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CAPILLARY CONDUCTION OF LIQUIDS THROUGH POROUS MEDIUMS

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Abstract

The flow of liquids in unsaturated porous mediums follows the ordinary laws of hydrodynamics, the motion being produced by gravity and the pressure gradient force acting in the liquid. By making use of Darcey's law, that flow is proportional to the forces producing flow, the equation $K\nabla^2\psi + \nabla K \cdot \nabla\psi + g\partial K/\partial z = -\rho_s A \partial\psi/\partial t$ may be derived for the capillary conduction of liquids in porous mediums. It is possible experimentally to determine the capillary potential $\psi = \int dp/\rho$, the capillary conductivity K, which is defined by the flow equation $q = K(g - \nabla\psi)$, and the capillary capacity A, which is the rate of change of the liquid content of the medium with respect to ψ. These variables are analogous, respectively, to the temperature, thermal conductivity, and thermal capacity in the case of heat flow. Data are presented and application of the equations is made for the capillary conduction of water through soil and clay but the mathematical formulations and the experimental methods developed may be used to express capillary flow for other liquids and mediums. The possible existance of a hysteresis effect between the capillary potential and moisture content of a porous medium is considered.

Introduction

THE term capillarity initially referred to the action of liquids in fine tubes but has since come to be used in connection with a wide variety of phenomena, including under certain conditions the flow of liquids through porous mediums.

Buckingham[1] was among the first to attempt a detailed analysis of capillary flow. He assumed capillary attraction to constitute a conservative force field and defined a capillary potential, the gradient of which was equal to the capillary force. Some years later Gardner and his associates[2] pointed out that the Buckingham potential was closely related to the pressure in the water films and showed that porous clay apparatus could be used for its measurement. A discussion of the relation between the so-called capillary force and the pressure in the film liquid has been given by the writer[3] and experimental data were presented to show that if a porous medium is wetted by a liquid and if the medium is unsaturated, the pressure in the liquid films is less than atmospheric pressure and depends on the amount of liquid present in the medium.

Whenever there is a difference in pressure between two points in a liquid film there is a tendency for water to move in the direction of the decrease in

[1] E. Buckingham, U. S. D. A. Bur. of Soils Bul. **38**, 61 (1907).

[2] Gardner, Israelsen, Edlefsen and Clyde, Phys. Rev. **20**, 196 (1922).

[3] L. A. Richards, Jour. of Ag. Res. **20**, 719 (1928).

pressure. A study of this phase of the problem indicates that the capillary conduction of liquids through porous mediums is similar to the flow of liquids through pipes or fine tubes to the extent that in both cases the flow may be expressed in terms of gravity and the pressure gradient in the liquid. There is, however, a significant difference between the two classes of flow phenomena and we are quite justified in continuing to use the term capillarity when dealing with liquid systems where the pressure distribution is determined by the surface tension and curvature of a gas-liquid interface.

Among the expressions which have been derived for capillary flow, those given by Gardner[4,5] are the most noteworthy but as yet a general theory in terms of known or measurable functions has not been published. The problem is complicated by a large number of variables and the complex configuration of the region occupied by the capillary liquid has been a stumbling block for theoretical analysis. Also, the lack of suitable apparatus has prevented accurate experimental measurements. In the present work some of the experimental difficulties have been overcome and the problem has been approached by methods which are closely analogous, both in theory and experiment, to those used in the study of thermal and electrical conduction in metals.

Forces Affecting Capillary Action

The forces acting in the boundary surfaces of liquids are directly responsible for all capillary phenomena and have their origin in the cohesive and adhesive attractions which are exerted between molecules. The most common cases of capillary action occur when one surface of a liquid is in contact with air. Because of the free energy possessed by the molecules of the liquid lying in such a surface there is a tendency for the surface to assume that configuration which makes the area a minimum. Mathematically a uniform surface density of free energy is equivalent to a uniform tension in the surface and for some purposes it is more convenient to express the physical properties of an air-liquid surface in terms of surface tension.

For instance, the pressure difference caused by a curved liquid surface is usually expressed by the relation,

$$p_W - p_A = T_1(1/R_1 + 1/R_2), \tag{1}$$

p_W and p_A being the pressures on the two sides of the surface, T_1 the surface tension, and $(1/R_1 + 1/R_2)$ the total curvature of the surface.

Because of the inequality between cohesive and adhesive forces at a liquid-solid interface, these surfaces also have a free energy which is important in determining the capillary behavior of liquids. If, for a given liquid and solid the surface densities of free energy for the air-liquid, air-solid and liquid-solid interfaces are respectively T_1, T_2, and T_{12}, it may be shown[6] that,

[4] W. Gardner, Soil Sci. **10**, 103 (1920).

[5] W. Gardner and J. A. Widtsoe, Soil Sci. **11**, 215 (1921).

