Anjali Yadav
1853.

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# Assignment 1

## Part A

Q1) In asymptotes if two values of m are real and same then write the formula to obtained c.

$$\frac{c^2}{2}\phi''_n(m) + \frac{c\phi'_{n-1}(m)}{1!} + \phi_{n-2}(m) = 0$$

Q2) Find the ∥ asymptotes of $a^2y^2 + b^2x^2 = x^2y^2$

$$a^2y^2 + b^2x^2 - x^2y^2 = 0$$
$$y^2(a^2 - x^2) + b^2x^2 = 0$$

$$a^2 - x^2 = 0$$
$$\boxed{x = \pm a}$$ asymptotes ∥el to y-axis

$$a^2y^2 + x^2(b^2 - y^2) = 0$$
$$b^2 - y^2 = 0$$
$$\boxed{y = \pm b}$$ asymptotes ∥el to x axis

Q3) Find the radius of curvature of $S = 4a\sin\psi$

$$S = 4a\sin\psi$$
$$\frac{ds}{d\psi} = \rho = 4a\cos\psi$$

Q4) Find radius of curvature of curve $r = a(1-\cos\theta)$

$$r = a(1 - \cos\theta)$$
$$\frac{dr}{d\theta} = r' = a\sin\theta$$

$$\frac{d^2 r}{d\theta^2} = r'' = a\cos\theta.$$

$$\rho = \frac{[r^2 + r'^2]^{3/2}}{r^2 + 2r'^2 - rr''}$$

$$\rho = \frac{\{[a(1-\cos\theta)]^2 + (a\sin\theta)^2\}^{3/2}}{[a(1-\cos\theta)]^2 + 2(a\sin\theta)^2 - a(1-\cos\theta)(a\cos\theta)}$$

$$\rho = \frac{[2 - 2\cos\theta]^{3/2}.}{[1 + 2(\cos^2\theta + \sin^2\theta) - 3\cos\theta]}$$

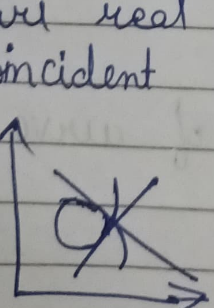
$$\rho = \frac{2\sqrt{2}a}{3}(1-\cos\theta)^{1/2}$$

$$\rho = \frac{2}{3}\sqrt{2ar} \qquad \frac{r}{a} = (1-\cos\theta)$$

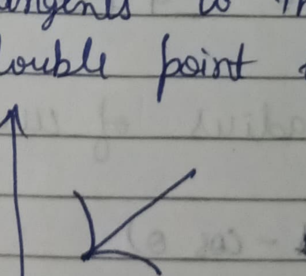
Q 5] Define node and cusp.

Node: If two branched through a double point on a curve are real and tangents to them are not coincident.

Cusp: If two branches through a double point are real and tangents to them are coincident, the double point is Cusp.



Node



Cusp

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Part B)

Q 1) Find the asymptotes of the Curve.

$$y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0$$

Coeff of highest degree terms of y and x are constant,
so there is no asymptote llel to y axis and x-axis.

For oblique Asymptotes,

$\phi_3(m) = m^3 - 6m^2 + 11m - 6.$

$\phi_2(m) = 0$

$\phi_1(m) = 1 + m.$

$\phi_3(m) = 0$ $\qquad (6 = 1, 2, 3)$

$m^3 - 6m^2 + 11m - 6 = 0$

$(m-1)(m^2 - 5m + 6) = 0$

$(m-1)(m-2)(m-3) = 0.$

$\boxed{m = 1, 2, 3}$

$$c = -\frac{\phi_{n-1}(m)}{\phi'_n(m)}$$

as $\phi_2(m) = 0$ value of $c = 0$

Eqn of asymptotes are $\quad y = mx + c$

$y = x$, $y = 2x$, $y = 3x$

Q2) Prove that radius of curvature at $(x, y)$ on the curve $y = \frac{ax}{a+x}$ is given by

$$\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2 \quad — (1)$$

$\Rightarrow$ $y = \frac{ax}{a+x}$

$$y' = \frac{a(a+x) - ax}{(a+x)^2} \Rightarrow \frac{a^2}{(a+x)^2}$$

$$y'' = \frac{(a+x)^2 - a^2 \times 2(a+x)}{(a+x)^4} \Rightarrow \frac{-2a^2}{(a+x)^3}$$

$$\rho = \frac{(1+y'^2)^{3/2}}{|y''|} \Rightarrow \frac{\left\{1 + \left[\frac{a^2}{(a+x)^2}\right]^2\right\}^{3/2}}{\left|-\frac{2a^2}{(a+x)^3}\right|}$$

Put $\rho$ in eq (1)

and $y = \frac{ax}{a+x}$ on right side of eq (1)

we will get

$$\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$$

Q3) Find radius of curvature of $a^2 y = x^3 - a^3$

$$y = \frac{1}{a^2}(x^3 - a^3)$$

$$y' = \frac{1}{a^2}(3x^2)$$

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$$y'' = \frac{6x}{a^2}$$

$$\rho = \frac{(1+y'^2)^{3/2}}{|y''|} \Rightarrow \frac{\left[1 + \left(\frac{3x^2}{a^2}\right)^2\right]^{3/2}}{\frac{6x}{a^2}}$$

$$\rho = \frac{\left[a^4 + 9x^4\right]^{3/2}}{\frac{6x}{a^2} \times a^6}$$

$$\boxed{\rho = \frac{(9x^4 + a^4)^{3/2}}{6xa^4}}$$

Part C

Q1) Show that asymptotes of curve $x^3 - 2y^3 - xy^2 + 2x^2y + xy - y^2 + 1 = 0$ cut the curve in 3 points which lies on line $x - y + 1 = 0$.

