Solving Sums

MOD 1

I) Signals

a) Periodic & Aperiodic

$x(n+N) = x(n)$

b) Even & Odd

$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

c) Energy & Power

$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$ if E = finite & P = 0 ⇒ Energy

$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$ if $E = \infty$ & P = finite ⇒ Power

II) System

a) Time invarient & Time varient

i) $n \Rightarrow n-m$

ii) $x(n) \Rightarrow x(n-m)$

iii) Compare (i) & (ii) (i) ≠ (ii) Time Varient

b) Linear & Non Linear

i) Sub $x_3(n) = a_1(x_1(n)) + a_2(x_2(n)) \Rightarrow y_3(n)$

ii) $y_3(n) = a_1y_1(n) + a_2y_2(n)$

iii) If True then Linear else Non Linear

c) Stable & Unstable

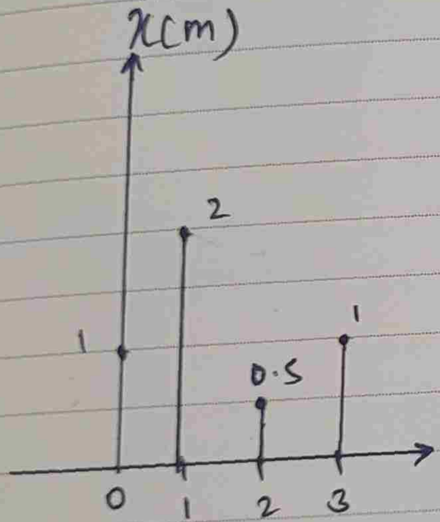
$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$

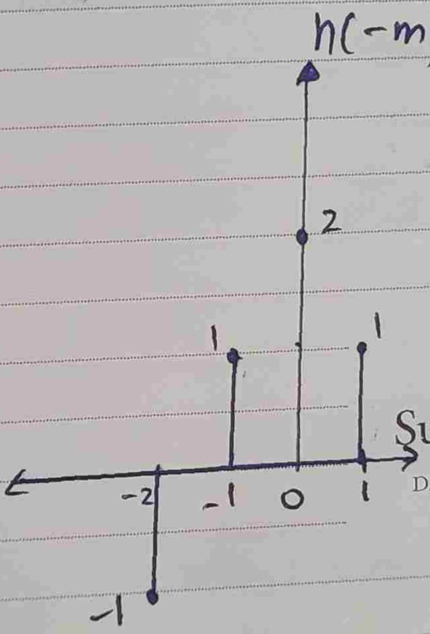
II Convolution

a) Linear

$y(n) = \sum_{m=-\infty}^{\infty} x(m)\, h(n-m)$

$x(n) = \{\underset{\uparrow}{1}, 2, 0.5, 1\}$, $h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$





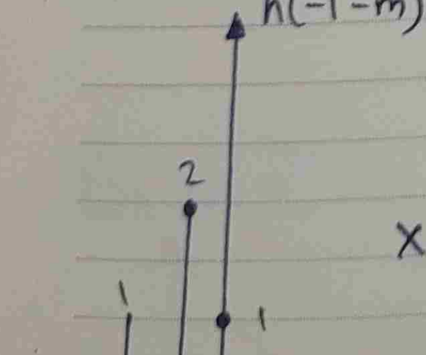
Sunday

4

Day (277 - 088)

$K = (0-1)$ to $(3+2) = -1$ to 5

for $K = -1$ ← shift left



$\times\ x(m) = 1$

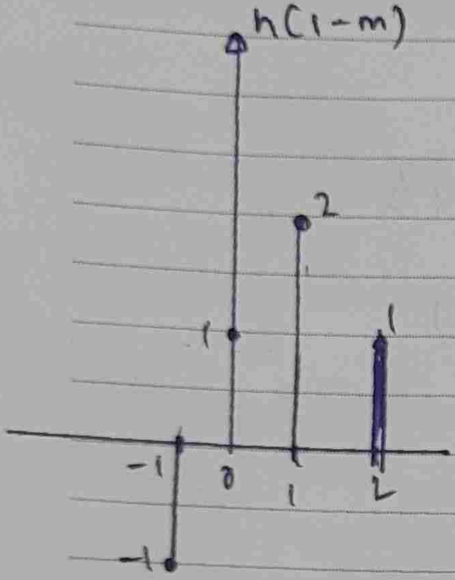
November

W	M	T	W	T	F	S	S
							1
44							8
45	2	3	4	5	6	7	15
46	9	10	11	12	13	14	22
47	16	17	18	19	20	21	29
48	23	24	25	26	27	28	
49	30						

For k = 0

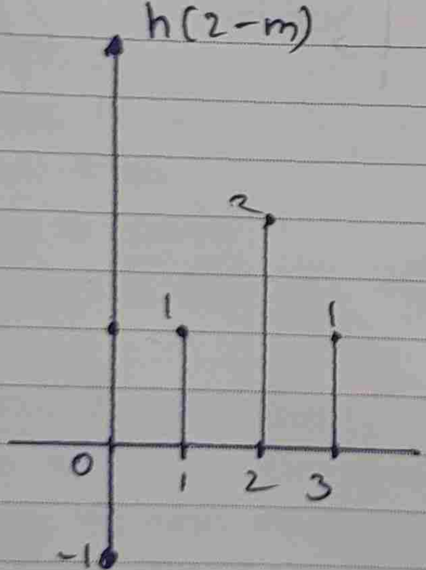
$x(m) \times h(-m) = 2\times 1 + 1\times 2 = 4$

