Assume.

$X|\mu \sim N(\mu, \sigma^2)$

$\mu \sim N(M, A)$

So that $\varepsilon := X-\mu \sim N(0, \sigma^2)$

Then mgf of ε is

$$
\begin{aligned}
E(e^{t\varepsilon}) &= E(e^{tx-t\mu}) \\
&= E(E(e^{tx-t\mu} \mid \mu)) \\
&= E(e^{-t\mu} E(e^{tx} \mid \mu)). \\
&= E(e^{-t\mu} \cdot e^{(\mu t + \frac{1}{2}\sigma^2 t^2)}). \\
&= E(e^{\frac{1}{2}\sigma^2 t^2})
\end{aligned}
$$

Next, we show that μ and ε is independent.

$$
\begin{aligned}
E(e^{t_1\mu + t_2\varepsilon}) &= E(e^{t_1\mu + t_2(X-\mu)}). \\
&= E(e^{(t_1-t_2)\mu + t_2 X}). \\
&= E(e^{(t_1-t_2)\mu}) \exp(\mu t_2 + \tfrac{1}{2}t_2^2\sigma^2). \\
&= E(e^{t_1\mu}) \exp(\tfrac{1}{2}t_2^2\sigma^2). \\
&= E(e^{t_1\mu}) E(e^{t_2\varepsilon}).
\end{aligned}
$$

$\therefore \mu \sim N(M, A) \perp \varepsilon \sim N(0, \sigma^2) \Rightarrow X = \mu + \varepsilon \sim N(M, A+\sigma^2)$