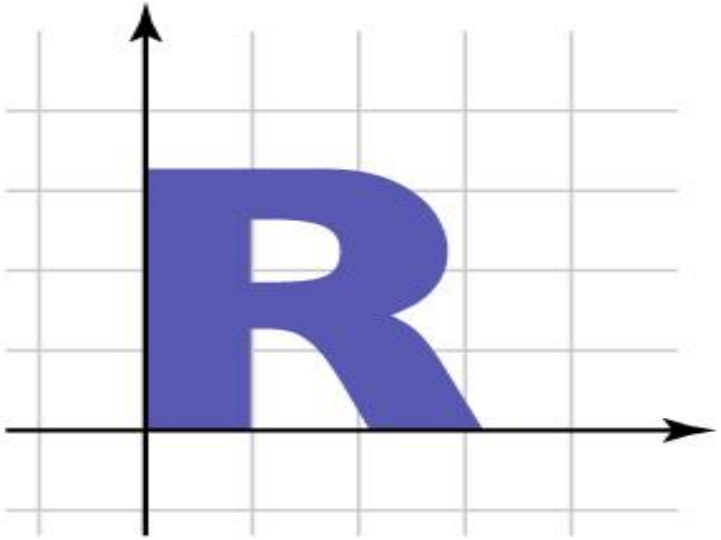
2 D transformations

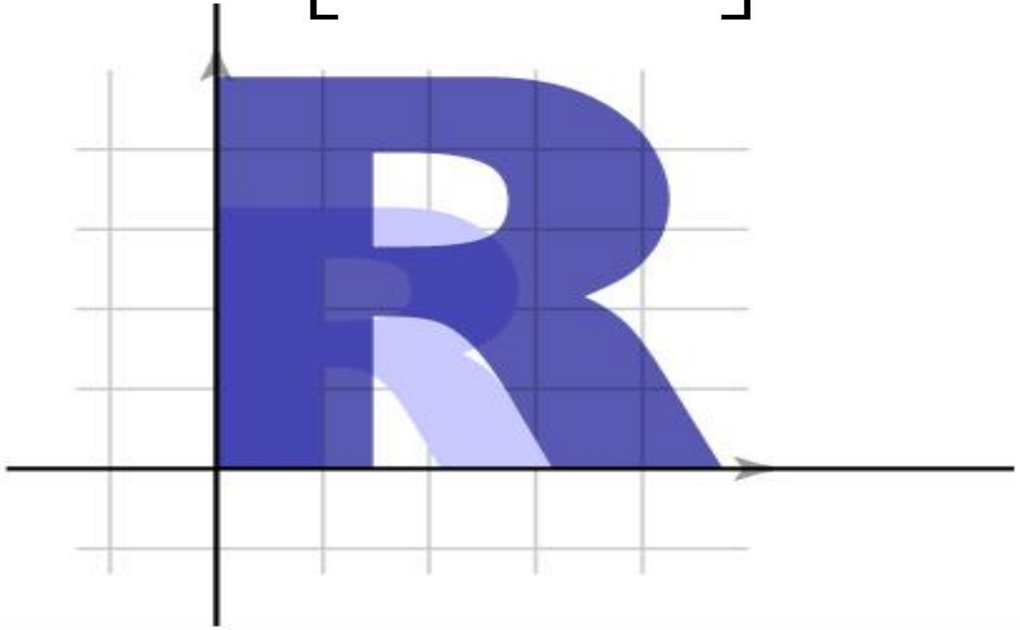
Linear transformation gallery

- Uniform scale

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} sx \\ sy \end{bmatrix}$$

$$\begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}$$





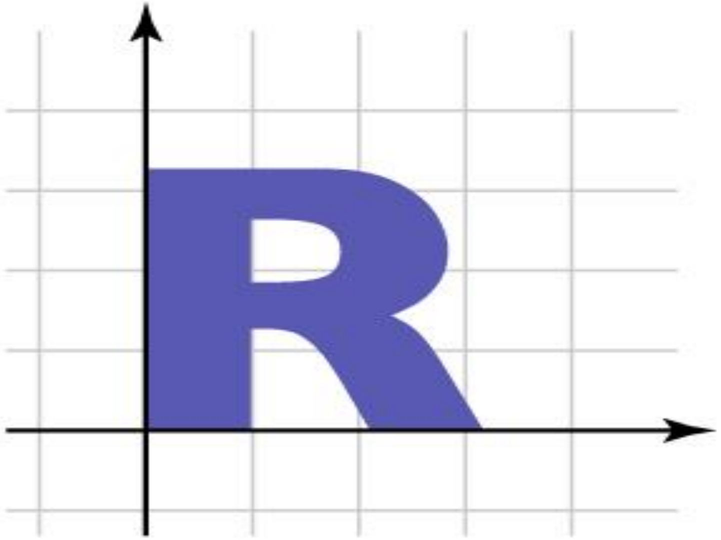
- Apply linear Transformation on triangle – Uniform (assume triangle coordinates)

