SOMAIYA
VIDYAVIHAR UNIVERSITY

Maximum Marks: 100	Semester: August 2022 – December 2022 Examination: ESE Examination	Duration:3 Hrs.
Programme code: 01 Programme: B. Tech Computer Engineering	Class: SY	Semester: III (SVU 2020)
Name of the Constituent College: K. J. Somaiya College of Engineering		Name of the department: Computer Engineering
Course Code: 116U01C301	Name of the Course: Integral transform and Vector Calculus.	
Instructions: 1) Draw neat diagrams 2) All questions are compulsory 3) Assume suitable data wherever necessary		

MARKING SCHEME

Que. No.	Question	Max. Marks
Q1	Solve any **Four** of the following	**20**
i)	Find $L\left[\frac{1}{t}(e^{-at}-e^{-bt})\right]$.	05
	$L[e^{-at}-e^{-bt}] = \frac{1}{s+a}-\frac{1}{s+b}$	02
	$\therefore L\left[\frac{e^{-at}-e^{-bt}}{t}\right] = \int_s^\infty \frac{1}{s+a}-\frac{1}{s+b}\,ds = \left[\log(s+a)-\log(s+b)\right]_s^\infty$	04
	$\boxed{=\log\left(\frac{s+b}{s+a}\right)}$	05
ii)	Find $L^{-1}\left\{\frac{4s+12}{s^2+8s+12}\right\}$. $= L^{-1}\left[\frac{4(s+3)}{(s+4)^2-4}\right] = L^{-1}\left[\frac{4[(s+4)-1]}{(s+4)^2-4}\right]$	05
		02
	$= 4e^{-4t}L^{-1}\left[\frac{s-1}{s^2-4}\right]$	04
	$= 4e^{-4t}\left[L^{-1}\left(\frac{s}{s^2-4}\right)-L^{-1}\left(\frac{1}{s^2-4}\right)\right] = \boxed{4e^{-4t}\left[\cosh 2t-\frac{1}{2}\sinh 2t\right]}$	05
iii)	Obtain half – range Fourier cosine series for $f(x)=x$ in $0<x<2$.	05
	$a_0 = \frac{2}{2}\int_0^2 x\,dx = \left[\frac{x^2}{2}\right]_0^2 = 2$	02
	$a_n = \frac{2}{2}\int_0^2 x\cos\frac{n\pi x}{2}\,dx = \left[x\frac{\sin\frac{n\pi x}{2}}{n\pi/2}+\frac{\cos\frac{n\pi x}{2}}{n^2\pi^2/4}\right]_0^2$	04
	$=\frac{4(-1)^n}{n^2\pi^2}-\frac{4}{n^2\pi^2}$ $\therefore \boxed{x = 1+\frac{4}{\pi^2}\sum_1^\infty \frac{[(-1)^n-1]}{n^2}\cos\frac{n\pi x}{2}}$	05
iv)	Find Z-Transform of $\sin\alpha k$ for $k\geq 0$	05
	$Z[\sin\alpha k] = \sum_{k=0}^{\infty}\left(\frac{e^{i\alpha k}-e^{-i\alpha k}}{2i}\right)z^{-k} = \frac{1}{2i}\sum_0^\infty\left(\frac{e^{i\alpha}}{z}\right)^k-\left(\frac{e^{-i\alpha}}{z}\right)^k$	02
	$=\frac{1}{2i}\left[\frac{z}{z-e^{i\alpha}}-\frac{z}{z-e^{-i\alpha}}\right] = \boxed{\frac{z\sin\alpha}{z^2-2z\cos\alpha+1}}$	05

