

28.12.2022(E)

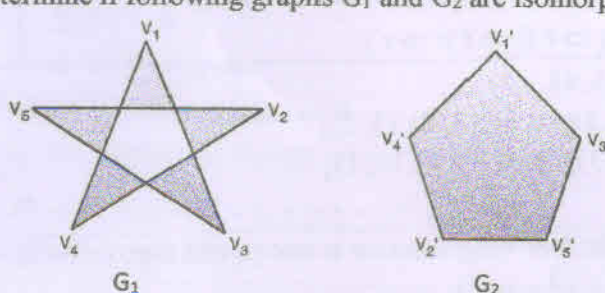

SOMAIYA
 VIDYAVIHAR UNIVERSITY

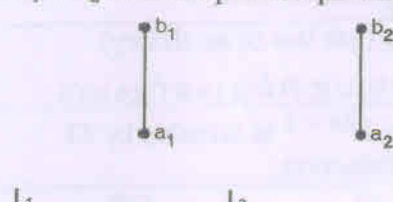
Semester: August 2022 – December 2022		
Maximum Marks: 100	Examination: ESE Examination	Duration:3 Hrs.
Programme code:01	Class: SY	Semester: III(SVU 2020)
Programme: B.Tech. Computer Engineering		
Name of the Constituent College: K. J. Somaiya College of Engineering		Name of the department: COMP
Course Code: 116U01C305	Name of the Course: Discrete Mathematics	
Instructions: 1)Draw neat diagrams 2) All questions are compulsory 3) Assume suitable data wherever necessary		

Que. No.	Question	Max. Marks
Q1	Solve any Four	20
i)	Determine number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5.	5
ii)	Prove by laws of logic (do not use truth table) that the following statement is a tautology. $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$	5
iii)	Let $A = \{1, 2, 3, 4\}$, $R = \{(1, 2), (1, 1), (1, 3), (2, 4), (3, 2)\}$ $S = \{(1, 4), (1, 3), (2, 3), (3, 1), (4, 1)\}$ Find $S \circ R$.	5
iv)	Test whether the following function is one-to-one onto or both. $f: Z \rightarrow Z, f(x) = x^2 + x + 1$	5
v)	How many edges must a planar graph have if it has 7 regions and 5 nodes? Draw one such graph.	5
vi)	Let $(A, *)$ be an algebraic system where $*$ is a binary operation. Such that for any a and b in A , $a * b = a$. (i) Show that $*$ is an associative operation. (ii) Can $*$ ever be a commutative operation.	5

Que. No.	Question	Max. Marks
Q2 A	Solve the following	10
i)	Prove the following (use law of set theory) $(A \cap B) \cup [B \cap ((C \cap D) \cup (C \cap \bar{D}))] = B \cap (A \cup C)$	5
ii)	Prove that $6^n + 2 + 7^{2n+1}$ is divisible by 43. Use mathematical induction.	5
OR		
Q2 A	Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 2), (2, 3), (3, 4), (2, 1)\}$. Find transitive closure of R using Warshall's algorithm.	10
Q2 B	Solve any One	10
i)	For the set $X = \{2, 3, 6, 12, 24, 36\}$, a relation \leq is defined as $x \leq y$ if x divides y . Draw the Hasse diagram for (X, \leq) . Answer the following. (i) What are the maximal and minimal elements? (ii) Give one example of chain and antichain.	10

	(iii) Is the poset a lattice ?	
ii)	Let $S = \{1,2,3,4,5\}$ and $A = S \times S$. Define the following relation R on $A : (a,b) R (c,d)$ if and only if $ad = bc$. Show that R is an equivalence relation.	10

Que. No.	Question	Max. Marks
Q3	Solve any Two	20
i)	The college catering service must decide if the mix of food that is supplied for receptions is appropriate. of 100 people questioned, 37 say they eat fruits, 33 say they eat vegetables, 9 say they eat cheese and fruits, 12 eat cheese and vegetables, 10 eat fruits and vegetables, 12 eat only cheese, and 3 report they eat all three offerings. How many people surveyed eat cheese? How many do not eat any of the offerings?	10
ii)	Determine if following graphs G_1 and G_2 are isomorphic or not. 	10
iii)	State pigeon hole principle and extended pigeon hole principle. What is the minimum number of students required in a discrete structures class to be sure that at least six will receive the same grade, if there are five possible grades A, B, C, D, E.	10

