

14-11-2023

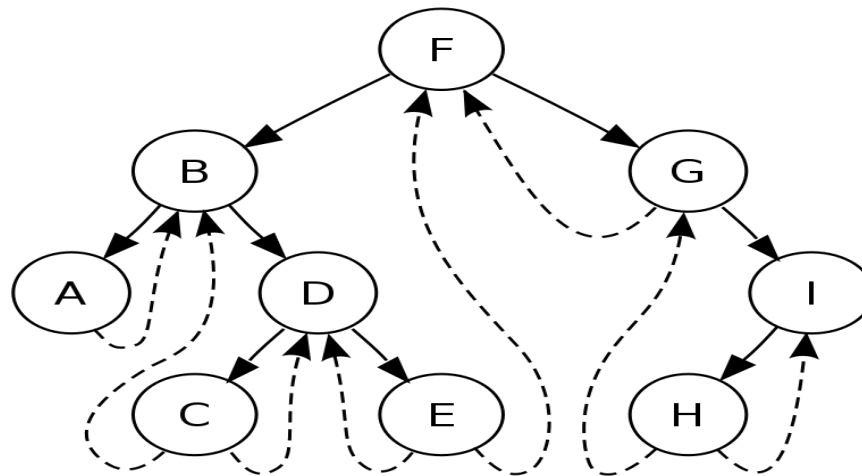
# Threaded Binary Tree

# Threads

- In linked representation of Binary tree,
- **Many of the nodes have NULL value in their left and right pointers.**
- It would be good to **use these pointer fields to keep some other information.**
- Useful for Traversal Operation

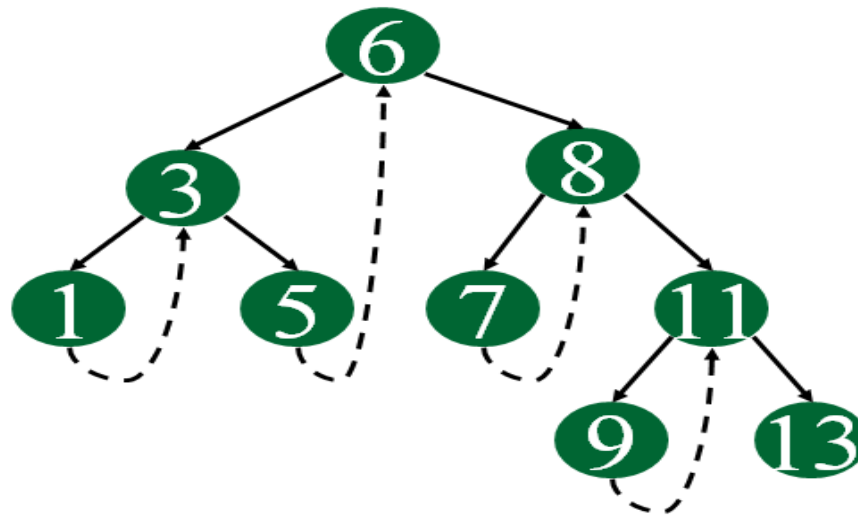
# Threads

- These NULL pointers can be used to point nodes higher in the tree.
- Such pointers are called Threads



# Threaded Binary Tree

- A binary tree which implements such pointers is called threaded Binary tree



The dotted lines  
represent threads

**Single Threaded Binary Tree**

# Threads

Threads corresponding to 3 Traversals:

- **Preorder**
- **Postorder**
- **Inorder**

# Inorder Threading

Two types-

- **One Way Inorder Threading/ Single Threaded**
- **Two Way Inorder Threading / Double Threaded**

# Inorder Threading-

## One Way Inorder Threading

- Each NULL right pointer is altered to contain a thread to point to that node's inorder successor.

## Two Way Inorder Threading

- Each NULL right pointer is altered to contain a thread to point to that node's inorder successor.

+

- Each NULL left pointer is altered to contain a thread to point to that node's inorder predecessor.

# Inorder Threading

Two types-

- **Right In-Threaded Binary Tree**
- **Left In-Threaded Binary Tree**
- **Fully In-Threaded Binary Tree**



## Inorder Threading-

### **Right In-Threaded Binary Tree**

- If we use right field of the node to take the thread

### **Left In-Threaded Binary Tree**

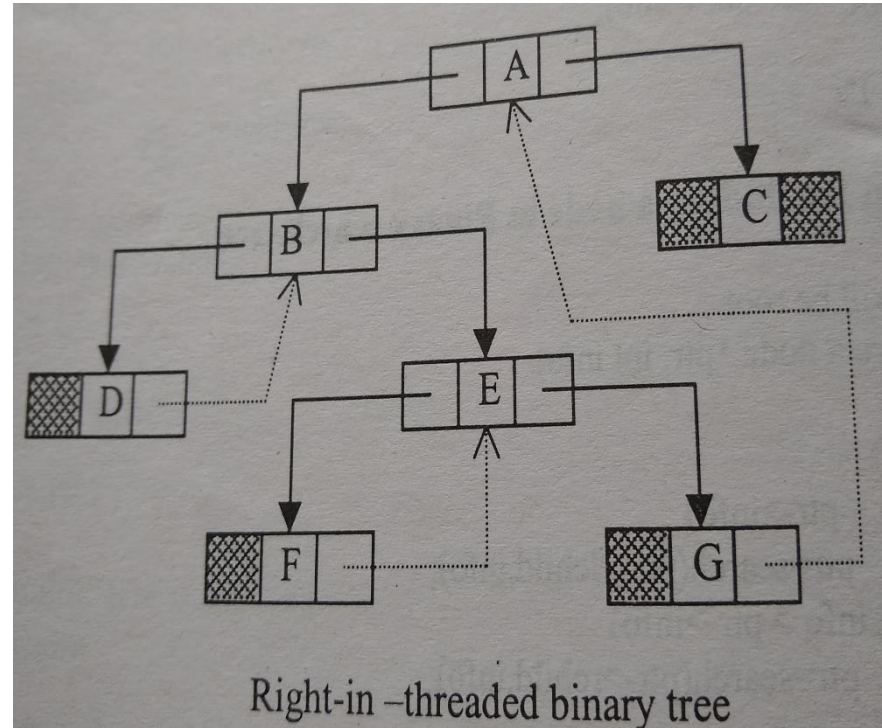
- If we use left field of the node to take the thread

### **Fully In-Threaded Binary Tree/In-Threaded Tree**

- If both left and right fields are used for threading

# Right In-Threaded Binary Tree

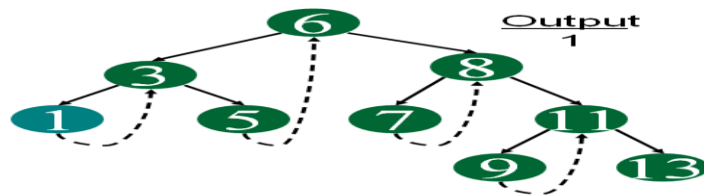
- Each NULL right pointer is altered to contain a thread to point to that node's inorder successor.
- For Eg-
- Thread of D points to B which is inorder successor of D
- Inorder Traversal-  
**DBFEGAC**



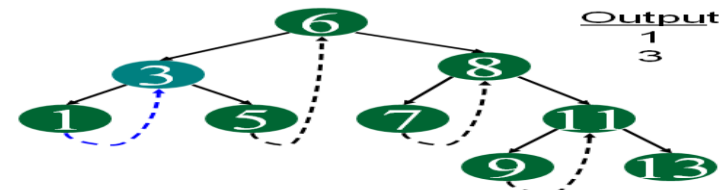
# Right In-Threaded Binary Tree

11

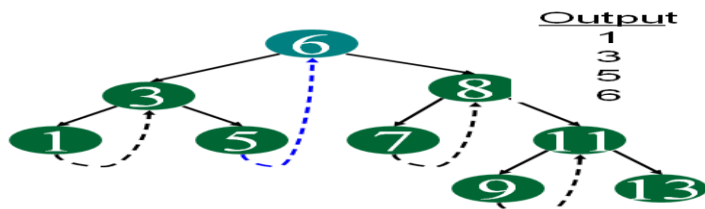
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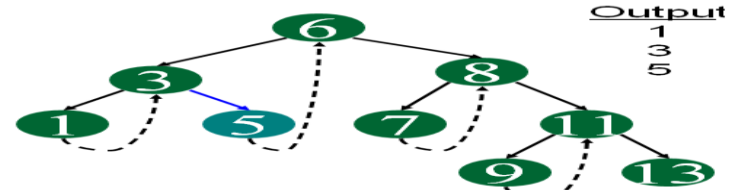
Start at leftmost node, print it



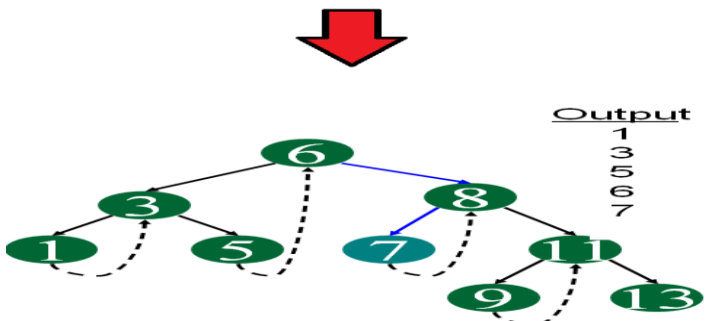
Follow thread to right, print node



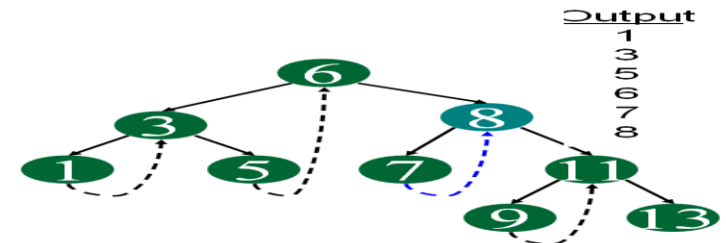
Follow thread to right, print node



Follow link to right, go to leftmost node and print



Follow link to right, go to leftmost node and print



Follow thread to right, print node

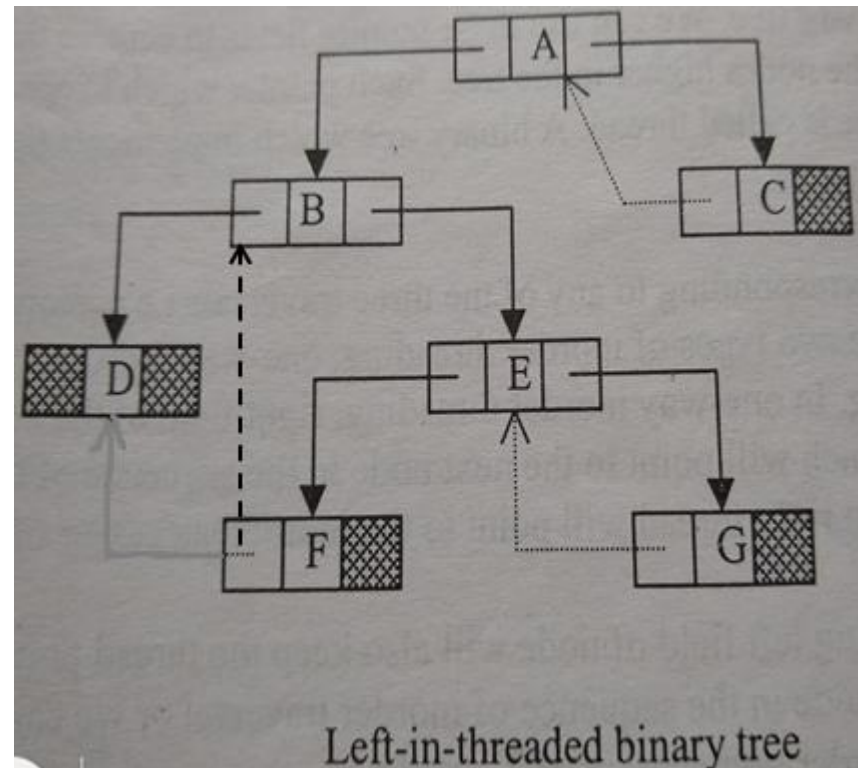
**continue same way for remaining node.....**

Prof. Shweta Dhawan Chachra

Courtesy: <https://www.geeksforgeeks.org/threaded-binary-tree/>

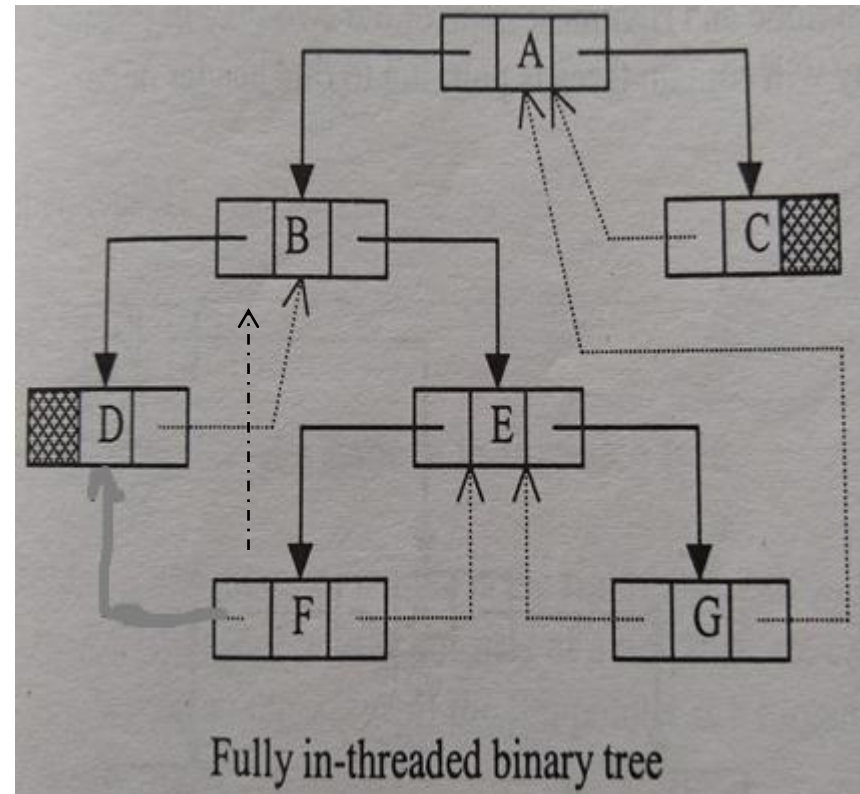
# Left In-Threaded Binary Tree

- Each NULL left pointer is altered to contain a thread to point to that node's inorder predecessor.
- For Eg-
- Thread of F points to B which is inorder predecessor of F
- Inorder Traversal-  
**DBFEGAC**



# Fully In-Threaded Binary Tree

- If both left and right fields are used for threading
- For eg-
- Left thread F points to B, inorder predecessor
- Right thread F points to E, inorder successor
- Inorder Traversal-  
**DBFEGAC**





## Preorder Threading-

- Similarly , we can have Right Pre-threaded and Left Pre-threaded Tree corresponding to the Preorder Traversal.

## Structure of a node in 2way In Threaded Tree

```
typedef enum {thread,link} boolean;  
struct node  
{  
    struct node *left_ptr;  
    boolean left;  
    int info;  
    struct node *right_ptr;  
    boolean right;  
};
```

## Structure of a node in 2way In Threaded Tree

```
typedef enum {thread,link} boolean;  
struct node  
{  
    struct node *left_ptr;  
    boolean left;   
    int info;  
    struct node *right_ptr;  
    boolean right;   
};
```

- Two boolean numbers
  - left
  - right
- to differentiate between a thread and link



# Enumeration

- An enumerated data type
- Enumeration
- Enum

# Enumeration

## What?

- Data type consisting of a set of named values called elements, members or enumerators of the type

# Enumeration

Eg-

```
enum e_tag {a,b,c,d=20,e,f,g=20,h}var;
```

In absence of initialization , the values assigned start at Zero and increase

If we do not explicitly assign values to enum names, the compiler by default assigns values starting from 0.