[6] N. K. Adam, Sci. Progress **21**, 431 (1927).

$$T_1 + T_2 - T_{12} = W = T_1(1 + \cos \theta), \tag{2}$$

W being the work necessary to separate unit area of liquid from solid and θ the angle of contact between liquid and solid. If $W < 2T_1$, θ may vary anywhere between 0° and 180°. If, however, $W \geqq 2T_1$ then θ is zero and the liquid will spread over the surface of the solid and is said to wet the solid.

Because of their effect on the angle of contact, adhesive forces are directly involved in an initial wetting process such as the spreading of a liquid in a dry porous medium, but after the medium is wetted adhesive forces are no longer effective in producing a motion of the liquid and influence capillary action only to the extent that they hold a thin film firmly in contact with the solid surface. The liquid lying outside the adsorbed films is free to move under the action of unbalanced forces. Capillary flow may be expressed in terms of the well-known laws of hydrodynamics and is distinguished from other cases of liquid flow only because of the relation of surface tension and curvature to the pressure and to the effective cross-sectional area of the liquid-transmitting region.

The equation of motion for a viscous fluid is given in various treatises on hydrodynamics[7,8] as,

$$\rho dv/dt = \rho F - \nabla p + \mu(\nabla \nabla \cdot v/3 + \nabla \cdot \nabla v), \tag{3}$$

where ρ is the mass of unit volume of the liquid, dv/dt is the acceleration and the right hand member is the expression for the forces acting per unit volume.

Dividing Eq. (3) by ρ we obtain the relation between the acceleration and forces for unit mass,

$$dv/dt = F - \nabla p/\rho + (\mu/\rho)(\nabla \nabla \cdot v/3 + \nabla \cdot \nabla v). \tag{4}$$

F represents the external or body forces and for most capillary problems gravity is the only external force which need be considered. The term $- \nabla p/\rho$ is the expression for the force due to the pressure gradient and the third term, being a function of the viscosity μ, and the velocity v, is the expression for the viscous retarding forces.

If the gravity force F is represented as the negative gradient of the potential $\phi = gz$, where z is the height from a reference level, then $F = - \nabla\phi = -g$. The variation of the gravity force with z is neglected, g being assumed constant. Also, if ρ is constant or depends on p alone, the force represented by the term $- \nabla p/\rho$ may be expressed as the gradient of the potential $\psi = \int dp/\rho$, the integral being taken from an arbitrarily chosen reference pressure to the pressure at the point in question. Eq. (4) may then be written,

$$dv/dt = - \nabla(\phi + \psi) + (\mu/\rho)(\nabla \nabla \cdot v/3 + \nabla \cdot \nabla v), \tag{5}$$

or,

$$dv/dt = - \nabla \Phi + (\mu/\rho)(\nabla \nabla \cdot v/3 + \nabla \cdot \nabla v), \tag{6}$$

[7] H. Lamb, Hydrodynamics, 5th edition, (1924).

[8] L. Page, Introduction to Theoretical Physics, (1928).

where $\Phi = (\phi + \psi)$ and $- \nabla \Phi$ is the total water-moving field or the total field tending to produce a motion of the water.

Expressions for the flow of viscous liquids through various channels have been derived from the general equations of motion. The equation of Poiseuille derived for flow through small tubes is in excellent agreement with experiment. Applications of the theory have been made by Slichter[9] for the flow of water through saturated porous mediums but the results are less definite in this case, because, even for the simplified idealized system made up of uniform spheres with regular packing, the complicated configuration of the water transmitting region makes necessary the introduction of various simplifying assumptions.

Liquids in Unsaturated Porous Mediums

The pore space region in sand or clay, or even a cotton wick forms a complicated system and the configuration arising when only part of this space is filled with a liquid is even more difficult to picture. If the condition for wetting, as expressed in connection with Eq. (2) is fulfilled, the liquid spreads over the solid surface and forms a continuous and connected configuration. Because of the action of surface tension, liquid tends to collect in small wedge and disk-shaped bodies in sharp corners of the pores or where the particles of the medium are close together, the size of these bodies and the thickness of the films connecting them depending on the amount of liquid present in the medium. It is through this connected configuration, bounded on one side by the adsorbed films in contact with the solid, and on the other by the curved air-liquid interface, that capillary flow takes place. The liquid occupying this region will be spoken of as capillary liquid.

Since water is the only liquid for which capillary flow data have been obtained, the discussion which follows is concerned primarily with water but the mathematical formulations given and the experimental methods presented may be used in expressing the capillary flow of other liquids in porous mediums whenever the condition of zero contact angle is fulfilled.

Pressure in the capillary water of a porous medium is determined by the tension and curvature of the air-water interface as given by Eq. (1), where p_W is the pressure in the water and p_A is the pressure in the air which, in general, will be atmospheric pressure. It is convenient when dealing with capillary phenomena to choose atmospheric pressure as the zero reference. Adopting this convention, p_A may be dropped from Eq. (1) and pressures less than atmospheric pressure will be considered negative.

If the liquid in a porous medium is in contact with free water and is at equilibrium under gravity, then, neglecting the variation in atmospheric pressure and the density of water with height z above the flat water surface, we may write,

$$p_W = T_1(1/R_1 + 1/R_2) = - g\rho z. \tag{7}$$

[9] C. S. Slichter, U. S. G. S. 19th An. Rept. **2**, 301 (1898).

