빅데이터 혁신공유대학

파이썬으로 배우는 데이터 구조

한동대학교 전산전자공학부
김영섭 교수

Data Structures in Python
Chapter 7 - 1

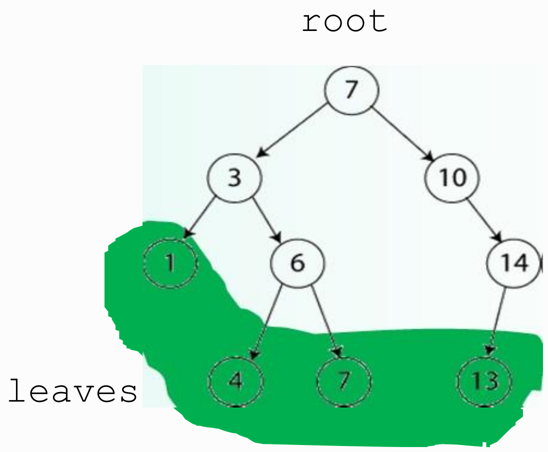
- **Tree Introduction**
- Tree Traversals
- Tree Algorithms

Agenda & Readings

- Agenda
  - Tree Terminology
  - Binary Tree Properties
  - Binary Tree and Node Representation
- Reference:
  - Problem Solving with Algorithms and Data Structures
  - Chapter 6 - Tree

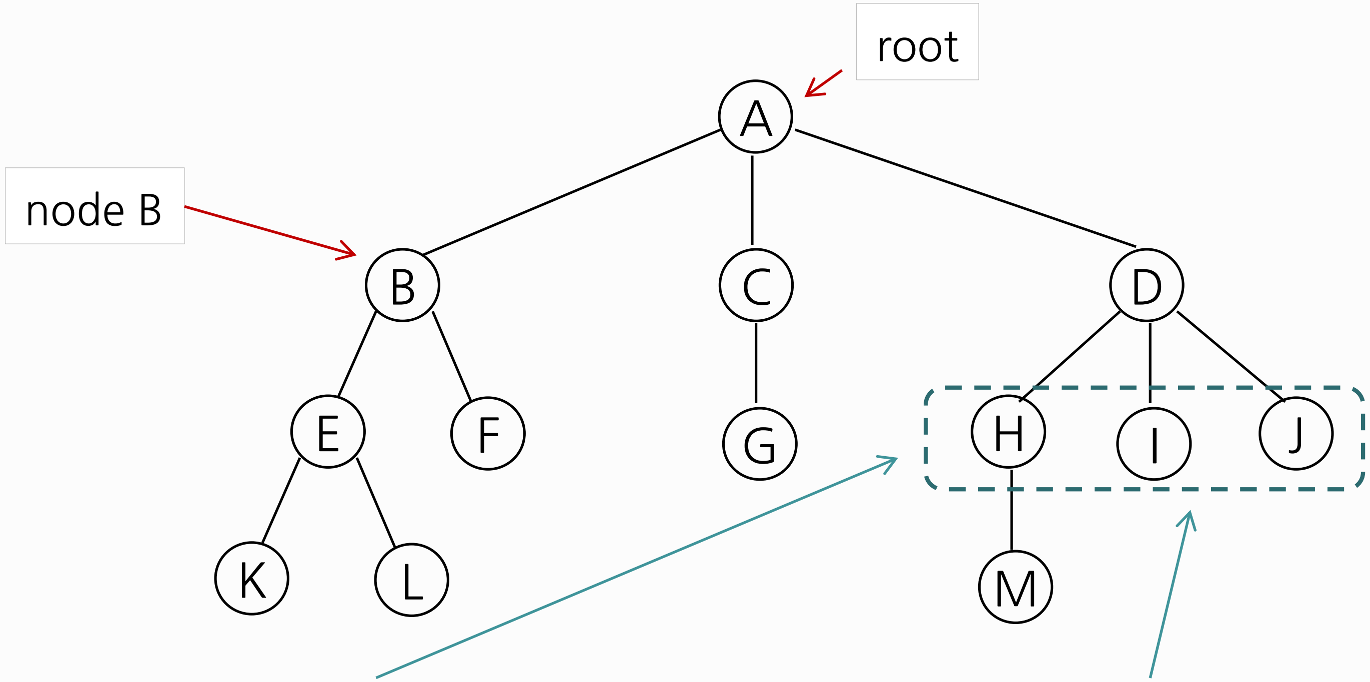
What is a Tree?

- A non-linear data structure
- An abstraction for a hierarchical structure
- It is defined as a set of points called nodes and a set of lines called edges where an edge connects two distinct nodes.



Introduction - Terminology

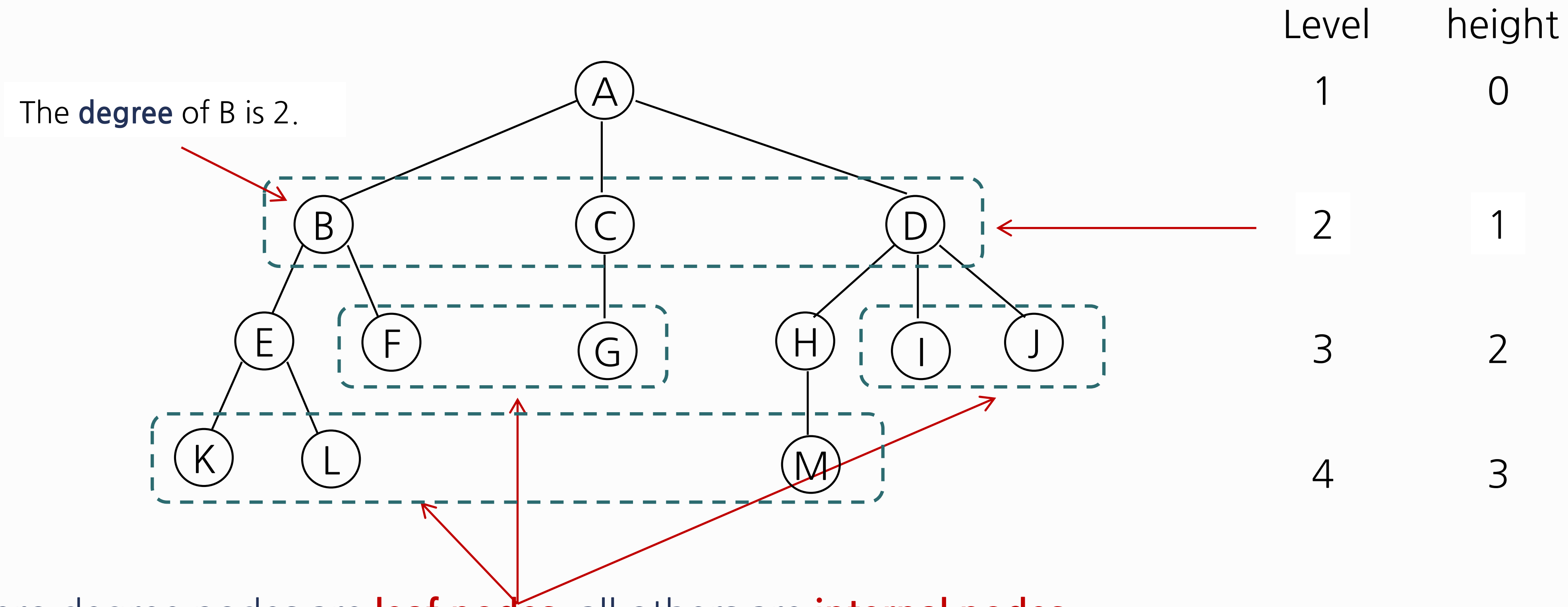
- **A tree data structure:** it is like a linked list that has a **first** node, this node is called as the **root** of the tree.
- **Example.** A **tree** with a root storing the value 'A'



- The **children** of **D** are **H, I, and J**; **H, I, and J** are **siblings**.
- The **parent** of **D** is **A**.

