HYCONS LAB
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LKA Guide

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1 LKA Info

1.1 Quick Summary

The Matlab LKA documentation utilizes a Simulink block to demonstrate the control objectives and constraints. The basic premise that the Lane Keeping uses is to use a given input and velocity to form a dynamic control system based on keeping the relative yaw angle(tangent to circle line) and lateral deviation(distance from the center of lane) as close to 0 as possible which ensure that the car can stay within the lane.

1.2 Inputs and Outputs

State variables: Lateral velocity V_y and yaw angle rate r
Input variable: Front steering angle
Output variables: Same as state variables

1.3 State/Output

Lateral velocity represents the sideways motion of the vehicle relative to its longitudinal axis. It accounts for the vehicle's movement perpendicular to the direction of travel. Changes in lateral velocity are influenced by factors such as steering angle and tire forces. Takes the output after computations and plugs back in to be reused at start The yaw angle rate describes the rate at which the vehicle's heading changes over time(based on the curvature equation) It measures the rotational speed of the vehicle around its vertical axis. Yaw angle rate is influenced by steering inputs and the vehicle's geometry. Also plugs its updated angle back in

1.4 Input

Front Steering Angle represents the angular displacement of the front wheels from the straight-ahead position. Adjusting the steering angle allows the vehicle to change direction based on a given state

1.5 State Space Model

The state equations describe how the state variables of the system evolve. In the case of the bicycle model, the state variables are the lateral velocity and yaw rate. The state equation in this case is X = Ax+Bu Here $x = [V_y, r]^T$ with the current curvature map over a given time variable $\dot{X}$ shows how this current state changes over time A is the state matrix, describing how the state variables evolve autonomously, without any control inputs. B is the input matrix, specifying how the control inputs affect the state variables C is an empty matrix in which the output is outputted to D can be given as a 0 matrix in this case there is no direct feed forward relationship

1.6 Curvature Calculation

The curvature of a parametric curve can be computed using the following formula:

$$\kappa = \frac{\dot{X}\ddot{Y} - \ddot{X}\dot{Y}}{(\dot{X}^2 + \dot{Y}^2)^{\frac{3}{2}}}$$

Where:

- $\dot{X}$ represents the first derivative of the desired X position with respect to time.
- $\ddot{X}$ represents the second derivative of the desired X position with respect to time.

- $\dot{Y}$ represents the first derivative of the desired Y position with respect to time.
- $\ddot{Y}$ represents the second derivative of the desired Y position with respect to time.

In the provided MATLAB function, these derivatives are approximated using the `gradient` function with a step size of 0.1. The Curvature Previewer block outputs the previewed curvature with a look-ahead time of one second. Therefore, given a sample time $Ts = 0.1$, the prediction horizon is 10 steps. The curvature used in this example is calculated based on trajectories for a double-lane change maneuver. So by Identifying the prediction horizon and obtaining the previewed curvature. In one example we can get PredictionHorizon = 10, time = 0:0.1:15; md = $getCurvature(V_x, time)$;

1.7 Dynamic Control System

The Lane Keeping Assist (LKA) system forms a dynamic control system that adjusts the vehicle's trajectory to keep it within the lane. This system takes the front steering angle input from the driver and the vehicle's velocity to compute the appropriate control actions in real-time. By monitoring the vehicle's position relative to the lane center line and the curvature of the road, the LKA system dynamically modulates the steering angle to correct deviations and maintain a desired path. It does so by Error Processing: The control system continuously monitors the lateral deviation as measured by onboard sensors. These error signals represent the discrepancies between the vehicle's actual position and orientation and the desired trajectory within the lane. Mathematically, the errors are defined as e_1 = deviation and e_2 as the yaw angle. The control system processes these error signals to compute the necessary steering adjustments required to minimize deviations from the lane center line.

1.8 Sensor Dynamics

The sensor dynamics module in the Simulink system takes in several inputs to compute the relative yaw angle and lateral deviation. The inputs include the current curvature (p), the longitudinal velocity (V_x), the lateral velocity (state variable V_y), and the yaw rate (state variable $\dot{\psi}$).

