Data Structures in Python
Chapter 5

- **Binary Search**
- Recursive Binary Search

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Agenda & Readings

- **Binary Search**
- Recursive Binary Search

Binary search

- Look at the following program that generates a random integer and then gives clues to a user trying to guess the number.

```
import random
RANGE = 128
secret = random.randrange(RANGE)
print('I am thinking of a secret number between 0 and', RANGE - 1)
guess = 0
while guess != secret:
    guess = int(input('What is your guess? '))
    if    guess < secret: print('Too low')
    elif  guess > secret: print('Too high')
    else:                 print('Congrat!')
```

- Guess the value of a secret number that is one of the n integers between 0 and n - 1.
- Each time that you make a guess, you are told whether your guess is equal to the secret number, too high, or too low.

0	1	2	.	.	.	76	77	78	.	.	.	125	126	127

Binary search

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- **Question 1:** Assume that n is a power of 2.
  - How many times can you guess to get to the answer all the time?
  - Can you express it in terms of n?
- **Question 2:**
  - What would be the value of the RANGE if you can guess 20 times?
  - Can you express it in terms of n?

Exercise: 스무고개 1

- Improve the program a bit:
  - Show the maximum number of guesses that the user can make.
  - Print the message "Nice try. I'm sure you'll do better next time" if the user fails.
- Sample Run:

```
I am thinking of a secret number between 0 and 127
What is your guess(chance:7)?  64
Too low
What is your guess(chance:6)?  96
Too low
What is your guess(chance:5)?  112
Too high
What is your guess(chance:4)?  94
Too low
What is your guess(chance:3)?  98
Too low
What is your guess(chance:2)?  100
Too low
What is your guess(chance:1)?  101
Too low
Nice try. I'm sure you'll do better next time.
```

