Data Structures in Python Chapter 2

1. Abstract Data Type(ADT)
2. **Performance Analysis**
3. Big-O Notation
4. Growth Rates

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그러므로 나의 사랑하는 자들아 너희가 나 있을 때 뿐 아니라 더욱 지금 나 없을 때에도 항상 복종하여 두렵고 떨림으로 너희 구원을 이루라 (Continue to work out your salvation with fear and trembling.) 빌2:12

나는 인애를 원하고 제사를 원하지 아니하며 번제보다 하나님을 아는 것을 원하노라 (호6:6)
하나님은 모든 사람이 구원을 받으며 진리를 아는데에 이르기를 원하시느니라 (딤전2:4)

그런즉 너희가 먹든지 마시든지 무엇을 하든지 다 하나님의 영광을 위하여 하라 (고전10:31)

Agenda & Reading

- **Performance Analysis**
  - **Introduction**
  - **Step Counts - Counting Operations**
- Big-O Notation - Asymptotic Analysis
  - Properties of Big-O
  - Calculating Big-O
- Growth Rates
  - Big-O Performance of Python Lists
  - Big-O Performance of Python Dictionaries
- References:
  - Textbook: Problem Solving with Algorithms and Data Structures
    - Chapter 3. Analysis
  - Textbook: www.github.idebtor/DSpy
    - Chapter 2.1 ~ 3

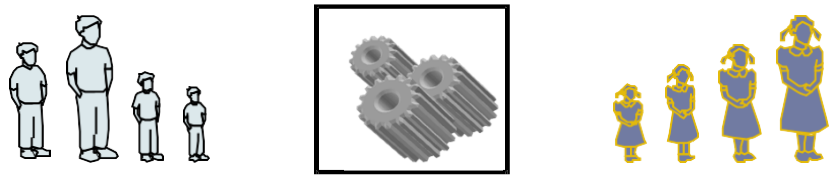
1 Introduction - What Is Performance Analysis?

- How to compare programs with one another?

- When two programs solve the same problem but look different, is one program better than the other?
- What criteria are we using to compare them?
  - Readability?
  - Efficiency? Time vs. Memory
- Why do we need Performance Analysis or Complexity Analysis?
  - Writing a working program is not good enough.
  - The program may be inefficient!
  - If the program runs on a large data set, then the running time may become an issue.

1 Introduction - Data Structures & Algorithm

- Data Structures:
  - A systematic way of **organizing** and **accessing** data.
  - No single data structure works well for **ALL** purposes.
- Algorithm
  - An algorithm is **a step-by-step procedure** for solving a problem in a finite amount of time.
- Program
  - A program is an algorithm that has been encoded into some programming language.
- **Program = data structures + algorithms**



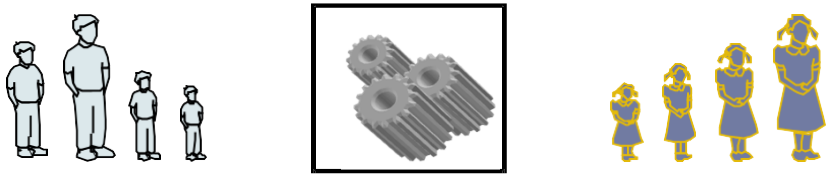
Input **Algorithm** **Output**

1 Introduction - Performance Analysis/Complexity

- When we analyze the **performance** of an algorithm, we are interested in how much of a given resource the algorithm uses to solve a problem.

- The most common resources are **time** (how many **steps** it takes to solve a problem) and **space** (how much memory it takes).

- We are going to be mainly interested in **how long** our programs take to run, as time is generally more precious resource than **space**.

