

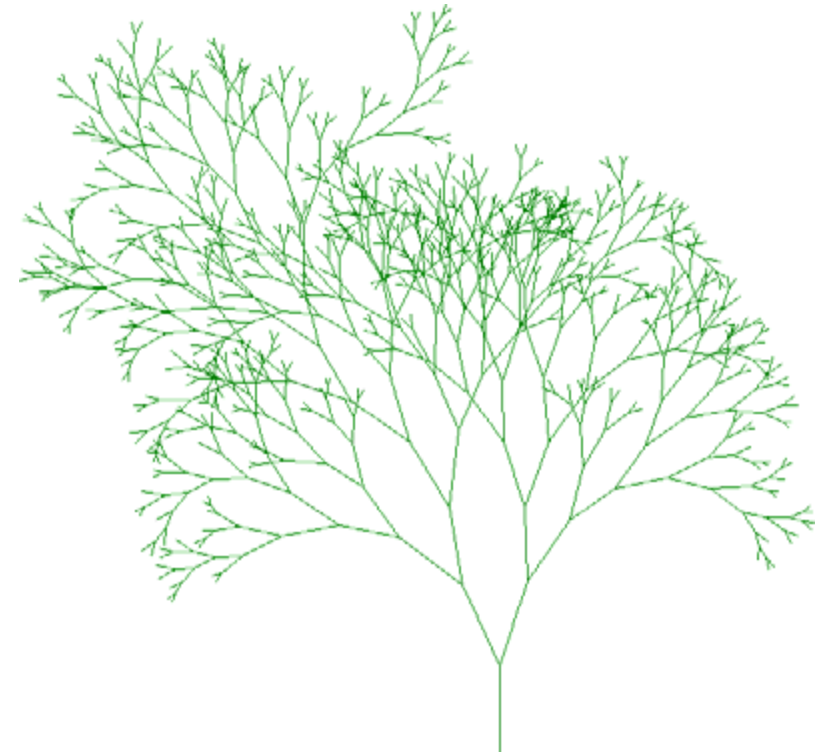
Data Structures in Python

Chapter 4

1. Recursion Concepts
2. Recursion Stack and Memoization
3. Recursive Algorithms
- 4. Recursive Graphics**

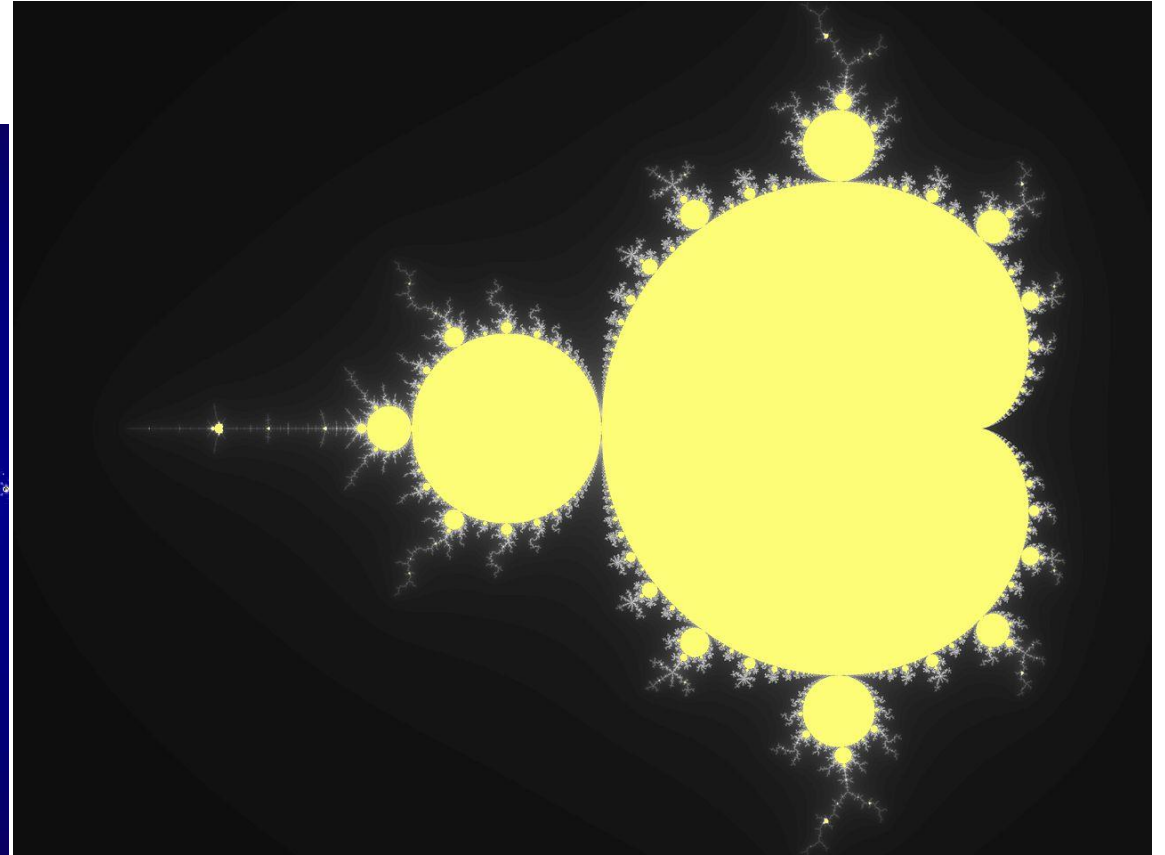
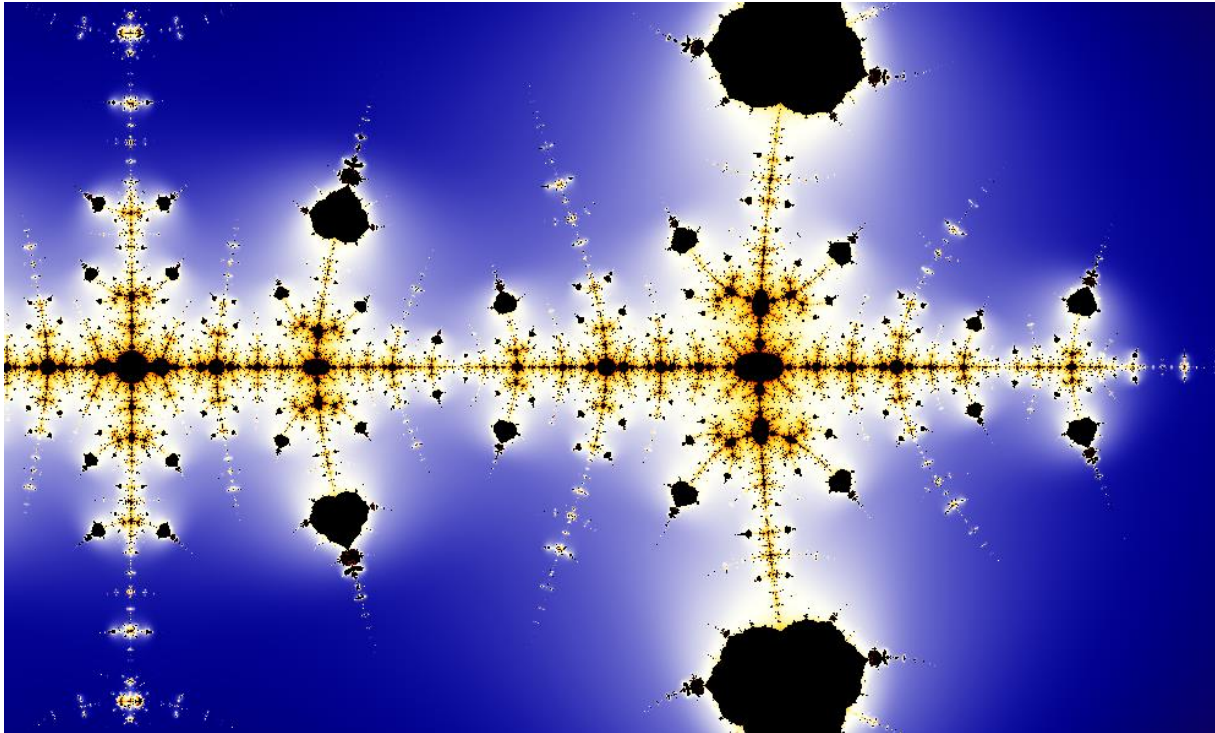
Agenda & Readings

- Introduction
- Using the Python Turtle
- Drawing the Koch Snowflake
- Using recursive drawing



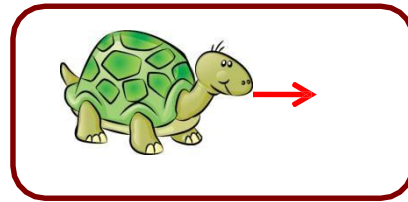
Introduction - Self-similarity

- A fractal is a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole.
 - This a property is called self-similarity (자기 유사성).