v)	If $\bar{a} = i + 2j - k$, $\bar{b} = 2i + j + 3k$, $\bar{c} = i - j + k$, $\bar{d} = 3i + j + 2k$, find $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d})$ $[\bar{a}\ \bar{c}\ \bar{d}] = \begin{vmatrix} 1 & 2 & -1 \\ 1 & -1 & 1 \\ 3 & 1 & 2 \end{vmatrix} = -5$, $[\bar{b}\ \bar{c}\ \bar{d}] = 7$ $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = \bar{b}[\bar{a}\ \bar{c}\ \bar{d}] - \bar{a}[\bar{b}\ \bar{c}\ \bar{d}]$ $= -5(2i + j + 3k) - 7(i + 2j - k)$ $= \boxed{-17i - 19j - 8k}$	05 02 04 05
vi)	Evaluate $\int_A^B y^2 dx + xy dy$ along $x = t^2, y = 2t$ from $A(1, -2)$ to $B(0, 0)$. $I = \int_A^B 4t^2(2t\,dt) + 2t^3 \cdot 2dt$ when $A \equiv (1, -2), t = -1$ $B \equiv (0, 0), t = 0.$ $\therefore I = \int_{t=-1}^{0} 8t^3 + 4t^3 dt = 12\left[\frac{t^4}{4}\right]_{-1}^{0}$ $= 3[-1] = \boxed{-3}$	05 02 04 05
Q2 A	Solve the following	10
i)	Evaluate $\int_0^\infty \frac{\sin^2 2t}{t} dt$. $L\left[\frac{\sin^2 2t}{t}\right] = \int_s^\infty L\left(\frac{1 - \cos 4t}{2}\right) ds = \frac{1}{2}\int_s^\infty \left[\frac{1}{s} - \frac{s}{s^2 + 16}\right] ds$ $= \frac{1}{2}\left[\log s - \frac{1}{2}\log(s^2 + 16)\right] = \frac{1}{4}\left[\log \frac{s^2}{s^2 + 16}\right]_s^\infty$ $= \frac{1}{4}\left[\log\left(\frac{s^2 + 16}{s^2}\right)\right]$ put $s = 0$ $= \boxed{\infty}$	05 03 04 05
ii)	Find $L^{-1}[\cot^{-1}(as)]$. $L^{-1}[\cot^{-1}(as)] = -\frac{1}{t} L^{-1}\left[\frac{d}{ds} \cot^{-1}(as)\right]$ $= -\frac{1}{t} L^{-1}\left[\frac{-a}{1 + a^2 s^2}\right]$ $= \frac{a}{t} L^{-1}\left[\frac{1}{1 + a^2 s^2}\right]$ $= a/t\ L^{-1}\left[\frac{1}{a^2} \frac{1}{s^2 + (1/a)^2}\right]$ $= 1/t\ L^{-1}\left[\frac{1/a}{s^2 + (1/a)^2}\right] = \boxed{\frac{1}{t} \sin t/a}$	05 02 04 05

	OR	
Q2 A	Find the Fourier Series of the function $(x) = e^{-x}, 0 < x < 2\pi$. Hence deduce the value of $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2+1}$	10

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} e^{-x}\,dx = -\frac{1}{\pi}\left[e^{-x}\right]_0^{2\pi} = \frac{-1}{\pi}\left[e^{-2\pi} - 1\right]$$ (02)

$$a_n = \frac{1}{\pi}\int_0^{2\pi} e^{-x}\cos nx\,dx = \frac{1}{\pi}\left[\frac{e^{-x}}{n^2+1}\left(-\cos nx + n\underbrace{\sin nx}_{=0}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{-2\pi}}{n^2+1}(-1) + \frac{1}{n^2+1}\right] = \frac{1}{\pi(n^2+1)}\left[1 - e^{-2\pi}\right]$$ (05)

$$b_n = \frac{1}{\pi}\int_0^{2\pi} e^{-x}\sin nx\,dx = \frac{1}{\pi}\left[\frac{e^{-x}}{n^2+1}\left(-\underbrace{\sin nx}_{=0} - n\cos nx\right)\right]_0^{2\pi}$$

$$= -\frac{n}{\pi(n^2+1)}\left[e^{-2\pi} - 1\right]$$ (08)

$$\therefore f(x) = \frac{1}{2\pi}(1 - e^{-2\pi}) + \frac{(1-e^{-2\pi})}{\pi}\sum_{n=1}^{\infty}\frac{\cos nx}{n^2+1} + \frac{(1-e^{-2\pi})}{\pi}\sum_{n=1}^{\infty}\frac{n\sin nx}{n^2+1}$$

put $x = \pi$

$$\Rightarrow e^{-\pi} = \frac{1}{2\pi}(1 - e^{-2\pi}) + \frac{(1-e^{-2\pi})}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+1} \quad \therefore \boxed{\sum_{n=2}^{\infty}\frac{(-1)^n}{n^2+1} = \frac{\pi e^{-\pi}}{(1-e^{-2\pi})}}$$ (10)

