

Data Structures in Python

1. Hash Table
2. Collision Resolution
- 3. Double Hashing & Rehashing**
4. HashMap Coding

Agenda & Readings

- Collision Resolution
 - Separate Chaining
 - Open Addressing
 - Linear Probing
 - Quadratic Probing
 - **Double Hashing**
- **Rehashing**
- Reference:
 - Problem Solving with Algorithms and Data Structures
 - Chapter 5 - Hashing

Collision Resolution by Open Addressing

1. Linear Probing (선형조사법)
2. Quadratic Probing (이차조사법)
3. Double Hashing (이중해싱법)

Collision Resolution - Double Hashing 이중해싱법

- Keep two hash functions, $h(x)$ and $h'(x)$.
- Use **a second hash function** for all tries i other than 0
 $f(i) = i * h'(x)$
- Good choices for $h'(x)$?

Hash function

$$h_i(k) = (h(k) + f(i)) \% \text{TableSize}$$

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$$h_0(49) = (h(49) + f(0)) \% 10 = 9 \text{ (collision)}$$

$$h_1(49) = (h(49) + 1 * (7 - 49 \% 7)) \% 10 = 6$$

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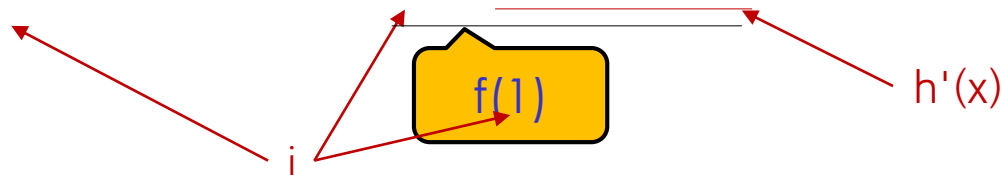
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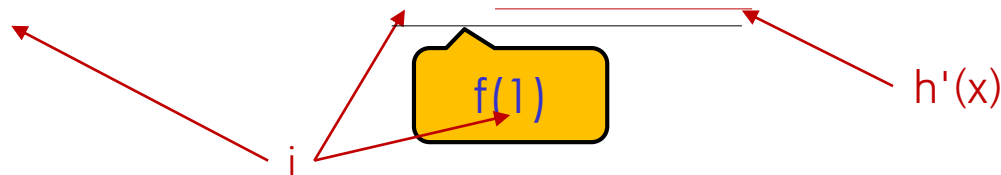
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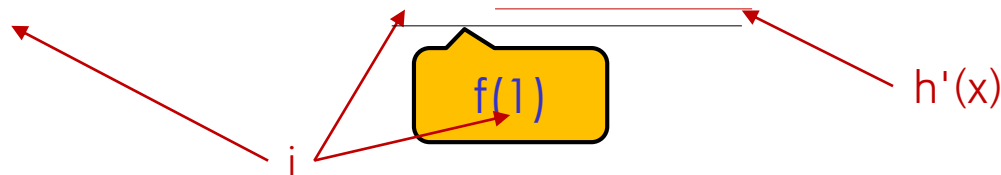
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$$h_2(49) = (h(49) + 2 * (7 - 49 \% 7)) \% 10 = 3$$

Collision Resolution Example - Double Hashing 이중해싱법

- Insert keys 43, 25 into the hash table below and find the probe sequence for each:
- Use $h(k) = k \% 13$ with $R = 7$.

0	1	2	3	4	5	6	7	8	9	10	11	12
26	None	54	94	17	31	None	None	None	None	None	None	17

$$h_0(43) = h(43) = 43 \% 13 = 4 \text{ (collision)}$$

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$$h_0(25) = h(25) = 25 \% 13 = 12 \text{ (collision)}$$

$$h_1(25) = (h(25) + 1 * (7 - 25 \% 7)) \% 13 = (12 + 3) \% 13 = 2$$

$$h_2(25) = (h(25) + 2 * (7 - 25 \% 7)) \% 13 = (12 + 6) \% 13 = 5$$

$$h_3(25) = (h(25) + 3 * (7 - 25 \% 7)) \% 13 = (12 + 9) \% 13 = 8, \text{ Probe sequence: } 12, 2, 5, 8$$

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0	1	2	3	4	5	6	7	8	9	10	11	12
26	None	54	94	17	31	None	None	25	None	43	None	17

