Problem Chosen	2023 MCM/ICM Summary Sheet	Team Control Number
B		12

Steel pipe ordering and transportation plan

Summary

Transportation problem is an optimization problem that often occurs in social and economic life. In this paper, we analyze the ordering, transportation and laying costs of natural gas steel pipes in reality. We establish a model to determine the ordering and transportation plan and find the optimal allocation of the minimum total cost.

First of all, we establish the ordering and transportation cost model of steel pipe model, and formulated the steel pipe ordering and transportation plan to minimize the total cost for seven steel pipe plants that have limited the maximum ordering quantity, different steel pipe prices, and two transportation conditions. **Floyd algorithm** is used to calculate the shortest railway distance between any two points, and the minimum transportation cost between any two points is calculated through the known table of the relationship between railway freight and distance. The model is solved through Lingo and other software programming forms, and the total cost is finally minimized. The total freight required is **12,786,316,000 yuan**. While simplifying the reality, the model retains the most critical part, optimizes the complex model in reality, and is compatible with the solution of more complex problems.

Secondly, by analyzing the factors that affect the purchase and transportation plan and the total cost, it is found that the change of the upper limit of production of steel plant S1 has the greatest impact on the purchase and transportation plan and the total cost. The change in the selling price of S1, S2, S3 and S5 steel pipes in the plant has a great impact on the purchase and transportation plan and the total cost. Since the order quantity of steel plant S4 and S7 is zero, the minimum total cost will not be affected when the steel pipe sales price of steel plant changes.

Finally, by improving the model of the first question, when the pipeline to be laid is not a line, but a tree diagram, and the railway, highway and pipeline form a network, by analyzing three special points with A9, A11 and A17 node degrees of 3 and six additional railway lines, the optimization model can find the optimal solution to make the model applicable to more general situations. The result of question 3 is: **14,066,314,000 yuan.**

In this paper, it is found that the important factors affecting the total cost include the layout of the transportation network, the production ceiling of the steel pipe plant, etc., while the impact of road transportation costs is minimal. This will help solve the real complex problems.

Keywords: Floyd algorithm；linear programming；Graph theory

Contents

1 Introduction

1.1 Problem Background

Due to the huge global energy consumption, some countries are short of energy. There is a large difference between the production and consumption areas of energy resources within the country. The eastern coastal areas are economically developed and have a huge demand for energy, but the energy is relatively poor, so that the economic advantages can not be fully played. Due to the limitation of economic level, the rich energy in the western region cannot be fully developed and utilized. Therefore, we need to establish an appropriate energy transportation system to solve the problem of energy shortage at home and abroad. The ordering and transportation of steel pipes have a significant impact on the investment cost of the construction project and the economic benefits after being put into use. The problem of steel pipe transportation is how to determine a transportation scheme that minimizes the total transportation cost when a certain material is transferred from a number of production places to a number of sales places, the supply quantity of each production place and the demand quantity of each sales place are known, and the transportation unit price between different places is known.

1.2 Restatement of the Problem

Considering the background information and restricted conditions identified in the problem statement, we need to develop a model to determine the ordering and transportation plan of trunk line of steel pipe to solve the following problems:

- Problem 1：Make an order and transportation plan for the steel pipes of the main pipeline to minimize the total cost
- Problem 2：Analyze which steel plant's steel pipe sales price change has the greatest impact on the purchase and transportation plan and total cost. Analyze which steel plant's upper limit of steel pipe production has the greatest impact on the purchase and transportation plan and total cost
- Problem 3：Provide a solution for this more general situation if the pipeline to be laid is not a line but a tree diagram, and the railway, highway and pipeline form a network,

1.3 Literature Review

The problem that this paper aims at is the optimization of steel pipe ordering and transportation, which has been discussed in previous studies. Babu S.et introduced optimization method to solve the problem and used the existing quadratic programming model to minimize the total transportation cost.[1] Wang Lina introduced Graph theory and Freud algorithm to solve the shortest path problem and minimize the total mileage of steel pipe transportation and reduce the total transportation cost. [2]

There is also a lot of algorithm on how to find the shortest path of the transportation problem. Heming Bing introduced Dijkstra Algorithm to find the path between any two points.[3] Simulated annealing algorithm is also used by Ch Leela Kumari to find the shortest path between two points.[4]

1.4 Our Work

We need a model that works for optimization of ordering and transportation of steel pipes for main pipelines. Our work mainly includes the following parts:

Firstly the objective function of the steel pipe transportation and ordering plan is established through the idea of mathematical programming, and the decision variables and constraints of the problem are known through analysis.

Next a weighted undirected graph is established to obtain the minimum path of transportation cost from the steel plant to the paving point using Floyd algorithm, and the global optimal solution is obtained using Lingo software.