For k = 1 ← Shift right



$\times\ x(m) = 1\times 1 + 2\times 2 + 0.5\times 1 = 5.5$

For k = 2

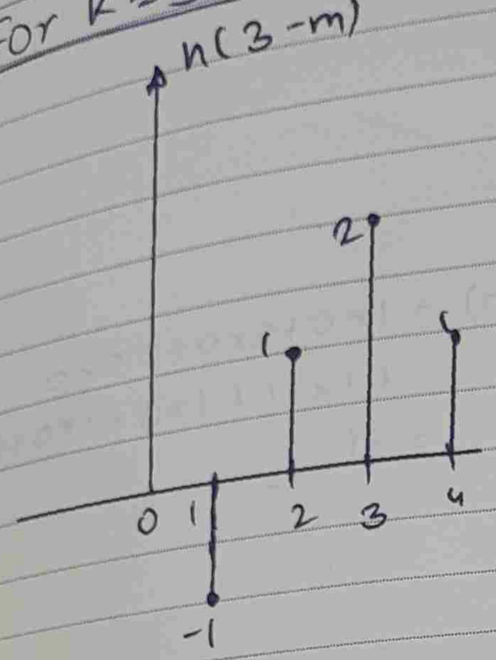


$\times\ x(m) = -\cancel{1\times 1} + 1\times 2 + 2\times 0.5 + \cancel{1\times 1}$
$= 3$

September

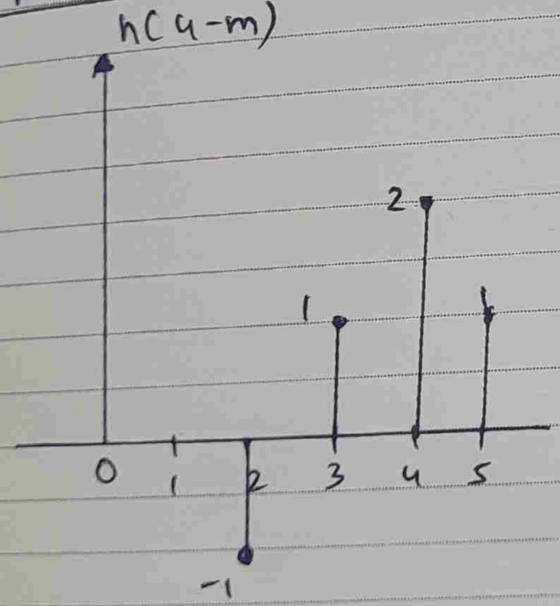
W	M	T	W	T	F	S	S
36		1	2	3	4	5	6
37	7	8	9	10	11	12	13
38	14	15	16	17	18	19	20
39	21	22	23	24	25	26	27
40	28	29	30				

For $K = 3$



$$\times x(m) = 1\times 0 + -\cancel{1}\times 2 \cancel{+} 1 \times 0.5 + 2\times 1$$
$$+ 0 \times 1$$
$$= 0.5$$

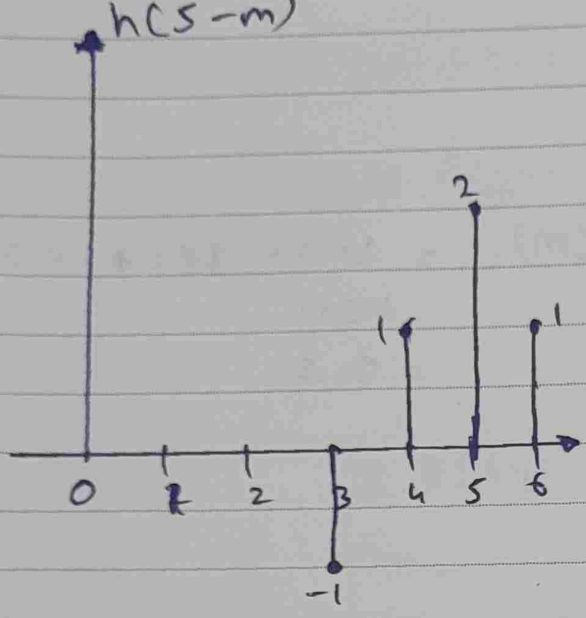
For $K = 4$



$$\times x(m) = \cancel{0\times 0}\; 1\times 0 + 2\times 0 + 0.5\times -1$$
$$+ 1\times 1 + 0\times 2 + 0\times 1$$
$$= 0.5$$