In order for the capillary water to attain the correct pressure for equilibrium at a given height, the moisture content of the medium changes until the curvature has the right value.

If the density of water is assumed to be constant and equal to unity, the potential $\psi = \int dp/\rho$ becomes numerically equal to the pressure. ψ is a potential due to pressure forces and when the pressure is determined by the surface tension and curvature of the air-liquid interface ψ will be called the capillary potential. For a given value of the pressure or capillary potential the amount of moisture present in a medium will depend on the number and kind of pore spaces. The curves in Fig. 1 are reproduced from an earlier paper by the
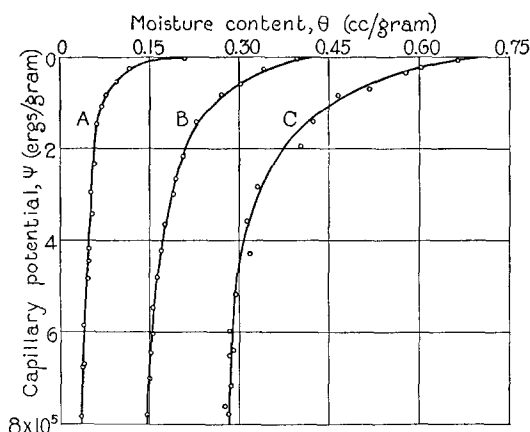

Fig. 1. Curves showing the relation between capillary potential and moisture content for, A—Bennet sandy soil, B—Greenville loam soil and C—Preston clay.

writer[3] and show the relation between moisture content and capillary potential for three mediums. To obtain these data a hollow capillary cell having a flat porous surface was filled with water at a controlled pressure. A layer of the medium to be studied was then spread on the porous surface and the moisture in the medium was allowed to come to pressure equilibrium with the water in the cell. Samples of the medium were removed at each pressure to determine the moisture content. The moisture content is given as the number of cubic centimeters of water per gram of dry medium and will be designated by θ. The capillary potential is expressed in ergs per gram and is numerically equal to the pressure expressed in dynes per square centimeter.

When the conditions for equilibrium under gravity, as expressed by Eq. (7), are fulfilled, the velocity and acceleration of the capillary liquid are everywhere zero and Eq. (5) becomes,

$$\nabla(\phi + \psi) = 0 \qquad (8)$$

which means that the force arising from the pressure gradient just balances gravity. If this condition does not obtain there will be a resultant water-moving force and in general there will be capillary flow.

Because of the complex configuration of the capillary liquid it would be

difficult to derive expressions for capillary flow from the general hydro-
dynamical equations but it is possible to deduce generalizations from ex-
perimental data which enable us to set up mathematical relations between
the flow and factors causing the flow. Fourier's law and Ohm's law are just
such generalizations. An experimental law, quite analogous to these two, and
attributed to Darcey, may be used in connection with the present problem.

Darcey, working with mediums under saturated conditions, found that
the flow of water through a column of soil is directly proportional to the
pressure difference and inversely proportional to the length of the column.
For low pressure gradients it has been found by numerous investigators[10,11]
that this law is in exact agreement with experiment and it is entirely analog-
ous to the well-known law of Poiseuille for the flow of liquids through capil-
lary tubes. However, both of these laws fail to hold for high pressure gradi-
ents. The limits within which they are true and the modifications which a
second approximation requires can be determined only by exhaustive ex-
periments on a wide range of materials. In view of the experimental data now
available it is assumed that Darcey's law holds for the low velocities and
pressure gradients dealt with in this paper.

Mathematically Darcey's law may be expressed by the equation

$$q = -K\nabla\Phi, \tag{9}$$

where q is the volume of water crossing unit area perpendicular to the flow,
in unit time and K is a proportionality factor which for a medium whose
pore spaces are filled with water will depend on the number and kind of pore
spaces and the viscosity. This factor has been variously known as the trans-
mission constant,[9] the coefficient of permeability,[10] and the conductivity.[1]
In c.g.s. units it is the number of cubic centimeters of water crossing one
square centimeter perpendicular to the flow in one second when there is a
water-moving force of one dyne per gram. From analogy with the thermal
and electrical cases it will here be called conductivity.

If there is a steady flow of liquid through a porous medium which is
only partially saturated, then the larger pore spaces contain air and the
effective cross-sectional area of the water conducting region is reduced. If
these air spaces could in some way be filled with solid, the condition of the
flow would be unchanged and the proportionality between the flow and the
water-moving force would still hold because Darcey's law is independent of
the size of particles or the state of packing. Hence the essential difference be-
tween flow through a porous medium which is saturated and flow through a
medium which is unsaturated is that under this latter condition the pressure
is determined by capillary forces and the conductivity depends on the mois-
ture content of the medium. The motion of liquids through unsaturated
porous mediums will be referred to as capillary flow and for this case K in
Eq. (9) will be called capillary conductivity.