$x^3 - 2y^3 - xy^2 + 2x^2y + xy - y^2 + 1 = 0$ — ①

No ||el asymptotes to x axis and y axis as there is a constant factor with highest degree of x & y.

Oblique Collision $\phi_2(m) = m - m^2$

$\phi_3(m) = 1 - 2m^3 + 2m - m^2$

Put $m = 1$ $\phi_3(m) = 0$

So $(m-1)$ is a factor of $\phi_3(m)$

$(m-1)(2m^2 + 3m + 1)$

$(m-1)(m+1)(2m+1)$

$m = 1, -1, -\frac{1}{2}$

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$$C = -\frac{\phi_2(m)}{\phi_3(m)} \Rightarrow -\left[\frac{-m^2-m}{(-6m^2-2m+1)}\right]$$

$m = 1 \quad \therefore \quad c = 0.$

$m = -1 \quad c = -1$

$m = -\frac{1}{2} \quad c = \frac{1}{2}$

Eqⁿ of Asymptotes = $y = -x - 1$

$y = x.$

$2y + x - 1 = 0$

Combined eq of all asymptotes =

$$(y-x)(2y+x-1)(y+x+1) = 0$$

$$2y^3 + xy^2 - y^2 + 2y^2 + xy - y - 2x^2y - x^3 + x^2$$
$$-2xy - x^2 + x = 0$$

$$\Rightarrow 2y^3 + xy^2 + y^2 - y - 2x^2y - x^3 - xy + x = 0 \quad (2)$$

highest degree of eq = 3.

Curve enterest at most $= n(n-2)$

$= 3(3-2) = \boxed{3}$

Now add eq (1) + (2)

we get $\boxed{x - y + 1 = 0}$

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Q2) Find the eqⁿ of circle of curvature of parabola $y^2 = 12x$ at point $(3,6)$

General Eq of Circle is

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

$$x_1 = x - \frac{y'}{y''}(1 + y'^2)^2$$

$$y_1 = y + \frac{1}{y''}(1 + y'^2)$$

$$y^2 = 12x$$

$$y' = \frac{6}{y}$$

$$y'' = -\frac{6}{y^2} \times y'$$

$$y'' = -\frac{36}{y^3}$$

$y'(3,6) = 1$

$y''(3,6) = -\frac{1}{6}$

$$r = \frac{[1 + (y')^2]^{3/2}}{|y''|} = \frac{(1 + 1)^{3/2}}{\frac{1}{6}}$$

$$r = 12\sqrt{2}$$

$$x_1 = 3 - \frac{1}{(-1/6)}(1 + (1)^2)^2$$

$$x_1 = 15$$

$$y_1 = 6 + \frac{1}{(-1/6)}(1 + (1)^2) \Rightarrow -6$$

Eqⁿ of circle will be

$(x-15)^2 + (y+6)^2 = (12\sqrt{2})^2$

$x^2 + 225 - 30x + y^2 + 36 + 12y - 288 = 0$

$\boxed{x^2 + y^2 - 30x + 12y - 27 = 0}$

Q3) Trace the curve $y^2(a+x) = x^2(a-x)$

$y^2 a + xy^2 - ax^2 + x^3 = 0$

Step 1: Symmetry)

y has even power, so curve is symmetrical about x-axis

Step 2) Origin (0,0)

Curve stasify the condition $0=0$ which states it passes through origin.

Step 3) Tangent = tangent at origin, putting lowest degree term in the eqⁿ of the curve to be 0.

$y^2 a - x^2 a = 0$

$y^2 = x^2 \Rightarrow \boxed{y = \pm x}$

tangents are real & distinct and not coincident

So it is Node

Step 4) Intersection point =

Put $y = 0$ then

$x^2(a-x) = 0$

$\boxed{x = 0, +a}$ Intersect at $(a,0), (0,0)$

Step 5) Asymptotes

highest degree $x$ has constant term so no ||el asymptote to $x$ axis

for $y$ axis, $a+x=0$

$$\boxed{x=-a}$$

~~Region~~ Oblique = $\phi_3(m) = m^2+1$

$\phi_2(m) = a(m^2-1)$

Imaginary also oblique asymptotes

Step 6) Region.

$$y^2 = \frac{x^2(a-x)}{a+x}$$

$$y = x\sqrt{\frac{a-x}{a+x}}.$$

| | |
|---|---|
| $x<a$ | $y=$ real |
| $x>a$ | $y=$ imaginary |
| $x=0$ | $y=0$ |

then $x \le a$