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Graph Analytics

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Adopted from: Mining of Massive Datasets
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Stanford University
<http://www.mmds.org>



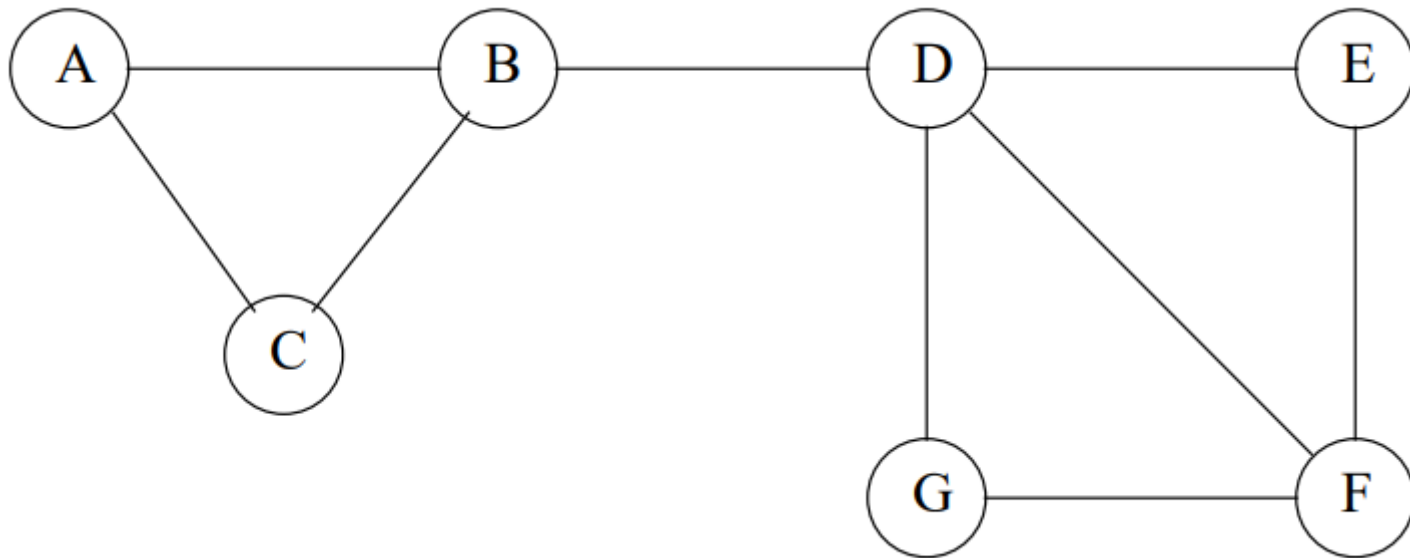
What is a Social Network?

- The essential characteristics of a social network are:
- There is a collection of entities that participate in the network.
- There is at least one relationship between entities of the network
- There is an assumption of non-randomness or locality.

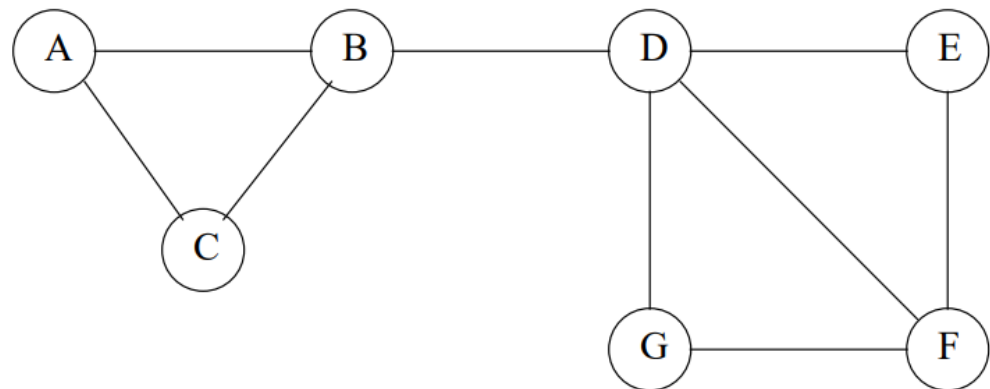
Social Graphs

- Social networks are naturally modeled as graphs, which we sometimes refer to as a **social graph**.
- The entities are the nodes, and an edge connects two nodes if the nodes are related by the relationship that characterizes the network.
- If there is a degree associated with the relationship, this degree is represented by labeling the edges.
- Often, social graphs are undirected, as for the Facebook friends graph.
- But they can be directed graphs, as for example the graphs of followers on X or Instagram.