Finally, the model is extended to all kinds of tree graphs or more complex shapes.。

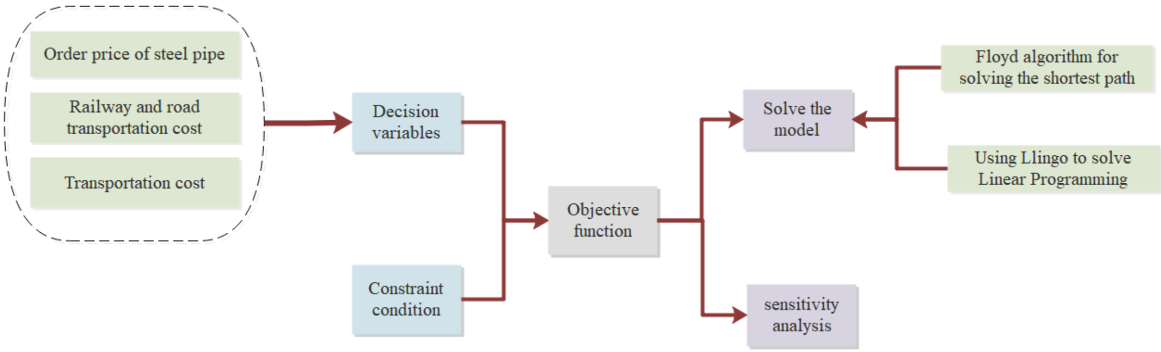

Figure 1： Overview of our work

2 Assumptions and Justifications

Considering that practical problems always contain many complex factors, first of all, we need to make reasonable assumptions to simplify the model, and each hypothesis is closely followed by its corresponding explanation.

- **Assumption 1: It is assumed that the steel pipes are laid according to the number of kilometers.**

The reason is that for the convenience of calculation, the steel pipe of 1km main pipeline is called 1 unit steel pipe, and the part of road transportation cost less than the whole kilometer is calculated as the whole kilometer. In order to reduce the cost, we assume that the laying points are evenly arranged according to the whole kilometer.

- **Assumption 2: It is assumed that the steel pipe has no loss during production, transportation and laying.**

In order to simplify the model and facilitate the calculation, we will ignore the losses caused by some special circumstances or accidental events, such as bad weather, traffic accidents, etc., and the steel pipes are all qualified with no loss at the connection points.

- **Assumption 3: There is a road on the side of the pipeline to be laid, which can transport the required steel pipes.**

According to the topic assumption, in order to simplify the calculation, it is unnecessary to consider the cost of road construction along the road during the laying process.

- **Assumption 4: The steel pipes required shall be provided by the steel plant.**

As we can see, there are steel plants that can produce such steel pipes for main pipes. It can be assumed that during the laying process, all steel pipes are produced by these seven steel plants and transported from the steel plants to the laying point.

3 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description	Unit
s_i	Upper limit of supply of S_i in steel plant	Unit steel pipe
p_i	Sales price of 1 unit steel pipe of steel plant S_i	ten thousand yuan
c_{ij}	Ordering and transportation costs from steel plant S_i to laying node A_j	ten thousand yuan
l_k	Quantity of steel pipes to be laid for pipe $\|A_kA_{k+1}\|$	Unit steel pipe
x_{ij}	Quantity of steel pipes transported from steel plant S_i to node A_j	Unit steel pipe
$y_j or z_j or q_j$	Quantity of steel pipes laid from node A_j to a certain direction	Unit steel pipe

4 Ordering and transportation cost model of steel pipe

We will now elaborate on our model. Based on the optimization problem, we establish a linear regression model to solve the objective function model with the lowest cost, and calculate the shortest path through Floyd algorithm to reduce the transportation cost.

4.1 The Establishment of Model

- **Decision variables**

The maximum supply of $i(i=1,2,\cdots,7)$ steel plants is s_i, the ordering and transportation costs from the ith steel mill to the laying node $j(j=1,2,\cdots,15)$ are c_{ij}.

Using $l_k=|A_kA_{k+1}|(k=1,2,\cdots,14)$ to indicate the quantity of steel pipes to be laid in section k. x_{ij} refers to the quantity of steel pipes S_i transported from the steel plant to the node j, y_j is the quantity of steel pipes laid from node j to the left, z_j is the quantity of steel pipes laid from node j to the right.