Q 2 B	Solve any **One** of the following	10
i)	Solve $(D^2 - 4)y = 3e^t, y(0) = 0, y'(0) = 3$ using Laplace transforms.	10

$$L(y'') - 4L(y) = 3\,L(e^t)$$

$$\Rightarrow s^2\bar{y} - sy(0) - y'(0) - 4\bar{y} = \frac{3}{s-1}$$ (03)

$$\Rightarrow (s^2-4)\bar{y} - 3 = \frac{3}{s-1}$$

$$\Rightarrow (s^2-4)\bar{y} = \frac{3}{s-1} + 3 = \frac{3s}{s-1}$$

$$\Rightarrow \bar{y} = \frac{3s}{(s-1)(s^2-4)} = \frac{3s}{(s-1)(s-2)(s+2)}$$ (06)

$$\therefore \bar{y} = \frac{3}{2}\,\frac{1}{s-2} - \frac{1}{2}\,\frac{1}{s+2} - \frac{1}{s-1}$$

$$\therefore y = \frac{3}{2}L^{-1}\left(\frac{1}{s-2}\right) - \frac{1}{2}L^{-1}\left(\frac{1}{s+2}\right) - L^{-1}\left(\frac{1}{s-1}\right)$$

$$= \boxed{\frac{3}{2}e^{2t} - \frac{1}{2}e^{-2t} - e^t}$$ (10)

ii)	If $f(x)=\begin{cases} x+\frac{\pi}{2}, -\pi\le x\le 0 \\ \frac{\pi}{2}-x, \quad 0\le x\le\pi\end{cases}$ Then Find the Fourier Series of $f(x)$ in $(-\pi,\pi)$ Hence deduce that $\frac{\pi^4}{96}=\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots$.	10

$$f(-x)=\begin{cases}\frac{\pi}{2}-x, & \pi\ge x\ge 0\\ \frac{\pi}{2}+x, & 0\ge x\ge -\pi\end{cases} = f(x)$$

$\therefore\ b_n=0$

$$a_0=\frac{2}{\pi}\int_0^\pi \frac{\pi}{2}-x\,dx=\frac{2}{\pi}\left[\frac{\pi}{2}x-\frac{x^2}{2}\right]_0^\pi=0$$ (03)

$$a_n=\frac{2}{\pi}\int_0^\pi\left(\frac{\pi}{2}-x\right)\cos nx\,dx$$

$$=\frac{2}{\pi}\left[\underbrace{\cancel{\left(\frac{\pi}{2}-x\right)\frac{\sin nx}{n}}}_{=0}-(-1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^\pi$$

$$=-\frac{2}{\pi}\left[\frac{(-1)^n}{n^2}-\frac{1}{n^2}\right]$$

$$=\frac{2}{\pi n^2}\left[1-(-1)^n\right]=\begin{cases}\frac{4}{\pi n^2} & \text{if } n \text{ odd}\\ 0 & \text{if } n \text{ even.}\end{cases}$$

$$\therefore\ \boxed{f(x)=\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}\cos(2k-1)x}$$ (07)

By Parseval's Id.

$$\frac{2}{\pi}\int_0^\pi f^2(x)\,dx=\frac{a_0^2}{2}+\sum_1^\infty a_n^2$$

$$\Rightarrow \frac{2}{\pi}\int_0^\pi\left(\frac{\pi}{2}-x\right)^2dx=\frac{16}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^4}$$

$$\Rightarrow \frac{2}{\pi}\left[\frac{\left(\frac{\pi}{2}-x\right)^3}{-3}\right]_0^\pi=\frac{16}{\pi^2}\sum_1^\infty\frac{1}{(2k-1)^4}$$

$$\Rightarrow \frac{\pi^2}{6}=\frac{16}{\pi^2}\sum_1^\infty\frac{1}{(2k-1)^4}$$

$$\Rightarrow \sum_{k=1}^{\infty}\frac{1}{(2k-1)^4}=\boxed{\frac{\pi^4}{96}}$$ (10)