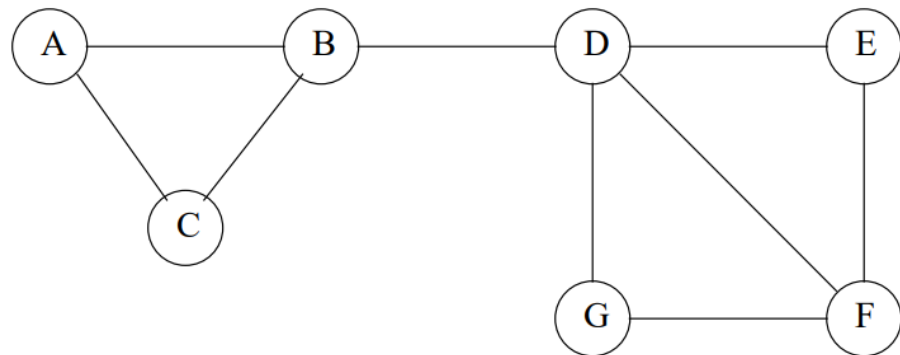
Example of a social graph



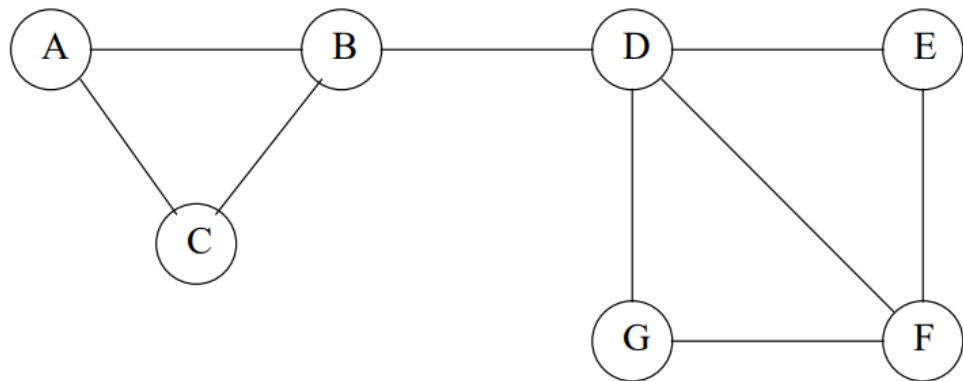
- Figure in the previous slide is an example of a tiny social network.
- The entities are the nodes A through G.
- B is friends with A, C, and D.
- Is this graph really typical of a social network, in the sense that it exhibits locality of relationships?



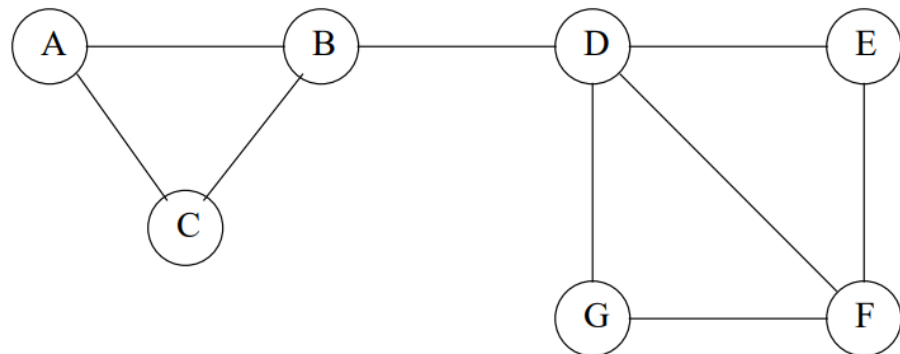
- First, note that the graph has nine edges out of the $7C2 = 21$ pairs of nodes that could have had an edge between them.
- Suppose X, Y , and Z are nodes, with edges between X and Y and also between X and Z .
- What would we expect the probability of an edge between Y and Z to be?



