

05-09-2023

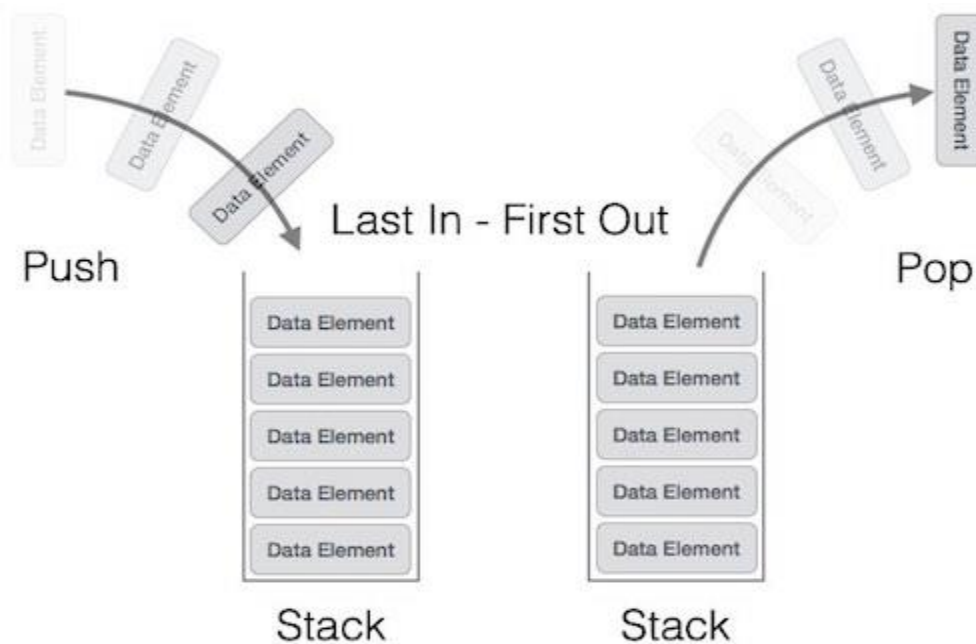
Stacks

Stacks

2

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- Stack is a linear data structure which follows a particular order in which the operations are performed.
- The order may be LIFO (Last In First Out) or FILO (First In Last Out).

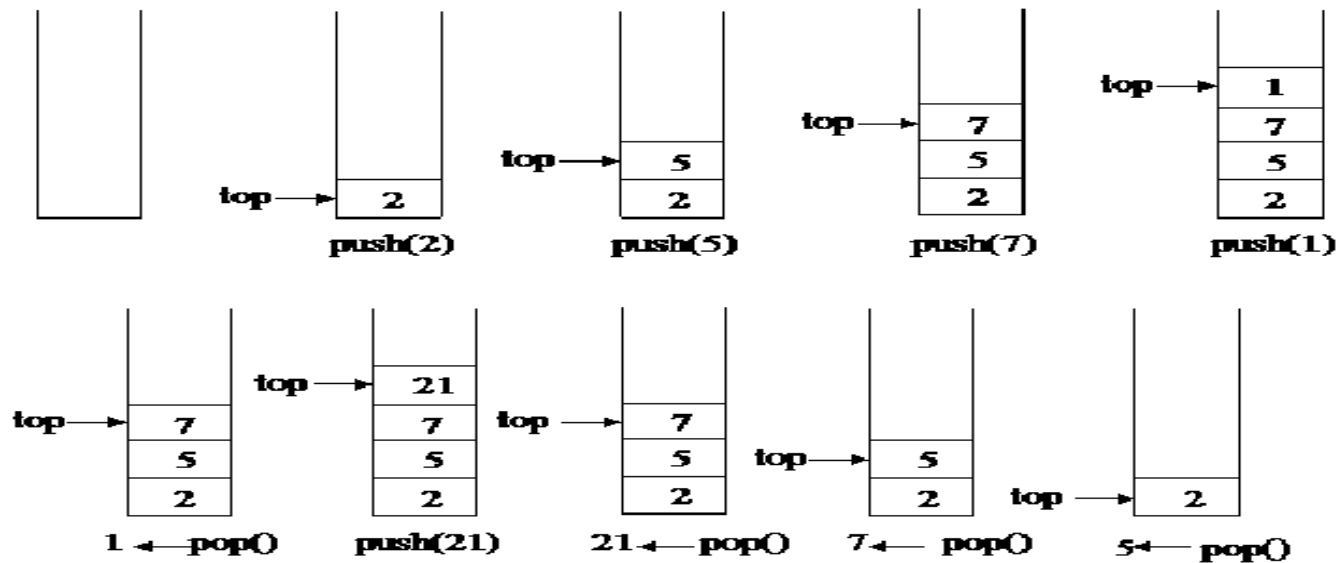


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Operation on Stacks³

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- **Push operation**-The insertion of an element into stack
- **Pop operation** - Deletion of an element from the stack
- In stack we always keep track of the last element present in the list with a pointer called top.



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Array Implementation of Stack

`top` Index of element at the top of stack

Stack is empty \rightarrow `top` is -1

Push operation

`top` is increased by 1

New element is placed at index `top`

Pop operation

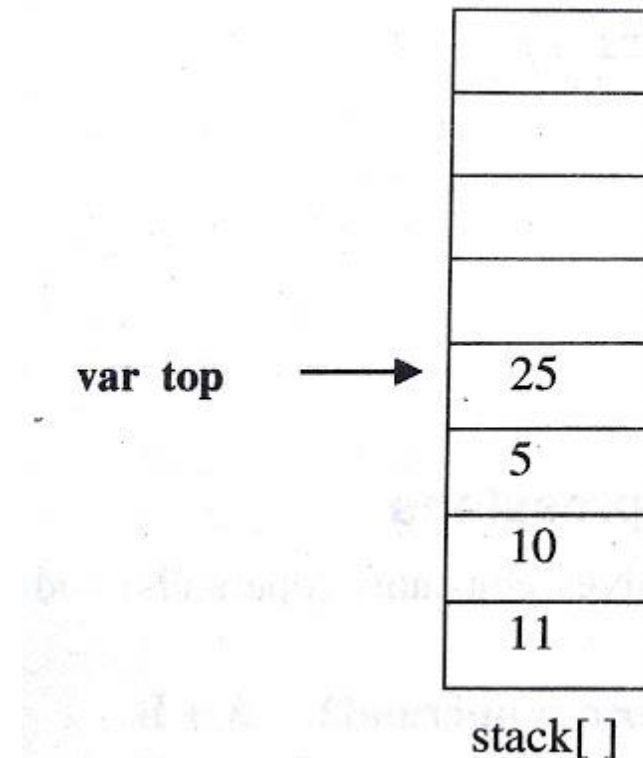
Element at index `top` is taken out

`top` is decreased by 1

Array Implementation of Stack

Array Implementation of Stack

- Push element one by one onto stack from 0th position to n-1th position.
- Variable top=position of the top element in the array
- If there is no element in the stack, value of top will be -1



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Stack Overflow /FullCondition

If($\text{top} == \text{MAX} - 1$)

Then print stack is full or overflowing

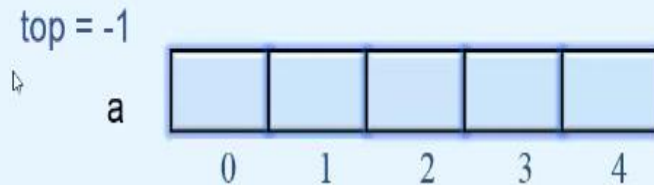
Stack Underflow /Empty Condition

If($\text{top} == -1$)