[10] Norah D. Stearns, U.S.G.S. Water Supply Paper **596F**, 121 (1927).
[11] F. H. King, U.S.G.S. 19th An. Rept. **2**, 67 (1898).

The equation of continuity for capillary flow may be written as

$$\nabla \cdot q = - \rho_s \partial\theta/\partial t \tag{10}$$

where $\nabla \cdot q$ is the divergence of the flow, ρ_s is the weight of the dry medium in unit volume and $\partial\theta/\partial t$ is the rate at which the moisture content is changing with the time. Now if θ is a single valued continuous function of ψ, $\partial\theta/\partial t = (d\theta/d\psi)\ (\partial\psi/\partial t) = A\partial\psi/\partial t$ and Eq. (10) becomes,

$$\nabla \cdot q = - \rho_s A \partial\psi/\partial t. \tag{11}$$

A is the rate of change of the moisture content with respect to the capillary potential and will be called the capillary capacity of the medium. The functional relation of A to ψ or θ may be determined from curves such as those shown in Fig. 1.

Putting Eq. (9) in cartesian coordinates we have,

$$q = - K\nabla(\phi + \psi) = - K\frac{i\partial(\phi + \psi)}{\partial x} + \frac{j\partial(\phi + \psi)}{\partial y} + \frac{k\partial(\phi + \psi)}{\partial z} \tag{12}$$

where i, j, and k are unit vectors along the x, y, and z axes respectively. If the z-axis is chosen positive upward along the vertical then $\partial\phi/\partial x = \partial\phi/\partial y = 0$, $\partial\phi/\partial z = g$, and

$$q = - K[i\partial\psi/\partial x + j\partial\psi/\partial y + k(g + \partial\psi/\partial z)]. \tag{13}$$

Substituting this value of q in Eq. (11) and further transposing to cartesian notation we have,

$$-\left[\frac{\partial K}{\partial x}\frac{\partial\psi}{\partial x} + \frac{K\partial^2\psi}{\partial x^2} + \frac{\partial K}{\partial y}\frac{\partial\psi}{\partial y} + \frac{K\partial^2\psi}{\partial y^2}\right.$$
$$\left. + \frac{\partial K}{\partial z}\left(g + \frac{\partial\psi}{\partial z}\right) + K\left(\frac{\partial g}{\partial z} + \frac{\partial^2\psi}{\partial z^2}\right)\right] = - \rho_s A \frac{\partial\psi}{\partial t}. \tag{14}$$

Dropping the term $\partial g/\partial z$ and rearranging gives a differential equation for the general case of capillary flow,

$$K\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + \frac{\partial K}{\partial x}\frac{\partial\psi}{\partial x} + \frac{\partial K}{\partial y}\frac{\partial\psi}{\partial y} + \frac{\partial K}{\partial z}\frac{\partial\psi}{\partial z} + g\frac{\partial K}{\partial z} = - \rho_s A \frac{\partial\psi}{\partial t}. \tag{15}$$

If from experimental data, K and A can be expressed as functions of the capillary potential, then ψ is the only dependent variable occurring in the equation.

Either ψ or θ may be used as the dependent variable and q, K, and A may be expressed in terms of either one. It seems that ψ will be the easier variable to use when experimentally investigating the nature of K. If ψ is a single valued function of θ, the choice is simply a matter of mathematical expediency. The moisture density, or the amount of water in unit volume

of the soil has been used as a variable in expressing capillary flow[5,12], but, for uniform packing this variable is proportional to what is here called the moisture content.

In order to make use of the above equations it is necessary to have information concerning the functions $\theta(\psi)$, and $K(\psi)$. There is already available in the literature[1,3,12,13,14,15] some data showing the relation between θ and ψ for soils and it is possible to determine, for a given porous medium, the way in which the capillary conductivity depends upon the capillary potential.

EXPERIMENTS WITH CAPILLARY CONDUCTION

If the pressure gradient and capillary flow are known for a given column of porous medium, the value of the conductivity may be calculated by sub-
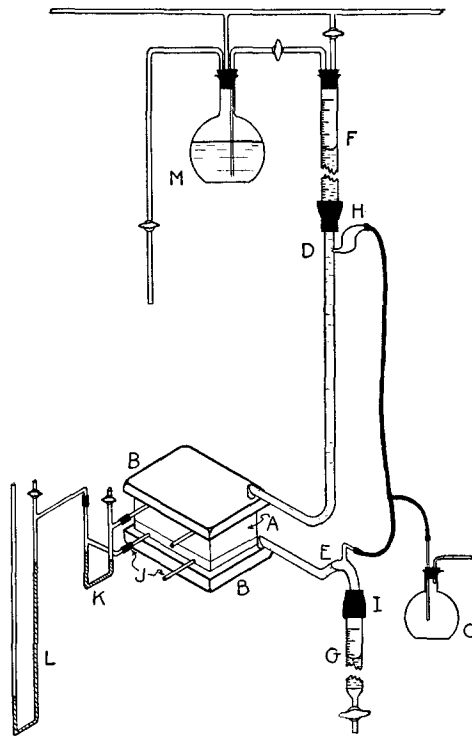

Fig. 2. Apparatus for measuring capillary conductivity.

stituting in Eq. (9). An arrangement of apparatus which has been used in making conductivity measurements is shown in Fig. 2.

