Factorio Optimization Problem

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1 Formalization

Let $p \in N^+$ be a natural number, $r \in Q^+$ be a rational number, $m \in N^+$ be a natural number, and $\vec{c} \in N^n s.t. \forall i \in N^+, i \leq n, c_i \geq 0$. Henceforth, r is the "rate", p is the "output quantity", m is the "multiplicity", and $\vec{c}$ is the "input". Then consider a directed acyclic graph G defined as

$$G = \{V, E\}$$
$$V = \{v_k \mid (p_k, r_k, \vec{c}_k, m_k),\ k \in N^+\}$$
$$E = \{e \mid E(v_a, v_b) \implies (\nexists f \in\ E\ s.t.\ f = E(v_a, v_c),\ v_b \neq v_c)\}$$
where $M(v_i) = m_i, P(v_i) = p_i$ for the multiplicities and output quantities of a vertex, respectively

Additionally, let a "source vertex" be defined as:

$$S \subset V = \{s \mid s = (p = 1, r = 0, \vec{c} = \vec{0}, m = 1)\}$$

Finally, we let $T \subset V = V - S$, the set of vertices that are not sources, and $|T| = Q$.

Notably, $Q = \sum_{i=1}^{n} M(v_i) \in N^+$. Then, let the Δ operator denote the differences between two connected vertices, v_i and v_j, as:

$$\Delta_{i \to j} = M(v_i) * P(v_i) - M(v_j) * \vec{c}_{C(v_j, v_i)}$$
where $C(v_j, v_i)$ returns the index of v_j for the unique v_i on the edge connecting the two.

We overload the operator for a vector representing multiplicities of indices, $\Delta\vec{z}$, to denote the same as the above across all such vertices and edges for a given graph, G, as:

$$\Delta\vec{z} = \sum_{i=0}^{|V|-1} \sum_{j=i+1}^{|V|} \Delta_{i \to j}$$
for index pairs $i, j, \in \vec{z}$

Then, let the optimizer $O(Q, G) : N \times \{V, E\} \to \vec{z} \in Z^{|V|}$ desire the following:

$$\vec{z} = \underset{\vec{z} \in N^{|V|}}{\arg} \Delta\vec{z}$$
for
$$0 \leq \min_{\vec{z} \in Z^{|V|}} \sum_{i=0}^{|V|-1} \sum_{j=i+1}^{|V|} \Delta_{i \to j}$$
where $z_i = M(v_i), i \in N^+ \leq |V|$