- **Objective function**

It can be seen from the analysis that we need to specify a purchase and transportation plan for the steel pipe of the main pipeline with the minimum total cost. That is, it is necessary to determine the quantity of steel pipes ordered from $S_i(i=1,2...,7)$ of each steel plant; The plan for transporting the ordered steel pipes to the laying place $A_1, A_2, ..., A_{15}$ (not only to the point of arrival, but to the whole pipeline), including the laying cost and transportation cost. It can be seen that the total cost consists of the following three parts:

$$\min \sum_{i=1}^{7}\sum_{j=1}^{15} x_{ij}c_{ij} + \frac{0.1}{2}\sum_{j=1}^{15}[y_j(y_j+1)+z_j(z_j+1)] \tag{1}$$

In the first part of transportation cost, we use Floyd algorithm to build the shortest path from S_i to A_i, then separate the railway and highway, transform the distance problem into the freight problem, construct weighted undirected graphs respectively, and calculate the minimum freight route respectively. Then we combine the two graphs to find the route with the minimum freight per unit steel pipe between any two points. Therefore, the transportation cost is equal to the sum of the ordered quantity multiplied by the total railway freight rate and the total highway freight rate

$$x_{ij} \times c_{ij} = x_{ij} \times (c_1 + c_2) = x_{ij} \times (w_1 \times d_1 + 0.1 d_2) \tag{2}$$

The second part is the ordering cost. Since the quantity of steel pipes is unchanged in the ordering and transportation links, the ordering and transportation cost c_{ij} is equal to the minimum freight of each factory plus the unit price of each factory：

$$x_{ij} \times p_i \tag{3}$$

The third part is the cost of pipeline laying. As the highway freight rate increases linearly, the farther the pipeline is from the node, the higher the price. Therefore, the method of laying the pipeline to both sides of the node is adopted. The paving cost is equal to the sum of the ordered quantity and the freight rate of the first kilometer road, the freight rate of the second kilometer road and the freight rate of the y_j kilometer road。

$$1 \times (0.1 + 0.2 + \cdots + 0.1 y_j) = \frac{0.1}{2} y_j(y_j+1) \tag{4}$$

$$1 \times (0.1 + 0.2 + \cdots + 0.1 z_j) = \frac{0.1}{2} z_j(z_j+1) \tag{5}$$

■ **Constraint**

When considering the plan of ordering and transporting steel pipes, because the sales prices of $S_{1,2,...,7}$ steel plant are inconsistent, each steel plant needs to produce at least 500 units and have the maximum supply, so we get the following set of constraints:：

$$\sum_{j=1}^{15} x_{ij} \in \{0\} \cup [500, s_i], i = 1, 2, \cdots, 7,$$

In the solution process, considering that the constraint condition is not easy to implement in the program, the 0-1 variable f_i is introduced. When f_i is 1, it means the factory supplies goods, and when F is 0, it means the factory does not supply goods

$$f_i = \begin{cases} 1, \text{factory i provides steel pipes} \\ 0, \text{factory i not provide steel pipes} \end{cases} \tag{6}$$

This constraint can be converted to:

$$500 f_i \le \sum_{j=1}^{15} x_{ij} \le s_i f_i, i = 1, 2, \cdots, 7 \tag{7}$$

In the process of pipeline laying, the transportation price of the highway is linearly related to the transportation mileage. In order to minimize the total cost, the pipeline can be laid from Node $A_{1,2,\ldots,15}$ to its left and right sides, and the length of the laid pipeline should be equal to the total number of steel pipes ordered; The sum of the right laying length of the i node and the left laying length of the i+1 node shall also be equal to the pipe length of the section; Node A_1 can only be laid to the right and node A_{15} can only be laid to the left.

$$\begin{cases} \sum_{i=1}^{7} x_{ij} = z_j + y_j, j = 1, 2, \cdots, 15 \\ z_j + y_{j+1} = l_j, j = 1, 2, \cdots 14 \\ y_1 = 0 \\ z_{15} = 0 \end{cases} \tag{8}$$

In this question, x_{ij} is the steel pipe quantity transported from the steel plant S_i to the node j, y_j is the steel pipe quantity laid from the node j to the left, z_j is the steel pipe quantity laid from the node to the right j and the steel pipe quantity, x_{ij}, y_j, z_j must be non negative.

$$\begin{cases} x_{ij} \ge 0, i = 1, 2, \cdots, 7, j = 1, 2, \cdots, 15, \\ y_j \ge 0, j = 1, 2, \cdots, 15, \\ z_j \ge 0, j = 1, 2, \cdots, 15 \end{cases} \tag{9}$$

■ Model expression

The expression of the model obtained from the above analysis is as follows:

$$\min \sum_{i=1}^{7} \sum_{j=1}^{15} x_{ij} c_{ij} + \frac{0.1}{2} \sum_{j=1}^{15} [y_j(y_j + 1) + z_j(z_j + 1)] \tag{10}$$

$$
s.t\begin{cases}
500f_i \le \sum_{j=1}^{15} x_{ij} \le s_i f_i, i=1,2,\cdots,7 \\
\sum_{i=1}^{7} x_{ij} = z_j + y_j, j=1,2,\cdots,15, \\
z_j + y_{j+1} = l_j, j=1,2,\cdots 14, \\
y_1 = 0, \\
z_{15} = 0, \\
x_{ij} \ge 0, i=1,2,\cdots,7, j=1,2,\cdots,15, \\
y_j \ge 0, j=1,2,\cdots,15, \\
z_j \ge 0, j=1,2,\cdots,15
\end{cases} \tag{11}
$$

4.2 The Solution of Model

MATLAB is used to establish a weighted undirected graph to solve the minimum purchase cost matrix and the objective function.

