SOMAIYA VIDYAVIHAR UNIVERSITY
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3D Computer Graphics

By

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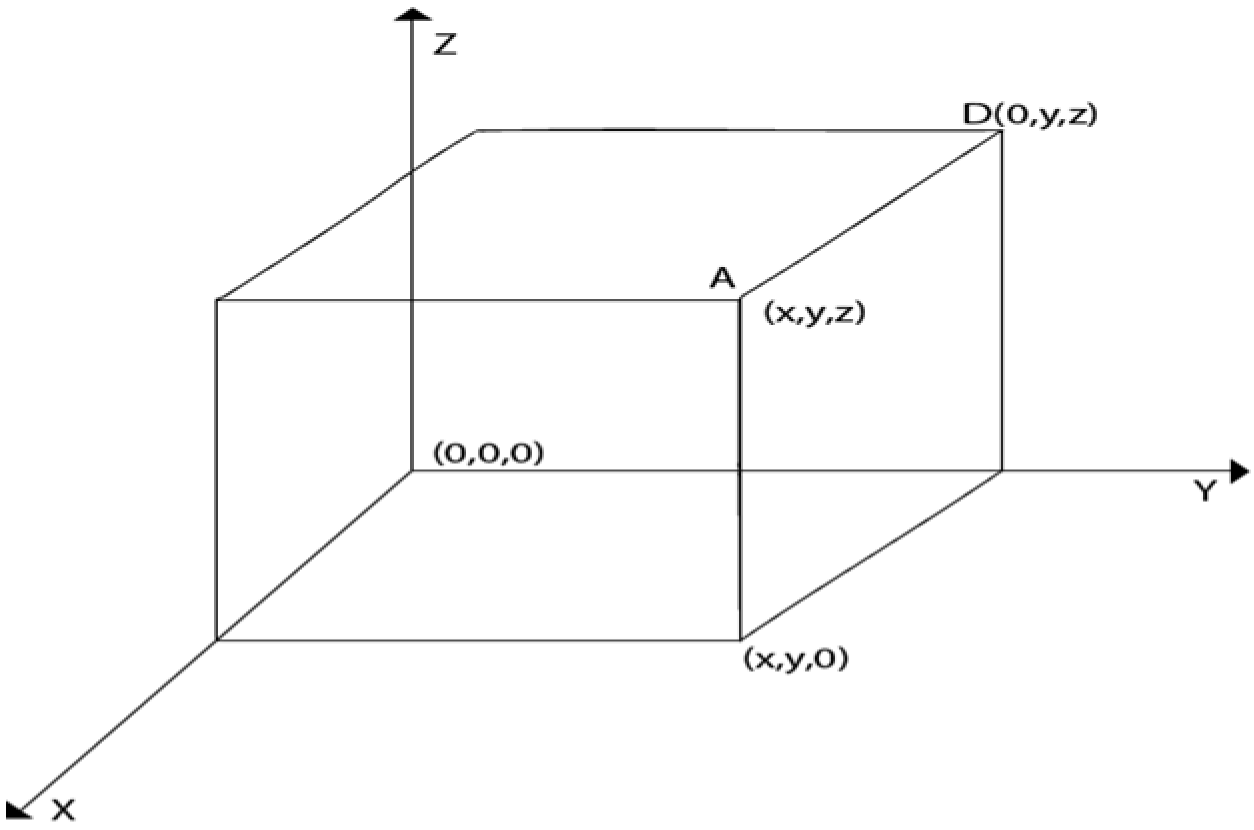
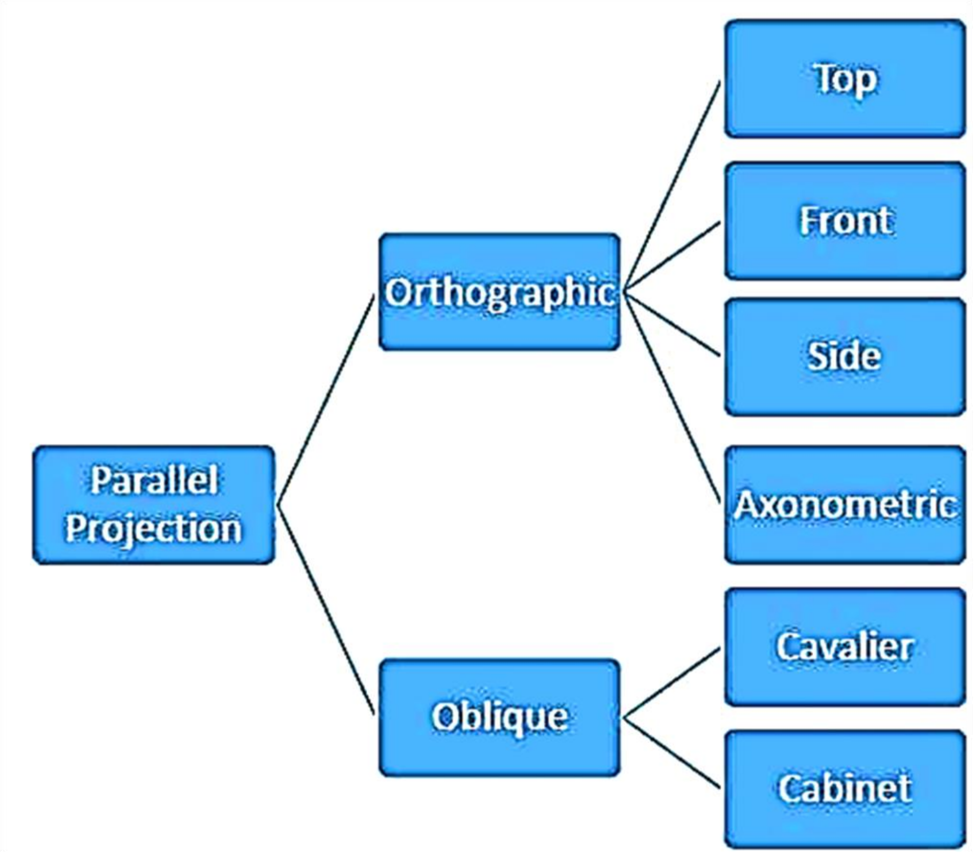
List of Application

- Entertainment
- Games
- Computer-aided design industries
- Scientific visualization.