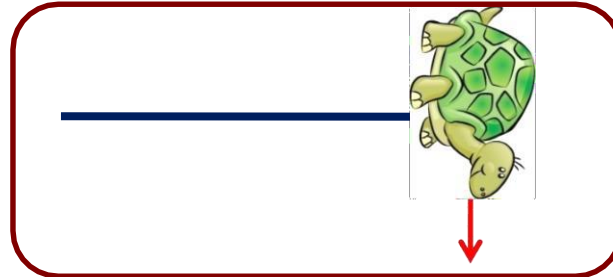


Python Turtle Class

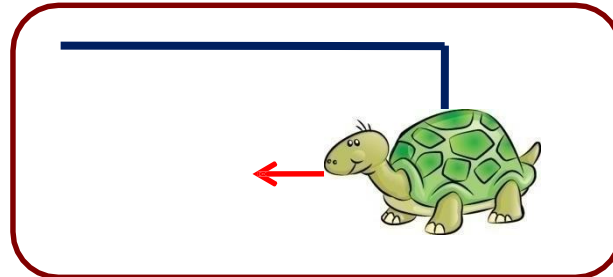
- Can be drawn using a “Turtle”
 - Named after Logo programming language
 - Pen location used to draw on the screen
 - Commands
 - Pen up
 - Pen down
 - Rotate ...



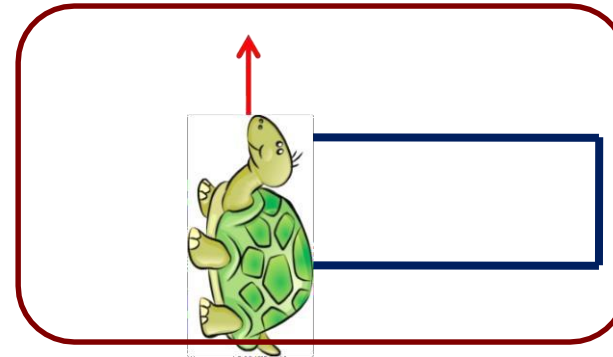
1. draw a line



2. rotate 90
3. draw a line



4. rotate 90
5. draw a line



6. rotate 90
7. draw a line

Python Turtle Class

- Steps:

- Import the turtle module which defines the Turtle and the Screen types. `import turtle`

- Create and open a window.

```
win = turtle.Screen()
```

- The window contains a canvas, which is the area inside the window on which the turtle draws.
- Create a turtle object which can move forward, backwards, turn left, turn right, the turtle can have its tail up/down.

```
tom = turtle.Turtle()
```

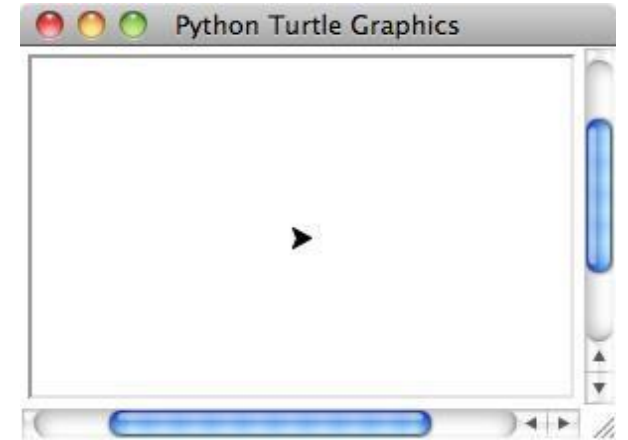
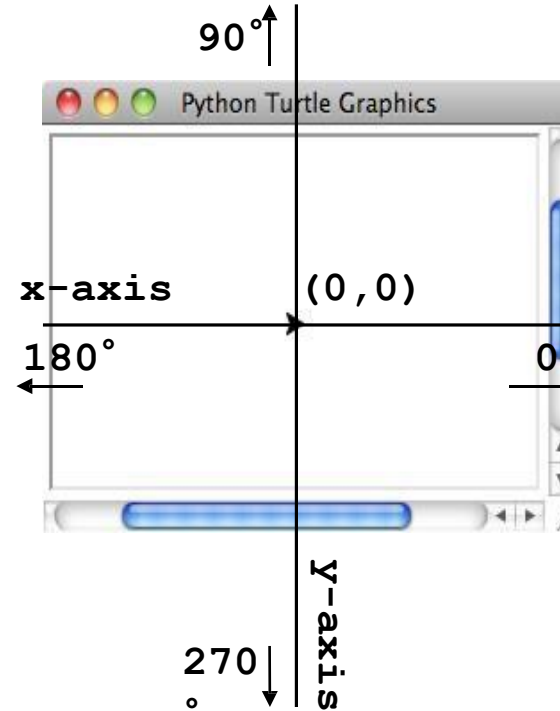
- If the tail is down, the turtle draws as it moves.
 - The width and color of the turtle tail can be changed.
- When the user clicks somewhere in the window, the turtle window closes and execution of the Python program stops.

```
turtle.exitonclick()
```