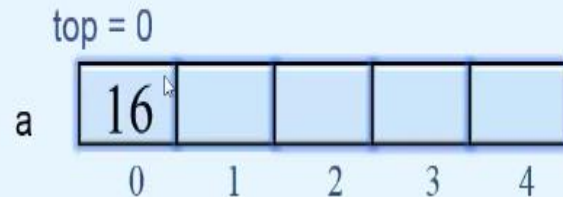
Then print stack is empty or underflowing

PUSH Operation on Stack

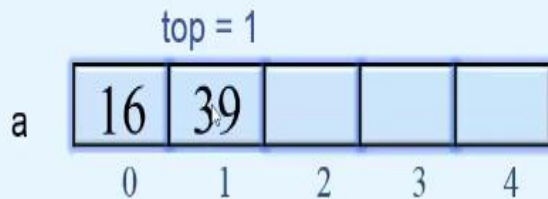
Empty stack



Push 16



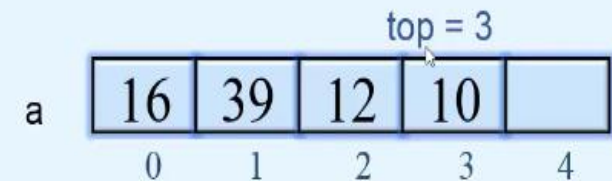
Push 39



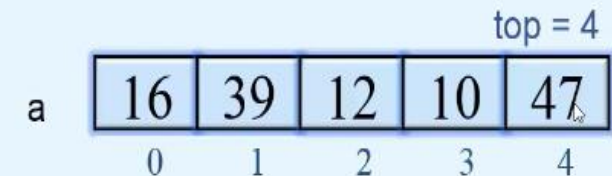
Push 12



Push 10



Push 47



Algorithm for PUSH Operation

- 1) Check Stack Full condition
If($\text{top} == \text{MAX} - 1$)
Then print stack is full or overflow
- 2) Otherwise increase the top value by 1
 $\text{top} = \text{top} + 1$
- 3) Input the value
- 4) Assign the item at top position
 $\text{stack_arr}[\text{top}] = \text{pushed_item}$

Try writing the code for Push Operation
function

PUSH Operation

```
push()
{
    int pushed_item;
    if(top == (MAX-1))
        printf("Stack Overflow\n");
    else
    {
        printf("Enter the item to be pushed in stack : ");
        scanf("%d",&pushed_item);
        top=top+1;
        stack_arr[top] = pushed_item;
    }
}/*End of push()*/
```

POP Operation on Stack

Empty stack

top = -1



a



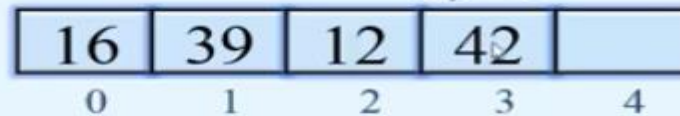
Stack Underflow
condition

If(top == -1)

Push 42

top = 3

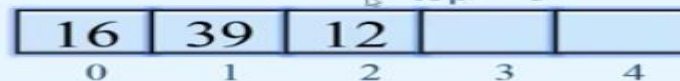
a



Pop

top = 3

a

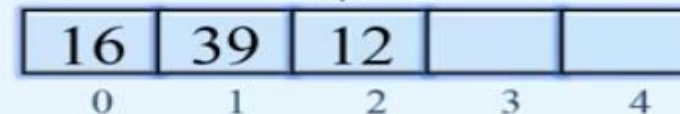


Popped item = 42

Pop

top = 2

a



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Algorithm for POP Operation

- 1) Check Stack Underflow condition
If($\text{top} == -1$)
Then print stack is underflow
- 2) Otherwise, delete the top position element
Popped_item = stack_arr[top]
- 3) Decrease the position of top
top = top - 1
- 4) Print the popped_item

Try writing the code for Pop Operation
function

POP Operation

```
pop()
{
    if(top == -1)
        printf("Stack Underflow\n");
    else
    {
        printf("Popped element is :
%d\n",stack_arr[top]);
        top=top-1;
    }
}/*End of pop()*/
```


Try writing the code for Display
Operation function

Display fn

```
display()
{
    int i;
    if(top == -1)
        printf("Stack is empty\n");
    else
    {
        printf("Stack elements :\n");
        for(i = top; i >= 0; i--)
            printf("%d\n", stack_arr[i] );
    }
}/*End of display()*/
```

Peek Operation

- An operation on certain abstract data types, specifically sequential collections such as stacks and queues,
 - which returns the value of the top ("front") of the collection
 - without removing the element from the collection.
- It thus returns the same value as operations such as "pop" or "dequeue", but does not modify the data.

Peek Operation

- The name "peek"
 - the name for this operation varies depending on data type and language.
- Peek is generally considered an inessential operation,
 - not included in the basic definition of these data types.

Peek Operation

- Sequential types for which peek is often implemented include:
 - Stack
 - Queue
 - Priority queue (such as a heap)
 - Double-ended queue (deque)
 - Double-ended priority queue (DEPQ)

Peek Operation

- Single-ended types, such as stack,
 - generally only admit a single peek, at the end that is modified.
- Double-ended types, such as dequeues,
 - admit two peeks, one at each end.
- Names for peek vary.
 - For queues "front" is common.
 - Dequeues have varied names, often "front" and "back" or "first" and "last".
 - The name "peak" is also occasionally found

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Linked List Representation of Stack



Linked List Representation of Stack

- The info field of the node holds the elements of the stack
 - Link fields hold pointers to the neighbouring element in the stack
- **The start pointer behaves as the TOP pointer variable of the stack**
- **Null pointer of the last node in the linked list signals the bottom of the stack**

Linked List Representation of Stack

```
struct node
{
    int info;
    struct node *link;
} *top=NULL;
```

Push Operation

- Inserting a node into the front or start of the list. i.e. at the top of the stack
- Stack after Before Push Operation,



- Stack after Push Operation, **Push www into stack, inserted at Top**



Linked List Representation of Stack

Beginning of linked list top of the stack

top Point to first node of the list

Stack is empty \rightarrow top is NULL

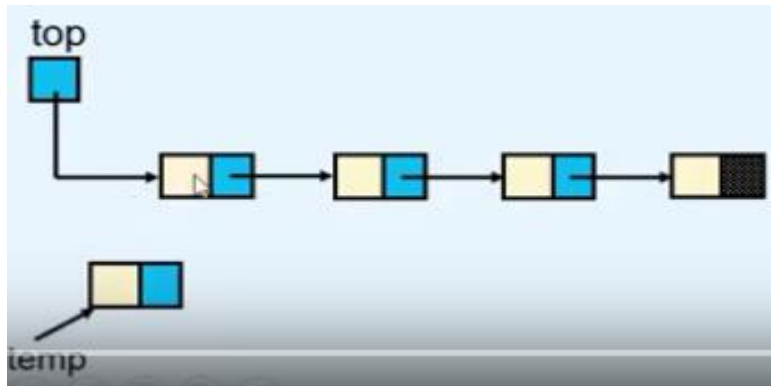
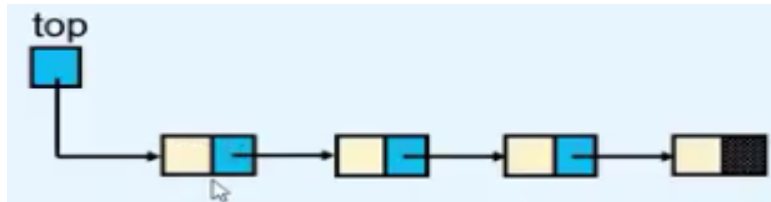
Push operation