3 D coordinate System



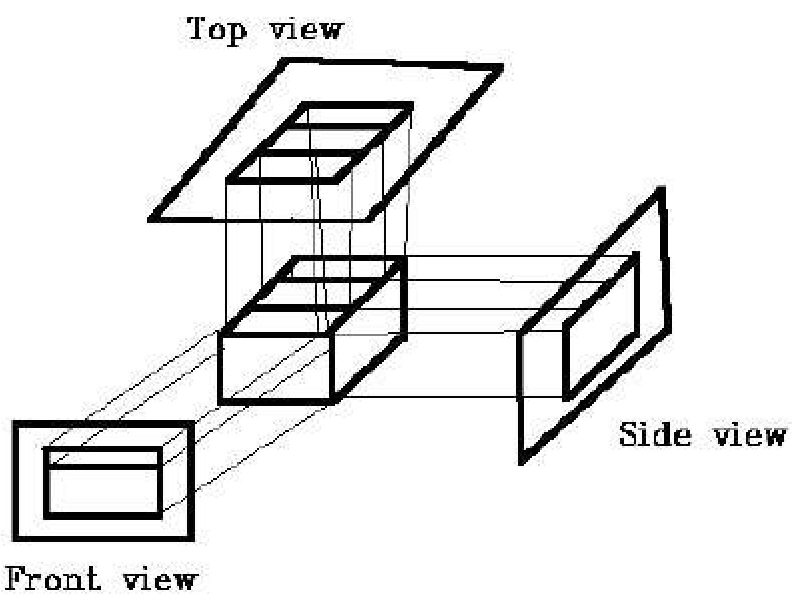
Parallel Projection



- Parallel projection discards z-coordinate
- Parallel lines from each vertex on the object are extended until they intersect the view plane.
- In parallel projection, we specify a direction of projection instead of center of projection.
- Parallel projections are less realistic, but they are good for exact measurements.

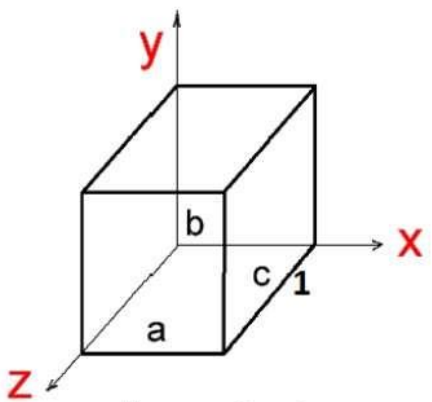
Orthographic Projection

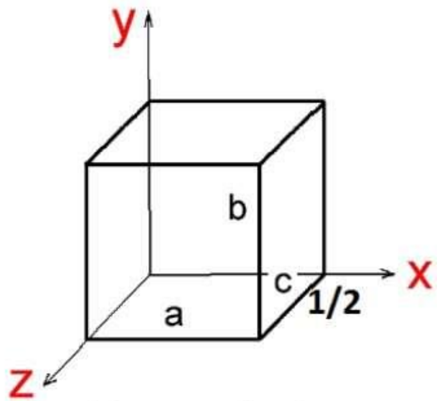
- Front Projection
- Top Projection
- Side Projection



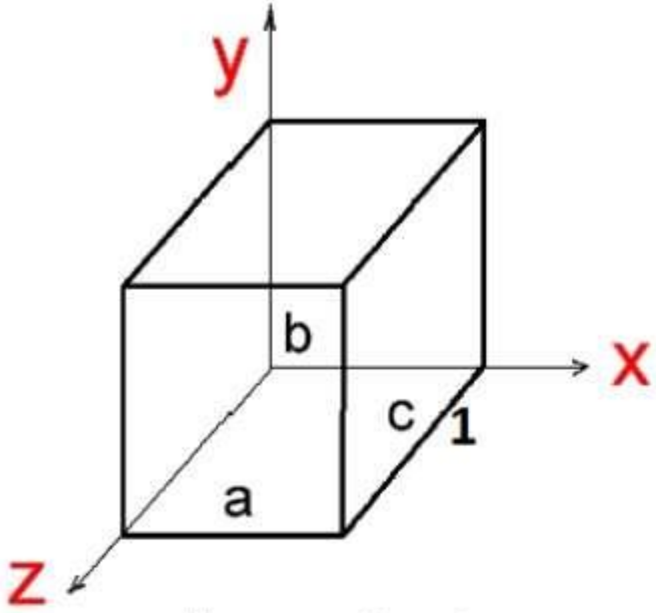
Oblique Projection

- the direction of projection is not normal to the projection of plane.
- the view of object better than orthographic projection.
- Types
  - **Cavalier** and **Cabinet**.

