HERIOT WATT UNIVERSITY

SCHOOL OF MATHEMATICAL AND COMPUTER SCIENCES

Department of Computer Science

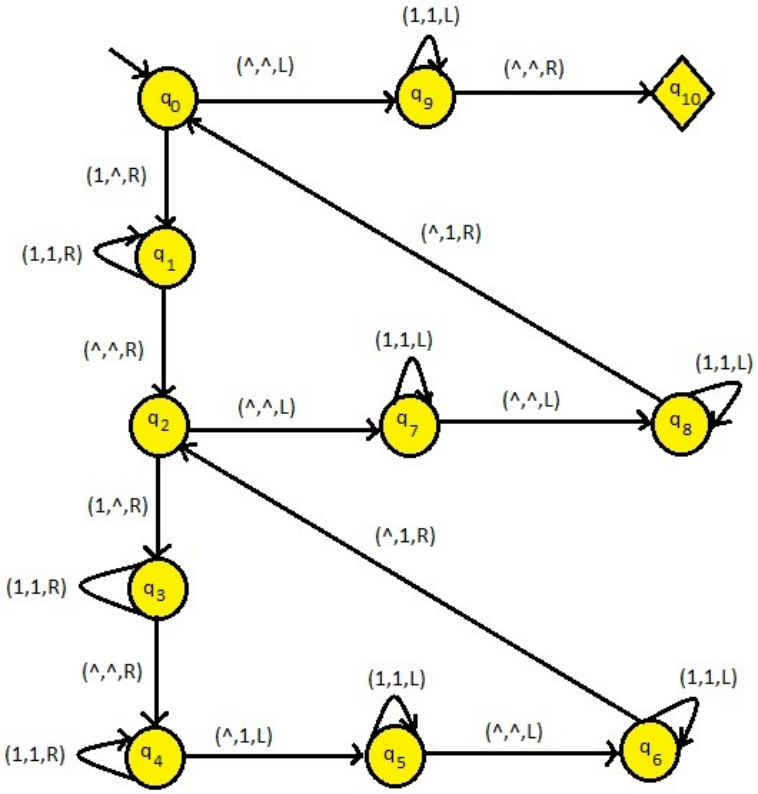
F29FB

FOUNDATIONS II

ASSIGNMENT— 2024/2025

Due Back Wednesday of week 8 (5 March)
ONLY SUBMIT ONE SINGLE PDF FILE.
NO HANDWRITTEN ASSIGNMENTS.
ALL NEEDS TO BE TYPED INCLUDING THE GRAPHS YOU GIVE.
Recall Plagiarism and Late submission guidelines. See your course's web site for more details on this.
Make sure you also complete the declaration form and submit it on canvas before you submit your assignment.

Consider the Turing machine whose graph is depicted here:



1. Give a new graph which is is as close as possible to the above graph in order to represent multiplication Turing machine which takes two numbers n and m written as sequences of 1s and returns the number n followed by the number m followed by their product. The numbers n, m and their product should be separated by the symbols of your choice (say $\diamondsuit$ and $\heartsuit$). Always the pointer should start in location 0 and returns back at the end of your computations to location 0. [2]

For example, input and output tapes may look as follows:

-1	0	1	2	3	4	5	6	7
$\vert\wedge\vert$	$1\vert$	$1\vert$	$\diamondsuit\vert$	$1\vert$	$1\vert$	$1\vert$	$\heartsuit\vert$	$\wedge\vert$
	$\uparrow$							
	q_0							

-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$\vert\wedge\vert$	$1\vert$	$1\vert$	$\diamondsuit\vert$	$1\vert$	$1\vert$	$1\vert$	$\heartsuit\vert$	$1\vert$	$1\vert$	$1\vert$	$1\vert$	$1\vert$	$1\vert$	$\wedge\vert$
	$\uparrow$													
	q_{10}													

2. Recall that the difference between your proposed graph and the above graph should be minimal. Clearly highlight this difference. Note that the most optimal change to the above graph necessary to give the required Turing machine is simply to change some triplet labels (s_i, s_j, X) between states q_l and q_r to (s'_i, s'_j, X'). In this case, the new graph is exactly the old graph but where the various (s_i, s_j, X) in question are replaced by the new (s'_i, s'_j, X') and in this question, you should list all these changes (i.e., all the changed (q_l, s_i, q_r, s_j, X) together with the new $(q_l, s'_i, q_r, s'_j, X')$ they changed to). [1]

3. Give the formal definition of your proposed graph as per the course's formal definition of a Turing machine. That is, you should identify all the states including the start state, you should also identify all the symbols and the action table. [2]

4. Give the logic of your graph (i.e., clearly explain how it works and say what each part of the graph actually does). [2]

5. Use the TM simulator at
`https://math.hws.edu/eck/js/turing-machine/TM.html`
to upload the Json file for your TM on canvas in the location marked assignment. [1]

6. Test your machine code on the numbers below and produce the following:

- ALL the tape computation sequences (i.e., all the tapes from the input one to the final output one).
- Your input/output and intermediate tapes in all the examples you run should, as in the above example tapes, have all the tape contents, all the locations, the current state, and the pointer pointing to the current symbol.

The examples you are asked to run your Turing machine on and produce all the tape configuration sequences (following the above guidelines) include:

(a) The unary inputs representing 0 and 3. [0.75]

(b) The unary inputs representing 2 and 0. [0.75]

(c) The unary inputs representing 3 and 2. [0.75]

(d) The unary inputs representing 1 and 5. [0.75]

7. Give the graph of a Turing machine that takes a number n in unary notation and returns the number, its square n^2 and its third power n^3 all separated by the symbols of your choice. Make sure you use updated versions of the copy Turing machine and the multiplication Turing machine and clearly explain why you have done so. [2]

Use the TM simulator at
`https://math.hws.edu/eck/js/turing-machine/TM.html`
to upload the Json file for your TM on canvas in the location marked assignment. [1]

Test your machine code on the input tape below to produce the output tape you see below it (remember you can use any symbols to separate n from n^2 from n^3). [1]

-1	0	1	2	3	4	5	6	7
$\vert\wedge\vert$	$1\vert$	$1\vert$	$\wedge\vert$	$\wedge\vert$	$\wedge\vert$	$\wedge\vert$	$\wedge\vert$	$\wedge\vert$
	$\uparrow$							
	q_0							

-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13				
$\vert\wedge\vert$	$1\vert$	$1\vert$	$\wedge\vert$	$1\vert$	$1\vert$	$1\vert$	$1\vert$	$\wedge\vert$	$1\vert$	$1\vert$	$1\vert$	$1\vert$	$1\vert$	$1\vert$	$1\vert$	$1\vert$	$\wedge\vert$	$\wedge\vert$
	$\uparrow$																	
	q_f																	

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