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20.(1)

$C_{11} = +\begin{vmatrix} -3 & 0 \\ 2 & -5 \end{vmatrix} = 15-0=15$

$C_{12} = -\begin{vmatrix} 1 & 0 \\ -4 & -5 \end{vmatrix} = -(-5-0) = 5$

$C_{13} = +\begin{vmatrix} 1 & -3 \\ -4 & 2 \end{vmatrix} = 2-12=-10$

$C_{21} = -\begin{vmatrix} 4 & -1 \\ 2 & -5 \end{vmatrix} = -(-20+2)=18$

$C_{22} = +\begin{vmatrix} 2 & -1 \\ -4 & -5 \end{vmatrix} = -10-4=-14$

$C_{23} = -\begin{vmatrix} 2 & 4 \\ -4 & 2 \end{vmatrix} = -(4+16) = -20$

$C_{31} = +\begin{vmatrix} 4 & -1 \\ -3 & 0 \end{vmatrix} = 0-3=-3$

$C_{32} = -\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = -(0+1)=-1$

$C_{33} = +\begin{vmatrix} 2 & 4 \\ 1 & -3 \end{vmatrix} = -6-4=-10$

$$adj(A) = \begin{pmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{pmatrix} = \begin{pmatrix} 15 & 18 & -3 \\ 5 & -14 & -1 \\ -10 & -20 & -10 \end{pmatrix}$$

정답: $\boxed{adj(A) = \begin{pmatrix} 15 & 18 & -3 \\ 5 & -14 & -1 \\ -10 & -20 & -10 \end{pmatrix}}$

(2) $$det(A) = \begin{vmatrix} 2 & 4 & -1 \\ 1 & -3 & 0 \\ -4 & 2 & -5 \end{vmatrix} \xrightarrow{-5R_1+R_3 \to R_3} \begin{vmatrix} 2 & 4 & -1 \\ 1 & -3 & 0 \\ -14 & -18 & 0 \end{vmatrix} = -1\begin{vmatrix} 1 & -3 \\ -14 & -18 \end{vmatrix}$$

$$= -(-18-42) = 60$$

$$A^{-1} = \frac{1}{det(A)} adj(A) = \frac{1}{60}\begin{pmatrix} 15 & 18 & -3 \\ 5 & -14 & -1 \\ -10 & -20 & -10 \end{pmatrix}$$

정답: $\boxed{A^{-1} = \frac{1}{60}\begin{pmatrix} 15 & 18 & -3 \\ 5 & -14 & -1 \\ -10 & -20 & -10 \end{pmatrix}}$

21.(1) $\det(-A) = \det(-1 \cdot A) = (-1)^4 \cdot \det(A) = 1 \cdot (-3) = -3$

정답! $\boxed{\det(-A) = -3}$

(2) $\det(2A) = (2)^4 \cdot \det(A) = 16 \cdot (-3) = -48$

정답! $\boxed{\det(2A) = -48}$

(3) $\det(A^T) = \det(A)$

$\det(A^2 A^T) = \det(A)\det(A)\det(A^T) = -3 \cdot -3 \cdot -3 = -27$

정답! $\boxed{\det(A^2 A^T) = -27}$

(4) $\det((-3A)^{-1}) = \dfrac{1}{\det(-3A)} = \dfrac{1}{(-3)^4 \cdot \det(A)} = -\dfrac{1}{243}$

정답! $\boxed{\det((-3A)^{-1}) = -\dfrac{1}{243}}$

(5) $A^{-1} = \dfrac{1}{\det(A)} \operatorname{adj}(A) \rightarrow \operatorname{adj}(A) = \det(A) A^{-1} = -3A^{-1}$

$\det(\operatorname{adj} A) = \det(-3A^{-1}) = (-3)^4 \cdot \det(A^{-1}) = \dfrac{81}{\det(A)}$

$= \dfrac{81}{-3} = -27$

정답! $\boxed{\det(\operatorname{adj} A) = -27}$

22.