- a=0
- b=1
- c=2
- d=20
- e=21
- f=22
- g=20
- h=21

# Enumeration

```
#include <stdio.h>

enum day {sunday, monday, tuesday, wednesday,
thursday, friday, saturday};

int main()
{
    enum day d = thursday;
    printf("The day number stored in d is %d", d);
    return 0;
}
```

Output-

```
The day number stored in d is 4
```

# Enumeration

## Enum in C

Declaration	<pre>enum days-of-week { Sun, Mon, Tue, Wed, Thu, Fri, Sat };</pre> <p>             Keyword ↑              enum variable ↑              state=0 ↑              state=1 ↑              state=6 ↑              Enumerators              (list of constants separated by commas)           </p>
Instantiation	<pre>enum days-of-week day;</pre> <p>Object of enum days-of-week</p>
Operation	<pre>day = wed;</pre> <p>             day  <b>3</b>              As state of wed=2           </p>



# Enumeration

```
#include <stdio.h>
enum day {sunday = 1, monday, tuesday = 5,
          wednesday, thursday = 10, friday, saturday};

int main()
{
    printf("%d %d %d %d %d %d %d", sunday, monday,
    tuesday, wednesday, thursday, friday, saturday);
    return 0;
}
```

Output-

```
1  2  5  6  10 11 12
```

## Enum used-

- **typedef enum {thread,link} boolean;**
- declares an enumeration datatype called boolean
- thread=0
- link=1

# Structure of a node in 2way In Threaded Tree

- These members can take values thread or link
  - **left = link**, pointer left\_ptr points to left child of the node
  - **left = thread**, pointer left\_ptr is a thread pointing to inorder predecessor of the node
  - **right = link**, pointer right\_ptr points to right child of the node
  - **right = thread**, pointer right\_ptr is a thread pointing to inorder successor of the node



## In-Threaded Tree

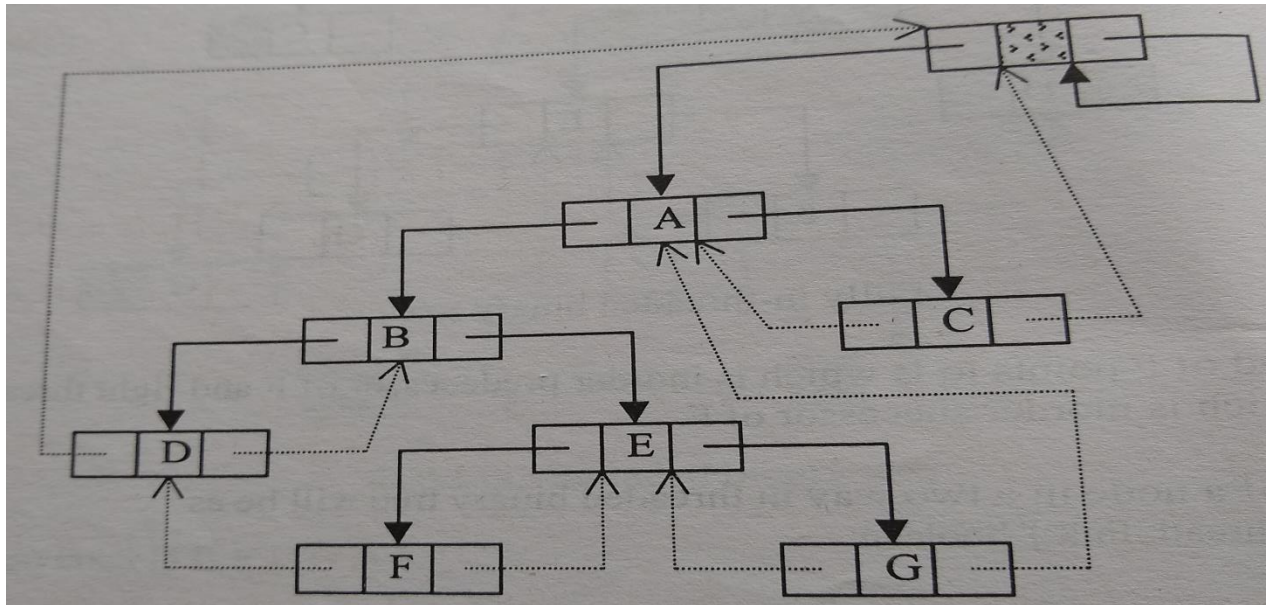
- In Inorder traversal
  - **1<sup>st</sup> Node** has no predecessor
  - **Last Node** has no Successor
  - **Left pointer of Leftmost Node/1<sup>st</sup> Node=NULL**
  - **Right Pointer of Rightmost Node/ Last Node =NULL**
- **Still NULL values.....**
- **Solution?**

**Solution= In-Threaded Tree with Header Nodes**

## Solution= In-Threaded Tree with Header Nodes

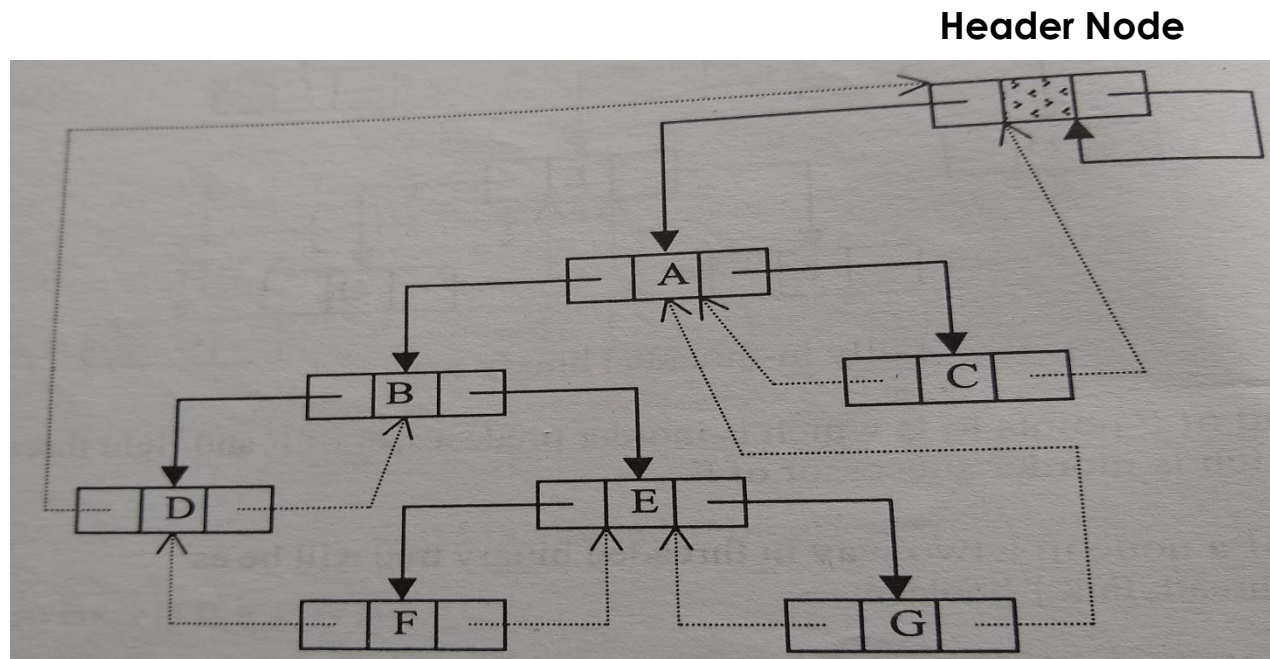
- A dummy node=Header Node is taken
- Our tree will be the left subtree of this Header Node.
- Left pointer of Header Node will point to the root node of our tree.

**Header Node**



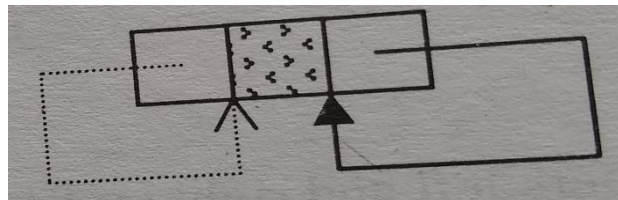
## Solution= In-Threaded Tree with Header Nodes

- Now, Leftmost/Rightmost Node will not contain NULL
- **Will contain threads pointing to this Header Node**



## Solution= In-Threaded Tree with Header Nodes

- When our tree will be empty then Left pointer of Header Node will be a thread pointing to itself.
- Condition for empty In-threaded Tree with Header Node-  
 $\text{head} \rightarrow \text{lchild} = \text{head}$



**Header Node**

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# Inorder Traversal in In-Threaded Binary Tree

## Finding Inorder Successor of a node in In-threaded Node

**The Inorder Successor of a node is the Leftmost node in the right subtree of that node**

- 1) If the Right pointer of a node consists of a link then we can traverse the right subtree and find the inorder successor.
- 2) If the right pointer is a thread, then that thread will point to the inorder successor

## Finding Inorder Successor of a node in In-threaded Node

```

struct node * in_succ(struct node *ptr)
{
    struct node *succ;
    if(ptr->right==thread)
        succ=ptr->right_ptr;
    else
    {
        ptr=ptr->right_ptr;
        while(ptr->left==link)
            ptr=ptr->left_ptr;
        succ=ptr;
    }
    return succ;
}

```

```

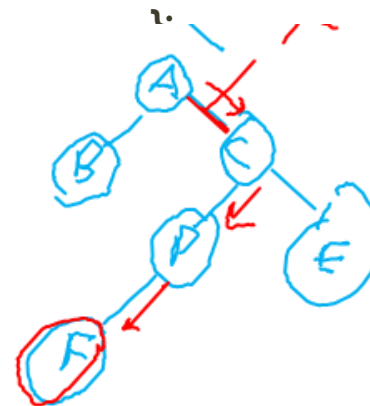
typedef enum {thread,link} boolean;
struct node

```

```

{
    struct node *left_ptr;
    boolean left;
    int info;
    struct node *right_ptr;
    boolean right;
}

```





## Finding Inorder Predecessor of a node in In-threaded Node

- **The Inorder Predecessor of a node is the Rightmost node in the left subtree of that node.**
- 1) If the Left pointer of a node consists of a link then we can traverse the left subtree and find the inorder predecessor.
  - 2) If the left pointer is a thread, then that thread will point to the inorder predecessor



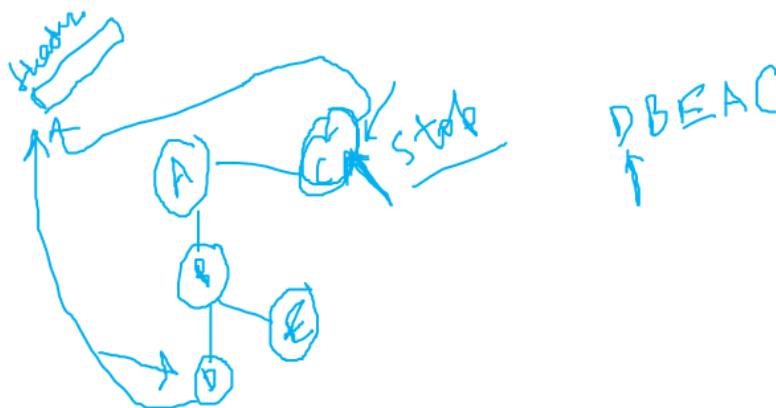
## Finding Inorder Predecessor of a node in In-threaded Node

```
struct node * in_pred(struct node *ptr)
{
    struct node *pred;
    if(ptr->left==thread)
        pred=ptr->left_ptr;
    else
    {
        ptr=ptr->left_ptr;
        while(ptr->right==link)
            ptr=ptr->right_ptr;
        pred=ptr;
    }
    return pred;
}
```

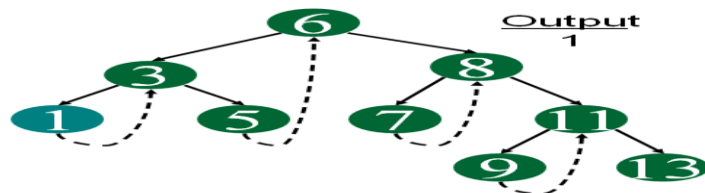
```
typedef enum {thread,link} boolean;
struct node
{
    struct node *left_ptr;
    boolean left;
    int info;
    struct node *right_ptr;
    boolean right;
};
```

## Inorder Traversal in Right In-threaded Node

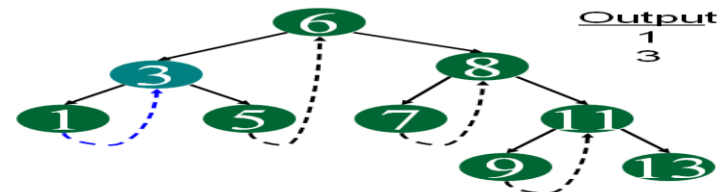
- **Traverse the leftmost node of the tree**
- With the help of `in_succ()` function, find the inorder successor of each node and traverse it
- **Rightmost node of the tree is the last node in the inorder traversal and**
- Its right pointer is a thread points to the header node,
- Hence we stop on reaching header node



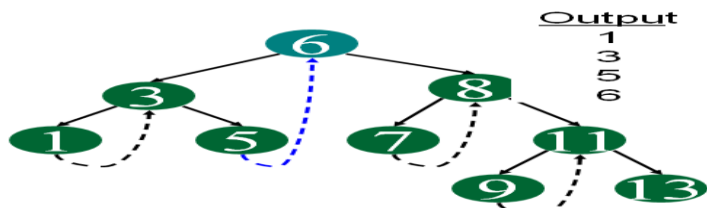
# Right In-Threaded Binary Tree



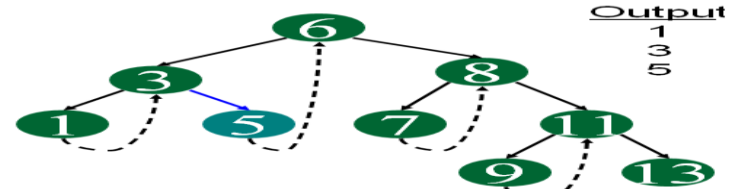
Start at leftmost node, print it



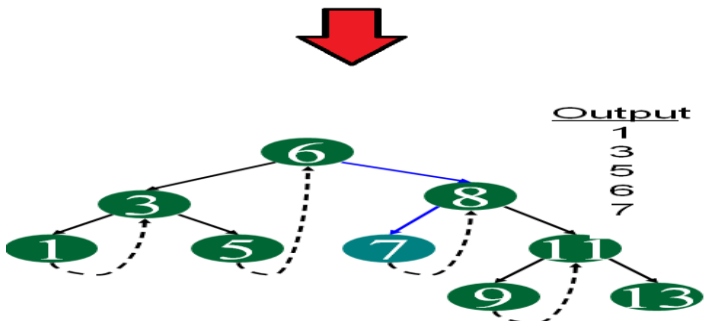
Follow thread to right, print node



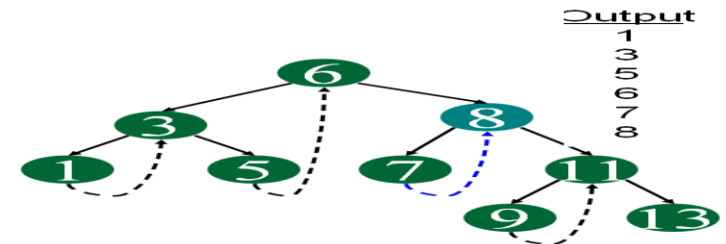
Follow thread to right, print node



Follow link to right, go to leftmost node and print



Follow link to right, go to leftmost node and print



Follow thread to right, print node

**continue same way for remaining node.....**

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Courtesy: <https://www.geeksforgeeks.org/threaded-binary-tree/>

## Inorder Traversal in In-threaded Node

```

inorder()
{
    struct node *ptr;
    if(head->left_ptr==head)
    {
        printf("Tree is empty");
        return;
    }
    ptr=head->left_ptr;
    /*Find the leftmost node and traverse it*/
    while(ptr->left==link)
        ptr=ptr->left_ptr;
    printf("%d",ptr->info);
    while(1)
    {
        ptr=in_succ(ptr);
        if(ptr==head) /*If last node is reached*/
            break;
        printf("%d",ptr->info);
    } /*end of while*/
}

```

- Rightmost node of the tree is the last node in the inorder traversal and
- Its pointer right pointer is a thread points to the header node,
- Hence we stop on reaching header node

14-11-2023

# AVL Trees

# AVL Trees

- Why is it called so?

# AVL Trees

- Why is it called so?
- Russian Mathematician **G.M. Adelson Velskii** and **E.M . Landis** came with a new technique for **balancing binary search tree**
- Called AVL trees on their names.