Python Turtle Class

- Instantiate a Turtle object:
- The turtle appears as an icon
 - Initial position: (0, 0)
 - Initial direction: East (0°)
 - Color: black
 - Line width: 1
 - pixel Pen: down (ready to draw)

```
tom = turtle.Turtle()
```



Python Turtle Class - Methods

- `forward(distance)` - move the turtle forward
- `backward(distance)` - move the turtle backwards
- `right(angle)` - turn the turtle clockwise
- `left(angle)` - turn the turtle anti-clockwise
- `up()` - puts the turtle tail/pen up, i.e., no drawing
- `down()` - puts the turtle tail/pen down, i.e., drawing
- `pencolor(color_name)` - changes the color of the turtle's tail
- `heading()` - returns the direction in which the turtle is pointing
- `setheading(angle)` - set the direction in which the turtle is pointing
- `position()` - returns the position of the turtle
- `goto(x, y)` - moves the turtle to position x, y
- `speed(number)` - set the speed of the turtle movement

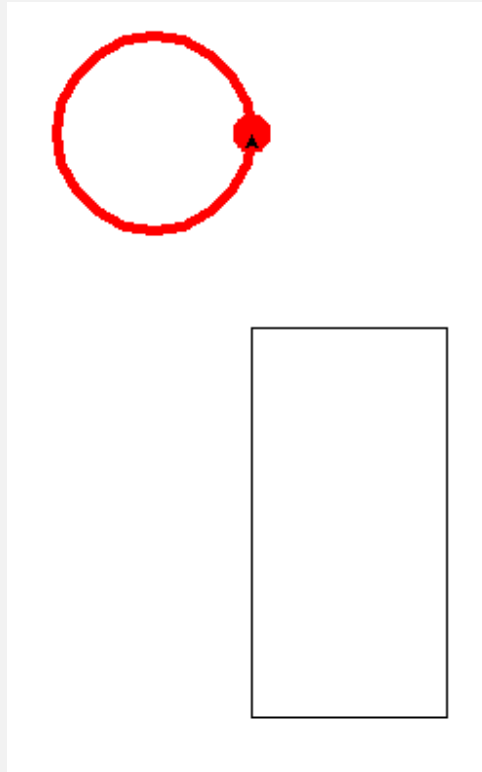
Python Turtle - Drawing Examples

```
import turtle
tom = turtle.Turtle()
tom.forward(100)
tom.right(90)
tom.forward(200)
tom.right(90)
tom.forward(100)
tom.right(90)
tom.forward(200)

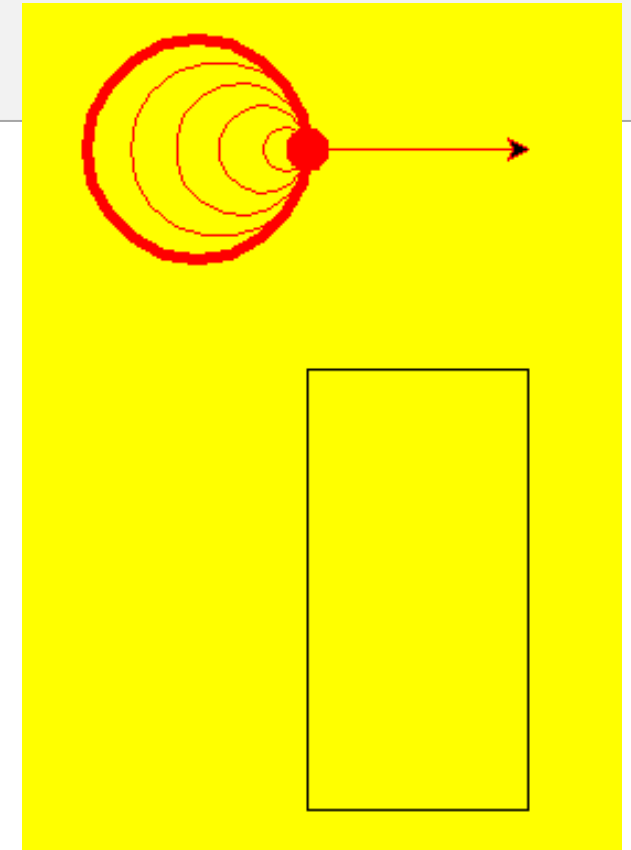
tom.up()      # pen up
tom.forward(100)

tom.down()    # pen down
tom.pensize(5)
tom.pencolor('red')
tom.circle(50) # radius=50

tom.dot(20)
```