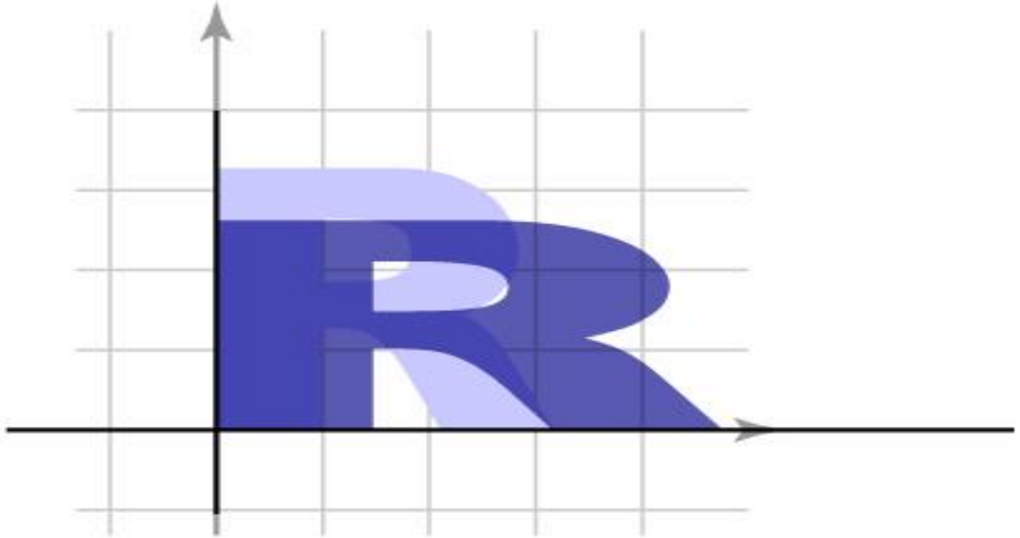
Linear transformation gallery

- Nonuniform scale

$$\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix}$$

$$\begin{bmatrix} 1.5 & 0 \\ 0 & 0.8 \end{bmatrix}$$



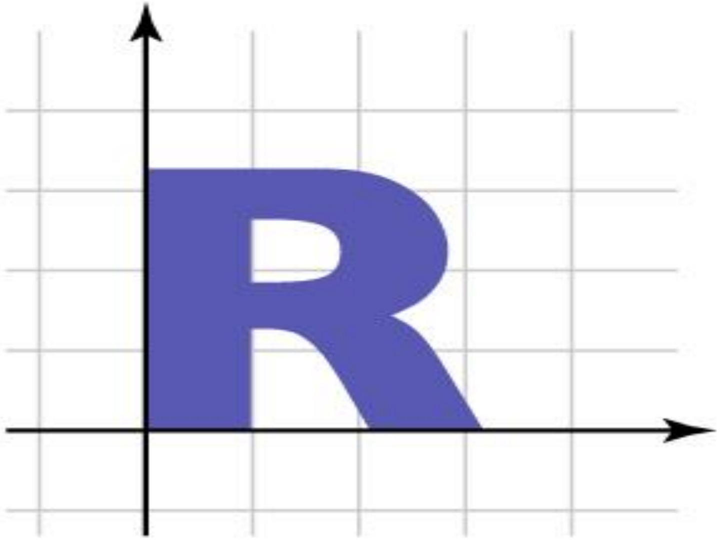


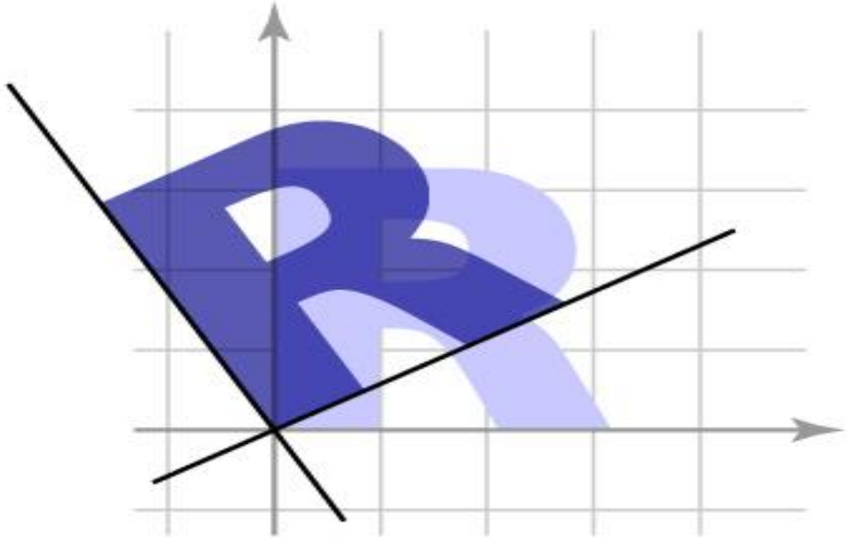
Linear transformation gallery

- Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$

$$\begin{bmatrix} 0.866 & -.05 \\ 0.5 & 0.866 \end{bmatrix}$$

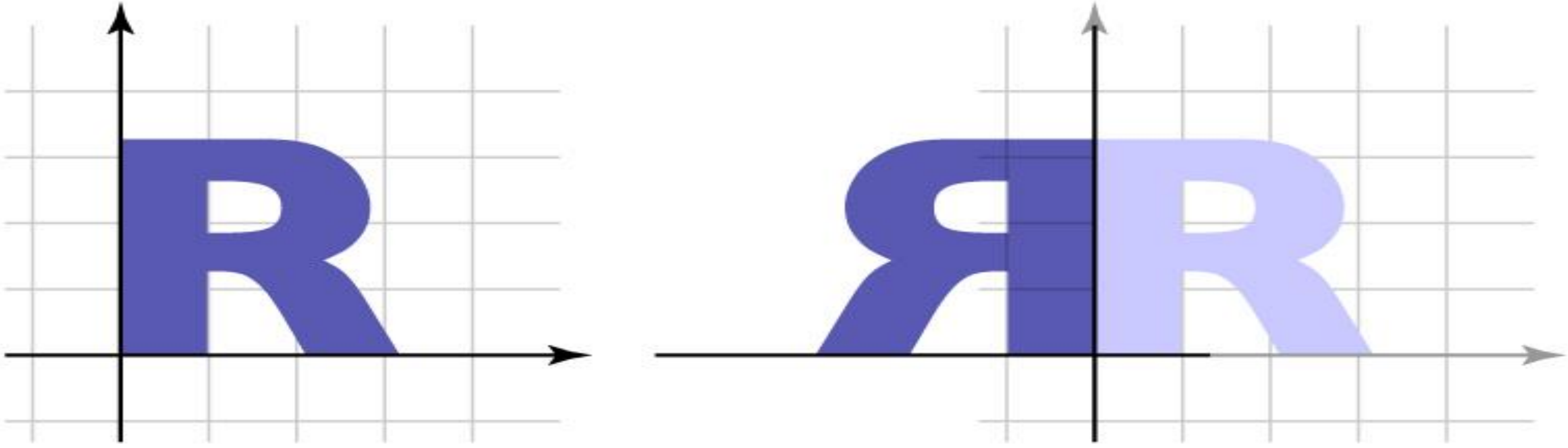




Linear transformation gallery

- Reflection
  - can consider it a special case of nonuniform scale

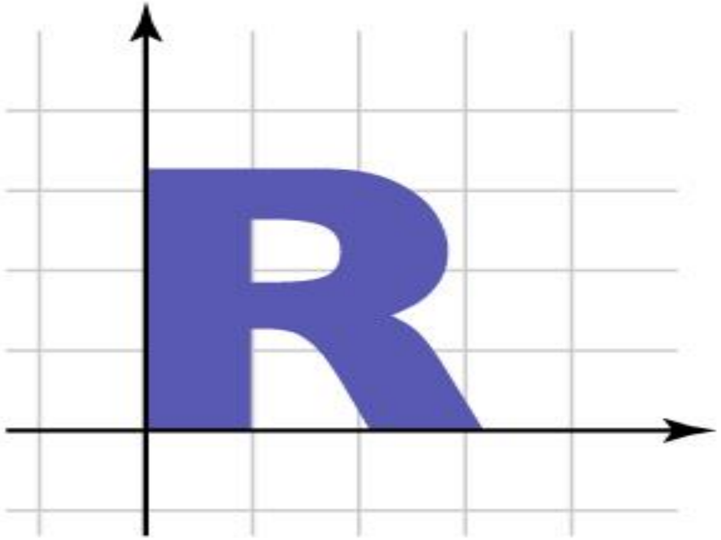
