Bayesian Integration Problem Set
86-631/42-631 Neural Data Analysis

Motivation

In the field of neuroscience, Bayesian integration is a crucial skill. In essence, it allows for the formulation of a prediction given two or more cues. For instance, in an experiment studying the neural responses of certain, tuned neurons within subjects to specific stimuli, real-time decision data can be collected. This information can then be used as training data to be able to predict the subject's decision based on the signal and noise defined by the stimulus. Another instance of the usefulness of Bayesian integration is the presentation of velocity or position data at previous timesteps to predict the next velocity/position.

Description

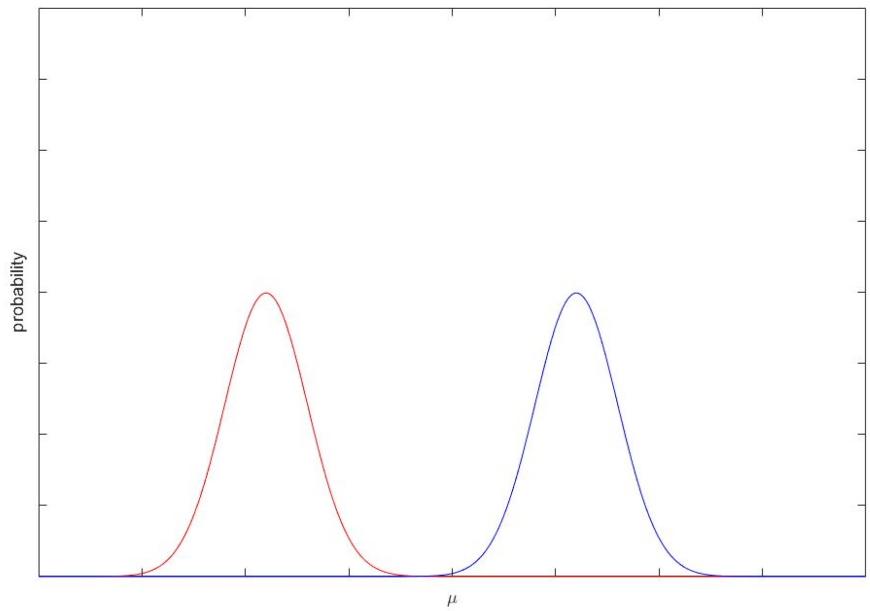
The objective of this problem set is to help you learn the concept of Bayesian integration. Some things to think about before beginning this problem set are "What probabilistic distributions are obtained upon combining two individual Gaussian distributions?" and "What parameters define this new distribution?" To aid you in completing this problem set, you will be using the Bayesian Integration Visualization Tool. Make sure you read the instructions detailing how to use the app, and thoroughly understand the mathematical premise behind the concept of integrating two distributions.

Section 1: 1D Case

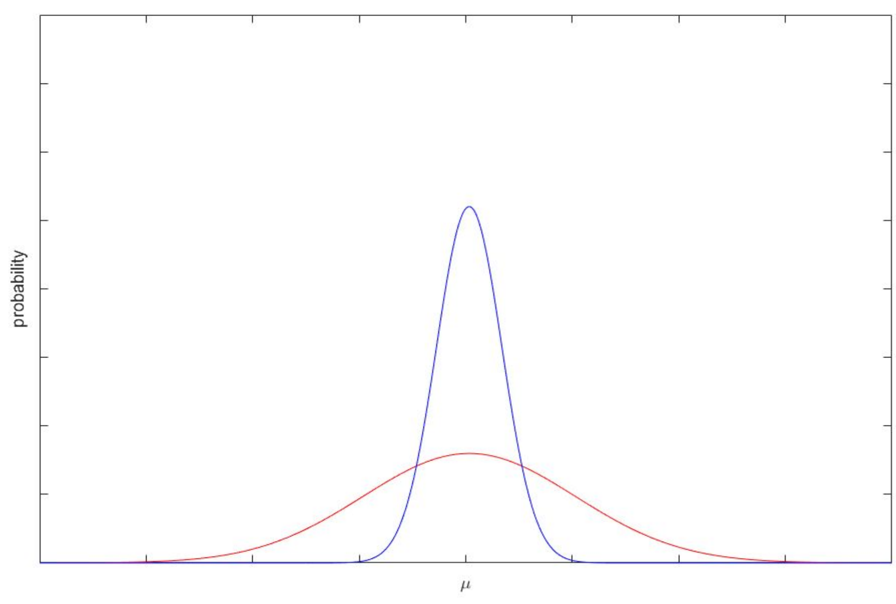
a. Select an arbitrary mean μ, $\{\mu \, \varepsilon \, \Re | \, -200 \leq \mu \, \leq 200\}$ and variance σ^2, $\{\sigma^2 \, \varepsilon \, \Re | \, 1 \leq \sigma^2 \, \leq 200\}$ for each of two different gaussian distributions. Using your understanding of the concept of integration, calculate the mean and variance of the integrated distribution by hand. Then, under the "1D Case" tab on the app, enter these selected means and variances for the two gaussian distributions into the textboxes. Observe the integrated curve. What is its mean and variance? Did they agree with the ones you found? If so, why was this the case, and what did you learn about the correct way to integrate probabilistic distributions?