Step1: Construct a weighted undirected graph of the railway network, whose weight is the distance between train stations. The shortest railway distance between any two points is calculated by Floyd algorithm, and the minimum transportation cost between any two points is calculated by the known relation table between railway freight and distance

Step 2: The weighted weighted undirected graph of the road network is 0.1 * the distance between stations, that is, the road transportation cost. Generates the adjacency matrix of a graph.

Step3: Calculate the minimum transportation cost between any two points, that is, the minimum value of the minimum transportation cost of railway and highway. In addition, transportation costs and procurement costs are combined.

Step 4: Import the known data, including the maximum capacity of the steel plant, the distance between laying nodes, and the generated minimum purchase and freight matrix into Lingo.

Step 5: Input the established objective function and constraint conditions to obtain the optimal solution

4.3 Result analysis

It can be seen from the results that 800 units of steel pipes are ordered from S1, among which 157 units are ordered and transported to A4178 units and A5200 units are transported to A6265 units and A7; A total of 800 units of steel pipes are ordered from S2, of which 179 units are ordered and transported to A2, and 5 units are transported to A3316 units and A5300 units to A8; A total of 999 units of steel pipes were ordered from S5, including 187 units of steel pipes ordered and transported to A3311 units of steel pipes and A4102 units of steel pipes and A5415 units of steel pipes and A11; A total of 1572 units of steel pipes are ordered from S6, including 315 units of steel pipes ordered and shipped to A10, 86 units of steel pipes shipped to A12333 units of steel pipes shipped to A14621 units of steel pipes shipped to A14165 units of steel pipes shipped to A15. The total freight required is 12786316000 yuan.

Table 2: Transportation Expenses

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	Total
S1	0	0	0	157	178	200	265	0	0	0	0	0	0	0	0	800
S2	0	179	5	0	316	0	0	300	0	0	0	0	0	0	0	800
S3	0	0	316	0	20	0	0	0	664	0	0	0	0	0	0	1000
S4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S5	0	0	187	311	102	0	0	0	0	0	415	0	0	0	0	999
S6	0	0	0	0	0	0	0	0	0	351	0	86	333	621	165	1572
S7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	0	179	508	468	616	200	265	300	664	351	415	86	333	621	165	
Price	1278631.6															

The pipe laying arrangement is shown in Table 3：

Table 3: Pipe laying arrangement

Specific path	Order quantity
S1→B4→A4	157
S1→B4→A5	178
S1→B6→A6	200
S1→A7	265
S2→B1→A2	179
S2→B2→A3	5
S2→B4→A5	316
S2→B8→A8	300
S3→B2→A3	316
S3→B4→A5	20
S3→B9→A9	664
S5→B2→A3	187
S5→B4→A5	311
S5→B4→A5→A4	102
S5→B11→A11	415
S6→B10→A10	351
S6→B13→A12	86
S6→B15→A13	333
S6→A14	621
S6→S7→A15	165

The right factory S_6, S_7, etc. cannot be transported to the left shop A_1, A_2, etc., so to reduce the cost, S_7 can only choose to lay $|A_{13}A_{14}|$ or $|A_{14}A_{15}|$ sections. For the paving point A_{14}, S_6 chooses steel mill S_7 to be superior. Therefore, S_7 will only lay $|A_{14}A_{15}|$

section. However, since the transportation cost from the point A_{15} to the point A_{14} is obviously higher than that from the point A_{14} to the point A_{15} separately, the quantity of steel pipes that S_7 needs to order is less than 500 kilometers of the length of $|A_{14}A_{15}|$, and also less than the title "steel pipe plants need to produce at least 500 units", so S_7 does not choose steel pipe plants to produce steel pipes. For the steel plant S_4, all the paving points are far away, so it is the same choice not to produce.

5 Factors affecting purchase and transportation plan

① **Analysis of the influence of the upper limit of steel pipe production on purchase and transportation plan and total cost：**

Through sensitivity analysis, keep the upper limit of output of other plants unchanged, and increase the upper limit of output of this plant by 2%, 4%, 6%, 8% and 10% respectively. The results are shown in Table 4. It can be seen that the change of the upper limit of production of steel plant S1 has the greatest impact on the purchase and transportation plan and the total cost.

