System Simulation and Modeling - Experiment 1

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Abstract

System dynamics simulation is adopted to investigate how to maximize hunters' gain from hunting foxes in a food chain that only consists of one producer trophic level and two consumer trophic levels, which are foxes and rodents, without bringing irreversible damage to the ecosystem of the island. This requires a balance between the number of consumers and the limitation of the food supply of the island ecosystem. To meet this requirement, the quantity of foxes to be hunted requires proper consideration according to the number of foxes and rodents and other factors. These factors for simulating the tendency of quantity variation of foxes and rodents include age composition and physiological phases composition of species, seasonal distributed birth rate, aging and disease, food demand, etc. A system dynamics model was constructed and series simulations and analyses were carried out. Based on the simulation results and analysis, 3 guidelines for hunting were proposed and verified, including avoiding overhunting, hunting in autumn and avoiding hunting infants.

1 Introduction

On an isolated island lives two animal species, including foxes and rodents, which are composites in a 3-trophic-level food chain. Hunters wish to hunt as many foxes as they can as long as not turn the ecological system into an irreversible situation (i.e. the extinction of foxes). This requires maintaining the balance of the ecological system by keeping the quantities of foxes and rodents to a relatively constant value according to the factors and environmental conditions.

The quantities of the two species are controlled by various complicated factors and conditions. Some factors vary according to seasons or months, e.g. birthrate reaches its peak in spring or summer and food demand comes to its trough in winter due to hibernation of species. Other factors vary according to the age compositions of species, e.g. an infant fox and an adult fox may have different food demands and probability of dying from illness. Besides seasons and age composition, factors may also be defined by other environmental conditions.

Forrester [1] initially developed the subject of system dynamics, which was originally for simulating the raw material supply, production, inventory, transportation, and market in the business management area using systematic and mathematical methods. System dynamics blazes a trail for simulating the above ecological system simulation problem. Blackwell[2] used dynamics to investigate the population of house mice, ship rats, and stoats in New Zealand. Anastácio[3] integrated dynamics into a system to investigate the population of crayfish in a crayfish and rice integrated system of production. However, in their research, the relatively large time-steps restricted the performance of their model. So in this paper, the system dynamics method is adopted to investigate how the hunter maximizes the benefit of hunting foxes on an isolated island considering the constraints of various factors and environmental conditions based on classic ecological commonsense, and age-distributed and seasonal-distributed parameters are selected as many as possible to illustrate accurate changes in micro time-steps.

2 Method

2.1 Problem Analysis

The quantity of foxes is selected as the core level variable as it will give direct guidance to hunters. Based on the general and classic commonsense of ecology quantity of foxes increase as birth-giving and immigrants; the quantity of foxes decreases as foxes are hunted, died of disease or decay, starvation, lack of nourishing, and emigration. The number of rodents should be studied respectively as it decides whether there is enough food resource to feed foxes from starvation. Those factors to the numbers of foxes and rodents will be elaborated on in detail in the following parts.

Once the parameters such as the seasonal and age distributions are settled as constants, the system boundaries shrink to the initial time, time-step size, initial numbers of species, and hunters' demands for foxes. The reduction of parameters simplified the difficulties in analyzing the simulation.

Figure 1 displays the casual relationships in a food chain, which is composed of two sets of interactions between food supplies and the number of rodents, and between the number of rodents and the number of foxes. These two sets of interactions constitute two feedback control loops.

As shown in figure 1, producers (i.e. plants) in the food chain will assimilate energy from solar radiation and the energy is transferred into the ecological system along the food chain, which means the variation of solar energy will constrain the assimilated energy by producers and thus limit the food supply for herbivores (i.e. rodents, the primary consumers). As for simulating the variation of solar radiation, the monthly distribution of energy is considered. Besides solar radiation, environmental capacity is considered a factor in the energy transferred to rodents due to the limitation of space for producers' growth.

Rodents and foxes share similar types of contributory factors including “breeding & growth”, “in-migration”, “hunted”, “disease & aging”, “starvation”, “lack of nourishing”, and “out-migration”. The meanings of these factors for a species are as follows:

- Breeding & growth: Newly bred infants increase the number of individuals in this species. The number of newborns is considered the product of the current number, the average birth rate, and the matting ratio, which is a function of seasons(months), age composition, and matting pattern. Of course, individuals of species will grow older over time.