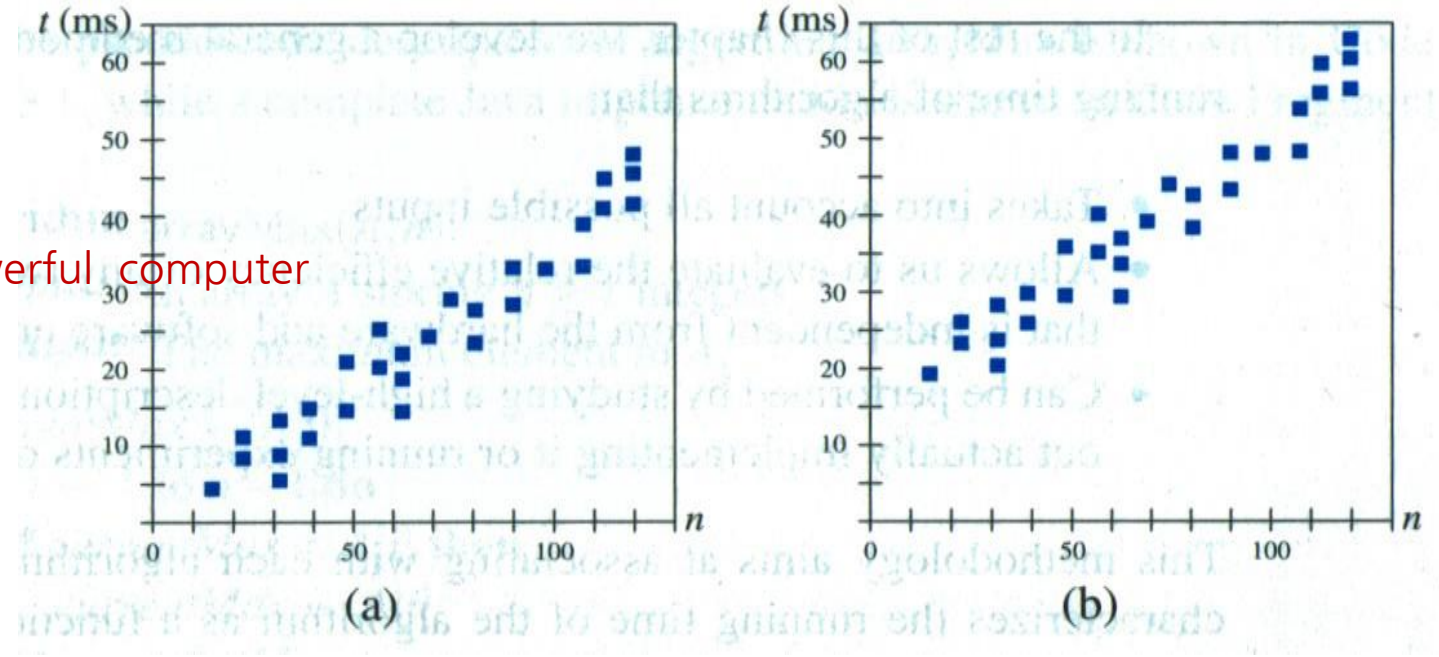


Input **Algorithm** **Output**

1 Introduction - Efficiency of Algorithms

- For example, the following graphs show the execution time, in milliseconds, against sample size, n of a given problem in **different computers**

More powerful computer



- The actual running time of a program depends **not only** on the efficiency of the algorithm, **but** on many other variables such as Processor speed & type, Operating system, … etc.

1 Introduction - Running-time of Algorithms

- In order to compare algorithm speeds experimentally
  - All other variables must be kept constant, i.e.
    - independent of specific implementations (C, C++ or Java),
    - independent of computers used, and,
    - independent of the data on which the program runs
  - Involved a lot of work (better to have some theoretical means of predicting algorithm speed)

1 Introduction - Example 1

- Task:
  - Complete the `sum_of_n()` function which calculates the sum of the first n natural numbers.
    - **Arguments:** an integer
    - **Returns:** the sum of the first n natural numbers