Introduction - Terminology

- **Definition.** child, parent, sibling, degree, leaf nodes, level, and internal node



- Zero degree nodes are **leaf nodes**, all others are **internal nodes.**
  - An ***internal node*** is any node that has at least one non-empty child.
- The **degree** of a node is the number of children.
- The **degree of a tree** is the **maximum of the degree of the nodes** in the tree.

Introduction - Representation of trees

- **Exercise.** The tree representing the HTML document below:

```
<html>
    <head>
        <title>Hello World!</title>
    </head>
    <body>
        <h1>This is a <u>Heading</u></h1>
        <p>This is a paragraph with some <u>underlined</u> text.</p>
    </body>
</html>
```

Introduction - Representation of trees

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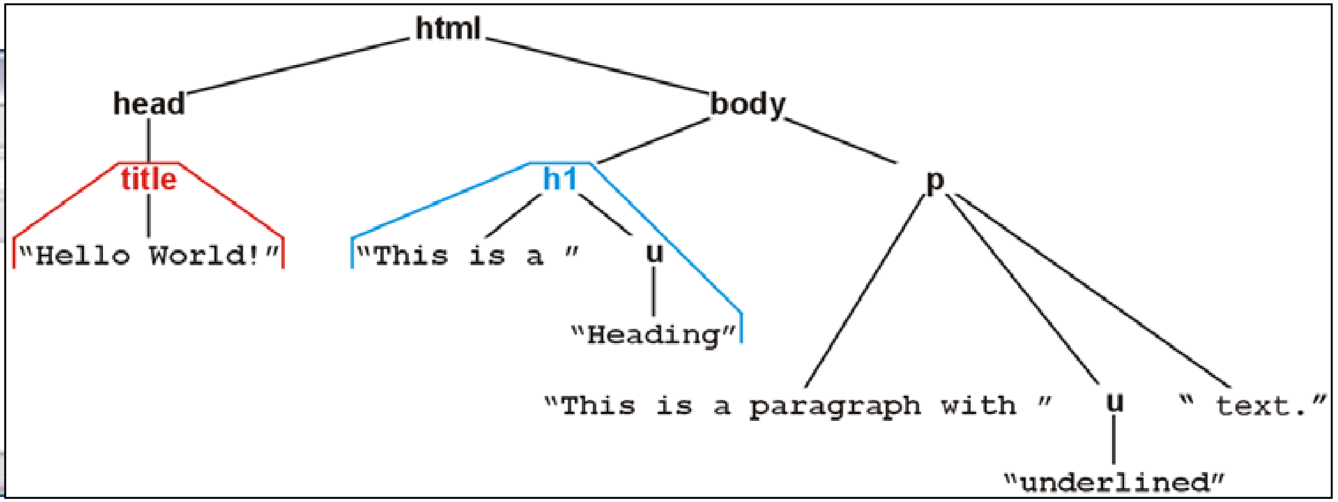
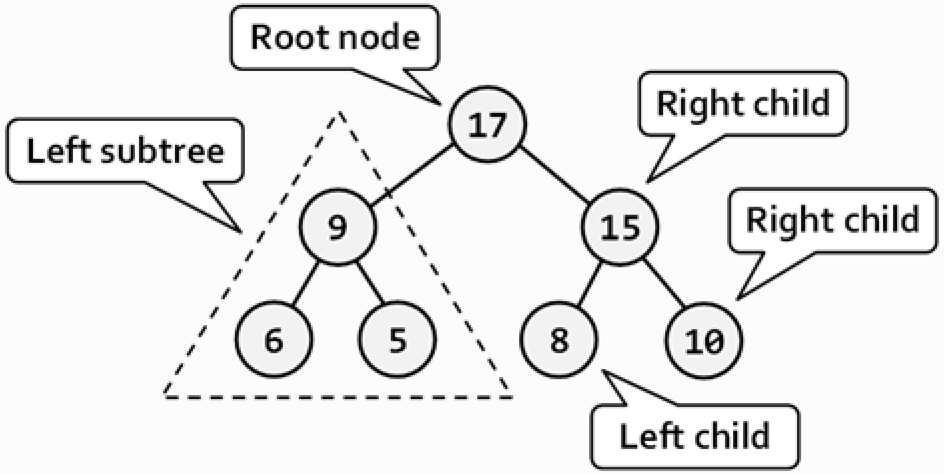


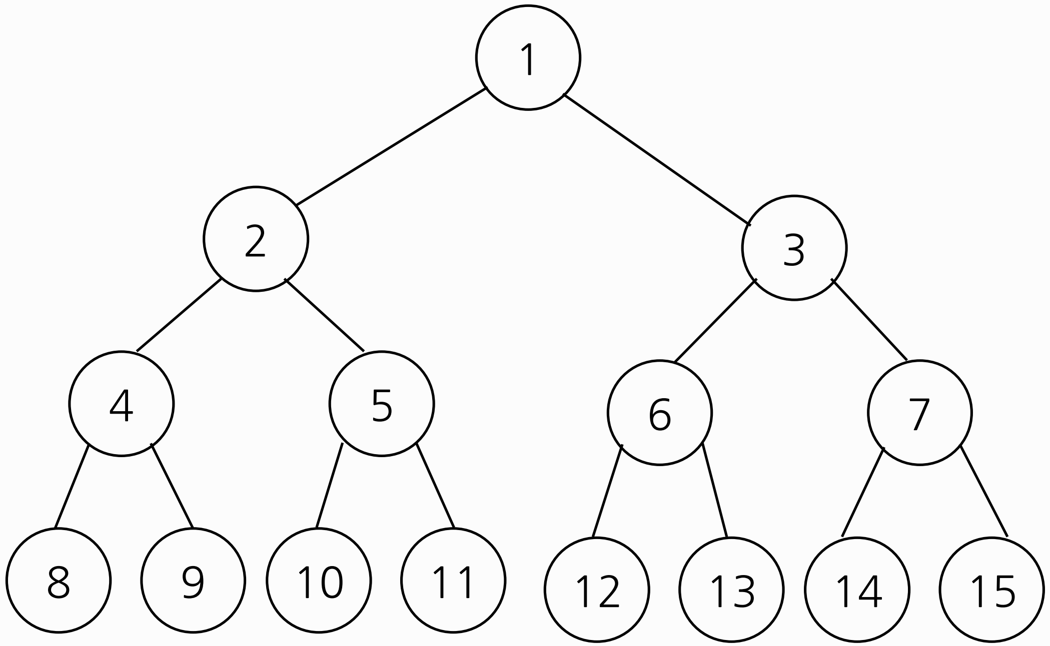
Binary trees

- **Definition:** A tree such that each node has *exactly* two children.
  - Notice, exactly two children - not up to two children! Because *exactly* two children means a left child **and/or** right child, no middle child.
  - Each child is either empty or another binary tree.
  - Given this constraint, we can label the two children as left and right nodes or subtrees.



Binary trees - Properties

- **Observation:**
  - **Q:** Maximum number of nodes in binary trees in each level and all levels?
  - **Q:** What is the max level k if there are n nodes? k(n) = ?