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

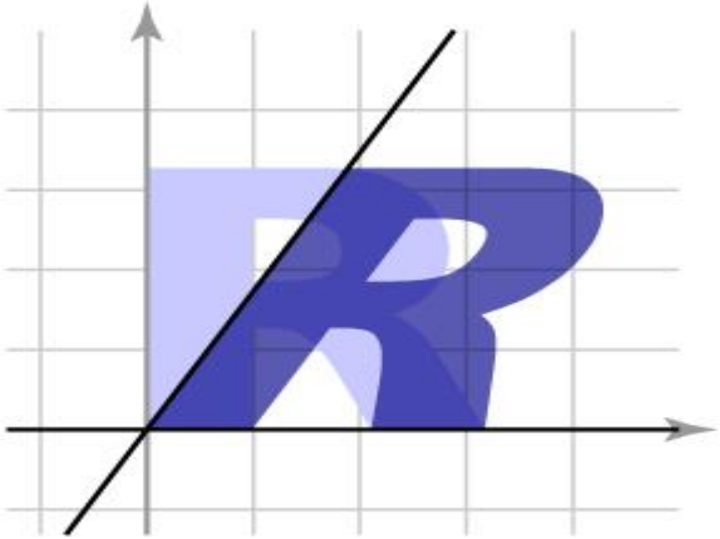


Linear transformation gallery

- Shear $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + ay \\ y \end{bmatrix}$

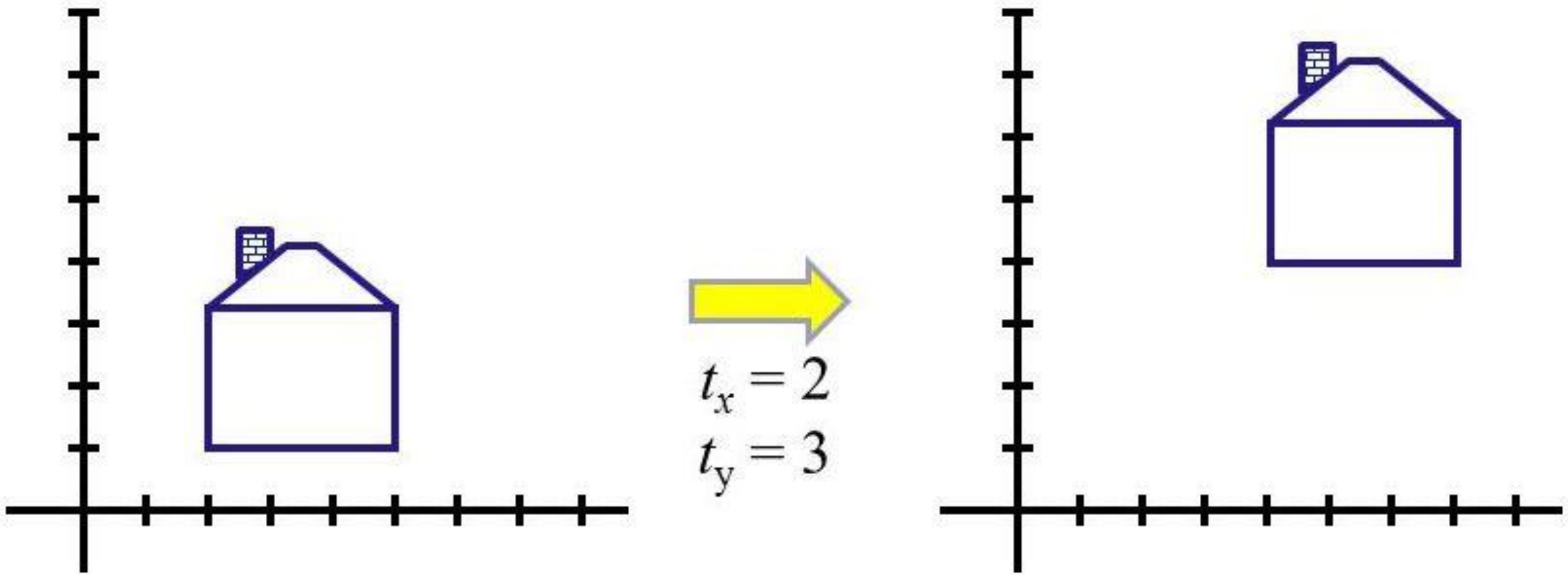
$$\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$$