A column of soil or other porous medium to be studied is mounted in the rectangular telescoping frame A between the two hollow fired clay cells, B,

[12] O. W. Israelsen, Hilgardia **2**, 480 (1927).
[13] Livingston, Hemmi and Wilson, Plant Physiol. **1**, 389 (1926).
[14] C. A. Shull, Bot. Gaz. **62**, 1 (1916).
[15] B. J. Korneff, Ann. Sci. Agronom. **43**, 480 (1927).

which have a flat porous wall of uniform thickness in contact with the column of soil. To insure good capillary contact these cells were held against the soil column with strong rubber bands. The cells are filled with water and connected as shown to the vacuum bottle C whose pressure is automatically controlled. It is the value of this pressure which determines the moisture content of the column. The pressure in the upper cell is greater than in the lower cell due to the weight of the hydrostatic column from D to E. With the arrangement shown there is a pressure and a gravity force tending to make the water move from the upper cell through the column to the lower cell. The water flowing in and out of the column is measured by the burettes F and G. By using pinch clamps at H and I the upper burette can be filled and the lower one emptied without disturbing the cell water pressure. The pressure difference at the two ends of the column is measured by means of four small porous clay cups J extending across the ends of the column. The cups are filled with water and connected to two mercurial manometers, K, only one of which is shown in the figure. These cups, which serve as capillary potentiometers, are 5 mm. in diameter and have a wall thickness of about 0.5 mm. Since the pressure drop along lines of flow passing through the cups is small, it is assumed these potentiometers indicate the pressure difference between the two ends of the soil column. If half of this difference is subtracted from the pressure difference indicated by manometer L, the average pressure in the water films of the soil column is obtained. Air in the column was at atmospheric pressure and evaporation from the cells and soil was prevented by enclosing them in a paraffined insulite box, the air in which was kept saturated by cloth wicks.

The porous clay cells, having the desired shapes and properties, were made up in the laboratory by the ordinary plaster of Paris casting process.[16] The hollow rectangular cells were cast in two pieces and cemented together with slip when partly dried. The clay used, when fired at 1000°C was readily permeable to water and a wall of the material 0.5 mm thick, when wet, will stand a pressure difference of one atmosphere without leaking air.

The liquid passing through a column was used over and over so that dissolved substances would not continue to change its surface tension and viscosity.

Since the conductivity was to be measured at a constant film thickness or constant curvature it was necessary to maintain a constant difference between the capillary water pressure and atmospheric pressure. Special apparatus[17] was divised for this purpose which would automatically maintain the pressure in flask C to within 0.5 mm of mercury of the desired value. Room temperature was controlled by means of a mercury ether thermo-regulator[18] and temperature fluctuations as read by a sensitive thermometer were less than 0.1°C. It was found necessary for some of the runs to store the ingoing liquid in a flask, M, at reduced pressure so as to prevent

[16] C. F. Binns, The Potter's Craft, New York (1922).
[17] L. A. Richards, Rev. Sci. Inst. 2, 50 (1931).
[18] L. A. Richards, J.O.S.A. and R.S.I. 18, 131 (1929).

dissolved air from coming out of solution under the low pressure in the burettes and cells. The connecting tubes were arranged so that any air collecting in the cells would expell itself. Four of the units shown in Fig. 2 were constructed.

When the rate of flow into a column becomes steady and equal to the flow out, a value of the conductivity may be calculated. If Q/t is the inflow in cc per sec., a, the area of the soil column, L its length and Δp the pressure difference, then, multiplying Eq. (9) by the column area to obtain the total flow gives,

$$Q/t = aq = -aK\nabla\Phi. \tag{16}$$

For the value of the water moving force we have

$$-\nabla\Phi = -g - \Delta p/L, \tag{17}$$

the negative signs of the right hand member indicating that the force acts in the negative z direction. Substituting (17) in (16) and solving for K gives

$$K = \frac{Q}{t} \cdot \frac{1}{a(g + \Delta p/L)}. \tag{18}$$

Conductivity data for two soils, A and B, and a ceramic clay C, are given in Table I and Fig. 3. The soil B and clay C are the same as those for which

TABLE I.

