Study on Interaction between Surface and Subsurface Flows using Conjunctive Flow Model

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ABSTRACT. The interaction between the surface and subsurface flow components plays an important role especially in initial loss and overland flow initiation at the early stage in rainfall events. Therefore, coupling of the surface and subsurface flow submodels is necessary in more comprehensive and sophisticated watershed modeling to deal with the interaction theoretically. We already proposed the conjunctive 2-D surface and 3-D subsurface flow model using an approximate version of the Saint-Venant equations to simulate the two-dimensional unsteady surface flow and a modified version of Richard's equation for the three-dimensional unsaturated and saturated unsteady subsurface flow [1, 2]. This paper shows a further development of the conjunctive model and focuses on the interaction between surface and subsurface flow components. The interaction of the two components is directly related to the estimation of effective rainfall or initial loss of hyetographs. In the conjunctive model the interaction is formulated using the comparison of the two parameters, "rainwater supply" and "infiltrability (infiltration capacity) ". The model reproduces the initiation of overland flow and initial loss process and contributes to more precise and reliable estimation of effective rainfall than the usual methods using the empirical formulas in watershed modeling.

KEY WORDS: conjunctive model, effective rainfall, infiltration capacity

INTRODUCTION

Initial loss, infiltration, and overland flow initiation are familiar to us and can easily be observed on pervious surfaces around our residences at the early stage in rainfall events. These hydrological processes are typical examples of an interaction between surface and subsurface flows. The interaction, however, has been disregarded in watershed modeling. Instead, most of the models separate the surface and subsurface flow components, calculating the effective rainfall using the empirical formula such as Horton's infiltration equation. For more sophisticated models using nonlinear partial differential equations for surface and subsurface flows, the common boundary condition as an interface of the two flow components has not been treated theoretically in the numerical calculations and the surface and subsurface flows have been ' eternally connected' [3].

In this paper, we show a conjunctive 2-D surface and 3-D subsurface flow model. The conjunctive model couples the two flow components with a common boundary condition set on the comparison between infiltrability (infiltration capacity) and rainwater supply at each time step in the numerical calculation. After formulating the conjunctive model, we applied the model to simulate the interaction between surface and subsurface flows: initial loss, infiltration and overland flow initiation.

GOVERNING EQUATIONS AND NUMERICAL METHODS

For the conjunctive model, we apply an approximated dynamic wave equation used in the diffusion flow model [4] – appropriately named the non-inertia model - for the two-dimensional

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surface flow, and a modified version of Richard's equation for the three-dimensional subsurface flow.

<Surface Flow>
The continuity equation for two-dimensional surface flow is described as

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(uh)+\frac{\partial}{\partial y}(vh)+i-r=0 \tag{1}$$

where h = flow depth normal to surface; u and v = cross sectional average velocities in the x and y directions ; i = infiltration rate; r = rainfall intensity.
Momentum conservations equation can be written in the non-inertia approximation form.

$$\frac{\partial h}{\partial x}-S_{0x}+S_{fx}=0 \tag{2}$$

$$\frac{\partial h}{\partial y}-S_{0y}+S_{fy}=0 \tag{3}$$

where S_{ox} and S_{oy} = bottom slopes in the x and y directions ; S_{fx} and S_{fy} = friction slopes in the x and y directions, respectively.
The friction slopes are obtained from the Darcy-Weisbach formula.

$$(S_{fx},S_{fy})=\left(f_d\frac{u^2}{8gh},\ f_d\frac{v^2}{8gh}\right) \tag{4}$$

The friction resistance coefficient f_d is calculated as a function of the Reynolds number : $\boldsymbol{R}= VR\,/\,\nu$; where V is the magnitude of flow velocity, R hydraulic radius, ν kinematic viscosity.

$$f_d=\frac{24}{\boldsymbol{R}} \qquad \text{for} \quad 0<\boldsymbol{R}<500 \tag{5}$$

$$f_d=\frac{0.223}{\boldsymbol{R}^{0.25}} \qquad \text{for} \quad 500<\boldsymbol{R}<30\,000 \tag{6}$$

$$f_d=\frac{1}{4}\left[-\log\left(\frac{k_s}{12R}+\frac{1.95}{\boldsymbol{R}^{0.95}}\right)\right]^{-2} \qquad \text{for} \quad \boldsymbol{R}>30\,000 \text{ and } k_s/R<0.05 \text{ (Yen [5])} \tag{7}$$