*A **full** binary tree*

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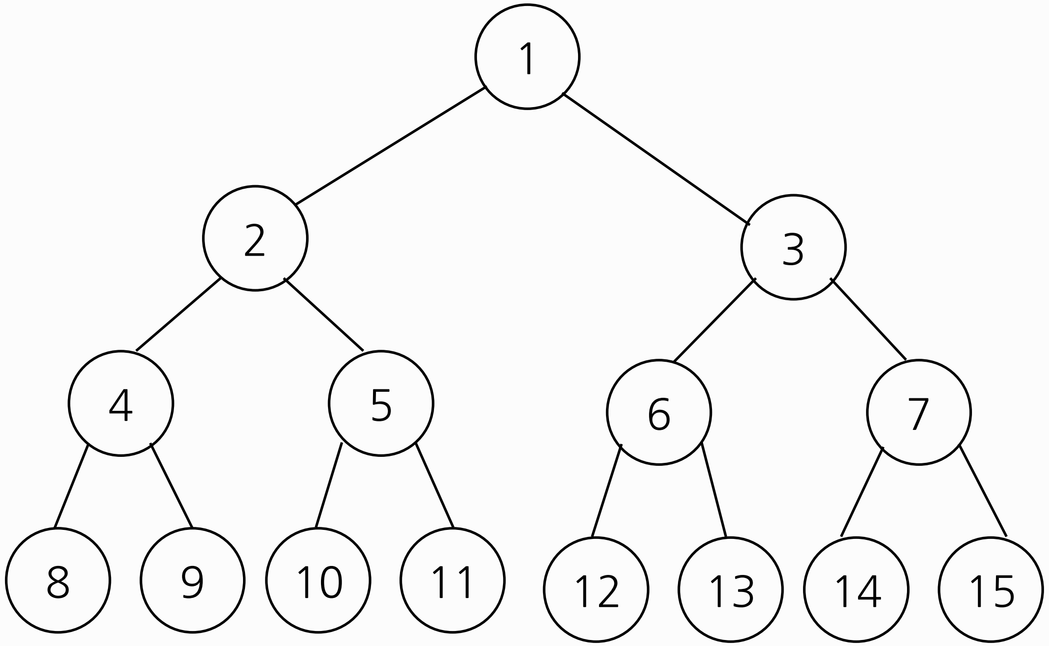

*A **full** binary tree*

Level	Node Numbers at Each Level	Total Numbers of Nodes
1	$1 = 2^0$	
2	$2 = 2^1$	
3	$4 = 2^2$	
4	$8 = 2^3$	
.	.	
11	$1024 = 2^{10}$	
.	.	
k		

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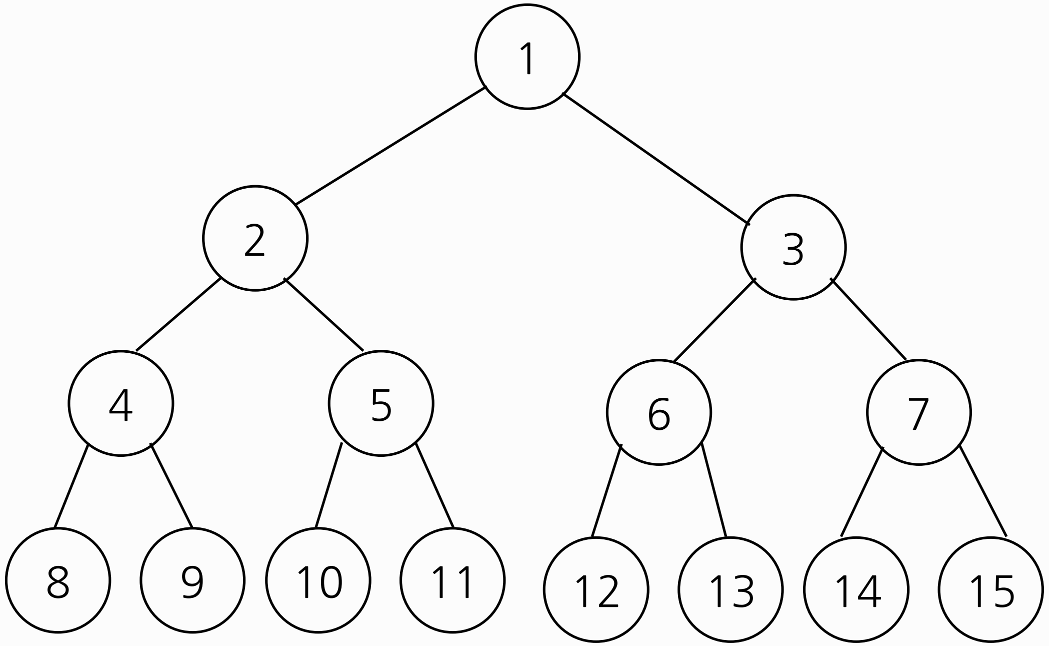

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Level	Node Numbers at Each Level	Total Numbers of Nodes
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2	$2 = 2^1$	$3 = 2^2 - 1$
3	$4 = 2^2$	$7 = 2^3 - 1$
4	$8 = 2^3$	$15 = 2^4 - 1$
.	.	.
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k		

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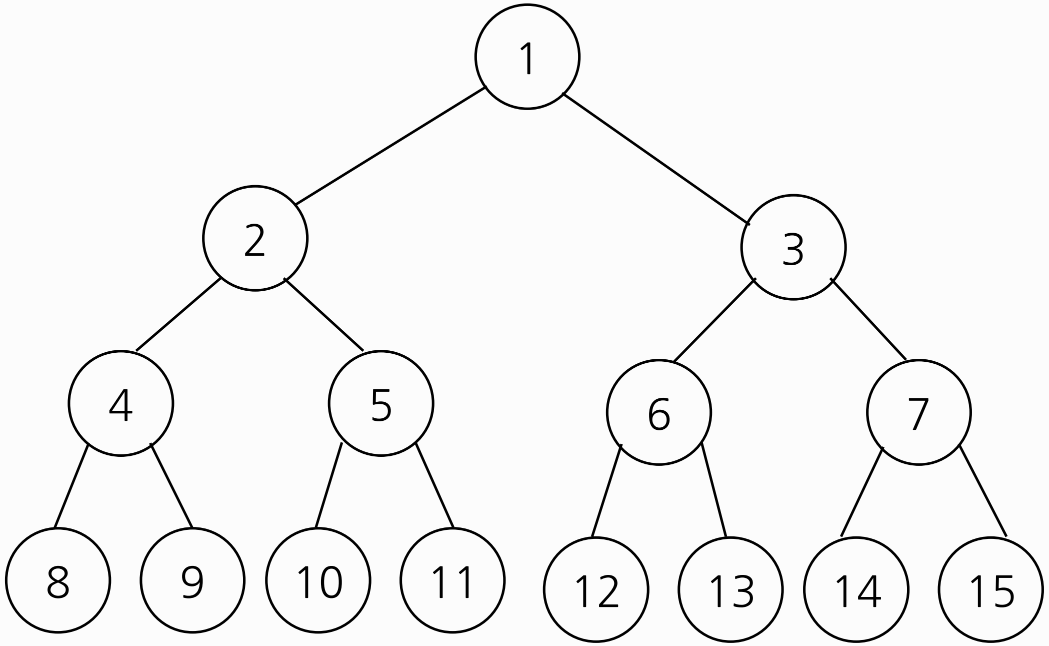

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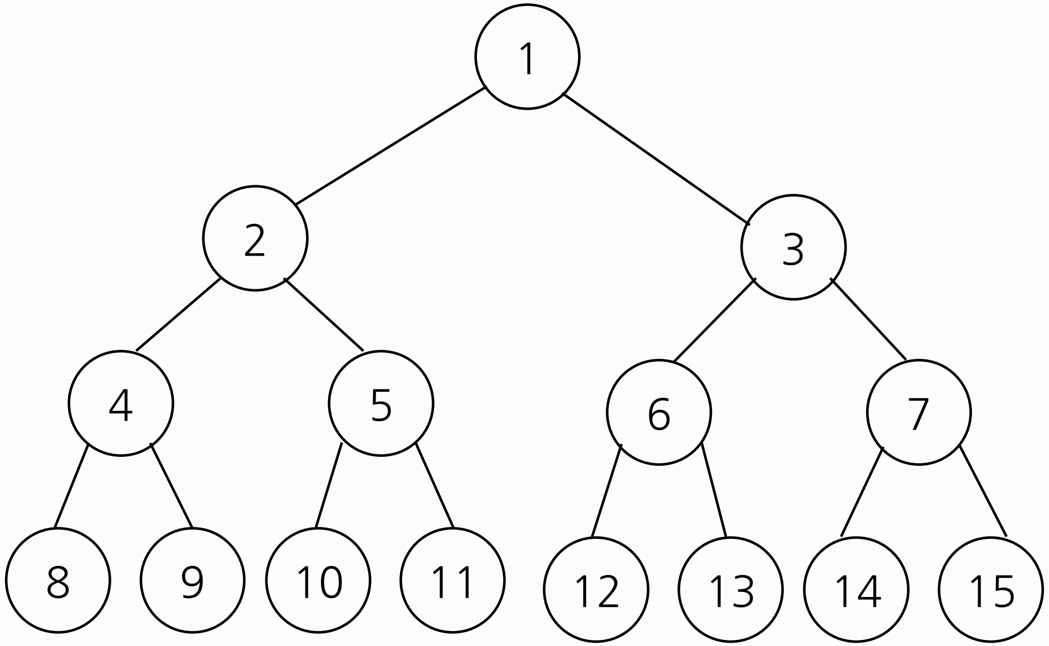
.	.	
k	$\mathbf{2^{k-1}}$	$\mathbf{2^k - 1}$
h	2^h	2^{h+1} - 1

Binary trees - Properties

- **Definition:** *A **full** binary tree* of level k is a binary tree having **2^k - 1 nodes**, $k \geq 0$.
- **Definition:** A binary tree with n nodes and level k is **complete** iff its nodes correspond to the nodes numbered from 1 to n in the full binary tree of level k .