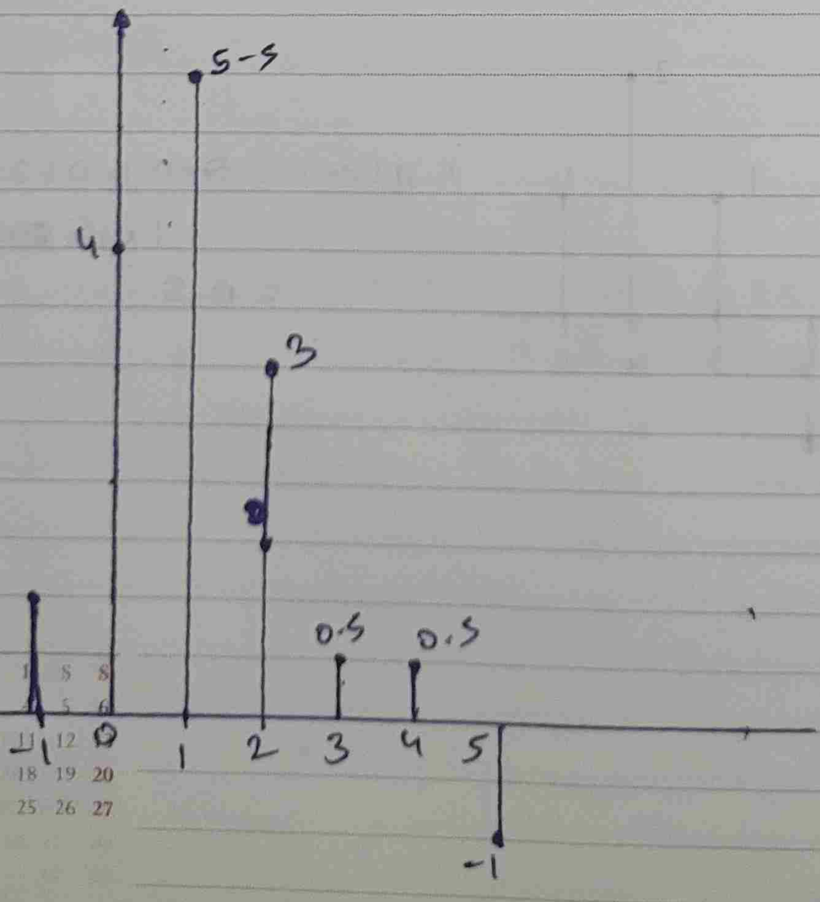
For $x = 5$

For $k=5$



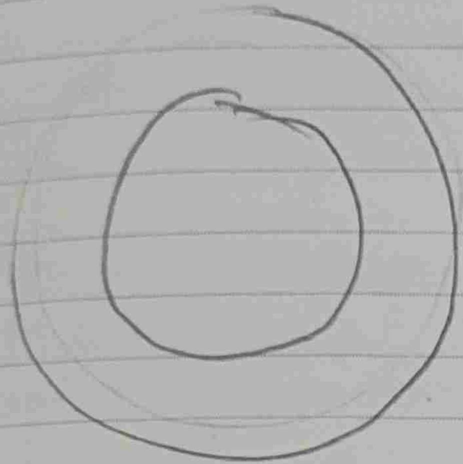
$\times\, x(m) = 1\times 0 + 2\times 0 + 0.5\times 0 + 1\times -1 + 1\times 0 + 2\times 0 + 1\times 0$
$= -1$

$y(n) = \{1, \underset{\overset{\uparrow}{0}}{4}, 5.5, 3, 0.5, 0.5, -1\}$



b) <u>Circular</u>

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m)\, x_2(n-m)$$



← inner circle enter the values clockwise & move anticlockwise for +ve

← Outside add anticlockwise does not move.

IV <u>Correlation</u>

a) <u>Auto</u>

$$r_{xx}(k) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-k) \qquad k = -(N-1) \text{ to } (N-1), \quad \text{Total} = 2N-1$$

$-$ ← shift Left

$+$ ← shift Right

No folding

b) <u>Cross</u>

$$r_{xy} = \sum_{n=-\infty}^{\infty} \underset{\substack{\downarrow \\ N}}{x(n)}\, \underset{\substack{\downarrow \\ M}}{y(n-k)} \qquad k = -(M-1) \text{ to } (N-1) \qquad \text{Total} = M+N-1$$

MOD2

I) ~~MOD2~~ Point Processing

1) Contrast streching

$\alpha = \frac{s_1}{r_1}$, $\beta = \frac{s_2 - s_1}{r_2 - r_1}$, $\gamma = \frac{(L-1) - s_2}{(L-1) - r_2}$

$$s = \begin{cases} \alpha \cdot r & 0 \le r < 3 \\ \beta(r - r_1) + s_1 & r_1 \le r < r_2 \\ \gamma(r - r_2) + s_2 & r_2 \le r < L-1 \end{cases}$$

2) Grey Level slicing

a) Without

$$s = \begin{cases} L-1 & r_1 \le r \le r_2 \\ 0 & \text{o.w.} \end{cases}$$

b) With

$$s = \begin{cases} L-1 & r_1 \le r \le r_2 \\ r & \text{o.w.} \end{cases}$$

3) Bit plane slicing

- 3 bit
  - MSB
  - Middle
  - LSB
- 4 bit
  - MSB
  - Middle Next to MSB
  - Middle Next to LSB
  - LSB

4) Dynamic Rang Compression (log Transform)

$S = C \log_{10}(1+r)$

5) Power - law transform

$S = Cr^{R}$

II) Neighbourhood Processing

a) Low Pass Averaging Filter (box)

→ Eliminates Gaussian noise

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

→ Limitation

- Leads to blurring of Image
- Impulse noise is attenuated & diffused not removed
- One Pixel with unrepresentative value can affect the mean value of the centre pixel

b) Weighted Average Filter

$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

→ The center pixel is given more importance

c) Median Filter

→ Non linear filter

→ Smoothens the image & ~~removed~~ minimizes salt & pepper noise

d) Gaussian filter

→ Class od linear smoothing Filters with weight chosen according to the shape

$$h(m,n) = \frac{1}{2\pi\sigma^2} e^{-(m^2+n^2)/2\sigma^2}$$

→ Properties

- Rotationall symmetric in 2-D
- FT od a Gaussian is itseld a Gaussian Function
- The degree od smoothening is governed by variance σ

e) High pass filter

$$\frac{1}{9} \times \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

f) High-boost Filter

→ Used to retain some od the Low frequency.

