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1853.

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Assignment 1

Part A

Q1) In asymptotes if two values of m are real and same then write the formula to obtained c.

$$\frac{c^2}{2}\phi''_n(m) + \frac{c\phi'_{n-1}(m)}{1!} + \phi_{n-2}(m) = 0$$

Q2) Find the ∥ asymptotes of $a^2y^2 + b^2x^2 = x^2y^2$

$a^2y^2 + b^2x^2 - x^2y^2 = 0$

$y^2(a^2 - x^2) + b^2x^2 = 0$

$a^2 - x^2 = 0$

$\boxed{x = \pm a}$ asymptotes ∥el to y-axis

$a^2y^2 + x^2(b^2 - y^2) = 0$

$b^2 - y^2 = 0$

$\boxed{y = \pm b}$ asymptotes ∥el to x axis

Q3) Find the radius of curvature of $S = 4a\sin\psi$

$S = 4a\sin\psi$

$\frac{ds}{d\psi} = \rho = 4a\cos\psi$

Q4) Find radius of curvature of curve $r = a(1-\cos\theta)$

$r = a(1 - \cos\theta)$

$\frac{dr}{d\theta} = r' = a\sin\theta$

$$\frac{d^2r}{d\theta^2} = r'' = a\cos\theta.$$

$$\rho = \frac{[r^2 + r'^2]^{3/2}}{r^2 + 2r'^2 - rr''}$$

$$\rho = \frac{\{[a(1-\cos\theta)]^2 + (a\sin\theta)^2\}^{3/2}}{[a(1-\cos\theta)]^2 + 2(a\sin\theta)^2 - a(1-\cos\theta)(a\cos\theta)}$$

$$\rho = \frac{[2 - 2\cos\theta]^{3/2}}{[1 + 2(\cos^2\theta + \sin^2\theta) - 3\cos\theta]}$$

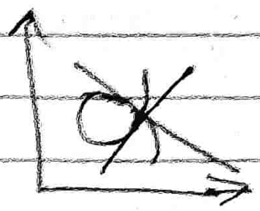
$$\rho = \frac{2\sqrt{2}\,a(1-\cos\theta)^{1/2}}{3}$$

$$\rho = \frac{2}{3}\sqrt{2ar} \qquad \frac{r}{a} = (1-\cos\theta)$$

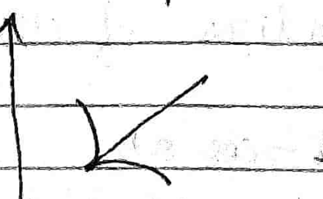
Q 5) Define node and cusp.

Node = If two branched through a double point on a curve are real and tangents to them are not coincident.

Cusp = If two branches through a double point are real and tangents to them are coincident, the double point is Cusp.



Node



Cusp

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Part B)

Q4 Find the asymptotes of the Curve.

$$y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0$$

Coeff of highest degree terms of y and x. are constant,
so there is no asymptote llel to y axis and x-axis.

For oblique Asymptotes,

$\phi_3(m) = m^3 - 6m^2 + 11m - 6$

$\phi_2(m) = 0$

$\phi_1(m) = 1 + m$.

$\phi_3(m) = 0$ $\quad$ $(6 = 1, 2, 3)$

$m^3 - 6m^2 + 11m - 6 = 0$

$(m-1)(m^2 - 5m + 6) = 0$

$(m-1)(m-2)(m-3) = 0$

$\boxed{m = 1, 2, 3}$

$$c = -\frac{\phi_{n-1}(m)}{\phi'_n(m)}$$

as $\phi_2(m) = 0$ $\quad$ value of $c = 0$

Eqn of asymptotes are $\quad y = mx + c$

$y = x$, $y = 2x$, $y = 3x$.