*A **full** binary tree*

Binary trees - Properties

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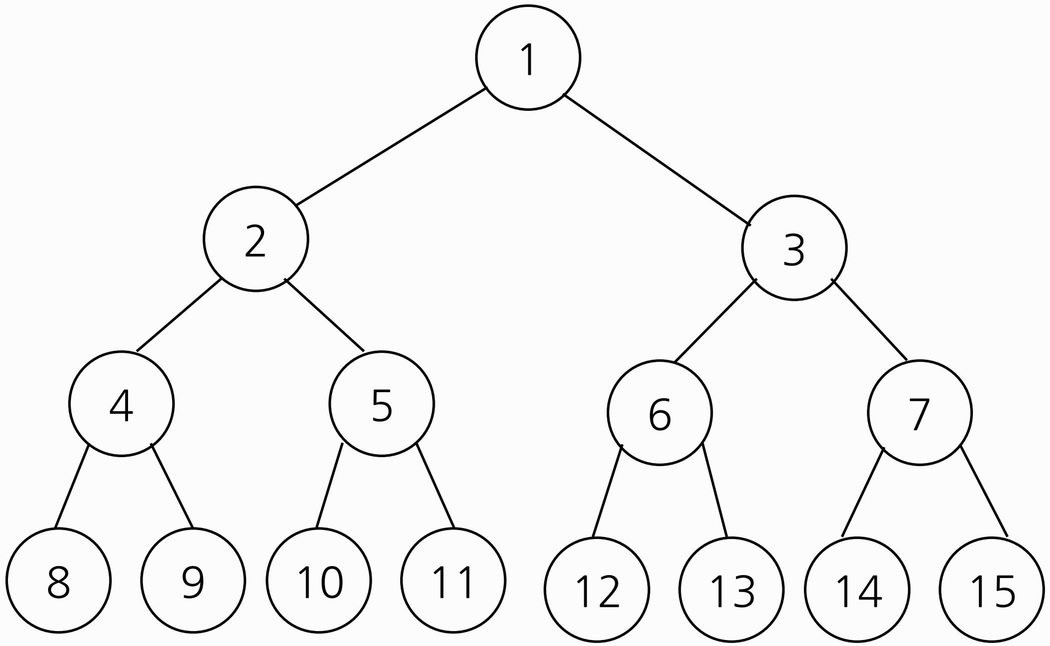

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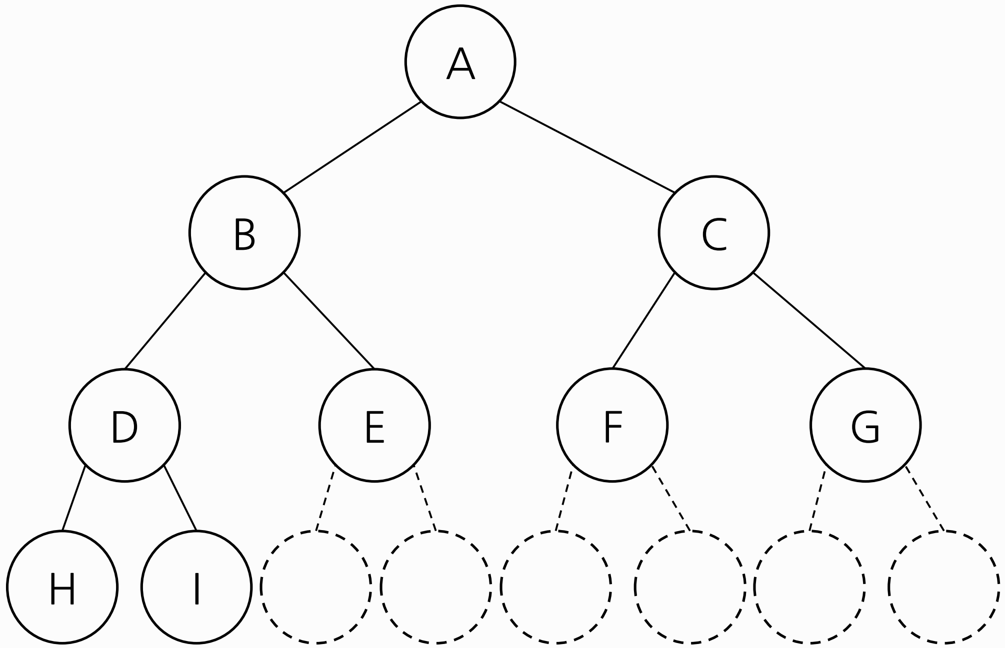
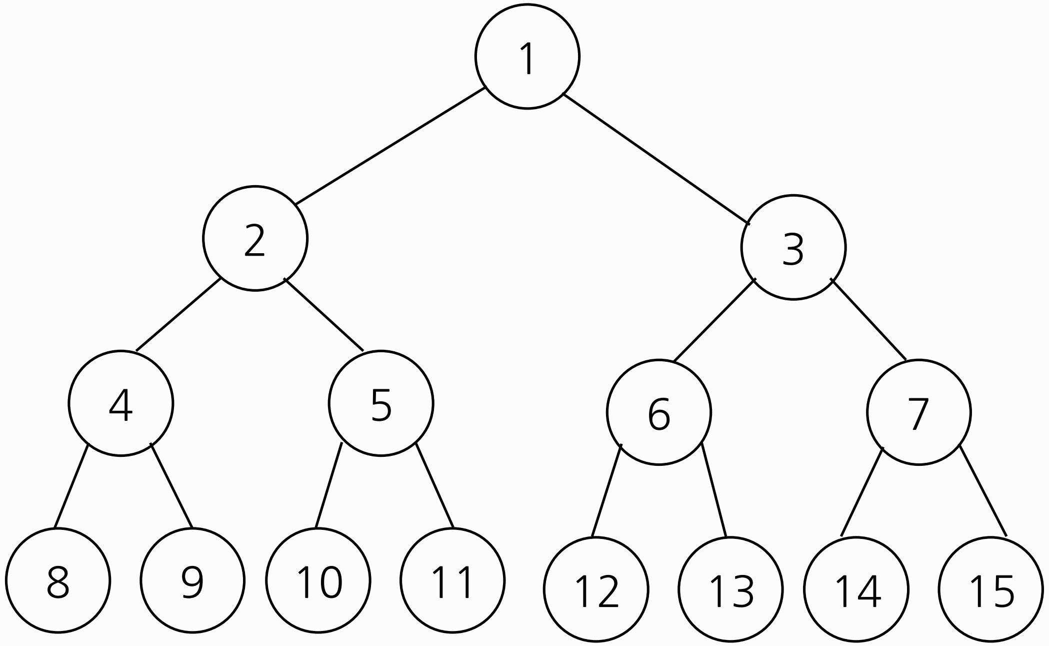


*A **complete** binary tree*

Binary trees - Array representation

- **Q:** Let's suppose that you have a **complete binary tree** in an array, how can we locate node x's parent or child?
- A **complete** binary tree with n nodes, any node index i, $1 \leq i \leq n$, we have
  - `parent(i) is at` $\lfloor i/2 \rfloor$ `If i = 1, i is at the root and has no parent`
  - `leftChild(i) is at 2i  if 2i <= n. If 2i > n, then i has no left child.`
  - `rightChild(i) is at 2i+1 if 2i+1 <= n. If 2i+1 > n, then i has no right child.`



Wow! Can we use this to all binary trees?
Why not?

