

Machine & Deep learning with Tensorflow

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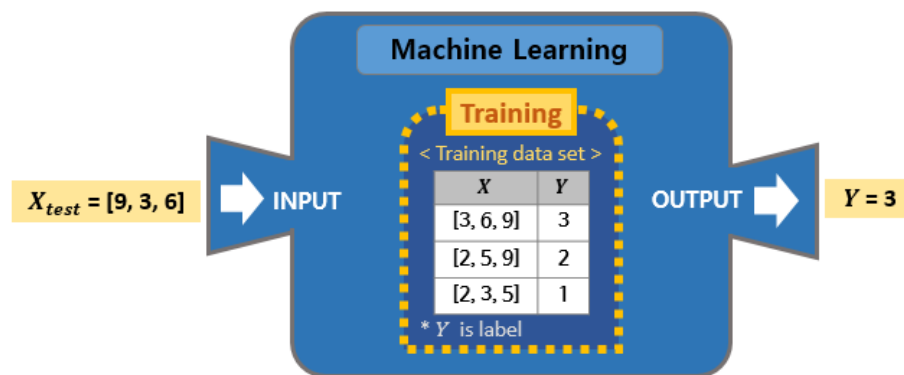
1 Machine Learning

- Limitations of explicit programming
 - Spam filter : many rules
 - Automatic driving : too many rules
- Machine learning : "Field of study that gives computers the ability to learn without being explicitly programmed" Arthur Samuel (1959)
- Method of learning (Supervised and Unsupervised learning)
 1. Supervised learning : learning with labeled examples - training set
ex) image labeling(cat, dog, car)
 2. Unsupervised learning : un-labeled data ex) Google news grouping, Word clustering
⇒ Can't give label because it's already grouping
⇒ Self learning !

2 Supervised learning

Most common problem type in Machine Learning

- Image labeling : learning from tagged images
 - Email spam filter : learning from labeled (spam or ham) email
 - Predicting exam score : learning from previous exam score and time spent
- Training data set



- Types of supervised learning
 - Predicting final exam score based on time spent → "**Regression**"
 - Pass/non-pass based on time spent → "**Binary Classification**"
 - Letter grade (A,B,C,D and E) based on time spent → "**Multi-label Classification**"

3 TensorFlow

TensorFlow is an open source software library for numerical computation using data flow graphs.

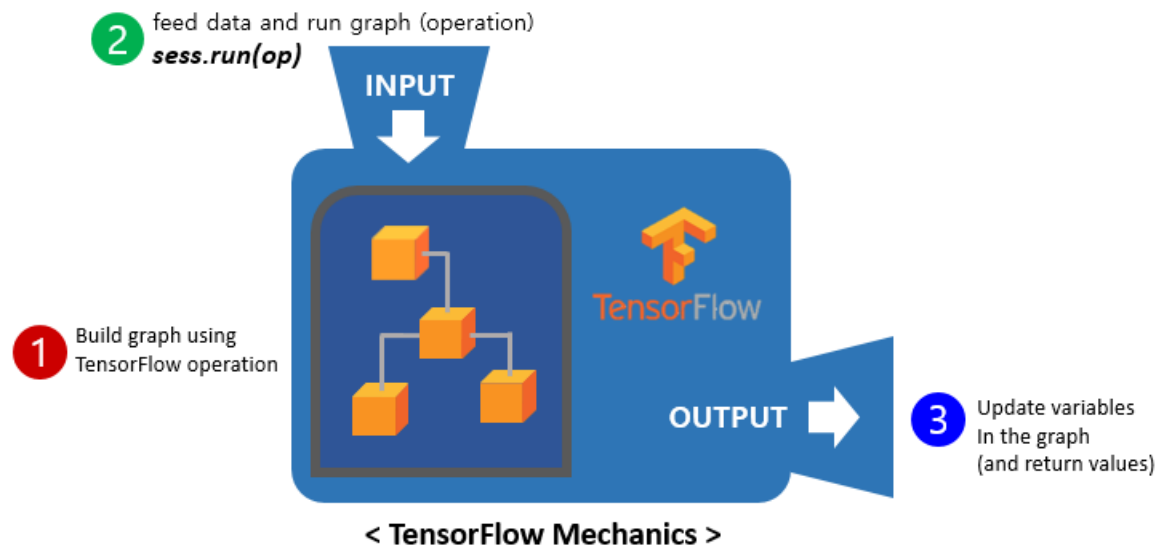
◇ What is Data Flow Graph?

- Nodes in the graph represent mathematical operations
- Edges represent the multi-dimensional data arrays(=tensors) communicated between them

`tf.__version__`: check installation and version

```
1 import tensorflow as tf
2
3 tf.__version__
```

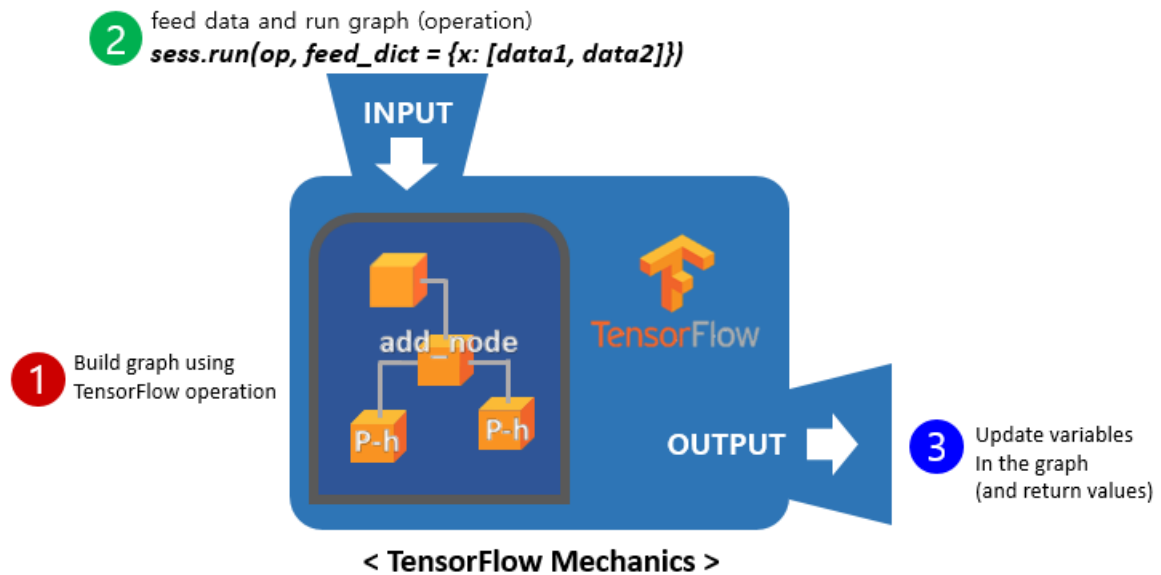
3.1 Computational Graph



MLlab 01 Computational Graph

```
1 node1 = tf.constant(3.0, tf.float32)
2 node2 = tf.constant(4.0)  ## also tf.float32 implicitly
3 node3 = tf.add(node1, node2)
4
5 print("node1:", node1, "node2:", node2)
6 print("node3:", node3)
7
8 sess = tf.Session()
9 print("sess.run(node1, node2):", sess.run([node1, node2]))
10 print("sess.run(node3):", sess.run(node3))
```

3.2 Placeholder



MLlab 01 Placeholder

```
1 a = tf.placeholder(tf.float32)
2 b = tf.placeholder(tf.float32)
3
4 add_node = a + b
5
6 print(sess.run(add_node, feed_dict={a: 3, b: 4.5}))
7 print(sess.run(add_node, feed_dict={a: [1, 3], b: [2, 4]}))
```