b. In each of these cases, given two gaussian distributions (in blue and red), draw the integrated distribution (your drawing does not have to be to scale but the patterns should be there) and check your answers with the GUI.

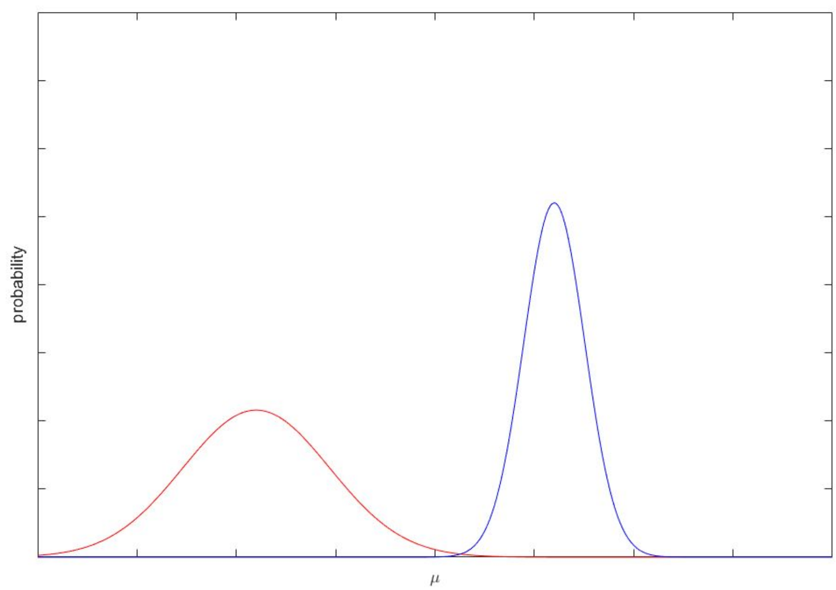
i.



ii.



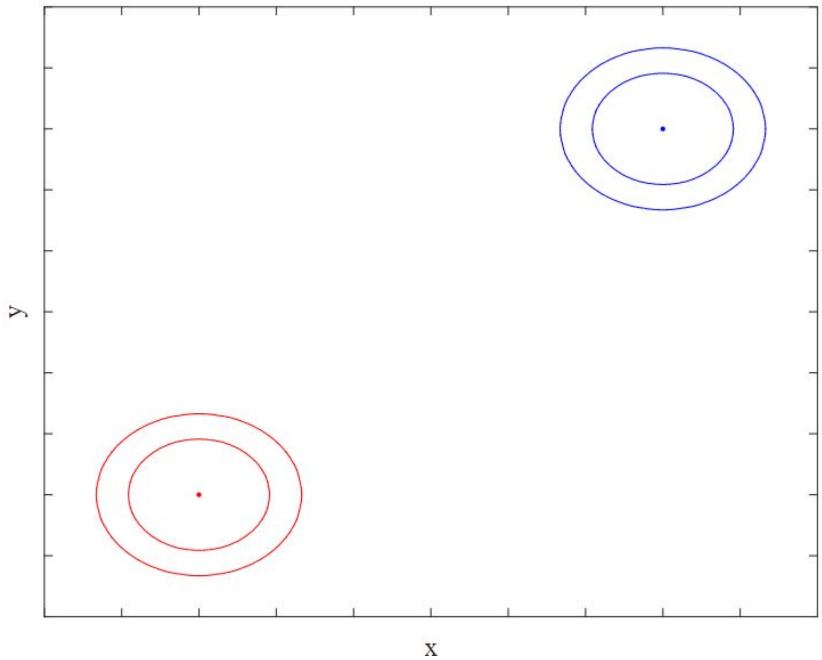
iii.