# AVL Trees

- **Balanced binary search tree**
- A binary search tree where height of left and right subtree of any node will be with maximum difference 1

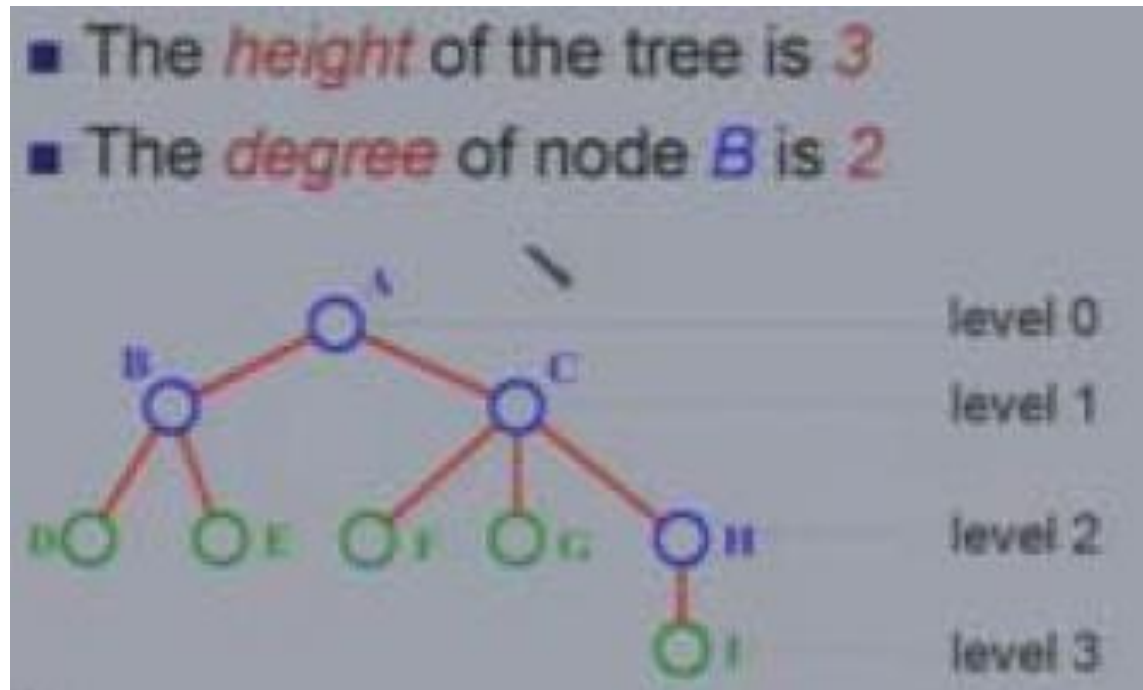
# AVL Trees

- Each node has a balance factor.
- **Balance Factor=Height of Left subtree-Height of Right Subtree**

# Tree

## Basic Terminology-

- Height -
  - Maximum level of any leaf in the tree.



# AVL Trees

- **Right Heavy Node/Right High**

- If Height of right subtree is one more than height of its left subtree.

- **Left Heavy Node/Left High**

- If Height of its left subtree is one more than height of its right subtree

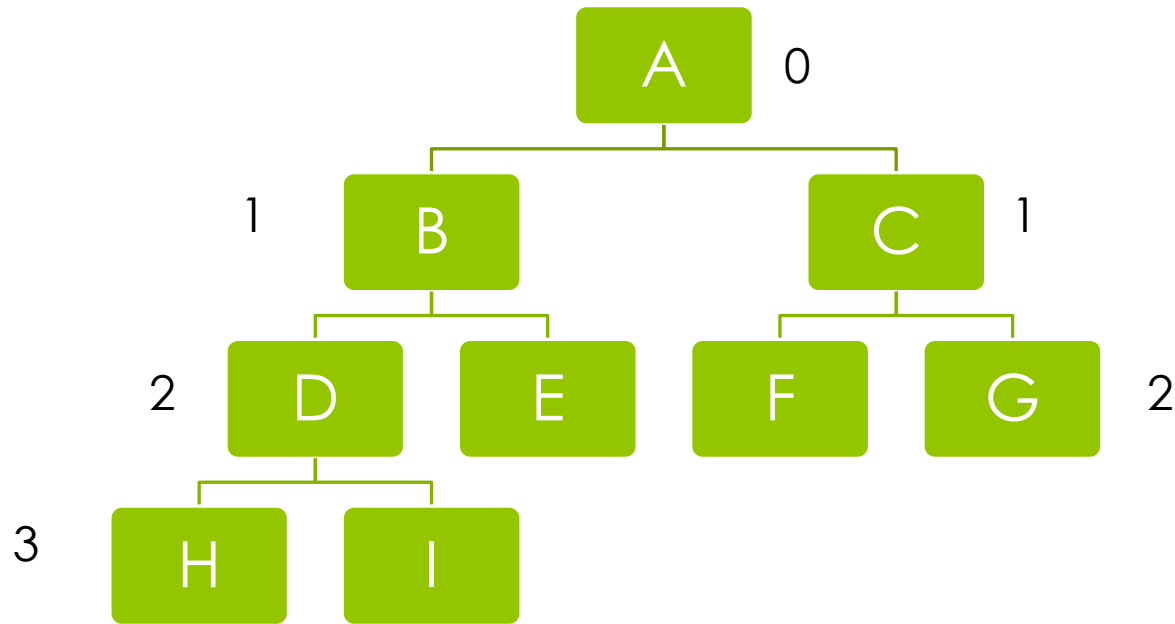
- **Balanced Node**

- If height of left subtree=Height of right subtree.

## Balance factor –

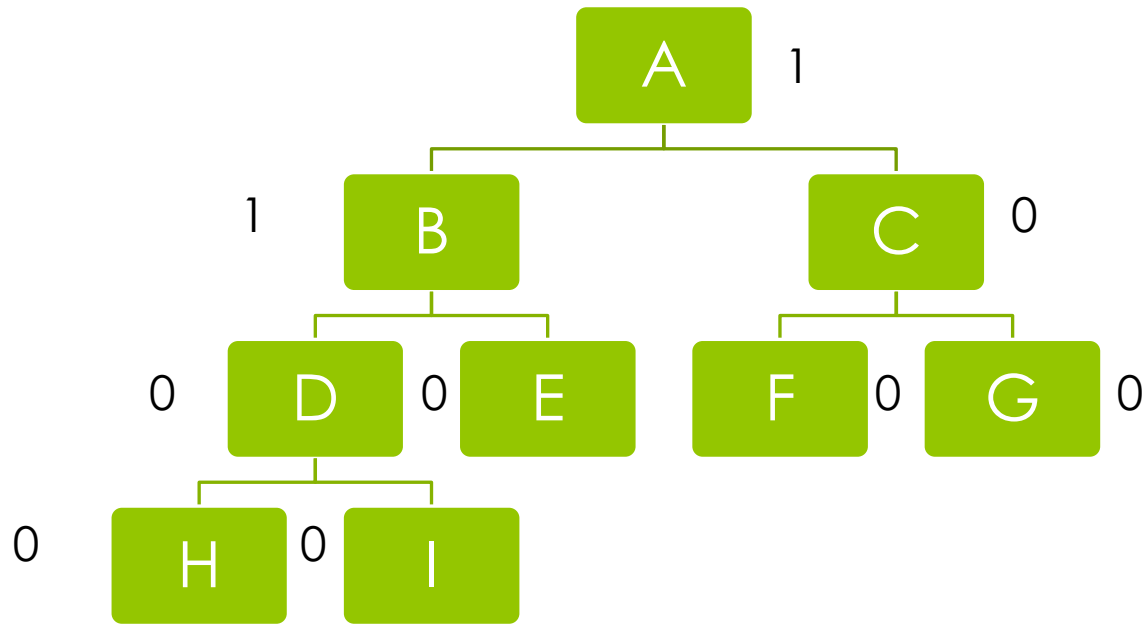
- Left High=1
- Right High=-1
- Balanced node=0

# AVL trees- Node with Levels



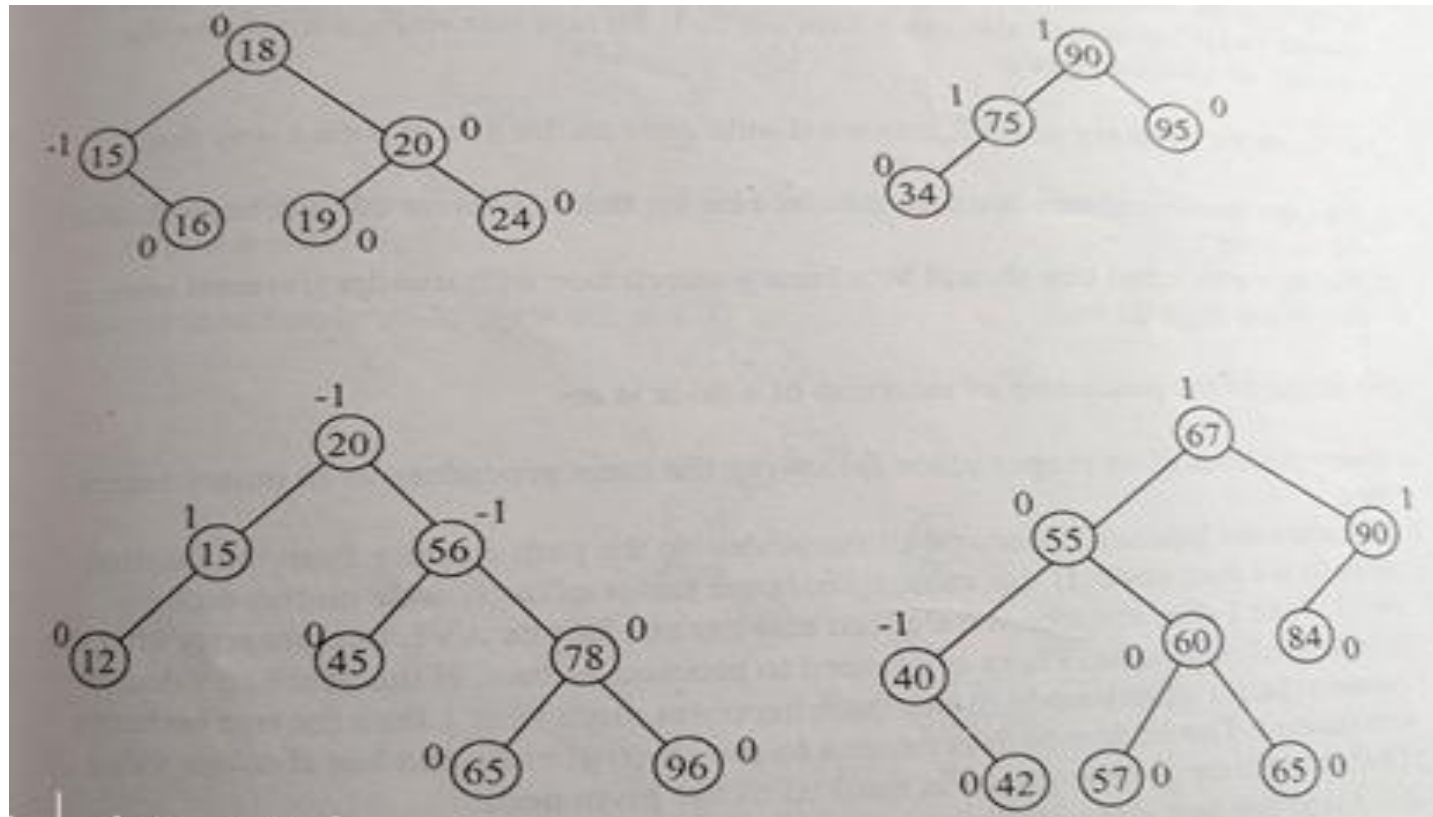
- For Node A=Left Heavy
- Balance Factor=Height of Left Subtree-Height of Right Subtree
- Balance Factor=3-2=1

# AVL tree with Balance Factor



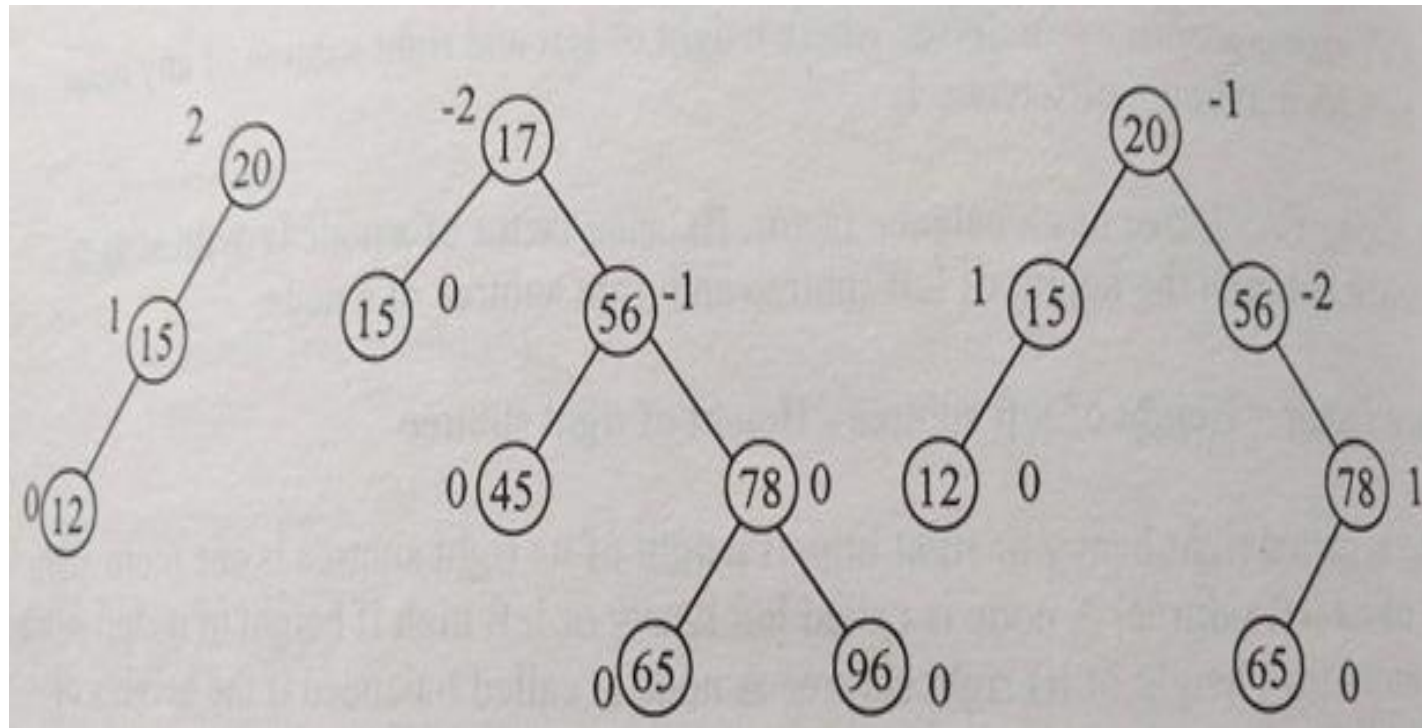
- For Node A=Left Heavy
- Balance Factor=Height of Left Subtree-Height of Right Subtree
- Balance Factor=3-2=1

## Binary Search Trees which are AVL Trees

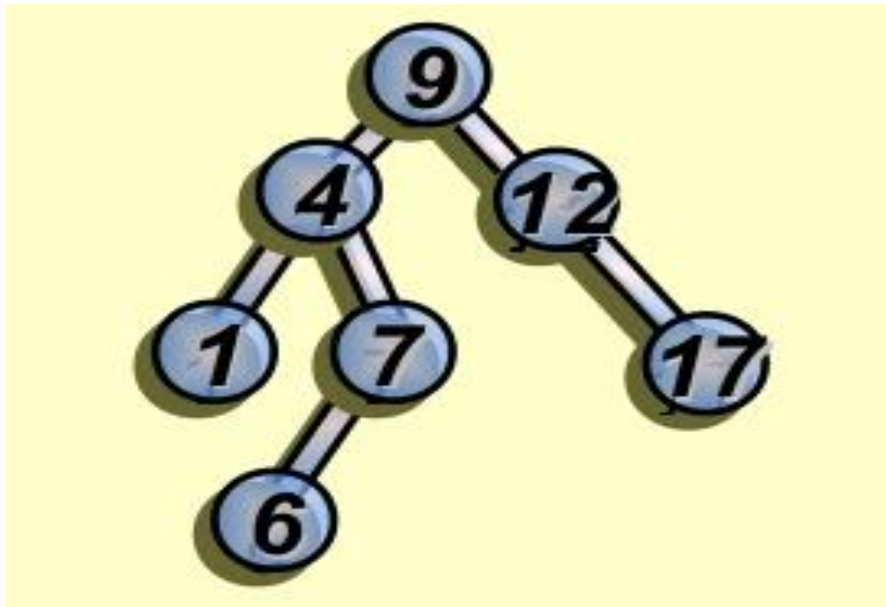




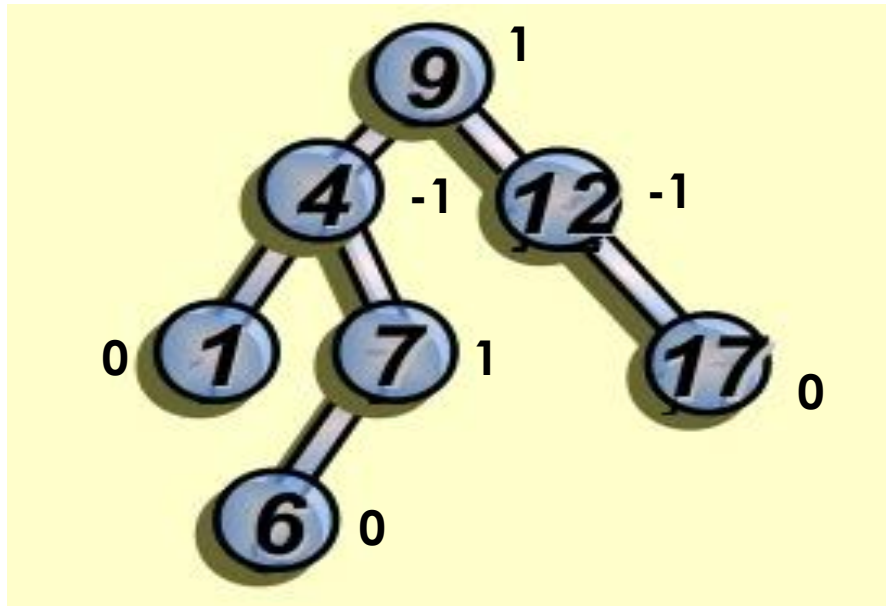
## Binary Search Trees which are not AVL Trees



