Analysis

Zach Mollatt - Notes from Stephen Abbott's Understanding Analysis 2nd edition

Contents

1 The Real Numbers

1.1 Irrationality

The Proof for $\sqrt{2}$ being Irrational
First, using the defintion for a rational number, assume $\sqrt{2}$ can be expressed in the form $\frac{p}{q}$ where p and q are integers.
Squaring both sides we then get:

$$\frac{p^2}{q^2} = 2 \tag{1.1}$$

Next, we will assume that p and q have no common factor as we could cancel it. As a result this implies

$$p^2 = 2q^2 \tag{1.2}$$

From this we can deduce that p^2 is even due to the being divisible by two, meaning p is also even. As a result we can represent $p = 2r$.

$$\therefore 2r^2 = p^2 \tag{1.3}$$

However, this implies p and q are both even. This is contradictory to our statement earlier assuming that they had no common factor. Hence equation (1.1) cannot hold and hence, the theorem of irrationality is proven.
Number Sets

- $\mathbb{N} = \{1, 2, 3, \ldots\}$ — Natural numbers
  Natural Numbers can perform addition perfectly well.
- $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ — Integers
  Extend to integers to have an additive identity (0) and subtraction.
- $\mathbb{Q} = \left\{\frac{p}{q} \mid p, q \in \mathbb{Z},\ q \neq 0\right\}$ — Rational numbers
  Multiplication and Division are now capable with this set.
- $\mathbb{R}$ — Real Numbers This accounts for any "gaps" on the number line where irrational components may be found.

$\mathbb{Q}$ defines a field (any set where, addition and multiplication are well-defined operations that are commutative, associative, and obey the distributive property $a(b + c) = ab + ac$).