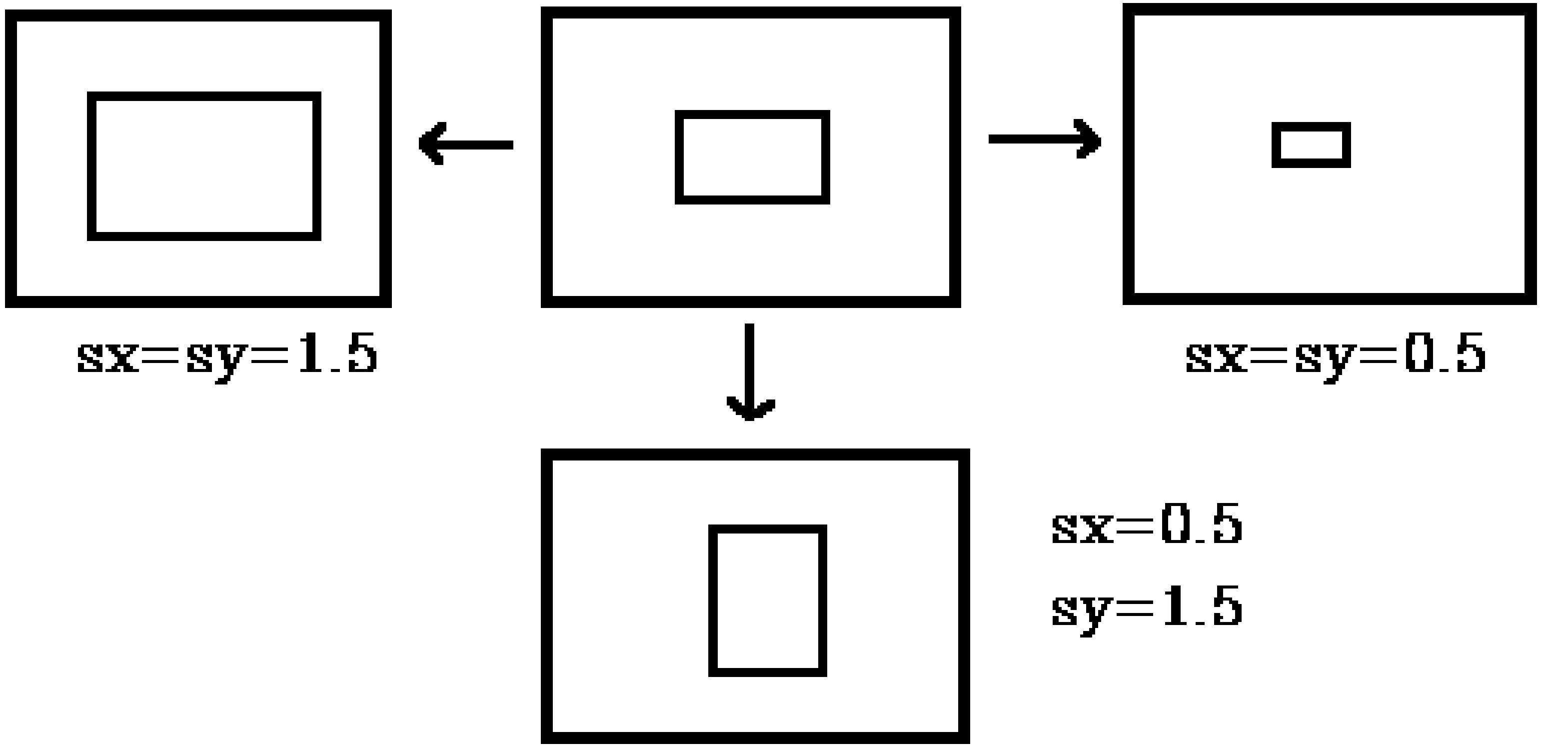


Translation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}. \qquad x' = x + T_x, \;\; y' = y + T_y$$