Problem remains:
The problem with storing an arbitrary binary tree using an array is the inefficiency *in memory usage*.

Binary trees - Properties

(1) The maximum number of **nodes on level k** of a binary tree is

$k \geq 1$

(2) The maximum number of **nodes in a binary tree of level k** is

$k \geq 1$

(3) The maximum level of a **complete binary tree** with **n** nodes is

$\lceil x \rceil$ is the smallest integer $\geq x$.

Binary trees - Properties

(1) The maximum number of **nodes on level k** of a binary tree is

$$2^{k-1}, \qquad k \geq 1$$

(2) The maximum number of **nodes in a binary tree of level k** is

$$2^{k} - 1, \quad k \geq 1$$

(3) The maximum level of a **complete binary tree** with **n** nodes is

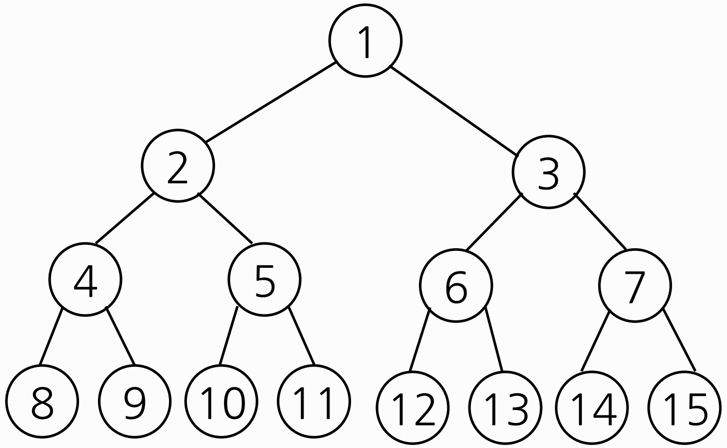
$k(n) = \lceil log_2\, (n+1) \rceil$, $\lceil x \rceil$ is the smallest integer $\geq x$.

$$n = 2^{k} - 1$$
$$n + 1 = 2^{k}$$
$$\log(n+1) = log 2^{k}$$
$$\log(n+1) = k$$
$$k(n) = \lceil \log(n+1) \rceil$$
$$k(n) = \lfloor \log(n) \rfloor + 1$$

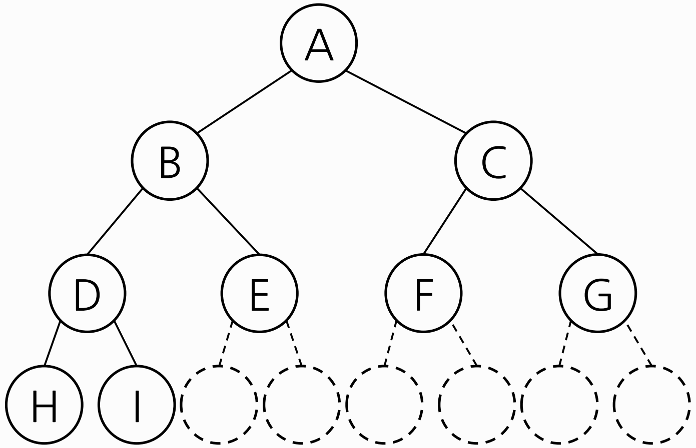
since k is an integer, and includes
the max level of complete binary tree.

Binary trees - Properties

- **Observation**: The max level of a full binary tree of *n* nodes is k = floor(log(n)) + 1 :
  - Many operations with trees have a run time that goes with the max level of some path within the tree;
  - If we have a full binary tree (or something *close* to it), we know that those operations run in $\boldsymbol{O}(\log \boldsymbol{n})$.



A ***full*** *binary tree*



A ***complete*** *binary tree*

Binary trees - Linked representation

- Node and Tree representations:

```
class Node:
    def __init__(self, key):
        self.key = key
        self.left = None
        self.right = None

    def insertLeft(self, key):
        ...
```

```
root = Node('A')
print(root)
```

key	
left	right

root

'A'	
None	None

Binary trees - Linked representation

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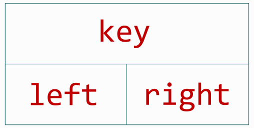


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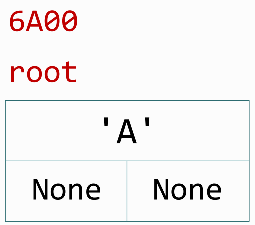
```
<__main__.Node object at
0x0000015985006A00>
```