4 Simple Linear Regression

- (Linear) Hypothesis

$$H(x) = Wx + b$$

- Cost/Loss function : How fit the line to our (training) data

$$cost = \frac{1}{m} \sum_{i=1}^m (H(x^{(i)}) - y^{(i)})^2$$

$$\Rightarrow cost(W, b) = \frac{1}{m} \sum_{i=1}^m (H(x^{(i)}) - y^{(i)})^2$$

- ◇ Goal : Minimize Cost function

$$\min_{W, b} cost(W, b)$$

4.1 Implementation of Simple Linear Regression

MLlab 02 method ① - Assign values

Step1. Build graph

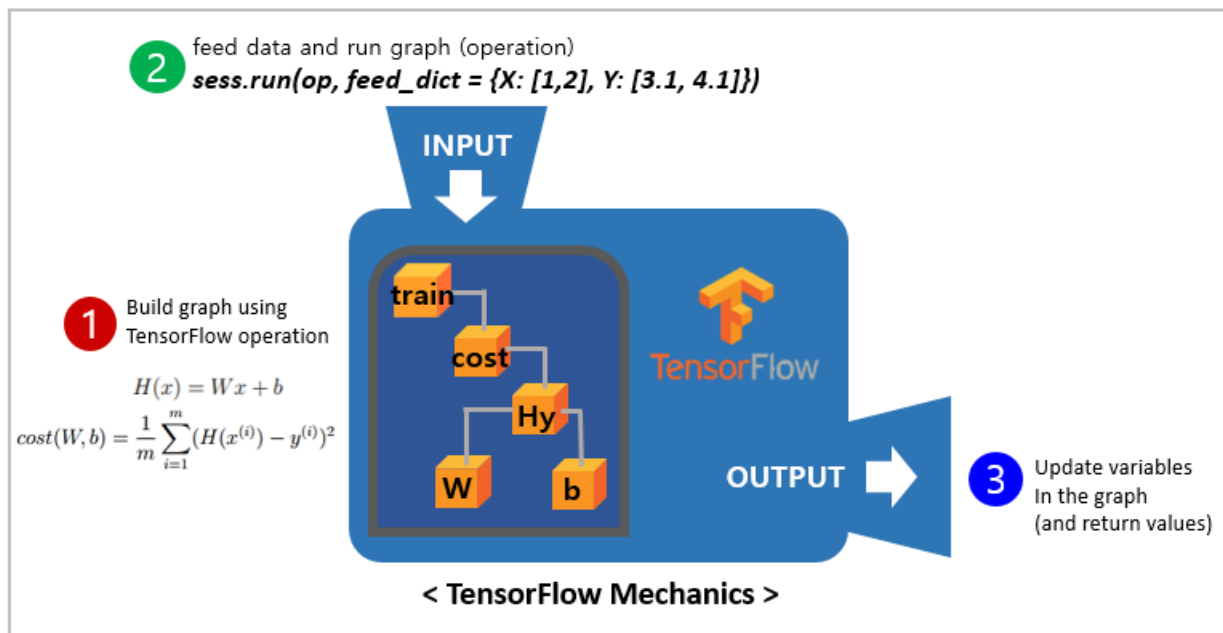
```
1 # X and Y data
2 x_train = [1, 2, 3]
3 y_train = [1, 2, 3]
4
5 w = tf.Variable(tf.random_normal([1]), name='weight')
6 b = tf.Variable(tf.random_normal([1]), name='bias')
7
8 # Our hypothesis XW+b
9 hypothesis = x_train * w + b
10
11 # Cost/Loss function
12 cost = tf.reduce_mean(tf.square(hypothesis - y_train))
13
14 # Gradient Descent_Minimize
15 optimizer = tf.train.GradientDescentOptimizer(learning_rate=0.01)
16 train = optimizer.minimize(cost)
```

Step2. Run & Update graph and get results

```
1 # Launch the graph in a session
2 sess = tf.Session()
3
4 # Initializes global variables in the graph
5 sess.run(tf.global_variables_initializer())
6
7 # Fit the line
8 for step in range(2001):
9     sess.run(train)
10     if step % 20 == 0:
11         print(step, sess.run(cost), sess.run(w), sess.run(b))
```

MLlab 02 method ② - Use placeholder

```
1 # Now we can use X and Y in place of x_data and y_data
2 ## Placeholders for a tensor that will be always fed using feed_dict
3 X = tf.placeholder(tf.float32, shape=[None])
4 Y = tf.placeholder(tf.float32, shape=[None])
5 W = tf.Variable(tf.random_normal([1]), name='weight')
6 b = tf.Variable(tf.random_normal([1]), name='bias')
7
8 # Our hypothesis
9 hypothesis = X * W + b
10 # Cost function
11 cost = tf.reduce_mean(tf.square(hypothesis - Y))
12 # Minimize
13 optimizer = tf.train.GradientDescentOptimizer(learning_rate=0.01)
14 train = optimizer.minimize(cost)
15
16 # Launch the graph in a session
17 sess = tf.Session()
18 # Initializes global variables in the graph
19 sess.run(tf.global_variables_initializer())
20
21 # Fit the line with new training data
22 for step in range(2001):
23     cost_val, W_val, b_val, _ = \
24         sess.run([cost, W, b, train],
25                 feed_dict={X: [1, 2, 3, 4, 5], Y: [2.1, 3.1, 4.1, 5.1, 6.1]})
26     if step % 20 == 0:
27         print(step, cost_val, W_val, b_val)
28
29 # Testing our model
30 print(sess.run(hypothesis, feed_dict={X: [5]}))
31 print(sess.run(hypothesis, feed_dict={X: [1.5, 3.5]}))
```



4.2 Gradient descent

One of the most popular and widely used method to find the minimum of a function for machine learning algorithms.

- Minimize cost function
- Gradient descent is used many minimization problems
- For a given cost function, $cost(W, b)$, it will find W, b to minimize cost
- It can be applied to more general function : $cost(w_1, w_2, \dots)$

1) How it works?

- Start with initial guesses
 - Start at 0 (or any other value)
 - Keeping changing W and b a little bit to try and reduce $cost(W, b)$
- Each time you change the parameters, you select the gradient which reduces $cost(W, b)$ the most possible
- Repeat
- Do so until you converge to a local minimum
- Has an interesting property
 - Where you start can determine which minimum you end up

2) Formal definition

- $cost(W) = \frac{1}{2m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$
- $W := W - \alpha \frac{\partial}{\partial W} cost(W)$ ($\alpha = 0.1$; learning rate)

$$\Rightarrow W := W - \alpha \frac{\partial}{\partial W} \frac{1}{2m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$

$$\Rightarrow W := W - \alpha \frac{1}{2m} \sum_{i=1}^m 2(Wx^{(i)} - y^{(i)})x^{(i)}$$

$$\Rightarrow W := W - \alpha \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})x^{(i)} : \text{“Gradient descent algorithm”}$$

4.3 Gradient descent algorithm

- Simplified Hypothesis

$$H(x) = Wx$$

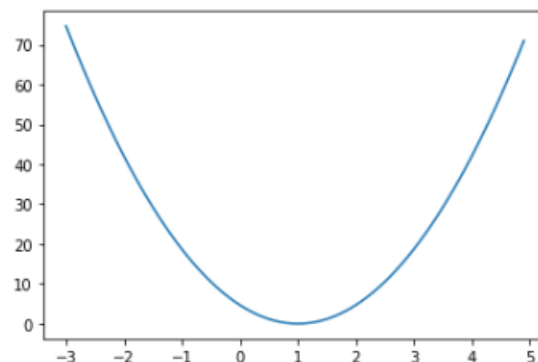
- Cost function

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m (Wx^{(i)} - y^{(i)})^2$$

MLlab 03 Plot of cost function

```
1 X = [1, 2, 3]
2 Y = [1, 2, 3]
3
4 W = tf.placeholder(tf.float32)
5
6 # Our hypothesis for linear model X*W
7 hypothesis = X * W
8
9 # cost/loss function
10 cost = tf.reduce_mean(tf.square(hypothesis - Y))
11
12 # Launch the graph in a session
13 sess = tf.Session()
14
15 # Initializes global variables in the graph
16 # (Open session to initialize variables)
17 sess.run(tf.global_variables_initializer())
18
19 # Variables for plotting cost function
20 W_val = []      # make empty list
21 cost_val = []
22
23 type(W_val)     # see type of variable
24
25 for i in range(-30, 50):    # start -30, cuz cannot be incremented by 0.1
26     feed_W = i * 0.1        # x-axis : 0.1 increments from -3 to 5
27     curr_cost, curr_W = sess.run([cost, W], feed_dict={W: feed_W})
28     W_val.append(curr_W)    # accumulate the list data
29     cost_val.append(curr_cost)
30
31 # Show the cost function
32 plt.plot(W_val, cost_val)
33 plt.show()
```