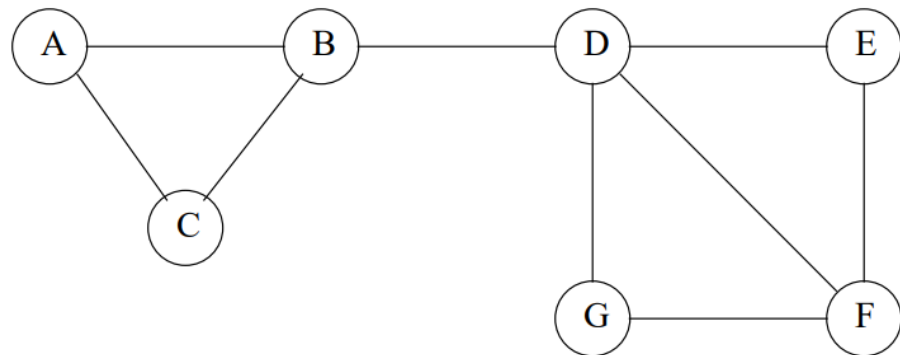
- Since we already know there are edges (X, Y) and (X, Z) , there are only seven edges remaining.
- Those seven edges could run between any of the 19 remaining pairs of nodes.
- Thus, the probability of an edge (Y, Z) is $7/19 = .368$



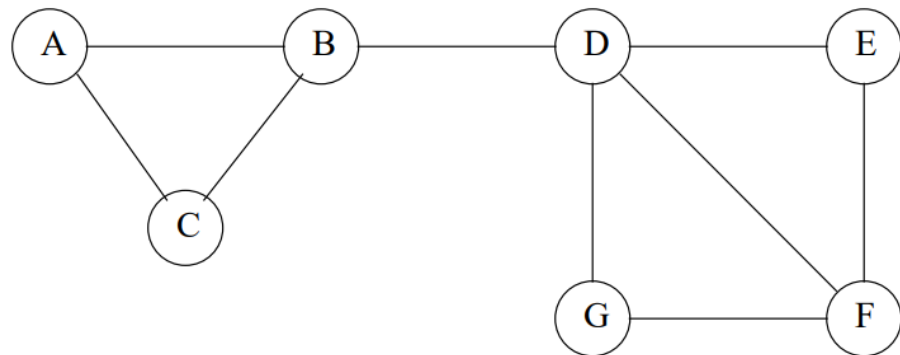
- Now, we must compute the probability that the edge (Y, Z) exists given that edges (X, Y) and (X, Z) exist.
- What we shall count is pairs of nodes that could be Y and Z , without worrying about which node is Y and which is Z



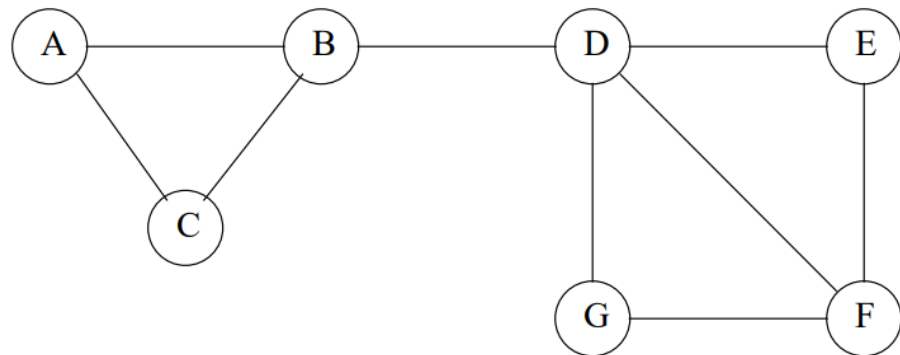
- If X is A , then Y and Z must be B and C , in some order.
- Since the edge (B, C) exists, $X=A$ contributes one positive example (where the edge does exist) and no negative examples (where the edge is absent).