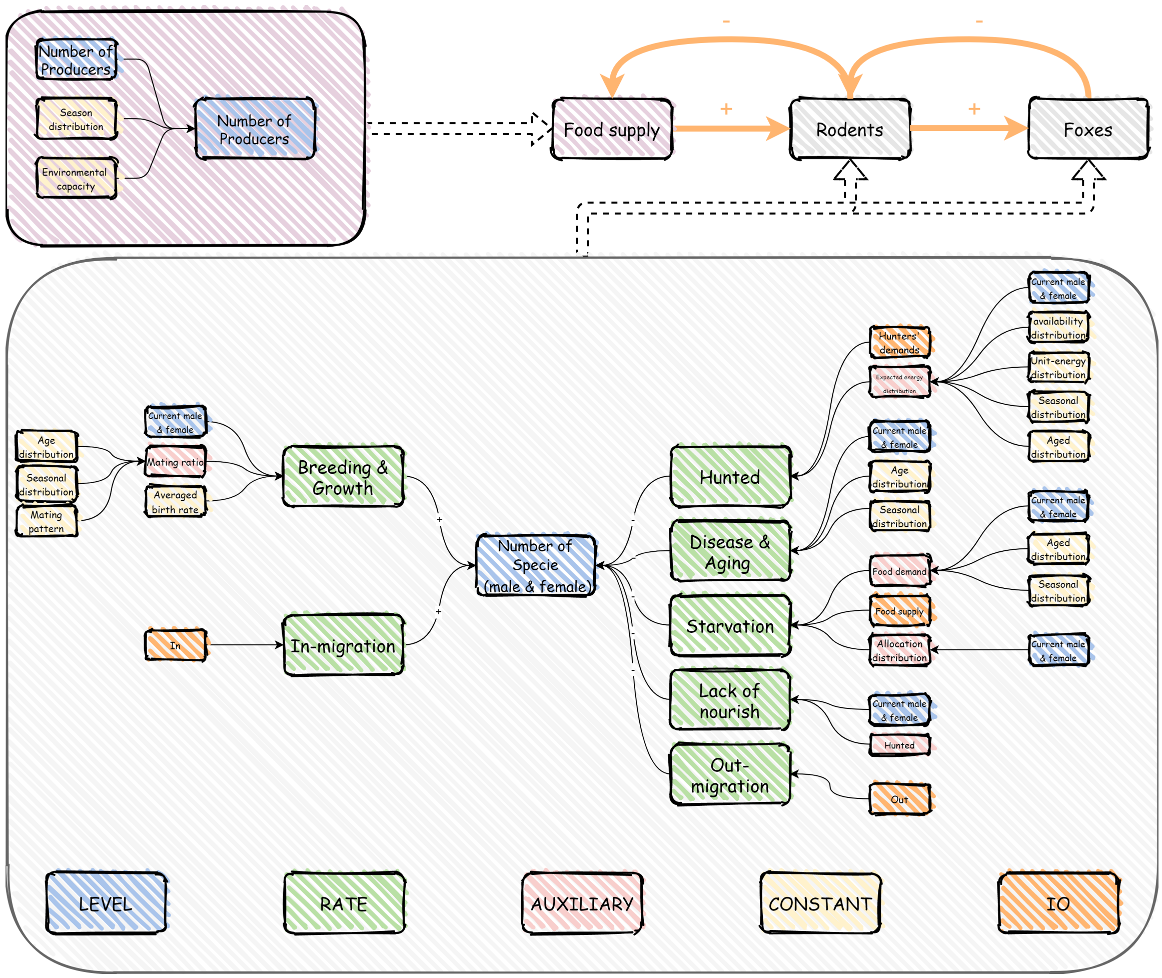


Figure 1: Causality of food supply, rodents and foxes.

- In-migration: The immigrants contribute to the number of species.
- Hunted: Some hunted individuals will decrease the number of species to produce as much energy as the hunters(or predators) demand under the constraints of current availability (with a normalized scale that denotes the likely hood to be captured by hunters) and energy of unit in age distribution, and a seasonal distributed energy scale.
- Disease & aging: Individuals who died of illness or decay decrease the number of species. The disease ratio varies along with season and age distributions.
- Starvation: Individuals who died from starvation decrease the number of species. This is controlled by three factors which are food demand, food supply, and allocation. The food demand varies with season and age distribution; the food supply is from the former trophic level in the food chain; the allocation is a scale-normalized age distribution, which denotes the different capabilities in intra-species competing for food allocation (i.e. hunting preys).

- Lack of nourishing: Infant individuals will die of a lack of nourishing and thus decrease the number of species. The mortality of mated couples is considered equal to that of the infants.

- Out-migration: The decrease in the number of species.

2.2 Definitions of Variables

Based on the analysis above, variables are listed in table 1. Here, the variable names are defined following the principles that: **Q** denotes the quantity of a variable distributed by age, which is represented by a vector; N denotes a real number; P denotes a probability function, and τ denotes a probability ratio vector. The parameters of the model are list listed in table 2 respectively.

2.3 Model construction

As shown in figure 1, the model is composed of 3 parts which are food supply(i.e. plants), rodents, and foxes. The evolution of energy of plants will be given in the followings.

The number of rodents and foxes in time-step $t + \Delta t$ is given in a united format by:

$$Q(t + \Delta t) = grow(Q(t)) + [R_H(Q(t)) + R_{DA}(Q(t)) + R_S(Q(t)) + R_{IM}(t) + R_{EM}(t)] \times \Delta t \qquad (1)$$

In the following sections, the construction of the model will be elaborated in detail. The components of the model will be given before the overall view of the model. The model of plants has a relatively simple structure which is described section 2.3.1, while the models of rodents and foxes share similar structure which is described in section 2.3.2 to section 2.3.7. The overview of the model will be given in section 2.4.