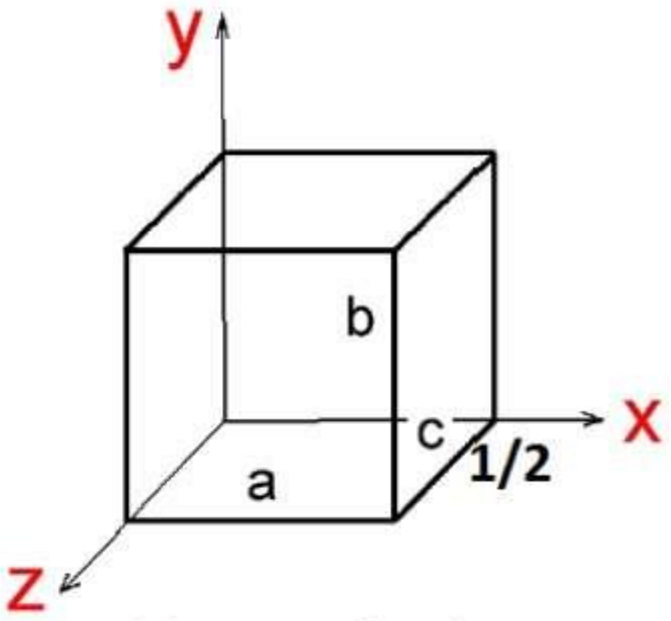




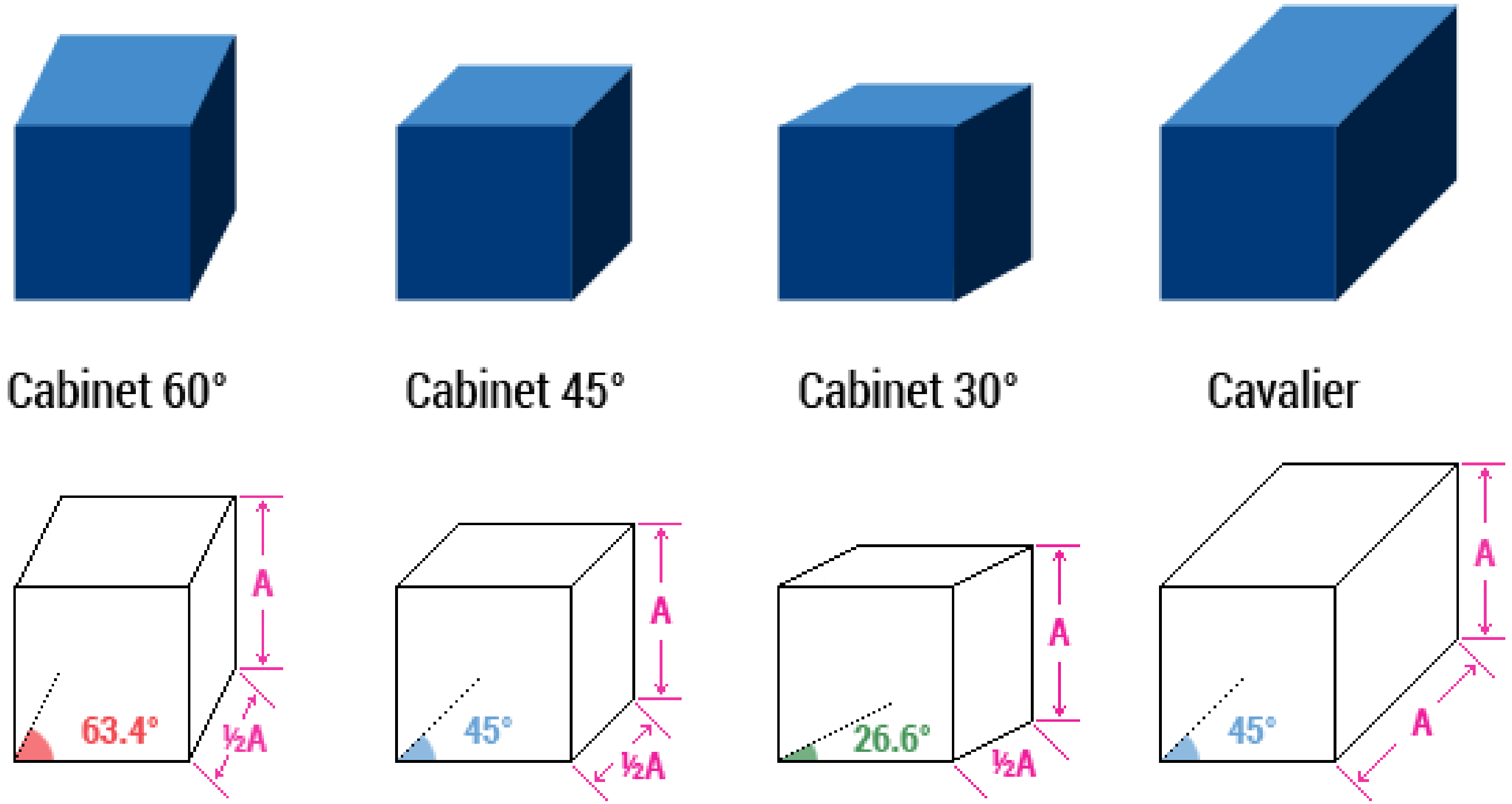
- The Cavalier projection makes 45° angle with the projection plane.
- The projection of a line perpendicular to the view plane has the same length as the line itself in Cavalier projection.
- The Cabinet projection makes 63.4° angle with the projection plane.
- In Cabinet projection, lines perpendicular to the viewing surface are projected at ½ their actual length.