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- Create 'B' and link with left of 'A'

```
root = Node('A')
node = Node('B')
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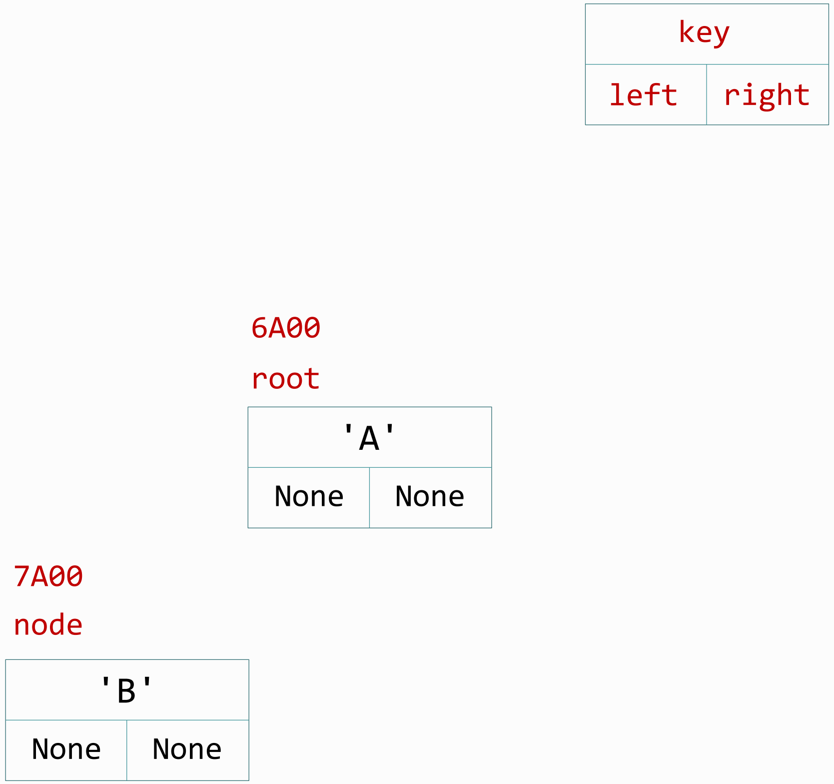

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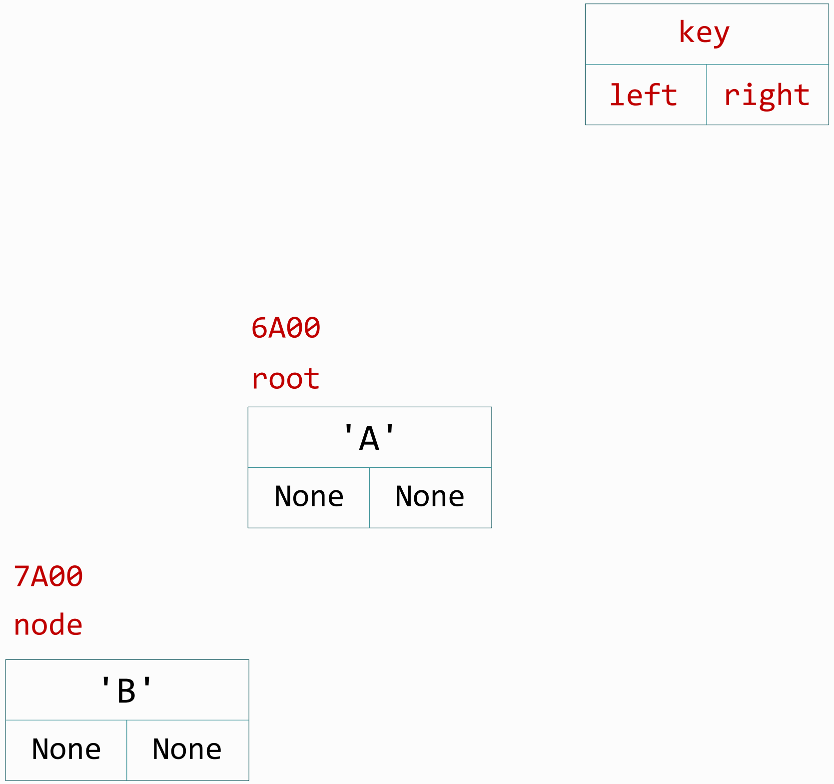

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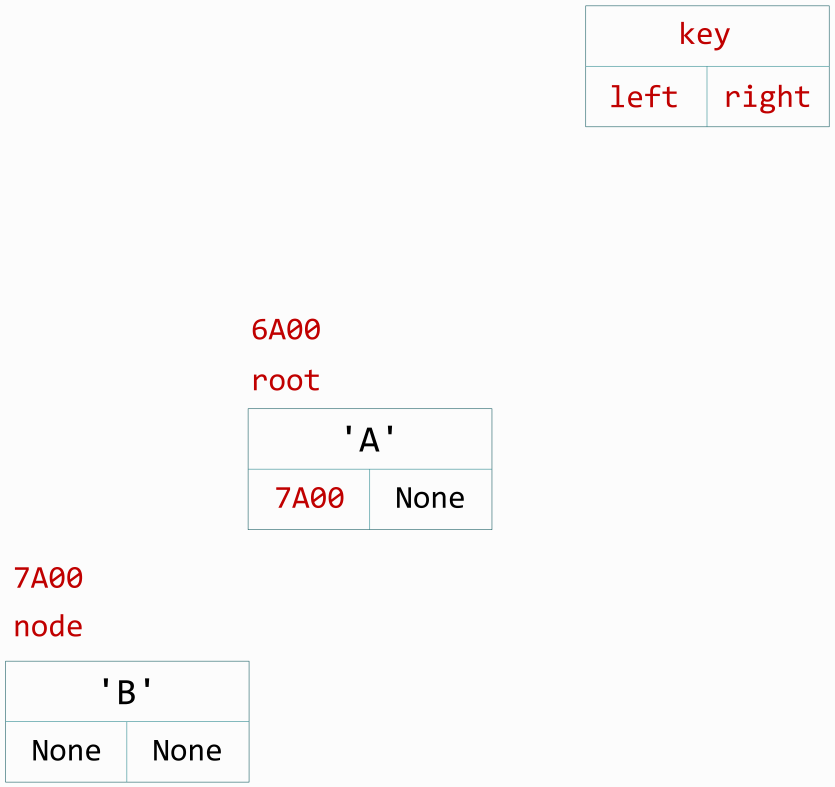
- Simplify the code above.



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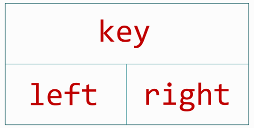


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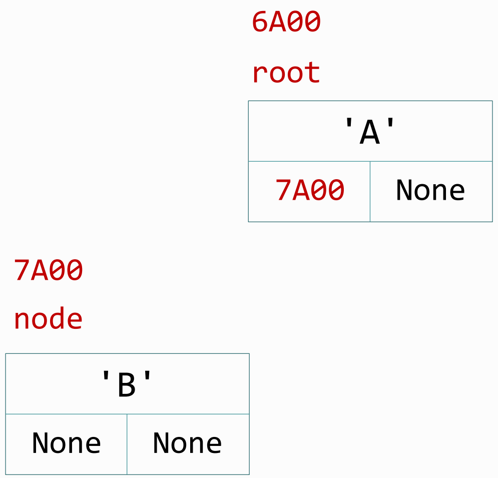


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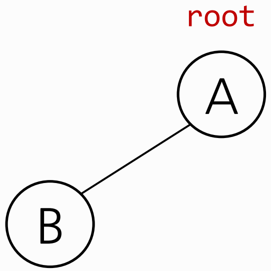
- Simplify the code and diagram.

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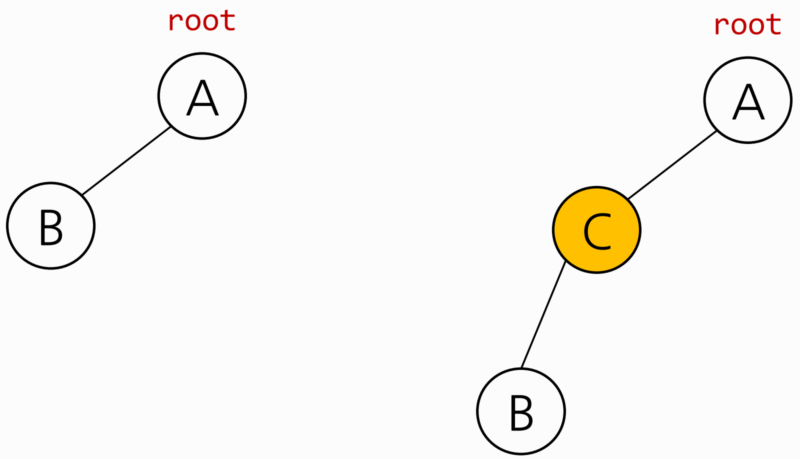

- Code **insertLeft()** and insert 'B' & 'C'.

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root = Node('A')
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root.insertLeft('C')
```

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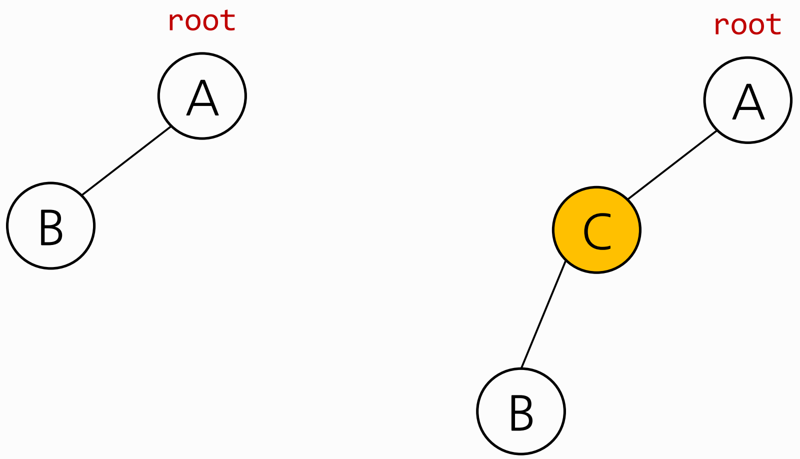

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root = Node('A')
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```
def insertLeft(self, key):
    if self.left == None:
        self.left = Node(key)
    else:
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```