$$\frac{1}{9} \times \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9k-1 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

III Spatial Enhancement

a) Histogram Streching

→ rmin & rmax decided, where the no° od pixel is 0 we ignore till we get a no° other then 0 the dirst is rmin the last is rmax

$$S = S_{min} + (r - r_{min}) \frac{S_{max} - S_{min}}{r_{max} - r_{min}}$$

b) Histogram Equalization

Grey level	No° od Pixel (nk)	$P_k = n_k / n$	ΣS_k (CDF	$S_k \times S_{max}$	Equalized Level
	n =				

→ it does not change id you apply it more than once.

MOD 3

I) DFT & IDFT

→ $x(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi n k}{N}}$ — DFT

→ $x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{j\frac{2\pi n k}{N}}$ — IDFT

→ Twiddle Factor ⇒ $e^{-j\frac{2\pi n k}{N}} = W_N^{nk}$

→ Kernel ⇒ $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$

→ 2D DFT ⇒ $T f T'$

→ 2D IDFT ⇒ $\frac{1}{N^2} (T f T')$

November

W	M	T	W	T	F	S	S
44							1
45	2	3	4	5	6	7	8
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47	16	17	18	19	20	21	22
48	23	24	25	26	27	28	29
49	30						

II) DCT & IDCT

→ $x(K) = \alpha(K) \sum_{n=0}^{N-1} x(n) \cos\left(\frac{(2n+1)\pi K}{2N}\right)$ — DCT

→ $x(n) = \sum_{K=0}^{N-1} \alpha(K)\, x(K) \cos\left(\frac{(2n+1)\pi K}{2N}\right)$ — IDCT

$\alpha(K) = \sqrt{\frac{1}{N}} \quad K = 0$

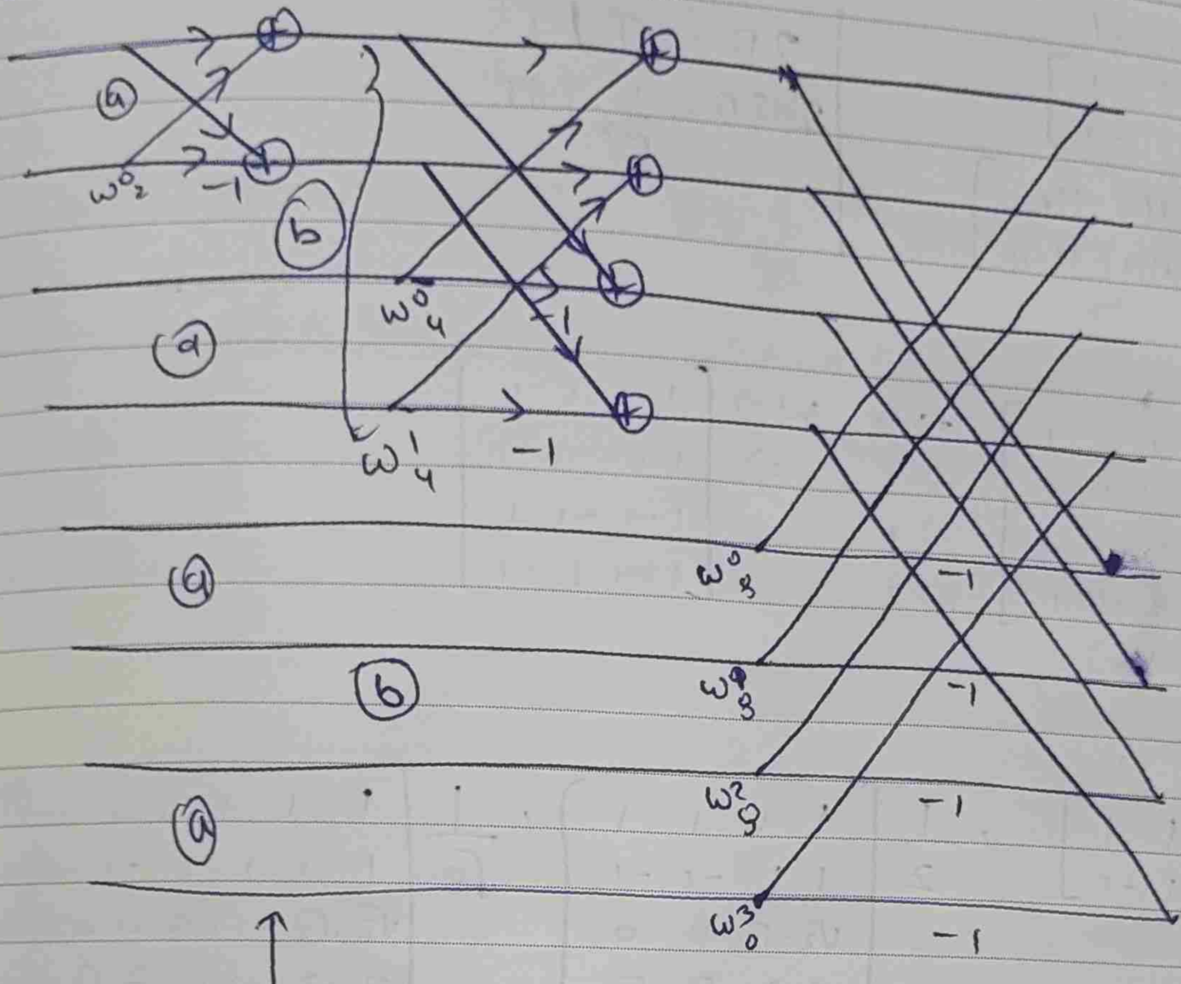
$= \sqrt{\frac{2}{N}} \quad K \neq 0$

→ Kernel ⇒

$$\begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.6532 & 0.2706 & -0.2706 & -0.6532 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.2706 & -0.6532 & 0.6532 & -0.2706 \end{bmatrix}$$