Cavalier Projection

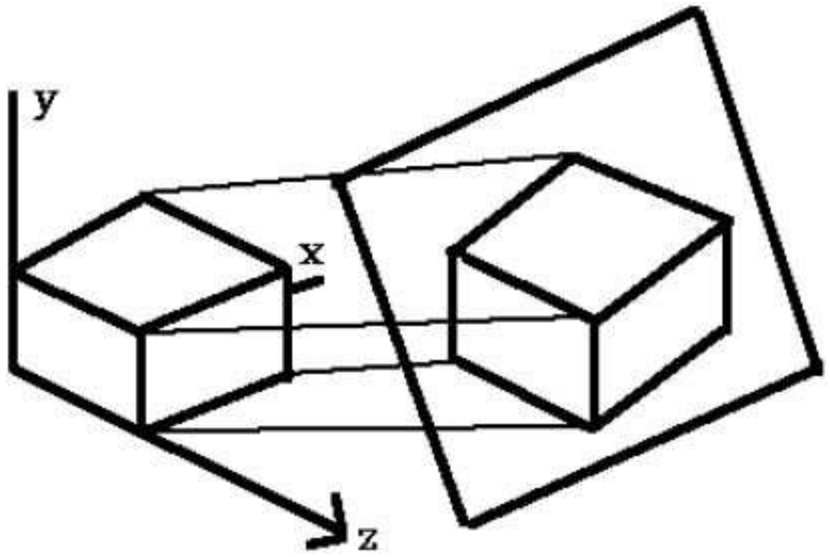


Cabinet Projection


Cabinet 60°
Cabinet 45°
Cabinet 30°
Cavalier
63.4°
45°
26.6°
45°
A
½A
A

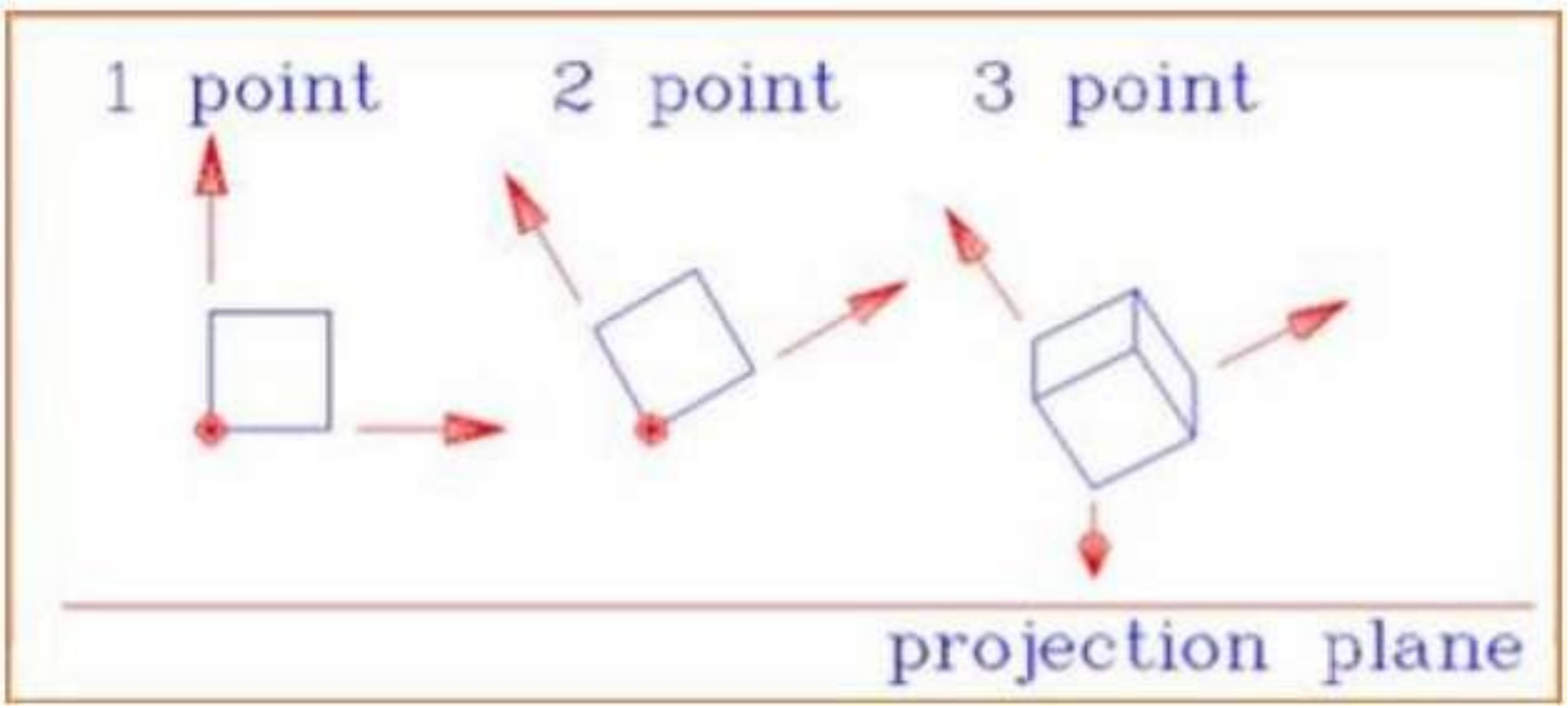
Isometric Projections

- the projection plane intersects each coordinate axis in the model coordinate system at an equal distance.
- In this projection parallelism of lines are preserved but angles are not preserved.