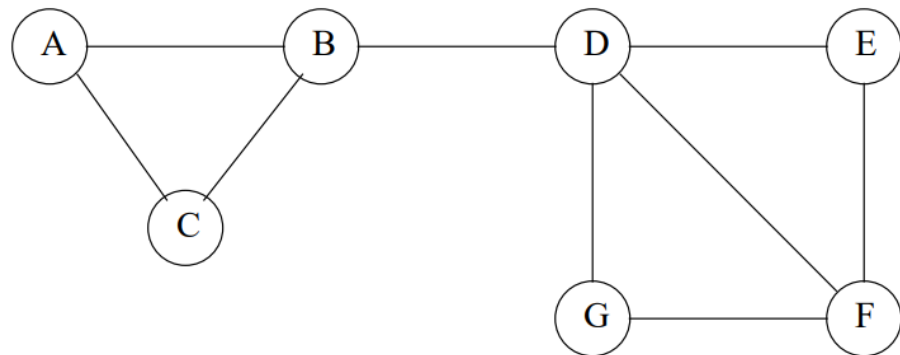
- The cases where X is C , E , or G are essentially the same.
- In each case, X has only two neighbors, and the edge between the neighbors exists.
- Thus, we have seen four positive examples and zero negative examples so far.



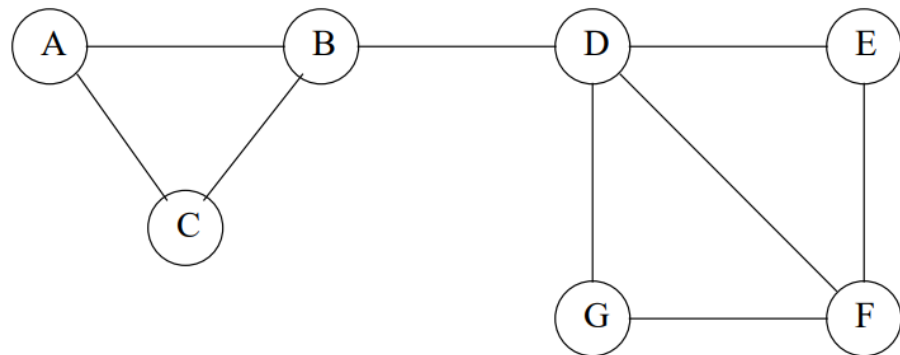
- Now, consider $X = F$.
- F has three neighbors, D, E, and G.
- There are edges between two of the three pairs of neighbors, but no edge between G and E.
- Thus, we see two more positive examples and we see our first negative example.



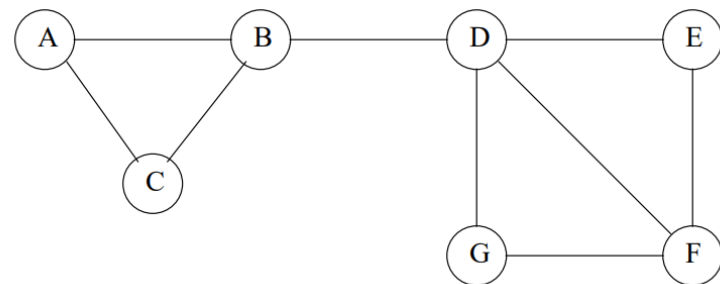
- If $X = B$, there are again three neighbors, A, C, D.
- But only one pair of neighbors, A and C, has an edge.
- Thus, we have two more negative examples, and one positive example.



- Finally, when $X = D$, there are four neighbors.
- Of the six pairs of neighbors, only two have edges between them.
- Negative += 4, Positive += 2