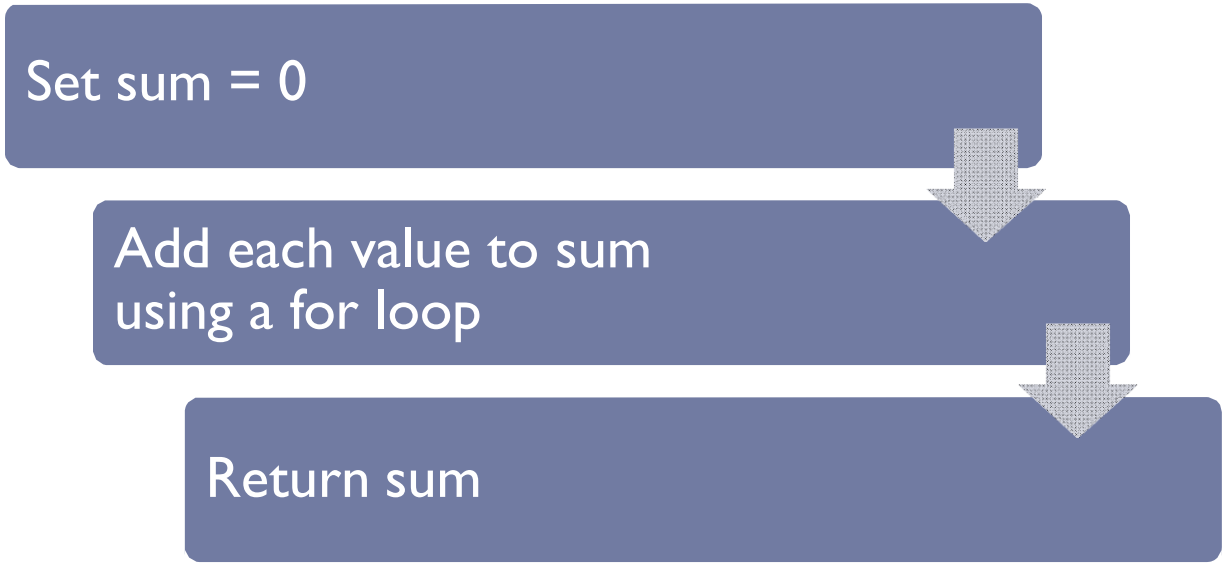
- Cases:

`sum_of_n(5)`	`15`

`sum_of_n(100_000)`	`5000050000`

1 Introduction - Algorithm 1

- sum_of_n

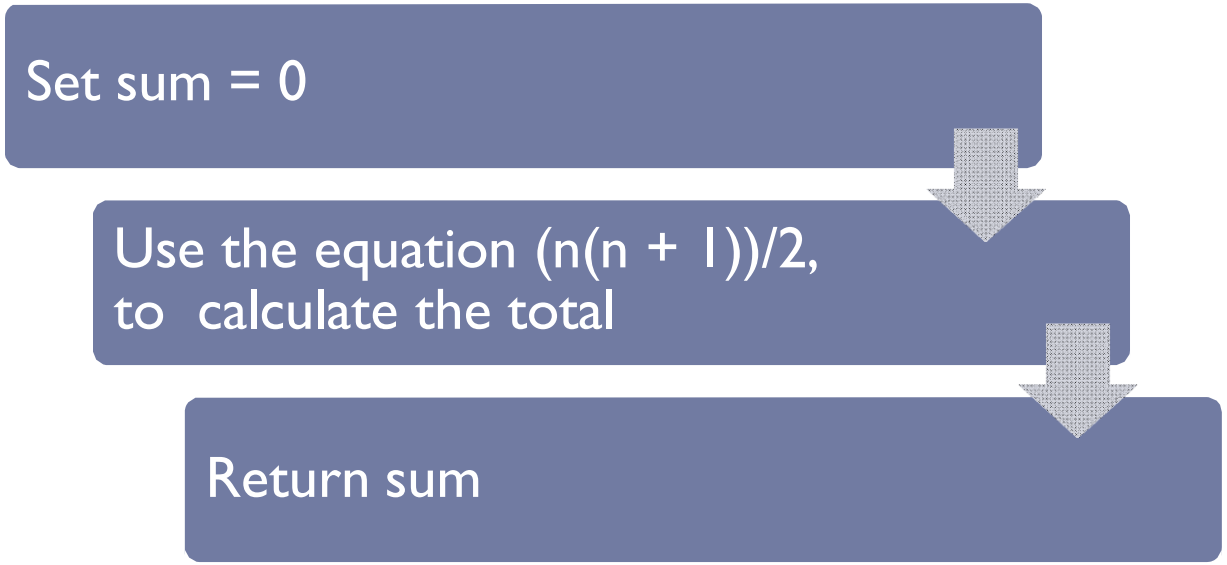



```
time_start = time.time()

sum = 0
for i in range(1,n+1):
    sum = sum + i

time_end = time.time()
time_taken = time_end - time_start
```