III) FFT

a) DIT



b) Hadmard

Same but not 'W' values

~~Eg a) DIT FFT~~

Note any Convolution sum using DFT, DIT FFT, etc steps ⇒ ~~x~~ If $x_1(n)$ & $x_2(n)$ are given & need to find convolution do this.

i) $x_1(n) \Rightarrow DFT[x_1(n)]$ or DIT-FFT$[x_1(n)] = x_1(k)$

ii) $x_2(n)$ or $h(n) \Rightarrow$ " " $x_2(k)$

iii) $x_1(k) \cdot x_2(k) = x_3(k)$ ← Actual multiplication

iv) $x_3(k) \Rightarrow IDFT[x_3(k)]$ or $\frac{1}{N}$ DIT-FFT$[x_3(k)]$

IV Transforms

a) Hadamard

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & H_{-N} \end{bmatrix}$$

$2D \Rightarrow T f T'$

$I \Rightarrow 2D \Rightarrow \frac{1}{N^2} T f T'$

b) Walsh

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{array}{l} \Rightarrow 0 \\ \Rightarrow 3 \\ \Rightarrow 1 \\ \Rightarrow 2 \end{array} \quad W \Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

c) Haar

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ \sqrt{2} & -\sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{bmatrix}, \quad \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

d) Slant Transform

$$\frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3/\sqrt{5} & 1/\sqrt{5} & -1/\sqrt{5} & -3/\sqrt{5} \\ 1 & -1 & -1 & 1 \\ 1/\sqrt{5} & -3/\sqrt{5} & 3/\sqrt{5} & -1/\sqrt{5} \end{bmatrix}$$

e) KL Transform

i) Find the mean vector & covariance

$$x = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \therefore x_0 = \begin{bmatrix} a \\ c \end{bmatrix}, x_1 \begin{bmatrix} b \\ d \end{bmatrix}, \bar{x} = \frac{1}{M}\sum_{k=0}^{M-1} x_k,$$

$$cov(x) = E[xx^T] - \bar{x}\bar{x}^T, \cancel{E = \frac{1}{M}} E[xx^T] = \frac{1}{M}\sum_{k=0}^{M-1} x_k x_k^T$$

ii) find Eigenvalues & then Eigen vector of the covariance

$$|cov(x) - \lambda I| = 0 \Rightarrow \lambda_0 = \alpha, \lambda_1 = \beta \Leftarrow \text{Eigen value}$$

$$[cov(x) - \lambda_0 I]\phi_0 = 0 \;\&\; (cov(x) - \lambda_1 I)\phi_1 = 0$$

$$\begin{bmatrix} \phi_{00} \\ \phi_{01} \end{bmatrix} \longrightarrow \overset{\overset{\text{assume}}{\downarrow}}{1} \longleftarrow \begin{bmatrix} \phi_{10} \\ \phi_{11} \end{bmatrix}$$

Then normalize,

$$\frac{\phi_0}{\|\phi_0\|} = \frac{1}{\sqrt{\phi_{00}^2 + \phi_{01}^2}}\phi_0, \quad \frac{\phi_1}{\|\phi_1\|} = \frac{1}{\sqrt{\phi_{10}^2 + \phi_{11}^2}}\phi_1$$

Sunday 18

Day (291 - 074)

iii) Form the transform matrix T such as row of T

$$T = [\underset{\lambda_{max}}{\underset{\downarrow}{\phi_{max}}}\ \phi_{\ldots}]$$

$y_0 = T[x_0]$

$y_1 = T[x_1]$

$$\therefore Y = \begin{bmatrix} y_0 & y_1 \end{bmatrix}$$ //

November

W M T W T F S S

44 1

Image Enchancement

i) Low pass filtering

→ $H(u,v) = \begin{cases} 1 & u^2+v^2 \le D_0^2 \\ 0 & \text{o.w.} \end{cases}$ ← cutoff frequency which determi... the amt of frequency components p...