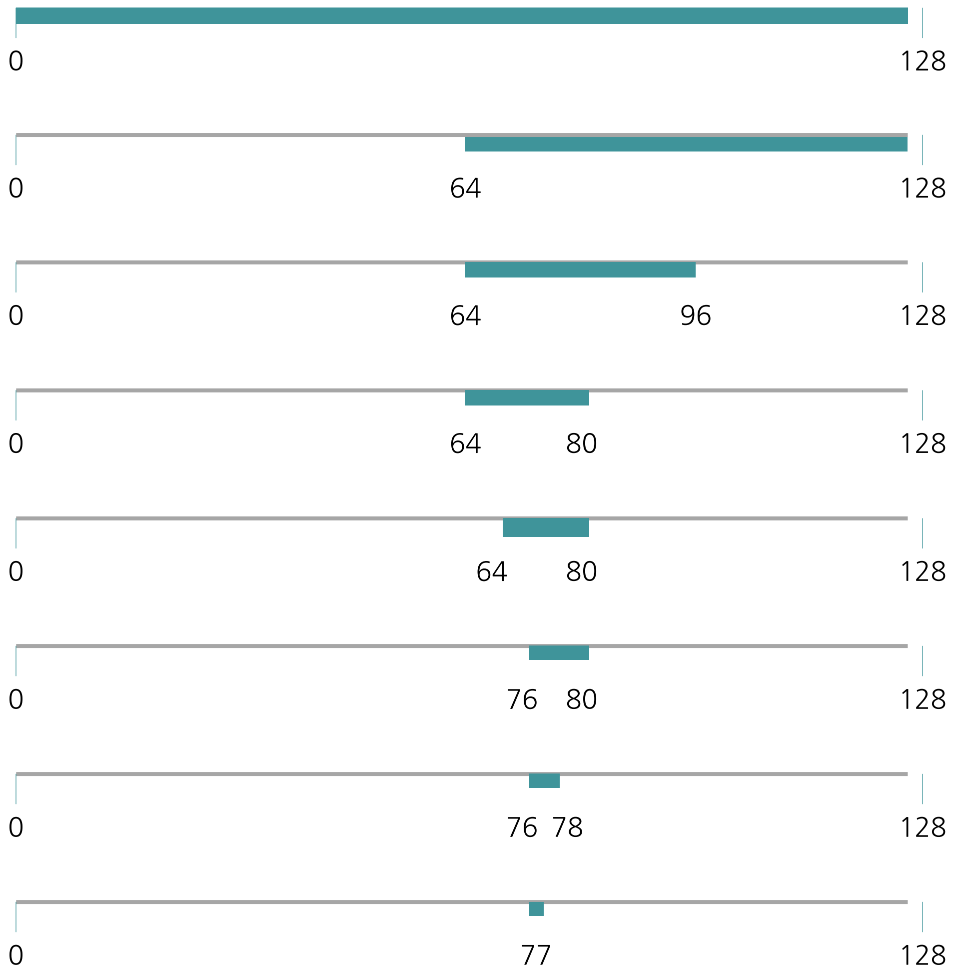
Exercise: 스무고개 2

- This script uses binary search to play the same game, but with the roles **reversed**: you choose the secret number, and the program guesses its value:
  - It asks the user to enter the number of guesses (or questions) k.
  - It displays the RANGE based on k such that the user can think of a number between 0 and 2^k - 1.
  - Then the computer always guesses the answer with k questions.

Exercise: 스무고개 2

- Finding a hidden number with **binary search**: Is the number greater than or equal to **m**?

search interval



length	question	response
128	>= 64 ?	true
64	>=96 ?	false
32	>= 80 ?	false
16	>= 72 ?	true
8	>= 76 ?	true
4	>= 78 ?	false
2	>= 77 ?	true
1	= 77	

An effective strategy is to guess the number in the middle of the interval **m**.

Use a half-open interval [lo, hi)
mid = (hi + lo) // 2

Exercise: 스무고개 2

- We use the notation `[lo, hi)` to denote all the integers greater than or equal to lo and less than (but not equal to) hi.
  - `[lo, hi)` is called a half-open interval which contains the left endpoint but not the right one.

- We start with `lo = 0` and `hi = n` and use the following **recursive** strategy.
  - **Base case**: If `hi - lo` equals 1, then the secret number is lo.
  - **Recursive step**: Otherwise, ask whether the secret number is greater than or equal to the number `mid = (hi + lo) // 2`. If so, look for the number in `[mid, hi)`; if not, look for the number in `[lo, mid)`.

Binary search: Analysis of running time

- Let n be the number of possible values. In Exercise 2, we have $\boldsymbol{n} = \boldsymbol{2^k}$, where $\boldsymbol{k} = \log_2 n$. Now, let $T(n)$ be the number of questions. The recursive strategy immediately implies that $T(n)$ must satisfy the following **recurrence relation**:

$$T(n) = T(n/2) + 1$$

  with $T(1) = 0$.

- Substituting 2^k for n, we can telescope the recurrence (apply it to itself) to immediately get a closed-form expression:

since $T(2^{k-1}) = T(2^{k-2}) + 1$

$$T(2^k) = T(2^{k-1}) + 1 = T(2^{k-2}) + 2 = \ldots = T(1) + k = k$$

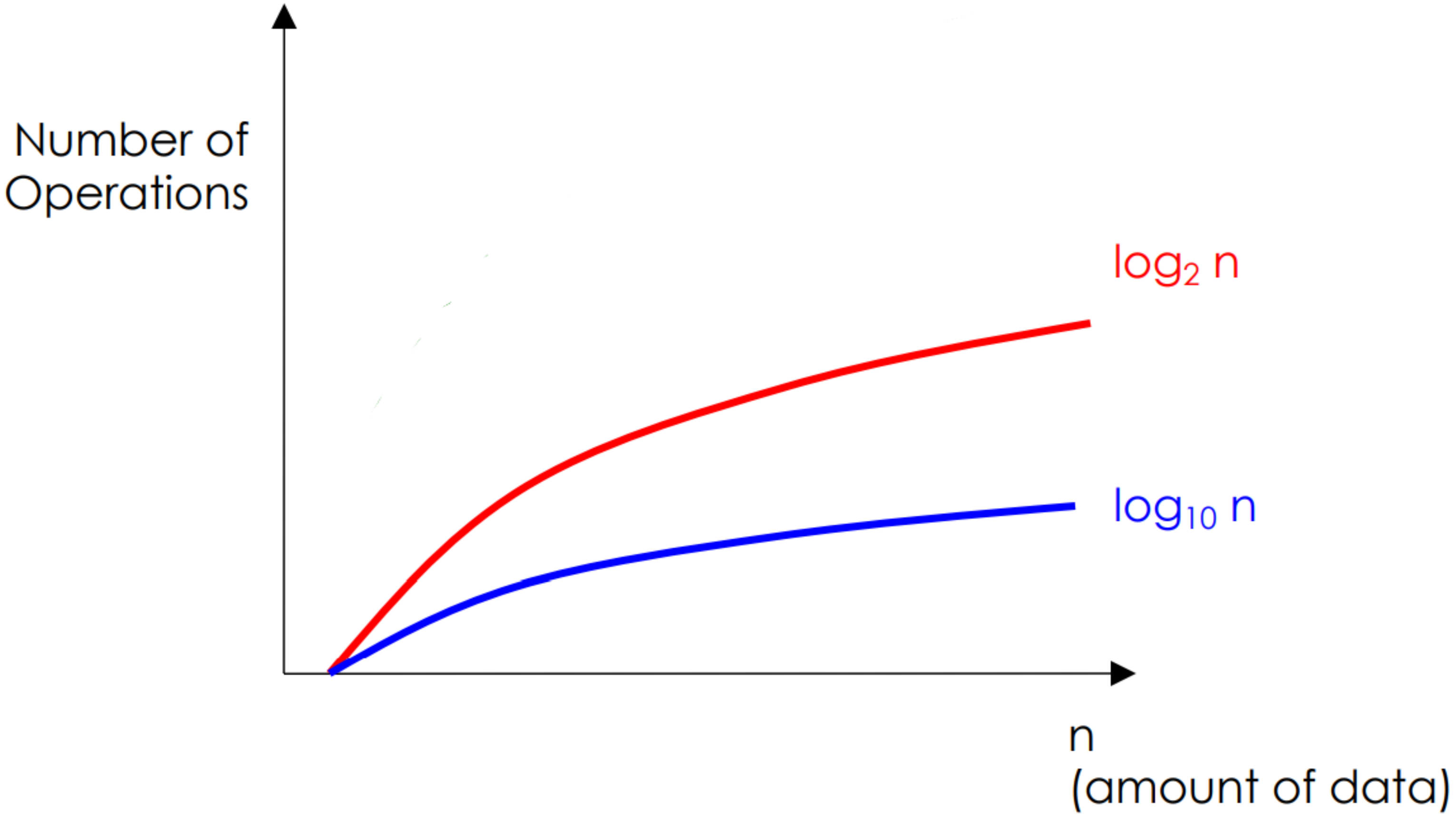
$T(2^{k-1}) = T(2^{k-2}) + 1$

- Substituting back n for 2^k (and $\log_2 n$ for k) gives the result

$$T(n) = \log_2 n$$

- We say Binary Search has the time complexity $\boldsymbol{O}(\log \boldsymbol{n})$.
  Note: Binary search work even when n is not a power of 2.

Time Complexity $O(\log n)$ "Logarithmic Time"