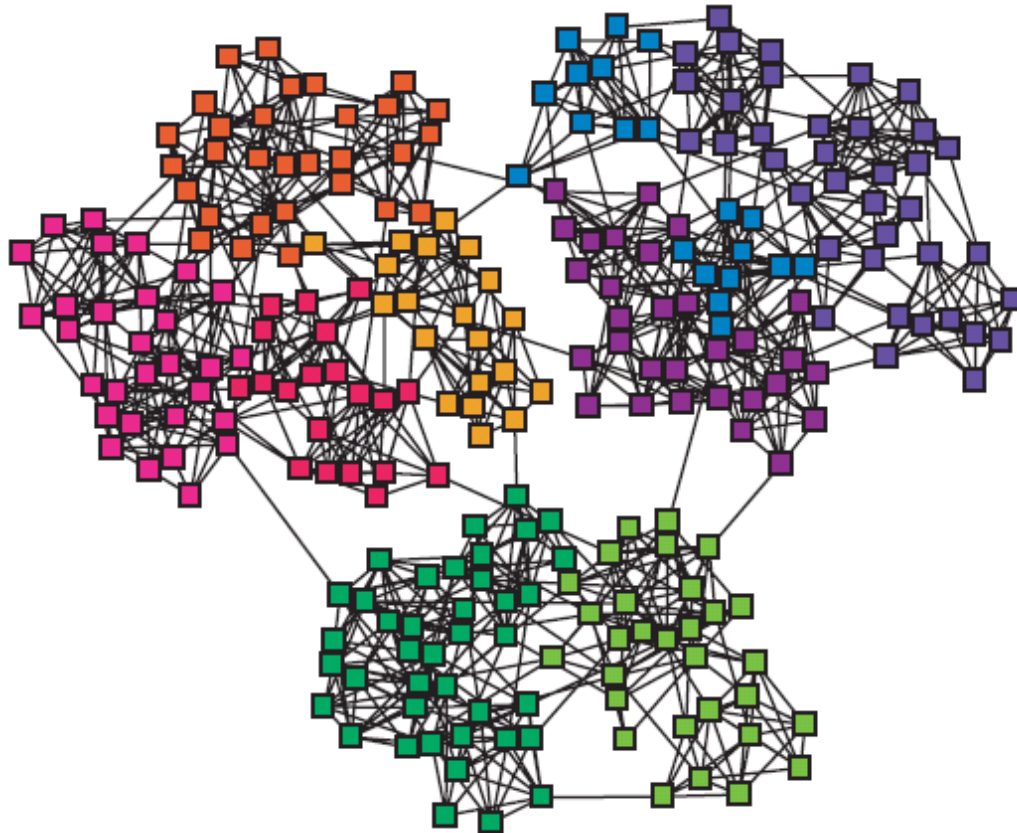


- Thus, the total number of positive examples is nine and the total number of negative examples is seven.
- $\text{Positive} / (\text{Negative} + \text{Positive}) = 9/16 = 0.563$
- This fraction is considerably greater than the .368 expected value
- We conclude that our network does indeed exhibit the locality expected in a social network.