→ D_0 controls the amount of blurring

→ Ringing Effect ⇒ Sharp cutoff frequencies produce an overshoot of image features whose frequency is close to the cutoff

→ Types

a) Ideal

b) butterworth

$$H(k,l) = \frac{1}{1+\left[\frac{\sqrt{k^2+l^2}}{D_0}\right]^{2n}}$$

→ Helps in lowering the ring effect.

c) Gaussian

$$H(u,v) = e^{-(u^2+v^2)/2\sigma^2} \Rightarrow e^{-(u^2+v^2)/2D_0^2}$$

2) Highpass filtering

→ Obtain from LP → $H_{HP}(u,v) = 1 - H_{LP}(u,v)$

→ Preserves high frequency

→ Enchances edges & fine details

~~→ Type~~ $H(u,v) = \begin{cases} 1 & u \ge D_0 \\ 0 & \text{o.w} \end{cases}$, $H(u,v) = \begin{cases} 1 & u^2+v^2 \ge D_0^2 \\ 0 & \text{o.w} \end{cases}$

→ Types

a) Ideal

b) Butterworth

$$H(u,v) = \frac{1}{1+[D_0/\sqrt{u^2+v^2}]^{2n}}$$

c) Gaussian

$$H(u,v) = 1 - e^{-(u^2+v^2)/2D_0^2}$$

3) Homomorphic Filtering

→ Enchances contrast

→ Reduce illumination artifacts

→ $f(x,y) = i(x,y)\, r(x,y)$

Illumination
- Varies slowly
- affects low freq

Reflection
- Varies faster
- affects high freq

→ Steps

i) Take $\ln(f(x,y)) = \ln(i(x,y)) + \ln(r(x,y))$

ii) Apply FT : $F(\ln(f(x,y)))$

iii) Apply $H(u,v) \Rightarrow Z(u,v)H(u,v) = Illum(u,v)H(u,v) + Refl(u,v)H(u,v)$

iv) Take inverse FT : $F^{-1}[Z(u,v)H(u,v)]$ or $S(x,y)$

v) Take exp $\Rightarrow e^{S(x,y)} = e^{i'(x,y)} e^{r'(x,y)}$

or $g(x,y) = i_0(x,y)\, r_0(x,y)$

October

W M T W T F S S

1 2 3 4

MOD 4

I) Thresholding

→ Global ⇒ $T = T[f(x,y)]$ ← Fails in uneven illumination

→ Local ⇒ $T = T[p(x,y), f(x,y)]$ ← subdivides

$T_{new} = \frac{M_1 + M_2}{2}$ ← average grey level value

II) Hough

$y = mx + c$

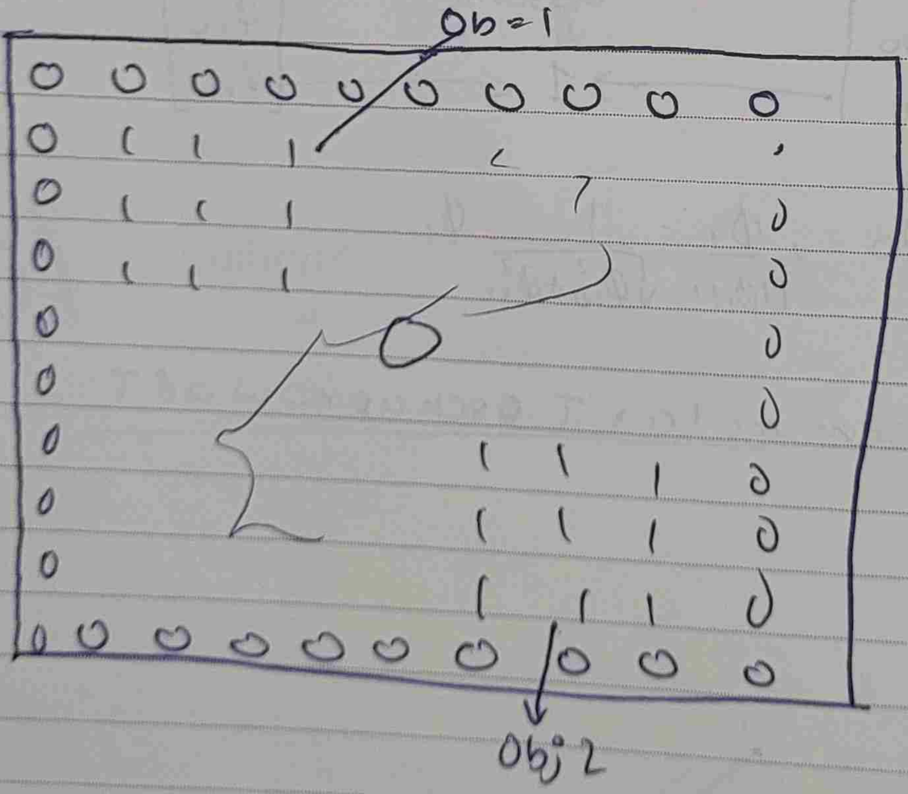
$c = -mx + y$

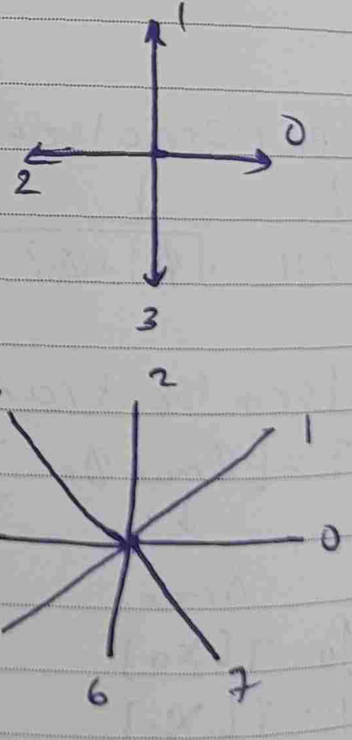
put $c = 0$, $m = ?$

put $m = 0$, $c = ?$

III Chain Coding

1 2 3





4D → Obj 1 = 0 0 3 3 2 2 1 1

Obj 2 = "