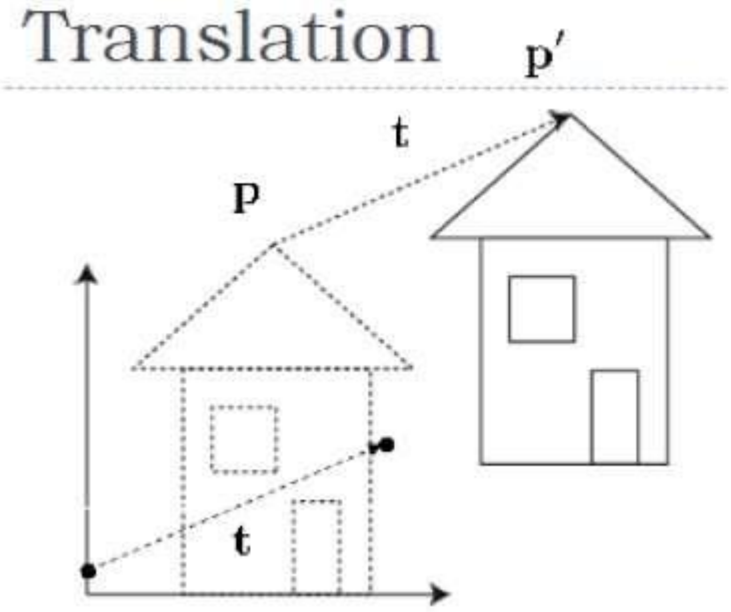
Perspective Projection

- the distance from the center of projection to project plane is finite and the size of the object varies inversely with distance which looks more realistic.
- The distance and angles are not preserved and parallel lines do not remain parallel. Instead, they all converge at a single point called **center of projection** or **projection reference point**.
- Types
  - **One point** perspective projection is simple to draw.
  - **Two point** perspective projection gives better impression of depth.
  - **Three point** perspective projection is most difficult to draw.


1 point
2 point
3 point
projection plane

Translation

- The Z coordinate transfer along with the X and Y coordinates.
- Similar to 2D translation.
- A translation moves an object into a different position on the screen.



- A point can be translated in 3D by adding translation coordinate (tx,ty,tz)(tx,ty,tz) to the original coordinate X,Y,Z to get the new coordinate X′,Y′,Z′.

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

P' = P·T

$$[X' \; Y' \; Z' \; 1] = [X \; Y \; Z \; 1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$

$$= [X + t_x \;\; Y + t_y \;\; Z + t_z \;\; 1]$$

Example

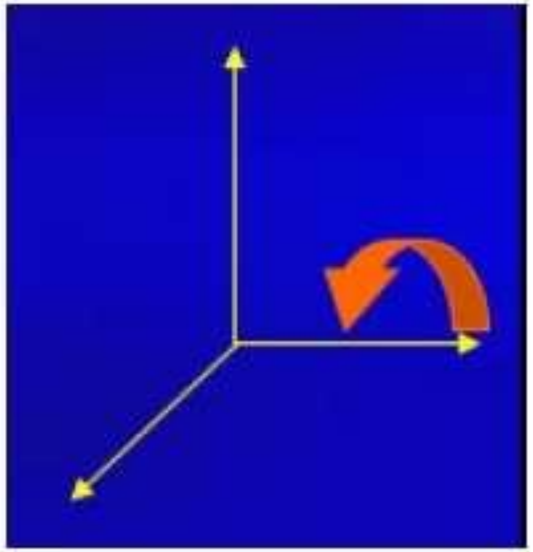
- For given line perform translation
  - A(2,4), B(7,8)
  - Translate with 2 unit distance

Rotation

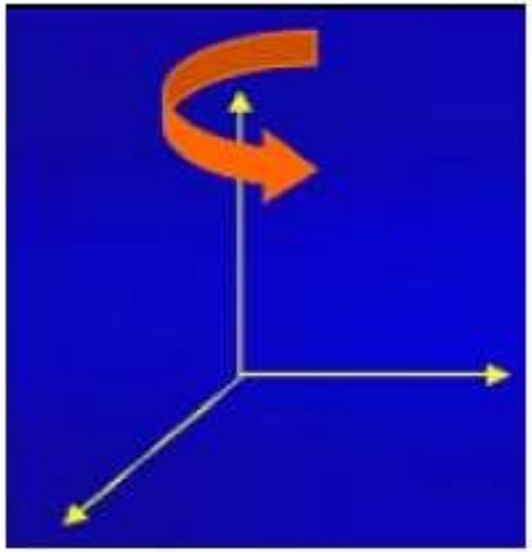
- 3D rotation is not same as 2D rotation. In 3D rotation, we have to specify the angle of rotation along with the axis of rotation.

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\theta & -sin\theta & 0 \\ 0 & sin\theta & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} R_y(\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -sin\theta & 0 & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} R_z(\theta)$$

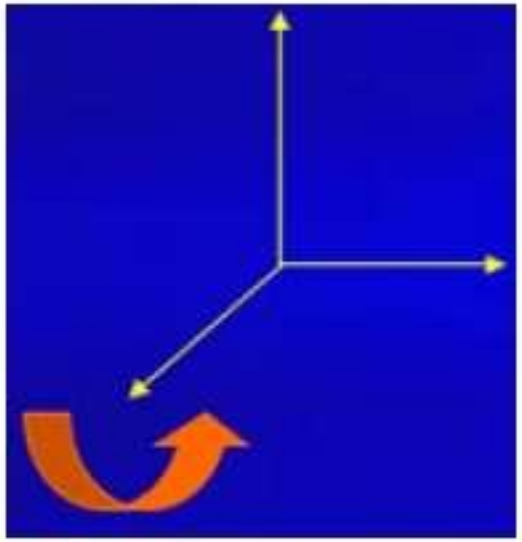
$$= \begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Rotation about x-axis