2.3.1 Plants

As the plants grow as an S-curve, the Logistic curve is adopted to depict the growth tendency of the plants. Let τ denote the time of growth phase of plants in the S-curve, the S-curve is given by:

$$E_p = \frac{1}{K + ab^{\tau}} \qquad (2)$$

where $\frac{1}{K}$ denotes the environmental energy capacity of certain area in certain month, and α, β are the growth rate parameters with $\alpha > 0, \beta > 1$.The parameters for eq 2 should be selected by fitting real data in different species, areas, and seasons. In this paper, these parameters are decided by given $1/K, t_{quarter}, t_{half}$, which means the environmental capacity, days of plants to grow from minute quantity and to reach a quarter of capacity, and half of the capacity respectively.

During the run-time of the simulation, τ can be retrieved by the given $E_p(t)$ according to the inverse function of eq 2, and thus the $E_p(t+\Delta t)$ can be calculated using eq 2 with $\tau + \Delta t$ as variable.

$$E_{plants}(t + \Delta t) = \frac{1}{K + \alpha\beta^{\tau+\Delta t}} \qquad (3)$$

2.3.2 Breeding & growth

This model component realizes two functions including the birth-giving of mated pairs and the growth of age of species. For the birth-giving of a species, the adult individuals mate in different patterns, and their mating behavior is constrained by seasons. For example, rodents may mate in monogamy or polygamy, and rodents may mate mostly in spring which it's their mating season. To illustrate this character, firstly, the adult's number is given by:

$$N_{adult} = \int Q(a) r_{adults}(a) \mathrm{d}a \tag{4}$$

Then the numbers of matted females in different mating patterns are given by:

$$N_{matted} = \begin{cases} min(N_{adult,male}, N_{adult,female}) & , monogamy \\ min(N_{adult,male} \cdot N_{mmp}, N_{adult,female}) & , polygamy \end{cases} \tag{5}$$

Then the number of newborn infants is given by:

$$N_{breeding} = N_{matted} \cdot BR_{monthly}(m) \cdot \frac{\Delta t}{30} \tag{6}$$

So the overall function of "Breeding & growth" is given by:

$$Q(t + \Delta t, a) = grow(Q(t, a)) = \begin{cases} N_{breeding}, a = 0 \\ Q(t - \Delta t, a), a \in [\Delta t, a_{max}] \end{cases} \tag{7}$$

2.3.3 Hunted

This model component realizes the reduction of a species brought about by its hunters/predators. The Hunters/predators will hunt for as much prey as possible till their demands are satisfied, which is usually embodied in the form of the energy criteria reached. Using $E_{predators}$ denotes the energy demands from predators.

In the hunting process, different prey may provide different amounts of energy, which varies with age and seasons. The distribution of energy is introduced as $E(a, m)$, given by:

$$E(a, m) = w_{energy}(m) \cdot E_{individual}(a) \cdot Q(a) \tag{8}$$

where $w_{energy}(m)$ denotes a weight factor varying by month, which is higher in winter and is lower in summer, $E_{individual}(a)$ denotes the energy per individual at different ages. $Q(a)$ is the vector of the number of males/females in a species.

Then the final rate of this component is given by:

$$R_H(a) = \frac{-E_{predators}}{E(a, m)} \cdot Q(a) \tag{9}$$

2.3.4 Disease & aging

This model component realizes the reduction of a species brought about by disease and aging. The following fact is considered. Firstly, Infants and seniors may face a higher probability of dying from the disease. Secondly, due to low environmental temperature, more individuals die in winter than in spring. Finally, the older an individual is the higher the risk of dying from aging. So this phenomenon is illustrated by introducing the probability ratio function $r_{DA}(a,m)$, which is the product of $r_{DA}(a) \times r_{DA}(m)$. Then the final rate of this component is given by:

$$R_{DA}(a) = -Q(a) \cdot r_{DA}(a,m) \tag{10}$$

2.3.5 Starvation

This model component realizes the reduction of a species brought about by starvation. A species will try to feed themselves by hunting prey or by grazing (i.e. by assimilating energy from their previous tropic level in the food chain). If their requirements for food are not satisfied, some individuals may be starved.