⇒ result graph : cost function



MLlab 03 Minimize cost function

```
1 x_data = [1, 2, 3]
2 y_data = [1, 2, 3]
3
4 W = tf.Variable(tf.random_normal([1]), name='weight')
5 X = tf.placeholder(tf.float32)
6 Y = tf.placeholder(tf.float32)
7
8 # Our hypothesis for linear model X*W
9 hypothesis = X * W
10 # cost/loss function
11 cost = tf.reduce_mean(tf.square(hypothesis - Y))
12
13 # Minimize: Gradient Descent using derivative: W -= learning_rate * derivative
14 learning_rate = 0.1
15 gradient = tf.reduce_mean((W * X - Y) * X)
16 descent = W - learning_rate * gradient
17 update = W.assign(descent)
18
19 # Launch the graph in a session
20 sess = tf.Session()
21 # Initializes global variables in the graph
22 sess.run(tf.global_variables_initializer())
23
24 for step in range(21):
25     sess.run(update, feed_dict={X: x_data, Y: y_data})
26     print(step, sess.run(cost, feed_dict={X: x_data, Y: y_data}), sess.run(W))
```

• Exercise: Using `tf.train.GradientDescentOptimizer()`

```
1 # tf Graph Input
2 X = [1, 2, 3]
3 Y = [1, 2, 3]
4
5 # Set wrong model weights
6 W = tf.Variable(5.0)
7
8 # linear model
9 hypothesis = X * W
10 # cost/loss function
11 cost = tf.reduce_mean(tf.square(hypothesis - Y))
12
13 # Minimize: Gradient Descent Magic
14 optimizer = tf.train.GradientDescentOptimizer(learning_rate=0.1)
15 train = optimizer.minimize(cost)
16
17 # Launch the graph in a session
18 sess = tf.Session()
19 # Initializes global variables in the graph
20 sess.run(tf.global_variables_initializer())
21
22 for step in range(100):
23     print(step, sess.run(W))
24     sess.run(train)
```

MLlab 03 Optional: compute_gradients() and apply_gradients()

```
1  # tf Graph Input
2  X = [1, 2, 3]
3  Y = [1, 2, 3]
4
5  # Set wrong model weights
6  W = tf.Variable(5.0)
7
8  # linear model
9  hypothesis = X * W
10
11 # Manual gradient
12 gradient = tf.reduce_mean((W * X - Y) * X) * 2
13
14 # cost/loss function
15 cost = tf.reduce_mean(tf.square(hypothesis - Y))
16 optimizer = tf.train.GradientDescentOptimizer(learning_rate=0.1)
17
18 # Get gradients
19 gvs = optimizer.compute_gradients(cost, [W])
20
21 # Apply gradients
22 apply_gradients = optimizer.apply_gradients(gvs)
23
24 # Launch the graph in a session
25 sess = tf.Session()
26 # Initializes global variables in the graph
27 sess.run(tf.global_variables_initializer())
28
29 for step in range(100):
30     print(step, sess.run([gradient, W, gvs]))
31     sess.run(apply_gradients)
```

5 Multi-variable Linear Regression

- Hypothesis

$$H(x_1, x_2, x_3, \dots, x_n) = w_1x_1 + w_2x_2 + w_3x_3 + \dots + w_nx_n + b$$

- Cost function

$$\text{cost}(W, b) = \frac{1}{m} \sum_{i=1}^m \left(H(x_1^{(i)}, x_2^{(i)}, x_3^{(i)}, \dots, x_n^{(i)}) - y^{(i)} \right)^2$$

5.1 Hypothesis using matrix

<Test Scores for General Psychology>

X_1	X_2	X_3	Y
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

① $H(x_1, x_2, x_3) = w_1x_1 + w_2x_2 + w_3x_3$

MLlab 04-1 ① Hypothesis using general tensorflow

```
1 x1_data = [73., 93., 89., 96., 73.]
2 x2_data = [80., 88., 91., 98., 66.]
3 x3_data = [75., 93., 90., 100., 70.]
4 y_data = [152., 185., 180., 196., 142.]
5
6 # placeholders for a tensor that will be always fed.
7 x1 = tf.placeholder(tf.float32)
8 x2 = tf.placeholder(tf.float32)
9 x3 = tf.placeholder(tf.float32)
10
11 Y = tf.placeholder(tf.float32)
12
13 w1 = tf.Variable(tf.random_normal([1]), name='weight1')
14 w2 = tf.Variable(tf.random_normal([1]), name='weight2')
15 w3 = tf.Variable(tf.random_normal([1]), name='weight3')
16 b = tf.Variable(tf.random_normal([1]), name='bias')
17
18 #Hypothesis
19 hypothesis = x1 * w1 + x2 * w2 + x3 * w3 + b
```

$$\textcircled{2} \quad H(X) = XW = (x_1 \quad x_2 \quad x_3) \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = (w_1x_1 + w_2x_2 + w_3x_3)$$

MLlab 04-1 $\textcircled{2}$ Hypothesis using matrix

```

1 x_data = [[73., 80., 75.], [93., 88., 93.], [89., 91., 90.], [96., 98., 100.], [73., 66., 70.]]
2 y_data = [[152.], [185.], [180.], [196.], [142.]]
3
4 # placeholders for a tensor that will be always fed.
5 X = tf.placeholder(tf.float32, shape=[None, 3])
6 Y = tf.placeholder(tf.float32, shape=[None, 1])
7
8 W = tf.Variable(tf.random_normal([3, 1]), name='weight')
9 b = tf.Variable(tf.random_normal([1]), name='bias')
10
11 #Hypothesis
12 hypothesis = tf.matmul(X, W) + b

1 #Simplified cost/loss function
2 cost = tf.reduce_mean(tf.square(hypothesis - Y))
3
4 # Minimize
5 optimizer = tf.train.GradientDescentOptimizer(learning_rate=1e-5)
6 train = optimizer.minimize(cost)
7
8 # Launch the graph in a session
9 sess = tf.Session()
10 # Initializes global variables in the graph
11 sess.run(tf.global_variables_initializer())
12
13 for step in range(2001):
14     cost_val, hy_val, _ = sess.run([cost, hypothesis, train], feed_dict={X: x_data, Y: y_data})
15     if step % 10 == 0:
16         print(step, "Cost:", cost_val, "\nPrediction:\n", hy_val)

```

5.2 Loading data from file

os.chdir() : Setting working directory

```

1 import os
2
3 print(os.getcwd())    # print working directory
4
5 os.chdir("../Data/") # set w.d.
6 os.getcwd()          # check working directory

```

✓ (disadvantage) Type of all data must be same.

MLlab 04-2 Loading data from file

```
1 import numpy as np
2
3 xy = np.loadtxt('data-01-test-score.csv', delimiter=',', dtype=np.float32)
4 x_data = xy[:, 0:-1]
5 y_data = xy[:, [-1]]
6
7 # Make sure the shape and data are OK
8 print(x_data.shape, x_data, len(x_data))
9 print(y_data.shape, y_data)
```

◇ Indexing, Slicing, Iterating

- Arrays can be indexed, sliced, iterated much like lists and other sequence types in Python
- As with Python lists, slicing in Numpy can be accomplished with the colon(:) syntax
- Colon instances(:) can be replaced with dots(...)