# Calculate the Balance factor



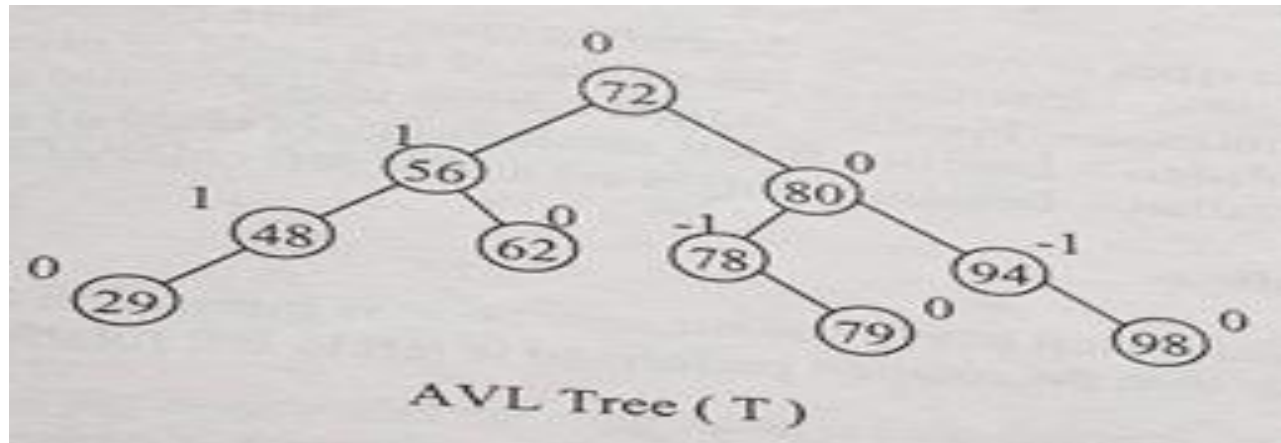
# Calculate the Balance factor



# Insertion in AVL Trees–

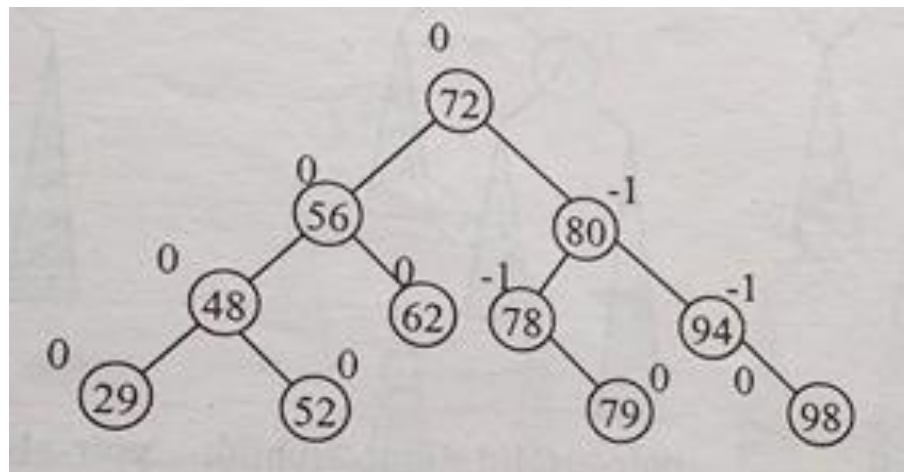
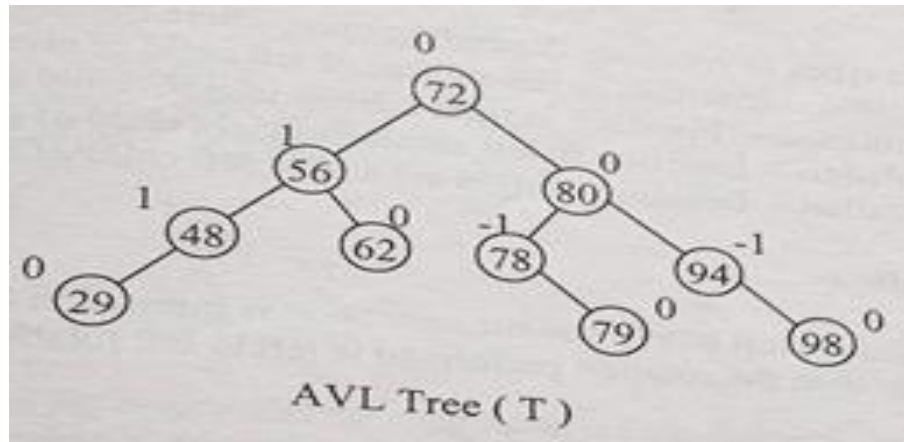
- Similar to Insertion in Binary Search Tree
- Steps-
  - 1) Insert the node at the proper place using same procedure as in BST
  - 2) Calculate the balance factors of all the nodes on the path starting from the inserted node to the root node.
    - a) **If the tree is balanced then there is no need to proceed further.**
    - b) **If absolute value of balanced factor of any node in this path  $>1$  then the tree becomes unbalanced.**
    - c) **The node which is nearest to the inserted node & has absolute value of balance factor  $>1$  is marked as Pivot node**
  - 3) We perform **rotations about the pivot node**

## Insertion in AVL Trees–



- Insert 52 in the given AVL tree

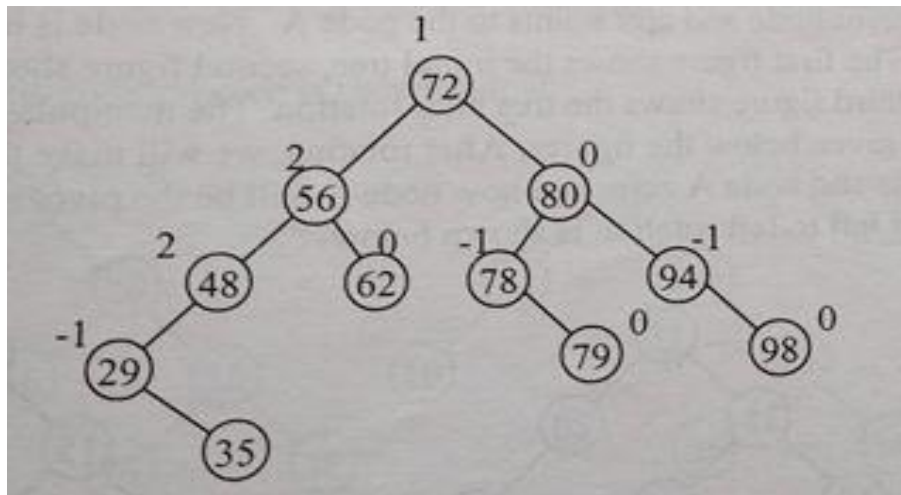
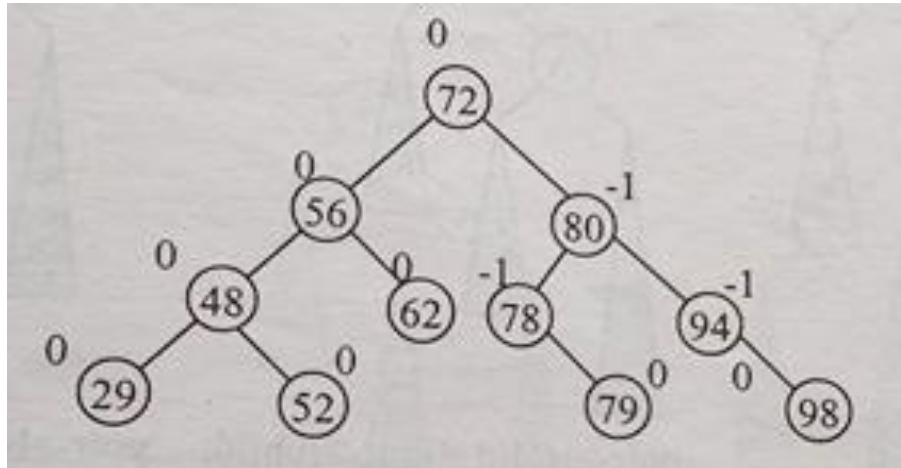
# Insertion in AVL Trees–



- After inserting 52,
  - the balance factors of some nodes changed
  - but tree is still balanced

- Now Insert 35

# Insertion in AVL Trees–



- After inserting 35,
  - Tree is unbalanced
  - Node 48 is unbalanced and
  - Is nearest to the inserted node, so it's the Pivot Node
- Rotations will be needed now

# AVL Rotations–

4 types of Rotation:

- Left to Left Rotation
- Right to Right Rotation
- Right to Left Rotation
- Left to Right Rotation



# AVL Rotations–

4 types of Rotation:

**C to S Rotation**

**Child to Subtree Rotation**

- Left to Left Rotation
- Right to Right Rotation
- Right to Left Rotation
- Left to Right Rotation

# AVL Rotations–

4 types of Rotation depending upon where the new node is inserted:

## Child to Subtree Relationship(Of Pivot Node)

- **Left to Left Rotation**

- Insertion in Left subtree of left child of Pivot Node

- **Right to Right Rotation**

- Insertion in Right subtree of right child of Pivot Node

- **Right to Left Rotation**

- Insertion in Left subtree of right child of Pivot Node

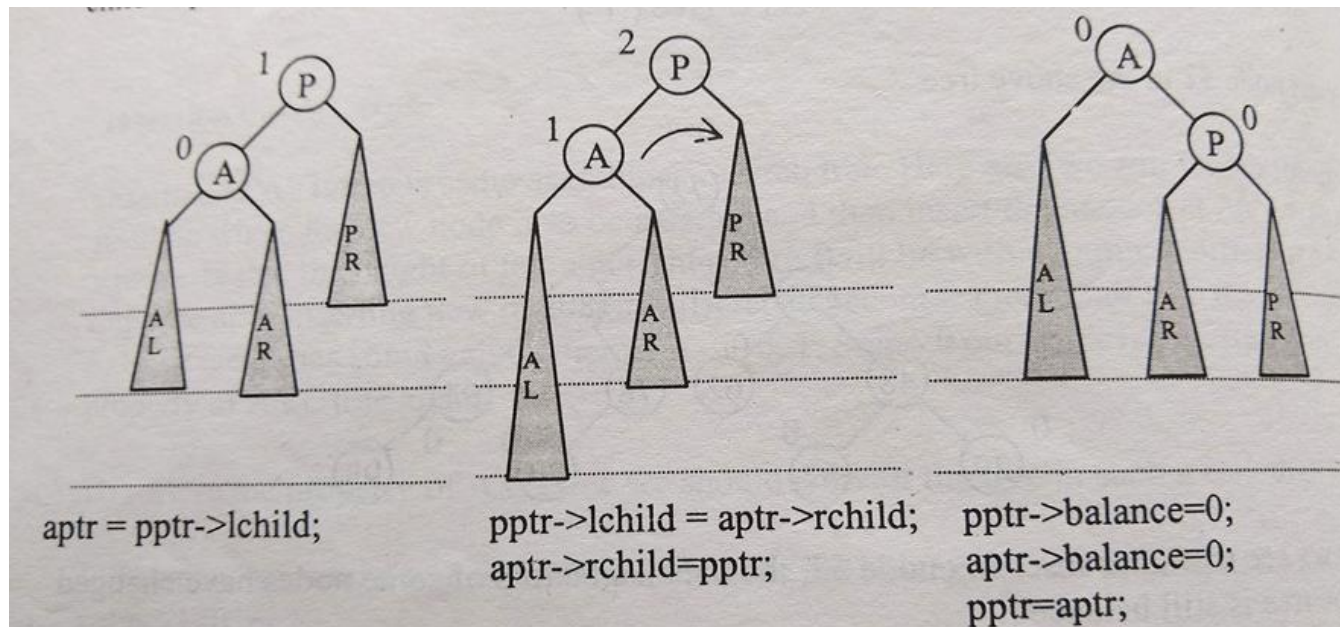
- **Left to Right Rotation**

- Insertion in right subtree of left child of Pivot Node

# AVL Rotations–

## ◉ Left to Left Rotation

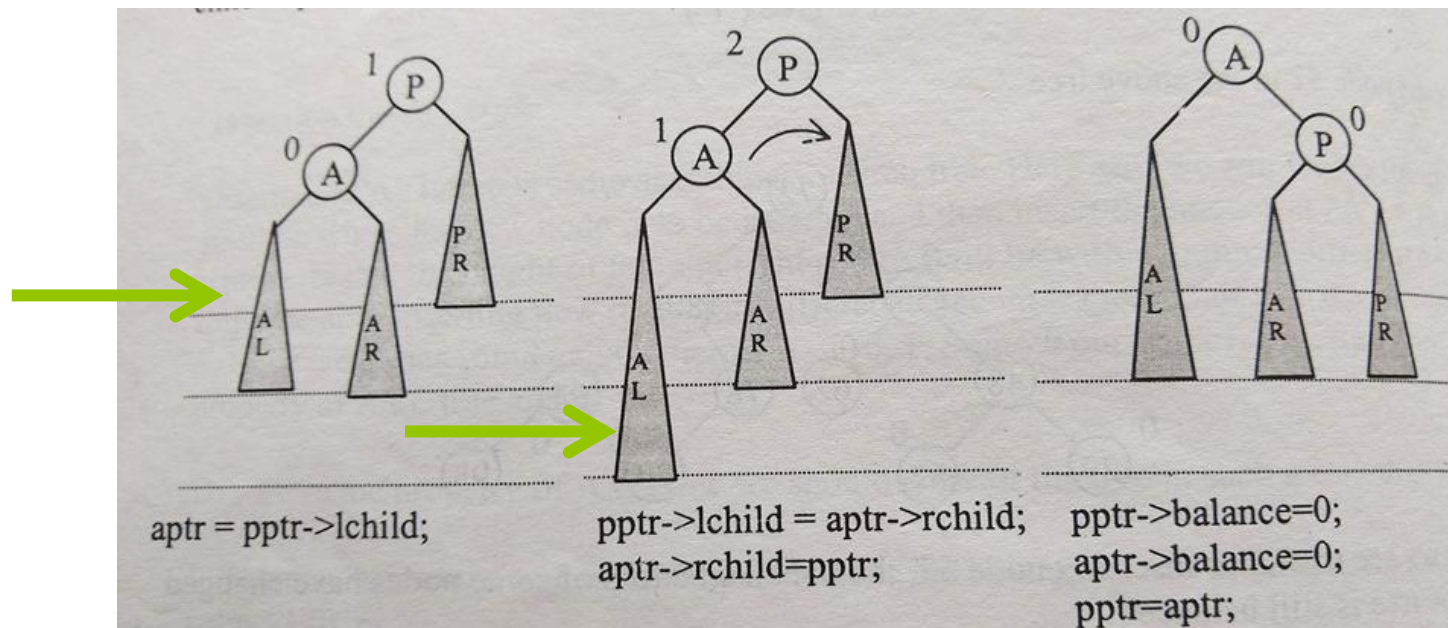
- ◉ P =Pivot node
- ◉ A =Left Child of the Pivot Node
- ◉ AL,AR=Left and Right Subtrees of Node A
- ◉ PR=Right Subtree of Node P