Scaling

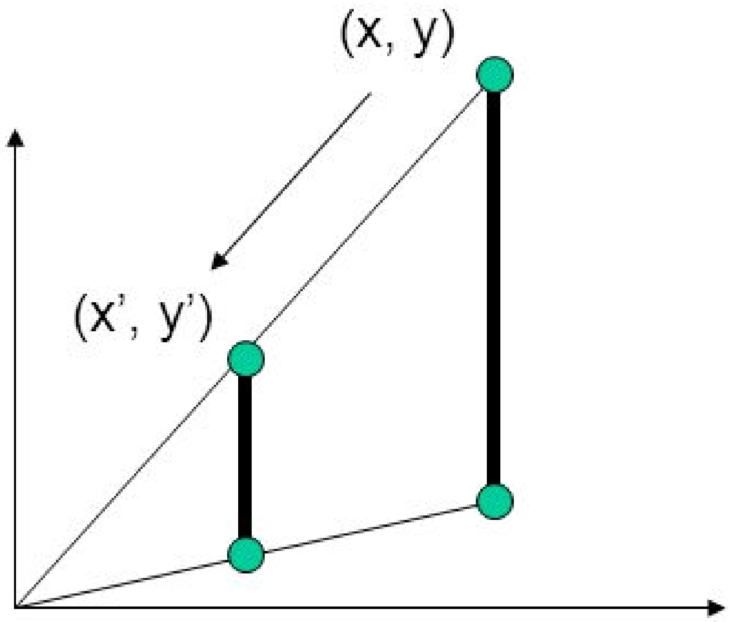


Scaling

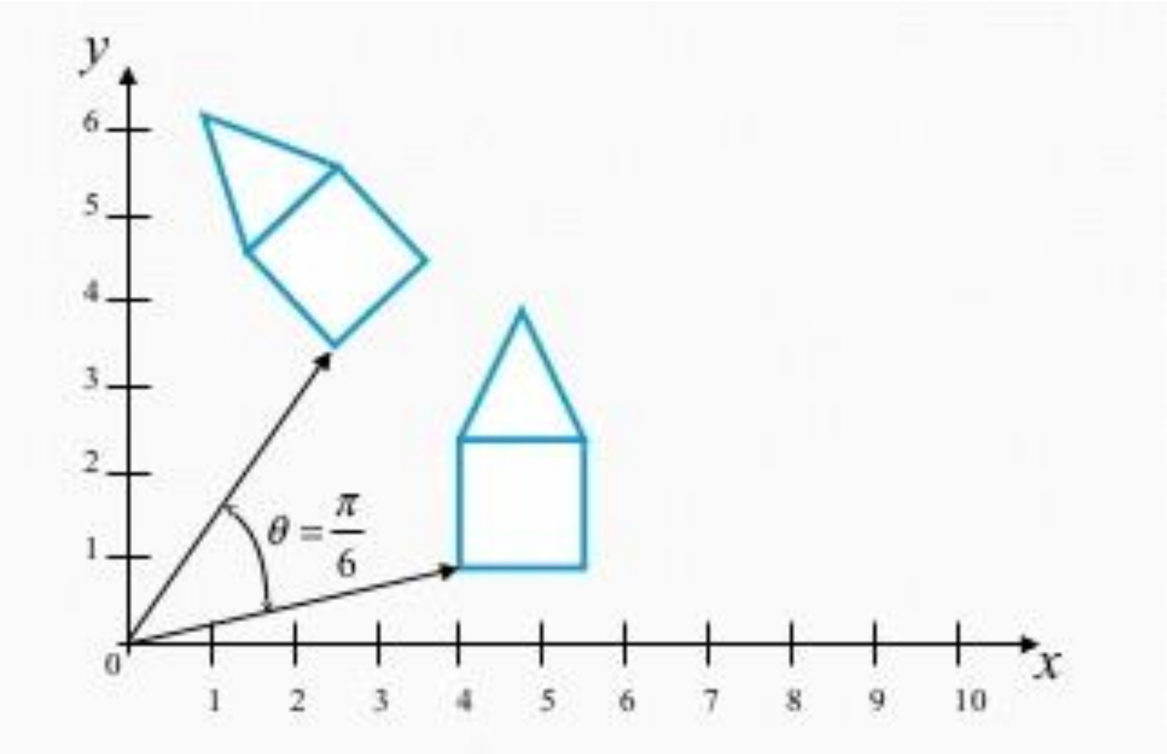
$x' = x\ s_x$

$y' = y\ s_y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