Node is inserted in the beginning of the list

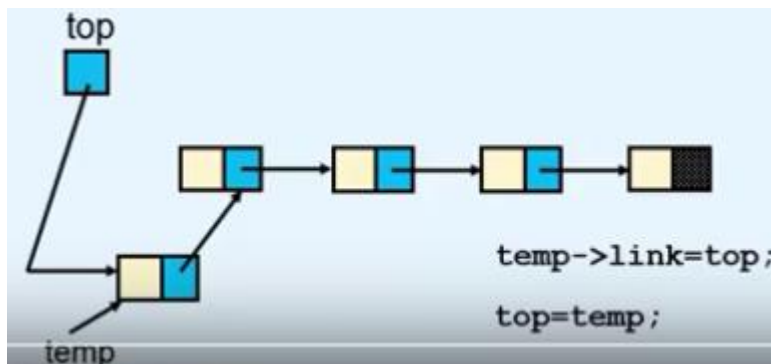
Pop operation

First node of the list is deleted

PUSH Operation on Stack



```
tmp = (struct node
*)malloc(sizeof(struct node));
```



```
tmp->link=top;
top=tmp;
```

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Push Operation

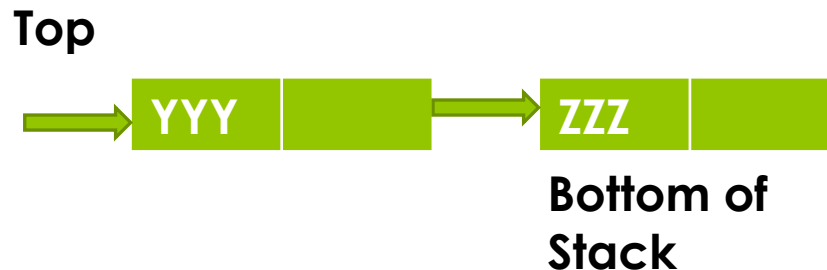
```
push()
{
    struct node *tmp;
    int pushed_item;
    tmp = (struct node *)malloc(sizeof(struct node));
    printf("Input the new value to be pushed on the stack : ");
    scanf("%d",&pushed_item);
    tmp->info=pushed_item;
    tmp->link=top;
    top=tmp;
}/*End of push()*/
```

Pop Operation

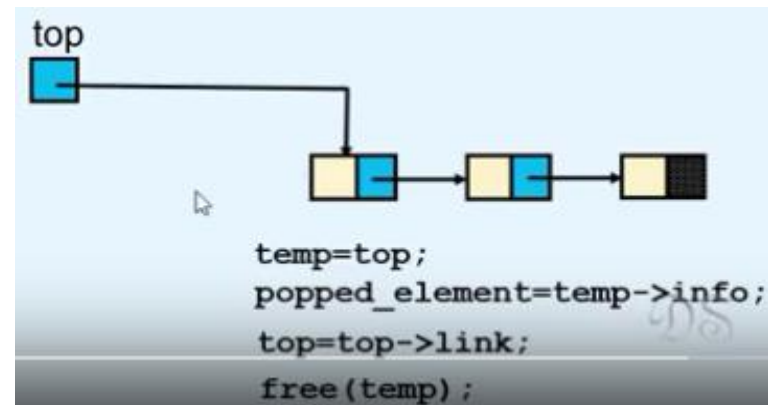
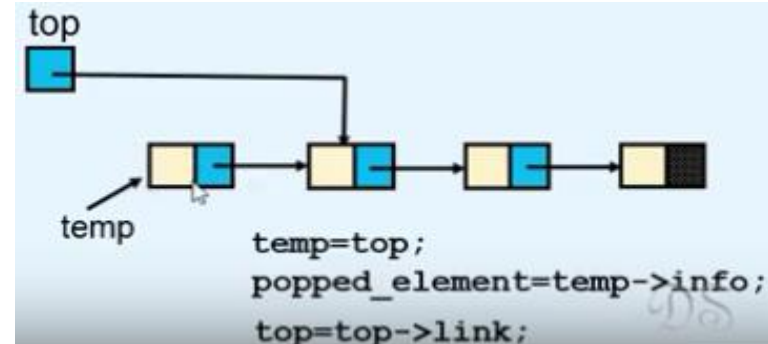
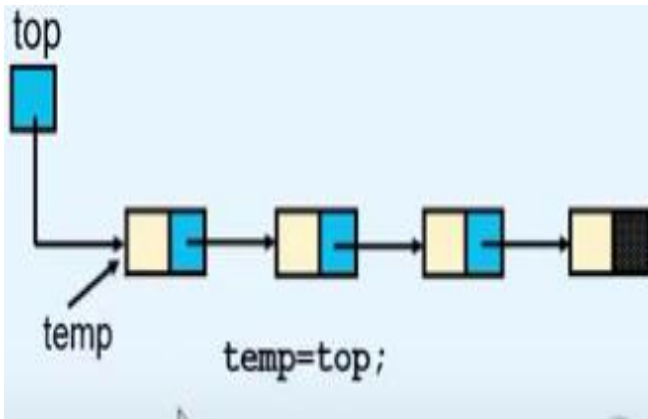
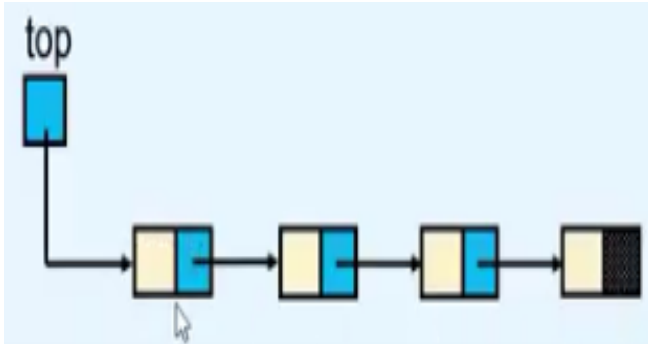
- Deleting a node pointed to by start pointer of the list.
i.e. at the top of the stack
- Stack after Before Pop Operation



- Stack after Pop Operation, element at top deleted



POP Operation on Stack



POP Operation

```
pop()
{
    struct node *tmp;
    if(top == NULL)
        printf("Stack is empty\n");
    else
    {
        tmp=top;
        printf("Popped item is %d\n",tmp->info);
        top=top->link;
        free(tmp);
    }
}

/*End of pop()*/
```


Display Operation

```
display()
{
    struct node *ptr;
    ptr=top;
    if(top==NULL)
        printf("Stack is empty\n");
    else
    {
        printf("Stack elements :\n");
        while(ptr!= NULL)
        {
            printf("%d\n",ptr->info);
            ptr = ptr->link;
        }
        /*End of while */
    }
    /*End of else*/
}
/*End of display()*/
```

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Application of Stack

Application of Stack –Reversal of a String

- Push each character of the string on the stack.
- When whole string is pushed on the stack
- Pop the characters from the stack to get the reversed string

Application of Stack –Reversal of a String

- Try Writing the Code snippet for it.....

Application of Stack –Polish Notation

- Polish notation (PN), also known as
 - normal Polish notation (NPN)
 - Łukasiewicz notation
 - Warsaw notation,
 - Polish prefix notation or
 - simply prefix notation,

Polish Notation-Prefix Notation

- Is a mathematical notation in which operators precede their operands,
- The description "Polish" refers to the nationality of logician Jan Łukasiewicz who invented Polish notation in 1924.