# AVL Rotations–

## ◦ Left to Left Rotation

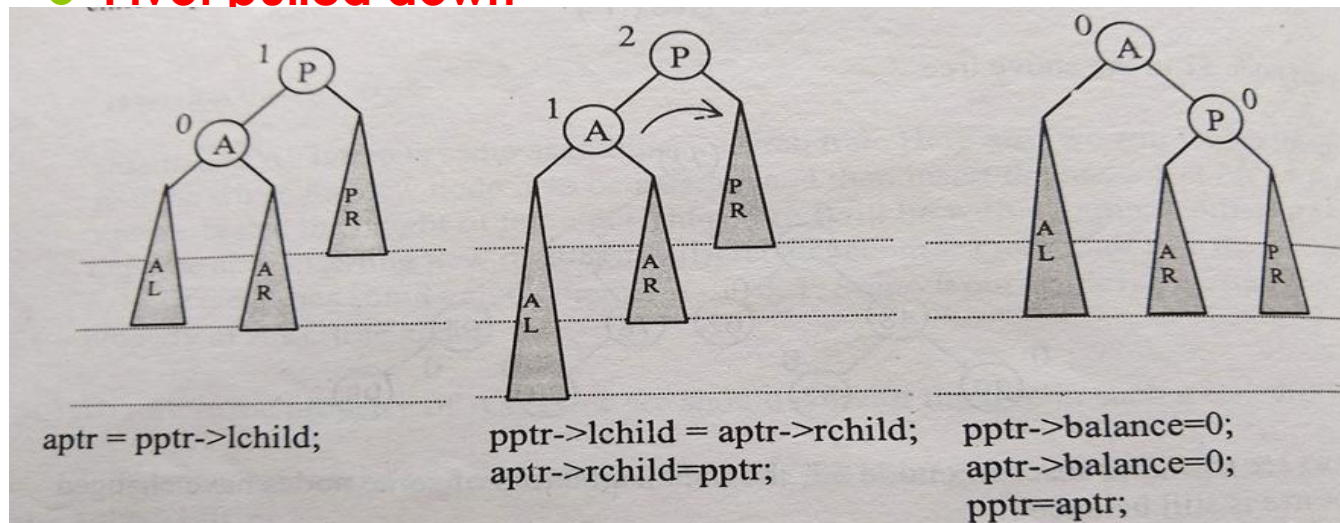
- When the Pivot node is left heavy and
- the new node is inserted in left subtree of the left child of pivot node then
- the rotation performed is left to left rotation



# AVL Rotations–

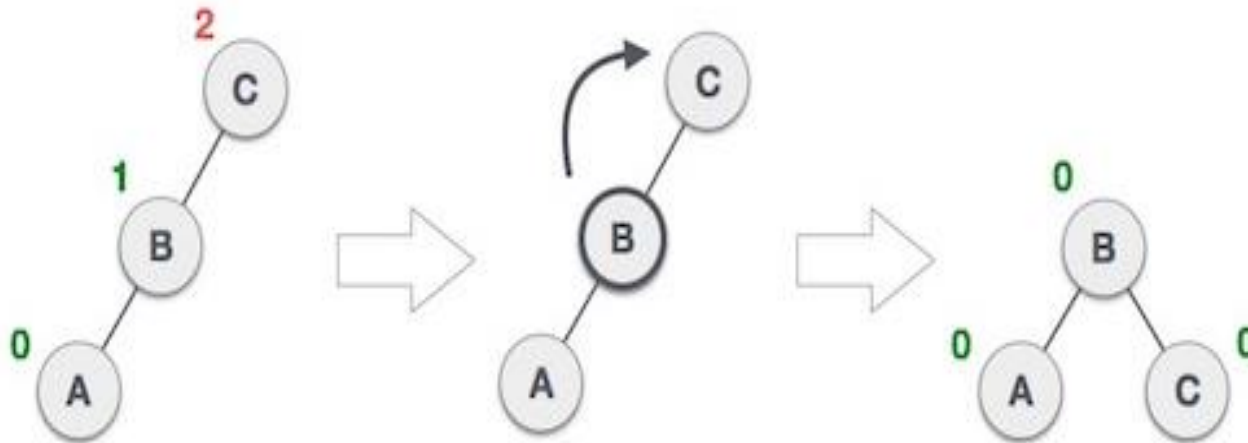
## ◦ Left to Left Rotation(Clockwise about Pivot)

- pointer pptr points to Pivot node
- Pointer aptr points to A node
- New node inserted in left Subtree of A
- **Clockwise Rotation about Pivot P is performed , Now A will be the pivot node**
- **Pivot pulled down**



# AVL Rotations—

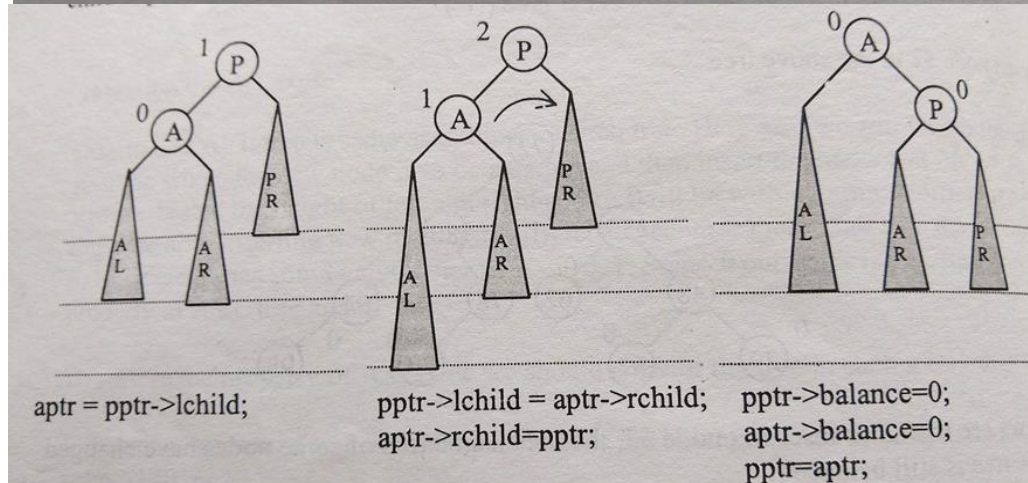
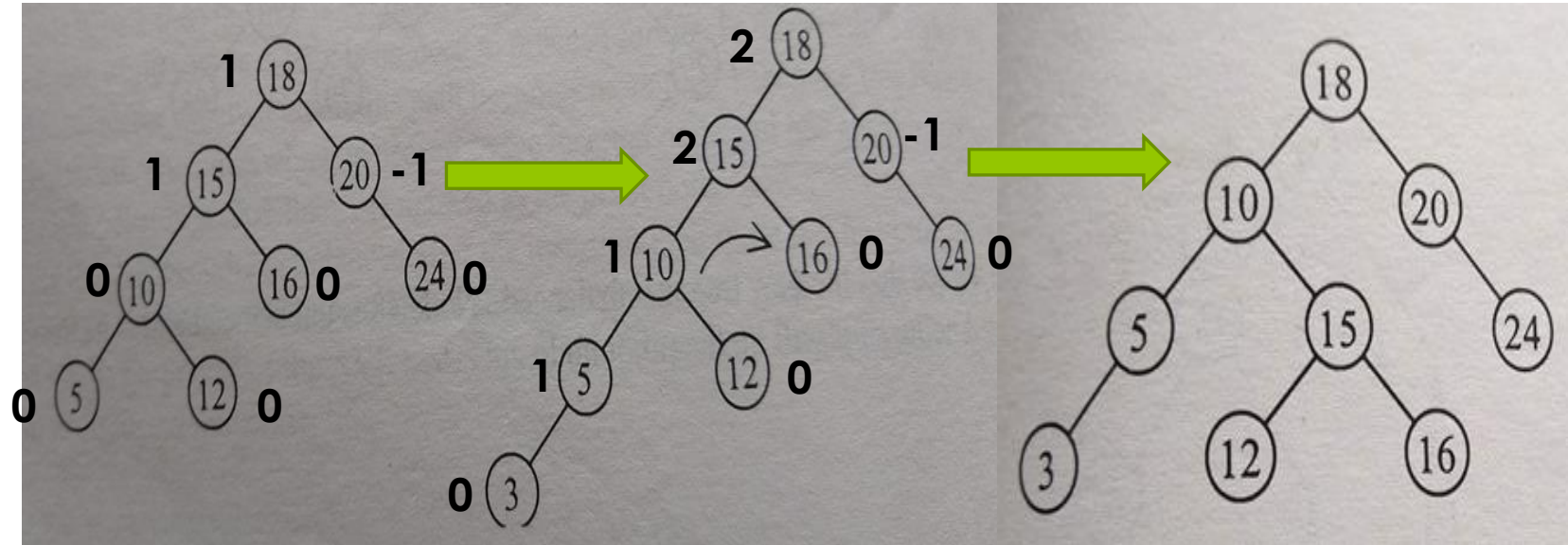
- Left to Left Rotation-Example
- Left to Left Rotation(Clockwise about Pivot)





# AVL Rotations–

## ◦ Left to Left Rotation-Example- Insert 3

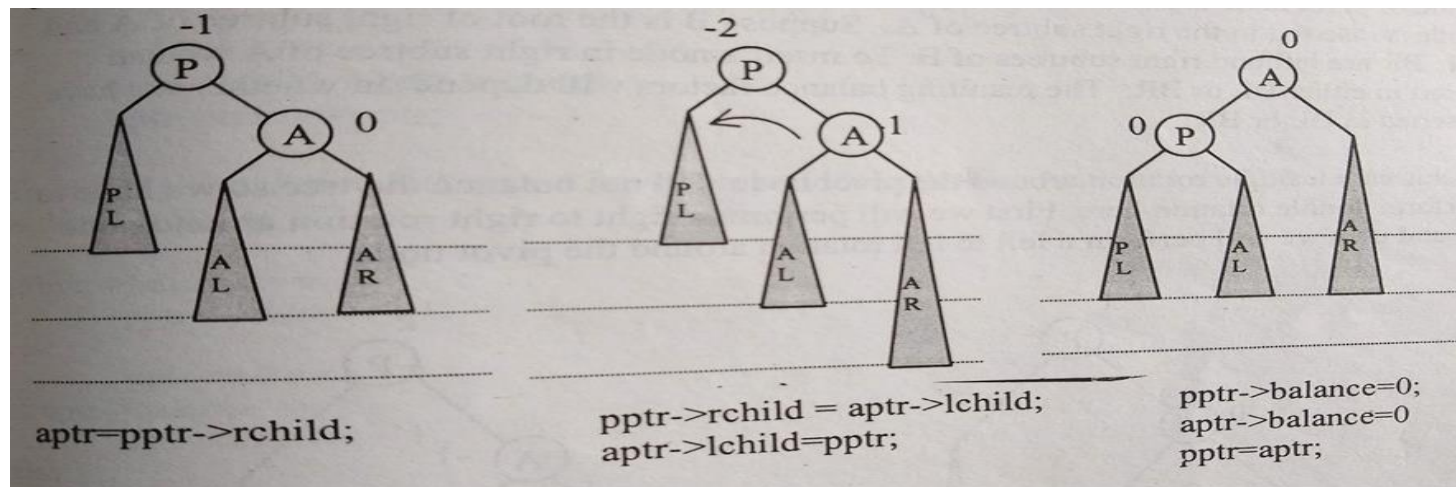


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# AVL Rotations–

## o Right to Right Rotation

- o P =Pivot node
- o A=Right Child of the Pivot Node
- o AL,AR=Left and Right Subtrees of Node A
- o PL=Left Subtree of Node P

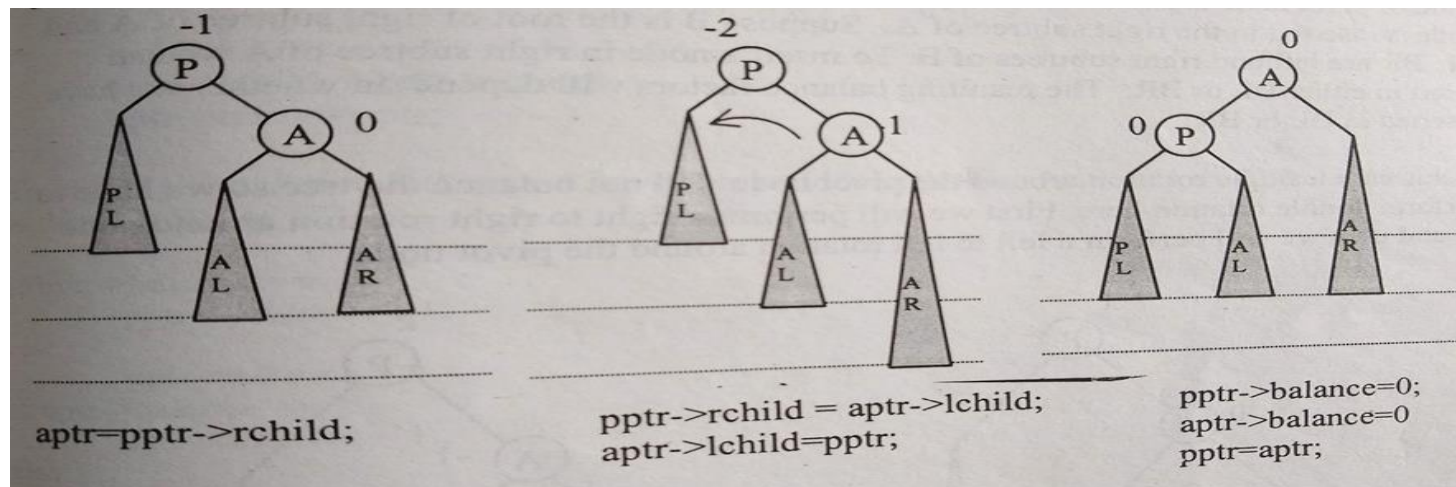




# AVL Rotations–

## o Right to Right Rotation(Anti –Clockwise about Pivot)

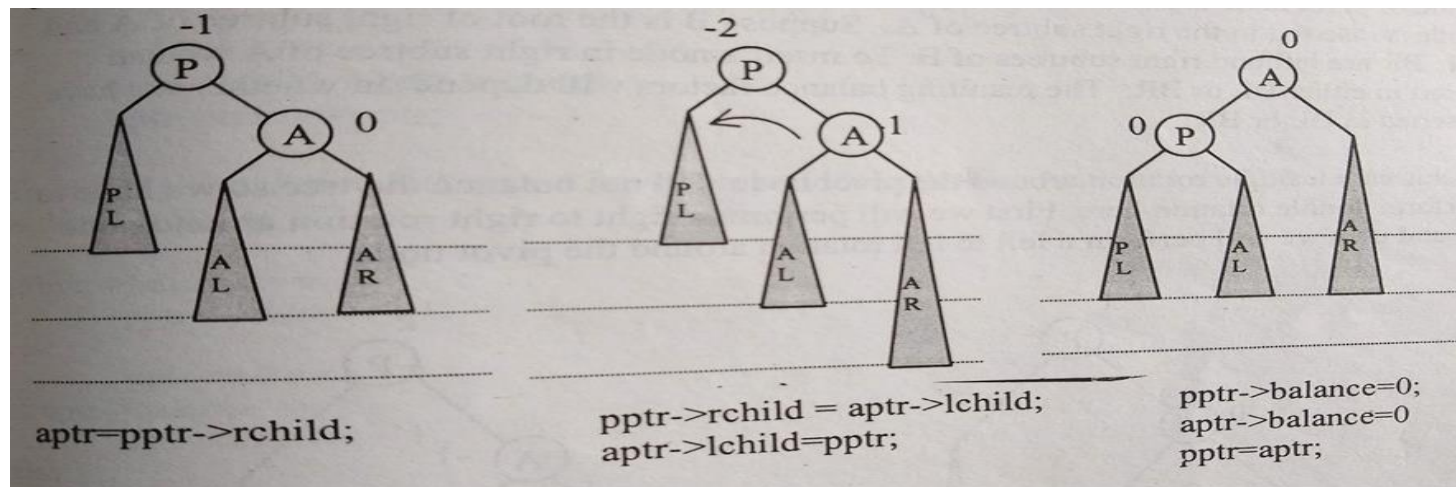
- o When the Pivot node is right heavy and
- o the new node is inserted in right subtree of the right child of pivot node then
- o the rotation performed is right to right rotation
- o Mirror image of Left to Left rotation



# AVL Rotations–

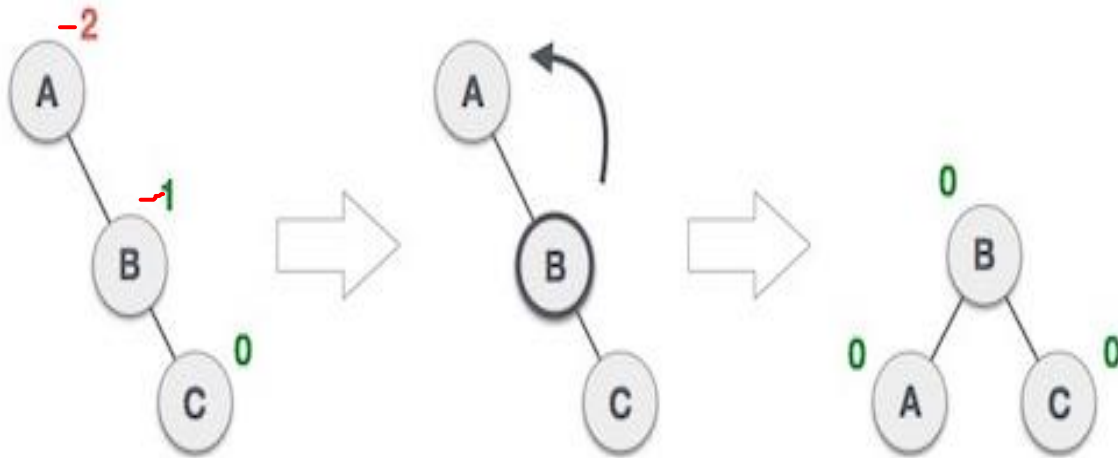
## o Right to Right Rotation

- o pointer pptr points to Pivot node
- o Pointer aptr points to A node
- o New node inserted in right Subtree of A
- o **Anticlockwise Rotation about Pivot P is performed, Now A will be the pivot node**