The timing calls embedded before and after the summation to calculate the time required for the calculation.

1 Introduction - Algorithm 2

- sum_of_n_2




```
time_start = time.time()

sum = 0
sum = n * (n + 1) / 2

time_end = time.time()
time_taken = time_end - time_start
```

The timing calls embedded before and after the summation to calculate the time required for the calculation.

1 Introduction - Experimental Result

- Using 4 different values for n: [10000, 100000, 1000000, 10000000]

n	sum_of_n (for loop)	sum_of_n_2 (equation)
10000	0.0033	0.00000181
100000	0.0291	0.00000131
1000000	0.3045	0.00000107
10000000	2.7145	0.00000123

Time Consuming Process!

Time increase as we increase the value of n.

NO impacted by the number of integers being added.

- We shall **count** the number of basic operations of an algorithm and **generalize** the count.

2 Counting Operations - Example 2A

- Example: Calculating a sum of the first 10 elements in the list

```
def count1(numbers):
    sum = 0                                  1 assignment
    index = 0                                1 assignment
    while index < 10:                        11 comparisons
        sum = sum + numbers[index]           10 plus/assignments
        index += 1                           10 plus/assignments
    return sum                               1 return
```

- Total = 34 operations (**steps**)

2 Counting Operations - Example 2B

- Example: Calculating **the sum of n elements** in the list.

```
def count2(numbers):
    n = len(numbers)
    sum = 0
    index = 0
    while index < n:
        sum = sum + numbers[index]
        index += 1
    return sum
```

```
1 assignment
1 assignment
1 assignment
n + 1 comparisons
n plus/assignments
n plus/assignments
1 return
```

- Total = **3n + 5** operations (steps)
- We need to measure an algorithm's time requirement as **a function of the problem size**, e.g., in the example above the problem size is the number of elements in the list.

2 Counting Operations - Problem size

- Performance is usually measured by the **rate** at which the running time increases as the problem size gets bigger,
  - i.e., we are interested in the relationship between the running time and the problem size.
  - It is very important that we identify **what the problem size is**.
    - For example, if we are analyzing an algorithm that processes a list, the problem size is the **size** of the list.
- In many cases, the problem size will be the **value** of a variable, where the running time of the program depends on how big that value is.

2 Counting Operations - Exercise 1

- How many operations are required to do the following tasks?
  - Adding an element to the end of a list
  - Printing each element of a list containing n elements

2 Counting Operations - Exercise 1 solution

- How many operations are required to do the following tasks?
  - Adding an element to the end of a list - one operation, a constant time, or O(1)
  - Printing each element of a list containing n elements - one operation, a constant time, or O(1)

2 Counting Operations - Example 3

- Consider the following two algorithms:
  - Algorithm A:
    - Outer Loop: n operations
    - Inner Loop: $\frac{n}{5}$ operations
    - Total = $(n * \frac{n}{5}) = \frac{n^2}{5}$ operations

```
for i in range(0, n):
    for j in range(0, n, 5):
        print(i, j)
```