The food demand of an individual is a ratio to its energy (i.e. the energy in its mass), which varies with age and season. So this feature is depicted by introducing probability ratios function $r_{demand}(a,m)$, which is the product of $r_{demand}(a) \times r_{demand}(m)$. Besides, considering there is a period of a duration time, in which the starving individuals can avoid death by ingestion, maximum duration days $N_{duration}$ is introduced. So the energy of demanding by a species $E_{demand}(a)$ is given by:

$$E_{demand}(a) = E(a,m) \cdot r_{demand}(a,m) \tag{11}$$

However, on the one hand, the demand is not always satisfied due to the limitation of food supply (i.e. E_{hunted}) from the previous trophic level, which is denoted by E_{supply}. On the other hand, the energy allocation inside a species varies with the age of individuals. More specifically, the seniors and infants may be less competitive than adults and juveniles in the access to the food supply. The allocation probability function $P_{alloc}(a)$ is introduced. As for the transition from food energy to internal energy, assimilation efficiency e_{asm} is introduced to depict this ratio of assimilated energy to ingested energy. So the actual energy supply $E_{supply}(a)$, distributed by age, is given by:

$$E_{supply}(a) = E^{e}_{supply asm} \cdot P_{alloc}(a) \tag{12}$$

The relative energy satisfied ratio $r_{satisfied}(a)$, which is distributed by age, is introduced to illustrate the Satisfaction of food demand at different ages in a species.$r_{satisfied}(a)$ is given by:

$$r_{satisfied}(a) = \begin{cases} 1, E_{demand}(a) \leq E_{supply}(a) \\ E_{supply}/E_{demand}, others \end{cases} \tag{13}$$

Here $N_{duration}$ is introduced to denote how many days an individual will survive without any food supply. The final rate of this component is then given by:

$$R_{ST}(a) = -Q(t) \cdot [1 - r_{satisfied}(a)]^{N_{duration}} \tag{14}$$

2.3.6 Lack of nourishing

This model component realizes the reduction of infants in a species brought about by the lack of nourishing. the infants of a species will die in the same death ratio as the adults. The mortality rate is given by:

$$R_{LN}(a) = \frac{\int [R_H(a) + R_{DA}(a) + R_{ST}(a)] \cdot r_{adults}(a)\mathrm{d}a}{\int Q(t,a)\mathrm{d}a} \cdot Q(a,t) \cdot r_{infant}(a) \tag{15}$$

2.3.7 Migration

The migration consists of immigrations $R_{IM}(a)$ and emigrations $R_{EM}(a)$. This function provides the possibility for further spatial distributed simulations, which is not considered in this paper.

2.4 Model overview

Based on the components above, the SD(System Dynamic) chart is shown in fig 2. It needs to be pointed out that the $grow()$ function is not a standard component of system dynamics as a shift is realized in it. But the introduction of $grow()$ makes a large extension to the performance of the model.

The key default boundary conditions and initial variables are listed in the table 3. The Environmental capacity is considered a "double-peaks" curve [4] distributed by months, and here, fitted with scale normalized Gaussian curves $N(\mu, \sigma)$, in where μ means mean month and σ means the standard deviation in months. $||Q||_{init}$ denotes the total number of the initial Q, which is considered distributed evenly in all ages. The probability ratios of $\{r_{stages}\}$ are fitted with a set of triangle and trapezium curves. Any curves will be fitted as long as they meet the requirement in table 2. So here only the threshold age of each stage is presented (e.g. in $(2, 4, 10, 15)$, 2 denotes all infants in a species will turn to juveniles in their 2 months old). The averaged birth rate $\bar{BR}_{monthly}$ denotes how many infants a pregnant female will give birth to in a month. The averaged unit energy of individuals $||\bar{E}_{individual}||$ denotes time and individual averaged energy of a species. Demanding ratio $||\bar{r}_{demand}||$ means the time and individual averaged ratios of demanding energy to their body energies in a day. The temporal distribution of mass and food intake of rodents were investigated by Zhao et al. [5], which provided instructions for the design of $E_{individual}(m)$ and $r_{demand}(m)$ in this paper.

The detailed default boundary conditions of rodents and foxes are shown in figure 3.

3 Result

The model is reliazed using Python (Appendix 5.1). Based on the model, different boundary conditions are selected to simulate the number of rodents and foxes in various ecological conditions.