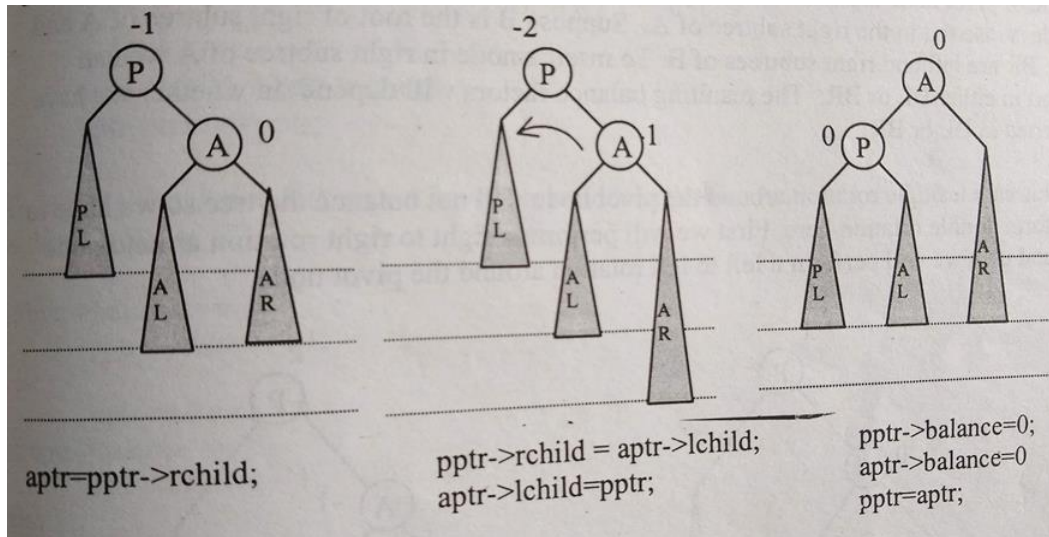
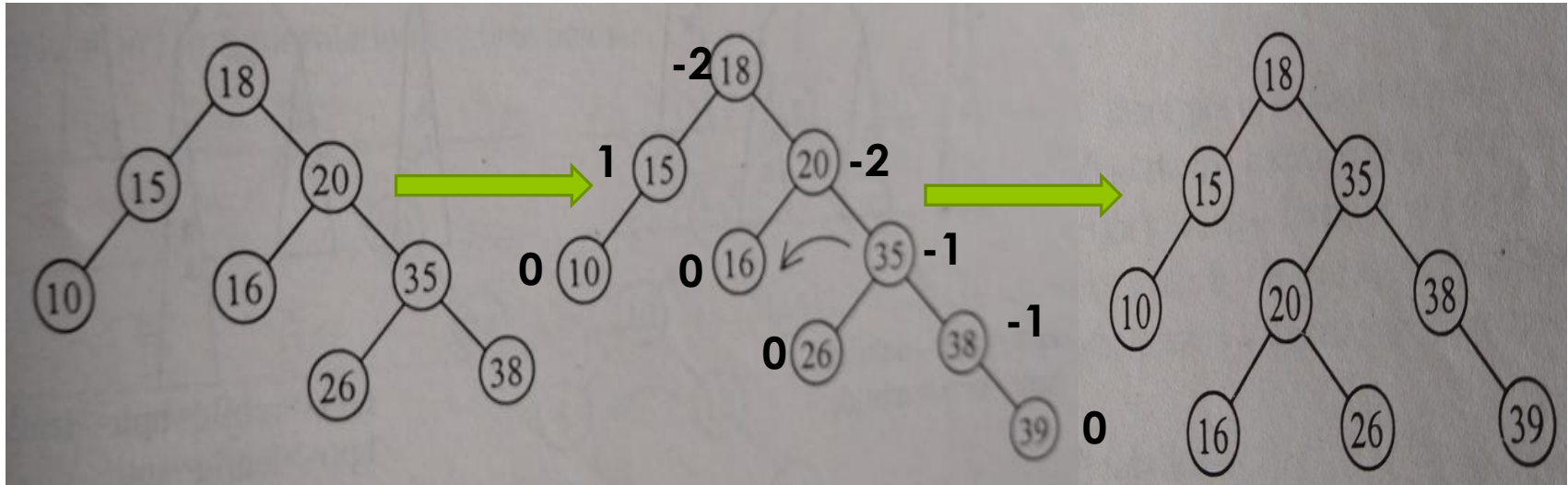
# AVL Rotations–

- Right to Right Rotation-Example
- Anti –Clockwise about Pivot



# AVL Rotations–

## Right to Right Rotation-Example-Insert 39



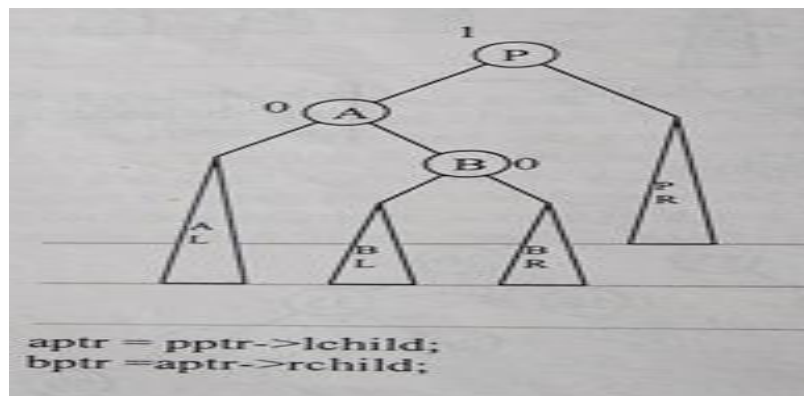
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# AVL Rotations–

## Child to Subtree Relationship(Of Pivot Node)

### o Left to Right Rotation

- o New node is inserted in the right subtree of Left child A.
- o B is the root of right subtree of A
- o BL, BR are left and right subtrees of B.
- o To insert a node in right subtree of A, **we can insert in either BL or BR.**
- o **The resulting balance factors depends on whether we have inserted in BL or BR**

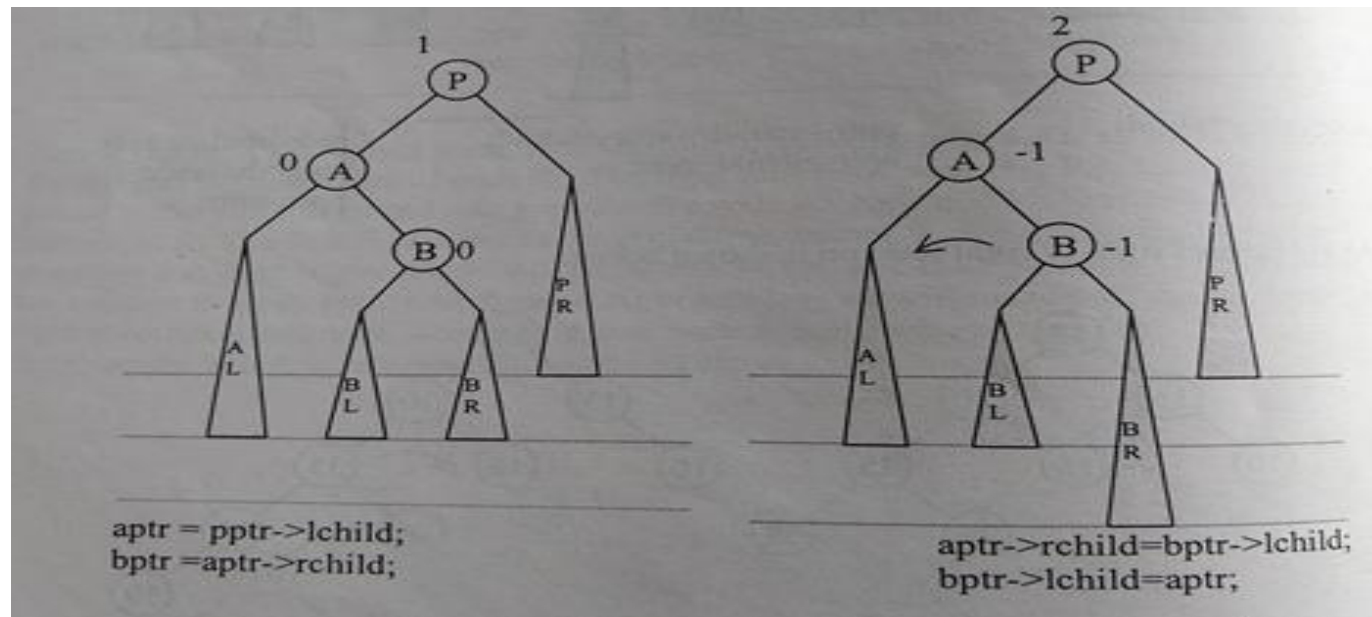


# AVL Rotations–

## Child to Subtree Relationship(Of Pivot Node)

### o Left to Right Rotation

- o When the Pivot node is left heavy and
- o the new node is inserted in right subtree of the left child of pivot node then
- o the rotation performed is left to right rotation

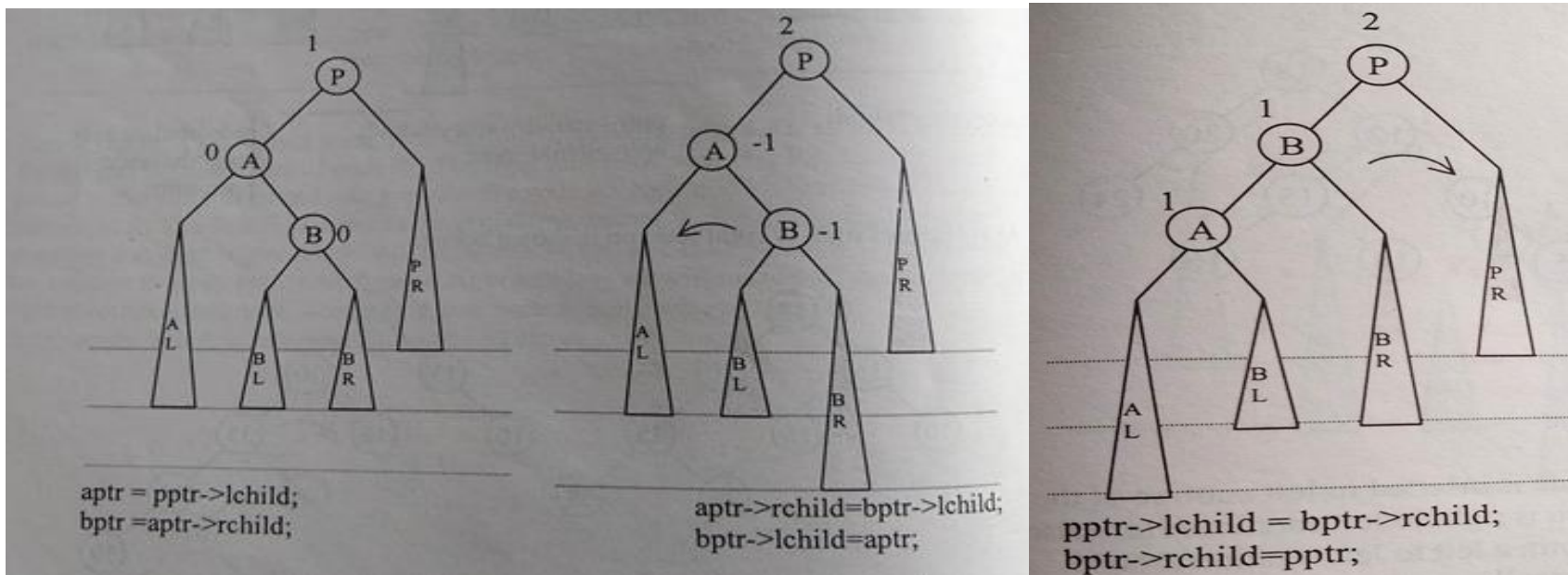




# AVL Rotations–

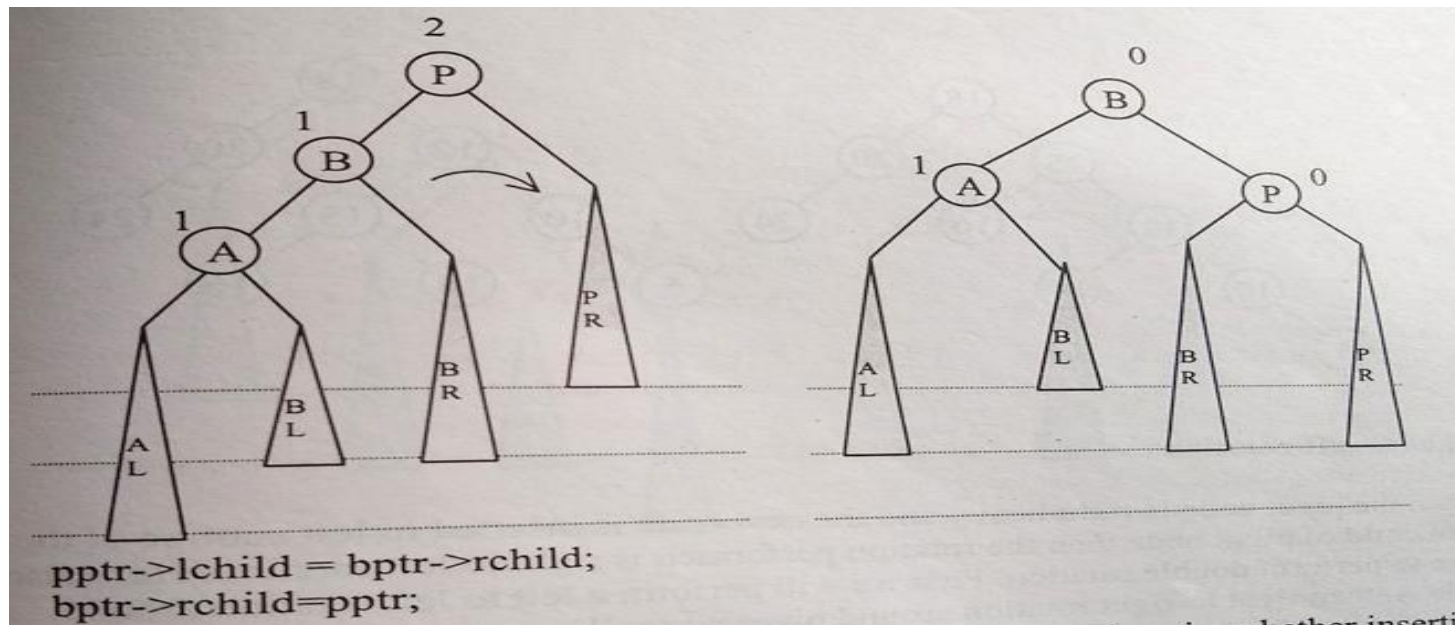
## ◉ Left to Right Rotation

- ◉ Single rotation will not balance the tree
- ◉ Double Rotation needed here
- ◉ **First Perform a Right to Right Rotation around node A**



# AVL Rotations–

- Left to Right Rotation
  - Then Perform a Left to Left rotation around the pivot node





# AVL Rotations–

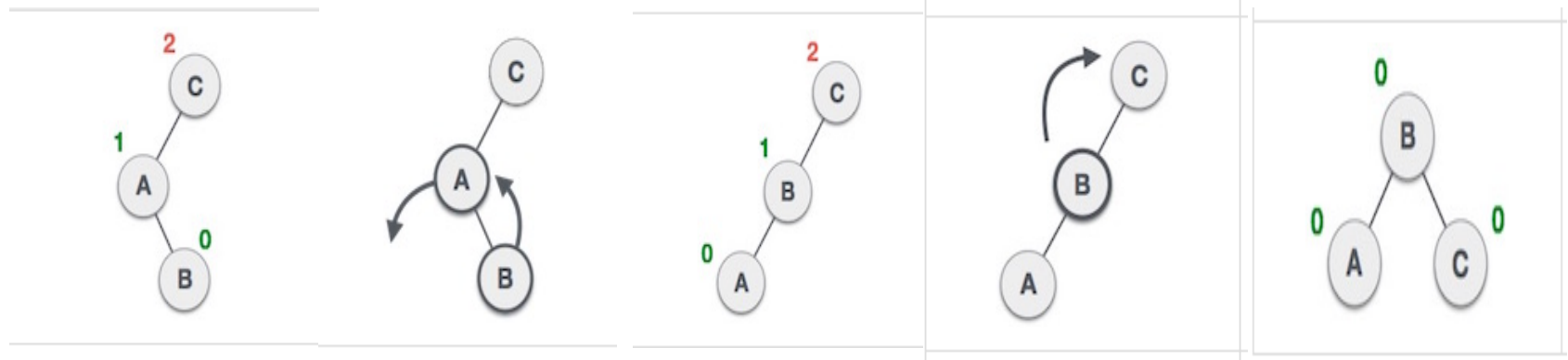
## Left to Right Rotation

1. First Perform a Right to Right Rotation around node A => Anti -Clockwise
2. Then Perform a Left to Left rotation around the pivot node => Clockwise