The process involves the following steps:

1.8.1 Relative Yaw Angle Calculation

The relative yaw angle (e_2) is computed using the equation:

$$e_2 = r - V_x \cdot p$$

where:

- r is the yaw rate.
- V_x is the longitudinal velocity.
- p is the current curvature.

This equation is integrated over time with the Laplace transform $\frac{1}{s}$.

1.8.2 Lateral Deviation Calculation

The lateral deviation (e_1) is computed using the equation:

$$e_1 = V_x \cdot e_2 + V_y$$

where:

- V_x is the longitudinal velocity.
- V_y is the lateral velocity.
- e_2 is the relative yaw angle calculated in the previous step.

This equation is integrated over time with the Laplace transform $\frac{1}{s}$.

In summary, the sensor dynamics module integrates the product of the current curvature and longitudinal velocity to compute the relative yaw angle, which is then combined with the lateral velocity to calculate the lateral deviation. These values are essential for the overall control and navigation of the vehicle.

1.8.3 Process Flow Diagram

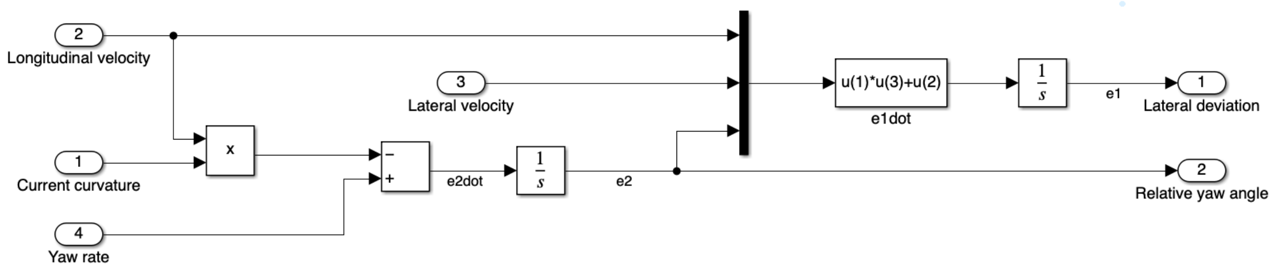
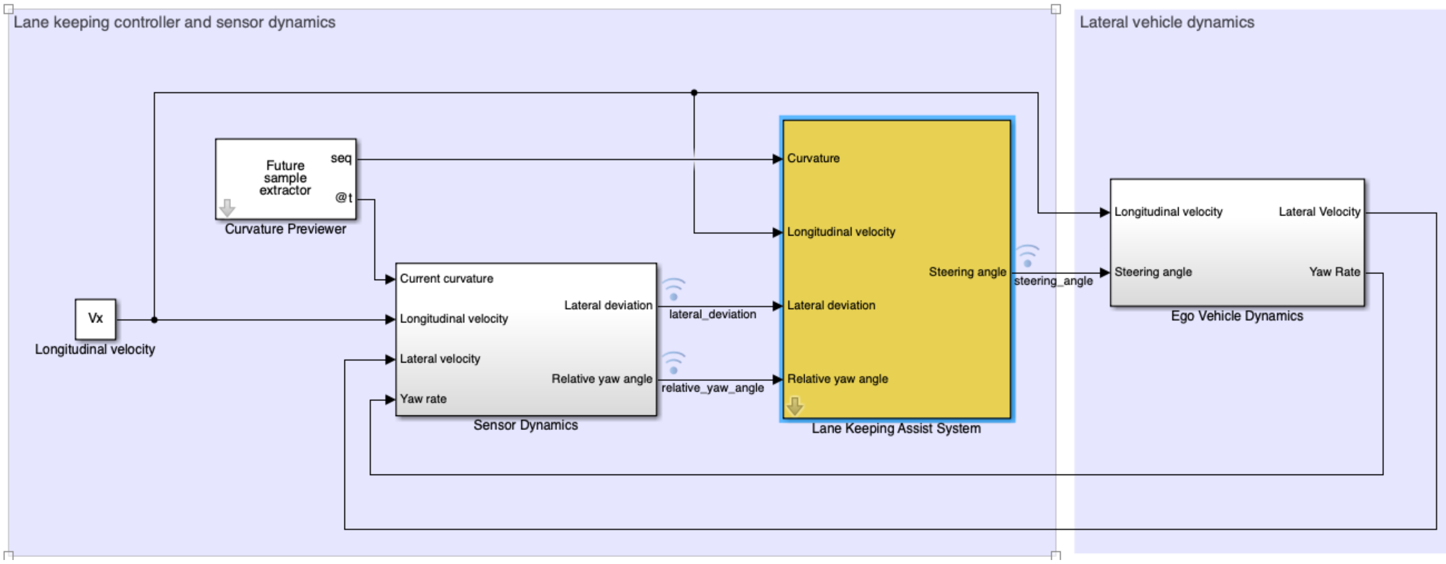