8D → Obj 1 = 0 0 6 6 4 4 2 2

Obj 2 = "

September

W	M	T	W	T	F	S	S
36		1	2	3	4	5	6
37	7	8	9	10	11	12	13
38	14	15	16	17	18	19	20
39	21	22	23	24	25	26	27
40	28	29	30				

IV) Moments

$M_{ij} = \sum_x \sum_y x^i y^j I(x,y)$, Central $\Rightarrow \sum_x \sum_y (x-\bar{x})^p (y-\bar{y})^q f(x,y) - M_{pq}$

MODS

I) IGS

{100, 110, 124, 124, 130, 110, 200, 210}

	Grey Level	Sum	IGS code
100	0110 0100	0110 0100	0110
110	0110 1110	0111 0010	0111
124	0111 1100	0111 1110	0111
124	0111 1100	1000 ϕ010	1000
130	1000 0010	1000 1100	1000
110	0110 1110	0111 1010	0111
200	0110 1000	0111 0010	0111
210	1101 0010	1101 0100	1101

II) Vector Quatization

No° od code vector = 2^{RxL} (Rate R & Dimension L)

Steps

1) Compute od dynamic range
2) Fixing the rate & Dimension
3) Determining the no° od code vector
4) " " Codo vector through centroid
5) Mapping input vectors to code vectors
6) Adjust input vector to dall into code vector
7) Transmission of indices
8) Reconstruction

November

W	M	T	W	T	F	S	S
44							1
45	2	3	4	5	6	7	8
46	9	10	11	12	13	14	15
47	16	17	18	19	20	21	22
48	23	24	25	26	27	28	29
49	30						

$$
\begin{bmatrix} 2 & 4 & 6 & 8 \\ 10 & 11 & 16 & 15 \\ 9 & 3 & 1 & 7 \\ 12 & 14 & 13 & 5 \end{bmatrix} - I
$$

Step 1 ⇒ $16-1=15$ ← Dynamic Range

Step 2 ⇒ R ⇒ 1 or 2 or 3 since we can represent the image in 4 bits.

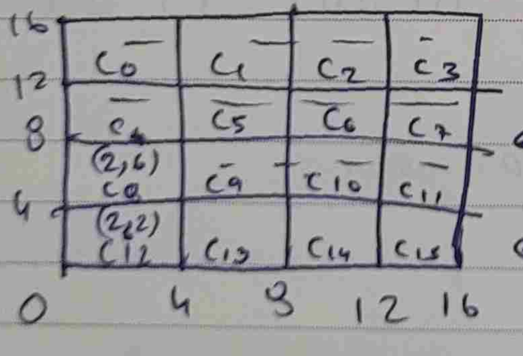
$\therefore R=2$

$L=2$

Step 3 ⇒ $N = 2^{RL} = 2^{2\times 2} = 16$

C_{00} to C_{15}

Step 4 ⇒ Interval $= \left\lfloor \frac{DR}{R\times L} \right\rfloor = \left\lfloor \frac{15}{2\times 2} \right\rfloor = 4$



Step 5 ⇒ Mapping I to C

$$
\begin{bmatrix} C_8 & C_9 \\ C_6 & C_3 \\ C_{14} & C_8 \\ C_3 & C_{11} \end{bmatrix}
$$

← Check the abs distance between the values eg (2,4) in I (circled)

$\delta(2,6) \to abs((2-2)+(4-6)) = 2$

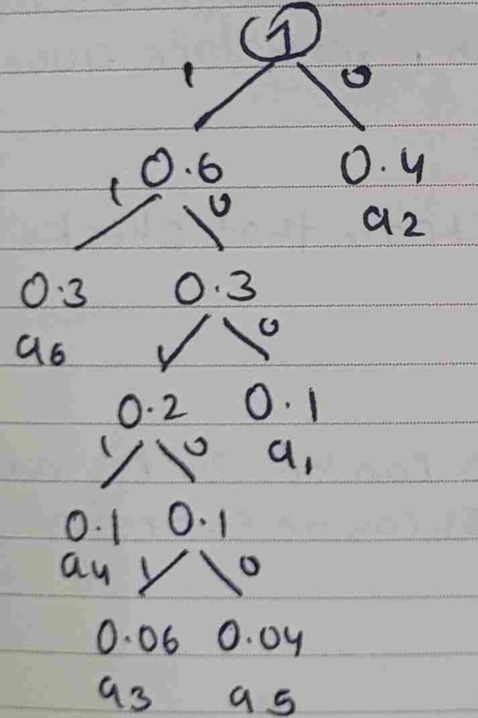
Step 8) ⇒ Reconstruct

2	6	6	6
10	10	14	14
10	2	2	6
14	14	14	6

Huffman Coding

(a) Eg) $a_1 = 10$, $a_2 = 40$, $a_3 = 6$, $a_4 = 10$, $a_5 = 4$, $a_6 = 30$

Symbols	Prob						
a_1	0.1	0.4	0.4	0.4	0.4	0.6	1
a_2	0.4	0.3	0.3	0.3	0.3	0.4	
a_3	0.06	0.1	0.1	0.2	0.3		
a_4	0.1	0.1	0.1	0.1			
a_5	0.04	0.06	0.1				
a_6	0.3	0.04					



$a_1 \Rightarrow 100 \Rightarrow 3$

$a_2 \Rightarrow 0 \Rightarrow 1$

$a_3 \Rightarrow 10101 \Rightarrow 5$

$a_4 \Rightarrow 1011 \Rightarrow 4$

$a_5 \Rightarrow 10100 \Rightarrow 5$

$a_6 \Rightarrow 11 \Rightarrow 2$

Avg Length $= 0.1 \times 3 + 0.4 \times 1 + 0.06 \times 5 + 0.1 \times 4 + 0.04 \times 5 + 0.3 \times 2 = 2.2$ bits/symbol.