Using surface elevation H instead of flow depth h in Equation (1) - (3), the governing equation for surface flow can be represented as follows:

$$\frac{\partial H}{\partial t}=\frac{\partial}{\partial x}K_x\frac{\partial H}{\partial x}+\frac{\partial}{\partial y}K_y\frac{\partial H}{\partial y}+r-i \tag{8}$$

where K_x and K_y are described by the equation (9) and calculated by the Reynolds number $\boldsymbol{R}$.

$$(K_x,K_y)=\left(\sqrt{\frac{8gh^3}{f_d}}\left|\frac{\partial H}{\partial x}\right|^{-\frac{1}{2}},\ \sqrt{\frac{8gh^3}{f_d}}\left|\frac{\partial H}{\partial y}\right|^{-\frac{1}{2}}\right) \tag{9}$$

In the non-linear equation (8), the diffusively coefficients, K_x and K_y change with flow depth and flow velocity.

<Subsurface Flow>
The equation for three-dimensional subsurface flow is represented on the basis of Richard's equation [6].

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\alpha_x(\theta)\frac{\partial H}{\partial x} + \frac{\partial}{\partial y}\alpha_y(\theta)\frac{\partial H}{\partial y} + \frac{\partial}{\partial z}\alpha_z(\theta)\frac{\partial H}{\partial z} \tag{10}$$

$$\alpha_i(\theta) = K_{si} K_r / (\partial\theta/\partial P) \tag{11}$$

where H = piezometric head; K_{sx}, K_{sy}, and K_{sz} = saturated permeabilities in the x, y, and z directions; K_r = relative permeability ; P = capillary pressure head; θ = volumetric moisture content ; $\theta = n\ S$; n = soil porosity ; S = saturation degree. Piezometric head H has the relation with P : $H = P + z$. Equation (10) can be applied to both saturated groundwater flow and unsaturated soil water flow. Except for the case of initial loss, the effect of surface flow on the subsurface flow can be treated by the surface boundary condition: flow depth.

The governing equations (8) and (10) for surface and subsurface flows are written in the form of heat diffusion equation. The numerical methods for two- and three-dimensional heat diffusion equations, therefore, can be applied to solve these equations. The ADE method of Larkin [7] was selected through the investigation of the numerical methods [2].

COUPLING OF SURFACE AND SUBSURFACE FLOW COMPONENTS

For coupling the surface and subsurface flow models together, we set the common boundary condition, comparing the rainwater supply and the demand of infiltration capacity or infiltrability I_p. The procedure of the conjunctive model calculation is described as follows:

1. Calculate the infiltrability I_p using the equation (12). The infiltrability is the potential infiltration rate under the condition of given surface water depth and soil water content just below the surface.

$$I_p = 0.5\ (K_s+K_1)(Y - H_1)\ /\ \Delta z \tag{12}$$

where K_s = saturated permeability ; K_1 and H_1 = permeability and piezometric head at the first node in the z direction ; Y = water depth at the surface calculated as

$$Y = h + r\ \Delta t \tag{13}$$

where h = water depth at a previous time step; r = rainfall intensity; Δt = time increment. The water depth h should be zero until overland flow occurs.

2. Calculate the modified rainfall intensity or rainwater supply as

$$R_s = Y/\Delta t = r + h\ /\ \Delta t \tag{14}$$

3. Compare the infiltrability I_p with the rainwater supply R_s.
4. If the infiltrability I_p is larger than the rainwater supply R_s, set the boundary condition at the surface for equation (10) as follows:

$$H_0 = H_1 + 2.0\ (R_s\ \Delta z\ )\ /\ (K_s+K_1) \tag{15}$$

In this case, infiltration rate equals to rainwater supply.

5. If the infiltrability I_p is less than the rainwater supply R_s, set the surface boundary condition as $H_0 = Y$.
6. Calculate the piezometric head H by equation (10) under the surface boundary condition from step 4 or step 5.
7. Go back to step 1 in the case of step 4. In the case of step 5, on the other hand, go on to surface flow calculation by equation (8) with the flow depth modified by the infiltration rate, and then go back to step 1 with new flow depth h.

At each time step, we compare the infiltrability and the rainwater supply and then set the right boundary condition for subsurface flow calculation. The infiltrability changes in time with surface water depth and soil moisture content just below the surface and thus controls the interaction between surface and subsurface flows.

SIMULATION OF INTERACTION BETWEEN SURFACE AND SUBSURFACE FLOWS

The conjunctive model was tested using the laboratory experimental data by Smith and Woolhiser[8]. Fig.1 shows the comparison between the observed and calculated hydrographs under constant rainfall intensity of 9.9 inch/hr. To save space, we omit the description of the experiment. Although the difference in the time of overland flow initiation is recognized between the two hydrographs, the calculated hydrograph reproduces the experimental data satisfactorily.