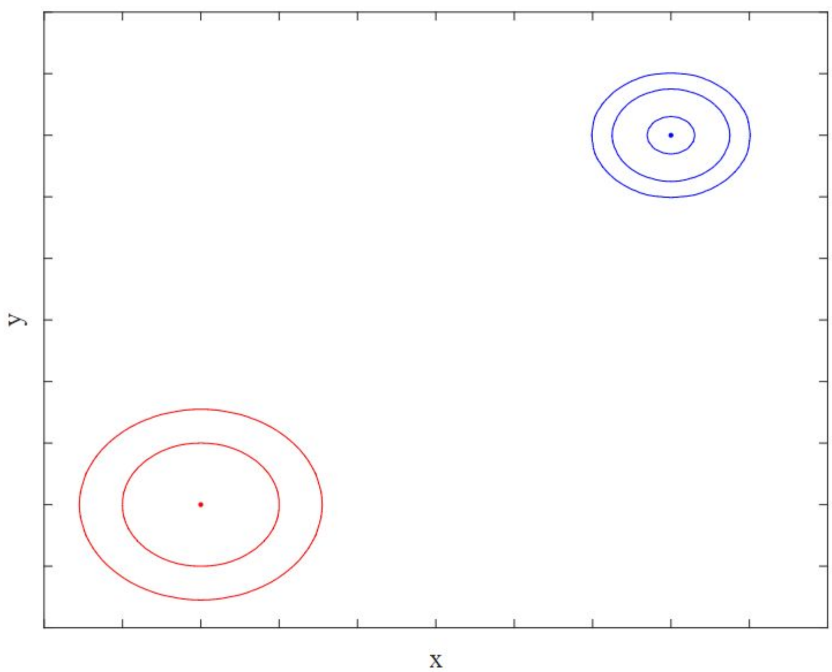
Section 2: 2D Case

a. Now, click on the “2D Case” tab. Similar to section 1b, draw out the integrated probability distribution, for each of these scenarios..

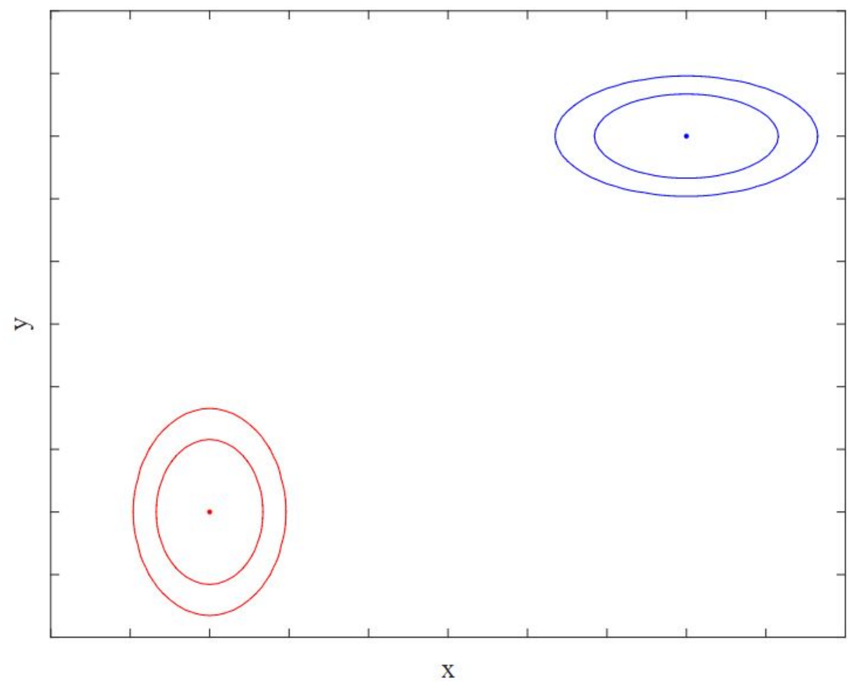
i.