Medium	Date	Column area cm²	Column length cm	Δp dyne cm⁻²	Q/t cc sec.⁻¹ 10⁵	ψ erg gram⁻¹	K 10⁻¹¹sec.
Sandy Soil A	May 15	144.0	4.0	0.0	61.0	0.494	415.0
	May 23	144.0	4.0	9750.0	12.1	1.45	24.6
	Oct. 23	144.0	4.0	865.0	160.0	0.201	930.0
	Nov. 11	144.0	4.0	16100.0	11.8	1.35	16.4
	Nov. 22	144.0	4.0	23300.0	6.82	2.38	6.95
Clay Loam Soil B	May 4	151.5	4.4	4740.0	172.5	0.214	555.0
	May 14	151.5	4.4	5630.0	159.0	0.868	465.0
	May 21	151.5	4.4	5920.0	141.0	1.70	400.0
	Oct. 23	151.5	4.4	8440.0	151.5	0.314	341.5
	Oct. 28	151.5	4.4	7650.0	140.0	0.835	340.0
	Nov. 12	151.5	4.4	7900.0	130.0	1.58	309.5
	Nov. 19	151.5	4.4	8970.0	117.6	2.43	257.0
	Dec. 1	151.5	4.4	11000.0	93.8	3.85	178.0
	Dec. 5	151.5	4.4	19600.0	59.8	5.82	72.7
Ceramic Clay C	May 10	112.0	4.5	9080.0	336.0	0.095	1000.0
	May 15	112.0	4.5	34300.0	57.0	0.904	59.0
	May 23	112.0	4.5	40400.0	32.8	1.53	29.4
	Oct. 23	112.0	4.5	16800.0	242.0	0.14	460.0
	Nov. 4	112.0	4.5	53500.0	4.35	1.46	3.28
	Nov. 11	112.0	4.5	28800.0	112.0	0.266	135.0

the curves with the corresponding letters in Fig. 1 were obtained. The A mediums in Figs. 1 and 3 are not the same but seem to have a similar mechanical composition.

In order to follow closely the changes in the rate of flow, readings of the burettes were taken several times a day and except in a few cases when difficulties with the apparatus developed the time intervals between the dates given in Table I indicate the time required for the flow to become steady.

As was stated above, K, which is the flow across unit area in unit time for unit potential gradient, depends on the mechanical composition and state of packing of the medium and on the viscosity and pressure in the water films. For a given determination of the conductivity all of these factors were
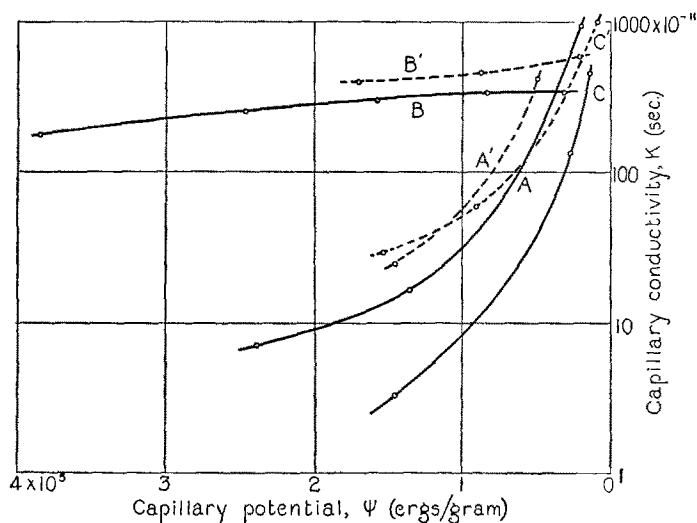

Fig. 3. Capillary conductivity curves for, *A*—a sandy soil,
B—Greenville loam soil and *C*—Preston clay.

held constant except the state of packing, which, under the compressive stress of the rubber bands, continued to become more dense with time. The effect of this change is indicated in the data.

When the apparatus was first set up the data shown in Fig. 3 as the dotted curves and designated by the primed letters was taken. The apparatus then stood for five months before obtaining the data shown by the full curves. In each case there is a shifting of the curve toward lower values of the conductivity.

APPLICATIONS OF THEORY AND DATA

The curves for the moisture content and conductivity shown in Figs. 1 and 3 have had all the factors except the capillary potential held constant and, before any general application of the flow equations can be made it would be necessary to know something about the relations among ψ, θ and K for different temperatures and different states of packing. However, as an example to illustrate the usefulness of such information, the data given here will be employed in the solution of a typical flow problem.

Let it be required to find for Greenville soil the moisture distribution and steady state upward flow from a water table one meter below the surface.

It will be assumed that the temperature and state of packing correspond to those for which the data were taken. Let us specify the further condition that due to various losses the capillary potential at the surface is $-6.\times10^5$ ergs per gram, this being the lowest value for which the conductivity data are available.