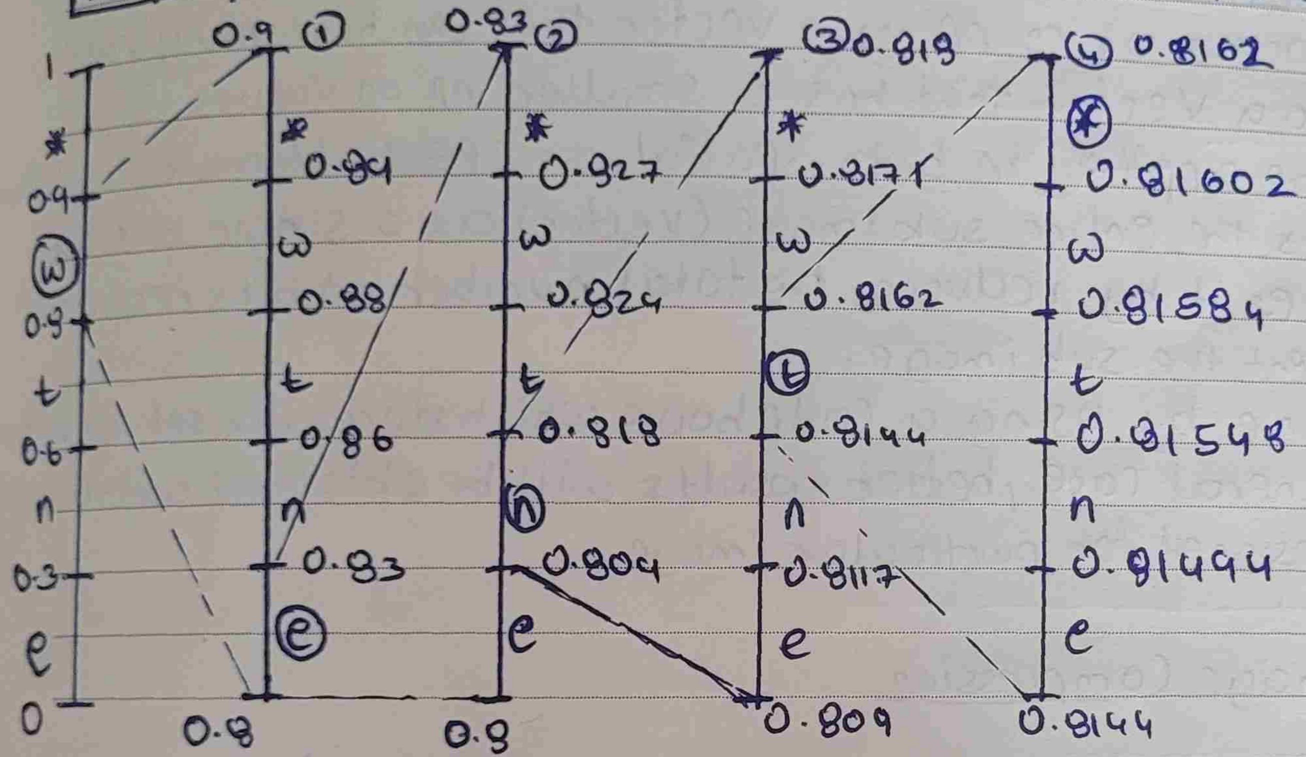
Arithmetic Coding

Encode msg "went*"

Sym	e	n	t	w	*
Prob	0.3	0.3	0.2	0.1	0.1

Formula $d = \text{UpperLimit} - \text{Lowerlimit}$

Range: $LL + d(\text{Prob of sym})$



For ①

$d = 0.9 - 0.8 = 0.1$

e: $0.8 + 0.1 \times 0.3 = 0.83$

n: $0.83 + 0.1 \times 0.3 = 0.86$

t: $0.86 + 0.1 \times 0.2 = 0.88$

w: $0.88 + 0.1 \times 0.1 = 0.89$

For ②

$d = 0.83 - 0.8$

$= 0.03$

e: $0.8 + 0.03 \times 0.3 = 0.80$

For ③

For ④

RLC

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

① Horizontal RLC

Run length vector $\Rightarrow$ (0,5)

No - 6 (0,3)(1,2)

Max length = 5 (1,5)

↓ (1,5)

3 bits (1,5)

No. of bits per pixel = 1 (0 or 1)

Total no. of Pixel = $6 \times (3+1) = 24$

No. of Og image - $5 \times 5 = 25$

$\therefore CR = \frac{25}{24} = 1.042 : 1$

② Vertical RLC

(0,2),(1,3)
(0,2),(1,3)
(0,2)(1,3)
(0,1)(1,4)
(0,1)(1,4)

$\}$ 10 Max length = 4 $\Rightarrow$ 3 bit

$\therefore$ Total no. of Pixel = $10(3+1) = 40$

$CR = \frac{25}{40} = 0.625 : 1$