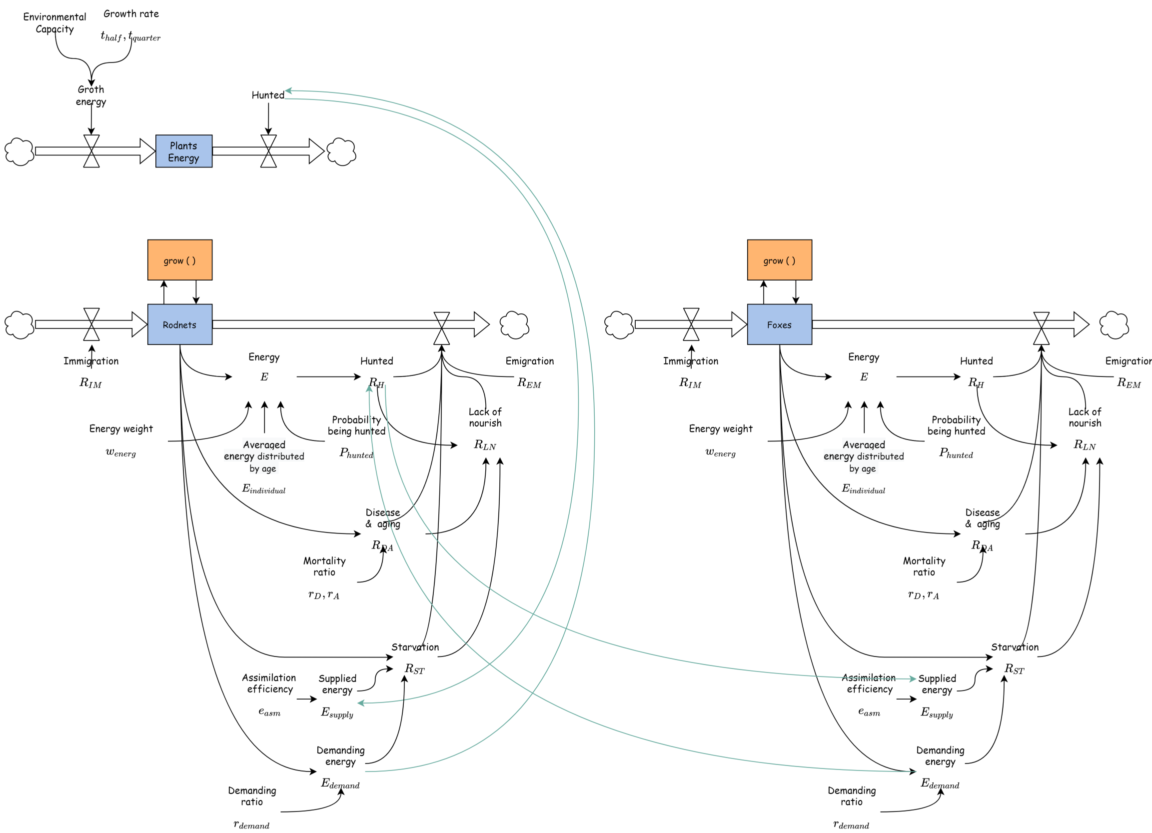


Figure 2: System dynamic chart of the model.

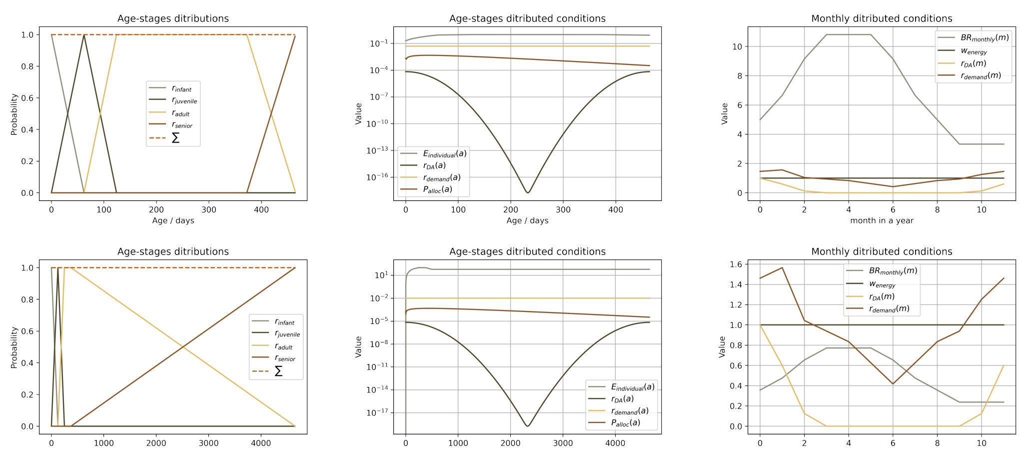
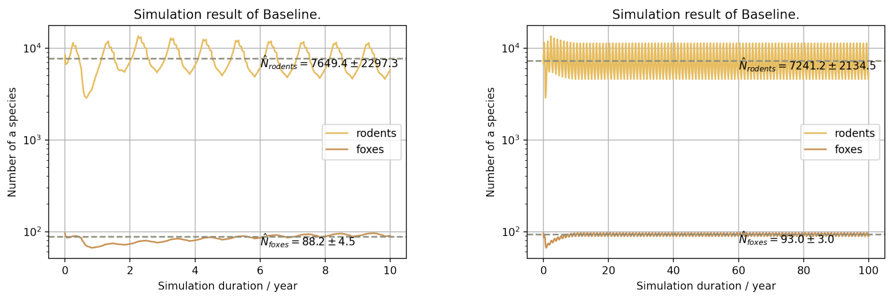
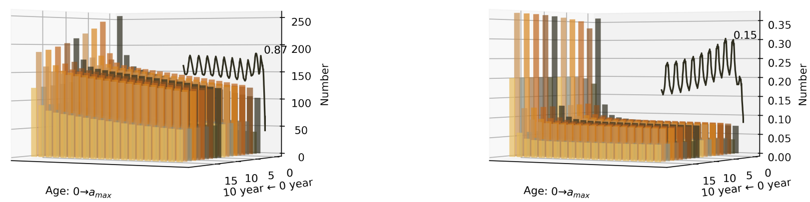


Figure 3: Initial boundary conditions. From left to right: r_{stages}; monthly distributed parameters, i.e. $BR_{monthly}(m)$,$w_{energy}(m)$, $r_{DA}(m)$ and $r_{demand}(m)$; age distributed parameters, i.e. $E_{individual}(a)$, $r_{DA}(a)$,$r_{demand}(a)$ and $P_{alloc}(a)$. The top row denotes the boundary conditions of foxes, and the bottom row of rodents, respectively.



