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78. $Q = X^T \underbrace{\begin{pmatrix} 2 & 2 & -2 \\ 2 & -1 & 4 \\ -2 & 4 & -1 \end{pmatrix}}_{A} X = Y^T D Y$

$$|\lambda I - A| = \begin{vmatrix} \lambda-2 & -2 & 2 \\ -2 & \lambda+1 & -4 \\ 2 & -4 & \lambda+1 \end{vmatrix} = \begin{vmatrix} \lambda-2 & -2 & 2 \\ 0 & \lambda-3 & \lambda-3 \\ 2 & -4 & \lambda+1 \end{vmatrix}$$

$$= (\lambda-2)\{(\lambda-3)(\lambda+1)+4(\lambda-3)\} + 2\{-2(\lambda-3)-2(\lambda-3)\}$$

$$= (\lambda-2)(\lambda-3)(\lambda+5) - 8(\lambda-3) = (\lambda-3)(\lambda^2+3\lambda-18) = (\lambda-3)^2(\lambda+6) = 0$$

$\lambda = 3$일때

$$\left(\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ -2 & 4 & -4 & 0 \\ 2 & -4 & 4 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \begin{array}{l} x = 2s-2t \\ y = s \\ z = t \end{array} \Rightarrow \overset{u_1}{\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}} \ \overset{u_2}{\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}} \Rightarrow \overset{v_1}{\begin{pmatrix} \frac{2}{\sqrt5} \\ \frac{1}{\sqrt5} \\ 0 \end{pmatrix}} \ \overset{v_2}{\begin{pmatrix} -\frac{2}{\sqrt{45}} \\ \frac{4}{\sqrt{45}} \\ \frac{5}{\sqrt{45}} \end{pmatrix}}$$

$$u_1 \xrightarrow{\text{정규화}} \begin{pmatrix} \frac{2}{\sqrt5} \\ \frac{1}{\sqrt5} \\ 0 \end{pmatrix} = v_1$$

$$v_2 = u_2 - (v_1 \cdot u_2) \cdot v_1 = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} + \frac{4}{\sqrt5} \begin{pmatrix} \frac{2}{\sqrt5} \\ \frac{1}{\sqrt5} \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{8}{5} \\ \frac{4}{5} \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{2}{5} \\ \frac{4}{5} \\ 1 \end{pmatrix}$$

$$v_2 \xrightarrow{\text{정규화}} \begin{pmatrix} -\frac{2}{\sqrt{45}} \\ \frac{4}{\sqrt{45}} \\ \frac{5}{\sqrt{45}} \end{pmatrix}$$

$\lambda = -6$일때

$$\left(\begin{array}{ccc|c} -8 & -2 & 2 & 0 \\ -2 & -5 & -4 & 0 \\ 2 & -4 & -5 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \begin{array}{l} x = \frac{1}{2}t \\ y = -t \\ z = t \end{array} \quad \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$$

$$X = \begin{pmatrix} \frac{2}{\sqrt5} & -\frac{2}{\sqrt{45}} & \frac{1}{3} \\ \frac{1}{\sqrt5} & \frac{4}{\sqrt{45}} & -\frac{2}{3} \\ 0 & \frac{5}{\sqrt{45}} & \frac{2}{3} \end{pmatrix} Y \qquad D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -6 \end{pmatrix}$$

$$Q = Y^T D Y = 3y_1^2 + 3y_2^2 - 6y_3^2$$

79. $3x^2-4xy+3y^2=6 \to (x\ \ y)\underbrace{\begin{pmatrix}3&-2\\-2&3\end{pmatrix}}_{A}\begin{pmatrix}x\\y\end{pmatrix}$

$|\lambda I-A|=\begin{vmatrix}\lambda-3&2\\2&\lambda-3\end{vmatrix}=\lambda^2-6\lambda+9-4=(\lambda-5)(\lambda-1)$

