqrt{21}}&-\frac{3}{\sqrt{14}}\\\frac{1}{\sqrt{21}}&\frac{2}{\sqrt{14}}\\-\frac{4}{\sqrt{21}}&-\frac{1}{\sqrt{14}}\end{pmatrix}$$

$$R=\begin{pmatrix}\langle u_1\cdot q_1\rangle&\langle u_2\cdot q_1\rangle\\0&\langle u_2\cdot q_2\rangle\end{pmatrix}=\begin{pmatrix}\sqrt{21}&-\frac{2\sqrt{21}}{7}\\0&\frac{\sqrt{14}}{7}\end{pmatrix}$$

정답!

$$\boxed{\begin{pmatrix}2&-1\\1&0\\-4&1\end{pmatrix}=\begin{pmatrix}\frac{2}{\sqrt{21}}&-\frac{3}{\sqrt{14}}\\\frac{1}{\sqrt{21}}&\frac{2}{\sqrt{14}}\\-\frac{4}{\sqrt{21}}&-\frac{1}{\sqrt{14}}\end{pmatrix}\begin{pmatrix}\sqrt{21}&-\frac{2\sqrt{21}}{7}\\0&\frac{\sqrt{14}}{7}\end{pmatrix}}$$

76.

$$|\lambda I - A| = \begin{vmatrix} \lambda-6 & 2 \\ 2 & \lambda-3 \end{vmatrix} = \lambda^2 - 9\lambda + 18 - 4 = \lambda^2 - 9\lambda + 14 = (\lambda-7)(\lambda-2)$$

① $\lambda = 2$

$$\left(\begin{array}{cc|c} -4 & 2 & 0 \\ 2 & -1 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 2 & -1 & 0 \\ -4 & 2 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$x = \frac{1}{2}t$, $y = t$

고유공간: $\left\{ t\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} \middle| t \in R \right\}$

정규화: $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

② $\lambda = 7$

$$\left(\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 4 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$x = -2t$, $y = t$

고유공간: $\left\{ t\begin{pmatrix} -2 \\ 1 \end{pmatrix} \middle| t \in R \right\}$

정규화: $\begin{pmatrix} -\frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}$

$$P = \begin{pmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \quad P^{-1} = P^T = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \quad D = \begin{pmatrix} 2 & 0 \\ 0 & 7 \end{pmatrix}$$

$$P^{-1}AP = \begin{pmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} 6 & -2 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$$

정답!

$$\boxed{P^{-1}AP = \begin{pmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} 6 & -2 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}}$$

77. (1)

정답!

$$\boxed{\begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}$$

(2)

정답!

$$\boxed{\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ -1 & 4 & \frac{5}{2} \\ 0 & \frac{5}{2} & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}$$