Reverse Polish Notation-Postfix Notation

- Reverse Polish notation (RPN), in which operators follow their operands.
- **The term Polish notation is sometimes taken (as the opposite of infix notation) to also include reverse Polish notation.**

Expression Representation

- An expression is defined as a number of operands or data items combined using several operators.
- 3 popular methods for representation:
 - Infix Notation
 - Prefix Notation
 - Postfix Notation

Infix Notation

- Used in General Mathematics
- Operator is written in between the operands
- Eg- $a+b$, $x+y*z$
- Called infix because of the position of the operator in expression.

Evaluation-

- Evaluated left to right but operator precedence must be taken into consideration
- **Not used inside computer, due to additional complexity of handling of precedence**

Prefix Notation

- Operator is written before the operands
- Also called Polish Notation
- Eg- $+ab$, $+x*yz$

Postfix Notation

- Operator is written after the operands
- Also called Reverse Polish or Suffix Notation
- Eg- $ab+$, xyz^*+

Polish Notation

- Prefix and Postfix are free from any precedence
- Computers use postfix form

Notation Conversion

- Scan the expression from left to right
- **Operator Precedence-**
 - Paranthesis evaluated first
 - After that evaluation is on the basis of operator precedence
- Logical Not
- Exponential Operator
- Multiplication/division/modulus
- Addition/Subtraction
- Left shift, Right Shift
- Relational
- Logical And
- Logical Or



Operator Precedence Table

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Description	Operator	Associativity
Function expression	()	Left to Right
Array Expression	[]	Left to Right
Structure operator	>	Left to Right
Structure operator	.	Left to Right
Unary minus	-	Right to left
Increment/Decrement	++ --	Right to Left
One's compliment	~	Right to left
Negation	!	Right to Left
Address of	&	Right to left
Value of address	*	Right to left
Type cast	(type)	Right to left
Size in bytes	sizeof	Right to left
Multiplication	*	Left to right
Division	/	Left to right
Modulus	%	Left to right
Addition	+	Left to right
Subtraction	-	Left to right
Left shift	<<	Left to right
Right shift	>>	Left to right
Less than	<	Left to right
Less than or equal to	<=	Left to right
Greater than	>	Left to right
Greater than or equal to	>=	Left to right
Equal to	==	Left to right
Not equal to	!=	Left to right
Description	Operator	Associativity
Bitwise AND	&	Left to right
Bitwise exclusive OR	^	Left to right
Bitwise inclusive OR		Left to right
Logical AND	&&	Left to right
Logical OR		Left to right
Conditional	? :	Right to left
Assignment	=	Right to left
	*= /= %=	Right to left
	+= -= &=	Right to left
	^= =	Right to left
	<<= >>=	Right to left
Comma	,	Right to left

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Manual Conversion -Infix to Postfix

- $A + B * C$ Given infix form
- Left to right scan, Brackets first then Operator Precedence
- $A + \underline{B C *}$ Convert the multiplication
- $A B C * +$ Convert the addition

Manual Conversion -Infix to Postfix

- $A + [(B + C) + (D + E) * F] / G$

Manual Conversion -Infix to Postfix

- $A + [(B + C) + (D + E) * F] / G$
- Left to right scan, Brackets first then Operator Precedence
- $A + [(\underline{BC+}) + (D + E) * F] / G$
- $A + [(\underline{BC+}) + (\underline{DE+}) * F] / G$
- $A + [(\underline{BC+}) + (\underline{DE+}) F *] / G$
- $A + [(BC+) (DE+) F * +] / G$
- $A + [(BC+) (DE+) F * +] G /$
- $A [(BC+) (DE+) F * +] G / +$
- $ABC + DE + F * + G / +$

Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

- 1) $A + B + C + D$
- 2) $(A + B) / (C - D)$
- 3) $(A + B) * C - (D - E) * (F + G)$
- 4) $((A+B) - C * (D/E)) + F$
- 5) $((A + B) * (C - D) + E) / (F + G)$

Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

1) $A + B + C + D$

$= \underline{AB+} + C + D$

$= \underline{AB+C+} + D$

$= AB+C+D+$

Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

1) $A + B + C + D = A B + C + D +$

2) $(A + B) / (C - D) = A B + C D - /$

Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

- 1) $A + B + C + D = A B + C + D +$
- 2) $(A + B) / (C - D) = A B + C D - /$
- 3) $(A + B) * C - (D - E) * (F + G) = AB + C * DE - FG + * -$

Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

- 1) $A + B + C + D = A B + C + D +$
- 2) $(A + B) / (C - D) = A B + C D - /$
- 3) $(A + B) * C - (D - E) * (F + G) = AB + C * DE - FG + * -$
- 4) $((A + B) - C * (D / E)) + F = A B + C D E / * - F +$

Manual Conversion -Infix to Postfix

Convert the following expressions from Infix to Postfix;

- 1) $A + B + C + D = A B + C + D +$
- 2) $(A + B) / (C - D) = A B + C D - /$
- 3) $(A + B) * C - (D - E) * (F + G) = AB + C * DE - FG + * -$
- 4) $((A + B) - C * (D / E)) + F = A B + C D E / * - F +$
- 5) $((A + B) * (C - D) + E) / (F + G) = A B + C D - * E + F G + /$

Manual Conversion -Infix to Prefix

- A/B^C+D

Manual Conversion -Infix to Prefix

- A/B^C+D
- Left to right scan, Brackets first then Operator Precedence
- $A/^BC+D$
- $/A^BC+D$
- $+/A^BCD$

Manual Conversion -Infix to Prefix

Convert the following expressions from Infix to Prefix;

- 1) $(P * Q \wedge R + S)$
- 2) $(A - B / C) * (D * E - F)$
- 3) $(A * B + (C / D)) - F$
- 4) $A + B * C - (D / E \wedge F) * G * H$
- 5) $(A + B) * C / D + E \wedge F / G$

Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1) $(P * Q \wedge R + S) = + * P \wedge Q R S$

Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1) $(P * Q^R + S) = + * P^R Q R S$

2) $(A - B / C) * (D * E - F) = * (- A / B C) (- * D E F)$

Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1) $(P * Q \wedge R + S) = + * P \wedge Q R S$

2) $(A - B / C) * (D * E - F) = * (- A / B C) (- * D E F)$

3) $(A * B + (C / D)) - F = - + * A B / C D F$

Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1) $(P * Q \wedge R + S) = + * P \wedge Q R S$

2) $(A - B / C) * (D * E - F) = * - A / B C - * D E F$

3) $(A * B + (C / D)) - F = - + * A B / C D F$

4) $A + B * C - (D / E \wedge F) * G * H$

$$= A + \underline{* B C} - (\underline{/ D \wedge E F}) * G * H$$

$$= A + * B C - \underline{*/ D \wedge E F G} * H$$

$$= A + \underline{* B C} - \underline{** / D \wedge E F G H}$$

$$= \underline{+ A * B C} - \underline{** / D \wedge E F G H}$$

$$= - + A * B C ** / D \wedge E F G H$$

Manual Conversion - Infix to Prefix

Convert the following expressions from Infix to Prefix;

1) $(P * Q \wedge R + S) = + * P \wedge Q R S$

2) $(A - B / C) * (D * E - F) = * (- A / B C) (- * D E F)$

3) $(A * B + (C / D)) - F = - + * A B / C D F$

4) $A + B * C - (D / E \wedge F) * G * H$

$$= A + * B C - (/ D \wedge E F) * G * H$$

$$= A + * B C - * / D \wedge E F G * H$$

$$= A + * B C - ** / D \wedge E F G H$$

$$= + A * B C - ** / D \wedge E F G H$$

$$= - + A * B C ** / D \wedge E F G H$$

5) $(A + B) * C / D + E \wedge F / G$

$$= + \underline{A B} * C / D + E \wedge F / G$$

$$= + \underline{A B} * C / D + \underline{\wedge E F} / G$$

$$= * + \underline{A B C} / D + \underline{\wedge E F} / G$$

$$= / * + \underline{A B C D} + \underline{\wedge E F} / G$$

$$= / * + \underline{A B C D} + / \underline{\wedge E F G}$$

$$= + / * + \underline{A B C D} / \underline{\wedge E F G}$$

Algorithm for Infix to Postfix using stack

Step 1-Put the opening bracket '(' on the start of the expression and add the closing bracket ')' at the end of the input expression 'P'

Step 2-Scan the Expression P from left to right and repeat steps 3 to 6 for each element of P until the stack is empty

Step 3-If an operand is encountered, add it to the output expression 'Q'

Step 4-If a left bracket is encountered push it onto the stack

Algorithm for Infix to Postfix using stack

Step 5-If an operator is encountered then

5a)Repeatedly pop from the stack and add to the output expression Q, **each operator which has the same or higher priority** compared to the operator just scanned.