MLlab 04-2 Slicing

```
1 nums = range(5)           # range() is a built-in function that creates a list of integers
2 print(nums)
3 print(nums[2:4])          # Get a slice form index 2 to 4 (exclusive)
4 print(nums[2:])           # Get a slice form index 2 to the end
5 print(nums[:2])           # Get a slice form the start to index 2 (exclusive)
6 print(nums[:])            # Get a slice of the whole list
7 print(nums[:-1])          # Slice indices can be negative
8
9 nums[2:4] = [8,9]         # Assign a new sublist to a slice
10 print(nums)
```

range(0, 5)

range(2, 4)

range(2, 5)

range(0, 2)

range(0, 5)

range(0, 4)

TypeError

<ipython-input-21-86ffeac3decc> in <module>

7 print(nums[:-1]) Slice indices can be negative

8

—> 9 nums[2:4] = [8,9] Assign a new sublist to a slice

10 print(nums)

TypeError: 'range' object does not support item assignment

MLlab 04-2 np.array()

```
1 import numpy as np
2
3 a = np.array([1, 2, 3, 4, 5])
4 # array([1, 2, 3, 4, 5])
5
6 a[1:3]
7 # array([2, 3])
8
9 a[-1]
10 # 5
11
12 a[0:2] = 9
13 a
14 # array([9, 9, 3, 4, 5])

1 b = np.array([[1, 2, 3, 4], [5, 6, 7, 8], [9, 10, 11, 12]])
2 # array([[ 1,  2,  3,  4],
3 #        [ 5,  6,  7,  8],
4 #        [ 9, 10, 11, 12]])
5
6 b[:, 1]
7 # array([ 2,  6, 10])
8
9 b[-1]
10 # array([ 9, 10, 11, 12])
11
12 b[-1,:]
13 # array([ 9, 10, 11, 12])
14
15 b[-1, ...]
16 # array([ 9, 10, 11, 12])
17
18 b[0:2,:]
19 # array([[1, 2, 3, 4],
20 #        [5, 6, 7, 8]])
```

MLlab 04-2 [Full Code] multi-variable linear regression

```
1 import tensorflow as tf
2 import numpy as np
3 tf.set_random_seed(777) # for reproducibility
4
5 #=====#
6 # Loading data from file
7 xy = np.loadtxt('data-01-test-score.csv', delimiter=',', dtype=np.float32)
8 x_data = xy[:, 0:-1]
9 y_data = xy[:, [-1]]
10
11 # Make sure the shape and data are OK
12 print(x_data.shape, x_data, len(x_data))
13 print(y_data.shape, y_data)
14
15 #=====#
16 # Placeholders for a tensor that will be always fed.
17 X = tf.placeholder(tf.float32, shape=[None, 3])
18 Y = tf.placeholder(tf.float32, shape=[None, 1])
19
20 W = tf.Variable(tf.random_normal([3, 1]), name='weight')
21 b = tf.Variable(tf.random_normal([1]), name='bias')
22
23 # Hypothesis
24 hypothesis = tf.matmul(X, W) + b
25
26 # Simplified cost/loss function
27 cost = tf.reduce_mean(tf.square(hypothesis - Y))
28
29 # Minimize
30 optimizer = tf.train.GradientDescentOptimizer(learning_rate=1e-5)
31 train = optimizer.minimize(cost)
32
33 # Launch the graph in a session
34 sess = tf.Session()
35 # Initializes global variables in the graph
36 sess.run(tf.global_variables_initializer())
37
38 #=====#
39 # Set up feed_dict variables inside the loop
40 for step in range(2001):
41     cost_val, hy_val, _ = sess.run([cost, hypothesis, train], feed_dict={X: x_data, Y: y_data})
42     if step % 10 == 0:
43         print(step, "Cost:", cost_val, "\nPrediction:\n", hy_val)
44
45 #=====#
46 # Ask my score
47 print("Your score will be", sess.run(hypothesis, feed_dict={X: [[100, 70, 101]]}))
48
49 print("Other scores will be", sess.run(hypothesis,
50                                     feed_dict={X: [[60, 70, 110], [90, 100, 80]]}))
```

5.3 Queue Runners

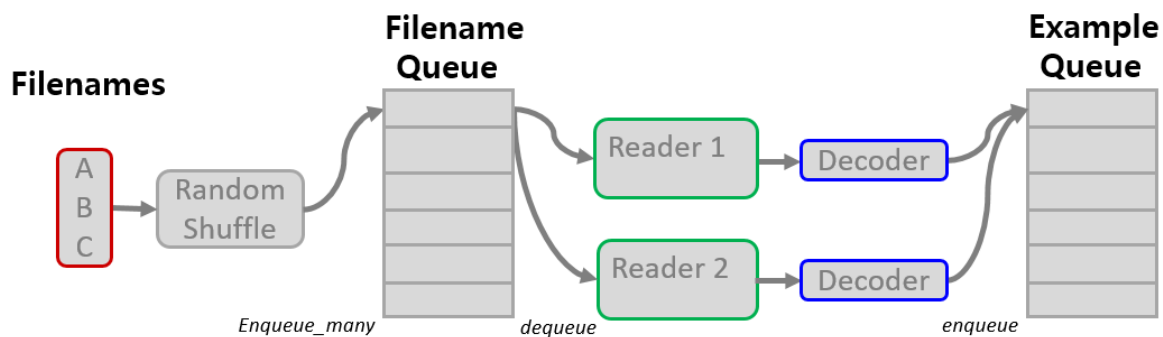
If the data volume is large, it becomes difficult to load the data into the memory. For example, when the data size is too large, using `Numpy` can cause out of memory problems. In this case, it is more efficient to load only the necessary data.

In the previous deep learning model, data was put into `feed_dict` argument in `tf.Session().Run()`. However, this method is slow because it copies data to a single thread.

To compensate for this drawback, TensorFlow has a "Queue Runners" system.

✓ Step of Queue Runners

- ① Make a list of files.
- ② Define a reader to read the file.
- ③ Set the data type with `tf.decode_csv()`. To determine parsing the file's value.



- 1

```
filename_queue = tf.train.string_input_producer(  
    ['data-01-test-score.csv', 'data-01-test-score.csv', ...],  
    shuffle=False, name='filename_queue')
```
- 2

```
reader = tf.TextLineReader()  
key, value = reader.read(filename_queue)
```
- 3

```
record_defaults = [[0.], [0.], [0.], [0.]]  
xy = tf.decode_csv(value, record_defaults = record_defaults)
```

MLlab 04-2 [Full Code] multi-variable linear regression using Queue Runners

```
1 import tensorflow as tf
2
3 ===Queue Runners=====
4 filename_queue = tf.train.string_input_producer(
5     ['data-01-test-score.csv'], shuffle=False, name='filename_queue')
6
7 reader = tf.TextLineReader()
8 key, value = reader.read(filename_queue)
9
10 # Default values, in cast of empty columns. Also specifies the type of the decoded result.
11 record_defaults = [[0.], [0.], [0.], [0.]]
12 xy = tf.decode_csv(value, record_defaults = record_defaults)
13
14 # collect batches of csv in
15 train_x_batch, train_y_batch = tf.train.batch([xy[0:-1], xy[-1:]], batch_size=10)
16
17 =====
18 # Placeholders for a tensor that will be always fed.
19 X = tf.placeholder(tf.float32, shape=[None, 3])
20 Y = tf.placeholder(tf.float32, shape=[None, 1])
21
22 W = tf.Variable(tf.random_normal([3, 1]), name='weight')
23 b = tf.Variable(tf.random_normal([1]), name='bias')
24
25 # Hypothesis
26 hypothesis = tf.matmul(X, W) + b
27
28 # Simplified cost/loss function
29 cost = tf.reduce_mean(tf.square(hypothesis - Y))
30
31 # Minimize
32 optimizer = tf.train.GradientDescentOptimizer(learning_rate=1e-5)
33 train = optimizer.minimize(cost)
34
35 # Launch the graph in a session
36 sess = tf.Session()
37 # Initializes global variables in the graph
38 sess.run(tf.global_variables_initializer())
39
40 ===Start populating the filename=====
41 coord = tf.train.Coordinator()
42 threads = tf.train.start_queue_runners(sess=sess, coord=coord)
43
44 for step in range(2001):
45     x_batch, y_batch = sess.run([train_x_batch, train_y_batch])
46     cost_val, hy_val, _ = sess.run([cost, hypothesis, train],
47                                     feed_dict={X: x_batch, Y: y_batch})
48     if step % 10 == 0:
49         print(step, "Cost:", cost_val, "Prediction:", hy_val)
50
51 coord.request_stop()
52 coord.join(threads)
```

MLlab 04-2 Shuffle Batch

```
1 # min_after_dequeue defines how big a buffer we will randomly sample
2 #   from -- bigger means better shuffling but slower start up and more memory used.
3 # capacity must be larger than min_after_dequeue and the amount larger
4 #   determines the maximum we will prefetch. Recommendation:
5 #   min_after_dequeue + (num_threads + a small safety margin) * batch_size
6
7 batch_size = 10
8 min_after_dequeue = 10000
9 capacity = min_after_dequeue + 3 * batch_size
10
11 example_batch, label_batch = tf.train.shuffle_batch([example, label],
12   batch_size=batch_size, capacity=capacity, min_after_dequeue=min_after_dequeue)
```

✓ Summary of Linear Regression

- Hypothesis $H(X) = WX$
- Cost function $cost(W) = \frac{1}{m} \sum_{i=1}^m (WX - y)^2$
- Gradient descent $W := W - \alpha \frac{\partial}{\partial W} cost(W)$

