Factorio Optimization Problem

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1 Formalization

Let $p \in \mathbb{N}^+$ be a natural number, $t \in \mathbb{Q}^+$ be a rational number, $m \in \mathbb{N}^+$ be a natural number, and $\vec{c} \in \mathbb{N}^n s.t. \forall i \in \mathbb{N}^+, i \leq n, 0 < c_i$. Henceforth, t is the "period", p is the "output quantity", m is the "multiplicity", and $\vec{c}$ is the "input". Then consider a directed acyclic graph G defined as:

$$
\begin{aligned}
&G = \{V, E\} \\
&V = \{v_k \mid v_k = (p_k, t_k, \vec{c}_k, m_k),\ k \in \mathbb{N}^+\} \\
&E = \{e \mid E(v_a, v_b) \implies (\nexists f \in\ E\, s.t.\ f = E(v_a, v_c),\ v_b \neq v_c)\} \\
&\quad \text{where } M(v_i) = m_i, P(v_i) = p_i \text{ for the multiplicities and output quantities of a vertex, respectively}
\end{aligned}
$$

Additionally, let a "source vertex" be defined as:

$$S \subset V = \{s \mid s = (p = 1, t = 1, \vec{c} = \emptyset, m \in \mathbb{N}^+)\}$$

Finally, we let $T \subset V = V - S$, the set of vertices that are not sources (i.e.: their $\vec{c} \neq \emptyset$,) and $|T| = Q$.

Notably, $Q = \sum_{i=1}^{n} M(v_i)$ for $v_i \in T$. Then, let the Δ operator denote the differences between two connected vertices, v_i and v_j, as:

$$
\begin{aligned}
&\Delta_{i \to j} = \begin{cases} M(v_i)\frac{P(v_i)}{T(v_i)} - M(v_j)\frac{\vec{c}_{j,i}}{T(v_j)} & \text{iff } E(v_i, v_j) \\ 0 & \text{otherwise} \end{cases} \\
&\text{where } \vec{c}_{j,i} \text{ denotes the } i^{th} \text{ component of the vector } \vec{c} \text{ belonging to vertex } v_j, \\
&\text{and } T(v_j) \text{ denotes the period of some } v_j
\end{aligned}
$$

We overload the operator when it acts on set of vertices (as opposed to only two), ΔV, to denote the same as the above across all such vertices and edges for a given graph, G, as:

$$
\Delta V = \sum_{i=1}^{|V|-1} \sum_{j=i+1}^{|V|} |\Delta_{i \to j}|
$$
$$
\text{for indices } i, j \text{ of some } v_i, v_j \in V
$$

Then, let the optimizer $\mathcal{O}(Q, G) : \mathbb{N} \times \{V, E\} \to \vec{z} \in \mathbb{N}^{|V|}$ desire the following:

$$
\vec{z} = \underset{\vec{z} \in \mathbb{N}^{|V|}}{\operatorname{argmin}} \Delta V
$$
$$
\text{for}
$$
$$
\Delta V = \min_{\vec{z} \in \mathbb{N}^{|V|}} \sum_{i=1}^{|V|-1} \sum_{j=i+1}^{|V|} \Delta_{i \to j}
$$
$$
\text{where } z_i = M(v_i)\ \forall v_i \in V
$$