

Group Meeting - November 6, 2020

Paper review & Research progress

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Confucius (Kung Fu Tzu)

- ① Wheresoever you go, go with all your heart.
- ② Choose a job you love, and you will never have to work a day in your life.



- 1 **Autofocused oracles for model-based design** (NeurIPS 2020) <https://arxiv.org/abs/2006.08052>
- 2 **Self-supervised Learning on Graphs: Deep Insights and New Directions**
<https://arxiv.org/abs/2006.10141>



Autofocused oracles for model-based design (NeurIPS 2020)

Clara Fannjiang and Jennifer Listgarten

<https://arxiv.org/abs/2006.08052>



Design problems (1)

Design problems

Design problems can be cast as seeking points in the design space, $\mathbf{x} \in \mathcal{X}$, that with high probability satisfy desired conditions on a property random variable, $y \in \mathbb{R}$. Solve:

$$\arg \max_{\mathbf{x}} P(y \in S | \mathbf{x})$$

where S is a constraint set.



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Model-based optimization (MBO)

MBO seeks the parameters θ of a **search model** $p_{\theta}(\mathbf{x})$ that maximizes an objective that bounds the original objective:

$$\max_{\mathbf{x}} P(y \in S | \mathbf{x}) \geq \max_{\theta \in \Theta} \mathbb{E}_{p_{\theta}(\mathbf{x})} [P(y \in S | \mathbf{x})] = \max_{\theta \in \Theta} \mathbb{E}_{p_{\theta}(\mathbf{x})} \left[\int_S p(y | \mathbf{x}) dy \right]$$



Design problems (2)

Oracle-based model-based design (MBD)

- Oracle-based MBD replaces costly and time-consuming queries of the ground truth $p(y|\mathbf{x})$, with calls to a trained regression model (i.e., **oracle**) $p_\beta(y|\mathbf{x})$ with parameters $\beta \in B$.
- Given access to a fixed dataset $\{(\mathbf{x}_i, y_i)\}_{i=1}^n$, the oracle is typically trained **once** using standard techniques and thereafter considered **fixed**.
- Optimize the lower bound:

$$\max_{\theta \in \Theta} \mathbb{E}_{p_\theta(\mathbf{x})} \left[\int_S p_\beta(y|\mathbf{x}) dy \right]$$



Design problems (3)

MBO problems are often tackled with an **Estimation of Distribution Algorithm** (EDA):

- Belongs to a class of iterative optimization algorithms
- Monte Carlo Expectation-Maximization

Given an oracle $p_\beta(y|\mathbf{x})$ and an initial search model $p_{\theta(t=0)}$:

① **E-step:**

- Sample from the current search model $\bar{\mathbf{x}}_i \sim p_{\theta(t-1)}(\mathbf{x})$ for all $i \in \{1, \dots, m\}$.
- Compute a weight for each sample $v_i = V(P_\beta(y \in S|\bar{\mathbf{x}}_i))$ where $V(\cdot)$ is a method-specific, monotonic transformation.

② **M-step:** Perform weighted MLE to yield an updated search model $p_{\theta(t)}(\mathbf{x})$ which tends to have more mass where $P_\beta(y \in S|\mathbf{x})$ is high:



Model-based design as a game (1)

Problem

Substituting the oracle $p_\beta(y|\mathbf{x})$ for the ground-truth $p(y|\mathbf{x})$ has a problem: **the oracle is only likely to be reliable over the distribution from which its training data were drawn.**

Solution

- An algorithmic strategy for iteratively updating the oracle within any MBO algorithm.
- Reformulate the MBD problem as a **non-zero-sum game**.



Model-based design as a game (2)

The oracle-based optimization

$$\arg \max_{\theta \in \Theta} \mathbb{E}_{p_{\theta}(\mathbf{x})} [P_{\beta}(y \in S|\mathbf{x})]$$

has the solution to be **sub-optimal** with respect to the original objective that uses the ground-truth $P(y \in S|\mathbf{x})$. But **the ground-truth is not accessible**. We introduce the **oracle gap**:

$$\mathbb{E}_{p_{\theta}(\mathbf{x})} [|P(y \in S|\mathbf{x}) - P_{\beta}(y \in S|\mathbf{x})|]$$



Model-based design as a game (3)

A non-zero-sum game with the coupled objectives of two players:

$$\arg \max_{\theta \in \Theta} \mathbb{E}_{p_{\theta}(\mathbf{x})} [P_{\beta}(y \in S|\mathbf{x})]$$

$$\arg \min_{\beta \in B} \text{ORACLEGAP}(\theta, \beta) = \arg \min_{\beta \in B} \mathbb{E}_{p_{\theta}(\mathbf{x})} [|P(y \in S|\mathbf{x}) - P_{\beta}(y \in S|\mathbf{x})|]$$

→ Search for a **Nash** equilibrium: a pair of values (θ^*, β^*) such that **neither** can improve its objective given the other.

→ An alternating ascent-descent algorithm.



Model-based design as a game (4)

An alternating ascent-descent algorithm:

- 1 **The Ascent step:** Fixing the oracle parameters and updating the search model parameters to increase the objective.
- 2 **The Descent step:** Fixing the search model parameters and updating the oracle parameters to decrease the objective.

Note

Isn't it just a variant of EM? I think it doesn't need a whole bunch of game theory (Nash equilibrium) machinery here.