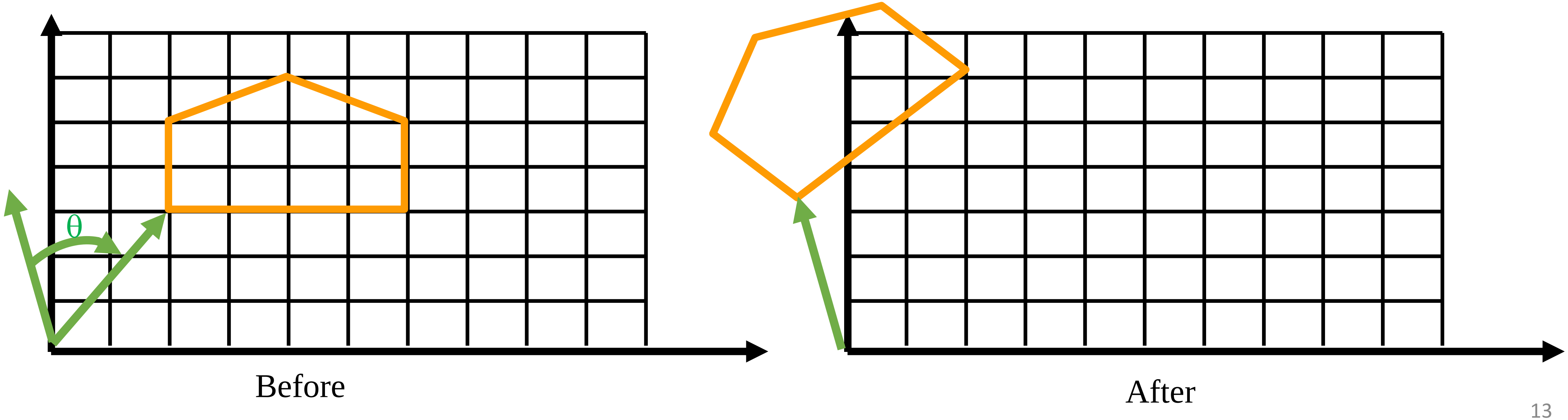


Rotation



Rotating in 2D

- New coordinates depend on *both* x and y
  - $x' = \cos\theta\, x - \sin\theta\, y$
  - $y' = \sin\theta\, x + \cos\theta\, y$



