23-01-2023 (E) 1 GPT

SOMAIYA VIDYAVIHAR UNIVERSITY

Maximum Marks: 100	Semester: August 2022 – December 2022 (Jan-2023) Examination: ESE Examination DSY (Reg + KT)		Duration: 3 Hrs.
Programme code: 01 Programme: B. Tech Computer Engineering		Class: SY	Semester: III (SVU 2020)
Name of the Constituent College: K. J. Somaiya College of Engineering		Name of the department: Computer Engineering	
Course Code: 116U01C301	Name of the Course: Integral transform and Vector Calculus.		
Instructions: 1) Draw neat diagrams 2) All questions are compulsory 3) Assume suitable data wherever necessary			

Que. No.	Question	Max. Marks
Q1	Solve any Four of the following	20
i)	Find $L(e^{-3t}\sin^2 t)$.	05
ii)	Find $L^{-1}\left(\frac{s}{(s-2)^6}\right)$	05
iii)	Obtain a half – range sine series in $(0,\pi)$ for $f(x)=x(\pi-x)$.	05
iv)	Find Z-Transform of $\cos\alpha k$ for $k\geq 0$	05
v)	Prove that $(\overline{a}\times\overline{b})\times(\overline{a}\times\overline{c})=[(\overline{a}\times\overline{b})\cdot\overline{c}]\overline{a}$	05
vi)	Evaluate $\int_A^B (x^2-y^2+x)dx-(2xy+y)dy$ along the parabola $y^2=x$ from $A(0,0)$ to $B(1,1)$.	05
Q2 A	Solve the following	10
i)	Evaluate using Laplace transform $\int_0^\infty \frac{e^{-2t}-e^{-3t}}{t}dt$.	05
ii)	Find $L^{-1}\left[\tan^{-1}\frac{a}{s}\right]$.	05
	OR	
Q2 A	Find Fourier series for $f(x)=x^2$ in $(0,2\pi)$. Hence deduce that $\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}+\cdots$.	10
Q 2 B	Solve any One of the following	10
i)	Solve $(D^2-D-2)y=20\sin 2t, y(0)=1\ and\ y'(0)=2$ using Laplace transforms.	10
ii)	Find Fourier Series of $f(x)=\begin{cases}\frac{\pi}{2}+x & -\pi<x<0\\ \frac{\pi}{2}-x & 0<x<\pi\end{cases}$ Hence deduce that a) $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$ b) $\frac{\pi^4}{96}=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots$	10

Q3	Solve any Two of the following	
i)	(a) Find the Fourier expansion of $f(x) = \begin{cases} -c, & -a < x < 0 \\ c, & 0 < x < a \end{cases}$ in the range $(-a, a)$.	20 05
	(b) Find Fourier Transform of $f(x) = \begin{cases} 1 \; for \; \lvert x\rvert < 1 \\ 0 \; for \; \lvert x\rvert > 1 \end{cases}$	05
ii)	Show that $\bar{F} = (2xyz^2)i + (x^2z^2 + z\cos yz)j + (2x^2yz + y\cos yz)k$ is conservative. Find the scalar potential Φ such that $\bar{F} = \nabla\Phi$ and hence find the work done by $\bar{F}$ in displacing a particle from $A(0,0,1)$ to $B(1, \pi/4, 2)$ along the straight line AB.	10
iii)	Verify Green's Theorem in the plane for $\oint_c [(xy + y^2)dx + x^2dy]$ Where C is the closed curve of the region bounded by $y = x$ and $y^2 = x$.	10
Q4	Solve any **Two** of the following	20
i)	(a) Find $Z[\frac{1}{K}]$	05
	(b) Prove that $\nabla \cdot \left[\frac{logr}{r}\bar{r}\right] = \frac{1}{r}[1 + 2logr]$	05
ii)	Prove that $\bar{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is solenoidal and determine the constants a, b, c if $\bar{F}$ is irrotational.	10
iii)	Using Stokes theorem evaluate $\int_c \bar{F}.d\bar{r}$ where $\bar{F} = 2y(1-x)\hat{\imath} + (x - x^2 - y^2)\hat{\jmath} + (x^2 + y^2 + z^2)\hat{k}$ and C is the boundary of the plane $x + y + z = 2$ Cut off by the coordinate planes.	10
Q5	Solve any **Four** of the following	20
i)	Evaluate $\int_0^\infty e^{-t}[1 + 2t - t^2 + t^3]H(t-1)dt$ using Laplace transform.	05
ii)	Find $L^{-1}\left(\frac{s^2}{(s^2+1)^2}\right)$ using convolution theorem.	05
iii)	Obtain the complex form of Fourier Series for $f(x) = e^{ax}$ in $(0, a)$.	05
iv)	Find $Z^{-1}[\frac{1}{(z-3)(z-2)}]$ for $\lvert z\rvert > 3$	05
v)	Find the angle between the surfaces $x\log z + 1 - y^2 = 0 \; and \; x^2y + z = 2$ at $(1,1,1)$.	05
vi)	Find the directional derivatie of $\emptyset = xy + yz + xz$ at (1,2,1) along the normal to the surface $x^2 + y^2 = z + 4$ at $(1,1,-2)$	05