Figure 1: Sensor Dynamics Simulink

Depicted In Figure 1 is the process flow of the sensor dynamics module. The module takes in the current curvature, longitudinal velocity, lateral velocity, and yaw rate as inputs. The (e_2) equation is mapped out to show how it is formed through V_x, md, and $YawRate$ and how it is combined with V_y to derive the e_1 error which is essential for the overall control and navigation of the vehicle.

1.9 Lane Keeping Assist System



The diagram above shows how the whole system comes together utilizing the parts above. The sensor dynamics element takes in these variables as described above and outputs the state variables in the form of the lateral deviation (V_y) and the relative yaw angle. Using this information the LKA uses a PID (Proportional-Integral-Derivative) controller, a feedback control mechanism commonly used in engineering systems. The PID controller continuously computes the steering angle adjustment based on the difference between the desired trajectory (lane center) and the actual vehicle position.

State Variables Monitoring: The LKA system monitors various state variables to determine the vehicle's position and orientation relative to the lane center. These include lateral deviation (distance from the center of the lane), yaw angle (vehicle heading), lateral velocity (sideways motion), and yaw rate (rate of change of heading).

Error Calculation: Error signals are computed by comparing the desired setpoint (typically zero deviation from the lane center) with the actual values of the state variables. These error signals represent deviations from the desired trajectory and serve as inputs to the PID controller.

PID Controller Operation:
Proportional Term (P): Adjusts the steering angle in proportion to the current error signal, providing an immediate response to deviations from the desired trajectory.
Integral Term (I): Accounts for accumulated errors over time by integrating the error signal, ensuring that any steady-state deviation is eliminated.
Derivative Term (D): Predicts future changes in the error signal by analyzing its rate of change, improving system stability and responsiveness.
Steering Adjustment: The output of the PID controller, representing the required correction to maintain the vehicle within the lane, is applied to the vehicle's steering system. This adjustment continuously adapts the steering angle based on changes in the state variables, running over some given time frame to adjust over and over.

In summary, the Lane Keeping Assist system utilizes a PID controller to autonomously adjust the vehicle's steering angle, based on real-time feedback from sensors, to ensure the vehicle remains within its lane and outputs this updated steering angle which is then given to the vehicle dynamics system in combination with the constant V_x to be ran through the state-space dynamics.

1.10 Vehicle Dynamics

The state variables represent the dynamic properties of the vehicle, such as lateral velocity (V_y) and yaw rate ($\dot{\psi}$). These variables describe the vehicle's motion and orientation in space, and the dynamic system describes their evolution in response to inputs over time

1.11 State Matrix (A)

The state matrix A describes how the state variables evolve over time in the absence of control inputs:

$$A = \begin{bmatrix} -(2C_f + 2C_r)/m & -(2C_f l_f - 2C_r l_r)/m \\ -(2C_f l_f - 2C_r l_r)/I_z & -(2C_f l_f^2 + 2C_r l_r^2)/I_z \end{bmatrix}$$

1.12 Input Matrix (B)

The input matrix B specifies how external control inputs affect the state variables:

$$B = \begin{bmatrix} 2C_f/m \\ 2C_f l_f/I_z \end{bmatrix}$$

m is the total vehicle mass (kg).
I_z is the yaw moment of inertia of the vehicle ($Kg * m^2$)
l_f is the longitudinal distance from the center of gravity to the front tires (m).
l_r is the longitudinal distance from the center of gravity to the rear tires (m).
C_f is the cornering stiffness of the front tires (N/rad).
C_r is the cornering stiffness of the rear tires (N/rad).

