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Experiment / assignment / tutorial No: 7
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Tutorial - 7

Q1) Find Fourier series for $f(x) = 2x - x^2$ in the interval $(0,3)$

A) $2l = 3$

$l = \frac{3}{2}$

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$= \frac{2}{3}\int_0^3 (2x - x^2)\,dx$$

$$= \frac{2}{3}\left[x^2 - \frac{x^3}{3}\right]_0^3$$

$$= \frac{2}{3}[9-9] = 0$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{3}\int_0^3 (2x - x^2)\cos\left(\frac{2n\pi x}{3}\right)dx$$

$$= \frac{2}{3}\left[(2x - x^2)\left[\frac{\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi x}{3}\right)}\right] - (2-2x)\left(\frac{-\cos\left(\frac{2\pi nx}{3}\right)}{\frac{4n^2\pi^2}{9}}\right) - (-2)\left(\frac{-\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{8n^3\pi^3}{27}\right)}\right)\right]_0^3$$

$$= \frac{2}{3} \times \frac{9}{4\pi^2 n^2} \left[(2-2\pi) \cdot \cos\left(\frac{2n\pi x}{3}\right) \right]_0^3$$

$$= \frac{3}{2\pi^2 n^2} [-4-2] = \frac{-9}{n^2\pi^2} = a_n$$

$$b_n = \frac{1}{l} \int_0^{2l} f(x) \cdot \sin\frac{n\pi x}{l} \, dx$$

$$= \frac{2}{3} \int_0^3 (2x - x^2) \cdot \sin\left(\frac{2n\pi x}{3}\right) dx$$

$$= \frac{2}{3} \left[(2x - x^2) \cdot \left(\frac{-\cos\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)} \right) - (2-2x) \cdot \left(\frac{-\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{4\pi^2 n^2}{9}\right)} \right) + (-2) \left(\frac{\cos\left(\frac{2nx\pi}{3}\right)}{\left(\frac{8n^3\pi^3}{27}\right)} \right) \right]_0^3$$

$$= \frac{2}{3} \left[(-3) \left(-\frac{\cos(2n\pi)}{(2n\pi)} \right) - (-4) + (-1) \cos\left(\frac{2n\pi}{8\pi^3}\right) \right]$$

$$b_n = \frac{3}{n\pi}$$

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$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{2n\pi x}{3}\right) + b_n \sin \frac{2n\pi x}{3} \right]$$

$$= \frac{-9}{n^2\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos \frac{2n\pi x}{3} + \frac{3}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{2n\pi x}{3}$$

Q2) Find Fourier Series for

A)
$$f(x) = \begin{cases} 0, & -2 < x < -1 \\ 1+x, & -1 < x < 0 \\ 1-x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$$

$$f(-x) = \begin{cases} 0, & 1 < x < 2 \\ 1-x, & 0 < x < 1 \\ 1+x, & -1 < x < 0 \\ 0, & -2 < x < -1 \end{cases}$$

$$= f(x).$$

f is even function

$\therefore b_n = 0$

$$a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\,dx$$

$$= \frac{2}{2}\int_{0}^{2} f(x)\,dx$$

$$= \int_0^1 (1-x)\,dx + \int_0^2 0\,dx$$

$$= \left(x - \frac{x^2}{2}\right]_0^1 \Rightarrow 1 - \frac{1}{2}$$

$$a_0 = 1/2$$

$$a_n = \int_{-2}^{2} f(x)\cos\frac{n\pi x}{2}\,dx$$

$$= \frac{2}{2}\int_0^2 f(x)\cos\frac{n\pi x}{2}\,dx$$

$$= \int_0^1 (1-x)\cos\frac{n\pi x}{2}\,dx + \int_{1}^{2} 0\,dx$$

$$= \left[(1-x)\cdot\left(\frac{\sin\left(\frac{n\pi x}{2}\right)}{\frac{n\pi}{2}}\right]^{0} - (-1)\cdot\left(\frac{-\cos\left(\frac{n\pi x}{2}\right)}{\frac{n^2\pi^2}{4}}\right)$$

$$= \frac{-4}{n^2\pi^2}\left[\cos\frac{n\pi x}{2}\right]_0^1$$

$$= \frac{4}{\pi n^2}\left[\cos\frac{n\pi}{2} - 1\right]$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{2}$$

$$= \frac{1}{4} - \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2}\left[\cos\frac{n\pi}{2} - 1\right] \cos\frac{n\pi}{2}$$