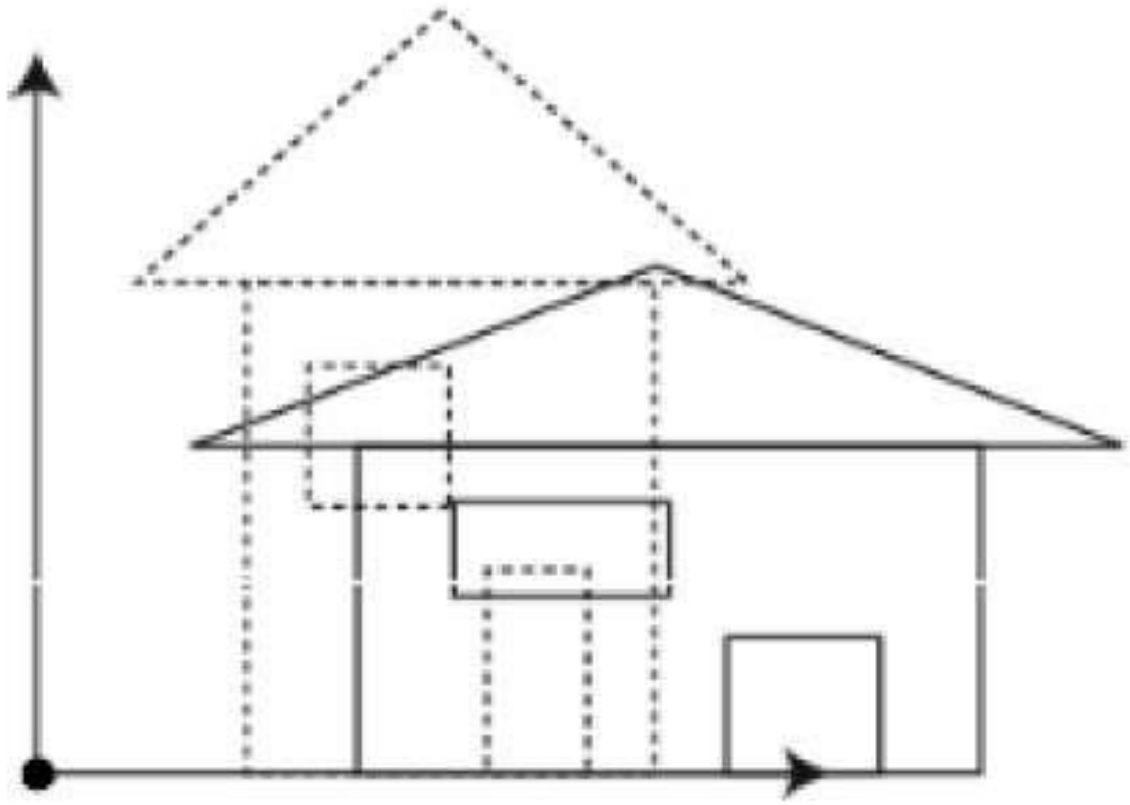


Rotation about y-axis



Rotation about z-axis

Scaling



- In 3D scaling operation, three coordinates are used. Let us assume that the original coordinates are X,Y,Z scaling factors are (Sx,Sy,Sz) respectively, and the produced coordinates are X′,Y′,Z′

$$S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

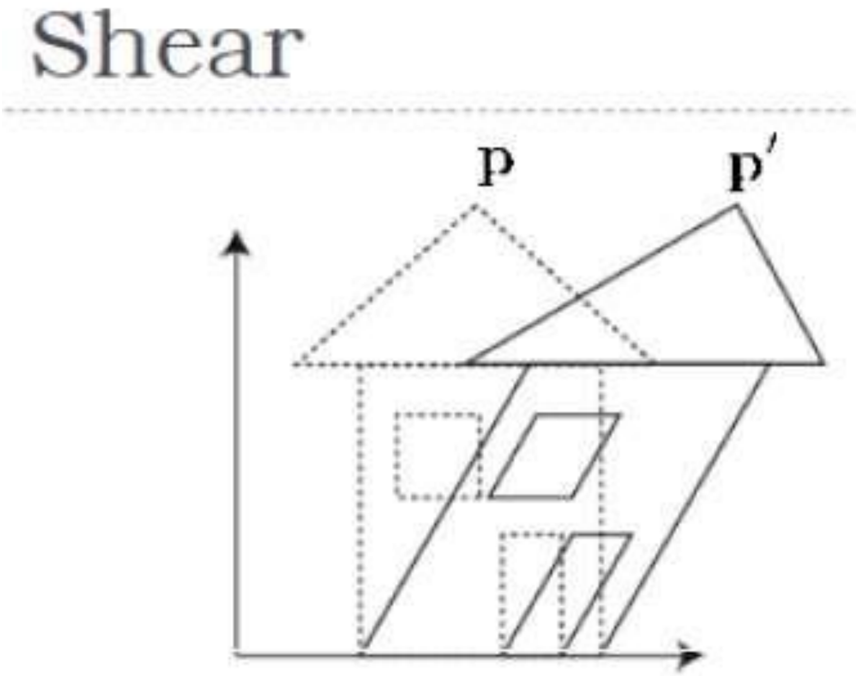
P' = P·S

$$[X' \ \ Y' \ \ Z' \ \ 1] = [X \ \ Y \ \ Z \ \ 1] \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= [X.S_x \ \ Y.S_y \ \ Z.S_z \ \ 1]$$

Shear

- A transformation that slants the shape of an object is called the **shear transformation**. Like in 2D shear, we can shear an object along the X-axis, Y-axis, or Z-axis in 3D.