6 Logistic Regression Classification

- Binary Classification \Rightarrow **0 & 1 encoding**
 - Spam Email Detection: Spam(**1**) or Ham(**0**)
 - Facebook feed: show(**1**) or hide(**0**)
 - Credit Card Fraudulent Transaction detection : legitimate(**0**) or fraud(**1**)

6.1 Logistic Hypothesis

- logistic function (sigmoid function) $g(z) = \frac{1}{(1 + e^{-z})}$
- let $z = WX$, then $H(x) = g(z)$

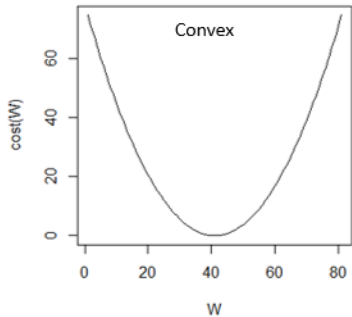
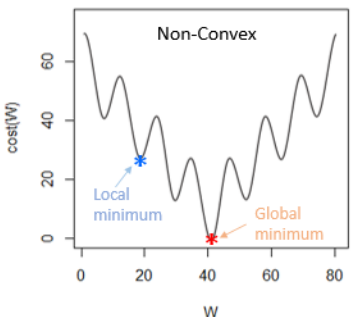
\Rightarrow Logistic Hypothesis

$$H(X) = \frac{1}{1 + e^{-W^T X}} \quad (0 < H(X) < 1)$$

6.2 Cost function

$$\text{cost}(W, b) = \frac{1}{m} \sum_{i=1}^m (H(x^{(i)}) - y^{(i)})^2$$

- What about the graph of the cost function in logistic regression?

	Linear regression	Logistic regression
Hypothesis	$H(X) = Wx + b$	$H(X) = \frac{1}{1 + e^{-W^T X}}$
Plot of cost function		

⇒ Because the graph of the cost function is **non-convex**, Global Minimum is not found and Local Minimum is found.

⇒ It is difficult to apply the gradient decent algorithm.

- New cost function for logistic

$$\text{cost}(W) = \frac{1}{m} \sum_{i=1}^m c(H(x), y)$$

$$c(H(x), y) = \begin{cases} -\log(H(x)) & : y = 1 \\ -\log(1 - H(x)) & : y = 0 \end{cases}$$

$$\Rightarrow c(H(x), y) = -y \log(H(x)) - (1 - y) \log(1 - H(x))$$

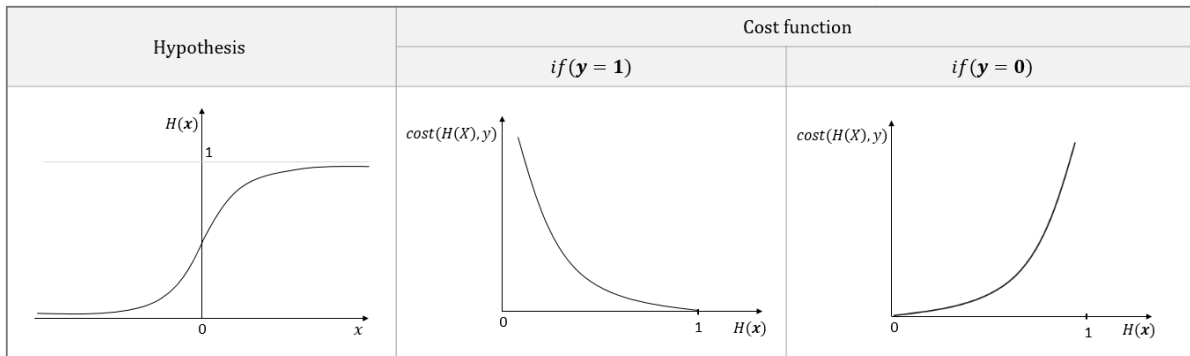
6.3 Minimize cost & Gradient descent

- (Logistic) Hypothesis

$$H(X) = \frac{1}{1 + e^{-W^T X}}$$

- (Logistic) Cost function

$$c(H(x), y) = -\frac{1}{m} \sum_{i=1}^m \{y \log(H(x)) + (1 - y) \log(1 - H(x))\}$$



- Gradient descent algorithm

$$W := W - \alpha \frac{\partial}{\partial W} cost(W)$$

MLlab 05 Gradient descent algorithm

```

1 # cost function
2 cost = - tf.reduce_mean(Y*tf.log(hypothesis) + (1-Y)*tf.log(1-hypothesis))
3
4 # Minimize
5 a = tf.Variable(0.1) # Learning rate; alpha
6 optimizer = tf.train.GradientDescentOptimizer(a)
7 train = optimizer.minimize(cost)

```

MLlab 05 [Full Code] Logistic Classification

```
1 import tensorflow as tf
2
3 #===Training Data=====#
4 x_data = [[1, 2], [2, 3], [3, 1], [4, 3], [5, 3], [6, 2]]
5 y_data = [[0], [0], [0], [1], [1], [1]] # binary 0,1
6
7 # Placeholders for a tensor that will be always fed.
8 X = tf.placeholder(tf.float32, shape=[None, 2])
9 Y = tf.placeholder(tf.float32, shape=[None, 1])
10
11 W = tf.Variable(tf.random_normal([2, 1]), name='weight') # 2 = number of input (X)
12                                                         # 1 = number of output (Y)
13 b = tf.Variable(tf.random_normal([1]), name='bias')
14
15 #=====#
16 # Hypothesis using sigmoid: tf.div(1., 1. + tf.exp(tf.matmul(X, W) + b))
17 hypothesis = tf.sigmoid(tf.matmul(X, W) + b)
18
19 # cost function.
20 cost = - tf.reduce_mean(Y * tf.log(hypothesis) + (1-Y) * tf.log(1-hypothesis))
21
22 # Gradient descent algorithm : Minimize cost.
23 train = tf.train.GradientDescentOptimizer(learning_rate=0.01).minimize(cost)
24
25 #===Accuracy computation=====#
26 # True if hypothesis>0.5 else False.
27 predicted = tf.cast(hypothesis > 0.5, dtype=tf.float32)
28 accuracy = tf.reduce_mean(tf.cast(tf.equal(predicted, Y), dtype=tf.float32))
29
30 #===Train the model=====#
31 # Launch the graph.
32 with tf.Session() as sess:
33     # Initializes TensorFlow variables
34     sess.run(tf.global_variables_initializer())
35
36     for step in range(10001):
37         cost_val, _ = sess.run([cost, train], feed_dict={X: x_data, Y: y_data})
38         if step % 200 == 0:
39             print(step, "Cost:", cost_val)
40         # Accuracy report
41         h, c, a = sess.run([hypothesis, predicted, accuracy],
42                             feed_dict={X: x_data, Y: y_data})
43         print("\nHypothesis: ", h, "\nCorrect (Y): ", c, "\nAccuracy: ", a)
```

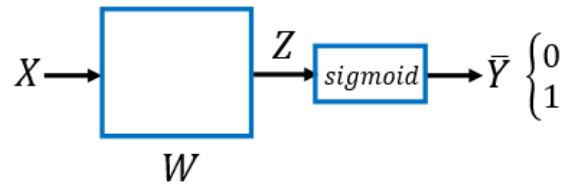
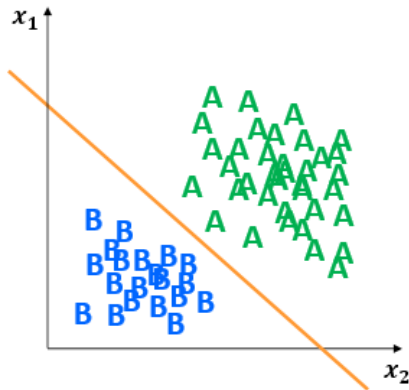
• Exercise : Classifying Diabetes.

```
1 import tensorflow as tf
2 import numpy as np
3 tf.set_random_seed(777) # for reproducibility
4
5 #===Load Data=====#
6 xy = np.loadtxt('data-03-diabetes.csv', delimiter=',', dtype=np.float32)
7 x_data = xy[:, 0:-1]
8 y_data = xy[:, [-1]]
9
10 print(x_data.shape, y_data.shape)
11
12 # Placeholders for a tensor that will be always fed.
13 X = tf.placeholder(tf.float32, shape=[None, 8])
14 Y = tf.placeholder(tf.float32, shape=[None, 1])
15
16 W = tf.Variable(tf.random_normal([8, 1]), name='weight')
17 b = tf.Variable(tf.random_normal([1]), name='bias')
```