5b) then Add the operator just scanned to the stack

Step 6-If a right bracket ')' is encountered then repeatedly pop from the stack and add to the output expression Q until a left bracket is encountered.

Pop the left bracket also

Infix to Postfix Conversion using stack

Eg- $P = (A+B)*C$

Infix to Postfix Conversion using stack

Eg- $P = (A+B)*C$

$P = ((A+B) * C)$

Scan	Stack	Expression Q
((
(((
A	((A
+	((+	A
B	((+	AB
)	(AB+
*	(*	AB+
C	(*	AB+C
)	NULL	AB+C*

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Infix to Postfix Conversion using stack

Convert $P/(Q-R)*S+T$

Infix to Postfix Conversion using stack

Convert $P/(Q-R)*S+T=(P/(Q-R)*S+T)$

Scan	Stack	Expression Q
((
P	(P
/	(/	P
((/ (P
Q	(/ (PQ
-	(/ (-	PQ
R	(/ (-	PQR
)	(/ Rule 5a)	PQR-
*	(*	PQR-/
S	(*	PQR-/S
+	(+	PQR-/S*
T	(+	PQR-/S*T
)	NULL	PQR-/S*T+

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Infix to Postfix Conversion using stack

Convert $(P/(Q-R)*S+T)$

$(P/(Q-R)*S+T)$		
Symbol	Stack	Expression
((—
P	(P
/	(/	P
((/(P
Q	(/(PQ
—	(/(—	PQ
R	(/(—	PQR
)	(/	PQR—
*	(*	PQR—/
S	(*	PQR—/S
+	(+	PQR—/S*
T	(+	PQR—/S*T
	null	PQR—/S*T+

Infix to Postfix Conversion using stack

Convert $P = a \ \&\& \ b \ || \ c \ || \ ! (e > f)$

Notation Conversion

- Scan the expression from left to right
- **Operator Precedence-**
 - Paranthesis evaluated first
 - After that evaluation is on the basis of operator precedence
 - Logical Not
 - Exponential Operator
 - Multiplication/division/modulus
 - Addition/Subtraction
 - Left shift, Right Shift
 - Relational
 - Logical And
 - Logical Or



Infix to Postfix Conversion using stack

Convert P=(a && b || c || ! (e > f))

Scan	Stack	Expression
((
a	(a
&&	(&&	a
b	(&& Rule 5a	ab
	(ab&&
c	(ab&&c
	(Rule 5a	ab&&c
!	(!	ab&&c
((!(ab&&c
e	(!(ab&&c e
>	(!(>	ab&&c e
f	(!(>	ab&&c ef
)	(!	ab&&c ef>
)	NULL	ab&&c ef>

- Operator Precedence
- Logical Not
- Exponential Operator
- Multiplication/division/modulus
- Addition/Subtraction
- Left shift, Right Shift
- Relational
- Logical And &&
- Logical Or ||

Infix to Postfix Conversion using stack

Convert $P = A + (B * C - (D / E \wedge F) * G * H)$

Infix to Postfix Conversion using stack

Convert $P = (A + (B * C - (D / E \wedge F) * G * H))$

Scan	Stack	Expression Q
((
A	(A
+	(+	A
((+ (AB
B	(+ (AB
*	(+ (*	AB
C	(+ (*	ABC
-	(+ (-	ABC*
((+ (- (ABC*
D	(+ (- (ABC*D
/	(+ (- (/	ABC*D
E	(+ (- (/	ABC*DE
^	(+ (- (/ ^	ABC*DE
F	(+ (- (/ ^	ABC*DEF
)	(+ (-	ABC*DEF^/
*	(+ (-*	ABC*DEF^/
G	(+ (-*	ABC*DEF^/G
*	(+ (-*	ABC*DEF^/G*
H	(+ (-*	ABC*DEF^/G*H
)	(+	ABC*DEF^/G*H*-
)	null	ABC*DEF^/G*H*-+

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Convert $P=(A+(B*C-(D/E^F)*G*H))$

Scan	Stack	Expression Q
((
A	(A
+	(+	A
((+ (AB
B	(+ (AB
*	(+ (*	AB
C	(+ (*	ABC
-	(+ (-	ABC*
((+ (- (ABC*
D	(+ (- (ABC*D
/	(+ (- (/	ABC*D
E	(+ (- (/	ABC*DE
^	(+ (- (/^	ABC*DE
F	(+ (- (/^	ABC*DEF
)	(+ (-	ABC*DEFA^/
*	(+ (-*	ABC*DEFA^/
G	(+ (-*	ABC*DEFA^/G
*	(+ (-*	ABC*DEFA^/G*
H	(+ (-*	ABC*DEFA^/G*H
)	(+	ABC*DEFA^/G*H*-
)	null	ABC*DEFA^/G*H*-+

Manual Evaluation of a Prefix notation

○ $+5*32$

Manual Evaluation of a Prefix notation

- Lets take an eg: $+5*32$
- Find an operator from left to right having 2 operands after it
- Multiplication of 3 and 2 is carried out
- Expression becomes $+56$
- Now $+$ has two operands so evaluated
- Exp=11

Manual Evaluation of a Postfix notation

- 532^{*+}

Manual Evaluation of a Postfix notation

- Lets take an eg: $532*+$
- Find first operator from left to right having 2 operands before it
- perform the operation
- Multiplication of 3 and 2 is carried out
- Expression becomes $56+$
- Now $+$ is evaluated
- $Exp=11$

Manual Evaluation

Evaluate the following expression-

$$P=653+9*+$$

Algorithm for Evaluation of Postfix using stack

Step 1-Scan the Expression P from left to right and repeat steps 2 and 3 for each element of 'P' until the last element

Step 2- If an operand is encountered, push it on the stack

1st operand say 'a' is pushed

2nd operand say 'b' is pushed

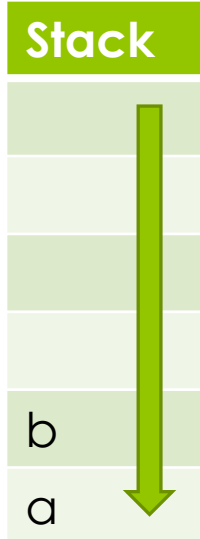
Step 3-If an operator is encountered then

3a) Remove the top two elements of stack and perform

a operator b

(where b is the top element and a is the next element to top element)