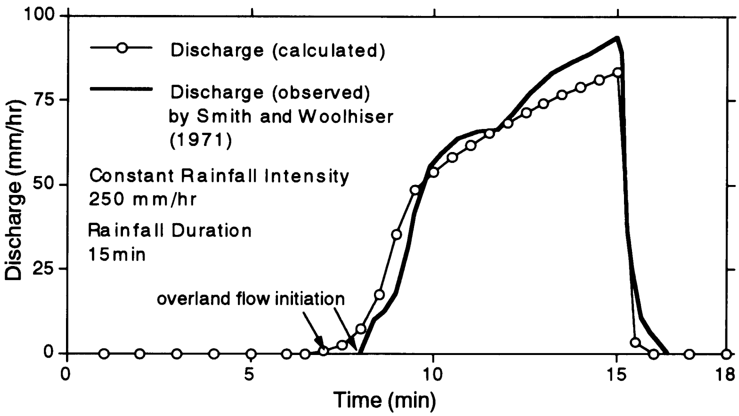

Fig.1 Comparison of Calculated Values with Experimental Data by R.E.Smith and D.A. Woolhiser

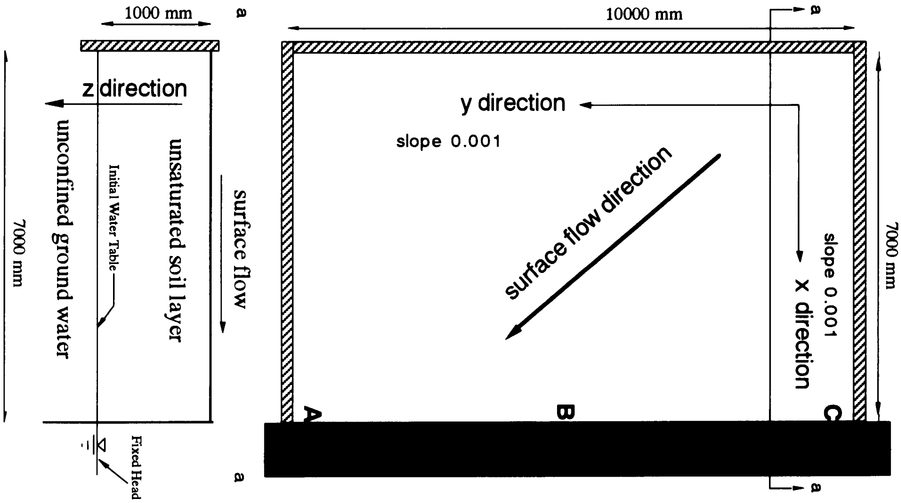

Fig.2 Surface Drainage Area for Conjunctive Model Simulation (right : top view, lower : cross sectional view)

After testing the model, we carried out simulation calculations of infiltration and overland flow. Fig.2 shows a top view of an analyzed drainage area. The surface flow area is 7.0 m by 10 m and has the slopes 0.001 in both of x and y directions. Under the top view, the cross sectional view features the soil layer and the water table of unconfined groundwater. For the application of the model, we assumed a homogeneous, isotropic porous body of stable soil layer. The soil has, according to the linearized equation of Philip [9], idealized hydraulic properties : $\alpha = K(dP/d\theta) = K_sK_r(dP/d\theta) = 2.5\text{mm}^2/\text{s}$, $k = dK/d\theta \approx (K_s/n)dK_r/dS = 0.05\text{mm/sec}$. These properties mean that the soil may be classified as fine silt rather than sand. Under the linearized condition the moisture diffusivity coefficient α is set to be constant, but the relative permeability K_r change with S and P. For the numerical calculation of the surface flow equation (8), the drainage area has the space increments $\Delta x = \Delta y = 0.5$ m, and the boundary conditions; top, leftmost, and bottom boundaries in the top view of Fig.2 are closed and rightmost boundary is open to the channel. In the calculation of subsurface flow, space increments are $\Delta x = \Delta y = 0.5$ m and $\Delta z = 0.01$ m. Besides we set $S = 0.2$ ($t = 0$, $0 < z < 1$m) as initial condition and $S = 1.0$ ($t > 0$, $z = 1$m) as bottom boundary condition. The hydrographs calculated under the triangular hyetograph with a peak rainfall intensity 100 mm/hr and under the rectangular hyetograph with a constant rainfall intensity 100 mm/hr are shown in Fig.3 and Fig4, respectively. The figures describe rainfall intensity, surface flow discharge, surface infiltration rate, infiltrability, and flow depth at point A,B,and C.