(a) 10 year (b) 100 year

Figure 4: Simulation result of baseline.



(a) (b)

Figure 5: Evolution of the age structures in 10 years. The bars in a row denote the averaged age structure in half year. The curve on the right are the infants ratios.

In the baseline section, the evolution of numbers in the default boundary conditions is simulated, which gives information about the temporal evolution pattern of total numbers and age structures. In the sensitivity analysis section, the variations of species numbers trend caused by small disturbances in boundary conditions or initial variables are simulated. Finally, based on the experiments before, some guiding principles for hunting are proposed and examined.

3.1 Baseline

The default boundary conditions are selected for the baseline simulation, which aims to discover whether the number of species will tend to converge to a stable state.

The evolution of numbers of two species in 10/100 years is shown in figure 4. As the 10-year result shown in figure 4 (a), the evolution of both species reaches the crests in the early autumn and trough in winter, with a pattern of yearly circle. In the very beginning of the simulation, both numbers of two species suffer from declines before growing to their first peaks in the early autumn of the first year. Then, both numbers decline to their troughs in the first winter, when their minimum values appear. Then in the following years, the yearly averaged numbers of rodents increase to their peaks in about 3 years and then decrease. On the opposite, in the following years, the yearly averaged numbers of foxes decline to their bottom in about 3 years and then increase. As it shown

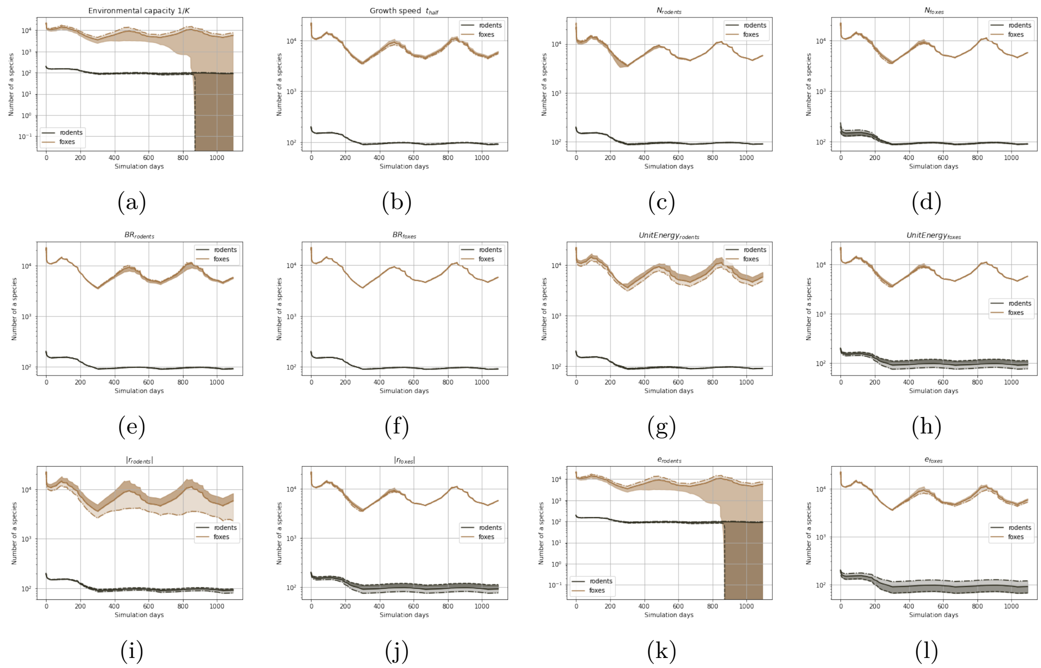


Figure 6: sensitivities to some key conditions. The solid lines represent the evolution results of the default conditions. The dashed lines represent the evolution results of the varied conditions, with darker color-filled space between the original line and the +10% dashed line, and lighter between the original line and the −10% line respectively.

in figure 4 (b), both curves finally converge to stable states in about 10 years with a ratio in about 0.00115.

The age structure of both species has the potential to predict the trends of the evolution of species numbers, thus can provide a better understand to guide the hunter activity. The age structures of two species in the first 10 years of the simulations are shown in figure 5. The age structures of rodents and foxes evolve from uniform distributions to trapezoidal distributions. The curves to the right sides of the bars display the infants ratios of the species, which indicates the breeding season of both species, and rodents have a much higher infants ratio, which gives the rodents shorter life cycles to generate more energy in a year.