3b)Push the result onto the stack



Algorithm for Evaluation of Postfix using stack

Evaluate the following expression-

$P = 653 + 9 * +$

Algorithm for Evaluation of Postfix using stack

P=653+9*+

P=53+9*+

P=3+9*+

P=+9*+

Evaluate $5+3=8$

Push 8

P=9*+

P=*+

Evaluate

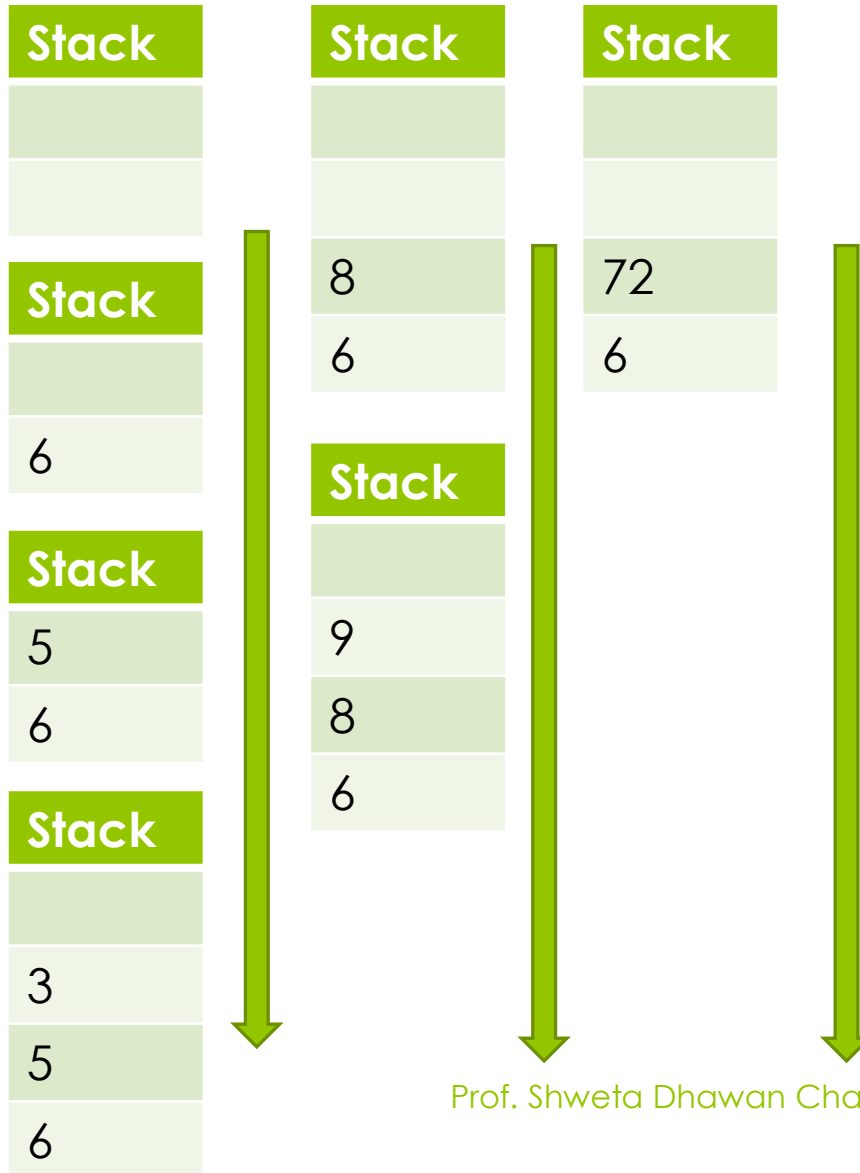
$8*9=72$

Push 72

P=+

Evaluate $6+72=78$

Ans=78



Algorithm for Evaluation of Postfix using stack

Evaluate the following expression-

- 1) 432^*+5-
- 2) 532^*+4-5+
- 3) $53+82-*$
- 4) Evaluate $562+*(12)4/-$
taking 12 as a single number

Evaluate 432^*+5-

Postfix \rightarrow a b c * + d - Let, a = 4, b = 3, c = 2, d = 5

Postfix \rightarrow 4 3 2 * + 5 -

Operator/Operand	Action	Stack
4	Push	4
3	Push	4, 3
2	Push	4, 3, 2
*	Pop (2, 3) and $3*2 = 6$ then Push 6	4, 6
+	Pop (6, 4) and $4+6 = 10$ then Push 10	10
5	Push	10, 5
-	Pop (5, 10) and $10-5 = 5$ then Push 5	5

Example

Evaluate the postfix expression
 $5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$

(a) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(b) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(c) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(d) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(e) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(f) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(g) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(h) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



(i) Input so far (shaded):

$5\ 3\ 2\ *\ +\ 4\ -\ 5\ +$



The result of the computation is 12.

Postfix Expression 5 3 + 8 2 - *

Infix Expression $(5 + 3) * (8 - 2)$
 Postfix Expression 5 3 + 8 2 - *

Below Postfix Expression can be evaluated by using Stack Data Structure as follows...

Reading Symbol	Stack Operations	Evaluated Part of Expression
Initially	Stack is Empty	Nothing
5	push(5)	Nothing
3	push(3)	Nothing
+	value1 = pop() value2 = pop() result = value2 + value1 push(result)	value1 = pop() value2 = pop() result = 5 + 3 Push(8) (5 + 3)

8	push(8)	<div> <div>8</div> <div>5</div> </div> (5 + 3)
2	push(2)	<div> <div>2</div> <div>8</div> <div>5</div> </div> (5 + 3)
-	value1 = pop() value2 = pop() result = value2 - value1 push(result)	value1 = pop() value2 = pop() result = 8 - 2 Push(6) (8 - 2) (5 + 3) * (8 - 2)
*	value1 = pop() value2 = pop() result = value2 * value1 push(result)	value1 = pop() value2 = pop() result = 6 * 8 Push(48) (6 * 8) (5 + 3) * (8 - 2)
\$ End of Expression	result = pop()	Display Result: 48 for final result

Infix Expression $(5 + 3) * (8 - 2) = 48$
 Postfix Expression 5 3 + 8 2 - * value is 48

Courtesy: http://btechsmartclass.com/data_structures/postfix-evaluation.html

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Evaluate $562+*(12)4/-$
taking 12 as a single number

