SOMAIYA VIDYAVIHAR UNIVERSITY

Semester: August 2021 – December 2021 Examination: ESE Examination		
Programme code: 01 **Programme: B.TECH**	**Class:** SY	**Semester: III (SVU 2020)**
Name of the Constituent College: **K. J. Somaiya College of Engineering**	**Name of the Department:** **COMP**	
Course Code: 116U01C301	**Name of the Course: Integral Transform and Vector Calculus**	
Duration : 1 Hour 45 Minutes(15 minutes extra for uploading)	**Maximum Marks : 50**	
Instructions: **1)Draw neat diagrams 2) Assume suitable data if necessary**		

Question No.		**Max Marks**
Q1 (A)	**Choose One correct option for the following questions (2 marks each)**	**10**
(i)	$\int_0^{\infty} \frac{\sin 3t}{t} dt$ is (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$	
(ii)	If $f(x) = sinh(px)$, p is not an integer then in the Fourier expansion of f(x) in interval (- π, π) value of a_0 is (a) $\frac{1}{p\pi}[e^{p\pi} - e^{-p\pi}]$ (b) $\frac{\pi}{4}$ (c) 0 (d) $\frac{1}{2p\pi}[e^{p\pi} + e^{-p\pi}]$	
(iii)	If $\bar{A} = \nabla(xy + yz + zx)$ then $\nabla \cdot \bar{A}$ is (a) 6 (b) 0 (c) $2xi + 2yj + 2zk$ (d) $(y+z)i + (x+z)j + (x+y)k$	
(iv)	convolution of sequences f(k) and g(k) is defined as (a) $\sum_{n=-\infty}^{\infty} f(n)g(k-n)$ (b) $\sum_{k=-\infty}^{\infty} f(n)g(k-n)$ (c) $\sum_{n=-\infty}^{\infty} f(k)g(k-n)$ (d) $\sum_{k=-\infty}^{\infty} f(k)g(k-n)$	
(v)	Stoke's theorem states that (a) $\iint_s \nabla \times (\bar{N} \cdot \bar{F})\, ds = \int_c \bar{F} \cdot d\bar{r}$ (b) $\iint_s \bar{N} \cdot (\nabla \times \bar{F})\, ds = \int_c \bar{F} \cdot d\bar{r}$ (c) $\iint_s \bar{N} \cdot (\nabla \times \bar{F})\, ds = \int_c \bar{F} \times d\bar{r}$ (d) $\iint_s \bar{N} \cdot (\nabla \cdot \bar{F})\, ds = \int_c \bar{F} \times d\bar{r}$	

Q1 (B)	**Attempt all the following questions. (2 marks Each)**	**10**
(a)	Find $L\left\{\int_0^t e^u \cosh u \ du\right\}$ **OR** Find $L[\{1+2t-t^2+t^3\}\{H(t-1)\}]$	
(b)	In Complex form of Fourier Series for $f(x) = e^{ax} \ in \ (-\pi, \pi)$, C_1 is	
(c)	Let $\emptyset = \mathrm{xy} + \mathrm{yz} + \mathrm{zx}$ Find directional Derivatives of $\emptyset$ at $\mathrm{P}(0,1,-1)$ in the direction of $3\mathrm{i} + 4\mathrm{j} + 5\mathrm{k}$. **OR** If $\emptyset = \mathrm{x}^3 + \mathrm{y}^3 + \mathrm{z}^3 - 3\mathrm{xyz}$ Find div(curl(grad$\emptyset$)).	
(d)	If a force $\vec{F} = 2x^2y\hat{\imath} + 3xy\hat{\jmath}$ displaces a particle in the xyplane from (0,0) to (1,4) along a curve $y = 4x^2$. Find the work done.	
(e)	If $\mathrm{F(z)} = \mathrm{Z\{f(k)\}} = \frac{1}{\mathrm{z}-2}$, $\lvert\mathrm{z}\rvert < 2$ then the sequence $\mathrm{f(k)}$ is	
Q. 2	**Attempt the following (6 marks Each)**	**12**
(a)	Obtain half range sine series for $f(x) = \frac{1}{4} - x, \ 0 < \mathrm{x} < 1/2$	
	OR	
	Express the function $f(x) = \begin{cases} 1, & \lvert x\rvert < 1 \\ 0, & \lvert x\rvert > 1 \end{cases}$ as a Fourier Integral	
(b)	Find Inv Laplace Transform of $\left(\frac{1}{(s-2)^4(s+3)}\right)$	
Q. 3	**Attempt any ONE question out of the following (6 marks Each)**	**6**
(a)	Find z-transform of $k^2 4^k, \ k \geq 0$	
(b)	If $\mathrm{F(z)} = \frac{1}{(\mathrm{z}-3)(\mathrm{z}-6)}$, $\lvert\mathrm{z}\rvert > 6$ then find inverse z-transform of $\mathrm{F(z)}$	
Q. 4	**Attempt any TWO questions out of the following (6 marks Each)**	**12**
(a)	Prove that $\nabla\left[\mathrm{a}.\nabla\frac{1}{r}\right] = -\frac{\bar{a}}{r^3} + \frac{3(\bar{a}.\bar{r})\bar{r}}{r^5}$	
(b)	Given $\bar{F} = (z^2+2x+3y)i + (3x+2y+z)j + (y+2zx)k$, **(i)** Prove that $\bar{F}$ is irrotational **(ii)** Find Scalar potential function $\emptyset$ such that $\bar{F} = \nabla\emptyset$ and $\emptyset(1,1,0) = 4$ **(iii)** Find the work done by $\bar{F}$ in moving a particle from $A(0,1,1)$ to $B(3,0,2)$	

(c)	Apply Greens Theorem to Evaluate $\int_c [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where c is boundary of the area enclosed by the x-axis and the upper half of the circle $x^2 + y^2 = a^2$	