17.11.2018 (E)

K. J. Somaiya College of Engineering, Mumbai-77
(Autonomous College Affiliated to University of Mumbai)
November 2018

Max. Marks: 100

End Semester Exam

Duration: 3 Hrs

Class: S.Y.BTech
Name of the Course: Applied Mathematics-III
Course Code: UCEC301 / UITC301

Semester: III
Branch: COMP/IT

Instructions:
(1) All Questions are Compulsory
(2) Figures to right indicate full marks. Each sub-question has equal marks.

Question No.			Max. Marks
Q.1	**A**	**Attempt the following**	
		Using Laplace transform solve $\frac{d^2y}{dx^2} + y = t, y(0) = 1, y'(0) = 0$	**07**
	B	**Attempt any THREE of the following**	**18**
	(a)	Find $L(erf\sqrt{t})$	
	(b)	Find the Laplace transform of $e^{-4t}\int_0^t u\sin 3u\,du$.	
	(c)	Using Convolution theorem, find the inverse Laplace transform of $\frac{s}{S^4+8s^2+16}$	
	(d)	Find inverse Laplace transform of $\frac{3s+1}{(s+1)(s^2+2)}$	
	(e)	Express the following function as Heaviside's unit step function and find its Laplace transform $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ \cos t, & t > \pi \end{cases}$	
Q.2	**A**	**Attempt the following**	**06**
		Obtain the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $0 \le x \le 2\pi$, and $f(x + 2\pi) = f(x)$.	
	B	**Attempt any THREE of the following**	**18**
	(a)	Find the Fourier series for $f(x) = \lvert\sin x\rvert$ in $(-\pi, \pi)$.	
	(b)	Obtain the expansion of $f(x) = x(\pi - x), 0 < x < \pi$ as a half-range cosine series. Hence show that, $\sum_1^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$.	
	(c)	Find the complex form of Fourier series of $f(x) = \cosh 3x + \sinh 3x$, in $(-\pi, \pi)$.	
	(d)	Find the Fourier transform of $f(x) = e^{-\lvert x\rvert}$.	
Q.3	**A**	**Attempt the following**	**08**
		Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$	

(P.T.O.)

	B	**Attempt any THREE of the following**	**18**
	(a)	Find the eigen values of **(i)** $A^3 - 3A^2 + A$ **(ii)** $6A^{-1} + 2I$, Where $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$.	
	(b)	Verify Cayley Hamilton theorem and find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$	
	(c)	If $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, prove that $A^{50} - A^{49} = \begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix}$.	
	(d)	Is the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ diagonalisable? If so find the diagonal form and the transforming matrix.	
	(e)	Determine whether matrix $A = \begin{bmatrix} 2 & -3 & 3 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ is derogatory and if so, find its minimal polynomial.	
Q.4	**A**	**Attempt the following**	**07**
		If $\bar{f} = x^2 i + xzj + yzk$ and $\bar{r} = xi + yj + zk$, find $div(\bar{f} \times \bar{r})$ and $curl(\bar{f} \times \bar{r})$.	
	B	**Attempt any THREE of the following**	**18**
	(a)	Find the directional derivative of $\varphi = xy^2 + yz^2$ at $(2, -1, 1)$ in the direction normal to the surface $x^2y + y^2x + yz^2$ at $(1,1,1)$.	
	(b)	Prove that $\nabla f(r) = f'(r)\frac{\bar{r}}{r}$, where $\bar{r} = xi + yj + zk$. Hence find f if $\nabla f(r) = -\frac{\bar{r}}{r^5}$ and $f(1) = 0$.	
	(c)	Show that the vector $\bar{f} = (x^2 - yz)i + (y^2 - xz)j + (z^2 - yx)k$ is irrotational and hence find its scalar potential.	
	(d)	Evaluate by Green's theorem $\oint (3x^2 - 8y^2)dx + (4y - 6xy)dy$, along C, where C is the boundary of the region bounded by $x = y^2$ and $y = x$.	
	(e)	Use Gauss's Divergence theorem to evaluate $\iint \bar{N}.\bar{F}\, ds$, where $\bar{F} = 2xi + xyj + zk$, over the region bounded by the cylinder $x^2 + y^2 = 4, z = 0, z = 6$.	