7 Softmax classification

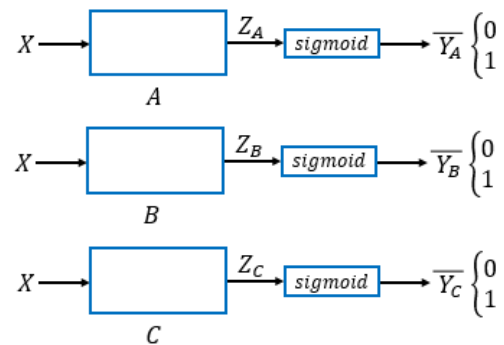
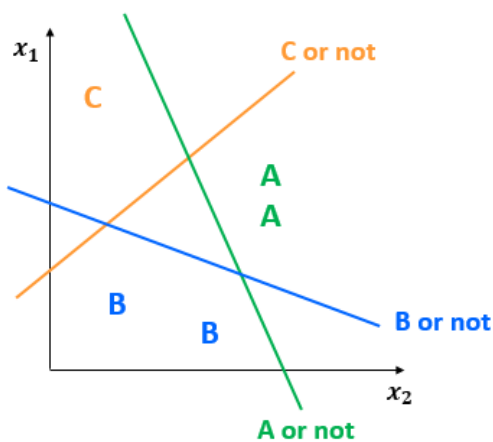
7.1 Multi-nomial classification

- Binary classification
 \Rightarrow Same as finding a line that divides into two (0 or 1).



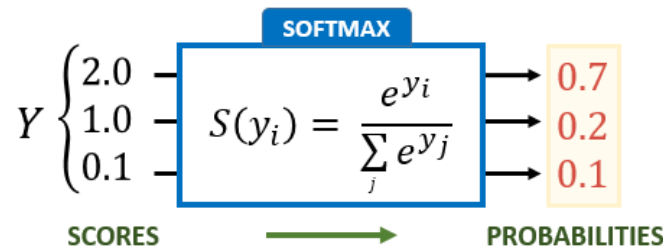
- Multi-nomial classification

X_1 (hours)	X_2 (attendance)	Y (grade)
10	5	A
9	5	A
3	2	B
2	4	B
11	1	C



$$\begin{bmatrix} w_{A1} & w_{A2} & w_{A3} \\ w_{B1} & w_{B2} & w_{B3} \\ w_{C1} & w_{C2} & w_{C3} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} w_{A1}x_1 + w_{A2}x_2 + w_{A3}x_3 \\ w_{B1}x_1 + w_{B2}x_2 + w_{B3}x_3 \\ w_{C1}x_1 + w_{C2}x_2 + w_{C3}x_3 \end{bmatrix} = \begin{bmatrix} Z_A \\ Z_B \\ Z_C \end{bmatrix} \rightarrow \text{sigmoid} \rightarrow \begin{bmatrix} \bar{y}_A \\ \bar{y}_B \\ \bar{y}_C \end{bmatrix} = \begin{bmatrix} 0 < H_A(X) < 1 \\ 0 < H_B(X) < 1 \\ 0 < H_C(X) < 1 \end{bmatrix}$$

7.2 Softmax



1. Like "sigmoid", it changes the result of the score to a value between 0 and 1.
2. It makes a total sum of the results = 1.

⇒ For example, if the result is $\overline{y_A} = 0.7$, this means that there is a 70% chance that A can be.

- one-hot encoding 을 이용해서, 가장 높은 확률의 선택지를 1.0으로, 나머지를 0.0으로 대치.
- one-hot encoding is argmax of tensorflow

⇒ 예측모델 완성!

- now 예측값과 실제값의 차이가 얼마인지 확인하는 Cost 함수를 완성해야됨

7.3 Cost function

- Cross-Entropy

$$D(S, L) = - \sum_i L_i \log(S_i) = \sum_i L_i (-\log(S_i))$$

$(S = \overline{Y}$ is predicted value, $L = Y$ is true value)

✓ Why using this cost function?

If the predicted value is the same as the true value, the cost value will be small.
When different, the cost value will be large.

- Logistic cost vs Cross-entropy cost ?

- Logistic cost

$$c(H(x), y) = -y \log(H(x)) - (1 - y) \log(1 - H(x))$$

$(H(x) \text{ is predicted value, } y \text{ is true value})$

- Cross entropy cost

$$D(S, L) = - \sum_i L_i \log(S_i) = \sum_i L_i (-\log(S_i))$$

$(S \text{ is predicted value, } L \text{ is true value})$

⇒ Let predicted value is \bar{Y} and true value is L ,

then it can see Logistic cost and Cross entropy cost are the same.

$$cost(\bar{Y}, L) = - \sum_i L_i \log(\bar{Y}_i) = -L \log(\bar{Y}) - (1 - L) \log(1 - \bar{Y})$$

7.4 Cross-Entropy Cost function

$$\begin{aligned} \mathcal{L} = cost(\bar{Y}, L) &= \frac{1}{m} \sum_{i=1} D(S_i, L_i) \\ &= \frac{1}{m} \sum_{i=1} D(Softmax(z_i), L_i) \\ &= \frac{1}{m} \sum_{i=1} D(Softmax(wx_i + b), L_i) \end{aligned}$$

($z_i = wx_i + b$, $\bar{Y} = S_i = Softmax(z_i)$ is predicted value, i is training set)

7.5 Gradient descent

Find W that minimizes the cost function $\mathcal{L}(W)$. (Same as for logistic)

$$W := W - \alpha \frac{\partial}{\partial W} \mathcal{L}(W)$$

7.6 Implementation of Softmax Classifier

MLlab 06-1 Softmax Classifier

```
1 import tensorflow as tf
2 tf.set_random_seed(777) # for reproducibility
3
4 #===Training Data=====#
5 x_data = [[1, 2, 1, 1],
6           [2, 1, 3, 2],
7           [3, 1, 3, 4],
8           [4, 1, 5, 5],
9           [1, 7, 5, 5],
10          [1, 2, 5, 6],
11          [1, 6, 6, 6],
12          [1, 7, 7, 7]]
13
14 # one-hot-encoding: y=[2,2,2,1,1,1,0,0]
15 y_data = [[0, 0, 1],
16          [0, 0, 1],
17          [0, 0, 1],
18          [0, 1, 0],
19          [0, 1, 0],
20          [0, 1, 0],
21          [1, 0, 0],
22          [1, 0, 0]]
23
24 X = tf.placeholder("float", [None, 4])
25 Y = tf.placeholder("float", [None, 3])
26
27 nb_classes = 3
28
29 W = tf.Variable(tf.random_normal([4, nb_classes]), name='weight')
30 b = tf.Variable(tf.random_normal([nb_classes]), name='bias')
31
32 #=====#
33 # Hypothesis using softmax
34 # tf.nn.softmax computes softmax activations
35 # softmax = exp(logits) / reduce_sum(exp(logits), dim)
36 hypothesis = tf.nn.softmax(tf.matmul(X, W) + b)
37
38 # Cross entropy cost/loss
39 cost = tf.reduce_mean(-tf.reduce_mean(Y*tf.log(hypothesis), axis=1))
40 optimizer = tf.train.GradientDescentOptimizer(learning_rate=0.1).minimize(cost)
41
42 #===Train the model=====#
43 # Launch graph
44 with tf.Session() as sess:
45     sess.run(tf.global_variables_initializer())
46
47     for step in range(2001):
48         _, cost_val = sess.run([optimizer, cost], feed_dict={X: x_data, Y: y_data})
49         if step % 200 == 0:
50             print(step, cost_val)
```