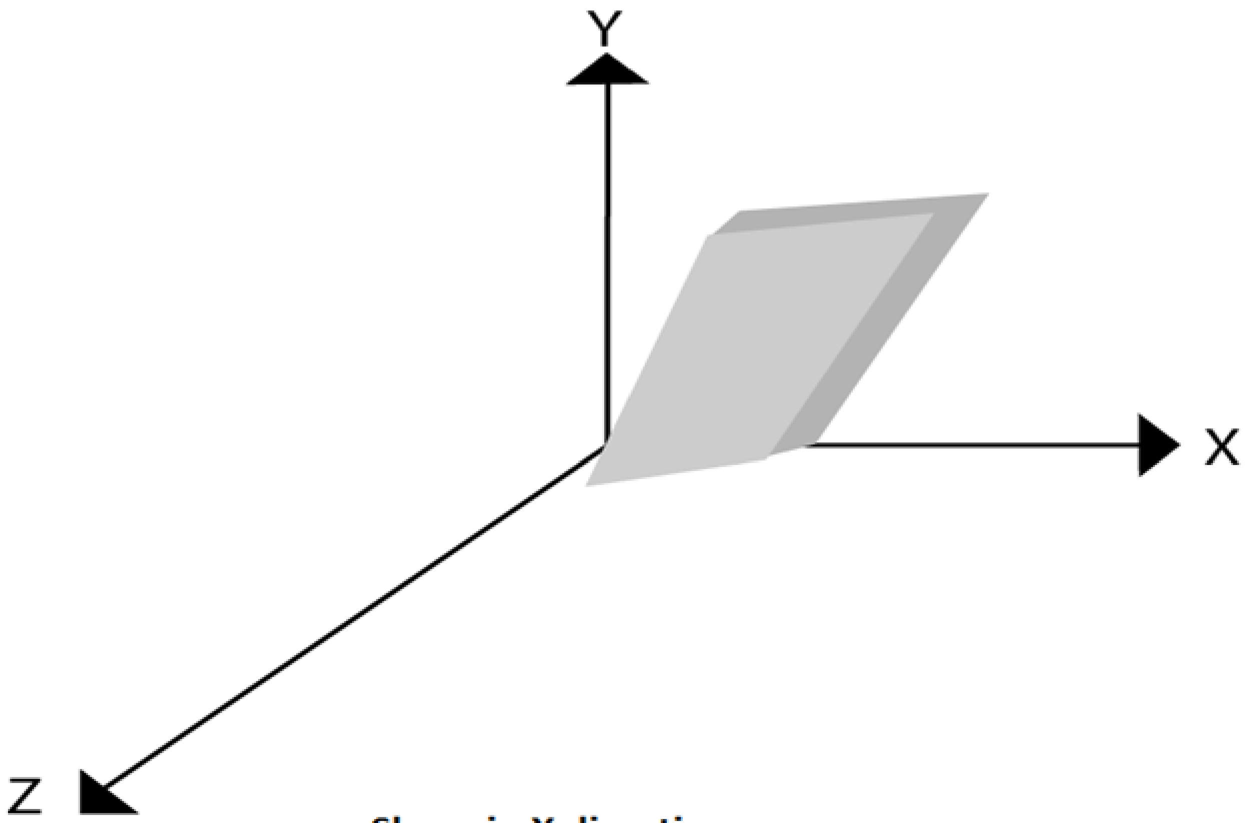
$$Sh = \begin{bmatrix} 1 & sh_x^y & sh_x^z & 0 \\ sh_y^x & 1 & sh_y^z & 0 \\ sh_z^x & sh_z^y & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