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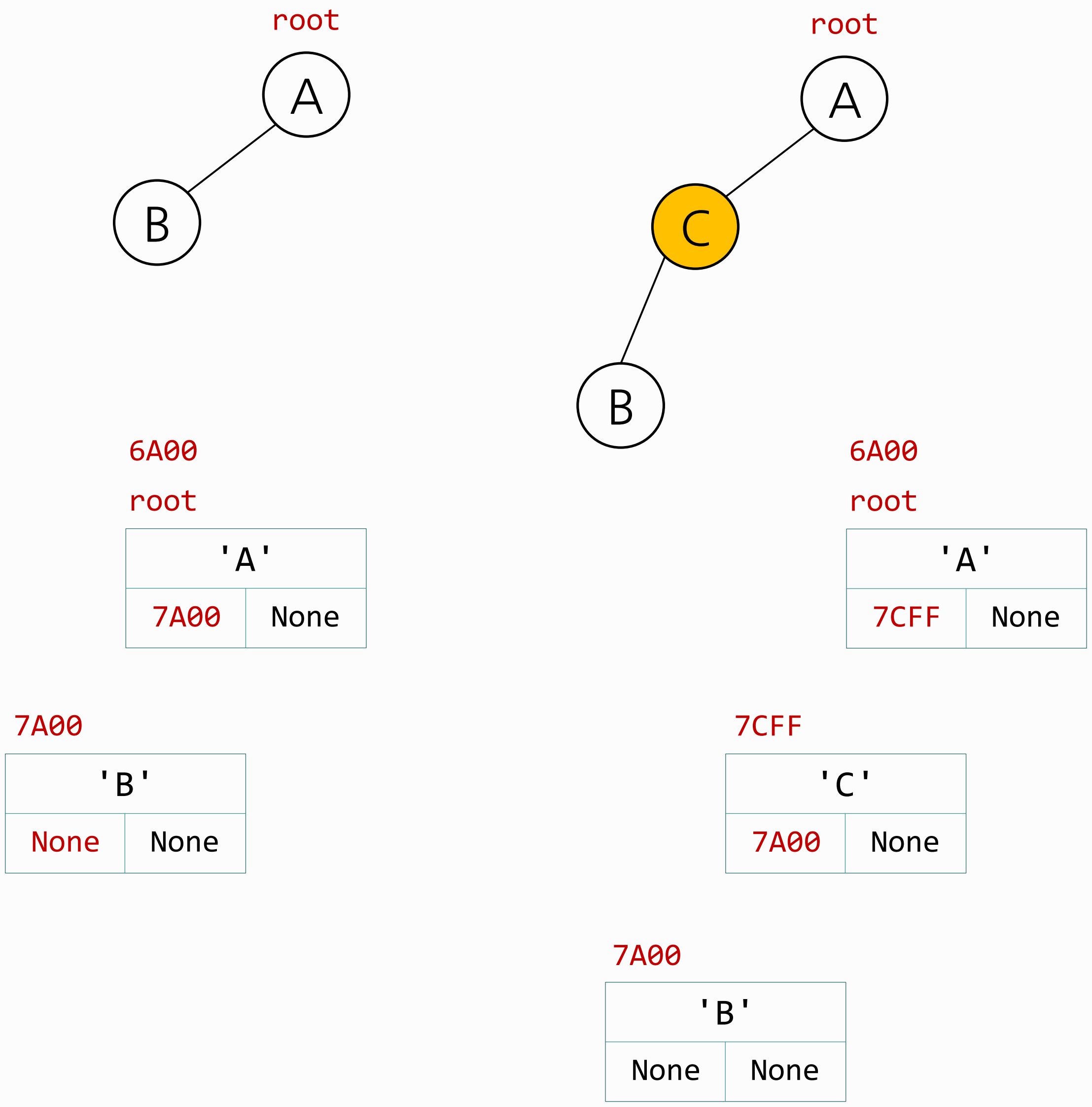

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def insertLeft(self, key):
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    else:
        node = Node(key)
        node.left = self.left
        self.left = node
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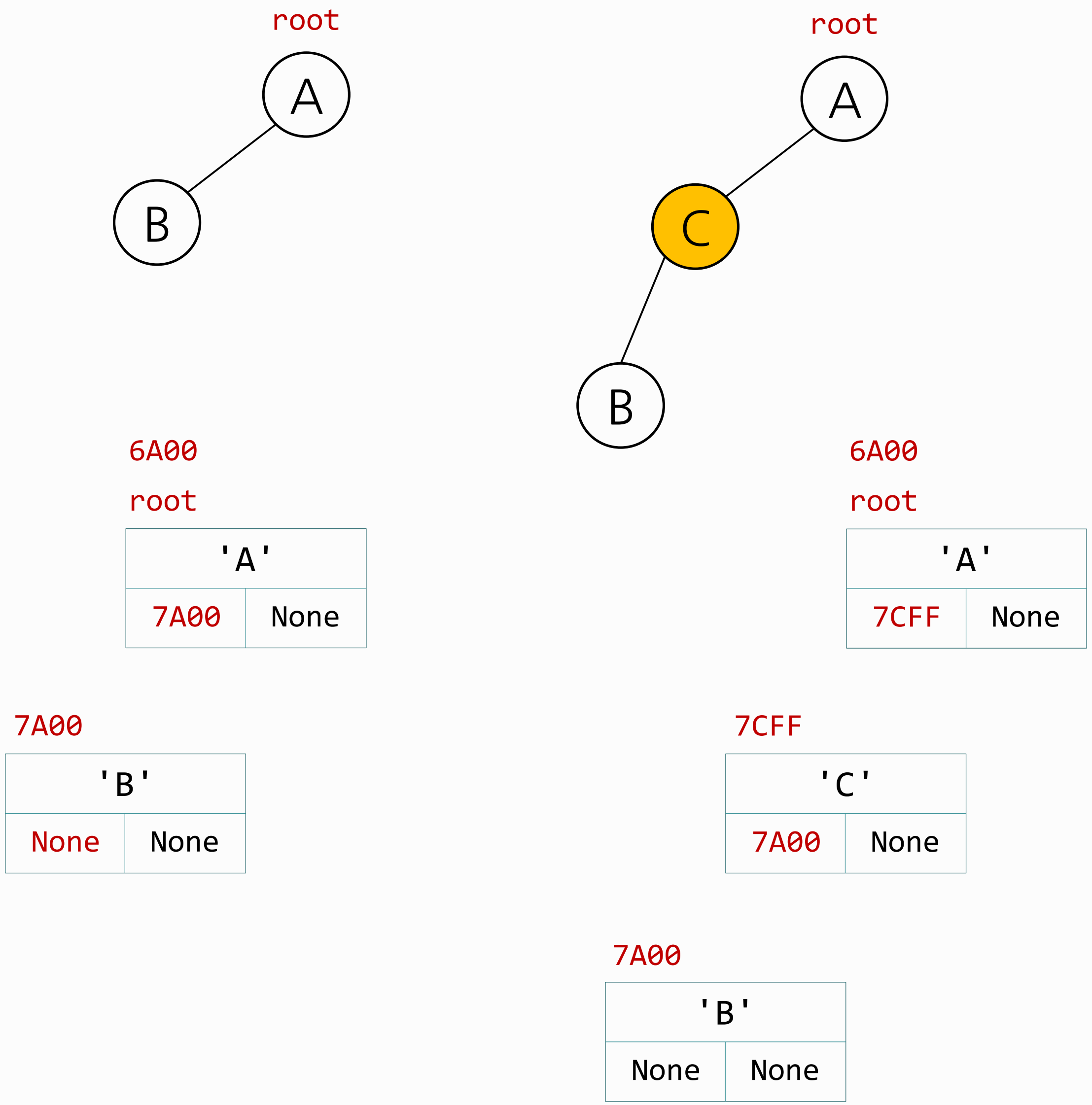

