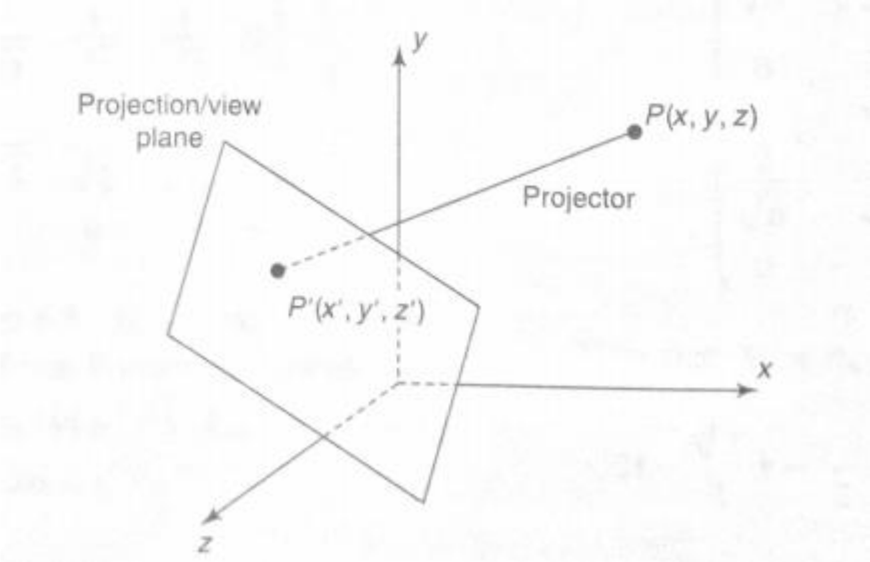
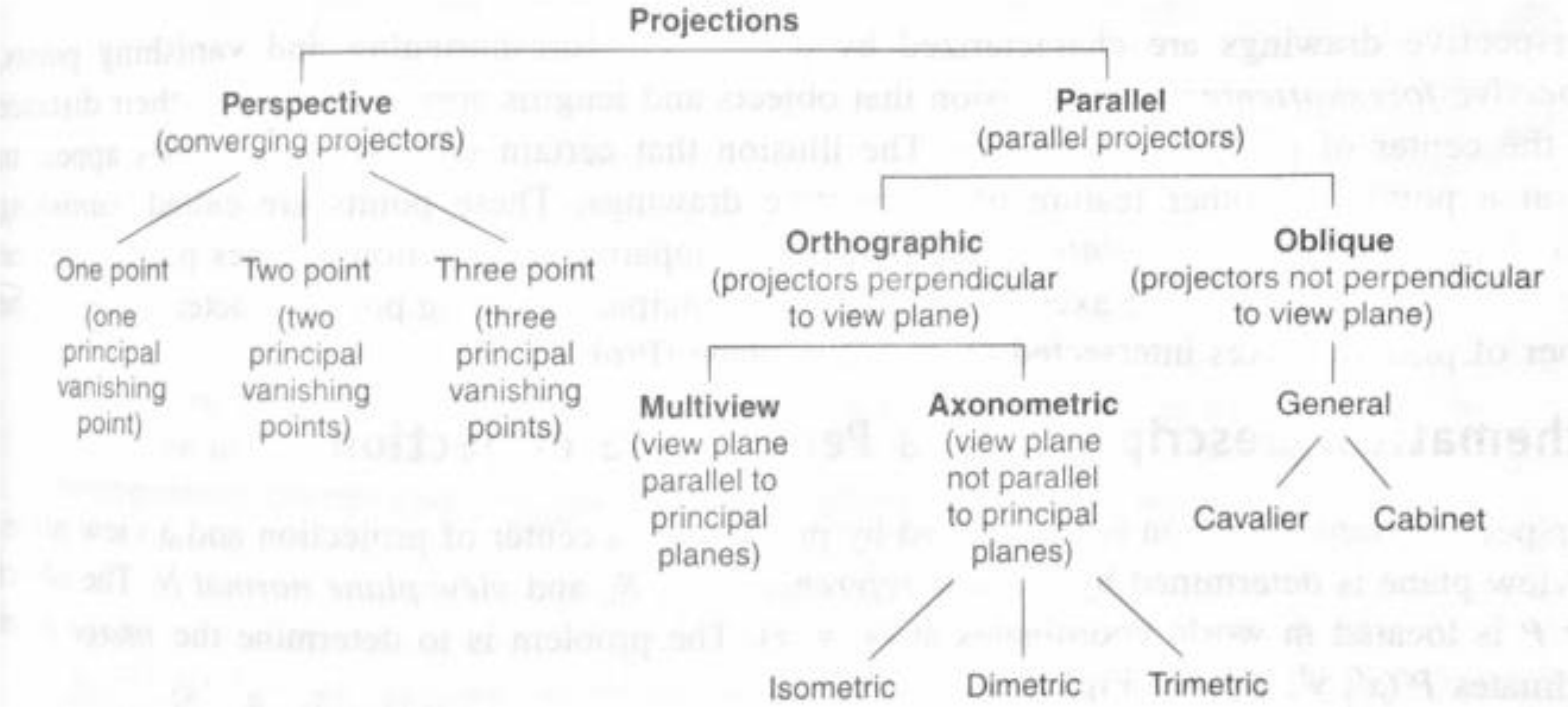
Introduction to Computer Graphics and Visualization