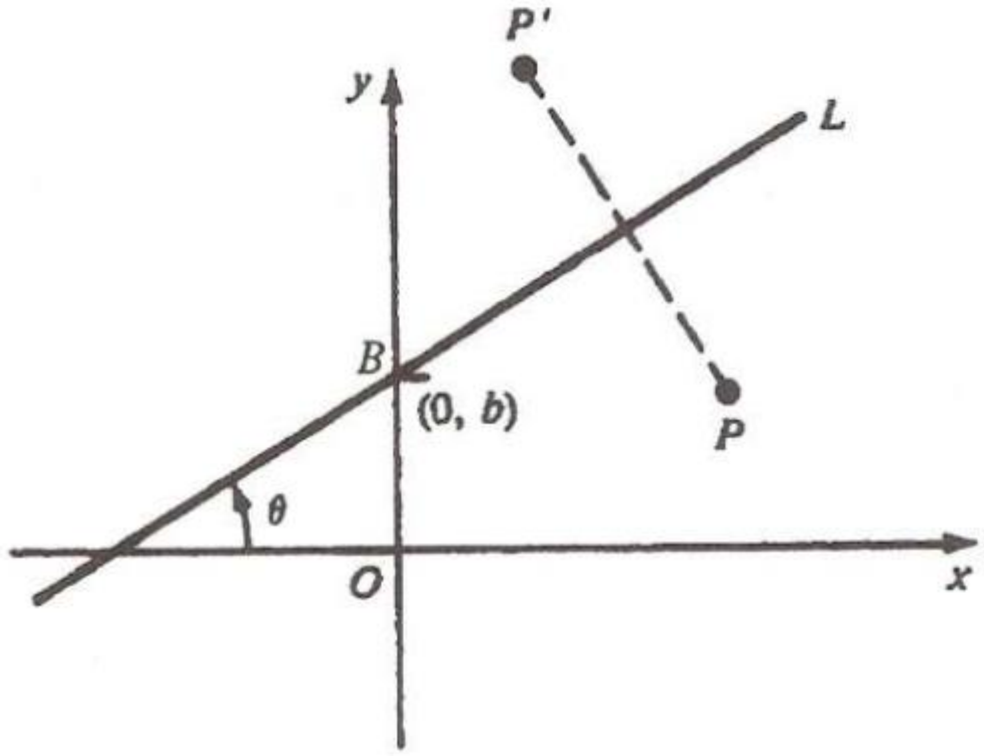
Rotating in 2D, matrix notation

- A rotation is a matrix multiplication:

P’=**R**P

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Give the sequence of transformation to reflect the object about the line . $y = mx + b.$
Also, derive the composite matrix for it.



- Let the line L in above figure have a y-intercept (0, b) and an angle of inclination θ (with respect to the x-axis). We do the following sequence of transformations:

1) Translate the intersection point B to the origin.
2) Rotate by θ so that line L aligns with the x-axis.
3) Mirror-reflect about the x-axis.
4) Rotate back by θ.
5) Translate B back to (0, b).

- In transformation notation, we have:

$$M_L = T(0,b).R_\theta.M_x.R_{-\theta}.T(0,-b)$$

- Where M_x is the reflection matrix about the x-axis, which is:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Now, the angle of inclination of a line is related to its slope m by the equation tan θ = m, we have:

$$M_L = T(0,b).R_\theta.M_x.R_{-\theta}.T(0,-b)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{pmatrix}$$

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Now, if tan θ = m, then, $\sin\theta = \frac{\mathrm{m}}{\sqrt{\mathrm{m}^2+1}}$ $\quad and \cos\theta = \frac{1}{\sqrt{\mathrm{m}^2+1}}$

Substituting these values for sin θ and cos θ after matrix multiplication, we have:

$$M_L = \begin{pmatrix} \dfrac{1-m^2}{m^2+1} & \dfrac{2m}{m^2+1} & \dfrac{-2bm}{m^2+1} \\ \dfrac{2m}{m^2+1} & \dfrac{m^2-1}{m^2+1} & \dfrac{2b}{m^2+1} \\ 0 & 0 & 1 \end{pmatrix}$$

Practice Question

- **Reflect the diamond-shaped polygon whose vertices are A(-1, 0), B(0, -2), C(1, 0), and D(0, 2) about the line y = x + 2.**

So, A’ = (-2, 1), B’=(-4, 2), C’ = (-2, 3), and D’ = (0, 2).