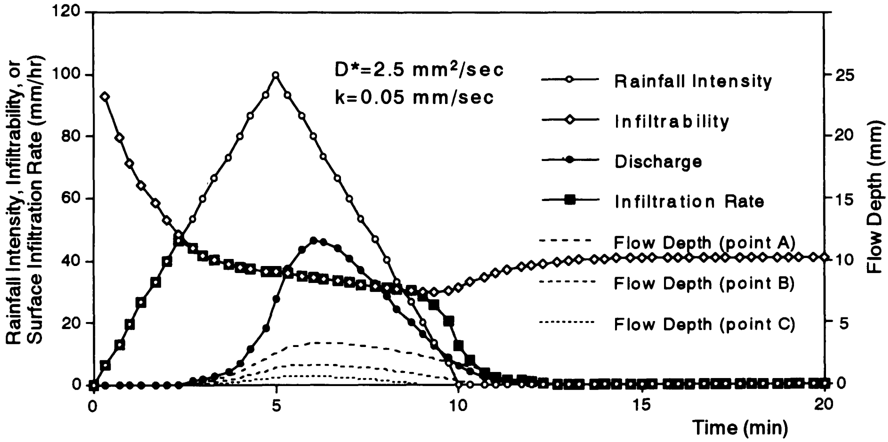

Fig.3 Calculated Hydrograph under Triangular Hyetograph

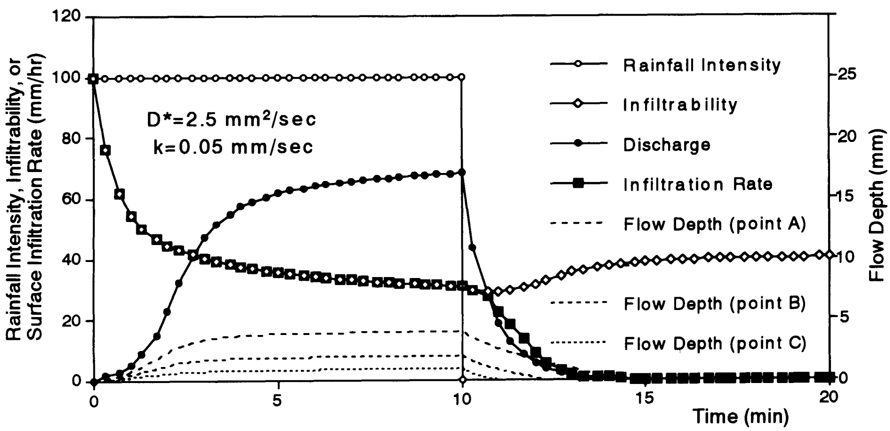

Fig.4 Calculated Hydrograph under Rectangular Hyetograph

In Fig.3 we easily recognize initial loss process in the first 2 minutes, when the infiltration rate is identical with the rainfall intensity or rainwater supply. During the initial loss process, the infiltrability is always larger than the rainwater supply. The infiltrability curve monotonously decreases in the same manner as Horton's empirical formula curve. At the time when the infiltrability curve crosses the rainfall intensity line of the hyetograph, the overland flow just initiates and has the peak discharge around 6 minutes just behind the peak rainfall intensity. Fig.3 also shows the surface flow depths at point A, B, and C. The flow depth at point A is the largest and the depths at point B and C follow in this order. The three flow depth curves indicate the initiations and terminations of overland flow are different in time at the three points. In the calculation, we found some part has overland flow and the other part has no water on the surface. The surface flow submodel thus simulated the two-dimensional overland flow.

The hydrograph calculated under the rectangular hyetograph with a constant rainfall intensity is also presented in Fig.4. The overland flow initiates simultaneously with rainfall and the infiltration rate monotonously decreases more rapidly than that in Fig.3. In the conventional approach such as Horton's equation, the same soil condition produces the same infiltration capacity. The conjunctive model, however, gives different infiltration capacities for different hyetographs as shown in Figs. 3 and 4. This is because the conjunctive model directly and theoretically deals with the interaction between surface and subsurface flows.

CONCLUDING REMARKS

We showed how to deal with the interaction between surface and subsurface flows, and introduced and defined the infiltrability I_p to compare with rainfall intensity. Using this coupling method, the 2-D surface and 3-D subsurface flow conjunctive model was formulated and the numerical calculation procedure of the model was also presented. Furthermore, we applied the conjunctive model to the drainage area after testing the model and thus simulated the 2-D surface and 3-D subsurface flows. The simulation results showed the infiltrability, the infiltration, and the overland flow and explained the mechanism of initial loss and overland flow initiation theoretically.

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