# AVL Rotations–

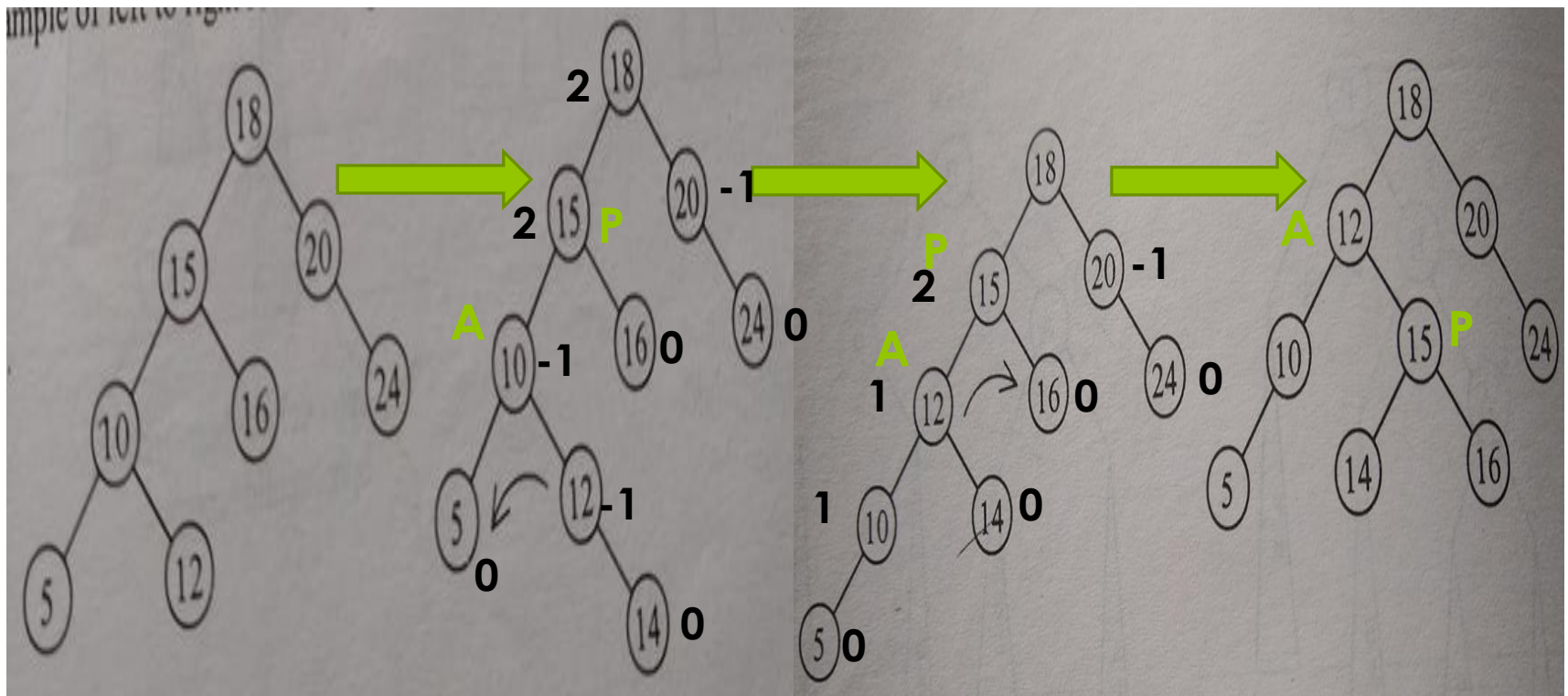
## ○ Left to Right Rotation-Example

- First Perform a Right to Right Rotation around node A=>Anti –Clockwise
- Then Perform a Left to Left rotation around the pivot node =>Clockwise



# AVL Rotations–

- **Left to Right Rotation-Example**
- First Perform a Right to Right Rotation around node A => Anti –Clockwise
- Then Perform a Left to Left rotation around the pivot node => Clockwise
- Insert 14

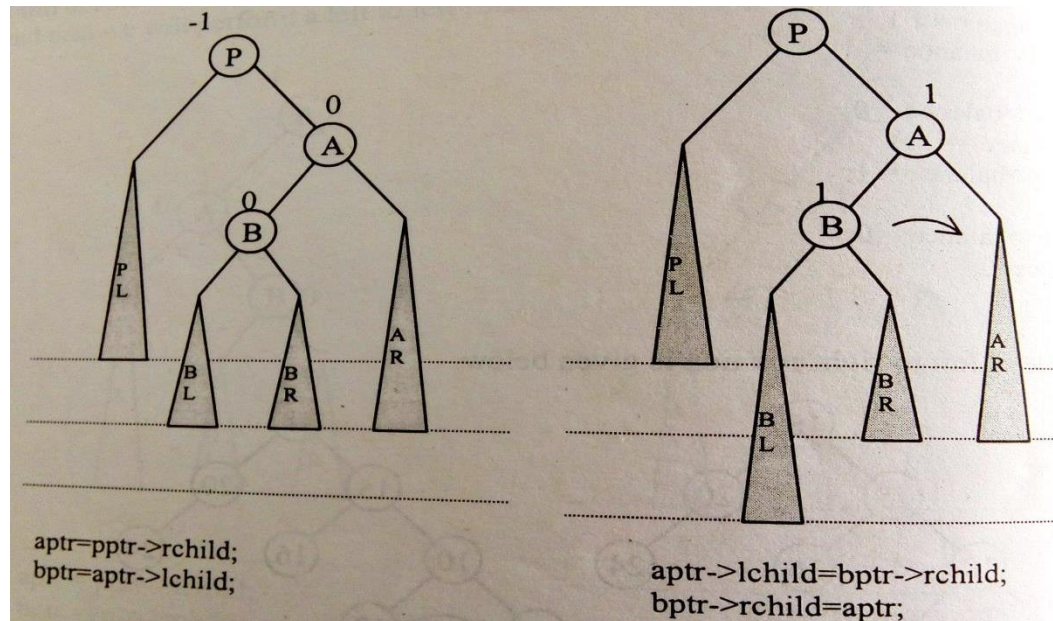


# AVL Rotations–

## Child to Subtree Relationship(Of Pivot Node)

### o Right to Left Rotation

- o When the Pivot node is right heavy and
- o the new node is inserted in left subtree of the right child of pivot node then
- o the rotation performed is right to left rotation



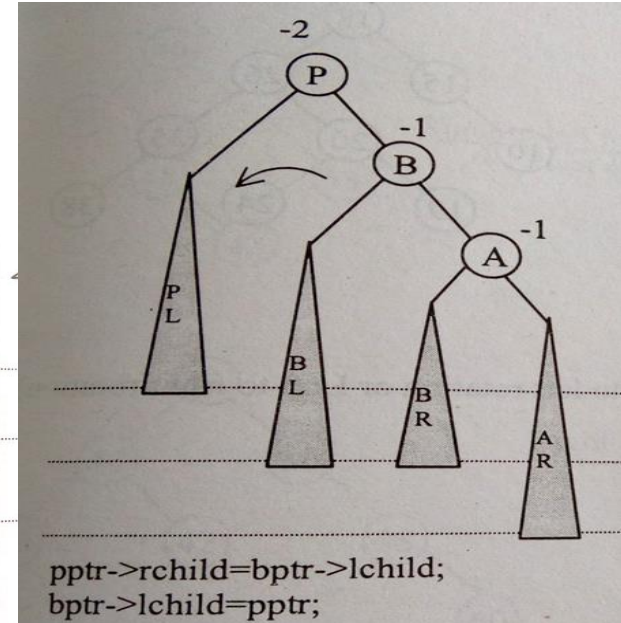
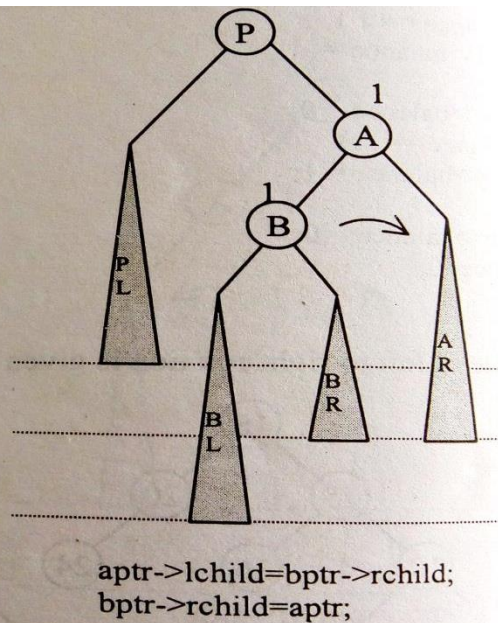
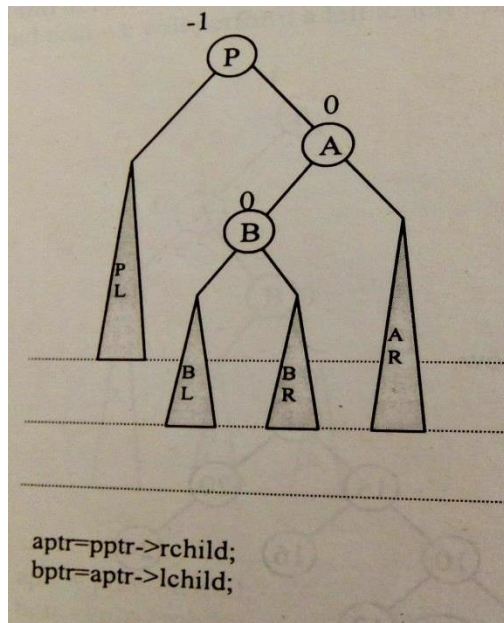
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# AVL Rotations–

## Child to Subtree Relationship(Of Pivot Node)

### o Right to Left Rotation

- o Double Rotation needed
- o First Perform Left to Left Rotation around node A

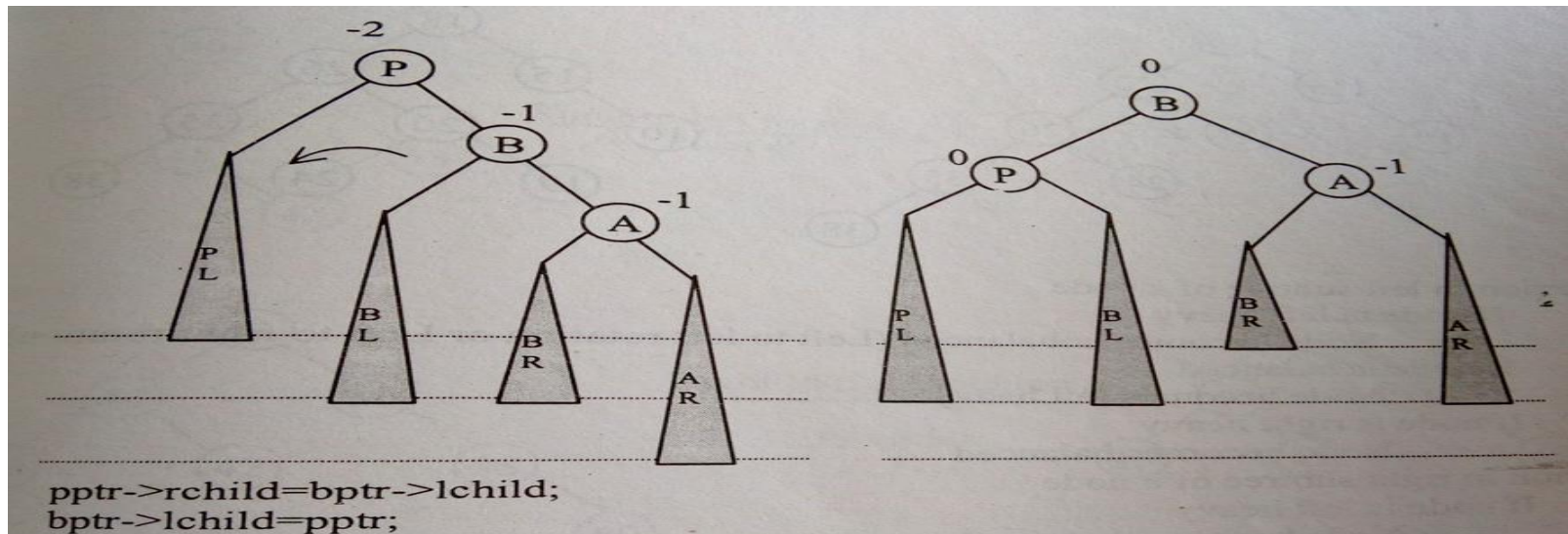


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# AVL Rotations–

- Right to Left Rotation
  - Then Right to Right rotation around pivot node P
  - Mirror image of Left to Right Rotation



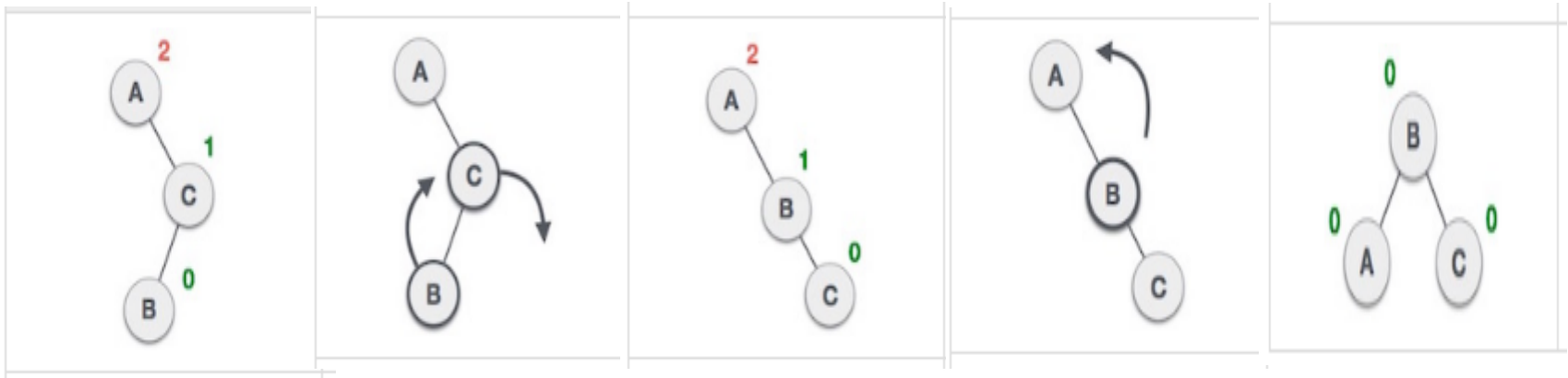
# AVL Rotations–

- **Right to Left Rotation**
  - First Perform Left to Left Rotation around node A=>Clockwise
  - Then Right to Right rotation around pivot node P=>Anti-Clockwise

# AVL Rotations–

## ○ Right to Left Rotation-Example

- First Perform Left to Left Rotation around node A=>Clockwise
- Then Right to Right rotation around pivot node P=>Anti-Clockwise