Q3	Solve any **Two** of the following	20
i)	(a) Find the Fourier coefficient a_n in Fourier expansion of $f(x)=\begin{cases}0, & -2<x<-1\\ 1+x, & -1<x<0\\ 1-x, & 0<x<1\\ 0, & 1<x<2\end{cases}$ in the range $(-2,2)$.	05

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx \quad , l=2$$

$$= \int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx = \int_0^1 (1-x)\cos\frac{n\pi x}{2}\,dx \quad 02$$

$$= \left[(1-x)\frac{2}{n\pi}\left(\sin\frac{n\pi x}{2}\right) - (-1)\frac{4}{n^2\pi^2}\left(-\cos\frac{n\pi x}{2}\right)\right]_0^1 \quad 04$$

$$= -\frac{4}{n^2\pi^2}\left[\cos\frac{n\pi}{2}-1\right] = \boxed{\frac{4}{n^2\pi^2}\left[1-\cos\frac{n\pi}{2}\right]} \quad 05$$

(b) Find Fourier Sine Transform of $e^{-|x|}$ 05

$$F_s(f(x)) = \int_0^\infty f(x).\sin sx\,dx \quad 02$$

$$= \int_0^\infty e^{-x}\sin sx\,dx$$

$$= \left[\frac{e^{-x}}{1+s^2}(-\sin sx - s\cos sx)\right]_0^\infty = \boxed{\frac{s}{1+s^2}} \quad 05$$

ii) If the vector function $\bar{F} = (x+2y+az)\hat{\imath} + (bx-3y-z)\hat{\jmath} + (4x+cy+2z)\hat{k}$ is irrotational find constants a, b, c. Find scalar potential function $\emptyset$ such that $\bar{F} = \nabla\emptyset$. Also find the work done of the moving partical in the same field from $(1,2,-4)$ to $(3,3,2)$ along the straight line joining these points. 10

$\because \bar{F}$ is irrotational, curl $\bar{F} = \bar{0}$

$\Rightarrow (c+1)\hat{i} - (4-a)\hat{j} + (b-2)\hat{k} = \bar{0}$

$\Rightarrow \boxed{c=-1,\ b=2,\ a=4}$ 03

$\therefore \exists\ \phi$ s.t $\bar{F} = \nabla\phi$

$\Rightarrow \frac{\partial\phi}{\partial x} = x+2y+4z, \quad \frac{\partial\phi}{\partial y} = 2x-3y-z$

$\frac{\partial\phi}{\partial z} = 4x-y+2z$

$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz$$

$$= x\,dx - 3y\,dy + 2z\,dz + 2(y\,dx + x\,dy) + 4(z\,dx + x\,dz) - (z\,dy + y\,dz)$$

$$= d\left(\frac{x^2}{2} - \frac{3y^2}{2} + z^2 + 2xy + 4xz - yz\right)$$

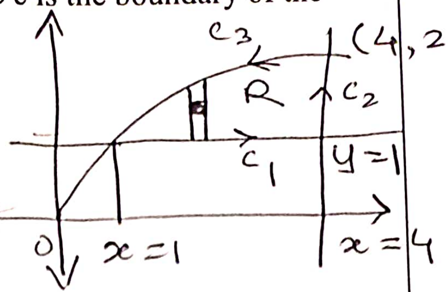
$$\therefore \boxed{\phi = \frac{x^2}{2} - \frac{3y^2}{2} + z^2 + 2xy + 4xz - yz + c} \quad 07$$

$$\text{work done} = \int_C \bar{F}\cdot d\bar{r} = \left[\frac{x^2}{2} - \frac{3y^2}{2} + z^2 + 2xy + 4xz - yz + c\right]_{(1,2,-4)}^{(3,3,2)}$$

$$= \boxed{49/2} \quad 10$$

iii)	Verify Green's Theorem for $\oint_c \left(\frac{1}{y}dx + \frac{1}{x}\,dy\right)$ where c is the boundary of the region defined by $x = 1, x = 4, y = 1, \& \ y = \sqrt{x}$.	10