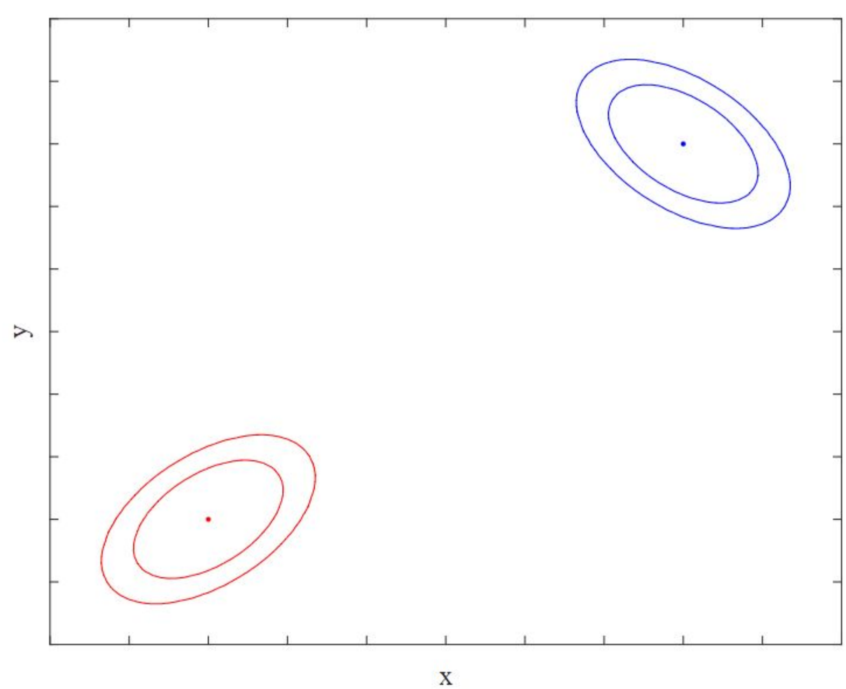
ii.



iii.



iv.



b. Set $\sigma^2{}_{1x}$ to 15, $\sigma^2{}_{1y}$ to 10 and Σ_{1x} to 12. Now change $\sigma^2{}_{1x}$ to 11. What happens, and why?

<u>Section 3: An Experimental Context of Bayesian Integration</u>

According to a study by Aslin et al. (2011), human performance during a categorical audio-visual speech perception (phonemic labeling) task is qualitatively consistent with the behavior of a Bayes-optimal observer. While each sensory modality provides information about the true stimulus, this model suggests that information provided by sensory cues are integrated and weighted based on their uncertainty in order to form an approximation of the true stimulus (along a spectrum of "Ba" and "Da"). Real-world sensory signals have uncertainty due to noise or variability in the environment and can thus be characterized by a Gaussian distribution over possible stimulus values.

Looking at the 1D case in the context of this study, the x-axis represents the location (value) of stimulus S and the y-axis represents the probability of likelihood. Assume that the red curve represents an audio signal and the blue curve represents a visual signal. The green curve represents the joint likelihood function of combined cues.

a. In the context of the experiment, what do the peaks of the red and blues curves represent? What does the peak of the green curve represent?

b. How is uncertainty of the signal represented in the 1D case?

c. Now, suppose that the auditory signal suggests a stimulus value of -100 and the visual signal suggests a stimulus value of 100. Each signal has an uncertainty of 50. What can you observe about the uncertainty of the combined cue estimate? Now, suppose that the uncertainty of the auditory signal is increased to 100. Did the uncertainty of the combined cues increase or decrease? What happened to its stimulus value estimate? Why do you think this happened?