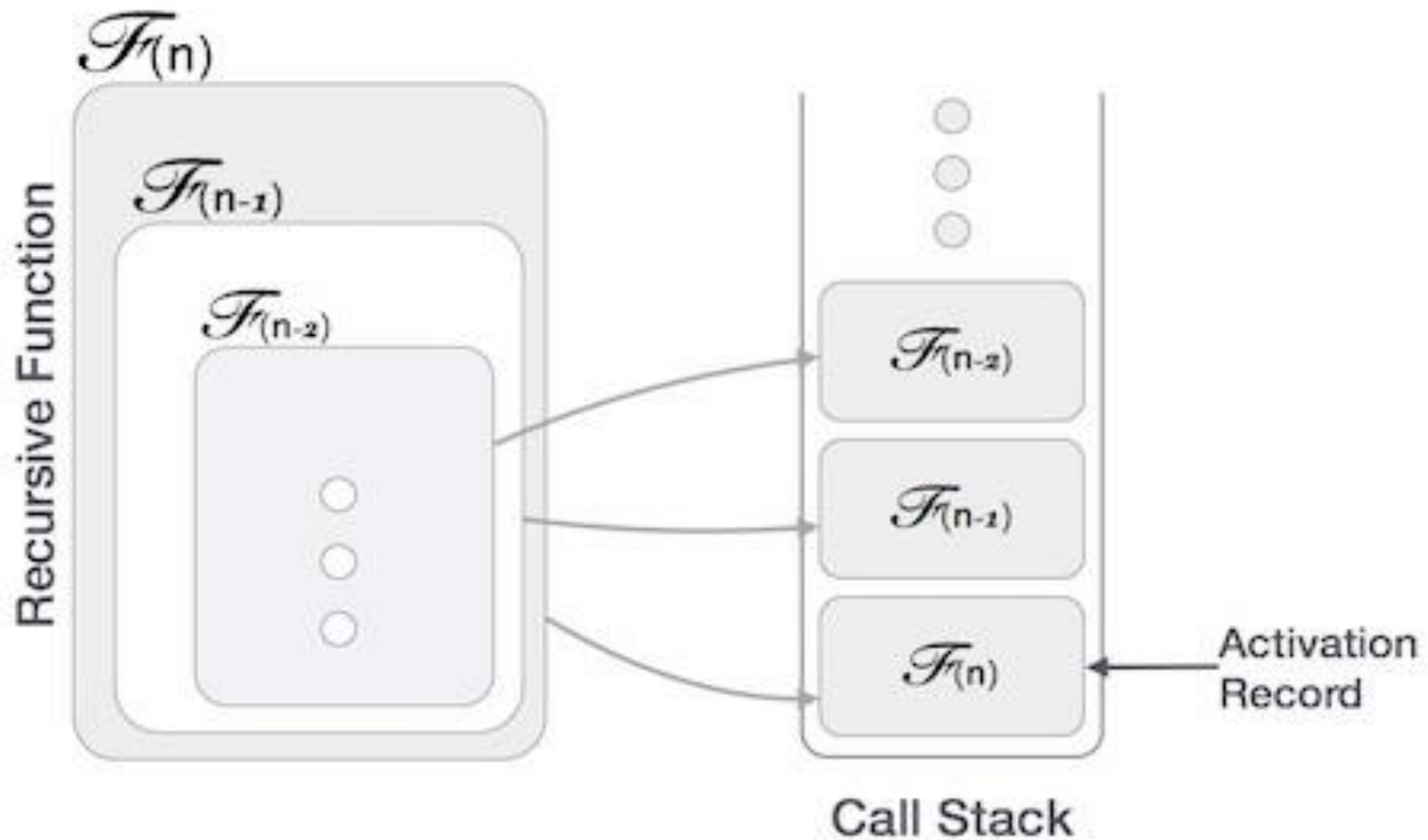
$562+*124/-$

Step	Input Symbol/Element	Stack	Intermediate Calculations Output
1	5 Push	5	
2	6 Push	5, 6	
3	2 Push	5, 6, 2	
4	+ Pop 2 elements and evaluate	5	$6 + 2 = 8$
5	Push result 8	5, 8	
6	* Pop 2 elements and evaluate	# empty	$5 \times 8 = 40$
7	Push result 40	40	
8	12 Push	40, 12	
9	4 Push	40, 12, 4	
10	/ Pop 2 elements and evaluate	40	$12 / 4 = 3$
11	Push result 3	40, 3	
12	- Pop 2 elements and evaluate	# empty	$40 - 3 = 37$
13	Push result 37	37	
14	No more elements		37

Courtesy: <https://unacademy.com/lesson/evaluation-of-a-postfix-expression-in-tabular-form/8Z5PDFL5>

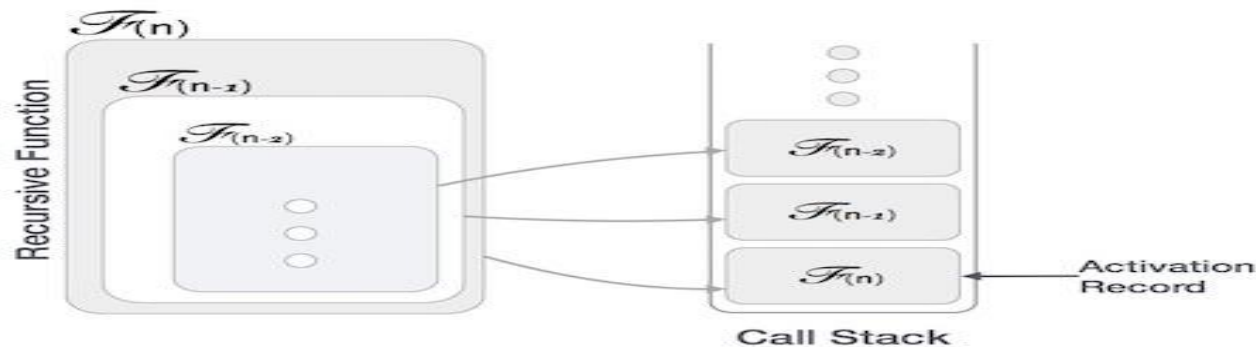
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Application of Stack – Recursion



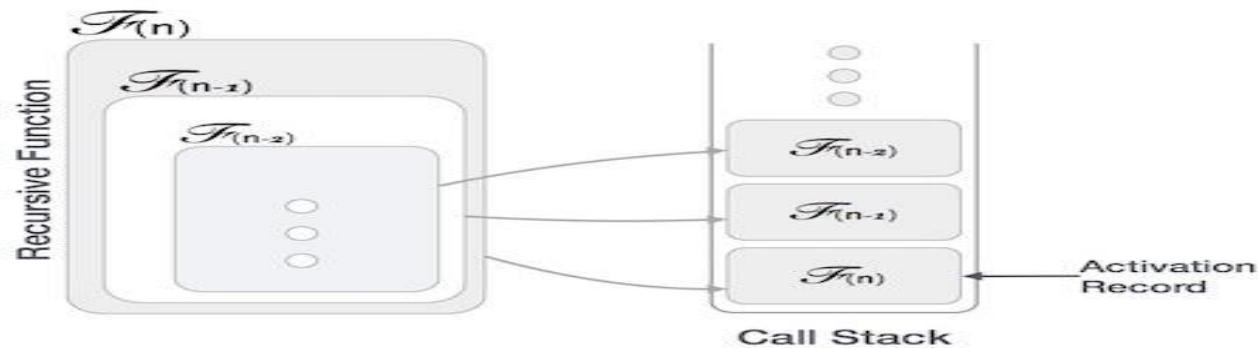
Application of Stack – Recursion

- Many programming languages implement recursion by means of **stacks**.
- A function (**caller**) calls another function (**callee**) or itself as callee,
 - The caller function transfers execution control to the callee.
 - This transfer process may also involve some data to be passed from the caller to the callee.



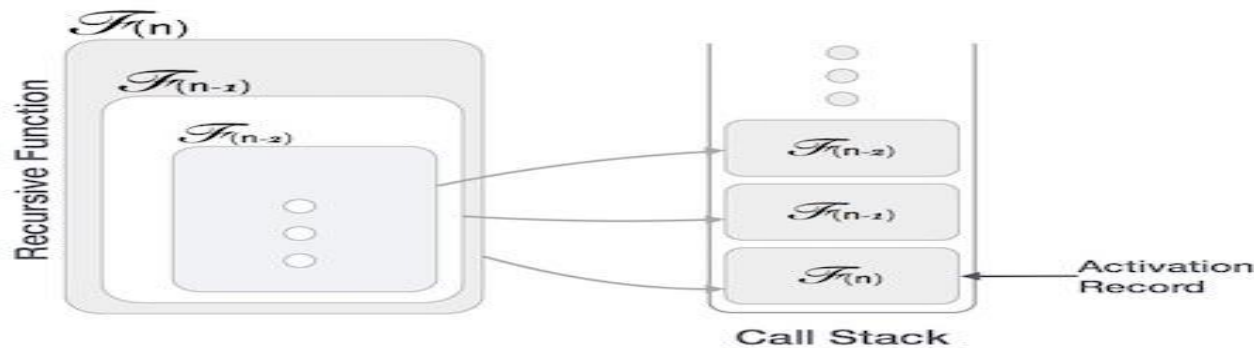
Application of Stack – Recursion

- The caller function has
 - to suspend its execution temporarily and
 - resume later
 - when the execution control returns from the callee function.