$P = \frac{1}{y}, \ Q = \frac{1}{x}$



$$\int_c P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$ (02)

Along C_1 : $y = 1, \ dy = 0$

$\therefore \int_{C_1} P\,dx + Q\,dy = \int_1^4 \frac{1}{y}\,dx = 3$

Along C_2 : $x = 4, \ dx = 0$

$\therefore \int_{C_2} P\,dx + Q\,dy = \int_1^2 \frac{dy}{4} = 1/4$

Along C_3 : $y = \sqrt{x}, \ dy = \frac{1}{2\sqrt{x}}\,dx$

$\therefore \int_{C_3} P\,dx + Q\,dy = \int_4^1 \frac{1}{\sqrt{x}} + \frac{1}{2x^{3/2}}\,dx = -5/2$ (06)

$\therefore \int_c P\,dx + Q\,dy = 3 + 1/4 - 5/2 = \boxed{3/4}$

$\& \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy = \int_1^4 \int_1^{\sqrt{x}} -\frac{1}{x^2} + \frac{1}{y^2}\,dx\,dy = \boxed{3/4}$ (10)

Q4	Solve any **Two** of the following	20
i)	(a) Find $Z^{-1}\left[\frac{1}{(z-3)(z-2)}\right]$ for $2 < \lvert z \rvert < 3$	05

$$F(z) = \frac{1}{(z-3)(z-2)} = \frac{1}{z-3} - \frac{1}{z-2}$$

$$= -\frac{1}{3}\frac{1}{(1 - z/3)} - \frac{1}{z(1 - 2/z)}$$ (02)

$$= -\frac{1}{3}\left(1 + \frac{z}{3} + \frac{z^2}{3^2} + \cdots \frac{z^k}{3^k}\right) - \frac{1}{z}\left(1 + \frac{2}{z} + \frac{2^2}{z^2} + \cdots\right)$$

$$= \sum_{k=0}^{\infty} -3^{-k-1} z^{+k} - \sum_{k=1}^{\infty} \frac{2^{k-1}}{z^k}$$

$$\therefore Z^{-1}[F(z)] = \boxed{\begin{cases} -3^{k-1}, & k \le 0 \\ -2^{k-1}, & k \ge 1 \end{cases}}$$ (05)

(b) Evaluate $\int_c \overline{F} \cdot d\overline{r}$, where $\overline{F} = \cos y\,\hat{\imath} - x \sin y\,\hat{\jmath}$ and c is the curve $y = \sqrt{1 - x^2}$ in the xy plane from (1,0) to (0,1). 05

$$\int_c \overline{F} \cdot d\overline{r} = \int_C \cos y\,dx - x \sin y\,dy$$

$$= \int d(x \cos y)$$ (03)

$$= [x \cos y]_{(1,0)}^{(0,1)} = \boxed{-1}$$ (05)

ii) Find $\phi(r)$ such that $\nabla\phi = -\frac{\bar{r}}{r^5}$ and $\phi(2) = 3$ — 10

$\nabla\phi = -(x^2+y^2+z^2)^{-5/2}(x\hat{i}+y\hat{j}+z\hat{k}) = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$

$\therefore \frac{\partial\phi}{\partial x} = -x(x^2+y^2+z^2)^{-5/2}, \quad \frac{\partial\phi}{\partial y} = -y(x^2+y^2+z^2)^{-5/2}$

$\frac{\partial\phi}{\partial z} = -z(x^2+y^2+z^2)^{-5/2}$

$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz = -(x^2+y^2+z^2)^{-5/2}(x\,dx + y\,dy + z\,dz)$

Let $x^2+y^2+z^2 = t \Rightarrow 2(x\,dx+y\,dy+z\,dz) = dt$ (05)