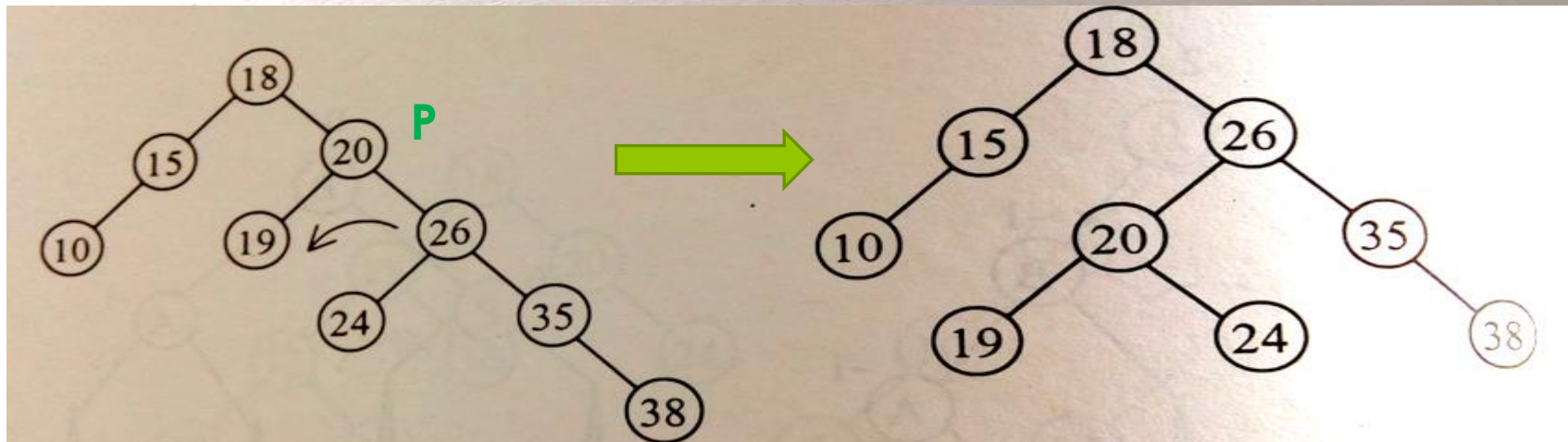
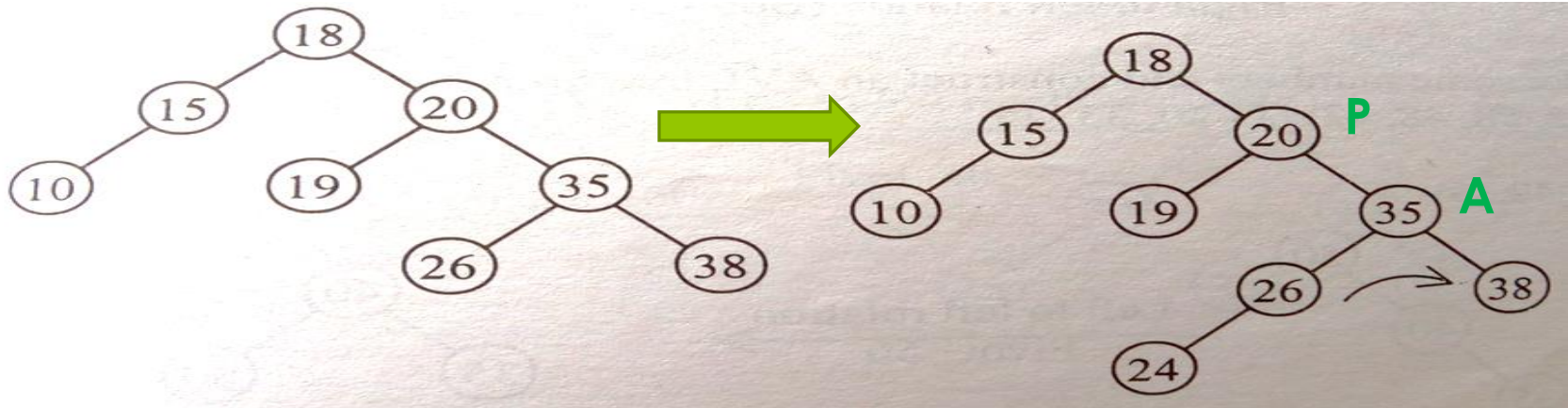




# AVL Rotations–

## o Right to Left Rotation

### o Example-Insert 24



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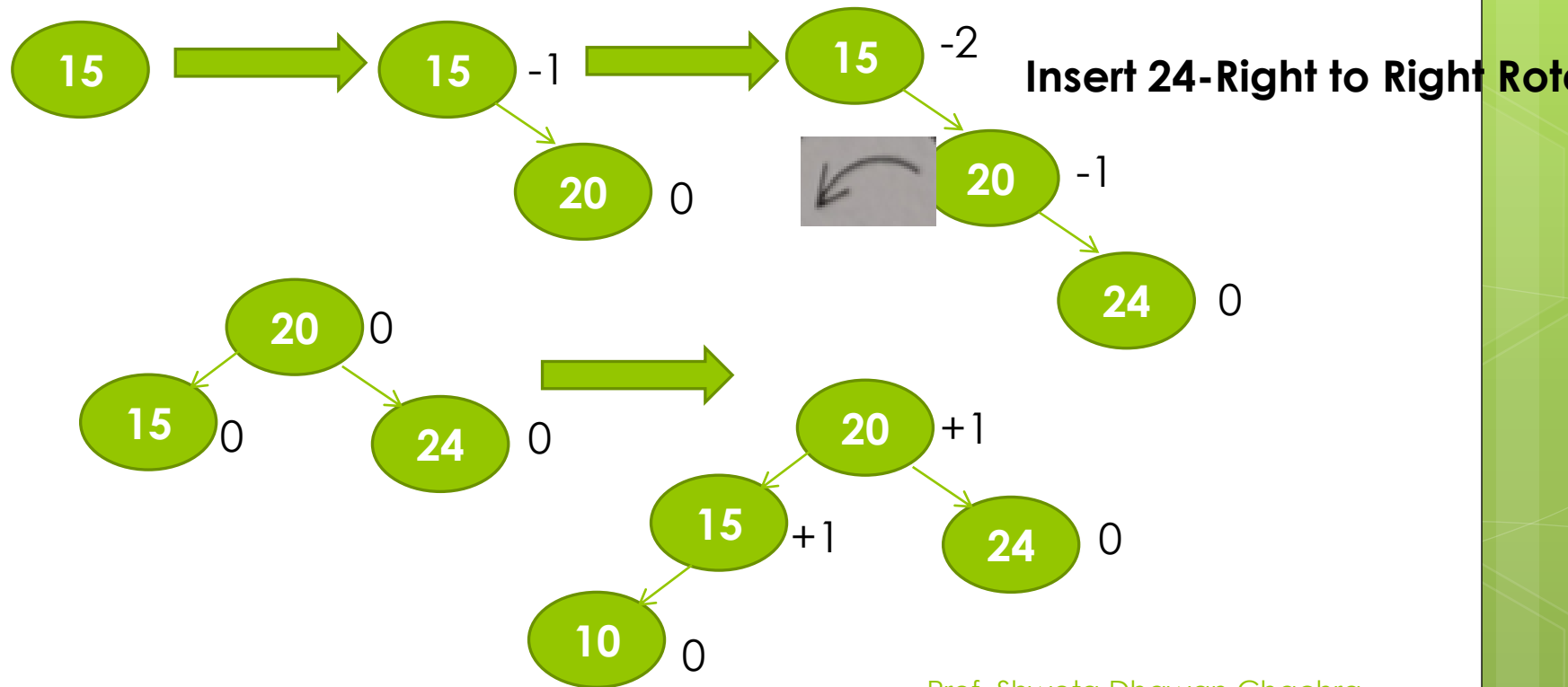
First Perform Left to Left Rotation around node A=>Clockwise  
Then Right to Right rotation around pivot node P=>Anti-Clockwise

**Construct an AVL tree by inserting the following numbers in the given sequence in an initially empty AVL tree.**

**15, 20, 24, 10, 13, 7, 30, 36**

Construct an AVL tree by inserting the following numbers in the given sequence in an initially empty AVL tree.

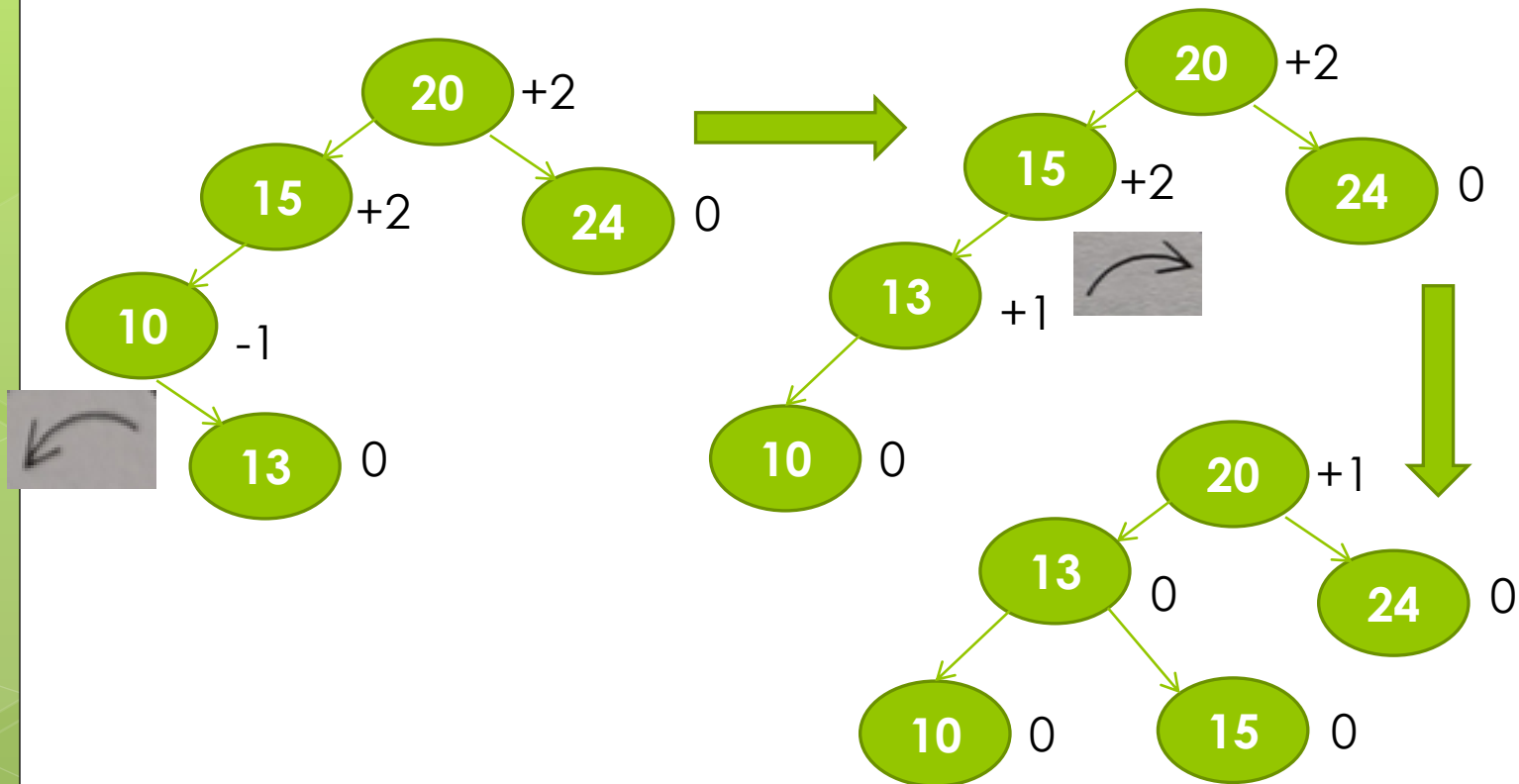
15, 20, 24, 10, 13, 7, 30, 36



Construct an AVL tree by inserting the following numbers in the given sequence in an initially empty AVL tree.

15, 20, 24, 10, 13, 7, 30, 36

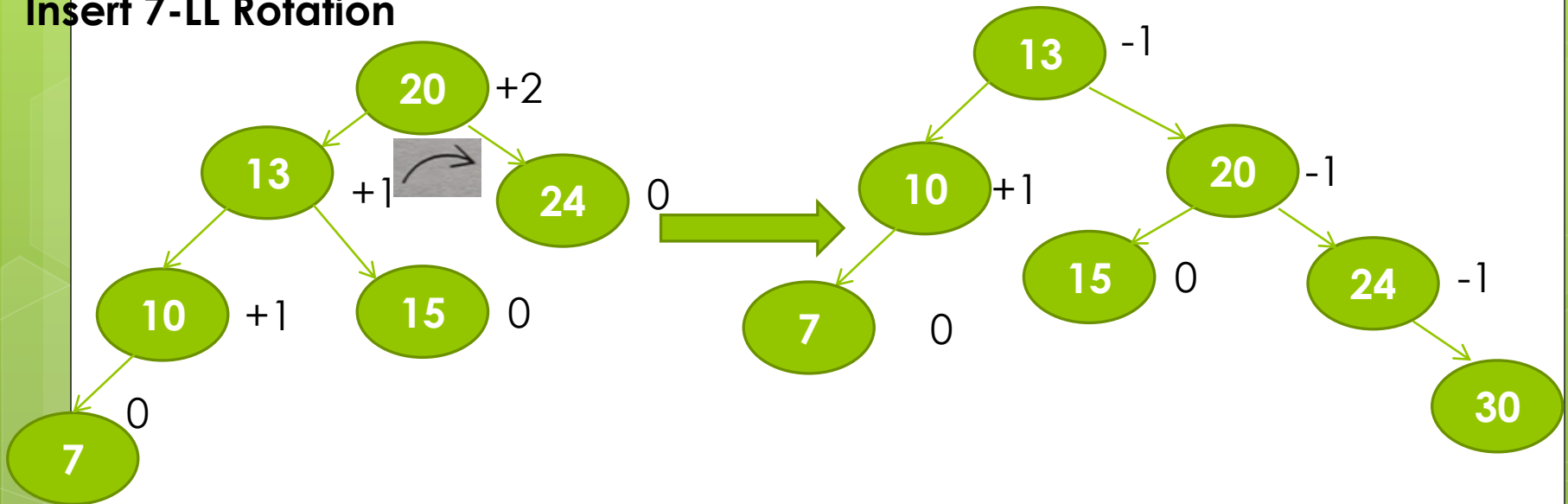
Insert 13-Left to Right Rotation



Construct an AVL tree by inserting the following numbers in the given sequence in an initially empty AVL tree.

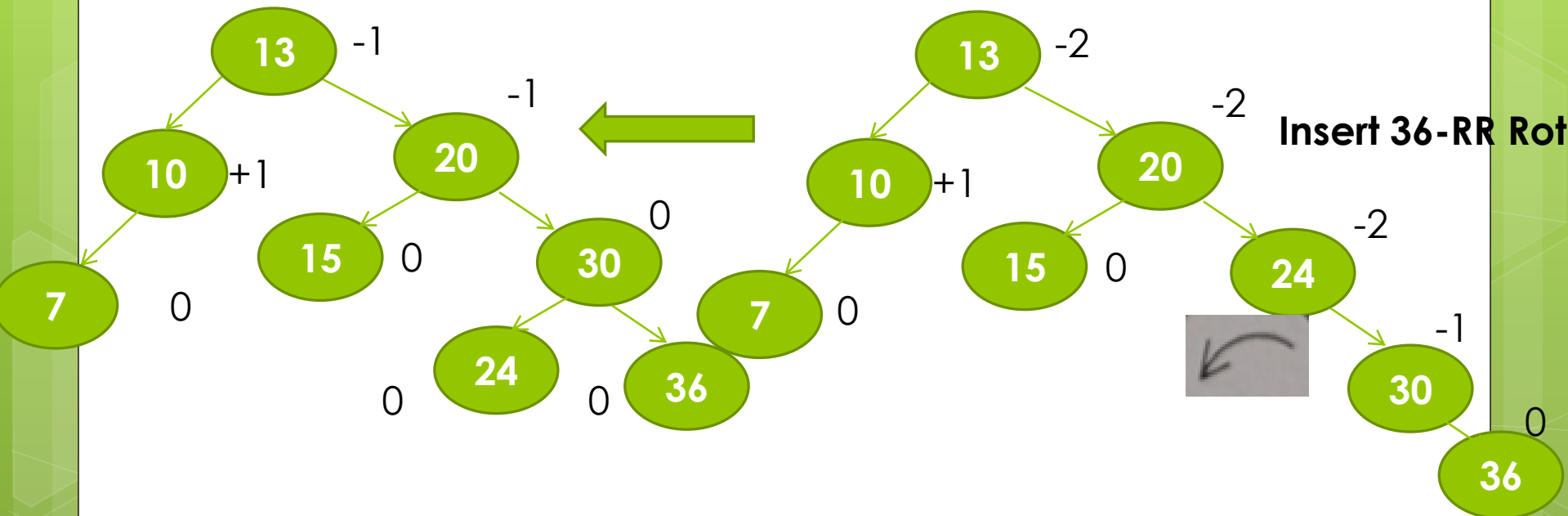
15, 20, 24, 10, 13, 7, 30, 36

### Insert 7-LL Rotation



Construct an AVL tree by inserting the following numbers in the given sequence in an initially empty AVL tree.

15, 20, 24, 10, 13, 7, 30, 36



**Final AVL Tree**

**Construct an AVL Tree for the following sequence**

**9,8,7,15,16,17,18**

**Further mention the type of Rotation and the balance factors in each intermediated step**

Construct an AVL Tree for the following sequence

9,8,7,15,16,17,18

Further mention the type of Rotation and the balance factors in each intermediated step