d. Suppose that a stimulus value greater than 0 indicates "Da" was produced, and a stimulus value less than 0 indicates "Ba" was produced. Based on this criterion, what would a Bayes-optimal observer say about the stimulus ("Ba" or "Da") if:

> i. The visual signal suggested a stimulus value of 100 and had an uncertainty of 70, and the auditory signal suggested a stimulus value of 50 and had an uncertainty of 50?
>
> ii. The visual signal suggested a stimulus value of -100 and had an uncertainty of 20, and the auditory signal suggested a stimulus value of 100 and had an uncertainty of 50?

iii. The visual signal suggested a stimulus value of 100 and had an uncertainty of 20, and the auditory signal suggested a stimulus value of -50 and had an uncertainty of 20?

e. Now try playing with the values. What affects the uncertainty of the combined cues estimate? What doesn't? Do you think this is an accurate reflection of the real world?

f. In a provisional normative model, cues are weighted solely in proportion to their sensory reliability, as estimated from single-cue performance. An ideal Bayes-optimal observer follows this normative model. What do you think may be a problem with this model? What do you think a more accurate model should look like?

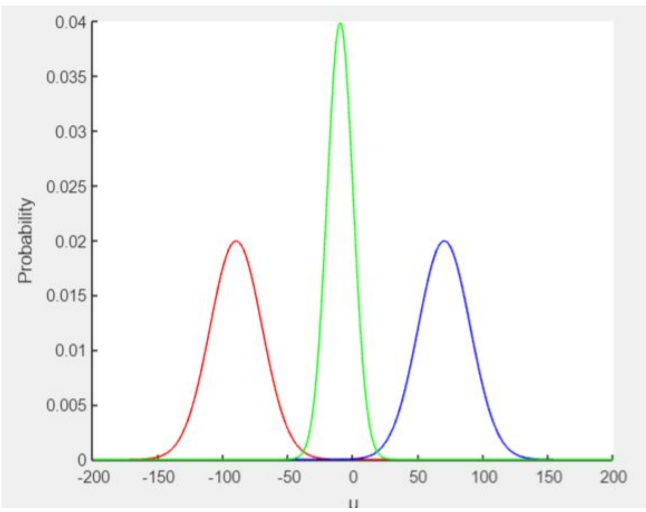
Reference:

Bejjanki, V. R., Clayards, M., Knill, D. C., & Aslin, R. N. (2011). Cue integration in categorical tasks: Insights from audio-visual speech perception. *PLoS One*, *6*(5), e19812.

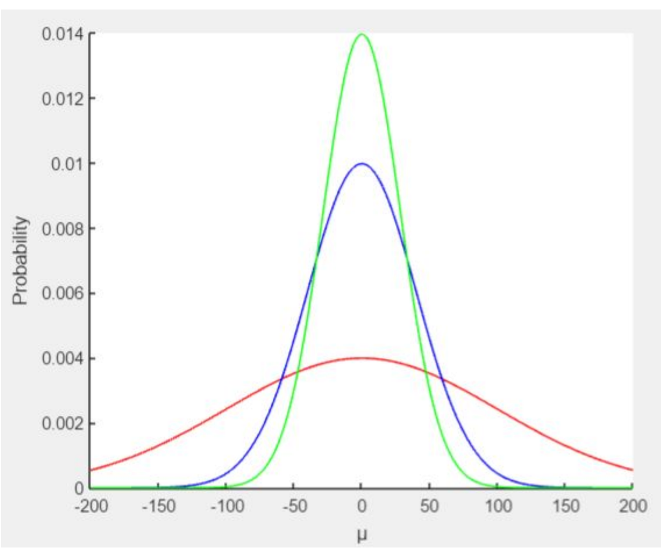
Answer Sheet

1a. Answers will vary based on selected means and variances. As an example, for μ_1=10, μ_2=28, σ^2_1=18, and σ^2_2=18, we obtain an integrated 1D-cue with mean and variance of 19 and 9, respectively.