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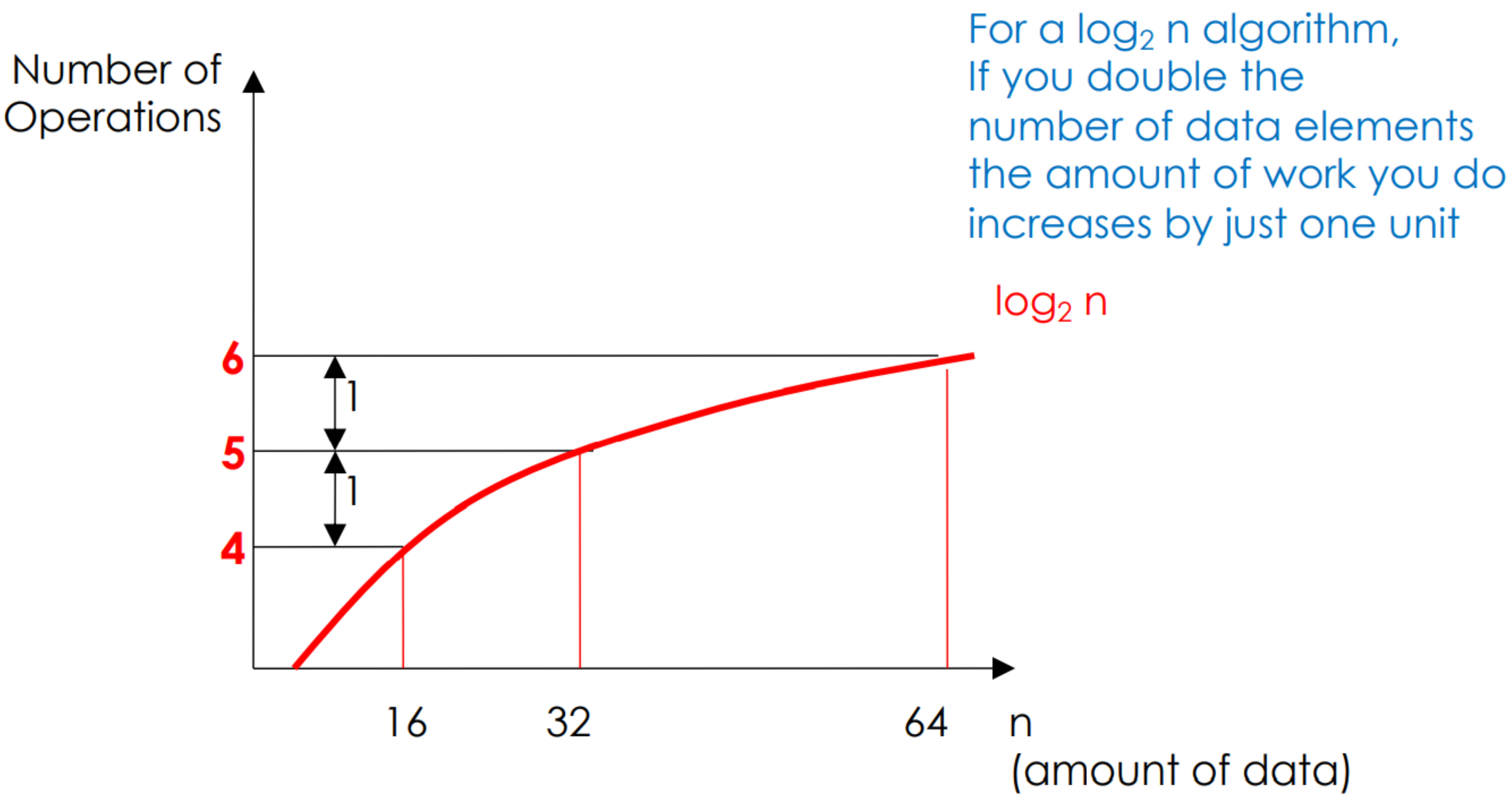

Binary Search (Worst Case)

- Finding an element in a list with one million elements requires only 20 guesses (questions or comparison)!
- But the list **must be sorted**.
  - What if we sort the list first using insertion sort?
    - Insertion sort $O(n^2)$ (worst case)
    - Binary search $O(\log n)$ (worst case)
    - Total time complexity $O(n^2) + O(\log n) = O(n^2)$
  - Fortunately, there are faster ways to sort.

Number of Elements	Number of Comparisons
16	4
32	5
64	6
128	7
256	8
1024	10
1,000,000	20

Summary

- Binary search is simple, but powerful!
- Binary search may be implemented using either iteration or recursion.
- Its time complexity is $O(\log n)$.