Application of Stack – Recursion

- Here, the caller function needs to start exactly from the point of execution where it puts itself on hold.
- It also needs the exact same data values it was working on.
- So, an activation record (or stack frame) is created for the caller function.
- Activation record keeps the information about
 - local variables,
 - formal parameters,
 - return address and
 - all information passed to the callee function.



Application of Stack –Recursion

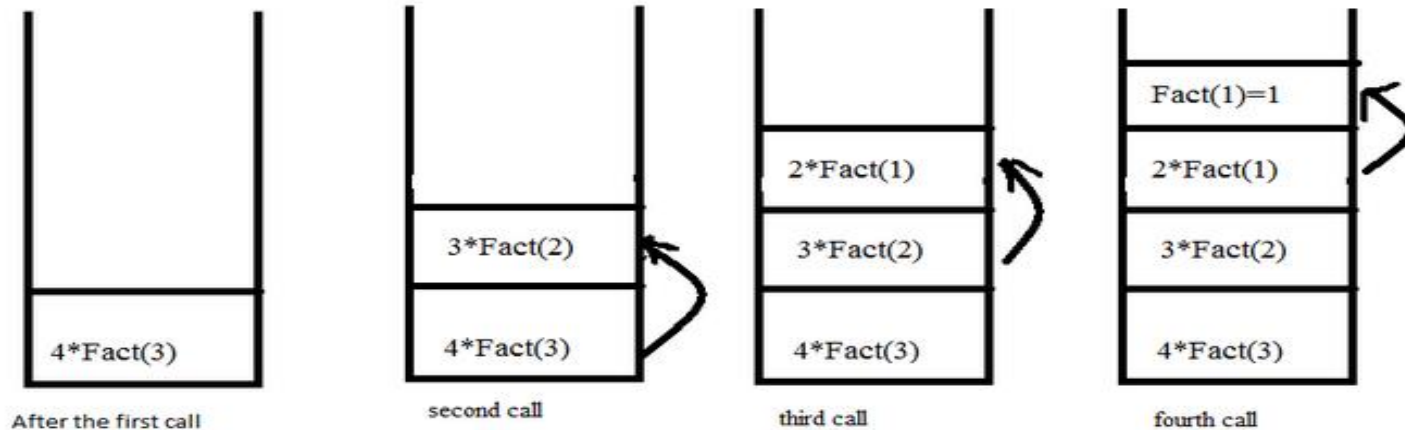
A recursive function to find the factorial of a positive whole number

```
int factorial(n)
{
    if (n == 1)
    {
        // base case
        return 1;
    }
    else
    {
        return n * factorial(n - 1);
        // function calls itself
    }
}
```

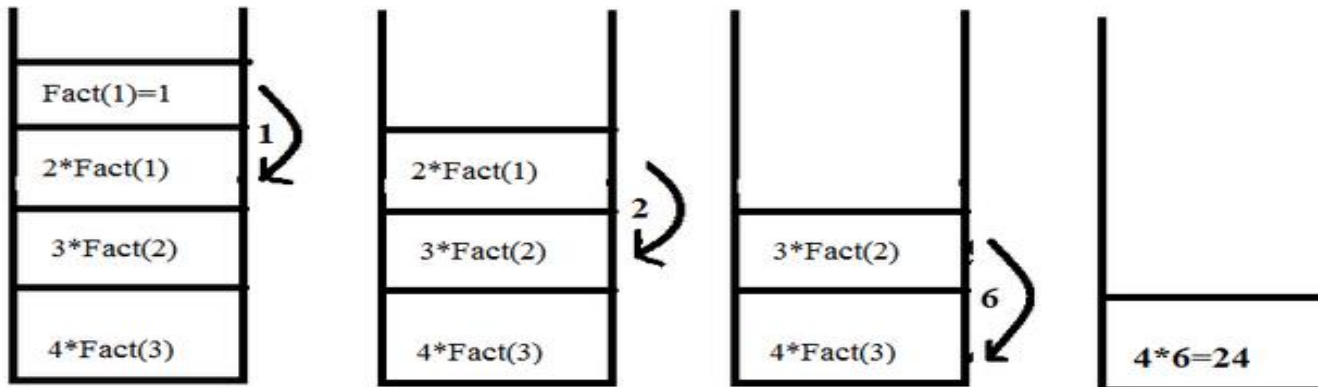
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Application of Stack – Recursion

When function call happens previous variables gets stored in stack



Returning values from base case to caller function

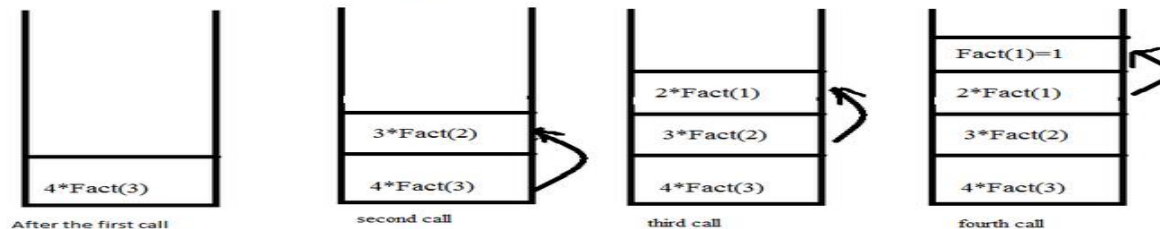


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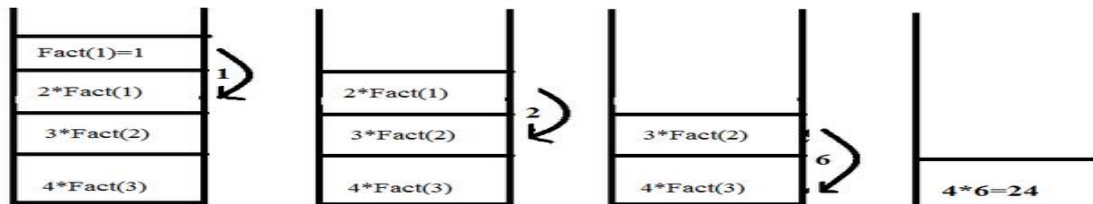
Application of Stack – Recursion

- Functions calls are 'stacked' one on top of the other,
- This is called the call stack (or execution stack)
- The call stack operates on a "Last In, First Out" basis. An item is "pushed" onto a stack on function call, and an item is "popped" off the stack when that function returns a value.

When function call happens previous variables gets stored in stack



Returning values from base case to caller function



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extra

Application of Stack – Recursion

Time Complexity

- In iterations,
 - we take number of iterations to count the time complexity.
- In recursion,
 - assuming everything is constant,
 - count the number of times a recursive call is being made.

Space Complexity

- Space complexity is counted as
 - what amount of extra space is required for a module to execute.
- In iterations,
 - the compiler hardly requires any extra space.
 - The compiler keeps updating the values of variables used in the iterations.

Application of Stack – Recursion

Time Complexity

- A call made to a function is $O(1)$,
- hence the (n) number of times a recursive call is made
- makes the recursive function $O(n)$.

Space Complexity

- In recursion,
 - the system needs to store activation record each time a recursive call is made.
- Thus, space complexity of recursive function
 - may go higher than that of a function with iteration.