

Fundamentals of Semigroup Theory

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2022-5-27

1 Introductory ideas

The definitions will be refer to all book, but 1.8 is referred to only in Section 3.5.

Throughout the book, mapping symbols are written on the right.

1.1 Basic definitions

Definition 1.1 (Semigroup). *A groupoid (S, μ) , $S \neq \emptyset$, μ is a map $: S \times S \rightarrow S$ and μ is associative*

$$\forall x, y, z \in S, ((x, y)\mu, z)\mu(x, (y, z)\mu)\mu \quad (1)$$

The notation of operator μ could be notated as multiplication.

$$(xy)z = x(yz)$$

When the multiplication of semigroup is clear from the context, we shall write simply S rather than (S, \cdot)

Definition 1.2 (Order of Set). *The cardinal number of set S , $|S|$.*

Definition 1.3 (Commutative (abelian) semigroup). $\forall x, y \in S, xy = yx$

Definition 1.4 (Identity). $1 \in S, \forall x \in S \rightarrow 1x = x1 = x$

S has at most one identity element

$$\forall x \in S, x1' = 1'x = x \rightarrow 1' = 11' = 1$$

Definition 1.5 (Monoid). $S, 1 \in S$

Remark 1 (S^1).

$$S^1 = \begin{cases} S & \text{if } 1 \in S \\ S \cup \{1\} & \text{if } 1 \notin S \end{cases}$$

1.2 Monogenic semigroups

1.3 Ordered sets, semilattices and lattices

1.4 Binary relations; equivalences

1.5 Congruences

1.6 Free semigroups and monoids; presentations

1.7 Ideals and Rees congruences

1.8 Lattices of equivalences and congruences

1.9 Exercises

1.10 Notes