$\lambda=5$ 일 때

$\left(\begin{array}{cc|c}2&2&0\\2&2&0\end{array}\right)\to\left(\begin{array}{cc|c}1&1&0\\0&0&0\end{array}\right)$ $\begin{matrix}x=-t\\y=t\end{matrix}$ $\begin{pmatrix}-1\\1\end{pmatrix}\to\begin{pmatrix}\frac{1}{\sqrt2}\\-\frac{1}{\sqrt2}\end{pmatrix}$

$\lambda=1$ 일 때

$\left(\begin{array}{cc|c}-2&2&0\\2&-2&0\end{array}\right)\to\left(\begin{array}{cc|c}1&-1&0\\0&0&0\end{array}\right)$ $\begin{matrix}x=t\\y=t\end{matrix}$ $\begin{pmatrix}1\\1\end{pmatrix}\to\begin{pmatrix}\frac{1}{\sqrt2}\\\frac{1}{\sqrt2}\end{pmatrix}$

$X=\begin{pmatrix}\frac{1}{\sqrt2}&\frac{1}{\sqrt2}\\\frac{1}{\sqrt2}&-\frac{1}{\sqrt2}\end{pmatrix}Y,\ 3x^2-4xy+3y^2=X^TAX=Y^TDY=y_1^2+5y_2^2=6$

$y_1^2+5y_2^2=6 \Rightarrow \frac{y_1^2}{6}+\frac{y_2^2}{\frac{6}{5}}=1$

∴ 장축의 길이가 $2\sqrt6$ 이고 단축의 길이가 $2\sqrt{\frac{6}{5}}$ 인 타원형 이다.

80. $x^2+8xy-5y^2=10 \Rightarrow (x\ y)\underbrace{\begin{pmatrix}1 & 4\\ 4 & -5\end{pmatrix}}_{A}\begin{pmatrix}x\\ y\end{pmatrix}$

$$|\lambda I-A|=\begin{vmatrix}\lambda-1 & -4\\ -4 & \lambda+5\end{vmatrix}=\lambda^2+4\lambda-5-16=\lambda^2+4\lambda-21=(\lambda+7)(\lambda-3)$$

$\lambda=-7$ 일때

$$\left(\begin{array}{cc|c}-8 & -4 & 0\\ -4 & -2 & 0\end{array}\right)\to\left(\begin{array}{cc|c}1 & \frac{1}{2} & 0\\ 0 & 0 & 0\end{array}\right)\quad \begin{matrix}x=-\frac{1}{2}t\\ y=t\end{matrix}\quad \begin{pmatrix}1\\ -2\end{pmatrix}\to\begin{pmatrix}\frac{1}{\sqrt5}\\ -\frac{2}{\sqrt5}\end{pmatrix}$$

$\lambda=3$ 일때

$$\left(\begin{array}{cc|c}2 & -4 & 0\\ -4 & 8 & 0\end{array}\right)\to\left(\begin{array}{cc|c}1 & -2 & 0\\ 0 & 0 & 0\end{array}\right)\quad \begin{matrix}x=2t\\ y=t\end{matrix}\quad \begin{pmatrix}2\\ 1\end{pmatrix}\to\begin{pmatrix}\frac{2}{\sqrt5}\\ \frac{1}{\sqrt5}\end{pmatrix}$$

$$X=\begin{pmatrix}\frac{2}{\sqrt5} & \frac{1}{\sqrt5}\\ \frac{1}{\sqrt5} & -\frac{2}{\sqrt5}\end{pmatrix}Y$$

$$x^2+8xy-5y^2=X^tAX=Y^tDY=3y_1^2-7y_2^2=10$$

$$\Rightarrow \frac{y_1^2}{\frac{10}{3}}-\frac{y_2^2}{\frac{10}{7}}=1$$

∴ 주축의 길이가 $2\sqrt{\frac{10}{3}}$ 인 쌍곡선이다.