3.2 Sensitivity analysis

In this section, the sensitivities of the evolution of numbers to some key conditions are tested. For each variable listed, variations of ±10% to the default initial value were applied, and the results are shown in figure 6. The variation of environmental capacity $\frac{1}{K}$ and $e_{rodents}$ lead to the most severe consequences, i.e. the crash of the ecological system. The variation of the initial quantity $N_{rodents}$ and N_{foxes} only make a small variation of the result in the very first of the simulation before the result curves converge to the curves of original conditions. The variation of birth rate $BR_{rodents}$ and BR_{foxes} will only reshape the curves in the breeding seasons significantly. The variation of individual energy $E_{individual}$ changes the scales of the curves slightly. The variation of food demand ratio r_d may cause considerable variations in the result curves, increasing r_d may lead to the potential of a system crash.

3.3 Enlightenment for hunting

Based on the result drawn above, the following guidelines for hunting is thus proposed: the evolution of species numbers tends to maintain a balance yet this ability is limited, which means overhunting may lead to the crash of the ecological system; the growth of fox numbers follow a seasonal growth pattern, an autumn oriented hunting activity may yield most products. To protect the reproductive potential, avoiding hunting young foxes is highly recommended. Using $R_{EM}(a,m)$ to depict the hunting activities, guidelines for hunting patterns are given by:

- Avoid overhunting: Hunting rate should match the growth speed of foxes (i.e. $R_{EM}(a,m)$ is supposed to be smaller than a threshold). overhunting may lead to a collapsing of the ecological system. $||R_{EM}(a,m)||$, the norm of $R_{EM}(a,m)$ is 10 for the control group, while 20 for the treatment group.

- Hunt in autumn: Hunt in the fall when fox numbers are at their peak of the year. Here monthly hunting quantity $||R_{EM}(m)||$ is represented by an array [0.028 0.028 0.028 0.028 0.028 0.028 0.056 0.139 0.278 0.278 0.056 0.028].

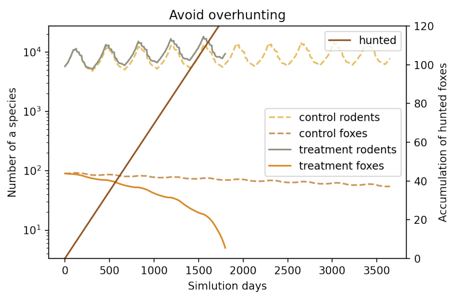
- Avoid hunting infants: The composition of hunted individuals distributed from $r_{stages(1)}$ to $r_{stages(3)}$, which is proportional to the original quantity distribution before emigration.

To verify these proposals, the following contrast experiment was carried out. The simulation results of baseline is adopted as the initial value for all experiments in this section. The original results continued running for 10 years with only ”Avoid overhunting” adapted played the role as the control group. Controlled trials with 4 sets were carried out and the experimental variables were ”Avoid overhunting”, ”Hunt in autumn”, ”Avoid hunting infants” and ”paradigm” (i.e. all of the above 3 suggestions were adopted).

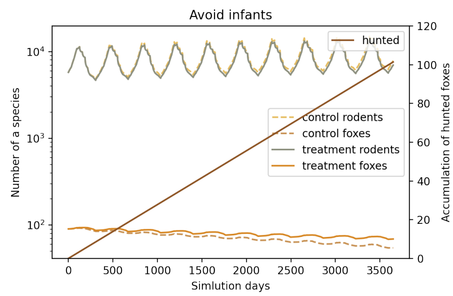
The results are shown in figure 7. As is shown overhunting may lead to a collapsing of the ecological system. Avoid hunting infants and autumn oriented hunting inhibits the decline of foxes compared to the control group. The paradigm group shows that the guidelines for hunting are workable, which makes it appliable to hunt as many foxes as possible, without binging permanent damage to the ecological system.

4 Conclusion and Discussion

A generalized model with various boundary conditions for the dynamics of plants, rodents and foxes in an ecological system was developed to guide the fox-hunting activity. The flexibility of the model's parameters makes the model more expandable by considering the age and seasonal distributions of parameters.



(a)



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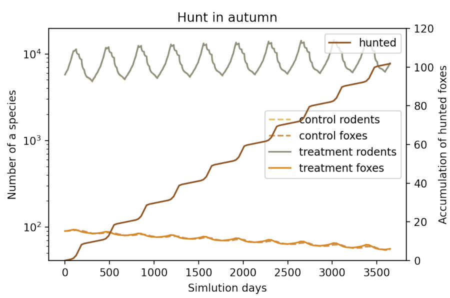
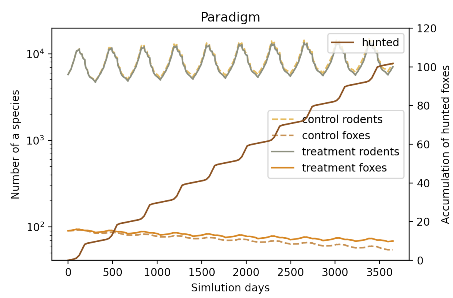


(c)



(d)

Figure 7: Results for verifing the guidelines. (a) avoid overhunting prevent the ecological system from collapsing; (b) Avoid hunting infants inhibits the decline of foxes largely; (c) hunt in autumn (autumn oriented hunt) inhibits the decline of foxes slightly
; (d) a paradigm with all 3 guidelines adopted yield maximum products with minimum decline.