MLlab 06-1 Testing & One-hot encoding

```
1  #==Testing & One-hot encoding==#####
2  with tf.Session() as sess:
3      sess.run(tf.global_variables_initializer())
4      a = sess.run(hypothesis, feed_dict={X: [[1, 11, 7, 9]]})
5      print(a, sess.run(tf.argmax(a, 1)))
6
7  print('-----')
8  with tf.Session() as sess:
9      sess.run(tf.global_variables_initializer())
10     b = sess.run(hypothesis, feed_dict={X: [[1, 3, 4, 3]]})
11     print(b, sess.run(tf.argmax(b, 1)))
12
13  print('-----')
14  with tf.Session() as sess:
15      sess.run(tf.global_variables_initializer())
16      c = sess.run(hypothesis, feed_dict={X: [[1, 1, 0, 1]]})
17      print(c, sess.run(tf.argmax(c, 1)))
18
19  print('-----')
20  with tf.Session() as sess:
21      sess.run(tf.global_variables_initializer())
22      all = sess.run(hypothesis, feed_dict={X: [[1, 11, 7, 9],
23                                              [1, 3, 4, 3],
24                                              [1, 1, 0, 1]]})
25      print(all, sess.run(tf.argmax(all, 1)))
```

7.7 Fancy Softmax Classifier

- `softmax_cross_entropy_with_logits()`

```
logits = tf.matmul(X, W) + b
hypothesis = tf.nn.softmax(logits)
```

```
## Cross entropy cost/loss
1 cost = tf.reduce_mean(-tf.reduce_mean(Y*tf.log(hypothesis), axis=1))
```

```
cost_i = tf.nn.softmax_cross_entropy_with_logits(logits=logits,
                                                  labels=Y_one_hot)
2 cost = tf.reduce_mean(cost_i)
```

- `tf.one_hot()` and `tf.reshape()`

```
Y = tf.placeholder(tf.int32, [None, 1]) # 0 ~ 6, shape=(?, 1)
Y_one_hot = tf.one_hot(Y, nb_classes) # one hot shape=(?, 1, 7)
Y_one_hot = tf.reshape(Y_one_hot, [-1, nb_classes]) # shape=(?, 7)
```

MLlab 06-2 Exercise: Predicting animal type based on various features

```
1 import tensorflow as tf
2 import numpy as np
3 tf.set_random_seed(777) # for reproducibility
4
5 ===Predicting animal type based on various features===#####
6 xy = np.loadtxt('data-04-zoo.csv', delimiter=',', dtype=np.float32)
7 x_data = xy[:, 0:-1]
8 y_data = xy[:, [-1]]
9
10 print(x_data.shape, y_data.shape)
11
12 nb_classes = 7 # 0 ~ 6
13
14 X = tf.placeholder(tf.float32, [None, 16])
15 Y = tf.placeholder(tf.int32, [None, 1]) # 0 ~ 6
16
17 # change to one-hot
18 Y_one_hot = tf.one_hot(Y, nb_classes)
19 Y_one_hot = tf.reshape(Y_one_hot, [-1, nb_classes])
20
21 W = tf.Variable(tf.random_normal([16, nb_classes]), name='weight')
22 b = tf.Variable(tf.random_normal([nb_classes]), name='bias')
23
24 =====#####
25 # tf.nn.softmax computes softmax activations
26 # softmax = exp(logits) / reduce_sum(exp(logits), dim)
27 logits = tf.matmul(X, W) + b
28 hypothesis = tf.nn.softmax(logits)
29
30 # Cross entropy cost/loss
31 cost_i = tf.nn.softmax_cross_entropy_with_logits(logits=logits, labels=Y_one_hot)
32 cost = tf.reduce_mean(cost_i)
33
34 optimizer = tf.train.GradientDescentOptimizer(learning_rate=0.1).minimize(cost)
35
36 ===Train the model===#####
37 prediction = tf.argmax(hypothesis, 1)
38 correct_prediction = tf.equal(prediction, tf.argmax(Y_one_hot, 1))
39 accuracy = tf.reduce_mean(tf.cast(correct_prediction, tf.float32))
40
41 # Launch graph
42 with tf.Session() as sess:
43     sess.run(tf.global_variables_initializer())
44
45     for step in range(2001):
46         _, cost_val, acc_val = sess.run([optimizer, cost, accuracy], feed_dict={X: x_data, Y: y_data})
47
48         if step % 100 == 0:
49             print("Step: {:5}\tCost: {:.3f}\tAcc: {:.2%}".format(step, cost_val, acc_val))
```

8 Application & Tip

8.1 Learning rate

- Large learning rate : Overshooting.
- Small learning rate : It takes too long and trapping in local minimum.

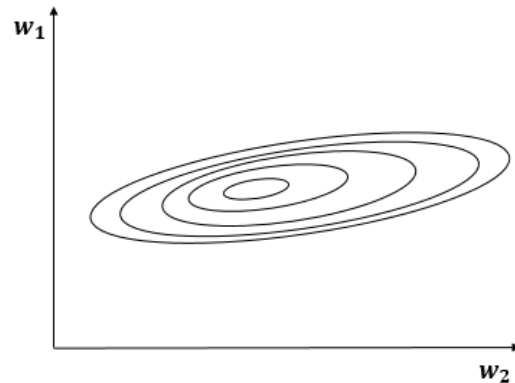
⇒ Try several learning rates. at first, try $\alpha=0.01$!

- ✓ Observe the cost function.
- ✓ Check it goes down in a reasonable rate.

8.2 Data Preprocessing

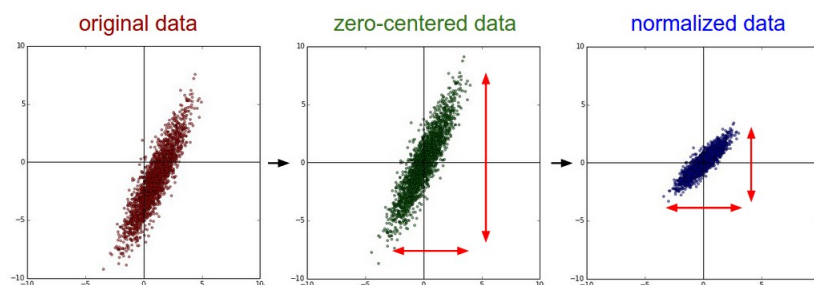
Preprocessing for gradient descent

X_1	X_2	Y
1	9000	A
2	-5000	A
4	-2000	B
6	8000	B
9	9000	C



x_1 변수는 10보다 작은 숫자, x_2 변수는 -5000에서 9000까지의 숫자라면 진짜 동그랗게 생긴 원의 모양이 아니라 한쪽으로 길게 늘어진 타원 모양이 된다. 이렇게 된다면 수평으로 이동할 때와 수직으로 이동할 때 엄청난 불균형이 발생하게 되어 gradient descent 알고리즘을 적용하기 어려운 상황이 될 수 있다.

등고선으로 표현할 때, 가장 좋은 형태는 완벽하게 둥근 원(circle)이다. 수평과 수직으로 동일한 범위를 갖게 만들면 가장 이상적인 원이 된다. gradient descent 알고리즘을 적용하기 전에 preprocessing 작업으로 데이터의 범위를 제한할 수 있다.



- Normalization

수식 : (요소값 - 최소값) / (최대값 - 최소값)

설명 : 전체 구간을 0 100으로 설정하여 데이터를 관찰하는 방법으로, 특정 데이터의 위치를 확인할 수 있게 해줌

sol of code: `xy = MinMaxScaler(xy)`

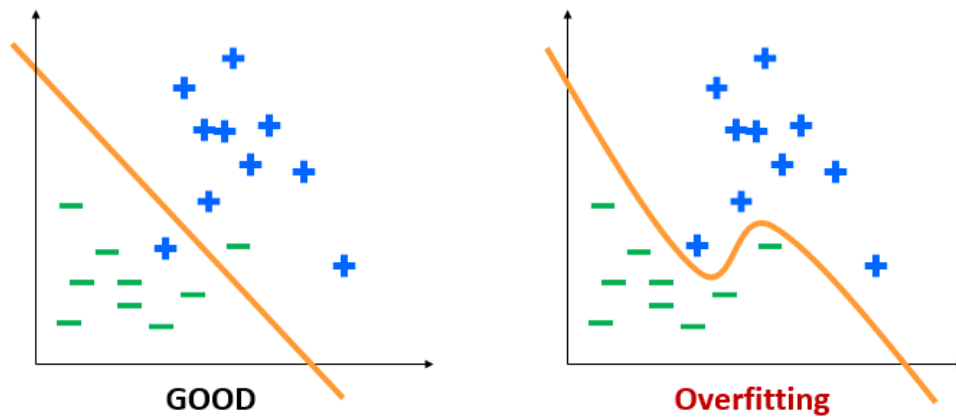
- Standardization

수식 : (요소값 - 평균) / 표준편차

설명 : 평균까지의 거리로, 2개 이상의 대상이 단위가 다를 때, 대상 데이터를 같은 기준으로 볼 수 있게 해줌

8.3 Overfitting

very good for training data set, but Not good at test data set or in real use.