$$2\vec{u}-5\vec{v}=2(3,-4,1)-5(5,-2,1)=(6,-8,2)+(-25,10,-5)$$
$$=(-19,2,-3)$$

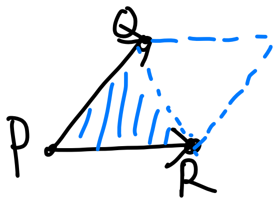
$$2\vec{u}-3\vec{v}=2(3,-4,1)-3(5,-2,1)=(6,-8,2)+(-15,6,-3)$$
$$=(-9,-2,-1)$$

$$(2\vec{u}-5\vec{v})\times(2\vec{u}-3\vec{v})=\begin{vmatrix} i & j & k \\ -19 & 2 & -3 \\ -9 & -2 & -1 \end{vmatrix}=i\begin{vmatrix} 2 & -3 \\ -2 & -1 \end{vmatrix}-j\begin{vmatrix} -19 & -3 \\ -9 & -1 \end{vmatrix}+k\begin{vmatrix} -19 & 2 \\ -9 & -2 \end{vmatrix}$$

$$=(-2-6)i-(19-27)j+(38+18)k=-8i+8j+56k$$

정답 : $\boxed{(2\vec{u}-5\vec{v})\times(2\vec{u}-3\vec{v})=(-8,8,56)}$

23. $\vec{PQ}=(2,1,-2),\ \vec{PR}=(-4,4,2)$



$$\vec{PQ}\times\vec{PR}=\begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ -4 & 4 & 2 \end{vmatrix}=\left(\begin{vmatrix} 1 & -2 \\ 4 & 2 \end{vmatrix},-\begin{vmatrix} 2 & -2 \\ -4 & 2 \end{vmatrix},\begin{vmatrix} 2 & 1 \\ -4 & 4 \end{vmatrix}\right)$$
$$=(10,4,12)$$

삼각형의 넓이 $=\frac{1}{2}(\|\vec{PQ}\times\vec{PR}\|)=\frac{1}{2}(\sqrt{100+16+144})=\sqrt{65}$

정답 : $\boxed{\sqrt{65}}$

24. 평행육면체의 부피 $= |\vec{u}\cdot(\vec{v}\times\vec{w})|$

$$\vec{v}\times\vec{w} = \begin{vmatrix} i & j & k \\ -3 & -1 & 0 \\ -1 & 2 & 4 \end{vmatrix} = \left(\begin{vmatrix} -1 & 0 \\ 2 & 4 \end{vmatrix}, -\begin{vmatrix} -3 & 0 \\ -1 & 4 \end{vmatrix}, \begin{vmatrix} -3 & -1 \\ -1 & 2 \end{vmatrix} \right)$$

$$= (-4, 12, -7)$$

$$\vec{u}\cdot(\vec{v}\times\vec{w}) = (4, 1, -3)\cdot(-4, 12, -7) = -16+12+21 = 17$$

정답: $\boxed{17}$

25. 사면체의 부피 $= \frac{1}{6}(|\vec{u}\cdot(\vec{v}\times\vec{w})|)$

$$\vec{v}\times\vec{w} = \begin{vmatrix} i & j & k \\ -1 & 1 & 2 \\ 1 & 3 & 2 \end{vmatrix} = \left(\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}, -\begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix}, \begin{vmatrix} -1 & 1 \\ 1 & 3 \end{vmatrix} \right) = (-4, 4, -4)$$

$$\vec{u}\cdot(\vec{v}\times\vec{w}) = (5, -1, 1)\cdot(-4, 4, -4) = -20-4-4 = -28$$

$$\frac{1}{6}(|\vec{u}\cdot(\vec{v}\times\vec{w})|) = \frac{1}{6}\times 28 = \frac{14}{3}$$

정답: $\boxed{\frac{14}{3}}$

26. $u\cdot(v\times w) = w\cdot(u\times v) = v\cdot(w\times u) = 5$

(1) $w\cdot(v\times u) = -(w\cdot(u\times v)) = -(u\cdot(v\times w)) = -5$

정답 : $\boxed{w\cdot(v\times u) = -5}$

(2) $v\cdot(w\times u) = u\cdot(v\times w) = 5$

정답 : $\boxed{v\cdot(w\times u) = 5}$

(3) $(w\times u)\cdot v$는 내적은 교환법칙이 성립하므로 $(w\times u)\cdot v = v\cdot(w\times u)$