By
Prof. Vaibhav P. Vasani
Assistant Professor
Department of Computer Engineering
K. J. Somaiya College of Engineering
Somaiya Vidyavihar University

Projection

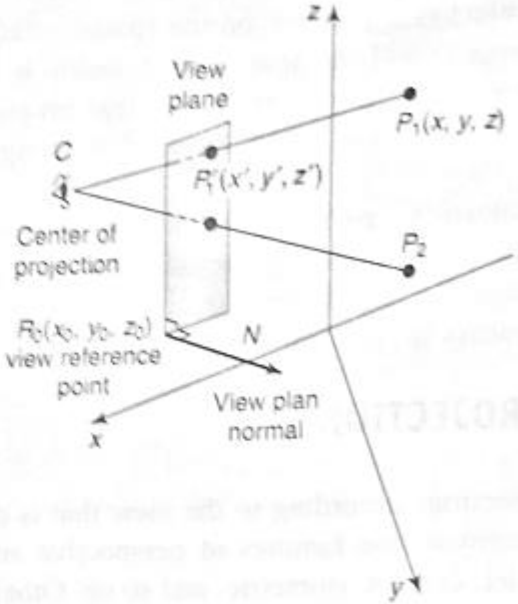


SOMAIYA VIDYAVIHAR UNIVERSITY
K J Somaiya College of Engineering



Somaiya TRUST

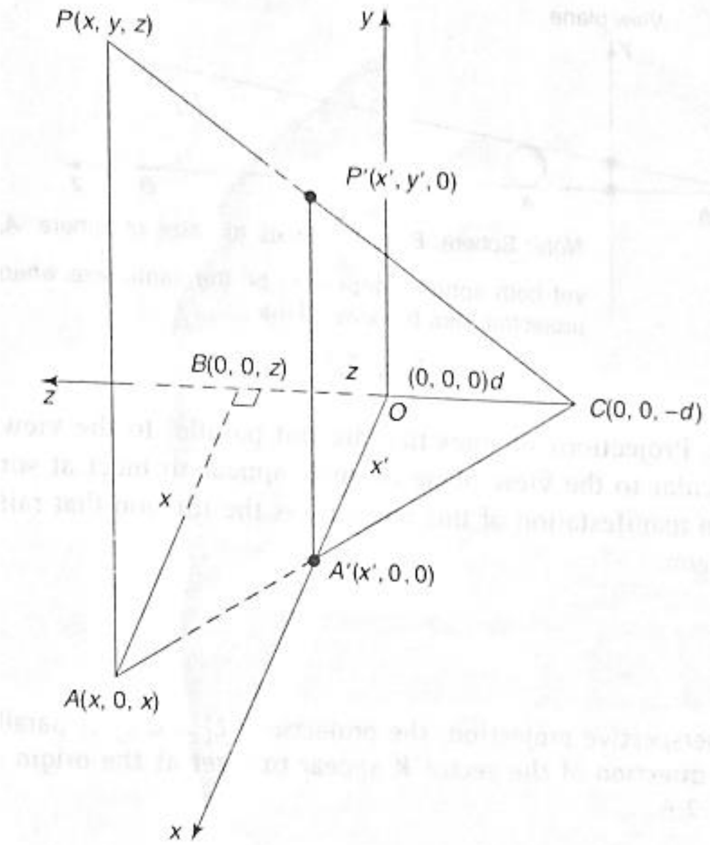
The techniques of perspective projection are generalizations of the principles used by artists in preparing perspective drawings of three-dimensional objects and scenes. The eye of the artist is placed at the *center of projection*, and the canvas, or more precisely the plane containing the canvas, becomes the view plane. An image point is determined by a projector that goes from an object point to the center of projection (see Fig. 7.3).



The standard perspective projection is shown in Fig. 7.4. Here, the view plane is the *xy* plane, and the center of projection is taken as the point $C(0, 0, -d)$ on the negative z axis.