$\therefore d\phi = -t^{-5/2}\,dt/2$

$\Rightarrow \phi = -\frac{1}{2}\frac{t^{-3/2}}{(-3/2)} + c = \frac{t^{-3/2}}{3} + c = \frac{1}{3}(x^2+y^2+z^2)^{-3/2} + c$

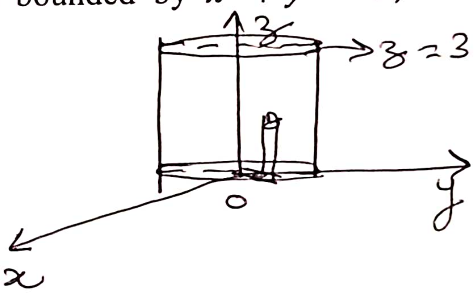
$= \frac{1}{3}\frac{1}{r^3} + c$

but $\phi(2) = 3 \Rightarrow c = 71/24$

$\therefore \boxed{\phi = \frac{1}{3r^3} + \frac{71}{24}}$ (10)

iii) Evaluate by Gauss's Divergence theorem $\iint_s \overline{N}.\bar{F}\,ds$ — 10

Where $\bar{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ & S is the region bounded by $x^2+y^2 = 4$, $z = 0, z = 3$.



$\iint_S \hat{N}\cdot\bar{F}\,ds = \iiint_V \nabla\cdot\bar{F}\,dv$ (02)

$\nabla\cdot\bar{F} = 4 - 4y + 2z$

put $x = r\cos\theta$

$y = r\sin\theta$

$z = z$, $dx\,dy\,dz = r\,dr\,d\theta\,dz$

$\therefore \iiint_V \nabla\cdot\bar{F}\,dv = \int_{\theta=0}^{2\pi}\int_{r=0}^{2}\int_{z=0}^{3}(4 - 4r\sin\theta + 2z)\,r\,dr\,d\theta\,dz$ (05)

$= 4\times3\times2\pi\int_{r=0}^{2} r\,dr - 4\cdot3\int_0^2 r^2\,dr\int_0^{2\pi}\sin\theta\,d\theta + 2\int_0^3 z\,dz\int_{r=0}^{2} r\,dr\int_0^{2\pi} d\theta$

$= 24\pi\times2 - 12\times\frac{8}{3}\times0 + 2\times\frac{9}{2}\,\frac{4}{2}\times2\pi$

$= 48\pi + 36\pi$

$= \boxed{84\pi}$ (05)

Q5	Solve any **Four** of the following	20
i)	Find Laplace transform of $(1+2t-3t^2+4t^3)H(t-2)$.	05

$L[f(t)H(t-a)] = e^{-as} L[f(t+a)]$

$L\{[4t^3 - 3t^2 + 2t + 1] H(t-2)\}$

$= e^{-2s} L[4(t+2)^3 - 3(t+2)^2 + 2(t+2) + 1]$ (02)

$= e^{-2s} L[4t^3 + 21t^2 + 38t + 25]$ (04)

$= \boxed{e^{-2s}\left[4 \cdot \frac{3!}{s^4} + 21 \cdot \frac{2!}{s^3} + \frac{38}{s^2} + 25\frac{1}{s}\right]}$ (05)

ii)	Using convolution theorem find inverse laplace transform of $\frac{1}{(s^2+1)(s^2+9)}$	05

$\phi_1(s) = \frac{1}{s^2+9}$, $\phi_2(s) = \frac{1}{s^2+1}$

$L^{-1}[\phi_1(s)] = \frac{1}{3}\sin 3u = f_1(u)$, $L^{-1}[\phi_2(s)] = \sin u = f_2(u)$ (02)

$L^{-1}[\phi_1\phi_2] = \int_0^t f_1(u) f_2(t-u)\,du = \frac{1}{3}\int_0^t \sin 3u \sin(t-u)\,du$

$= \frac{1}{6}\int_0^t \cos(4u-t) - \cos(2u+t)\,du$ (04)

$= \frac{1}{6}\left[\frac{\sin(4u-t)}{4} - \frac{\sin(2u+t)}{2}\right]_0^t = \frac{1}{3}\left[\frac{\sin 3t}{4} - \frac{\sin 3t}{2} + \frac{\sin t}{4} + \frac{\sin t}{2}\right]$