2 State Equation

The state equation is represented as $\dot{X} = AX + BU$, where:

- X is the state vector containing the state variables (V_y and $\dot{\psi}$).
- $\dot{X}$ represents the rate of change of the state vector over time.
- A is the state matrix, describing how the state variables evolve autonomously, without any control inputs.
- B is the input matrix, specifying how the control inputs affect the state variables.

3 State Matrix (A)

In the provided system, the matrix A has the following form:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Each entry in the matrix A represents the rate of change of one state variable concerning another.

- a_{11}: Represents the rate of change of lateral velocity (V_y) with respect to itself. This term may involve factors such as tire forces and vehicle mass.
- a_{12}: Represents the rate of change of lateral velocity (V_y) with respect to yaw rate (r). This term may involve factors such as tire stiffness coefficients (C_f and C_r) and the distances from the center of mass to

the front and rear axles (l_f and l_r).
- a_{21}: Represents the rate of change of yaw rate (r) with respect to lateral velocity (V_y). This term may involve factors such as tire forces and vehicle geometry.
- a_{22}: Represents the rate of change of yaw rate (r) with respect to itself. This term may involve factors such as tire stiffness coefficients (C_f and C_r) and the distances from the center of mass to the front and rear axles (l_f and l_r).

Overall, the entries in matrix A are determined based on the physical properties of the vehicle and its dynamic behavior, and they represent how the state variables V_y and r evolve over time in the absence of control inputs.

4 Input Matrix (B)

In the provided system, the matrix B has the following form:

$$B = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}$$

Each entry in the matrix B represents how the control input affects the rate of change of a state variable. Specifically:

- b_{11}: Represents how the front steering angle input affects the rate of change of lateral velocity (V_y).
- b_{21}: Represents how the front steering angle input affects the rate of change of yaw rate (r).

Overall, the entries in matrix B are determined based on the relationship between the control input and the dynamics of the system, quantifying how this input influences the rates of change of the state variables V_y and r.

5 Output Calculation

Once the state space model is defined, the system's outputs (e.g., V_y and $\dot{\psi}$) can be computed from the state variables using appropriate output equations.

- The outputs represent the vehicle's lateral velocity and yaw rate, which are essential for controlling its motion and stability.

In summary, the state space model describes how the vehicle's state variables evolve over time in response to control inputs and external forces. By defining the state and input matrices, the model quantifies the relationship between inputs and outputs, enabling the prediction and control of the vehicle's dynamic behavior.

5.1 Summary

The lane-keeping system operates by processing a single input alongside two crucial state variables, employing an error equation and curvature calculation to determine the lateral velocity and relative yaw of the vehicle. These values are then utilized in a PID control system to generate continuous adjustments to the steering angle, thereby ensuring the vehicle remains within its designated lane.

Moving forward, it's imperative to integrate safety constraints within the system to align with SCOTS functionality. SCOTS operates within a predefined safe space, allowing a system to run until it's no longer feasible. Therefore, incorporating parameters defining acceptable steering angle limits becomes essential. These constraints serve as safeguards, enabling SCOTS to detect deviations beyond the permissible range and prompt appropriate safety measures.

Additionally, in adapting the system, particular attention must be paid to integrating the error calculation component from the sensor dynamics section of the Simulink. This ensures that the updated velocity and yaw rate, crucial inputs for the PID system, incorporate real-time error analysis to enhance overall system accuracy and performance.

In summary, the lane-keeping system integrates various components, including error analysis, PID control, and safety constraints, to facilitate dynamic and precise steering adjustments for optimal vehicle trajectory maintenance and safety assurance, with the next steps involving alignment with SCOTS principles and refinement of error analysis for enhanced performance.