Data Structures in Python Chapter 5

- Binary Search
- **Recursive Binary Search**

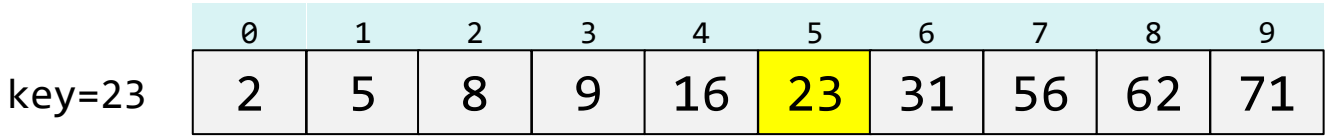
Prof. Youngsup Kim, idebtor@gmail.com, CSEE Dept., Grace School Rm204 Handong Global University

Agenda & Readings

- Binary Search
- **Recursive Binary Search**

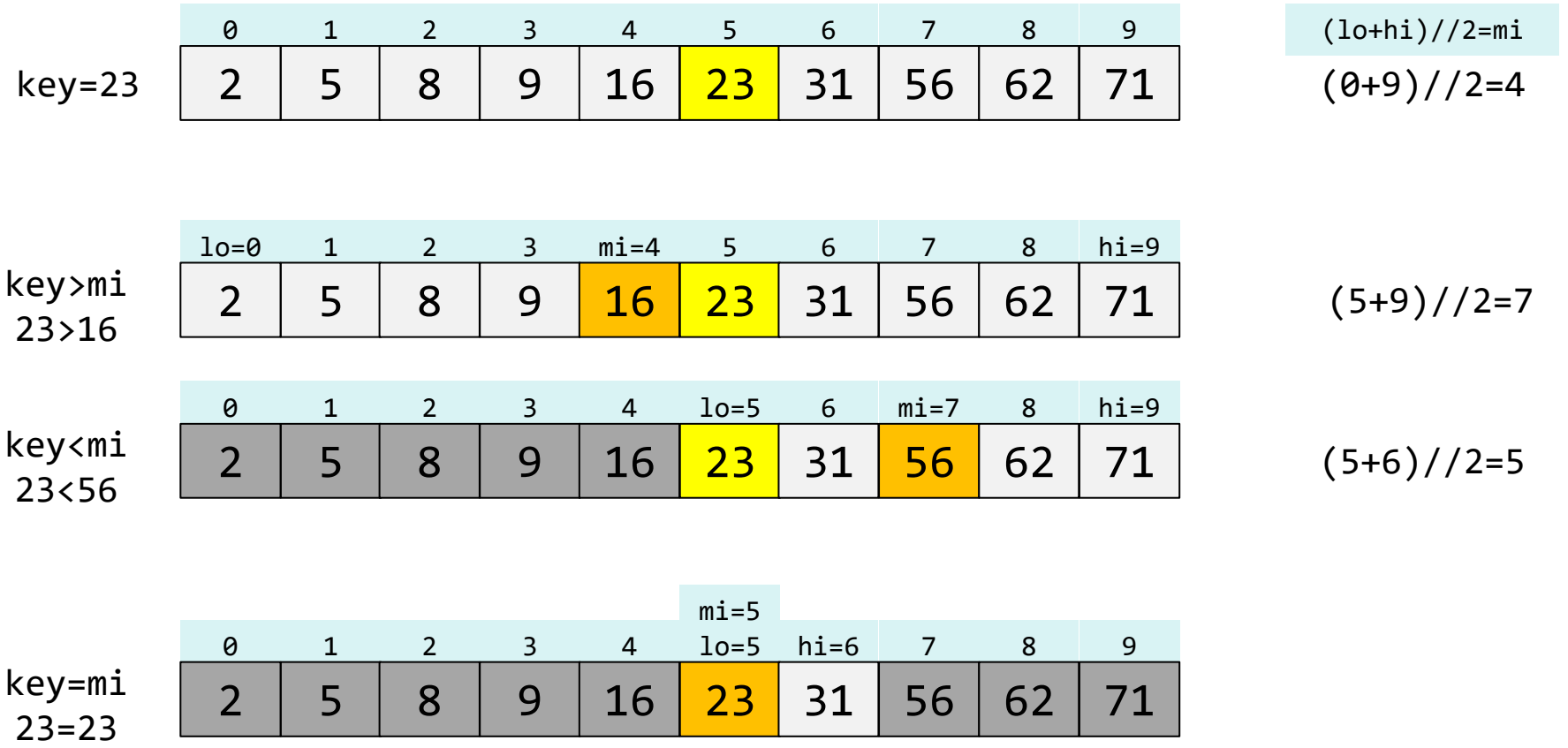
Recursive Binary Search

- Binary search is an efficient algorithm for finding an item from **a sorted list** of items.
  - It works by repeatedly **dividing in half the portion of the list** that could contain the item, until you've **narrowed down the possible locations to just one.**



Recursive Binary Search

- For instance, we want to search "23" from the array. If we find it, we return its array index; otherwise, -1 or something else.



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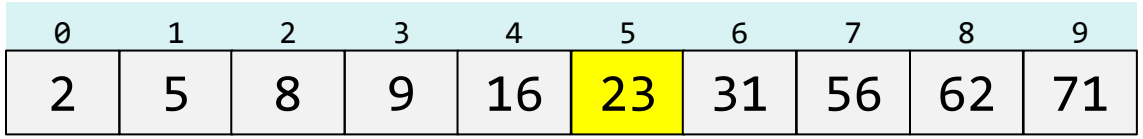

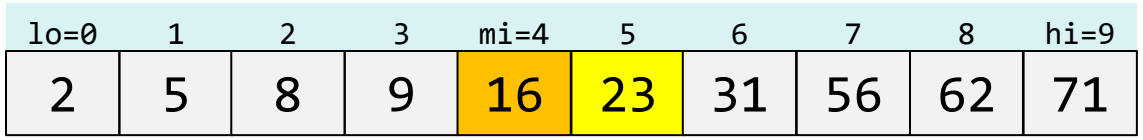

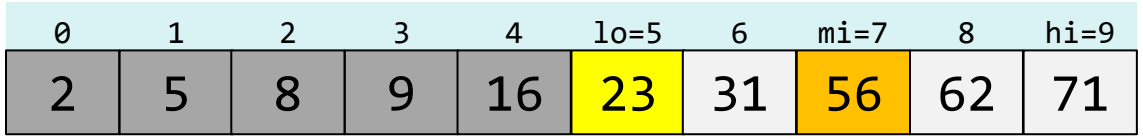

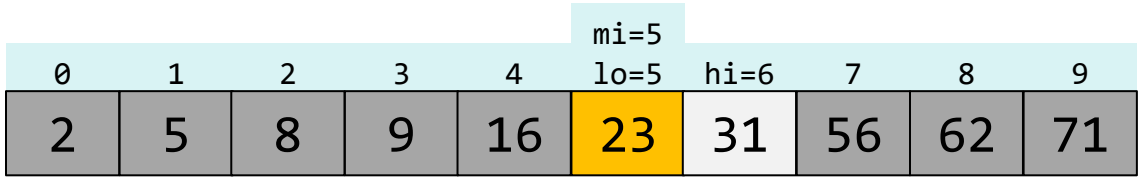

key	key sought
list	**sorted array**
mi	midpoint
lo	smallest possible index
hi	largest possible index

```
def binary_search(list, key, lo, hi):
  if lo > hi: return -1

  mi = (lo + hi) // 2
  if key == list[mi]: return mi
  if key <  list[mi]:                    same      new
                                         for low   for high
    return binary_search(list, key, lo,    mi-1)
  else:
    return binary_search(list, key, mi+1,  hi )
                                         new       same
                                         for low   for high
```

Recursive Binary Search

- Given the numbers 1 to 100, what is **the number of guesses** at most needed to find a specific number if you are given the hint 'higher' or 'lower' for each guess you make?
  - Since the numbers are sequential (or sorted), we can use binary search.
  - Look at the middle element: if it's after than the number we're looking for, search the first half. If it's before the number we're looking for, look at the second half.
  - Each check cuts the size of the list numbers in half; how many times can we do this?
  - If we think backwards, in terms of doubling the list, we'll need n doublings to generate a list of length $2^n = 100$. What is the value of n?
  - Since $2^6 = 64$ and $2^7 = 128$ (or $\log_2 64 = 6, \log_2 128 = 7$), $n = 6.x$
    Therefore $n = 7$ guesses will be enough.