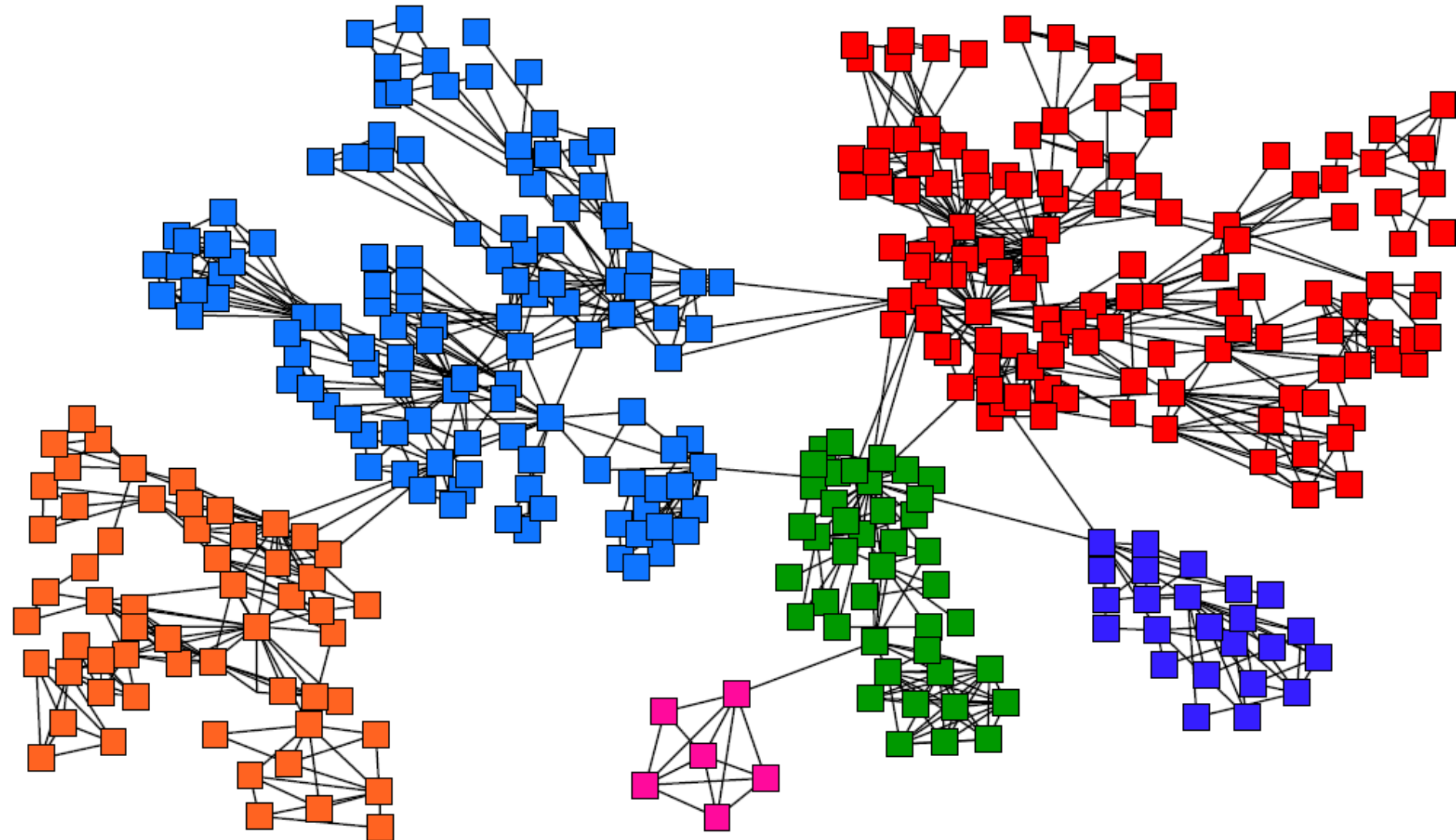


Networks & Communities

- We often think of networks being organized into **modules, cluster, communities**:

