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Paper 2

Topic: What is the value-range of a function? Why does Frege need a special notation to talk about value ranges? How are value-ranges introduced as objects? What is one special use value ranges can be put to?

Value-range is a new axiom Frege proposes in the Foundations to help build on the logical system that he had been working on in the Foundations. In the essay below, I would go over the definition of the value-range, its properties, and the necessity of it in Frege's logical system.

Frege identifies value-range as the mappings from the input to the output of a function. For example, if we consider function f(x) = y, the collection of the pairs <x,y=F(x)> is the value-range of the function f(x). Extending this definition to concepts, because concepts are functions that map objects to truth value, the value-range of the concepts are the mappings of objects to truth values. Moreover, extensions are objects which fall under the corresponding concepts. Thus, given the objects of the extensions, a concept would return the True value. This corresponds to the definition of value-range proposed by Frege. Therefore, extensions are value-ranges of the concepts (functions).

Having introduced the definition of value-ranges, Frege also presents a new notation for them in the Foundations. Value-range is the collection of the arguments and output value mappings which corresponds to the function. Because functions take in objects as parameters and

return either truth value or objects, functions' arguments and the returning values can only be objects. Therefore, value-ranges are objects since they are consisting of objects. Having established that value-ranges are objects, it is necessary to introduce the names to signify the objects, hence why Frege introduces new notation for value-ranges. Objects are identified and comparable based on their names, for example, 'a cow' as an object is different of 'a horse' because their names differ. Concepts like "is a positive number" and "is a negative number" are impossible to identify and compare on their own since metaphysically, they are incomplete and unsaturated. Frege's notation of value-ranges separates the object 'value-ranges' from functions. This leads to a special application of value-ranges. In contrast to objects, functions (concepts) are not comparable entities. Functions do not have names to signify them hence they cannot be equated as objects. Since a value-range identifies the function, through comparing the value-ranges of functions, which is possible because they are objects, it is possible now to compare functions. Similarly, extensions, which are shown to be value-ranges of concepts, become comparable by using the special notation, hence, concepts are comparable. Overall, the new notation of the value-ranges is introduced as the defining names of the objects and through the new notation of value-ranges, Frege uses them to identify and compare the concepts, which were formerly impossible.

Value-range is an important part of the Foundations, when it sets the criterion of identity to identify abstract identity such as functions. Frege proposed value-range as a collection of argument-value pairs as an object-like entity that functions possess, therefore through its notation, functions, or concepts, now becomes comparable. However, the logic behind value-range is flawed and Russell's Paradox exemplifies this problem. This further shows that the goal to achieve a completely logically fluent language system is still to be longed for.