Correction:

$$\frac{\partial P_i}{\partial \tilde{\mathbf{x}}} = \mathbf{R}_{L_i L_{i-1}} \mathbf{R}_{LI} \cdot [-\mathbf{R}_{IW}[({}_W P_i -{}_W \mathbf{p}_{WI})]_\times \quad -\mathbf{R}_{IW} \quad 0]$$

The robot state is defined as $\mathbf{x} = [\mathbf{R}_{WI}, {}_W\mathbf{p}_{WI}, {}_W\mathbf{v}_I, \mathbf{b}_a, \mathbf{b}_g, \mathbf{g}]$

1. Coordinate Transformation

The 3D point ${}_W P_i$ is first transformed from the **world frame** to the **IMU frame**, then to the **LiDAR frame**:

$$ {}_L P_i = \mathbf{R}_{LI} \cdot \mathbf{R}_{IW} \cdot ({}_W P_i -{}_W \mathbf{p}_{WI}) \tag{1}$$

where:

- $\mathbf{R}_{IW}$: Rotation from world to IMU
- $\mathbf{R}_{LI}$: LiDAR-to-IMU extrinsic rotation
- ${}_W\mathbf{p}_{WI}$: IMU position in world frame

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2. Perturbation Analysis for Rotation

For a small perturbation $\delta\theta$ in attitude $\mathbf{R}_{IW}$, we use the **right perturbation model** on $SO(3)$:

$$\mathbf{R}_{IW} \to \mathbf{R}_{IW} \cdot \exp([\delta\theta]_\times) \tag{2}$$

The perturbed point becomes:

$$ {}_L P_i' = \mathbf{R}_{LI} \cdot \mathbf{R}_{IW} \cdot \exp([\delta\theta]_\times) \cdot ({}_W P_i -{}_W \mathbf{p}_{WI}) \tag{3}$$

Using first-order approximation $\exp([\delta\theta]_\times) \approx \mathbf{I} + [\delta\theta]_\times$:

$$\begin{aligned} {}_L P_i' &\approx \mathbf{R}_{LI} \cdot \mathbf{R}_{IW} \cdot (\mathbf{I} + [\delta\theta]_\times) \cdot ({}_W P_i -{}_W \mathbf{p}_{WI}) \\ \Rightarrow \delta L P_i &= \mathbf{R}_{LI} \cdot \mathbf{R}_{IW} \cdot [\delta\theta]_\times \cdot ({}_W P_i -{}_W \mathbf{p}_{WI}) \end{aligned} \tag{4}$$

Using the property $[a]_\times b = -[b]_\times a$:

$$\delta_L P_i' = -\mathbf{R}_{LI} \cdot \mathbf{R}_{IW} \cdot [({}_W P_i -{}_W \mathbf{p}_{WI})]_\times \cdot \delta\theta \tag{5}$$

As derived previously, the rotation perturbation leads to:

$$\frac{\partial_L P_i}{\partial \delta\theta} = -\mathbf{R}_{LI} \cdot \mathbf{R}_{IW} \cdot [({}_W P_i -{}_W \mathbf{p}_{WI})]_\times \tag{6}$$

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3. Position Perturbation

$$ {}_W\mathbf{p}'_{WI} ={}_W \mathbf{p}_{WI} + \delta\mathbf{p} \tag{7}$$

For a position perturbation $\delta\mathbf{p}$, the derivative is straightforward according to equation (1):

$$\frac{\partial_L P_i}{\partial \delta\mathbf{p}} = -\mathbf{R}_{LI} \cdot \mathbf{R}_{IW} \tag{8}$$

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4. Full Matrix

Combining the rotation and position terms, we get:

$$\frac{\partial P_i}{\partial \tilde{\mathbf{x}}} = \mathbf{R}_{L_i L_{i-1}} \mathbf{R}_{LI} \cdot [-\mathbf{R}_{IW}[({}_W P_i -{}_W \mathbf{p}_{WI})]_\times \quad -\mathbf{R}_{IW} \quad 0] \tag{9}$$

where:

- $\mathbf{R}_{L_i L_{i-1}}$: Relative rotation between consecutive LiDAR frames