Under the condition of steady linear flow involved in the problem, $\partial\psi/\partial x = \partial\psi/\partial y = \partial\psi/\partial t = 0$ and Eq. (15) becomes

$$K\frac{\partial^2\psi}{\partial z^2} + \frac{\partial K}{\partial z}\frac{\partial\psi}{\partial z} + g\frac{\partial K}{\partial z} = 0. \tag{19}$$

The conductivity data given by curve B, Fig. 3, may be represented analytically by the expression,

$$K = a\psi + b \tag{20}$$

where $a = 5.35\times10^{-15}$ and $b = 3.85\times10^{-9}$. Eq. (19) may now be written

$$(a\psi + b)\frac{\partial^2\psi}{\partial z^2} + a\left(\frac{\partial\psi}{\partial z}\right)^2 + ag\frac{\partial\psi}{\partial z} = 0. \tag{21}$$

The first integration of this equation gives,

$$\frac{\partial\psi}{\partial z} = \frac{1 - gac_1(a\psi + b)}{ac_1(a\psi + b)}. \tag{22}$$

Integrating again to obtain a general solution of (21) we have,

$$\frac{1}{g^2a^2c_1}\left[1 - gac_1(a\psi + b) - \log\,(1 - gac_1(a\psi + b))\right] = z + c_2. \tag{23}$$

If coordinates are chosen such that $z = 0$ at the water table, then $\psi = 0$ when $z = 0$, and $\psi = -6.\times10^5$ when $z = 100$. Substituting these conditions in (23) gives two new equations from which the values of the integration constants were found to be, $c_1 = -1.66\times10^{19}$ and $c_2 = -2288$.

The equation for the flow, under the conditions of the problem, becomes

$$q = -K\nabla\Phi = K(-\partial\psi/\partial z) - g). \tag{24}$$

Substituting the values of K and $\partial\psi/\partial z$ given by Eqs. (20) and (22) gives

$$q = (a\psi + b)\left[\frac{-1 + gac_1(a\psi + b)}{ac_1(a\psi + b)} - g\right] \tag{25}$$

which reduces to

$$q = -1/ac_1 = 1/(1.66\times10^{19}\times5.35\times10^{-15}) = 1.13\times10^{-5} \text{ cc sec.}^{-1}\text{ cm}^{-2} \tag{26}$$

or $q = 0.97$ cc per day per cm².

Using Eq. (23), the values of ψ for various heights above the water table were calculated and are represented by curve A, Fig. 4. Curve B gives the

moisture content corresponding to the value of the capillary potential at each height under the flow conditions of the problem and was determined by making use of curve B, Fig. 1.
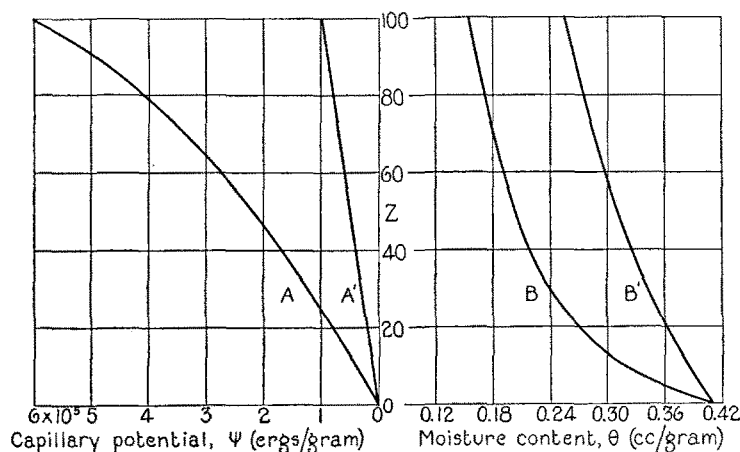

Fig. 4. Curves showing the distribution of capillary potential and moisture content with height above a water table 100 cm below the surface of Greenville soil. A and B are for upward capillary flow while A' and B' are for static equilibrium.

The curves A' and B', Fig. 4, give the distribution of pressure and moisture with height when there is no surface loss. That is, when the capillary liquid is at rest under gravity and the flow is zero.

Relation between Capillary Potential and Moisture Content

It is of considerable importance in connection with the use of the equations for capillary flow in a porous medium to know, for a certain temperature and state of packing, whether the moisture content is a single valued function of the capillary potential. Haines[19],[20] and others[21] have found there is a decided hysteresis effect for a porous medium made up of small spheres. The analogy is made that a medium composed of spheres acts like a capillary tube having a succession of bulges and contractions, because the height to which a liquid stands in such a tube depends on whether the liquid is first raised and allowed to sink to its equilibrium level or whether the liquid rises of itself to the rest position.

If it were possible to set up a system of capillary tubes which would be equivalent to a mass of soil as regards their ability to hold and to transmit moisture in a given direction, it would seem likely that some of these tubes would have a varying cross-section and hence a hysteresis effect might be expected. However, because of the lateral interconnections between soil

[19] W. B. Haines, Jour. of Agric. Sci. 17, 264 (1927).
[20] W. B. Haines, Jour. of Agric. Sci. 20, 97 (1930).
[21] Smith, Foote and Busang, Phys. Rev. 37, 1015 (1931).

capillaries and because the cellular pore spaces between soil particles have various sized openings, it is difficult to predict whether all porous mediums made up of particles will show a hysteresis effect.