Table 4: Total cost table obtained by changing parameter S

Change parameters：s	**2%**	**4%**	**6%**	**8%**	**10%**
S1	1276983.6	1275335.6	1273687.6	1272039.6	1270391.6
S2	1278071.6	1277511.6	1276951.6	1276391.6	1275831.6
S3	1278131.6	1277631.6	1277131.6	1276631.6	1276131.7
S4	1278631.6	1278631.6	1278631.6	1278631.6	1278631.6
S5	1278631.6	1278631.6	1278631.6	1278631.6	1278631.6
S6	1278631.6	1278631.6	1278631.6	1278631.6	1278631.6
S7	1278631.6	1278631.6	1278631.6	1278631.6	1278631.6

② **Analysis of the influence of the change of steel pipe selling price on purchase and transportation plan and total cost：**

When the selling price of steel pipes in steel mills changes, keep the selling price of steel pipes in six steel mills unchanged, and increase the selling price of steel pipes in steel mills by 2%, 4%, 6%, 8% and 10% respectively. The corresponding minimum total cost is shown in the following table. It can be seen that the change of steel pipe sales price of steel plants S1, S2, S3 and S5 has a great impact on the purchase and transportation plan and the total cost. Since the order quantity of steel plant S4 and S7 is zero, the minimum total cost will not be affected when the steel pipe sales price of steel plant changes.

Table 5: Total cost table obtained by changing parameter S

Change parameters：p	**2%**	**4%**	**6%**	**8%**	**10%**
S1	1281191.6	1283751.6	1286311.9	1288871.6	1291431.7
S2	1281111.6	1283591.7	1286071.6	1288551.6	1291031.6
S3	1281731.6	1284831.6	1287931.8	1291031.6	1294131.9
S4	1278631.6	1278631.6	1278631.6	1278631.6	1278631.6
S5	1281706.1	1284204.6	1285974.4	1287744.5	1288359.1
S6	1281768.1	1283830.6	1285848.1	1287820.6	1289640.1
S7	1278631.6	1278631.6	1278631.6	1278631.6	1278631.7

6 Optimization of steel pipe ordering and transportation cost model

6.1 The Establishment of Model

When the pipeline to be laid is not a line but a tree diagram, railways, highways and pipelines form a network. It can be seen from Figure 2 that due to the appearance of the tree diagram, there will be multiple branches at some pipelines. The degree of A9, A11 and A17 nodes is 3. We need to consider these three special nodes and the six additional railway lines. The objective function in the model built by Question 1 can be updated as follows:

$$\min \sum_{i=1}^{7}\sum_{j=1}^{21} x_{ij}c_{ij} + \frac{0.1}{2}\sum_{j=1}^{21}[y_j(y_j+1)+z_j(z_j+1)+q_j(q_j+1)] \tag{12}$$

Constraint condition：

$$s.t\begin{cases} f_i = \begin{cases} 1, \text{factory i provides steel pipes} \\ 0, \text{factory i not provide steel pipes} \end{cases} \\ 500 f_i \le \sum_{j=1}^{21} x_{ij} \le s_i f_i, i = 1,2,\cdots,7, \\ \sum_{i=1}^{7} x_{ij} = z_j + y_j, j = 1,2,\cdots,21 and j \ne 9,11,17, \\ \sum_{i=1}^{7} x_{ij} = y_j + z_j + q_j, j = 9,11,17, \\ z_j + y_{j+1} = l_j, j = 1,2,\cdots,14, \\ q_9 + y_{16} = 42, q_{11} + z_{17} = 10, \\ y_{17} + y_{18} = 130, q_{17} + y_{19} = 190, \\ z_{19} + y_{20} = 260, z_{20} + y_{21} = 100, \\ y_1 = 0, \\ z_j = 0, j = 15,16,18,21, \\ q_j = 0, j = 1,2,\cdots,21 and j \ne 9,11,17, \\ x_{ij} \ge 0, y_j \ge 0, z_j \ge 0, q_j \ge 0, i = 1,2,\cdots,7, j = 1,2,\cdots,21 \end{cases} \tag{13}$$

6.2 The Solution of Model

Write MATLAB code to build a map to find the shortest path and use Lingo to solve the model.

Step 1: Build the weighted undirected graphs of railways and highways respectively. Note that S6 and A21 overlap in the third question. You need to connect an edge with a distance of 0 between two nodes. The edges of both points must be equal.

Step 2: Calculate the minimum transportation cost between any two points, that is, the minimum value of the minimum transportation cost of railway and highway. In addition, transportation costs and procurement costs are combined.