Based on the results and analysis of the model, 3 guidelines were proposed and verified.

In further research, the setting and choices of some boundary conditions still require better and more professional consideration. Besides, more features (e.g. the age distribution of being hunted) is also required to be integrated into the model.

5 Appendix

5.1 Source code

The source code can be download here(https://hyder.top/2022/05/02/Ecological-Simulation-Based-on-System-Dynamics/.).

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Table 1: Definitions of key variables used in this paper.

Variable	Unit	Description
Status		
t	days	Time-steps from the beginning of the simulation. $t \in [0, t_{max}]$.
Δt	days	Size of time-step.
$Month(t)$		Month at time-step of t.
a	days	Age, $a \in [0, a_{max}]$
$\mathbf{Q}(t)$	individuals	Individuals of a species which varies with time. A $(a_{max} + 1) \times 2$ vector, in where a_{max} denotes the maximum age in days of the species.
Breeding & growth		
N_{matted}	pairs	Number of matted pairs.
$N_{breeding}$	individuals/Δt	The number of new born infants.
Hunted		
$E_{predators}$	unit energy	Energy demand of predators.
R_H	individuals/Δt	Number of hunted individuals distributed by ages in a time-step. A $(a_{max} + 1) \times 2$ vector.
Disease & aging		
R_{DA}	individuals/Δt	Number of individuals died of disease and aging distributed by ages in a time-step. A $(a_{max} + 1) \times 2$ vector.
Starvation		
$E(a, m)$	unit energy	The energy of an individual of a species in a month. A $(a_{max} + 1) \times 2$ vector.
E_{demand}	unit energy	Energy demand of individuals in different ages. A $(a_{max} + 1) \times 2$ vector.
E_{supply}	unit energy	Ingested energy. A $(a_{max} + 1) \times 2$ vector.
R_S	individuals/Δt	Number of individuals died from starvation in a time-step. A $(a_{max} + 1) \times 2$ vector.
Lack of nourishing		
R_{LN}	individuals/Δt	Number of infants died from starvation in a time-step. A $(a_{max} + 1) \times 2$ vector.
Migration		
R_{IM}	individuals/Δt	Number of immigrates in a time-step. A $(a_{max}+1)\times 2$ vector.
R_{EM}	individuals/Δt	Number of emigrates in a time-step. A $(a_{max}+1)\times 2$ vector.
Plants		
E_{plant}	unit energy	Energy of plants.

Parameter	Unit	Description
$\{r_{stages}\}$		Probability ratios of being in the phase as on of its stages with respect to age, composed with $\{r_{stages}\} = \{r_{infant}(a), r_{juvenile}(a), r_{adult}(a), r_{senior}(a)\}$. These variables need to meet the the following constraint given by $r_{infant}(a) + r_{juvenile}(a) + r_{adult}(a) + r_{senior}(a) = 1$
$r_{matting}(a, m, pattern)$		Ratio of matted female in total female. The parameter *pattern* denotes the matting pattern. $r_{matting,a} \in [0, 1]$
$BR_{monthly}$	individuals/month	Averaged birth rate of a pair of couples per month.
$E_{individual}(a)$	unit energy	The energy per individual.
$w_{energy}(m)$		Weight factor for $E_{individual}(a)$ in different months.
$r_{DA}(a, m)$		Probability ratios of individuals of a specified age died of disease and aging in a certain month.
e_{Asm}		Assimilation efficiency, a well-known ecological concept, is the ratio of ingested energy to assimilated energy.
$r_{demand}(a, m)$		Probability ratios of energy demanding ratios to internal energy.
$P_{alloc}(a)$		Probability of supplied allocation among individuals with various ages.
K, α, β		Parameter of logistic curve for the growth of plants.

Table 2: Definitions of parameters.

Conditions or variables	value	
Initial date	2022-05-01	
Δt	1 day	
$\frac{1}{K}$	$1000 \times [0.3 + 0.7N(6, 1) + 0.1N(9, 0.5)]$	
$t_{quarter}, t_{half}$	15 days, 20 days	
Conditions or variables	Values for rodents	Values for foxes
$\|\|Q\|\|_{init}$	$(10^4, 10^4)$	$(10^3, 10^3)$
r_{stages}	$(2, 4, 10, 15)$	$(4, 8, 80, 120)$
$\bar{BR}_{monthly}$	7	0.5
$\|\|\bar{E}_{individual}\|\|$	1	10
$\|\|\bar{r}_{demand}\|\|$	0.1	0.01
e_{Asm}	0.1	0.2

Table 3: The default boundary conditions and initial variables.