- Binary search guarantees to take no more than $\log \boldsymbol{n}$ guesses or comparison.
- More precisely, $\lceil log_2 n \rceil$ if n is not a power of 2.

- https://jwoop.tistory.com/9

Exercise: Exception Filter

- **The exception filter** reads in a sorted array of strings from a file(which we refer to as the `whitelist`) and an arbitrary sequence of strings from standard input and write those in the sequence that are **not** in the `whitelist`.

- **For example:**
  - When checking the spelling of a word, you need to know only whether your word is in the dictionary (or `whitelist`) and are not interested in the definition.
- **Another examples:**
  - Your email application might use an **exception filter** to reject any messages that are not on a `whitelist` that contains the email addresses of your friends.
  - Your operating system might have an **exception filter** that disallows network connections to your computer from any device having an IP address that is not on a preapproved `whitelist`.

  - In a computer search, we keep the whitelist sorted in order of the key.
  - The array length n need not be a power of 2.

Exercise: Exception Filter

- **Task:** Find and print all misspelled words in `words.txt` using binary search algorithm. The correct words in `dict.txt` are listed in sorted order. A skeleton code, `dict.txt` and `words.txt` files are provided.
  - When checking the spelling of a word, you need to know only whether your word is in the dictionary (or **`whitelist`**) and are not interested in the definition.
  - We keep the **`whitelist`** sorted in order of the key.
  - The array length **`n`** need not be a power of 2.

whitelist in sorted order

```
$ more dict.txt
accommodate
broccoli
conscience
definitely
embarrass
necessary
occurred
publicly
receive
separate
until
```

word list to check misspelled

```
$ more words.txt
definately
occured
occurred
untill
until
recieve
acommodate
seperate
separate
```

redirect standard input through the file

```
$ python exfilter.py < words.txt
definately
occured
untill
recieve
acommodate
seperate
```

misspelled words, not in dict.txt (or whitelist)

Exercise: Exception Filter

- Skeleton Code:

```
%%writefile exfilter.py
import sys

def binary_search(arr, key, lo, hi):
    pass

def search(arr, key):
    pass

if __name__ == "__main__":
    with open('dict.txt') as f:
        arr = f.read().splitlines()
    for line in sys.stdin:
        key = line.rstrip()
        if search(arr, key) < 0:
            print(key)              # not found in dict
```

```
%%cmd
python exfilter.py < words.txt
```

i/o reference: https://stackoverflow.com/questions/1450393/how-do-you-read-from-stdin

Exercise: Exception Filter

- **Sample Runs in Jupyter Lab and in Windows cmd shell:**

```
Overwriting exfilter.py
```

```
[2]: %%cmd
     python exfilter.py < words.txt
```

```
Microsoft Windows [Version 10.0.19042.1288]
(c) Microsoft Corporation. All rights reserved.

C:\GitHub\DSpyx\jupyter>python exfilter.py < words.txt
definately
occured
untill
recieve
acommodate
seperate

C:\GitHub\DSpyx\jupyter>
```

```
[3]: %%bash
     python exfilter.py < words.txt
```

```
definately
occured
untill
recieve
acommodate
seperate
```

Note: This does not work in Windows PowerShell. Use cmd.

```
C:\GitHub\DSpyx\jupyter>cmd
Microsoft Windows [Version 10.0.19042.1288]
(c) Microsoft Corporation. All rights reserved.

C:\GitHub\DSpyx\jupyter>python exfilter.py < words.txt
definately
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seperate

C:\GitHub\DSpyx\jupyter>
```

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Summary

- Binary search is simple, but powerful!
- Binary search may be implemented using either iteration or recursion.
- Its time complexity is $O(\log n)$.