Using similar triangles ABC and $A'OC$, we find

$$x' = \frac{d \cdot x}{z + d} \qquad y' = \frac{d \cdot y}{z + d} \qquad z' = 0$$



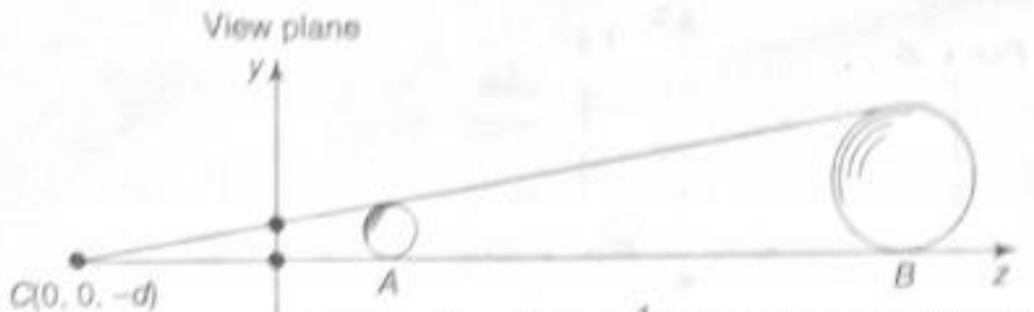
The perspective transformation between object and image point is nonlinear and so cannot be represented as a 3×3 matrix transformation. However, if we use homogeneous coordinates, the perspective transformation can be represented as a 4×4 matrix:

$$(x' \quad y' \quad z' \quad 1) = (d \cdot x \quad d \cdot y \quad 0 \quad z + d) = (x \quad y \quad z \quad 1)\begin{pmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & d \end{pmatrix}$$

Perspective Anomalies

The process of constructing a perspective view introduces certain anomalies which enhance realism in terms of depth cues but also distort actual sizes and shapes.

1. *Perspective foreshortening.* The farther an object is from the center of projection, the smaller it appears (i.e. its projected size becomes smaller). Refer to Fig. 7.5.