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Q2) Prove that radius of curvature at (x, y) on the curve $y = \frac{ax}{a+x}$ is given by

$$\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2 \quad -(1)$$

$\Rightarrow$ $y = \frac{ax}{a+x}$.

$$y' = \frac{a(a+x) - ax}{(a+x)^2} \Rightarrow \frac{a^2}{(a+x)^2}$$

$$y'' = \frac{(a+x)^2 - a^2 \times 2(a+x)}{(a+x)^4} \Rightarrow \frac{-2a^2}{(a+x)^3}$$

$$\rho = \frac{(1+y'^2)^{3/2}}{|y''|} \Rightarrow \frac{\left\{1 + \left[\frac{a^2}{(a+x)^2}\right]^2\right\}^{3/2}}{\left|-\frac{2a^2}{(a+x)^3}\right|}$$

Put ρ in eq (1)
and $y = \frac{ax}{a+x}$ on right side of eq (1)

we will get:

$$\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$$

Q3) Find radius of curvature of $a^2y = x^3 - a^3$

$$y = \frac{1}{a^2}(x^3 - a^3)$$

$$y' = \frac{1}{a^2}(3x^2)$$

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$$y'' = \frac{6x}{a^2}$$

$$\rho = \frac{(1+y'^2)^{3/2}}{|y''|} \Rightarrow \frac{\left[1 + \left(\frac{3x^2}{a^2}\right)^2\right]^{3/2}}{\frac{6x}{a^2}}$$

$$\rho = \frac{(a^4 + 9x^4)^{3/2}}{\frac{6x}{a^2} \times a^6}$$

$$\boxed{\rho = \frac{(9x^4 + a^4)^{3/2}}{6xa^4}}$$

Part C

Q1) Show that asymptotes of curve $x^3 - 2y^3 - xy^2 + 2x^2y + xy - y^2 + 1 = 0$ cut the curve in 3 points which lies on line $x - y + 1 = 0$.

$$x^3 - 2y^3 - xy^2 + 2x^2y + xy - y^2 + 1 = 0 \quad \text{--- (1)}$$

No ||el asymptotes to x axis and y axis as there is a constant factor with highest degree of x & y.

Oblique Collision $\phi_2(m) = m - m^2$

$\phi_3(m) = 1 - 2m^3 + 2m - m^2$

Put $m = 1$ $\quad \phi_3(m) = 0$

So $(m-1)$ is a factor of $\phi_3(m)$

$(m-1)(2m^2 + 3m + 1)$

$(m-1)(m+1)(2m+1)$

$$m = 1, -1, -\frac{1}{2}$$

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$$C = -\frac{\phi_2(m)}{\phi_3(m)} \Rightarrow = \left[\frac{-m^2-m}{(-6m^2-2m+1)}\right]$$

$m = 1$ $\quad C = 0$

$m = -1$ $\quad C = -1$

$m = -\frac{1}{2}$ $\quad C = \frac{1}{2}$

Eq^n of Asymptotes = $\quad y = -x - 1$

$y = x.$

$2y + x - 1 = 0$

Combined eq of all asymptotes =

$(y-x)(2y+x-1)(y+x+1) = 0$

$2y^3 + xy^2 - y^2 + 2y^2 + xy - y - 2x^2y - x^3 + x^2$
$-2xy - x^2 + x = 0$

$\Rightarrow 2y^3 + xy^2 + y^2 - y - 2x^2y - x^3 - xy + x = 0$ — (2)

highest degree of eq = 3.

Curve interest at most $= n(n-2)$

$= 3(3-2) = \boxed{3.}$

Now add eq (1) + (2)

we get $\boxed{x - y + 1 = 0}$

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Q2) Find the eq^n of circle of curvature of parabola $y^2 = 12x$ at point $(3,6)$