$(w\times u)\cdot v = v\cdot(w\times u) = u\cdot(v\times w) = 5$

정답 : $\boxed{(w\times u)\cdot v = 5}$

(4) $v = (v_1, v_2, v_3)$ 가정하면 $v\times v = \begin{vmatrix} i & j & k \\ v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = 0$

$u\cdot(v\times v) = u\cdot 0 = 0$

정답 : $\boxed{u\cdot(v\times v) = 0}$

(5) $w = (w_1, w_2, w_3)$, $u = (u_1, u_2, u_3)$으로 가정하면

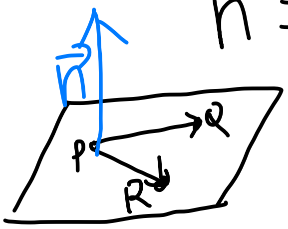
$$w\times u = \begin{vmatrix} i & j & k \\ w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = \left(\begin{vmatrix} w_2 & w_3 \\ u_2 & u_3 \end{vmatrix}, -\begin{vmatrix} w_1 & w_3 \\ u_1 & u_3 \end{vmatrix}, \begin{vmatrix} w_1 & w_2 \\ u_1 & u_2 \end{vmatrix} \right)$$

$$(w\times u)\cdot w = w_1\begin{vmatrix} w_2 & w_3 \\ u_2 & u_3 \end{vmatrix} - w_2\begin{vmatrix} w_1 & w_3 \\ u_1 & u_3 \end{vmatrix} + w_3\begin{vmatrix} w_1 & w_2 \\ u_1 & u_2 \end{vmatrix} = \begin{vmatrix} w_1 & w_2 & w_3 \\ w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \end{vmatrix}$$

$= 0$

정답 : $\boxed{0}$

27. $\overrightarrow{PQ}=(5,2,3)$ $\overrightarrow{PR}=(-2,2-4)$



$$\vec{n}=\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix} i & j & k \\ 5 & 2 & 3 \\ -2 & 2 & -4 \end{vmatrix}=\left(\begin{vmatrix} 2 & 3 \\ 2 & -4 \end{vmatrix},-\begin{vmatrix} 5 & 3 \\ -2 & -4 \end{vmatrix},\begin{vmatrix} 5 & 2 \\ -2 & 2 \end{vmatrix}\right)$$

$$=\left((-8-6),-(-20+6),(10+4)\right)=(-14,14,14)$$

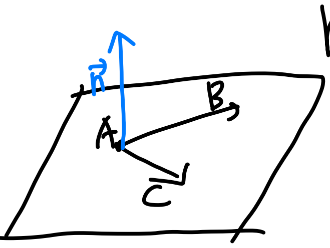
$$-14(x-1)+14(y-0)+14(z-1)=0$$

$$-14x+14+14y+14z-14=0$$

$$-14x+14y+14z=0 \rightarrow x-y-z=0$$

정답: $\boxed{x-y-z=0}$

28. $\overrightarrow{AB}=(1,3,0)$ $\overrightarrow{AC}=(-2,-2,2)$



$$\vec{n}=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix} i & j & k \\ 1 & 3 & 0 \\ -2 & -2 & 2 \end{vmatrix}=\left(\begin{vmatrix} 3 & 0 \\ -2 & 2 \end{vmatrix},-\begin{vmatrix} 1 & 0 \\ -2 & 2 \end{vmatrix},\begin{vmatrix} 1 & 3 \\ -2 & -2 \end{vmatrix}\right)$$

$$=(6,-2,4)$$

$$6(x-0)-2(y+3)+4(z-1)=0 \rightarrow 6x-2y+4z-10=0$$

$$6x-2y+4z=10 \rightarrow \frac{3}{5}x-\frac{1}{5}y+\frac{2}{5}z=1$$

정답: $\boxed{\frac{3}{5}x-\frac{1}{5}y+\frac{2}{5}z=1}$