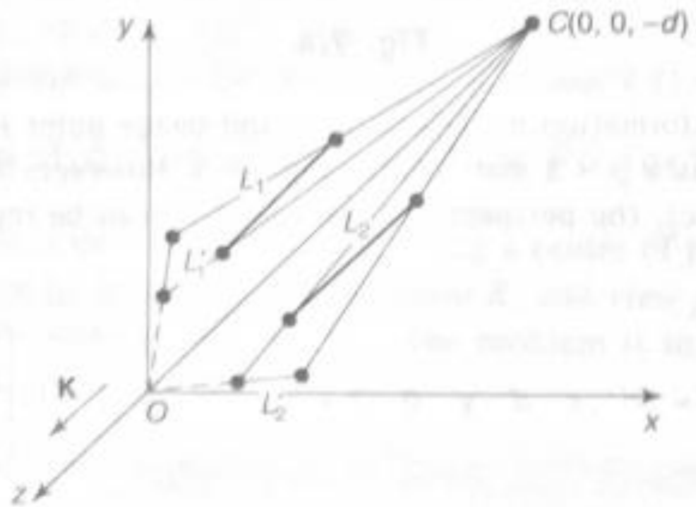


Note: Sphere B is $2\frac{1}{2}$ times the size of sphere A, yet both spheres appear to be the same size when projected onto the view plane

2. *Vanishing points.* Projections of lines that are not parallel to the view plane (i.e. lines that are not perpendicular to the view plane normal) appear to meet at some point on the view plane. A common manifestation of this anomaly is the illusion that railroad tracks meet at a point on the horizon.

Example 7.2

For the standard perspective projection, the projections L_1' and L_2' of parallel lines L_1 and L_2 having the direction of the vector **K** appear to meet at the origin (Problem 7.8). Refer to Fig. 7.6.



3. *View confusion.* Objects behind the center of projection are projected upside down and backward onto the view plane. Refer to Fig. 7.7.

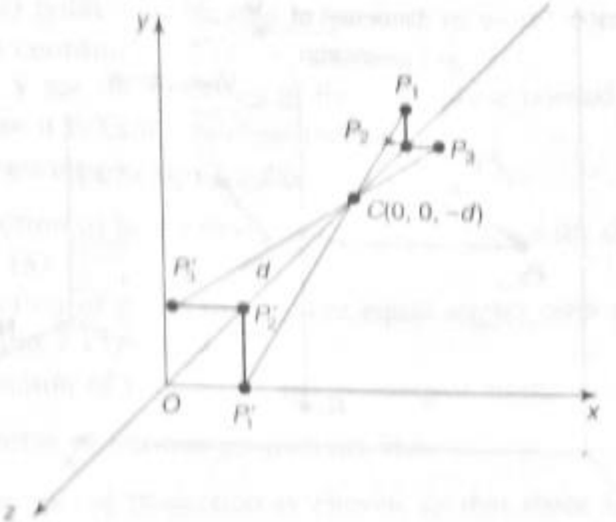


Fig. 7.7

SOMAIYA VIDYAVIHAR UNIVERSITY
K J Somaiya College of Engineering

4. *Topological distortion.* Consider the plane that passes through the center of projection and is parallel to the view plane. The points of this plane are projected to infinity by the perspective transformation. In particular, a finite line segment joining a point which lies in front of the viewer to a point in back of the viewer is actually projected to a broken line of infinite extent (Problem 7.2) (see Fig. 7.8).