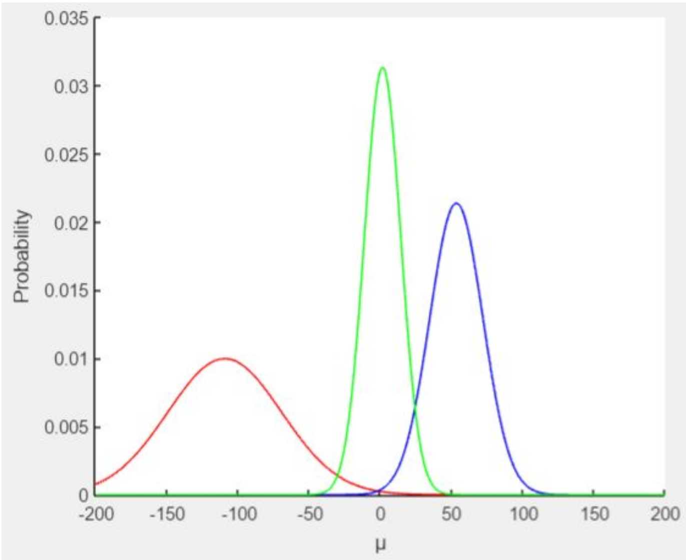
1bi.



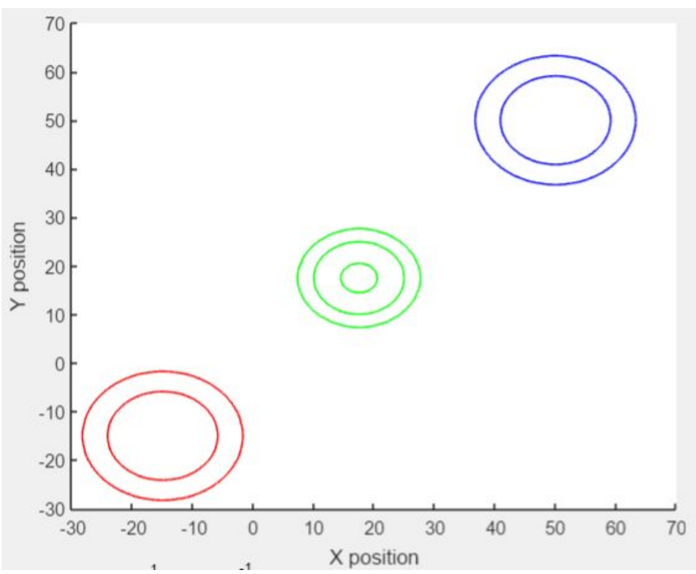
1bii.



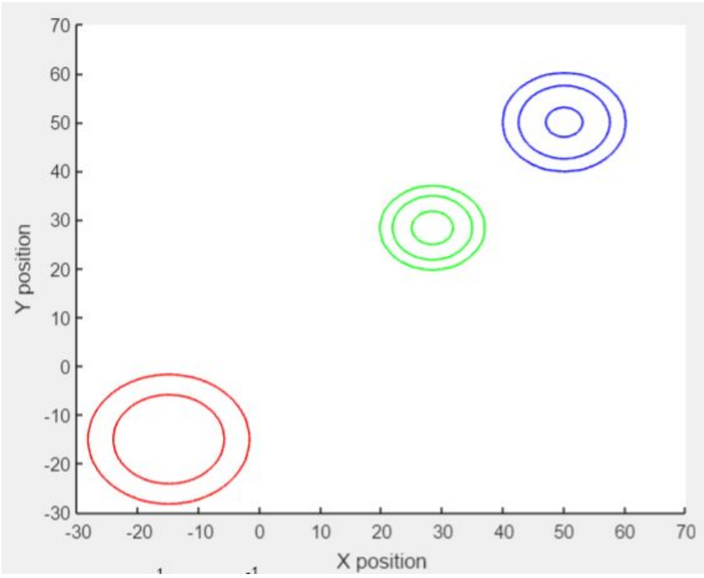
1biii.