$= \boxed{\frac{1}{6}\left[-\frac{\sin 3t}{4} + \frac{3\sin t}{4}\right]}$ (05)

iii)	Obtain the complex form of Fourier Series for $f(x) = \cosh 3x + \sinh 3x$ in $(-3,3)$.	05

$C_n = \frac{1}{2l}\int_{-l}^{l} f(x) e^{-\frac{in\pi x}{l}}\,dx$, $l = 3$, $f(x) = e^{3x}$

$C_n = \frac{1}{6}\int_{-3}^{3} e^{3x} e^{-\frac{in\pi x}{3}}\,dx = \frac{1}{6}\int_{-3}^{3} e^{\left(3 - \frac{in\pi}{3}\right)x}\,dx$

$= 1/6 \left[\frac{3e^{\left(\frac{9-in\pi}{3}\right)x}}{(9-in\pi)}\right]_{-3}^{3}$ (03)

$= \frac{1}{(9-in\pi)2}\left[e^{(9-in\pi)} - e^{-(9-in\pi)}\right] = \frac{\sinh(9-in\pi)}{(9-in\pi)}$

$\therefore \boxed{f(x) = \sum_{n=-\infty}^{\infty} \frac{\sinh(9-in\pi)}{(81+n^2\pi^2)} e^{\frac{in\pi x}{3}}(9-in\pi)}$ (05)

iv)	Using convolution theorem find $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z-1)}\right]$	05

$\frac{8z^2}{(2z-1)(4z-1)} = \frac{8z^2}{8(z-\frac{1}{2})(z-1/4)} = \frac{z^2}{(z-a)(z-b)}$, $a = \frac{1}{2}$, $b = 1/4$

$Z^{-1}\left[\frac{8z^2}{(2z-1)(4z-1)}\right] = Z^{-1}\left[\frac{z}{z-a} \times \frac{z}{z-b}\right]$ (03)

$= \sum_{m=0}^{n} a^m b^{n-m} = b^n\left[\frac{\left(\frac{a}{b}\right)^{n+1} - 1}{a/b - 1}\right]$

$= \frac{a^{n+1} - b^{n+1}}{a-b} = \boxed{4\left[\left(\frac{1}{2}\right)^{n+1} - \left(\frac{1}{4}\right)^{n+1}\right]}$ (05)

v) Find the unit normal to the surface $xy^3z^2 = 4$ at$(-1,-1,2)$. 05

$$\nabla\phi = i\frac{\partial}{\partial x}(xy^3z^2) + j\frac{\partial}{\partial y}(xy^3z^2) + k\frac{\partial}{\partial z}(xy^3z^2)$$

$$= i\,y^3z^2 + 3xy^2z^2\,j + 2xy^3z\,k$$

$$= -4i - 12j - 4k \text{ at } (-1,-1,2)$$ 03

$\therefore$ unit normal to $xy^3z^2 = 4$ at $(-1,-1,2)$

$$= \frac{\nabla\phi}{|\nabla\phi|} = \frac{-4i-12j+4k}{\sqrt{16+144+16}} = \boxed{-\frac{1}{\sqrt{11}}(i+3j-k)}$$ 05

vi) In what direction from the point $(2,1,-1)$ is the directional derivative of $\Phi = x^2yz^3$ Maximum? What is its magnitude? 05

$$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$$

$$= 2xyz^3\,i + x^2z^3\,j + 3x^2yz^2\,k$$

at $(2,1,-1)$, $\nabla\phi = -4i - 4j + 12k$

Dirⁿ derivative is max in the dirⁿ of $\nabla\phi = -4i - 4j + 12k$ 03

max magnitude $= |\nabla\phi|$ $\cancel{\frac{\nabla\phi}{|\nabla\phi|}}$ $= \cancel{\frac{-4i-4j+12k}{\sqrt{16+16+144}}}$

$= \sqrt{176}$ $= \boxed{\cancel{\frac{-4i-4j+12k}{\sqrt{176}}}}$ 05