Hyder Pressuala
SY Comps B2-2

Batch: B2 Roll No.: 16010122151
Experiment / assignment / tutorial No. 9
Grade: AA / AB / BB / BC / CC / CD / DD
Signature of the Faculty In-Charge with date

Q1) Show that the velocity given by $\bar{V} = (y+z)i + (z+x)j + (x+y)k$ is irrotational & to find its scalar potential. Find work done in moving particle from A(0,0,0) to B(1,2,3)

A) $$\nabla \times \bar{V} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y+z & z+x & x+y \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(x+y) - \frac{\partial}{\partial z}(z+x)\right] - j\left[\frac{\partial}{\partial x}(x+y) - \frac{\partial}{\partial z}(y+z)\right] + k\left[\frac{\partial}{\partial x}(z+x) - \frac{\partial}{\partial y}(y+z)\right]$$

$$= i(1-1) - j(1-1) + k(1-1)$$
$$= 0$$

Since, curl $\bar{v}$ is 0, the velocity given by $\bar{v}$ is irrotational.

To find scalar potential,

$$\bar{V} = \nabla\phi$$

$$(y+z)i + (z+x)j + (x+y)k = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$$

$$\Rightarrow \frac{\partial\phi}{\partial x} = y+z \Rightarrow \phi = xy + xz$$

$$\frac{\partial \phi}{\partial y} = z + x \Rightarrow \phi = yz + xy$$

$$\frac{\partial \phi}{\partial z} = x + y \Rightarrow \phi \; xz + yz$$

$$\therefore \phi = xy + yz + xz + c$$

work done $= \int_c \bar{v} \cdot \bar{dr}$

$$= \int_c d\phi$$

$$= [\phi]_{(0,0,0)}^{(1,2,3)}$$

$$= [xy + yz, + zx + c]_{(0,0,0)}^{(1,2,3)}$$

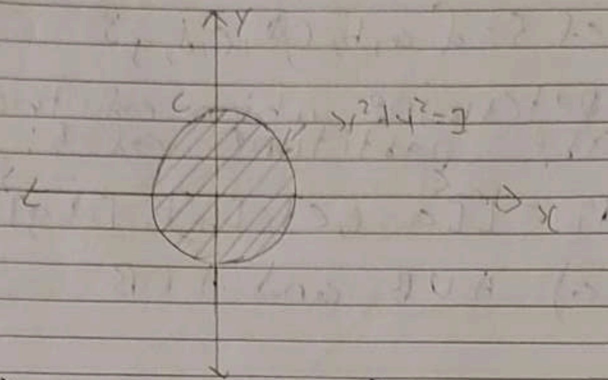
$$= [2 + 6 + 3 + c - c]$$

$$= 11$$

Q2) Compute $\int_c \bar{F} \cdot \bar{dr}$ where $\bar{F} = \frac{yi - xj}{x^2 + y^2}$

and c is the circle $x^2 + y^2 = 1$ traversed counter clockwise.

A) $\int_c \bar{F} \cdot \bar{dr} = \int_c \frac{y}{y^2 + x^2} dx - \frac{x}{x^2 + y^2} dy$



$x^2+y^2=1$ $\theta : 0$ to 2π

$\therefore r = 1$

Put $x = r\cos\theta = \cos\theta$
$y = r\sin\theta = \sin\theta$

$dx = -\sin\theta \, d\theta$
$dy = \cos\theta \, d\theta$

$$\therefore \int_C \bar{F}_0 \, d\bar{r} = \int_C \frac{\sin\theta}{\cos^2\theta + \sin^2\theta} (-\sin\theta) \, d\theta$$

$$= \int_C \cancel{-\sin^2\theta \, d\theta} \; \frac{\cos\theta \quad \cos\theta \, d\theta}{\cos^2\theta + \sin^2\theta}$$

$$= \int_0^{2\pi} -\sin^2\theta \, d\theta - \cos^2\theta \, d\theta$$

$$= -\int_0^{2\pi} (\sin^2\theta + \cos^2\theta) \, d\theta$$

$$= -\int_0^{2\pi} d\theta$$

$$= -[\theta]_0^{2\pi}$$

$$= -2\pi.$$

Q3) Green Theorem For
$F = (x^2 - xy)i + (x^2 - y^2)j$, & C is triangle having vertices at $(0,0)$, $(1,1)$, $(1,-1)$