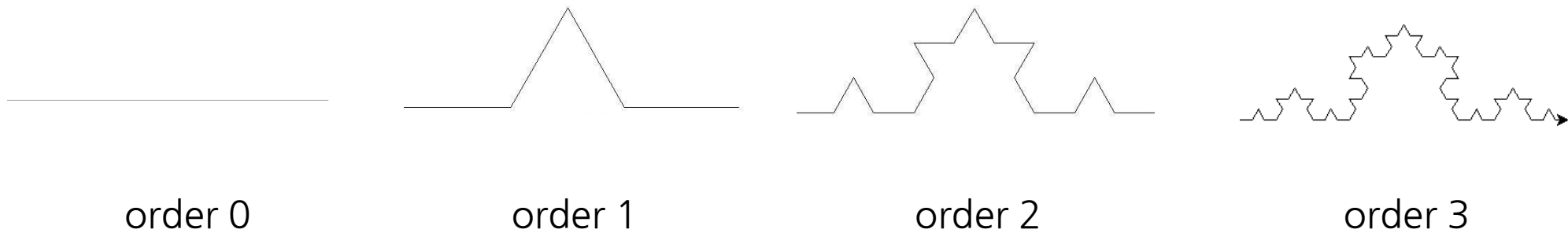


```
turtle.bgcolor('yellow')
tom.pensize(1)
n = 10
while n < 50:
    tom.circle(n)
    n += 10
tom.rt(90)
tom.fd(100)
```



Recursive graphics - Koch curves

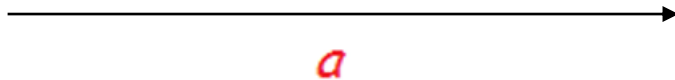
- A Koch curve of order 0 is a straight line.
- To form a Koch curve of order n , draw a Koch curve order $n - 1$, turn left 60 degrees, draw a second Koch curve of order $n - 1$, turn right 120 degrees (left - 120 degrees), draw a third order $n - 1$.
- These recursive instructions lead immediately to turn into a turtle code.
- These recursive schemes have proved useful in modeling self-similar patterns found in nature, such as snowflakes, trees, etc.



Recursive graphics - Koch curves

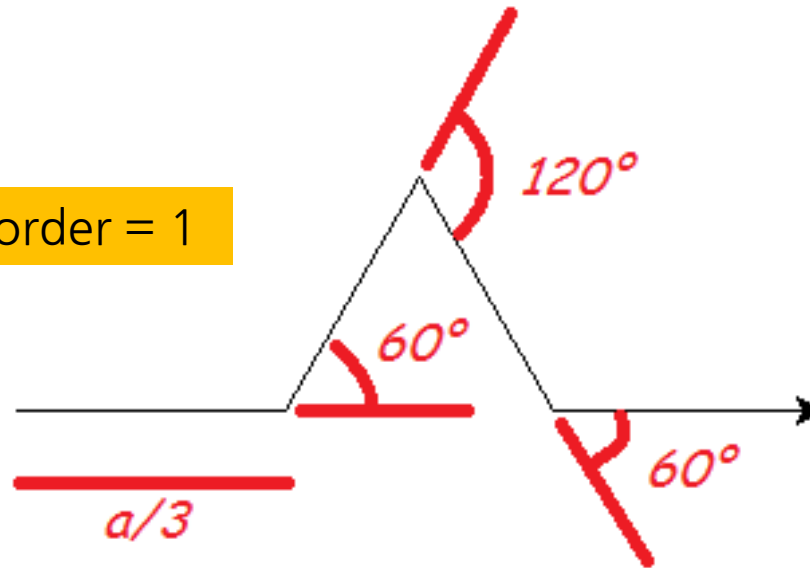
- We're going to define a function that either draws a line with a kink in it or draws a straight line the same length.
 - Step 1 - Draw the basic unit
 - Step 2 - Coding using a function
 - Step 3 - Coding using a recursion

order = 0



forward(a)