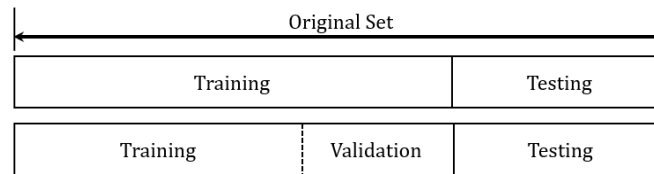
⇒ Solution

- ✓ More training data!
- ✓ Reduce the number of duplicate features(x).
- ✓ **Regularization** : Let's not have too big numbers in the weight.
(λ is regularization strength.)

$$\mathcal{L} = \frac{1}{m} \sum_{i=1} D(S(wx_i + b), L_i) + \lambda \sum_{i=1} w_i^2$$

```
# Regularization
lambda = 0.001 # regularization strength
l2reg = lambda * tf.reduce_mean(tf.square(W))
```

8.4 Training & Validation & Test datasets



MLlab 07-1 Training and Test datasets

```
1 import tensorflow as tf
2 tf.set_random_seed(777) # for reproducibility
3
4 #==Training Data=====#
5 x_data = [[1, 2, 1], [1, 3, 2], [1, 3, 4], [1, 5, 5],
6           [1, 7, 5], [1, 2, 5], [1, 6, 6], [1, 7, 7]]
7
8 y_data = [[0, 0, 1], [0, 0, 1], [0, 0, 1], [0, 1, 0],
9           [0, 1, 0], [0, 1, 0], [1, 0, 0], [1, 0, 0]]
10
11 #==Test Data=====#
12 # Evaluation our model using this test dataset
13 x_test = [[2, 1, 1], [3, 1, 2], [3, 3, 4]]
14
15 y_test = [[0, 0, 1], [0, 0, 1], [0, 0, 1]]
16
17 #=====#
18 X = tf.placeholder("float", [None, 3])
19 Y = tf.placeholder("float", [None, 3])
20
21 W = tf.Variable(tf.random_normal([3, 3]))
22 b = tf.Variable(tf.random_normal([3]))
23
24 # tf.nn.softmax computes softmax activations
25 # softmax = exp(logits) / reduce_sum(exp(logits), dim)
26 hypothesis = tf.nn.softmax(tf.matmul(X, W) + b)
27
28 # Cross entropy cost/loss
29 cost = tf.reduce_mean(-tf.reduce_sum(Y * tf.log(hypothesis), axis=1))
30 # Try to change learning_rate to small numbers
31 optimizer = tf.train.GradientDescentOptimizer(learning_rate=0.1).minimize(cost)
32
33 #=====#
34 # Correct prediction Test model
35 prediction = tf.argmax(hypothesis, 1)
36 is_correct = tf.equal(prediction, tf.argmax(Y, 1))
37 accuracy = tf.reduce_mean(tf.cast(is_correct, tf.float32))
38
39 # Launch graph
40 with tf.Session() as sess:
41     # Initialize TensorFlow variables
42     sess.run(tf.global_variables_initializer())
43
44     for step in range(201):
45         cost_val, W_val, _ = sess.run([cost, W, optimizer],
46                                       feed_dict={X: x_data, Y: y_data})
47         print(step, cost_val, W_val)
48
49     # predict
50     print("Prediction:", sess.run(prediction, feed_dict={X: x_test}))
51     # Calculate the accuracy
52     print("Accuracy: ", sess.run(accuracy, feed_dict={X: x_test, Y: y_test}))
```


8.5 Exercise: MNIST Dataset

MLlab 07-2 Check MNIST Data



① Load 'MNIST Data' from library

```
1 from tensorflow.examples.tutorials.mnist import input_data
2 # Check out http://www.tensorflow.org/get_started/mnist/beginners for
3 # more information about the mnist dataset
4
5 mnist = input_data.read_data_sets("MNIST_data/", one_hot=True)
```

② $28 \times 28 \times 1$ image

```
1 # MNIST data image of shape 28 * 28 = 784
2 X = tf.placeholder(tf.float32, [None, 784])
3
4 # 0 ~ 9 digits recognition => 10 classes
5 Y = tf.placeholder(tf.float32, [None, nb_classes])
```

• source : [\[THE MNIST DATABASE\] http://yann.lecun.com/exdb/mnist/](http://yann.lecun.com/exdb/mnist/)

- Training epoch / batch

- one **epoch** : one forward pass and one backward pass of all the training examples.
- **batch size** : the number of training examples in one forward/backward pass.
- number of **iterations** : number of passes, each pass using [batch size] number of examples. To be clear, one pass = one forward pass + one backward pass

- ✓ Example.

if you have 1000 training examples, and your batch size is 500, then it will take 2 iterations to complete 1 epoch.

MLlab 07-2 [Full Code] MNIST Dataset

```
1 import tensorflow as tf
2 tf.set_random_seed(777) # for reproducibility
3
4 === Load MNIST Data from html =====#
5 from tensorflow.examples.tutorials.mnist import input_data
6 # Check out http://www.tensorflow.org/get\_started/mnist/beginners for
7 # more information about the mnist dataset
8
9 mnist = input_data.read_data_sets("MNIST_data/", one_hot=True)
10
11 === MNIST Data =====#
12 nb_classes = 10
13
14 # MNIST data image of shape 28 * 28 = 784
15 X = tf.placeholder(tf.float32, [None, 784])
16
17 # 0 ~ 9 digits recognition => 10 classes
18 Y = tf.placeholder(tf.float32, [None, nb_classes])
19
20 W = tf.Variable(tf.random_normal([784, nb_classes]))
21 b = tf.Variable(tf.random_normal([nb_classes]))
22
23 === Softmax! =====#
24 # Hypothesis using softmax
25 hypothesis = tf.nn.softmax(tf.matmul(X, W) + b)
26
27 # Cross-entropy cost
28 cost = tf.reduce_mean(-tf.reduce_sum(Y*tf.log(hypothesis), axis=1))
29 optimizer = tf.train.GradientDescentOptimizer(learning_rate=0.1).minimize(cost)
30
31 === Test model =====#
32 is_correct = tf.equal(tf.argmax(hypothesis, 1), tf.argmax(Y, 1))
33
34 # Calculate accuracy
35 accuracy = tf.reduce_mean(tf.cast(is_correct, tf.float32))
36
37 =====#
38 # Parameters
39 training_epochs = 15
40 batch_size = 100
41
42 with tf.Session() as sess:
43     # Initialize TensorFlow variables
44     sess.run(tf.global_variables_initializer())
45     # Training cycle
46     for epoch in range(training_epochs):
47         avg_cost = 0
48         total_batch = int(mnist.train.num_examples / batch_size)
49
50         for i in range(total_batch):
51             batch_xs, batch_ys = mnist.train.next_batch(batch_size)
52             c, _ = sess.run([cost, optimizer], feed_dict={X: batch_xs, Y: batch_ys})
53             avg_cost += c / total_batch
54
55         print('Epoch:', '%04d' % (epoch + 1), 'cost =', '{:.9f}'.format(avg_cost))
56
57 === Report results on test dataset =====#
58 # Test the model using "test sets".
59 print("Accuracy: ", accuracy.eval(session=sess,
60     feed_dict={X: mnist.test.images, Y: mnist.test.labels}))
```

MLlab 07-2 Exercise: Sample image show and prediction

```
1 import matplotlib.pyplot as plt
2 import random
3
4 # Get one and predict
5 r = random.randint(0, mnist.test.num_examples - 1)    # select random number.
6
7 with tf.Session() as sess:
8     sess.run(tf.global_variables_initializer())
9
10    print("Label:", sess.run(tf.argmax(mnist.test.labels[r:r+1], 1)))
11    print("Prediction:", sess.run(tf.argmax(hypothesis, 1),
12                                   feed_dict={X: mnist.test.images[r:r + 1]}))
13
14 plt.imshow(
15     mnist.test.images[r:r+1].reshape(28, 28), cmap='Greys', interpolation='nearest')
16 plt.show()
```