A) $$\int_C F \cdot dr = \int_C (x^2 - xy)dx + (x^2 - y^2)dy$$

$$= \int_C Pdx + Qdy$$

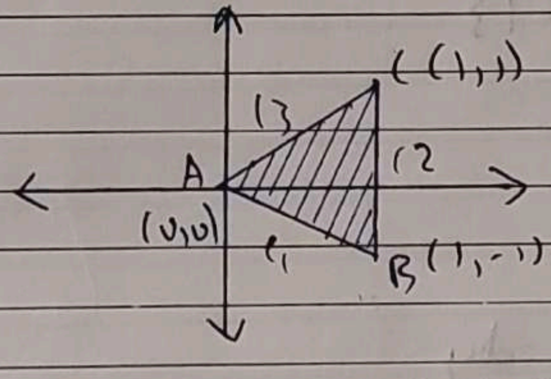
$P = x^2 - xy$ $\qquad$ $Q = x^2 - y^2$

$\frac{\partial P}{\partial y} = -x$ $\qquad$ $\frac{\partial Q}{\partial x} = 2x$

By Greens Theorum

$$\int_C Pdx + Qdy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

$$= \iint_R 3x \, dx\,dy.$$



Equation of AB = $x+y=0$
Equation of CA = $x=y$

Taking vertical strip

y : AB to CA
$-x$ to x

x : 0 to 1.

$$\int_{x=0}^{1}\int_{y=-x}^{x} 3x\,dx\,dy = 3\int_0^1 x\,(y)_{-x}^{x}\,dx$$

$$= 3\int_0^1 x\,(x+x)\,dx$$

$$= 3\int_0^1 2x^2\,dx$$

$$= \frac{6}{3}\left[x^3\right]_0^1$$

$$= 2(1-0) = 2-(1)$$

b) $C = C_1 + C_2 + C_3$

Along C_1

$x+y=0$
$x=-y$
$dx = -dy$

$y \to 0$ to -1

$$\int_C (x^2 - xy)\,dx + (x^2 - y^2)\,dy$$

$$= \int_0^{-1} (y^2 + y^2)(-dy) + (y^2 - y^2)\,dy$$

$$= -\int_0^{-1} \cancel{y}\ 2y^2\,dy$$

$$= \frac{-2}{3}\left[y^3\right]_0^{-1}$$

$$= \frac{-2}{3}[-1] = \frac{2}{3}$$

Along C_2

$x = 1$

$dx = 0 \qquad y \to -1$ to 1

$$\int_C (x^2 - xy)\,dx + (y^2 - y^2)\,dy$$

$$= \int_{-1}^{1} (1-y)\,0 + (1-y^2)\,dy$$

$$= \int_{-1}^{1} (1-y^2)\,dy$$

$$= \left[y - \frac{y^3}{3} \right]_{-1}^{1} = \left[1 - \frac{1}{3} - \left(-1 + \frac{1}{3}\right) \right]$$

$$= 2 - \frac{2}{3} = \frac{4}{3}.$$

~~Along~~ Along C_3

$x = y$

$dx = dy$

$y \to 1$ to 0

$$\int_{C_3} (x^2 - xy)\,dx + (x^2 - y^2)\,dy$$

$$\int_{1}^{0} (y^2 - y^2)\,dy + (y^2 - y^2)\,dy = 0$$

$$C_1 + C_2 + C_3 = \frac{2}{3} + \frac{4}{3} = 2 \quad -(2)$$

$(1) = (2)$

Hence, Green theorem verified

Q4) Evaluate $\int_C \bar{F}\, d\bar{r}$ by Stoke's Theorem for $\bar{F} = x^2 i + xy j$ and C is the boundary of rectangle $x=0, y=0, \cancel{x} y=b$

$x=a, y=b$

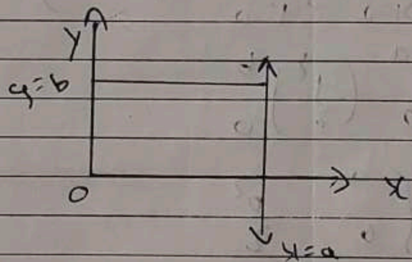
A) $\nabla \times \bar{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & xy & 0 \end{vmatrix}$

$= i\left[\frac{\partial(0)}{\partial y} - \frac{\partial}{\partial z}(xy)\right] - j\left(\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial z}(x^2)\right)$

$+ k\left[\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial y}(x^2)\right]$

$= i(0) - j(0) + k(y)$

$= y\hat{k}$



Taking vertical strip

$y: 0$ to b

$x: 0$ to a

$\hat{N} = \hat{k}$

$$\hat{N} \cdot (\nabla \times \bar{F}) = y$$
$$ds = dx\,dy$$

By stokes theorum

$$\therefore \int_C \bar{F} \cdot \bar{dr} = \iint_S \hat{N} \cdot (\nabla \times \bar{F})\, ds$$

$$= \iint_S y\, dx\, dy$$

$$= \int_{x=0}^{a} \int_{y=0}^{b} y\, dx\, dy$$

$$= \int_0^a \left(\frac{y^2}{2}\right)_0^b$$

$$= \int_0^a \frac{b^2}{2}$$

$$= \frac{b^2}{2} (x)_0^a$$

$$= \frac{ab^2}{2}$$