81. $-6x_1^2+4x_1x_2-3x_2^2=(x_1\ x_2)\underbrace{\begin{pmatrix}-6 & 2\\ 2 & -3\end{pmatrix}}_{A}\begin{pmatrix}x_1\\ x_2\end{pmatrix}$

$$|\lambda I-A|=\begin{vmatrix}\lambda+6 & -2\\ -2 & \lambda+3\end{vmatrix}=\lambda^2+9\lambda+18-4=\lambda^2+9\lambda+14$$

$$=(\lambda+7)(\lambda+2)=0 \quad \therefore \lambda=-7 \text{ 또는 } \lambda=-2$$

① $\lambda=-7$ 일때

$$\left(\begin{array}{cc|c}-1 & -2 & 0\\ -2 & -4 & 0\end{array}\right)\to\left(\begin{array}{cc|c}1 & 2 & 0\\ 0 & 0 & 0\end{array}\right)\quad \begin{matrix}x=-2t\\ y=t\end{matrix}\quad \begin{pmatrix}-2\\ 1\end{pmatrix}\xrightarrow{\text{정규화}}\begin{pmatrix}\frac{2}{\sqrt5}\\ -\frac{1}{\sqrt5}\end{pmatrix}$$

② $\lambda=-2$ 일때

$$\left(\begin{array}{cc|c}4 & -2 & 0\\ -2 & 1 & 0\end{array}\right)\to\left(\begin{array}{cc|c}1 & -\frac12 & 0\\ 0 & 0 & 0\end{array}\right)\quad \begin{matrix}x=\frac12 t\\ y=t\end{matrix}\quad \begin{pmatrix}1\\ 2\end{pmatrix}\xrightarrow{\text{정규화}}\begin{pmatrix}\frac{1}{\sqrt5}\\ \frac{2}{\sqrt5}\end{pmatrix}$$

$$X=\begin{pmatrix}\frac{2}{\sqrt5} & \frac{1}{\sqrt5}\\ -\frac{1}{\sqrt5} & \frac{2}{\sqrt5}\end{pmatrix}Y \Rightarrow Y=\begin{pmatrix}\frac{2}{\sqrt5} & -\frac{1}{\sqrt5}\\ \frac{1}{\sqrt5} & \frac{2}{\sqrt5}\end{pmatrix}X$$

$$-6x_1^2+4x_1x_2-3x_2^2=X^TAX=Y^TDY=(y_1,y_2)\begin{pmatrix}-7 & 0\\ 0 & -2\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}$$

$$=-7y_1^2-2y_2^2$$

정답! $\boxed{①}$

82. $-x_1^2+6x_1x_2+7x_2^2=(x_1\ x_2)\underbrace{\begin{pmatrix}-1&3\\3&7\end{pmatrix}}_{A}\begin{pmatrix}x_1\\x_2\end{pmatrix}$

$$|\lambda I-A|=\begin{vmatrix}\lambda+1&-3\\-3&\lambda-7\end{vmatrix}=\lambda^2-6\lambda-7-9=\lambda^2-6\lambda-16$$

$$=(\lambda-8)(\lambda+2)=0 \quad \therefore \lambda=8 \text{ 또는 } \lambda=-2$$

① $\lambda=8$ 일 때

$$\left(\begin{array}{cc|c}9&-3&0\\-3&1&0\end{array}\right)\rightarrow\left(\begin{array}{cc|c}1&-\frac{1}{3}&0\\0&0&0\end{array}\right)\quad \begin{array}{l}x=\frac{1}{3}t\\ y=t\end{array},\quad \begin{pmatrix}1\\3\end{pmatrix}\xrightarrow{\text{정규화}}\begin{pmatrix}\frac{1}{\sqrt{10}}\\ \frac{3}{\sqrt{10}}\end{pmatrix}$$