0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Probability
-200
-150
-100
-50
0
50
100
150
200
μ

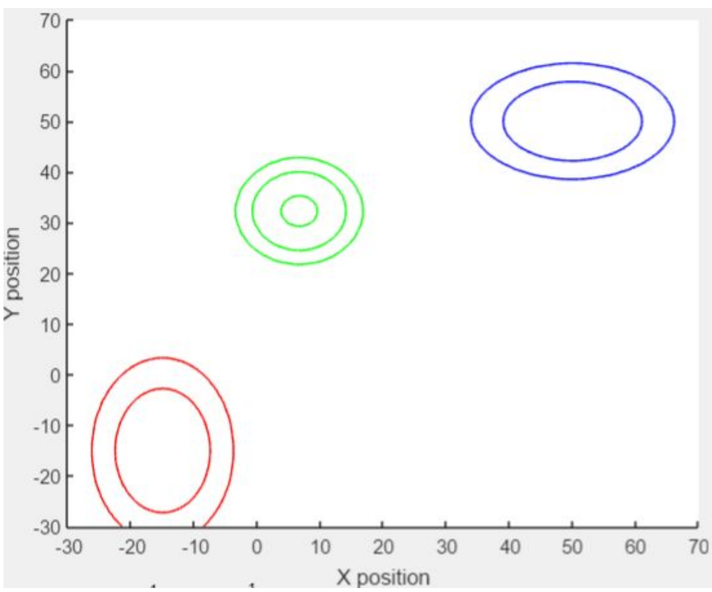
2ai.


70
60
50
40
30
20
10
0
-10
-20
-30
Y position
-30
-20
-10
0
10
20
30
40
50
60
70
X position

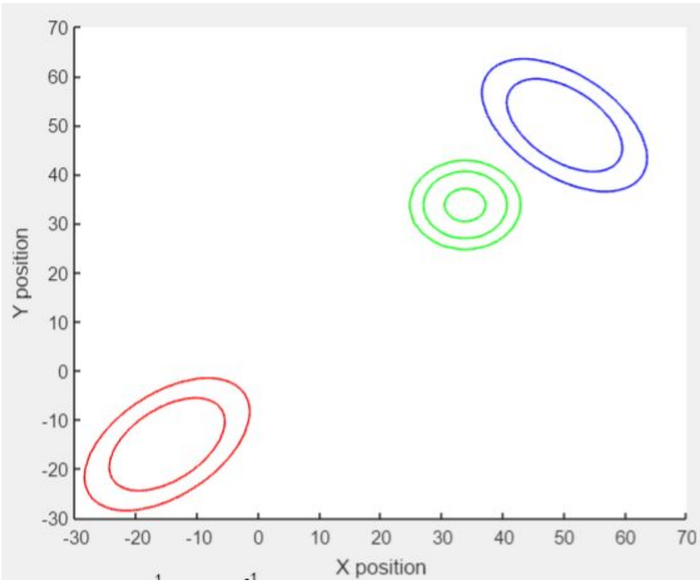
2aii.



2aiii.



2aiv.



2b. The GUI does not allow for such a value to be entered. The reason for this is that the absolute value of the covariance cannot be more than the product of the square root of the variances.

3a. The peak of the red curve represents the stimulus value suggested by the audio signal, and the peak of the blue curve represents the stimulus value suggested by the visual signal. The peak of the green curve represents the estimate of stimulus value by a Bayes-optimal observer.

3b. Uncertainty is represented by the variance.

3c. The uncertainty of the combined cues increased because the weight of the information provided by the auditory signal has decreased. Thus the stimulus value estimate shifts towards the stimulus suggested by the visual signal.

3di. Say “Da”
3dii. Say “Ba”
3diii. Say “Da”

3e. The variance of either signal affects the uncertainty of the combined cues, but the means don’t affect. The later question is more open-ended and any answer with substantial reasoning can be considered correct. For example, if the stimulus values suggested by each signal are more similar with each other, then the uncertainty of the estimated stimulus should decrease. This is not reflected in this model.

3f. In tasks defined over categorical perceptual dimensions, optimal cue weights should depend not only on the sensory variance affecting the perception of each cue but also on the environmental variance inherent in each task-relevant category. That is to say, participants may have some inherent bias towards certain cue modalities. Humans may have an internal model of within-phoneme environmental variability that they factor into their categorization. By accounting environmental variability in addition to sensory reliability, a more accurate model would be “flatter” (have greater sensory uncertainty) than the provisional normative model.