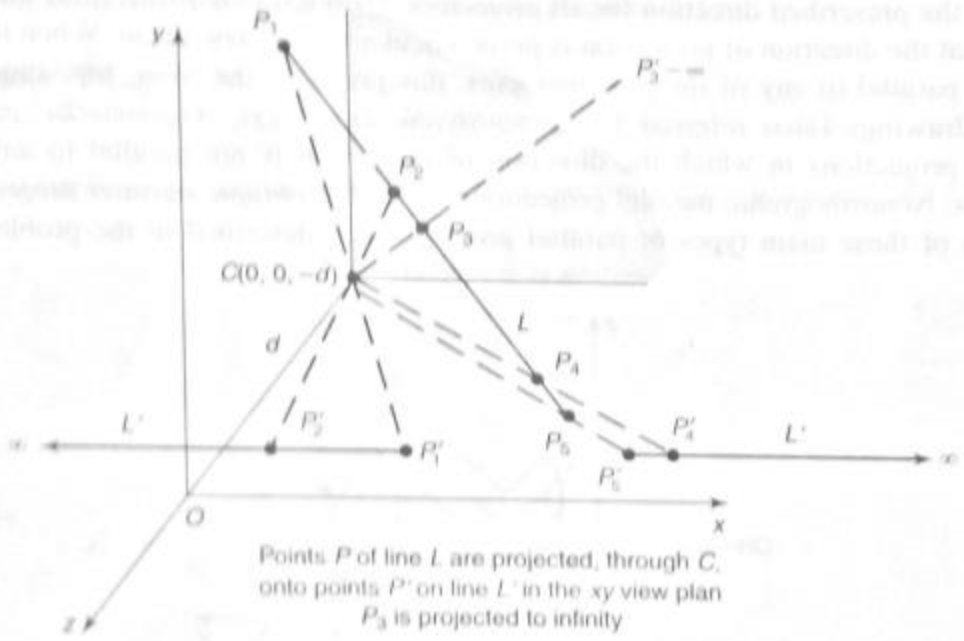


Fig. 7.8

Somaiya TRUST

Example 7.3

For orthographic projection onto the *xy* plane, from Fig. 7.11 it is easy to see that

$$Par_K: \begin{cases} x' = x \\ y' = y \\ z' = 0 \end{cases}$$

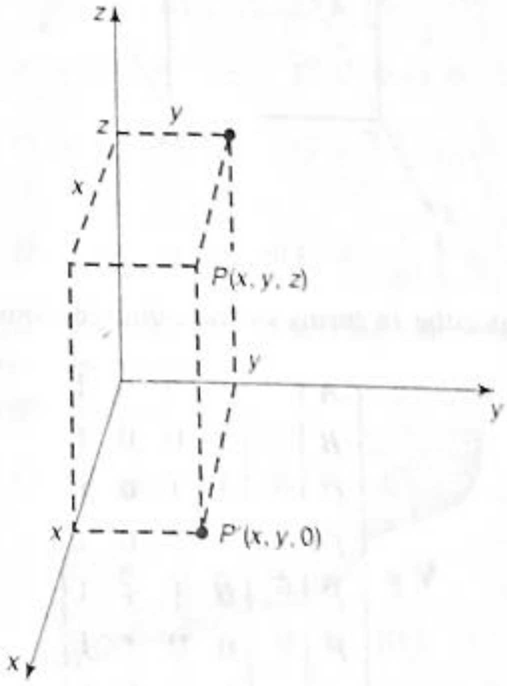


Fig. 7.11

The matrix form of Par_K is

$$Par_K = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The general parallel projective transformation is derived in Solved Problem 7.11.

7.1 The unit cube (Fig. 7.12) is projected onto the xy plane. Note the position of the x, y, and z axes. Draw the projected image using the standard perspective transformation with (a) $d = 1$ and (b) $d = 10$, where d is distance from the view plane.

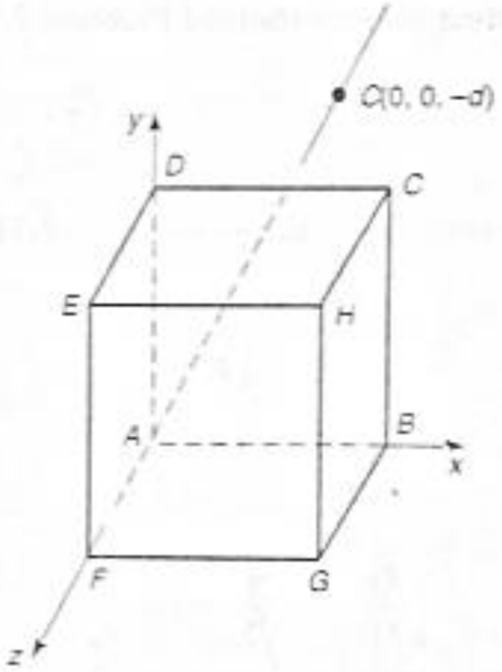


Fig. 7.12

We represent the unit cube in terms of the homogeneous coordinates of its vertices:

$$\mathbf{V} = \begin{pmatrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

From Example 7.1 the standard perspective matrix is

$$Per_{\mathbf{K}} = \begin{pmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & d \end{pmatrix}$$

(a) With $d = 1$, the projected coordinates are found by applying the matrix $Per_{\mathbf{K}}$ to the matrix of coordinates $\mathbf{V}$. Then

$$\mathbf{V} \cdot Per_{\mathbf{K}} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 2 \\ 1 & 1 & 0 & 2 \end{pmatrix}$$

If these homogeneous coordinates are changed to three-dimensional coordinates, the projected image has coordinates:

$$A' = (0, 0, 0) \qquad E' = \left(0, \frac{1}{2}, 0\right)$$

$$B' = (1,0,0) \qquad F' = (0, 0, 0)$$

$$C' = (1,1,0) \qquad G' = \left(\frac{1}{2}, 0, 0\right)$$

$$D' = (0,1,0) \qquad H' = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$$

We draw the projected image by preserving the edge connections of the original object (see Fig. 7.13). [Note the vanishing point at (0, 0, 0).]

SOMAIYA VIDYAVIHAR UNIVERSITY
K J Somaiya College of Engineering

(b) With $d = 10$, the perspective matrix is

$$Per_{\mathbf{K}} = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 10 \end{pmatrix}$$

Then

$$\mathbf{V} \cdot Per_{\mathbf{K}} = \begin{pmatrix} 0 & 0 & 0 & 10 \\ 10 & 0 & 0 & 10 \\ 10 & 10 & 0 & 10 \\ 0 & 10 & 0 & 10 \\ 0 & 10 & 0 & 11 \\ 0 & 0 & 0 & 11 \\ 10 & 0 & 0 & 11 \\ 10 & 10 & 0 & 11 \end{pmatrix}$$

is the matrix image coordinates in homogeneous form. The projected image coordinates are then

$$A' = (0, 0, 0) \qquad E' = \left(0, \frac{10}{11}, 0\right)$$

$$B' = (1, 0, 0) \qquad F' = (0, 0, 0)$$

$$C' = (1, 1, 0) \qquad G' = \left(\frac{10}{11}, 0, 0\right)$$

$$D' = (0, 1, 0) \qquad H' = \left(\frac{10}{11}, \frac{10}{11}, 0\right)$$

Somaiya TRUST

Note the different perspectives of the face $E'F'G'H'$ in Figs 7.13 and 7.14. (To a viewer standing at the center of projection $(0, 0, -d)$, this face is the back face of the unit cube.)