Model-based design as a game (5)

Some bound

For any search model $p_{\theta}(\mathbf{x})$, if the oracle parameters β satisfy:

$$\mathbb{E}_{p_{\theta}(\mathbf{x})}[\mathcal{D}_{\text{KL}}(p(y|\mathbf{x})||p_{\beta}(y|\mathbf{x}))] = \int_{\mathcal{X}} \mathcal{D}_{\text{KL}}(p(y|\mathbf{x})||p_{\beta}(y|\mathbf{x}))p_{\theta}(\mathbf{x})d\mathbf{x} \leq \epsilon$$

then the following bound holds:

$$\mathbb{E}_{p_{\theta}(\mathbf{x})}[|P(y \in S|\mathbf{x}) - P_{\beta}(y \in S|\mathbf{x})|] \leq \sqrt{\frac{\epsilon}{2}}$$



Auto-focusing

Generally, we don't have access to the ground-truth $p(y|\mathbf{x})$, we do have labeled training data, $\{(\mathbf{x}_i, y_i)\}_{i=1}^n$, whose labels come from the ground-truth distribution $y_i \sim p(y|\mathbf{x} = \mathbf{x}_i)$. Practical oracle parameter update:

$$\beta^{(t)} = \arg \max_{\beta \in B} \frac{1}{n} \sum_{i=1}^n \frac{p_{\theta^{(t)}}(\mathbf{x}_i)}{p_0(\mathbf{x}_i)} \log p_{\beta}(y_i|\mathbf{x}_i)$$

Auto-focusing strategy: The oracle is retrained on re-weighted training data according to the importance weights:

$$w_i = p_{\theta}(\mathbf{x}_i)/p_0(\mathbf{x}_i)$$



Discussion

I find the experiments in this paper **not** related to our current work. But the open question is how to apply it into our work of graph/molecule generation?

Some ideas

We want to generate molecules with certain properties:

- In our case, the **oracle model** $p_{\beta}(y|\mathbf{x})$ is a molecular-properties-predicting model, \mathbf{x} is the molecular graph, y is the property, and β is the learnable parameters of a GNN.
- The **search model** $p_{\theta}(\mathbf{x})$ **cannot** be directly applied to VAE. But I think of a way around as follows. Let $p_{\theta}(\mathbf{z})$ be the learnable/adaptive prior of a VAE (or GAN), where \mathbf{z} is the latent. From \mathbf{z} , we reconstruct the molecule \mathbf{x} by the decoder.
- The game theory machinery can be applied similarly.

Self-supervised Learning on Graphs: Deep Insights and New Directions

Wei Jin, Tyler Derr, Haochen Liu, Yiqi Wang, Suhang Wang, Zitao Liu,
Jiliang Tang

<https://arxiv.org/abs/2006.10141>



Proposals

Self-supervised learning (SSL): creates domain specific pretext tasks on unlabeled data – SelfTask.

Note

- In my opinion, the authors haven't applied the famous **label propagation** (neighborhood aggregation) that propagates labels from labeled nodes into un-labeled nodes.
- However, that label propagation carries **uncertainty** → I have an idea of a **probabilistic model** addressing this uncertainty.



Local structure information (1)

- **Node Property:** The aim is to predict the property for each node in the graph such as vertex degree, local node importance, and local clustering coefficient.

$$\mathcal{L}_{\text{self}}(\theta, \mathbf{A}, \mathbf{X}, \mathcal{D}_U) = \frac{1}{|\mathcal{D}_U|} \sum_{v_i \in \mathcal{D}_U} (f_{\theta}(\mathcal{G})_{v_i} - d_i)^2$$

where \mathcal{D}_U represents the set of unlabeled nodes.



Local structure information (2)

- **Edge Mask:** Randomly mask some edges and then the model is asked to reconstruct the masked edges:

$$\begin{aligned}\mathcal{L}_{\text{self}}(\theta, \mathbf{A}, \mathbf{X}, \mathcal{D}_U) &= \frac{1}{|\mathcal{M}_e|} \sum_{(v_i, v_j) \in \mathcal{M}_e} \ell(f_w(|f_\theta(\mathcal{G})_{v_i} - f_\theta(\mathcal{G})_{v_j}|), 1) \\ &+ \frac{1}{|\bar{\mathcal{M}}_e|} \sum_{(v_i, v_j) \in \bar{\mathcal{M}}_e} \ell(f_w(|f_\theta(\mathcal{G})_{v_i} - f_\theta(\mathcal{G})_{v_j}|), 0)\end{aligned}$$

where \mathcal{M}_e is the edge set while $\bar{\mathcal{M}}_e$ is the set of non-edges.



- **Pairwise Distance:** Predict the shortest path pairwise distance. Discretize into 4 categories: $p_{ij} = 1$, $p_{ij} = 2$, $p_{ij} = 3$, and $p_{ij} \geq 4$.
- **Distance to clusters:** First, partitioning the graph to get k clusters. For each cluster, assign the node with the highest degree to the center of the corresponding cluster. Create a cluster distance $\mathbf{d}_i \in \mathbb{R}^k$ for node v_i where the j -th element is the distance from v_i to the center of C_j .



Label propagation

Note

- The authors missed the **label propagation** (message passing) that propagates the labels from labeled nodes into un-labeled nodes.
- However, the propagated labels carry **uncertainty**. We need to address this too.

Ideas:

- Suppose we have discrete labels of N types. Initialize each node with a one-hot vector of size N indicating its labels. Un-labeled nodes just have a zero vector.
- Run message passing iterations.
- For each un-labeled nodes, we normalize its vector to sum up to 1. This vector **might** indicate the probability the node belongs to each type \rightarrow addresses **uncertainty**.
- Train the pretext model to predict each node's distribution. We can use KL or Jensen-Shannon distance.

