Chapter10

BY 定积分的应用

1 Type

1. 平面图形的面积

$$\begin{array}{cc} f(x)\text{连续} & S=\int_a^b f(x)\mathrm{d}x \\ \left\{\begin{array}{ll} y(t) & \text{连续} \\ x(t) & \text{连续可微}\wedge x'\neq 0 \end{array}\right. & S=\int_\alpha^\beta |y(t)\cdot x'(t)|\mathrm{d}t \\ r(\theta)\text{连续}, \beta-\alpha\leqslant 2\pi & S=\dfrac{1}{2}\int_\alpha^\beta r^2(\theta)\mathrm{d}\theta \end{array}$$

2. 平行截面体求体积

$$A(x)\text{是在}x\text{轴上的截面的面积, 且}A\text{连续}$$

$$V=\int_a^b A(x)\mathrm{d}x$$

$$\begin{array}{cc} & \text{旋转体的体积，绕}x\text{轴旋转} \\ f(x)\text{连续} & \text{截面面积为}\pi f^2(x) \\ & \Rightarrow V=\pi\int_a^b f^2(x)\mathrm{d}x \\ \text{参数方程} & \left\{\begin{array}{l} x(t) \\ y(t) \end{array}\right. x\text{连续可微，}y\text{连续.} \\ & V=\pi\int_\alpha^\beta (y(t))^2\mathrm{d}(x(t))=\int_\alpha^\beta y^2x'\mathrm{d}t \\ \text{极坐标方程} & r(\theta)\text{带入}x=r\cos\theta; y=r\sin\theta; \\ \text{一般取}0-\pi & V=\pi\int_\alpha^\beta r^2\sin^2\theta\mathrm{d}(r\cos\theta)=\pi r^3\int_\alpha^\beta 1-\cos^2\theta\mathrm{d}(\cos\theta) \\ & f(x)\text{绕}y\text{轴旋转} \\ & V=2\pi\int_a^b x\cdot f(x)\mathrm{d}x \end{array}$$

3. 平面曲线的弧长与曲率

$$\begin{array}{cc} & \text{弧微分}\mathrm{d}s=\sqrt{\mathrm{d}x^2+\mathrm{d}y^2} \\ f(x)\text{连续可微} & s=\int_a^b\sqrt{1+(f'(x))^2}\mathrm{d}x \\ \left\{\begin{array}{l} x(t) \\ y(t) \end{array}\right. \text{连续可微} & s=\int_a^b\sqrt{(x')^2+(y')^2}\mathrm{d}t \\ r(\theta)\text{光滑} & s=\int_\alpha^\beta\sqrt{r^2+(r')^2}\mathrm{d}\theta \end{array}$$

$$\text{曲率：}\lim_{\Delta t\to 0}\frac{\Delta\alpha}{\Delta s}.\alpha\text{是转过的角度，}s\text{是弧长.}$$

$$\text{为曲线在该点处的密切圆的半径的倒数}$$

$$\alpha(t)=\arctan\frac{y'(t)}{x'(t)}; s(t)=\int_\alpha^t\sqrt{(x'(p))^2+(y'(p))^2}\mathrm{d}p$$

$$\text{曲率}K=\frac{\mathrm{d}\alpha}{\mathrm{d}s}=\frac{x'y''-x''y'}{((x')^2+(y')^2)^{3/2}}=\frac{|x'y''-x''y'|}{((x')^2+(y')^2)^{3/2}}$$

$$\text{对于普通函数:}K=\frac{|y''|}{(1+(y')^2)^{3/2}}$$

4. 旋转曲面的面积

$$f\text{光滑} \qquad S=2\pi\int_a^b f(x)\sqrt{1+[f'(x)]^2}\mathrm{d}x$$

$$\text{参数方程}s \quad S=2\pi\int_a^b y(t)\sqrt{[x'(t)]^2+[y'(t)]^2}\mathrm{d}t$$