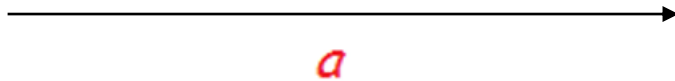
order = 1



Recursive graphics - Koch curves

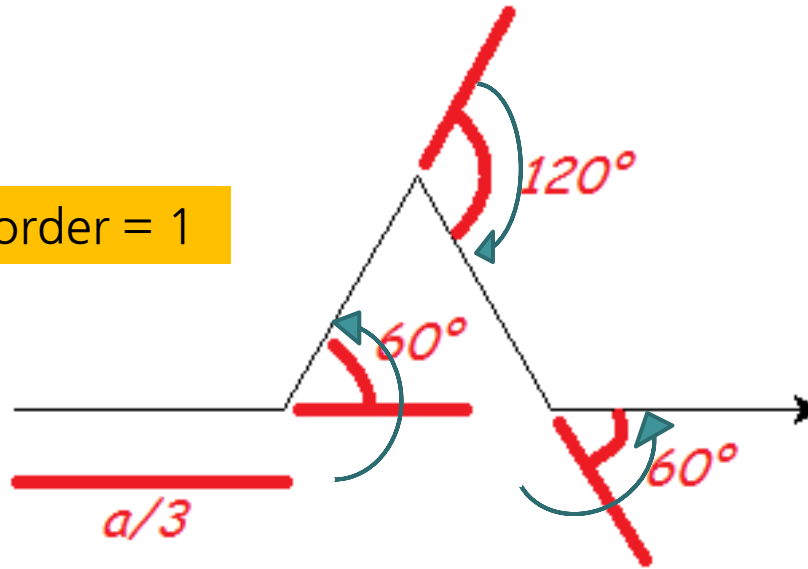
- We're going to define a function that either draws a line with a kink in it or draws a straight line the same length.
 - Step 1 - Draw the basic unit
 - Step 2 - Coding using a function
 - Step 3 - Coding using a recursion

order = 0



```
forward(a)
```

order = 1



```
from turtle import *  
a = 100
```

```
forward(a/3)  
left(60)
```

```
forward(a/3)  
right(120)
```

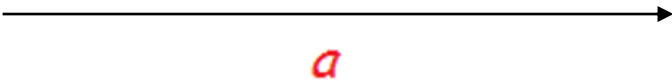
```
forward(a/3)  
left(60)
```

```
forward(a/3)
```

Recursive graphics - Koch curves

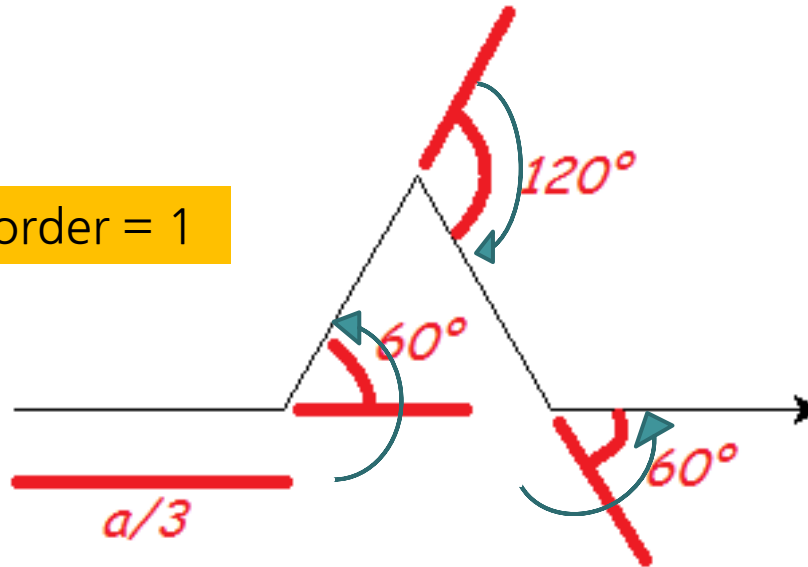
- Simplify the "order=1" case using for loop:

order = 0



```
forward(a)
```

order = 1



```
from turtle import *  
a = 100  
  
for t in [60, -120, 60, 0]:  
    forward(a/3)  
    left(t)
```