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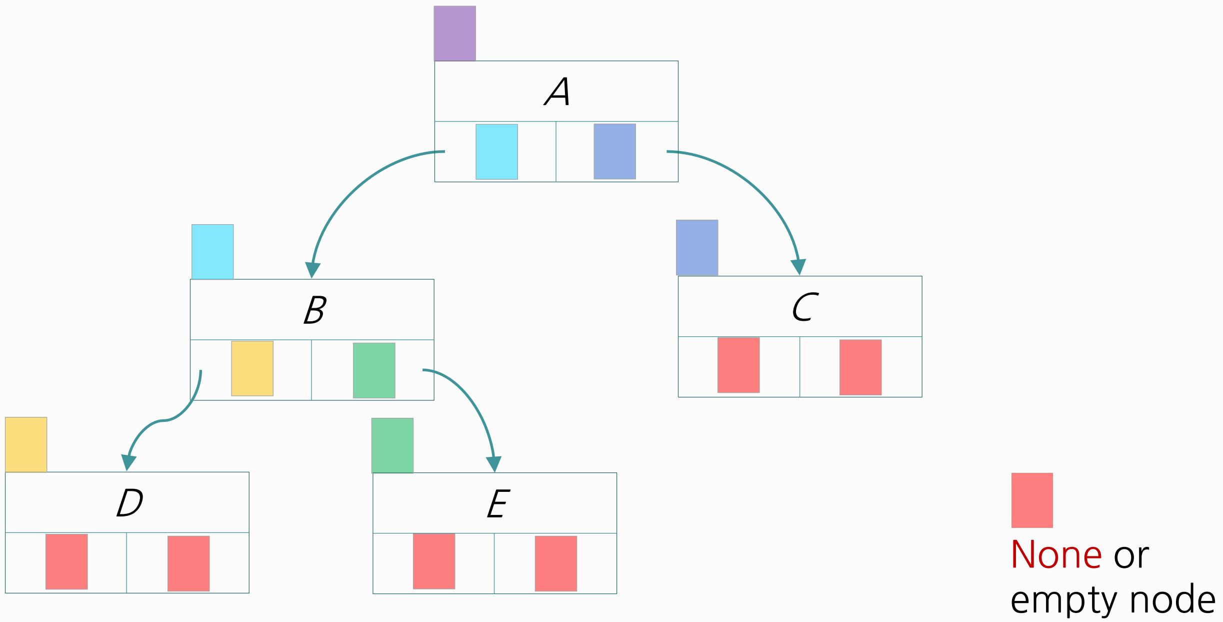

- **Q.** Is this node structure good enough?
  - Not easy to find its parent node. A parent field could be added if necessary.

Binary trees - Linked representation

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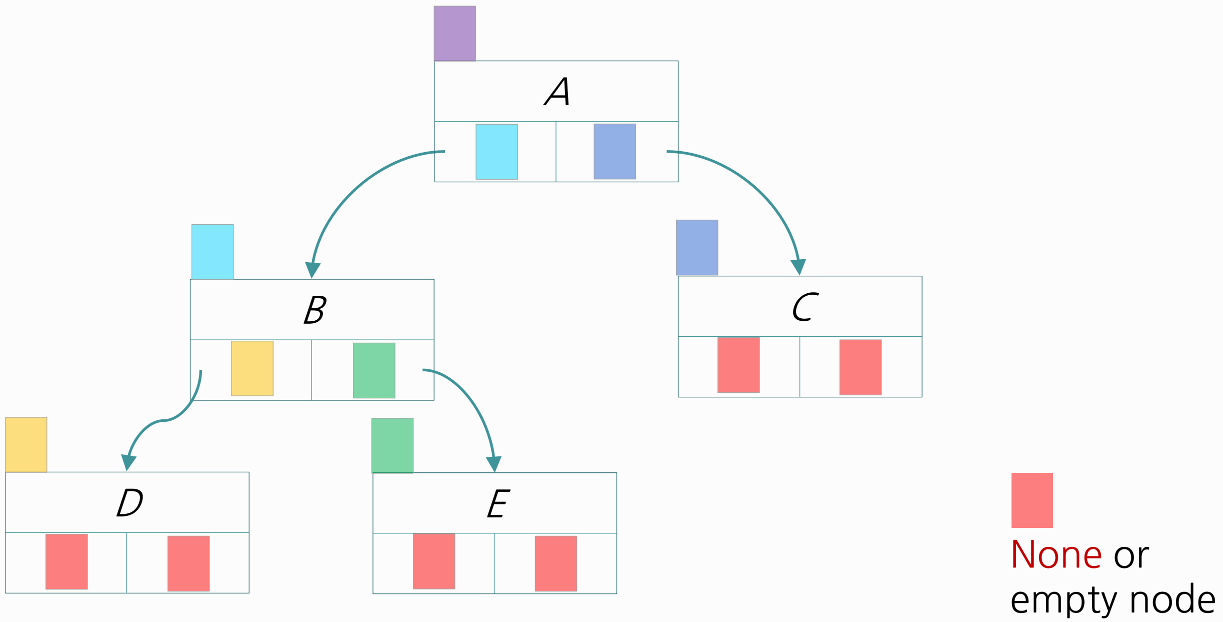


- Q. Is this node structure good enough?
  - Not easy to find its parent node. A parent field could be added if necessary.
- Q. It is similar to a doubly-linked list(DLL). What is different?
  - One head, but many tails. None points empty node conceptually.

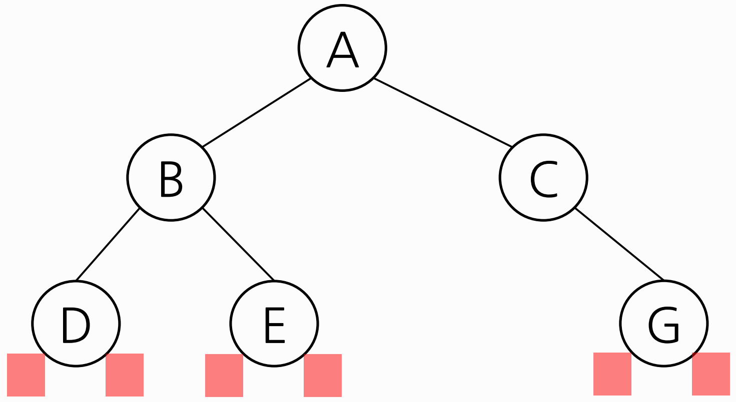
Binary trees - Exercise 1

- Build a tree shown below using insertLeft() and insertRight().

```
class Node:
    def __init__(self, key):
        self.key = key
        self.left = None
        self.right = None

    def insertLeft(self, key):
        ...
    def insertRight(self, key):
        ...

if __name__ == '__main__':
    root = Node('A')
    root.insertLeft('B')
    root.insertRight('C')
    # your code here
```



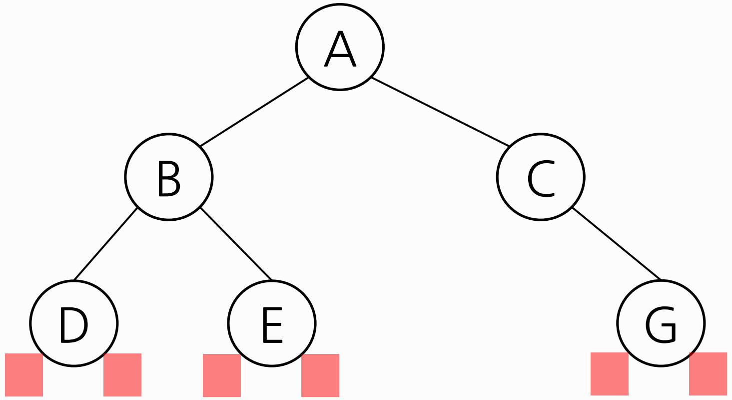
Binary trees - Exercise 2

- Extend Exercise 1 such that it exactly reproduces the output using the as shown below.

```
class Node:
    def __init__(self, key):
        self.key = key
        self.left = None
        self.right = None

    def insertLeft(self, key):
        ...
    def insertRight(self, key):
        ...

if __name__ == '__main__':
    ...
    for node in [a, b, c, d, e, g]
        if node.key:
            print(...
        ...
```




```
A:(B, C)
B:(D, E)
C:(None, G)
D:(None, None)
E:(None, None)
G:(None, None)
```

Data Structures in Python
Chapter 7 - 1

- **Tree Introduction**
- Tree Traversals
- Tree Algorithms

Prof. Youngsup Kim, idebtor@gmail.com, CSEE Dept., Grace School Rm204 Handong Global University