Q3) Find complex form of fourier series

$$f(x) = \begin{cases} a, & 0 < x < l \\ -a, & l < x < 2l \end{cases}$$

A) $$c_n = \frac{1}{2l}\left[\int_0^l a \cdot e^{-\frac{in\pi x}{l}} dx + \int_l^{2l} -a \cdot e^{-\frac{in\pi x}{l}} dx\right]$$

$$= \frac{a}{2l}\left[\left(\frac{e^{-\frac{in\pi x}{l}}}{-\frac{in\pi}{l}}\right)_0^l - \left(\frac{e^{-\frac{in\pi x}{l}}}{-\frac{in\pi}{l}}\right)_l^{2l}\right]$$

$$= \frac{a}{2l} \times \frac{l}{-in\pi}\left[(e^{-in\pi} - 1) - (e^{-in\pi 2} - e^{-in\pi})\right]$$

$$= \frac{ai}{2n\pi}\left[(-1)^n - 1 - 1 + (e^{-1})^n\right]$$

$$= \frac{ai}{n\pi}\left[(-1)^n - 1\right] \quad n \neq 0$$

$$C_0 = \frac{1}{2l}\int_0^{2l} f(x)\,dx$$

$$= \frac{a}{2l}\left[\int_0^{l} dx - \int_l^{2l} dx\right]$$

$$= \frac{a}{2l}\left[l - l\right] = 0$$

$$f(x) = \frac{ai}{\pi}\sum_{n=-\infty}^{\infty}\frac{1}{n}\left[(-1)^n - 1\right]e^{\frac{in\pi x}{l}}$$

$n = \infty$ $\quad\therefore n \neq 0$

$= 0$

Now $C_0 = \frac{a_0}{2}$

$a_0 = 0$

$C_n + C_{-n} = a_n$

$$= \frac{ai}{n\pi}\left[(-1)^n - 1\right] - \frac{ai}{n\pi}\left[(-1)^n - 1\right]$$

$\therefore a_n = 0$

$C_n - C_{-n} = ib_n$

$$b_n = \frac{1}{i}\left[C_n - C_{-n}\right]$$

$$= \frac{1}{i}\left(\frac{ai}{n\pi}\left[(-1)^n - 1\right] + \frac{ai}{n\pi}\left[(-1)^n - 1\right]\right)$$

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$$f = \frac{a_0}{2} \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n x \pi}{l}$$

Q4) Express the following function

$$f(x) = \begin{cases} 1 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$$

as fourier integral & evaluate

$$\int_0^{\infty} \frac{\sin w \cos w x}{w}\, dw$$

A) we see that f is even

$$f(x) = \frac{2}{\pi} \int_{w=0}^{\infty} \cos wx \left[\int_{s=0}^{\infty} f(s) \cos ws \, ds \right]$$

$$= \frac{2}{\pi} \int_{w=0}^{\infty} \cos wx \left[\frac{\sin ws}{w} \right]_0^1 dw$$

$$= \frac{2}{\pi} \int_0^{\infty} \cos wx \cdot \frac{\sin w}{w}\, dw$$

$$= \int_{w=0}^{\infty} \frac{\sin w}{w} \cos wx \, dw$$

we observe that f is discontinous at $x = 1, 1$

$$f(1) = \frac{1}{2}\left[\lim_{x \to 1^-} f(x) + \lim_{x \to 1^+} f(x) \right]$$

$= \frac{1}{2}[1+0] = \frac{1}{2} \Rightarrow f(-1)$

Q5) Find fourier transform of

$$f(x) = \begin{cases} 1 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \frac{\sin x}{x}\,dx$

A) $F[f(x)] = F(s)$

$$= \int_{-\infty}^{\infty} f(x)\, e^{isx}\,dx$$

$$= \int_{-1}^{1} 1 \cdot e^{isx}\,dx$$

$$= \left[\frac{e^{isx}}{is}\right]_{-1}^{1}$$

$$= \frac{1}{is}\left[e^{is} - e^{-is}\right] \frac{2}{2}$$

$$= \frac{2}{s}\left[\frac{e^{is} - e^{-is}}{2i}\right]$$

$$= \frac{2}{s}\sin s$$

$$F(s) = \begin{cases} \frac{2\sin s}{s} & \text{for } s \neq 0 \\ 2 & \text{for } s = 0 \left(\lim_{s \to 0} \frac{\sin s}{s} = 1\right) \end{cases}$$

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Now $f(x_1) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2\sin s}{s} e^{isx} ds$

$f(0) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin s}{s} ds$

$= \frac{2}{\pi}\int_{0}^{\infty} \frac{\sin s}{s} ds$

$\int_{0}^{\infty} \frac{\sin s}{s} ds = \frac{\pi}{2} \cdot f(0)$

$\int_{0}^{\infty} \frac{\sin s}{s} ds = \frac{\pi}{2}$