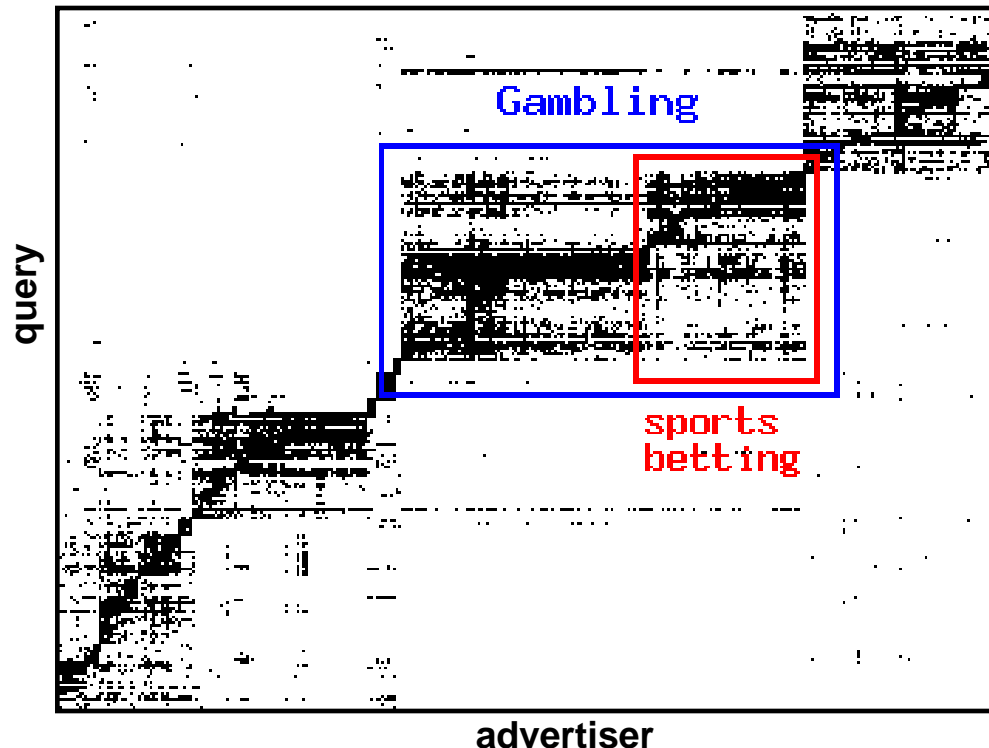


Goal: Find Densely Linked Clusters



Micro-Markets in Sponsored Search

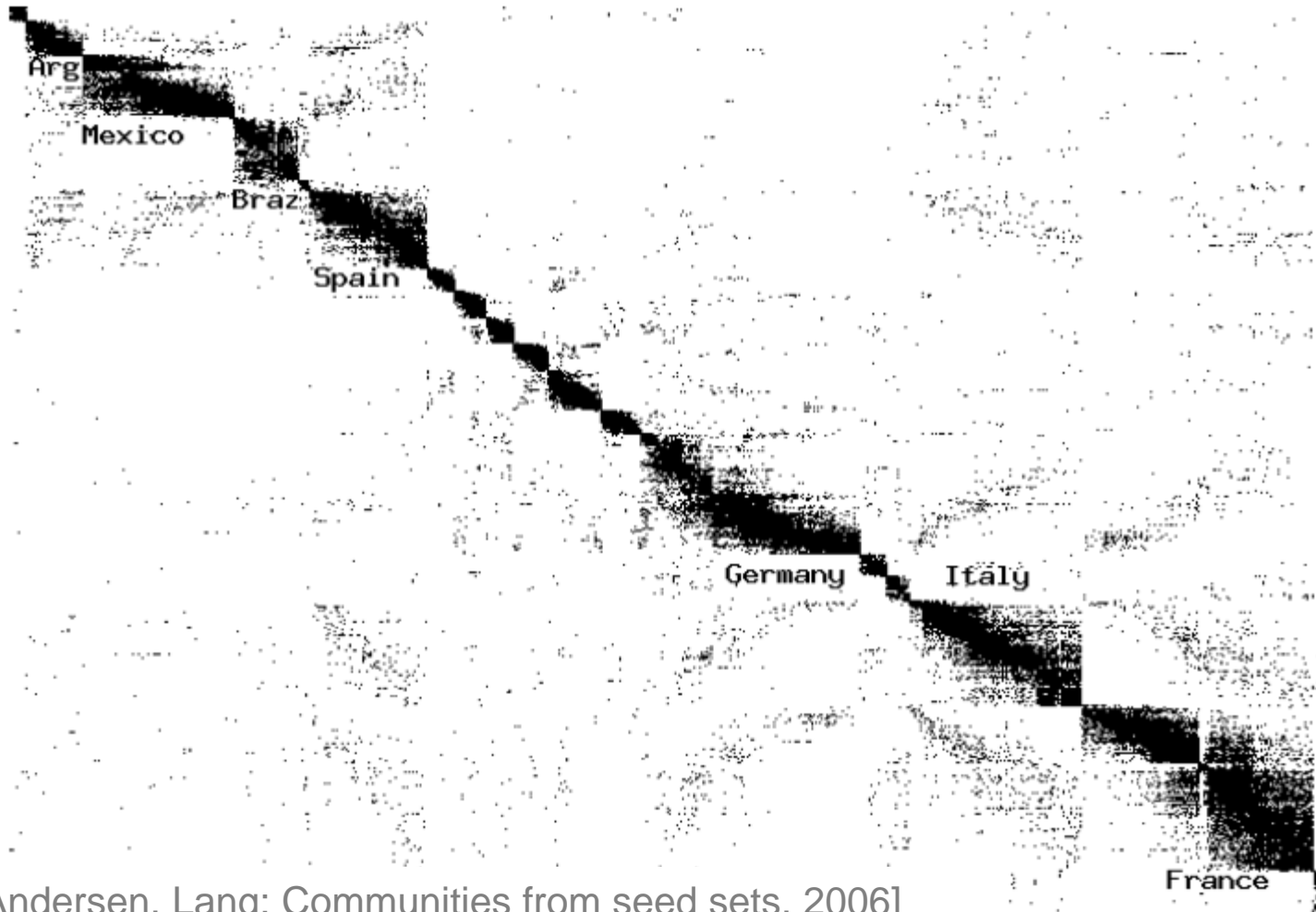
- Find micro-markets by partitioning the query-to-advertiser graph:



[Andersen, Lang: Communities from seed sets, 2006]