The capillary-potential vs. moisture content data shown in Fig. 1 were obtained by having a one half inch layer of soil in contact through a porous plate with water which is at controlled pressure. This method is open to the objections that it is impossible to tell when the soil water is at pressure equilibrium with the water in the cell and also each point on the curve is taken for a different mass of soil. To overcome these undesirable features a fired clay cell like those used in the transmission experiments was suspended on a balance and provided with a lid for covering the upper surface which was porous. Connections to the cell were made by means of a glass tube three meters long and heavy rubber tubing in such a way that when the cell contains water any air accumulating inside will expel itself. The weight of the cell system was made practically independent of changes in the water pressure and a sensitivity of 5 mg in the weighings was attained. When ready for an experiment, a known weight of dry soil is spread on the plate and the amount of moisture in the soil at any later time may be determined from the balance reading.

TABLE II.

	Date		Capillary potential 10^5 erg gram^{-1}	Moisture content cc gram^{-1}
1	Jan.	15	0.072	0.2669
2	"	17	0.930	0.2347
3	"	19	1.50	0.2264
4	"	20	2.93	0.2112
	"	22	6.50	—
5	"	24	4.47	0.1908
6	"	26	2.31	0.2037
7	"	29	0.990	0.2188
8	"	31	0.175	0.2415
9	Feb.	2	0.072	0.2618
10	"	4	0.950	0.2274
11	"	5	1.97	0.2139
12	"	7	0.950	0.2215
13	"	8	0.071	0.2577
14	"	12	0.940	0.2268
15	"	17	2.08	0.2162
16	"	18	3.48	0.2075

The only data yet available with this apparatus were taken for Greenville soil and are presented in Table II and Fig. 5. The cell was filled with soil solution so that transfers of moisture between soil and plate would not cause changes in surface tension and the temperature was controlled for the experiment. Balance readings were taken at frequent intervals after a change in pressure and except for 9 and 13, the points shown represent equilibrium values of the moisture content for the corresponding pressures. It is apparent for a 4 mm layer of soil in a rather loose state of packing there is a considerable hysteresis effect over the pressure range studied.

These results, however, are not conclusive because under the conditions

of the experiment it is probable changes in the state of packing accompanied the changes in moisture content. These changes in packing if present would be in the right direction and might be of sufficient magnitude to account for the observed hysteresis effect.
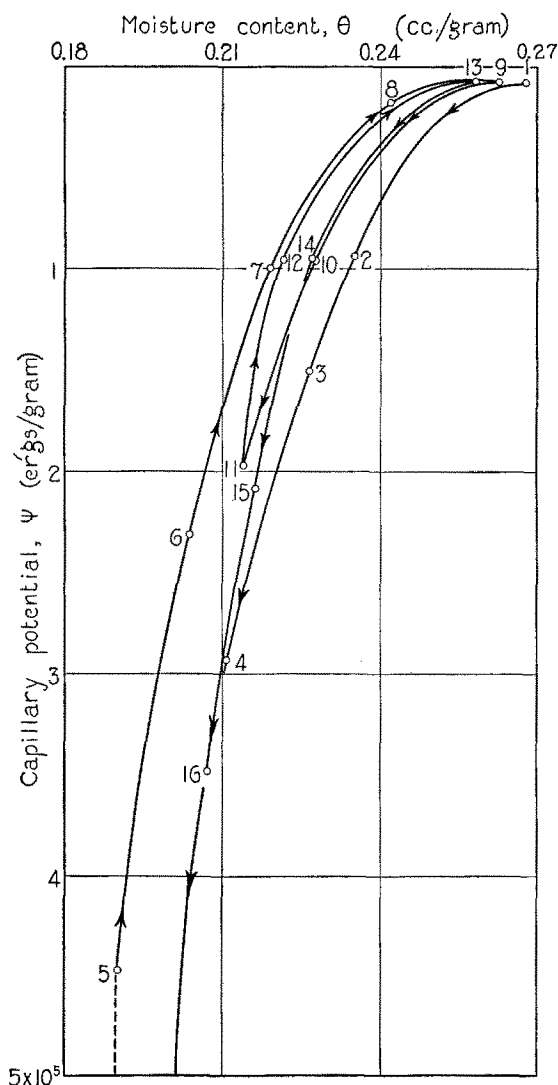

Fig. 5. Curves showing the variation in the moisture content of Greenville soil for cyclic variations in the capillary potential.

To avoid the possibility of the structural change, another experiment using a piece of sandstone as the porous medium has been set up. This material, being composed of loosely cemented particles of various sizes has a rigid structure and should give a good index as to what would happen in a

soil or clay if the pore structure were fixed. As yet no data are available from this experiment.

Another source of information on the hysteresis effect lies in the conductivity experiments. If curves like those shown in Fig. 3 retrace themselves as the potential is increased to zero, it would be good evidence that no hysteresis effect exists. With but one exception the curves shown were obtained by starting with the porous medium wet and proceeding to lower pressures and moisture contents. For curve C, as indicated by the dates in Table I, the data for the second point on the curve were taken after the low conductivity determination was made. The consistant relation between the dotted and full curves for the three mediums would seem to indicate that if a hysteresis effect exists for the Preston clay, it does not have a very large effect on the relation between the capillary potential and the conductivity.

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