Step 3: Import the known data, including the maximum capacity of the steel plant, the distance between laying nodes, and the generated minimum purchase and freight matrix into Lingo.

Step 4: Input the updated objective function and constraint conditions to obtain the optimal solution

6.3 Result analysis

It can be seen from the results that 800 units of steel pipes are ordered from S1, among which 281 units are ordered and transported to A4, 53 units are transported to A5200 units and A6266 units are transported to A7; A total of 800 units of steel pipes are ordered from S2, of which 179 units are ordered and transported to A2124 units and A4197 units to A5300 units and A8 units; A total of 1000 units of steel pipes are ordered from S3, of which 231 units are ordered and shipped to A3, 63 units are shipped to A4664 units and shipped to A9, 42 units are shipped to A16; A total of 1303 units of steel pipes were ordered from S5, of which 277 units were ordered and transported to A3365 units and to A5126 units and to A10380 units and to A11155 units and to A17. A total of 2000 units of steel pipes were ordered from S6, of which 225 units were ordered to transport to A10111 units of steel pipes to A12393 units of steel pipes to A13571 units of steel pipes to A14165 units of steel pipes to A15, 80 units of steel pipes to A18, 95 units of steel pipes to A19260 units of steel pipes to A20,100 units of steel pipes to A21, and the total freight required was 14066314000 yuan.

Table 6: Transportation Expenses

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	Total
S1	0	0	0	281	53	200	266	0	0	0	0	0	0	0	0	0	0	0	0	0	0	800
S2	0	179	0	124	197	0	0	300	0	0	0	0	0	0	0	0	0	0	0	0	0	800
S3	0	0	231	63	0	0	0	0	664	0	0	0	0	0	0	42	0	0	0	0	0	1000
S4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S5	0	0	277	0	365	0	0	0	0	126	380	0	0	0	0	0	155	0	0	0	0	1303
S6	0	0	0	0	0	0	0	0	0	225	0	111	393	571	165	0	0	80	95	260	100	2000
S7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total	0	179	508	468	615	200	266	300	664	351	380	111	393	571	165	42	155					
Price	1406631.4																					

The pipe laying arrangement is shown in Table 7:

Table 7: Pipe laying arrangement

Specific path	
S1→B4→A5→A4	S5→B3→A3
S1→B5→A5	S5→B5→A5
S1→B6→A6	S5→B9→A10
S1→A7	S5→A17→A11
S2→B1→A2	S5→A17
S2→B5→A5→A4	S6→A20→B9→A10
S2→B8→A8	S6→A20→A19→A12
S3→B3→A3	S6→A20→A13
S3→B5→A5→A4	S6→A14
S3→B16→A9	S6→S7→A15
S3→A16	S6→A20→A18
S6→A20→A19	S6→A20
S6→A21	

7 Sensitivity Analysis

t can be seen from the analysis that the upper limit of supply of each steel plant, the selling price of each steel plant's unit steel pipe, the railway freight rate per unit steel pipe, and the road freight rate per unit steel pipe will have a corresponding impact on the purchase and transportation plan and the total cost. Therefore, we will conduct sensitivity analysis on these four variables. Since the first two variables have been discussed in the second question, we will not analyze them here.

The handling method for railway freight rate of unit steel pipe can be as follows: based on the freight rate of 200000 yuan less than 300 km, record the increased freight rate for each mileage, and then fluctuate in proportion. For example, the freight rate of 401-450 km is 290000 yuan, which is 145% higher than the freight rate of less than 300 km. When - 8% fluctuation sampling occurs, the freight rate of less than 300 km becomes 184000 yuan, and the freight rate of 401-450 km becomes 266800 yuan. The results are as follows. It can be seen from the results that the total price of railway freight increases linearly with the change of parameters. Therefore, the impact of the change of railway freight rate on the purchase and transportation plan and the total cost is excluded.

Table 8: sensitivity analysis

Change parameter range	2%	4%	6%	8%	10%
Total freight	1283580.88	1288508.96	1293415.98	1298302.5	1303154.5

Similarly, it can be seen from the results that the total price of highway freight increases linearly with the change of parameters, so the impact of the change of highway freight on the purchase and transportation plan and the total cost is excluded.

Table 9: sensitivity analysis

Change parameter range	2%	4%	6%	8%	10%
Total freight	1279545.04	1280450.59	1281347.67	1282236.62	1283118.21

8 Model Evaluation and Further Discussion

8.1 Strengths

•**Accuracy of results.** We relied on data and literature for model simplification. To ensure the accuracy of results through verification of different methods.

•**Universality.** On the basis of the first question, we give a more general and universal tree graph model for ordering and transportation. In addition, the sensitivity analysis of the solution results of the model verifies the accuracy of the results.