General Eq of Circle is

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

$$x_1 = x - \frac{y'}{y''}(1 + y'^2)^2$$

$$y_1 = y + \frac{1}{y''}(1 + y'^2)$$

$y^2 = 12x$	$y'' = \frac{-6}{y^2} \times y'$
$y' = \frac{6}{y}$	$y'' = -\frac{36}{y^3}$

$$y'(3,6) = 1$$

$$y''(3,6) = -\frac{1}{6}$$

$$r = \frac{(1 + (y')^2)^{3/2}}{|y''|} = \frac{(1+1)^{3/2}}{\frac{1}{6}}$$

$$r = 12\sqrt{2}$$

$$x_1 = 3 - \frac{1}{(-1/6)}(1 + (1)^2)^2$$

$$x_1 = 15$$

$$y_1 = 6 + \frac{1}{(-1/6)}(1 + (1)^2) \Rightarrow -6$$

Eq^n of circle will be

$(x-15)^2 + (y+6)^2 = (12\sqrt{2})^2$

$x^2 + 225 - 30x + y^2 + 36 + 12y - 288 = 0$

$\boxed{x^2 + y^2 - 30x + 12y - 27 = 0}$

Q3) Trace the curve $y^2(a+x) = x^2(a-x)$

$y^2a + xy^2 - ax^2 + x^3 = 0$

Step 1: Symmetry)
y has even power, so curve is symmetrical about x-axis

Step 2) Origin (0,0)
Curve satisfy the conditions $0=0$
which states it passes through origin

Step 3) Tangent = tangent at origin, putting lowest degree term in the eq^n of the curve to be 0.

$y^2a - x^2a = 0$

$y^2 = x^2 \Rightarrow \boxed{y = \pm x}$

tangents are real & distinct and not coincided
So it is Node

Step 4) Intersection point =
Put $y=0$ then

$x^2(a-x) = 0$

$\boxed{x = 0, +a}$ intersect at $(a,0), (0,0)$

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Step 5) Asymptotes

highest degree x has constant term so no ||el asymptote to x axis

for y axis, $a + x = 0$

$$\boxed{x = -a}$$

~~Region~~ Oblique = $\phi_3(m) = m^2 + 1$

$\phi_2(m) = a(m^2 - 1)$

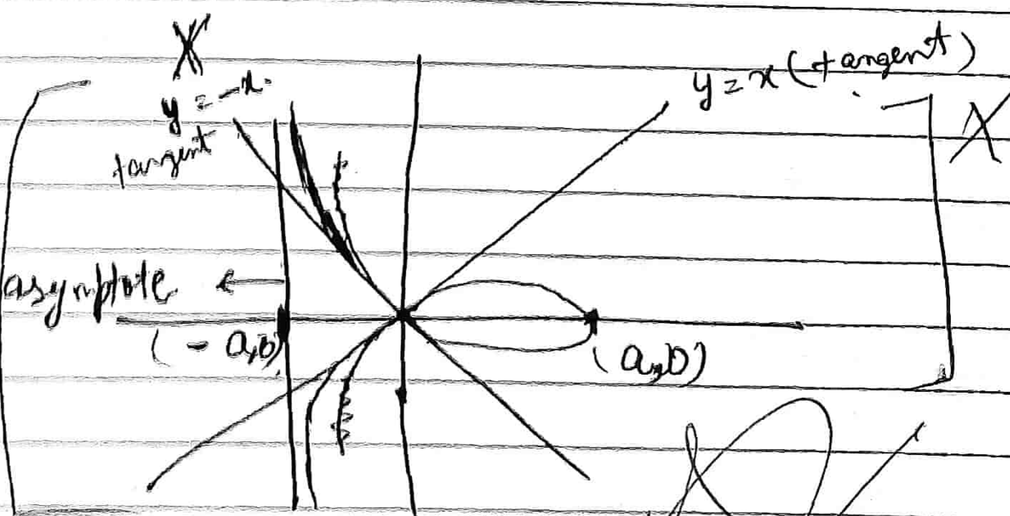
Imaginary so oblique asymptotes

Step 6)
Region. $y^2 = \dfrac{x^2(a-x)}{(a+x)}$

$y = x\sqrt{\dfrac{a-x}{a+x}}$.

$x < a$	$y =$ real
$x > a$	$y =$ imaginary
$x = 0$	$y = 0$

then $\cancel{0 \le} x \le a$



Curve of the ques^n