P' = P · Sh

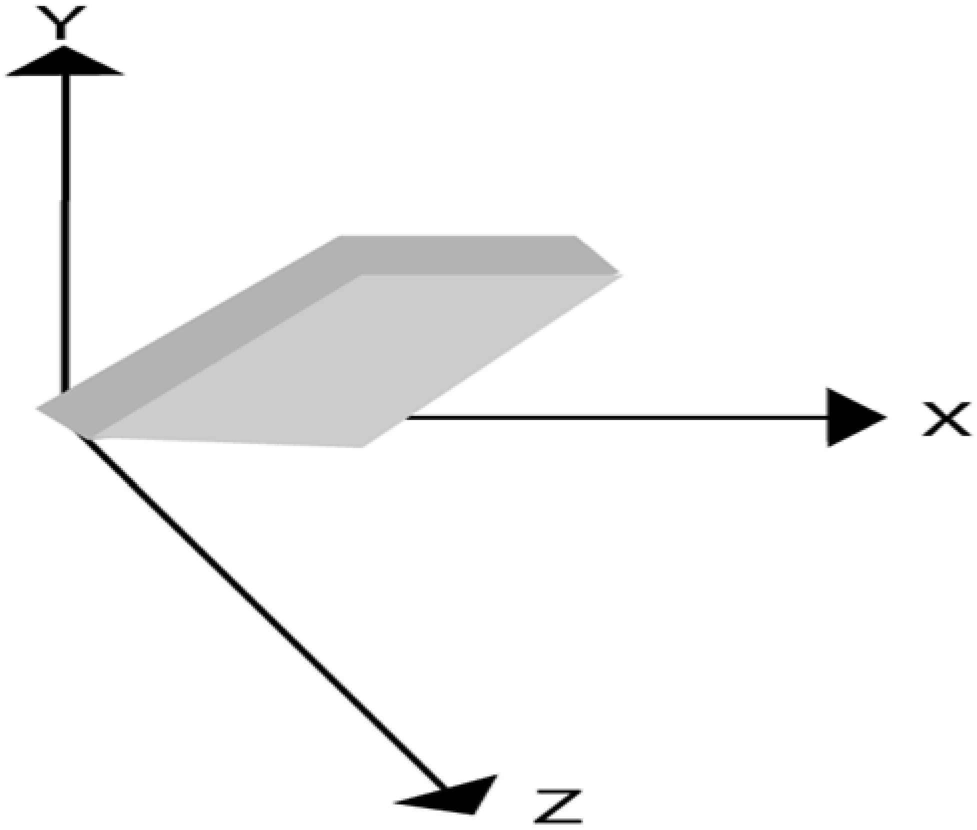
$$X' = X + Sh_x^y Y + Sh_x^z Z$$

$$Y' = Sh_y^x X + Y + sh_y^z Z$$

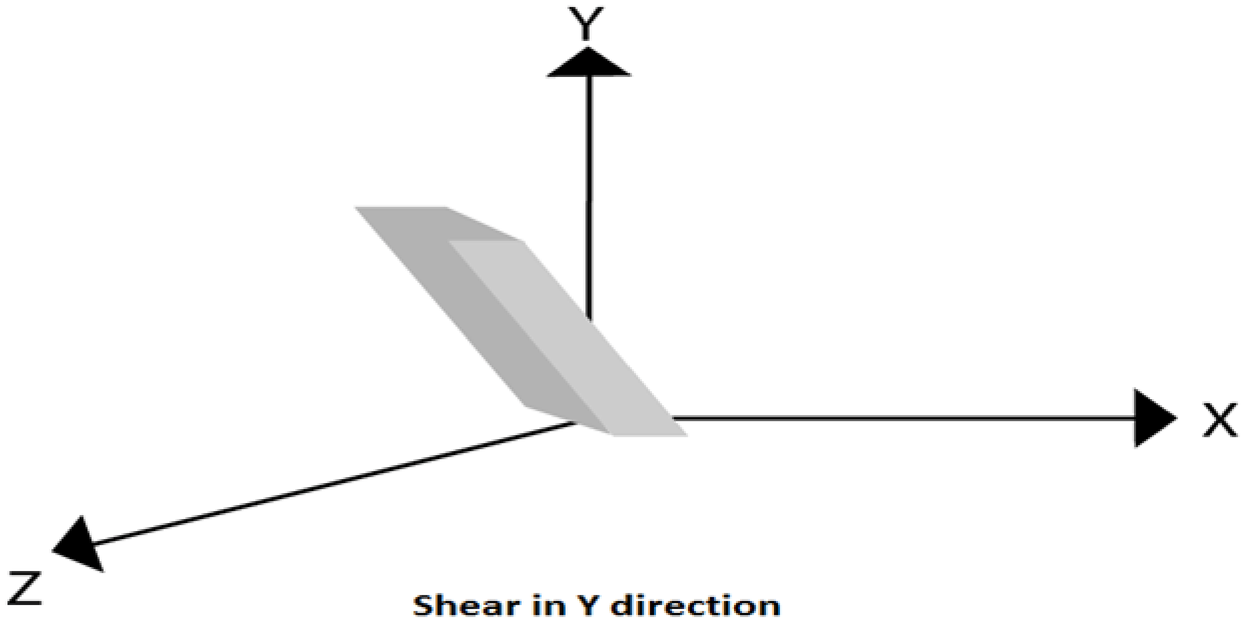
$$Z' = Sh_z^x X + Sh_z^y Y + Z$$



Shear in X direction
(a)



Shear in X, Y direction
(b)



Shear in Y direction
(c)

Inverse Transformations

- If T is a translation matrix than inverse translation is representing using T^{-1}. The inverse matrix is achieved using the opposite sign.

- **Translation matrix**

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{pmatrix}$$

- **Inverse translation matrix**

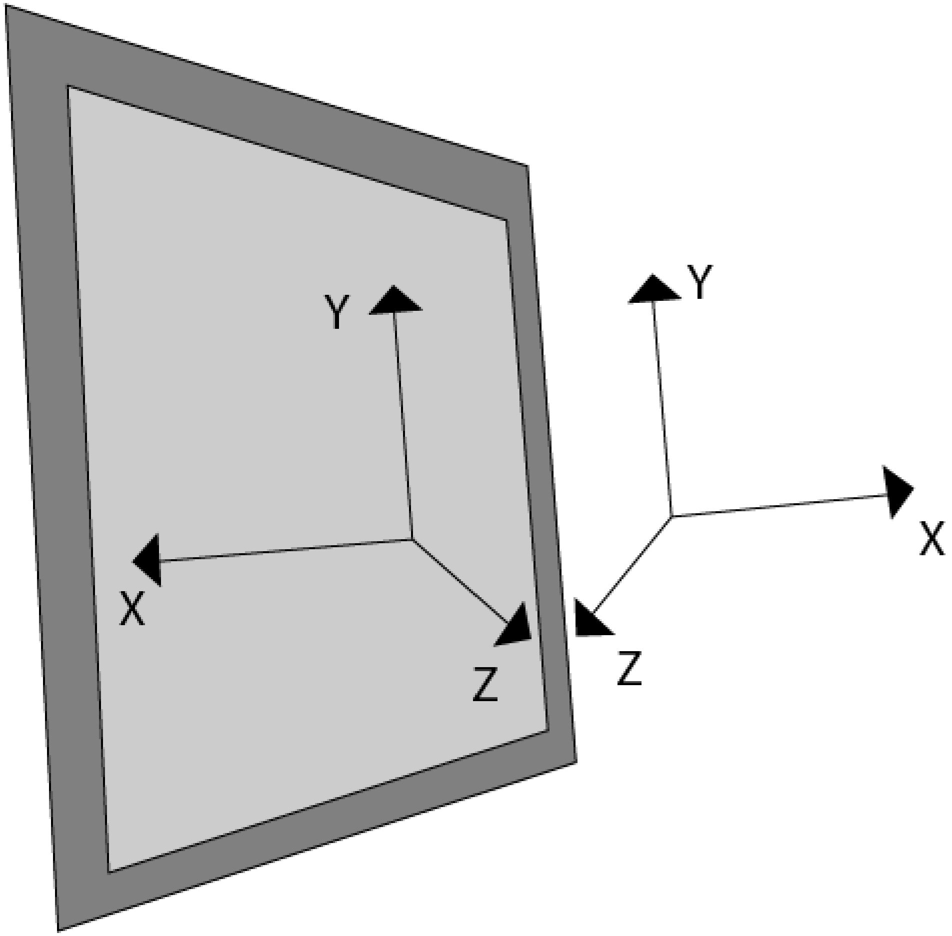
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -T_x & -T_y & -T_z & 1 \end{pmatrix}$$

Rotation and its inverse matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} -\cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & -\cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Reflection

- A mirror image of an object.



Reflection relative to XY plane

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Reflection relative to YZ plane

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Reflection relative to ZX plane

Thank you