•**Stability and high fault tolerance.** Our model was tested for sensitivity and its stability was verified. Keeping other plant variables unchanged, analyze the impact of the upper limit of steel pipe production and the change of the selling price on the purchase and transportation plan and the total cost, and verify the accuracy of the results by fluctuating the parameters by 10%. These steps also improve the fault tolerance of the model.

8.2 Weaknesses

•Lack of adversary factor. In the objective function, there is some carelessness in dealing with the cost of pipeline laying. If more accurate expressions can be used to approximate the cost of highway and railway, and the problem can still be solved, a better solution can be obtained.

9 Conclusion

In order to solve the problem of steel pipe ordering and transportation, this paper establishes a multivariate nonlinear optimization model through reasonable analysis, and gradually optimizes the model using enumeration method, graph theory and other methods. The process is rigorous and theoretical. Under the condition that the natural gas pipeline can be laid normally, the scheme of minimizing the total cost of ordering and transportation of steel pipes is given. Through the establishment of weighted undirected graph, Floyd algorithm is used to obtain the minimum path of transportation cost from the steel plant to the paving point, the idea of mathematical programming is used to establish the objective function and determine the constraint conditions, and the Lingo software is used to obtain the global optimal solution. And the model is extended to all kinds of tree graphs or more complex shapes, which can be solved through the idea of this model and has strong universality. Through the corresponding sensitivity analysis, we can see that the upper limit of production of steel plant S1 has the greatest impact on the purchase and transportation plan and the total cost, which is helpful for people to solve similar problems in real life.

References

[1]Babu S. Suresh,Jayasudha K.. A Simplex Method-Based Bacterial Colony Optimization Algorithm for Data Clustering Analysis[J]. International Journal of Pattern Recognition and Artificial Intelligence,2000,36(12).
[2]Wang Lina,Wang Hejing,Yang Xin,Gao Yanfeng,Cui Xiaohong,Wang Binrui. Research on smooth path planning method based on improved ant colony algorithm optimized by Floyd algorithm [J]. Frontiers in Neurorobotics,2000.
[3]Heming Bing,Lu Lai. Improvement and Application of Dijkstra Algorithms[J]. Academic Journal of Computing & Information Science,2000,5.0(5.0).
[4]Ch Leela Kumari,Kamboj Vikram Kumar,Bath S K. Hybridizing slime mould algorithm with simulated annealing algorithm: a hybridized statistical approach for numerical and engineering design problems.[J]. Complex & intelligent systems,2022.
[5]Renjie Hao,Qingyu Ge,Songyuan Huang. Optimization of C4 olefin preparation process based on control variables and simulated annealing algorithm[J]. Academic Journal of Engineering and Technology Science,2022,5.0(9.0).

Appendices

Appendix 1

Introduce: Floyd algorithm

```
% 生成结点名字
Node_s = strcat('S', cellstr(int2str((1:7)')));
Node_a = strcat('A', strtrim(cellstr(int2str((1:15)'))));
Node_b = strcat('B', strtrim(cellstr(int2str((1:17)'))));

Node = [Node_s(:)', Node_a(:)', Node_b(:)'];

G = graph;
G = G.addnode(Node);

% 铁路图边
edge1 = {
    'B1', 'B3', 450; 'B2', 'B3', 80; 'B3', 'B5', 1150;
    'B5', 'B8', 1100; 'B4', 'B6', 306; 'B6', 'B7', 195;
    'B7', 'S1', 20; 'S1', 'B8', 202; 'B8', 'S2', 1200;
    'B8', 'B9', 720; 'B9', 'S3', 690; 'B9', 'B10', 520;
    'B10', 'B12', 170; 'B12', 'S4', 690; 'B12', 'B14', 160;
    'B12', 'B11', 88; 'B11', 'S5', 462; 'B14', 'B13', 70;
    'B14', 'B15', 320; 'B15', 'B16', 160; 'B16', 'S6', 70;
    'B16', 'B17', 290; 'B17', 'S7', 30;
    };

% 添加边到图中
G1 = G.addedge(edge1(:, 1), edge1(:, 2), cell2mat(edge1(:, 3)));
d1 = distances(G1);

% 将最短路转化为价格
c1 = inf * ones(size(d1));

c1(d1 == 0) = 0;
c1(d1 > 0 & d1 <= 300) = 20;
c1(d1 > 300 & d1 <= 350) = 23;
c1(d1 > 350 & d1 <= 400) = 26;
c1(d1 > 400 & d1 <= 450) = 29;
c1(d1 > 450 & d1 <= 500) = 32;
c1(d1 > 500 & d1 <= 600) = 37;
c1(d1 > 600 & d1 <= 700) = 44;
```

```
c1(d1 > 700 & d1 <= 800) = 50;
c1(d1 > 800 & d1 <= 900) = 55;
c1(d1 > 900 & d1 <= 1000) = 60;
idx = d1 > 1000 & d1 < inf;
c1(idx) = 60 + 5*ceil(d1(idx)/100 - 10);

% 公路图边
edge2 = {
    'A1', 'A2', 104; 'A2', 'B1', 3; 'A2', 'A3', 301; 'A3', 'B2', 2;
    'A3', 'A4', 750; 'A4', 'B5', 600; 'A4', 'A5', 606; 'A5', 'B4', 10;
    'A5', 'A6', 194; 'A6', 'B6', 5; 'A6', 'A7', 205; 'A7', 'B7', 10;
    'A7', 'S1', 31; 'A7', 'A8', 201; 'A8', 'B8', 12; 'A8', 'A9', 680;
    'A9', 'B9', 42; 'A9', 'A10', 480; 'A10', 'B10', 70; 'A10', 'A11', 300;
    'A11', 'B11', 10; 'A11', 'A12', 220; 'A12', 'B13', 10; 'A12', 'A13', 210;
    'A13', 'B15', 62; 'A13', 'A14', 420; 'A14', 'S6', 110; 'A14', 'B16', 30;
    'A14', 'A15', 500; 'A15', 'B17', 20; 'A15', 'S7', 20;
    };

% 添加边到图中
G2 = G.addedge(edge2(:, 1), edge2(:, 2), cell2mat(edge2(:, 3)));
% 生成邻接矩阵
c2 = full(adjacency(G2, 'weighted'));

c2 = c2 * 0.1;
c2(c2 == 0) = inf;

% 生成整个图，边权为价格
c = min(c1, c2);
G3 = graph(c);
% 求最短路
dist = distances(G3);

% 将运输费用和采购费用合并
p = [160; 155; 155; 160; 155; 150; 160];
res = dist(1:7, 8:22) + p;

% 写入文件
writematrix(res, 'Data.xlsx');
```