② $\lambda=-2$ 일 때

$$\begin{pmatrix}-1&-3\\-3&-9\end{pmatrix}\rightarrow\left(\begin{array}{cc|c}1&3&0\\0&0&0\end{array}\right)\quad \begin{array}{l}x=-3t\\ y=t\end{array}\quad \begin{pmatrix}-3\\1\end{pmatrix}\xrightarrow{\text{정규화}}\begin{pmatrix}-\frac{3}{\sqrt{10}}\\ \frac{1}{\sqrt{10}}\end{pmatrix}$$

$$X=\begin{pmatrix}\frac{1}{\sqrt{10}}&-\frac{3}{\sqrt{10}}\\ \frac{3}{\sqrt{10}}&\frac{1}{\sqrt{10}}\end{pmatrix}Y \Rightarrow Y=\begin{pmatrix}\frac{1}{\sqrt{10}}&\frac{3}{\sqrt{10}}\\ -\frac{3}{\sqrt{10}}&\frac{1}{\sqrt{10}}\end{pmatrix}X$$

$$-x_1^2+6x_1x_2+7x_2=X^TAX=Y^TDY=(y_1\ y_2)\begin{pmatrix}8&0\\0&-2\end{pmatrix}\begin{pmatrix}y_1\\y_2\end{pmatrix}$$
$$=8y_1^2-2y_2^2$$

정답! $\boxed{②}$

83. (1) $3x_1^2+4x_1x_2-3x_2^2 = (x_1\ x_2)\underbrace{\begin{pmatrix}3 & 2\\ 2 & -3\end{pmatrix}}_{A}\begin{pmatrix}x_1\\ x_2\end{pmatrix}$

$$|\lambda I - A| = \begin{vmatrix}\lambda-3 & -2\\ -2 & \lambda+3\end{vmatrix} = \lambda^2-9-4 = \lambda^2-13=0$$

$\therefore$ $\lambda=\sqrt{13}$ 또는 $\lambda=-\sqrt{13}$ 이것을 통해서 부정인걸 알 수 있습니다.

정답: $\boxed{\text{부정}}$

(2) $5x^2+4x_1x_2+4x_2 = (x_1\ x_2)\underbrace{\begin{pmatrix}5 & 2\\ 2 & 4\end{pmatrix}}_{A}\begin{pmatrix}x_1\\ x_2\end{pmatrix}$

$$|\lambda I - A| = \begin{vmatrix}\lambda-5 & -2\\ -2 & \lambda-4\end{vmatrix} = \lambda^2-9\lambda+20-4 = \lambda^2-9\lambda+16=0$$

$\therefore$ $\lambda = \dfrac{9\pm\sqrt{81-64}}{2} = \dfrac{9\pm\sqrt{17}}{2}$ 이므로 모든 고유값이 양수 이므로 양한정이다.

정답: $\boxed{\text{양한정}}$

84. $2x_1^2+2x_2^2+2x_3^2-4x_1x_2-4x_2x_3$

$$= (x_1\ x_2\ x_3)\underbrace{\begin{pmatrix} 2 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 2 \end{pmatrix}}_{A}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda-2 & 2 & 0 \\ 2 & \lambda-2 & 2 \\ 0 & 2 & \lambda-2 \end{vmatrix} = (\lambda-2)\{(\lambda-2)(\lambda-2)-4\} - 4(\lambda-2)$$

$$= (\lambda-2)\{\lambda^2-4\lambda+4-4-4\} = (\lambda-2)(\lambda^2-4\lambda-4)$$

$\therefore$ $\lambda=2$ 또는 $\lambda=2+2\sqrt{2}$ 또는 $\lambda=2-2\sqrt{2}$

여기서 $\lambda=2-2\sqrt{2}<0$ 므로 $\lambda=2$ 또는 $\lambda=2+2\sqrt{2}$ 는 양수이므로 고유값이 양수 2개와 음수 1개이므로 부정이다.

정답: $\boxed{\text{부정}}$