Movies and Actors

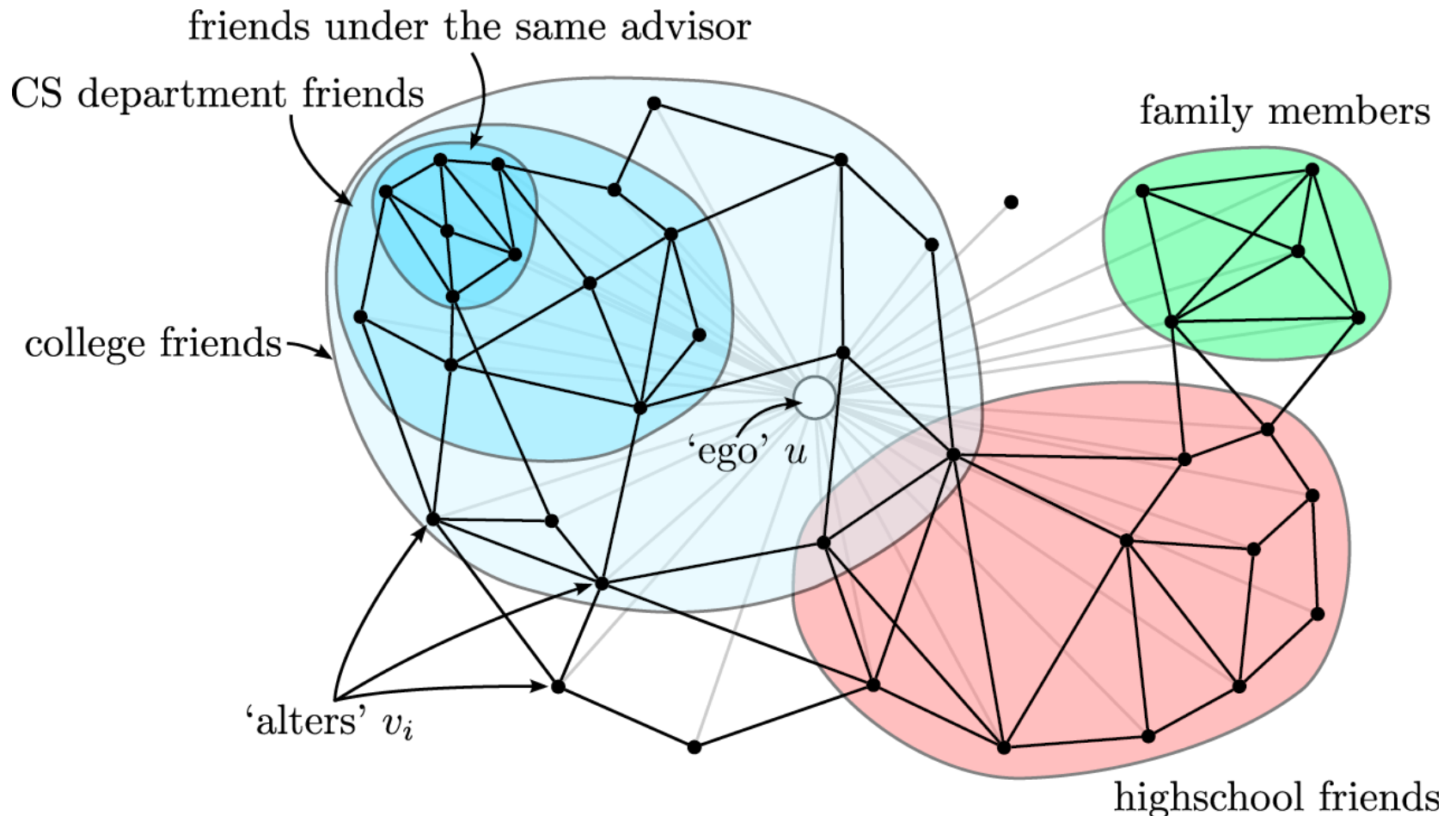
- Clusters in Movies-to-Actors graph:



[Andersen, Lang: Communities from seed sets, 2006]

Twitter & Facebook

■ Discovering social circles, circles of trust:



[McAuley, Leskovec: Discovering social circles in ego networks, 2012]

Questions?