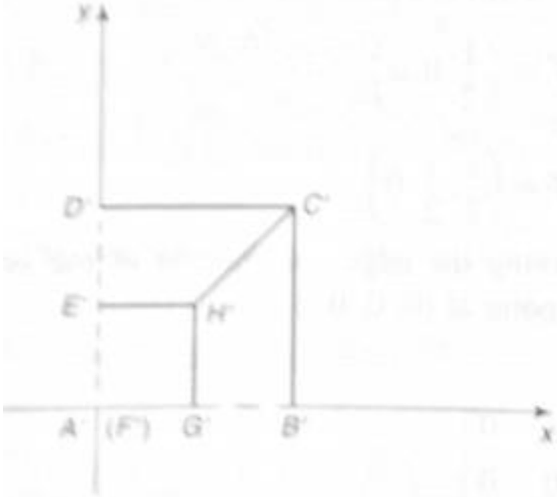


Fig. 7.13

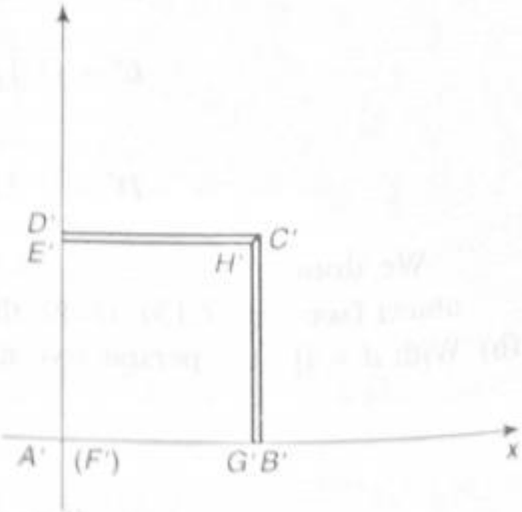


Fig. 7.14

7.2 Under the standard perspective transformation Per_K, what is the projected image of (a) a point in the plane $z = -d$ and (b) the line segment joining $P_1(-1, 1, -2d)$ to $P_2(2, -2, 0)$? (See Fig. 7.15.)

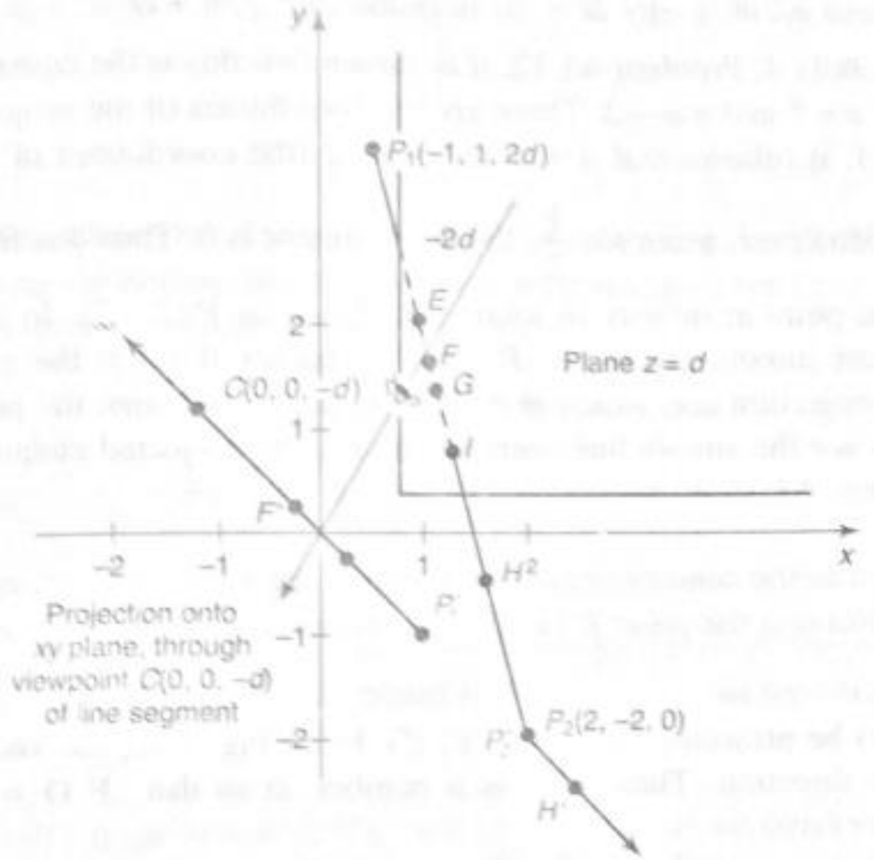


Fig. 7.15

Thank you