Factorio Optimization Problem

September 27, 2023

1 Formalization

Let $p \in N^+$ be a natural number, $r \in Q^+$ be a rational number, $m \in N^+$ be a natural number, and $\vec{c} \in N^n s.t. \forall i \in N^+, i \leq n, c_i > 0$. Henceforth, r is the "rate", p is the "output quantity", m is the "multiplicity", and $\vec{c}$ is the "input". Then consider a directed acyclic graph G defined as

$$
\begin{aligned}
&G = \{V, E\} \\
&V_i \in V = \{v_k \mid (p_k, r_k, \vec{c}_k, m_k),\ k \in N^+\} \\
&E_i \in E = \{e \mid E(v_a, v_b) \implies (\exists j \in N^+ \leq dim(\vec{c_b}),\ M(v_a) * P(v_a) \geq M(v_b) * j, j = C(v_b, v_a) \\
&\qquad\qquad \wedge\ \nexists (f \in E\ s.t.\ f = E(v_a, v_c),\ v_b \neq v_c)
\end{aligned}
$$

where $M(v_i) = m_i, P(v_i) = p_i$ for the multiplicities and output quantities of a vertex, respectively, and $C(v_j, v_i)$ returns the index of $\vec{c_j}$ that $P(v_i)$ corresponds to

Additionally, let a "source vertex" be defined as:

$$S \subset V = \{s \mid s = (p = 1, r = 0, \vec{c} = \emptyset, n_k \in N^+\}$$

Finally, we let $T \subset V = V - S$, the set of vertices that are not sources, and $|T| = Q$. Notably, $Q = \sum_{i=1}^{n} M(v_n) \in N^+$. Then, let the Δ operator denote the differences between two connected edges, v_i, v_j as:

$$
\begin{aligned}
\Delta_{i \to j} &= M(v_j) * \vec{c}_{C(v_j, v_i)} - M(v_i) * P(v_i) \\
&\text{where } \Delta_{i \to j} \text{ is defined iff } E(v_i, v_j) \wedge \Delta_{i \to j} > 0
\end{aligned}
$$

Then, let the optimizer $O(Q, G) : N \times \{V, E\} \to \vec{z} \in Z^{|V|}$ desire the following:

$$
\begin{aligned}
\vec{z} &= \underset{z \in Z^{|V|}}{\operatorname{argmin}} \sum_{i=0}^{|V|-1} \sum_{j=i+1}^{|V|} \Delta_{i \to j} \\
&\text{where } z_i = M(v_i), i \in N^+ \leq |V|
\end{aligned}
$$