

Factorio Optimization Problem

September 28, 2023

1 Formalization

Let $p \in N^+$ be a natural number, $r \in Q^+$ be a rational number, $m \in N^+$ be a natural number, and $\vec{c} \in N^n$ s.t. $\forall i \in N^+, i \leq n, c_i \geq 0$. Henceforth, r is the “rate”, p is the “output quantity”, m is the “multiplicity”, and \vec{c} is the “input”. Then consider a directed acyclic graph G defined as

$$G = \{V, E\}$$

$$V = \{v_k \mid (p_k, r_k, \vec{c}_k, m_k), k \in N^+\}$$

$$E = \{e \mid E(v_a, v_b) \implies (\nexists f \in E \text{ s.t. } f = E(v_a, v_c), v_b \neq v_c)\}$$

where $M(v_i) = m_i, P(v_i) = p_i$ for the multiplicities and output quantities of a vertex, respectively

Additionally, let a “source vertex” be defined as:

$$S \subset V = \{s \mid s = (p = 1, r = 0, \vec{c} = \emptyset, m = 1)\}$$

Finally, we let $T \subset V = V - S$, the set of vertices that are not sources, and $|T| = Q$.

Notably, $Q = \sum_{i=1}^n M(v_i) \in N^+$. Then, let the Δ operator denote the differences between two connected vertices, v_i and v_j , as:

$$\Delta_{i \rightarrow j} = M(v_i) * P(v_i) - M(v_j) * \vec{c}_{C(v_j, v_i)}$$

where $C(v_j, v_i)$ returns the index of v_j for the unique v_i on the edge connecting the two.

We overload the operator for a vector representing multiplicities of indices, $\Delta \vec{z}$, to denote the same as the above across all such vertices and edges for a given graph, G , as:

$$\Delta \vec{z} = \sum_{i=0}^{|V|-1} \sum_{j=i+1}^{|V|} \Delta_{i \rightarrow j}$$

for index pairs $i, j \in \vec{z}$

Then, let the optimizer $O(Q, G) : N \times \{V, E\} \rightarrow \vec{z} \in Z^{|V|}$ desire the following:

$$\vec{z} = \arg_{\vec{z} \in Z^{|V|}} \Delta \vec{z}$$

for

$$0 \leq \min_{\vec{z} \in Z^{|V|}} \sum_{i=0}^{|V|-1} \sum_{j=i+1}^{|V|} \Delta_{i \rightarrow j}$$

where $z_i = M(v_i), i \in N^+ \leq |V|$