29.11.2018 (E)

K. J. Somaiya College of Engineering, Mumbai-77
(Autonomous College Affiliated to University of Mumbai)

End Semester Exam
Nov- Dec 2018

Max. Marks: 100

Class : S . Y . BTECH

Name of the Course: **Discrete Structure and Graph Theory.**

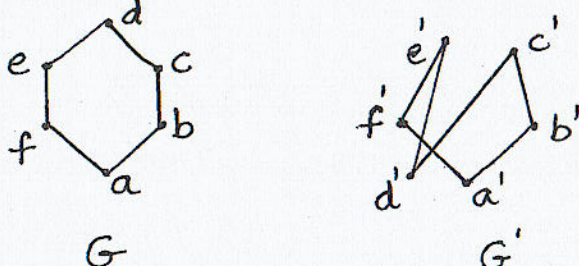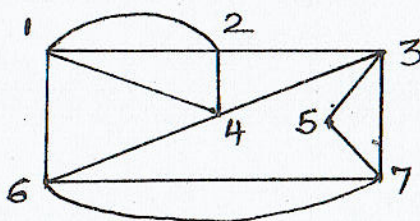Course Code: UCEC305

Duration: 3Hours

Semester: III

Branch: COMP

Instructions:
1. All Questions are Compulsory
2. Assume suitable data wherever necessary.

Question No.		Max. Marks
Q 1	a) Show that $1^2 + 2^2 + 3^2 + \ldots + n^2 = n(n+1)(2n+1)/6$, $n \geq 1$ by mathematical induction.	5
	b) If f:R → R is given by y=2x+1, Prove that f is one to one and onto.	5
	c) Find how many integers between 1 and 60 are not divisible by 2,3 and 5 respectively.	5
	d) Solve the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$, $n \geq 2$ with initial conditions $a_0 = 0$, $a_1 = 1$.	5
Q 2	a) Let R be a relation on A. Prove that i) If R is reflexive, so is R^{-1}. ii) R is symmetric if and only if $R = R^{-1}$. iii) R is antisymmetric if and only if $R \cap R^{-1} \subseteq I_A$.	10
	b) If 11 numbers between 1 and 20 are chosen, show that at least two of them will be multiples of each other.	5
	c) Show that the maximum number of edges in a simple graph with n vertices is n(n-1)/2.	5
Q 3	a) Let A={1,2,3,4 }and relation R={(1,4),(2,1),(2,3),(3,1),(3,4) and (4,3)} State Warshall's algorithm and Find the transitive closure of R by Warshall's Algorithm.	10
	b) Test whether the following function is injective F: Z→Z, $f(x) = x^2 + x + 1$	05

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	c) Let A be a set of integers and let R be a relation on AxA defined by (a,b) R (c,d) if a+d=b+c. Prove that R is an equivalence relation.		05
	OR		
Q 3	a) If R is a relation from A to B and S is a relation from B to C. Show that $(RoS)^{-1} = S^{-1} o R^{-1}$.		5
	b) Consider the poset A=({1, 2, 3, 4, 6, 9, 12, 18,36}, /) find the greatest lower bound and the least upper bound of the sets {6,8} and {4,6,9}.		5
	c) Obtain the conjunctive normal form of $(\sim p \wedge q \wedge r) \vee (p \wedge q)$		5
	d) Check whether A={2,4,12,16} is lattice under divisibility.		5
Q 4	a) Determine whether the following graphs are isomorphic?		5
	b) A connected planar graph has 10 vertices each of degree 3. Into how many regions, does a representation of this planar graph split the plane?		5
	c) If eight persons are chosen from any group, show that at least two of them will have the same birthday in a week.		5
	d) Define Eulerian path and circuit. Find Eulerian path and circuit.		5

Q 5	a) Consider the (2, 6) group encoding function $e : B^2 \rightarrow B^6$ defined by	10
	$e(00) = 000000 \quad e(01) = 011110$	
	$e(10) = 101101 \quad e(11) = 110011$	
	Decode the following relative to maximum likelihood decoding function.	
	i) 001110 ii) 111101 iii) 110010	
	b) Prove that if $a^2 = a$, then a=e, a being an element of a group.	05
	c) Show that the set {1,2,3,4,5} is not a group under addition and multiplication modulo 6.	05

OR

Q 5	a) Show that the (3, 6) encoding function $e : B^3 \rightarrow B^6$ defined by	10
	$e(000) = 000000 \quad e(100) = 100101$	
	$e(001) = 000110 \quad e(101) = 100011$	
	$e(010) = 010010 \quad e(110) = 110111$	
	$e(011) = 010100 \quad e(111) = 110001$	
	is a group code.	
	b) Let H = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix, Determine the group code $e_H : B^3 \rightarrow B^6$	10