A Conjecture on Symmetric Additions in Bases of the Form $n^2 + 1$

Hyacinthe Hervé

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Abstract

We present a conjecture on an intriguing property of numeral systems with bases of the form $b = n^2 + 1$. We show that by arranging the digits from 1 to n^2 in an $n \times n$ square resembling a numeric keypad, the sums of symmetric pairs always yield the same result: a number composed of n occurrences of the digit 1 followed by a 0 in the given base. We propose an explanation for this phenomenon and suggest avenues for a rigorous proof.

1 Introduction

Numeral systems have long fascinated mathematicians due to their unique arithmetic and algebraic properties. While manipulating numbers in different bases, we discovered a surprising regularity in specific bases of the form $b = n^2 + 1$.

2 Definition and Empirical Observation

Consider a base $b = n^2 + 1$. We construct an $n \times n$ grid containing the digits from 1 to n^2, arranged as follows:

We then observe that the sums of symmetric number pairs relative to the center always yield a number of the form:

The results for some specific bases are:

Base 5 ($n = 2$): 110_5

Base 10 ($n = 3$): 1110_{10}

Base 17 ($n = 4$): 11110_{17}

Base 26 ($n = 5$): 111110_{26}

3 Formulation of the Conjecture

Conjecture: Let $b = n^2 + 1$ be a numeral base. If we arrange the digits from 1 to n^2 in an $n \times n$ grid, imitating a numeric keypad layout, then:

this implies that the total sum always yields a number of the form $111\ldots10_b$.

4 Justification and Proof Approaches

The numbers are organized in a symmetric grid where each element x has an opposite element y such that their sum equals $b - 1$.

Due to the square structure, there are always $n^2/2$ pairs of symmetric numbers.

Since $b - 1$ is the largest number that can be represented with n digits of 1 in base b, the sum follows directly.

A rigorous proof could be developed by generalizing these observations to all bases of the form $n^2 + 1$, using combinatorial and arithmetic arguments.

5 Conclusion and Perspectives

We have highlighted an intriguing numerical regularity in certain bases and formulated a conjecture that appears to hold empirically. We hope this observation will be further explored by the mathematical community.