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SOMAIYA VIDYAVIHAR UNIVERSITY

28 NOV 2023 (E)

Maximum Marks: 100 **Programme code: 01/40** **Programme: B. Tech Computer Engineering/EXCP**	**Semester: August 2022 – December 2022** **Examination: ESE Examination** **Class: SY**	**Duration:3 Hrs.** **Semester: III (SVU 2020)**
Name of the Constituent College: K. J. Somaiya College of Engineering		**Name of the department:** Computer Engineering/Electronics and Computer engineering.
Course Code: 116U01C301/116U40C301	**Name of the Course:** Integral transform and Vector Calculus.	
Instructions: 1) Draw neat diagrams 2) All questions are compulsory 3) Assume suitable data wherever necessary		

Que. No.	Question	Max. Marks
Q1	Solve any **Four** of the following	20
i)	Find the Laplace transform of $t\int_0^t e^{-u}\sin u\,du$	05
ii)	Find the inverse Laplace transform of $\frac{e^{-3s}}{(s+4)^3}$	05
iii)	Obtain Fourier Series for $f(x) = x^2, -\pi < x < \pi$	05
iv)	Find Z transform of $f(k)$ indicating the region of convergence where $f(k) = \begin{cases} a^k \; for \; k < 0 \\ b^k \; for \; k \geq 0 \end{cases}$ and a > b	05
v)	Prove that $\bar{d} \circ \{\bar{a} \times [\bar{b} \times (\bar{c} \times \bar{d})]\} = (\bar{b} \circ \bar{d})[\bar{a}\,\bar{c}\,\bar{d}]$	05
vi)	Find the work done in moving a particle in the force field $\bar{F} = 3x^2 i + (2xz - y)j + zk$ along the straight line $x = 2t, y = t, z = 3t$ joining the points O(0,0,0) and P(2,1,3)	05
Q2 A	Solve the following	10
i)	Find Laplace Transform of $\frac{\sin^2 t}{t}$	05
ii)	Find Inverse Laplace Transform of $\log\left(\frac{s+a}{s+b}\right)$	05
	OR	
Q2 A	Obtain Fourier Series for $f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x - \pi, & \pi < x < 2\pi \end{cases}$ State the value of $f(x)$ at $x = \pi$. Hence, deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots$	10
Q 2 B	Solve any **One** of the following	10
i)	Using Laplace Transform Solve $(D^2 + 9)y = 18t$ When $y(0) = 0$ and $y(\pi/2) = 0$.	10

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iii)	Show that half range sine series for the function $\cos x$ in $0 < x < \pi$, is $\cos x = \frac{8}{\pi}\sum_{m=1}^{\infty}\left(\frac{m}{4m^2-1}\right)\sin 2mx$	10
Q3	Solve any **Two** of the following	20
i)	Show that in $(-2,2)$, $\cosh 2x + \sinh 2x = \sinh 4\sum_{n=-\infty}^{\infty}\frac{(-1)^n(4+in\pi)}{16+n^2\pi^2}e^{\frac{in\pi x}{2}}$ using Complex form of Fourier series.	10
ii)	Prove that $\nabla\left[\frac{(\bar{a}\cdot\bar{r})}{r^n}\right] = \frac{\bar{a}}{r^n} - \frac{n(\bar{a}\cdot\bar{r})\bar{r}}{r^{n+2}}$	10
iii)	Show that $\bar{F} = (ye^{xy}\cos z)i + (xe^{xy}\cos z)j - (e^{xy}\sin z)k$ is irrotational and find the scalar potential for $\bar{F}$ and evaluate $\int \bar{F}\cdot d\bar{r}$ along the curve joining the points $(0,0,0)$ and $(-1,2,\pi)$.	10
Q4	Solve any **Two** of the following	20
i)	(a)Using Convolution find $z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$	05
	(b) Use Gauss's divergence theorem to evaluate $\iint_s \bar{N}\cdot\bar{F}\,ds$ where $\bar{F} = 4xzi - y^2j + yzk$ and s is the surface of the cube bounded by $x = 0, x = 1$, $y = 0, y = 1, z = 0, z = 1$.	05
ii)	Find the values of a, b, c if the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at (1,2,-1) has maximum magnitude 64 in the direction parallel to the z-axis.	10
iii)	Verify Green's theorem for $\bar{F} = (xy + y^2)i + x^2j$ and C is the closed curve of the region bounded by $y = x$ and $y = x^2$.	10
Q5	Solve any **Four** of the following	20
i)	Using Laplace Transform evaluate $\int_0^{\infty} e^{-t}(1+2t)\,H(t-2)dt$	05
ii)	Find Inverse Laplace transform of $\frac{1}{s^2(s+a)^2}$ using Convolution theorem.	05
iii)	Find Fourier Cosine transform of $f(x) = e^{-2x} + 4e^{-3x}$, $x > 0$	05
iv)	Find $z^{-1}\left[\frac{4z}{(z-a)}\right]$ for $\|z\| > \|a\|$	05
v)	Find the angle between the surfaces $x\log z + 1 - y^2 = 0, x^2y + z = 2$ at $(1,1,1)$.	05
vi)	Use Stoke's theorem to evaluate $\int_C \bar{F}\cdot d\bar{r}$ where $\bar{F} = x^2\hat{\imath} + xy\hat{\jmath}$ and c is the boundary of the rectangle $x = 0, y = 0, x = a, y = b$.	05