Appendix 2
Introduce: Lingo

```
sets:
factory/1..7/: s, f; ! s 表示每个钢厂的最大供应量，f 表示每个钢厂是否生产;
node/1..15/:y, z, l; ! y 表示向左铺设的长度，z 表示向右铺设的长度，l 表示两个结点之间的距离;
link(factory, node):x, c; ! x 表示采购方案，c 表示价格;
endsets

data:
s = 800  800 1000      2000      2000      2000      3000;

l = 104, 301, 750, 606, 194, 205, 201, 680, 480, 300, 220, 210, 420, 500, 0;
c = @ole("C:\Users\ASUS\OneDrive - njupt.edu.cn\Mathematical Contest in Modeling\国赛 B_钢管订购与运输\program\Data.xlsx", "res");

enddata

! 目标函数 ;
min = @sum(link(i, j):x(i, j)*c(i, j)) + 0.05 * @sum(node(j): y(j)^2 + y(j) + z(j)^2 + z(j));

! 不超过产能;
@for(factory(i): @sum(node(j):x(i, j)) <= s(i) * f(i));
! 最少订购 500;
@for(factory(i): @sum(node(j):x(i, j)) >= 500 * f(i));
! 左右铺设的长度应等于总订购量;
@for(node(j):@sum(factory(i):x(i, j)) = z(j) + y(j));
! 某结点向右的和下一个结点向左的距离应等于两个结点的距离;
@for(node(j)|j#ne#15:z(j) + y(j + 1) = l(j));
! 第一个结点不能向左铺;
y(1) = 0;
! 最后一个结点不能向右铺;
z(15) = 0;
! f 为 0-1 变量数组;
@for(factory: @bin(f));
! 限制为整数;
@for(link: @gin(x));
@for(node: @gin(y));
@for(node: @gin(z));
res = @sum(link(i, j):x(i, j)*c(i, j)) + 0.05 * @sum(node(j): y(j)^2 + y(j) + z(j)^2 + z(j));
```

```
! 写入数据到文件;
data:
@ole("C:\Users\ASUS\OneDrive - njupt.edu.cn\Mathematical Contest in Modeling\国赛B_钢管订购与运输\res.xlsx", "res") = x;
@ole("C:\Users\ASUS\OneDrive - njupt.edu.cn\Mathematical Contest in Modeling\国赛B_钢管订购与运输\res.xlsx", "price") = res;
@ole("C:\Users\ASUS\OneDrive - njupt.edu.cn\Mathematical Contest in Modeling\国赛B_钢管订购与运输\res.xlsx", "Y") = y;
@ole("C:\Users\ASUS\OneDrive - njupt.edu.cn\Mathematical Contest in Modeling\国赛B_钢管订购与运输\res.xlsx", "z") = z;
enddata
end
```