  - Algorithm B:
    - Outer Loop: n operations
    - Inner Loop: 5 operations
    - Total = $(n * 5) = 5 * n$ operations

```
for i in range(0, n):
    for j in range(0, 5):
        print(i, j)
```

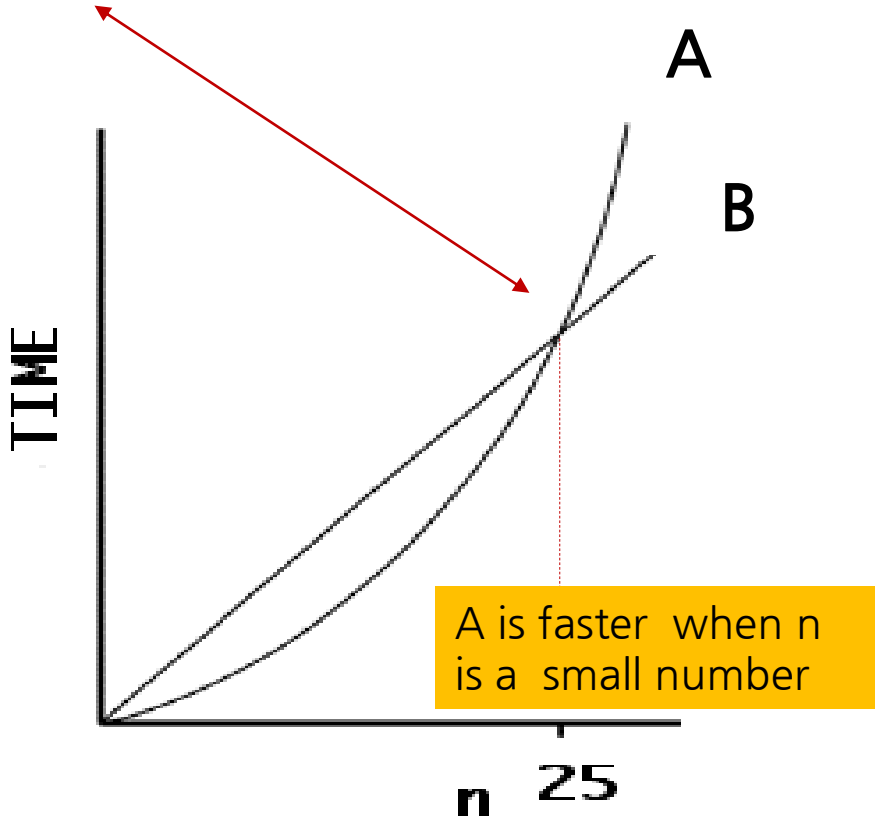
2 Counting Operations - Growth Rate Function - A or B?

- Consider the following two algorithms:
  - Algorithm A: $\frac{n^2}{5}$
  - Algorithm B: $5 * n$

n	5	10	15	20	24	25	26	30
A	5	20	45	80	115	125	135	180
B	25	50	75	100	120	125	130	150

- If n is 10^6 ,
  - Algorithm A's time requirement is
    - $\frac{n^2}{5} = \frac{10^{12}}{5} = 2 * 10^{11}$
  - Algorithm B's time requirement is
    - $5 * n = 5 * 10^6$

- What does the **growth rate** tell us about the running time of the program?



2 Counting Operations - Growth Rate Function - A or B?

- **For smaller values of n**, the differences between algorithm A $(n^2/5)$ and algorithm B (5n) are not very big. But the differences are very evident for larger problem sizes such as for n > 1,000,000

- $2 * 10^{11}$ vs. $2 * 10^{6}$

- Bigger problem size, produces bigger differences
- **Algorithm efficiency is a concern for large problem sizes**

Summary

- Performance Analysis measure an algorithm's time requirement **as a function of the problem size n** by **using a growth-rate** function.
- It is an **implementation-independent** (including hardware and coded language) way of measuring an algorithm.
- Performance(Complexity) analysis focuses on large problems.