Que. No.	Question	Max. Marks
Q4	Solve any Two	20
i)	If L_1 and L_2 are the lattices shown in following figure. Draw the Hasse diagram of $L_1 \times L_2$ with the product partial order. 	10
ii)	Define Euler path, Euler circuit, Hamiltonian Path and Hamiltonian Circuit. Determine Hamiltonian Cycle and path in graph shown in fig. (a)	10

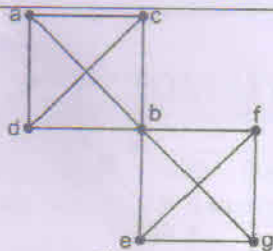


Figure a

Determine Euler Cycle and path in graph shown below fig. (b)

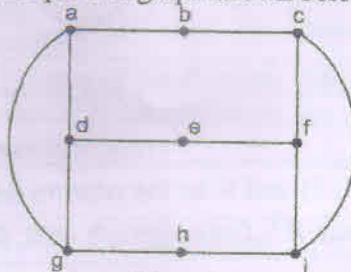
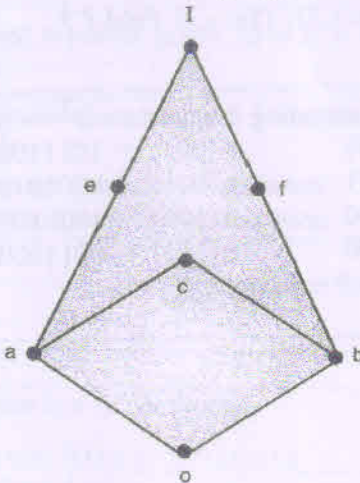
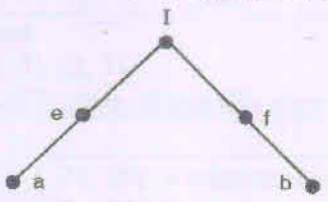


Figure b

iii)	Consider the set $A = \{1, 2, 3, 4, 5, 6\}$ under the multiplication modulo 7. (a) Find the multiplication table for the above (b) Find the inverses of 2, 3 and 5, 6 (c) Prove that it is a cyclic group (d) Find the orders and the subgroups generated by $\{3, 4\}$ and $\{2, 3\}$	10
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Que. No.	Question	Max. Marks
Q5	Solve any four	20
i)	Consider the lattice shown in the following figure  Determine whether or not each of the following is a sublattice of L.  Fig. a	5

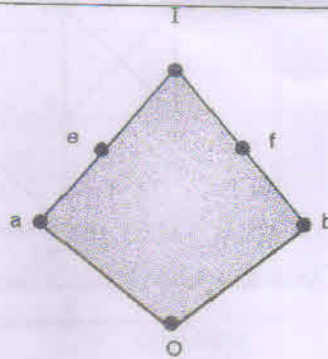


Fig. b

ii)	Let $A = \{1, 2, 3, 4, 5\}$ and R be the relation defined by $a R b$ if and only if $a < b$ compute R , R^2 and R^3 . Draw digraph of R , R^2 and R^3 .	5
iii)	Let $A = \{a, b, c, d, e\}$ and let R be the relation whose matrix is $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$ <p>(i) Find the reflexive closure of R (ii) Find the symmetric closure of R.</p>	5
iv)	Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2 - 1$, $g(x) = 4x^2 + 2$ find (i) $f \circ (g \circ f)$ (ii) $g \circ (f \circ g)$.	5
v)	Function $f(x) = (4x + 3) / (5x - 2)$. Find f^{-1} .	5
vi)	Consider the $(3, 8)$ encoding function $e: B^3 \rightarrow B^8$ defined by $e(000) = 00000000$ $e(001) = 10111000$ $e(010) = 00101101$ $e(011) = 10010101$ $e(100) = 10100100$ $e(101) = 10001001$ $e(110) = 00011100$ $e(111) = 00110001$ How many errors will e detect and correct?	5