Collision Resolution Exercise - Double Hashing 이중해싱법

Insert sequence: 89, 18, 49, 58, 69, 23

Empty Table

After 89

After 18

After 49

After 58

After 69

After 23

0

1

2

3

4

5

6

7

8

9

49

18

89

49

18

89

49

18

89

49

18

89

49

18

89

49

18

89

0

0

1

2

2

Unsucessful
no. of probes

$h(x) = x \% 10$

$h'(x) = R - (x \% R)$
R is prime number less than TableSize


$h_0(49) = (h(49)+f(0)) \% 10 = 9$ (collision)
 $h_1(49) = (h(49)+1*(7 - 49 \% 7)) \% 10 = 6$

$h_0(58) =$
 $h_1(58) =$

$h_0(69) =$
 $h_1(69) =$

$h_0(23) =$
 $h_1(23) =$
:

Collision Resolution Analysis - Double Hashing

- Imperative that **TableSize is prime**
 - e.g., insert 23 into previous table
- Empirical tests show **double hashing** close to random hashing.  Is it good or bad?
- Extra hash function takes extra time to compute.

Rehashing

- Rehashing is the reconstruction of the hash table:

0	6
1	15
2	
3	24
4	
5	
6	13



0	
1	
2	
3	
4	
5	
6	6
7	23
8	24
9	
10	
11	
12	
13	13
14	
15	15
16	

Rehashing

- Rehashing is the reconstruction of the hash table:
 - All the elements in the container **are rearranged** according to their hash value into the new set of buckets. This may alter the order of iteration of elements within the container.
- Increases the size of the hash table when load factor becomes "too high" (defined by a cutoff)
 - Anticipating that collisions would become higher
- Typically expand the table to **twice** its size (**but still prime**)
 - $\text{TableSize}_{\text{new}} = \text{nextprime}(2 * \text{TableSize}_{\text{old}})$
 - e.g., $2 \rightarrow 5$, $5 \rightarrow 11$, $11 \rightarrow 23$
- Need to **reinsert all existing elements** into new hash table

Rehashing Example

$$h(x) = x \% 7$$
$$\lambda = 0.57$$

0	6
1	15
2	
3	24
4	
5	
6	13

Insert 23



$$\lambda = 0.71$$

0	6
1	15
2	
3	24
4	
5	
6	13

TableSize = 7

$$\lambda_{max} = 0.6$$

Rehashing
since $\lambda > \lambda_{max}$



0	
1	
2	
3	
4	
5	
6	6
7	23
8	24
9	
10	
11	
12	
13	13
14	
15	15
16	

TableSize = 17
nextprime(TableSize * 2)

$$h(x) = x \% 17$$
$$\lambda = 0.29$$

Hashing Analysis

- The **load factor** (λ) of the hash table is the number of items in the table divided by the size of the table.
- If λ is small then keys are more likely to be mapped to slots where they belong and searching will be $O(1)$.
- If λ is large then collisions are more likely and more comparisons (is the slot available or not) are needed to find an empty slot.

Rehashing Analysis

- If the load factor goes high, the performance slows down significantly. In that case the easiest solution is to copy the entire hash table into a larger table. This process is called rehashing.
- When to rehash
 - For separate chaining, the load factor should not exceed **0.75**.
For open addressing, the load factor should not exceed **0.5**.
- Rehashing a table is expensive (since elements must be inserted using the new hash function) - do only occasionally, e.g. double size of table each time, but make sure that the size is a prime number.

Rehashing - Exercise

- Rehash the following table into a new hash table below using the hash function: Use $\text{hash}(\text{key}) = \text{key} \% 13$ and quadratic probing to resolve the collisions. Show your computation, collision and resolution. Compute the load factors before and after rehashing .

0	1	2	3	4	5	6
56	43	30	None	None	26	13

0	1	2	3	4	5	6	7	8	9	10	11	12

Hashing Applications

- Symbol table in compilers
- Accessing tree or graph nodes by name
 - e.g., city names in Google maps
- Maintaining a transposition table in games
 - Remember previous game situations and the move taken (avoid re-computation)
- Dictionary lookups
 - Spelling checkers
 - Natural language understanding (word sense)
- Heavily used in text processing languages
 - e.g., Perl, Python, etc.

Summary

- The hash table size uses a prime number in general.
 - The table size is larger than number of inputs (to maintain $\lambda \ll 1.0$)
 - It helps its performance and prevents it from rehashing.
- The collision cannot be avoided.
 - Collision resolution strategies are required.
 - There are some trade-offs between chaining vs. probing
 - Collision chances decrease in this order:
linear probing \rightarrow quadratic probing \rightarrow double hashing
- Rehashing is recommended when the load factor λ exceeds 0.5 in general.