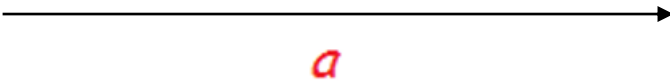
```
from turtle import *  
a = 100  
  
forward(a/3)  
left(60)  
  
forward(a/3)  
right(120)  
  
forward(a/3)  
left(60)  
  
forward(a/3)
```

Recursive graphics - Koch curves

- Simplify the "order=1 & 2" cases using a function:

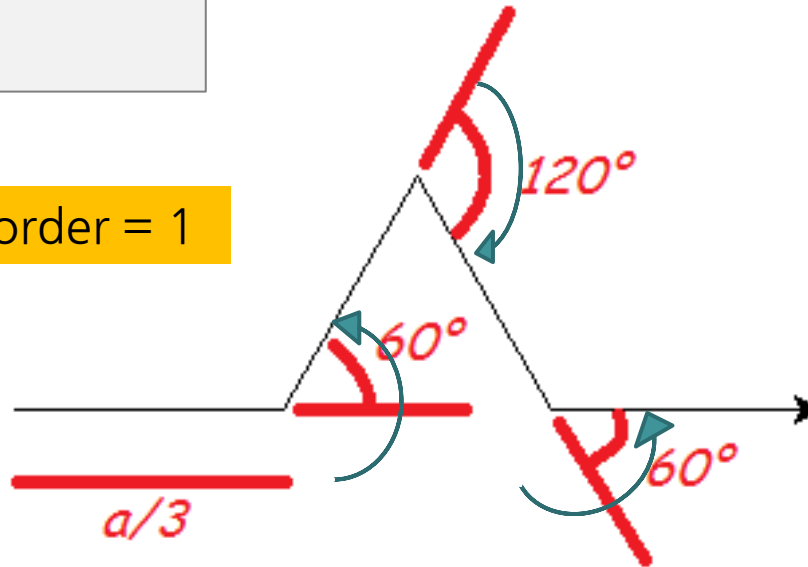
```
from turtle import *  
  
def koch(a, order):  
    if order > 0:  
        for t in [60, -120, 60, 0]:  
            forward(a/3)  
            left(t)  
    else:  
        forward(a)
```

order = 0



forward(a)

order = 1



```
from turtle import *  
a = 100  
  
for t in [60, -120, 60, 0]:  
    forward(a/3)  
    left(t)
```



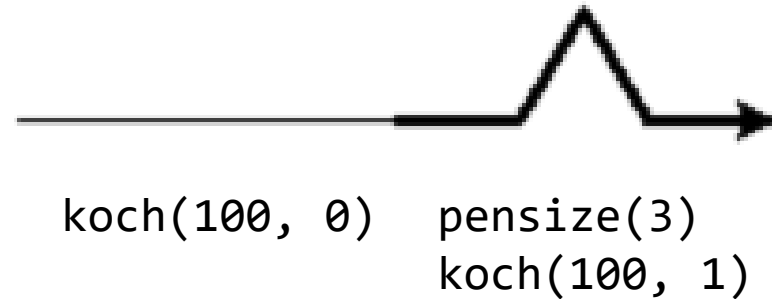
```
from turtle import *  
a = 100  
  
forward(a/3)  
left(60)  
  
forward(a/3)  
right(120)  
  
forward(a/3)  
left(60)  
  
forward(a/3)
```

Recursive graphics - Koch curves

- Sample Run:

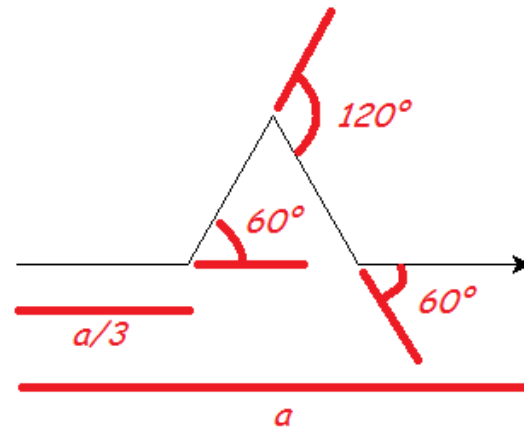
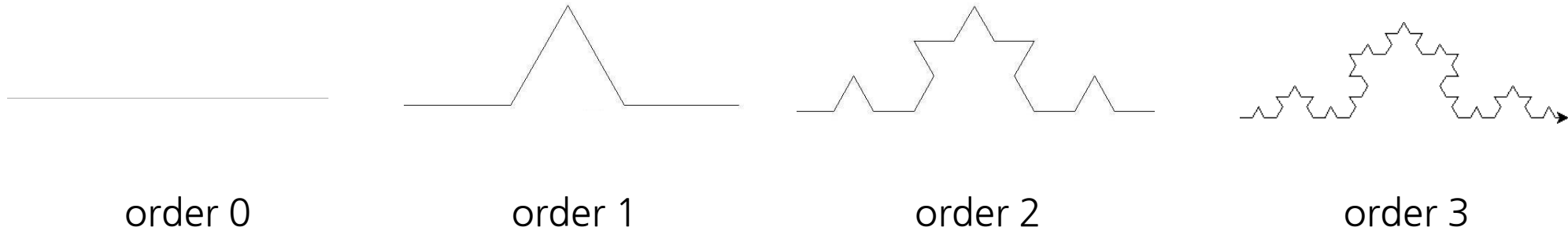
```
from turtle import *  
  
def koch(a, order):  
    if order > 0:  
        for t in [60, -120, 60, 0]:  
            forward(a/3)  
            left(t)  
    else:  
        forward(a)
```

```
# Sample Run  
koch(100, 0)  
pensize(3)  
koch(100, 1)
```



Recursive graphics - Koch curves

- Idea: recursively applying a simple rule to each of the triangles' sides
- Examples:
 - The pattern:



Each of the 4 lines is $a/3$ long, and because of the choice of angles, the distance between the ends of the line is a .

Recursive graphics - Koch curves

- **Idea:** recursively applying a simple rule to each of the triangles' sides
 - Replace the `forward(a/3)` with another call to `koch`, to draw a kinked line of the same length.
 - The variable `order` goes down by one each time, and when it hits zero, we just draw a line. Change the function **koch**, just a little:

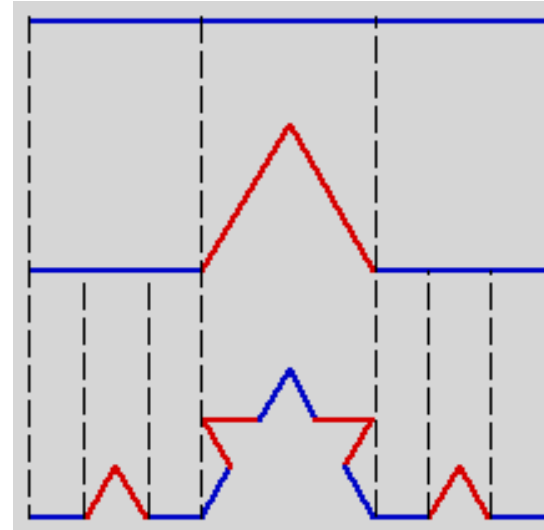
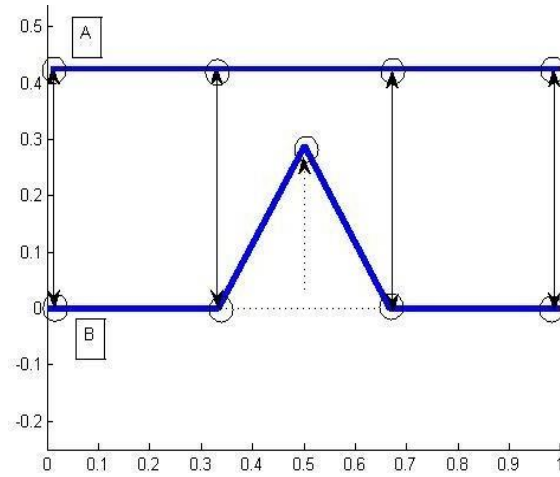
```
from turtle import *  
  
def koch(a, order):  
    if order > 0:  
        for t in [60, -120, 60, 0]:  
            koch(a/3, order - 1)  
            left(t)  
    else:  
        forward(a)
```

```
# Sample Run  
reset()  
clear()  
koch(300, 2)
```



Recursive graphics - Koch curves

- Idea: recursively applying a simple rule to each of the triangles' sides
- Examples:

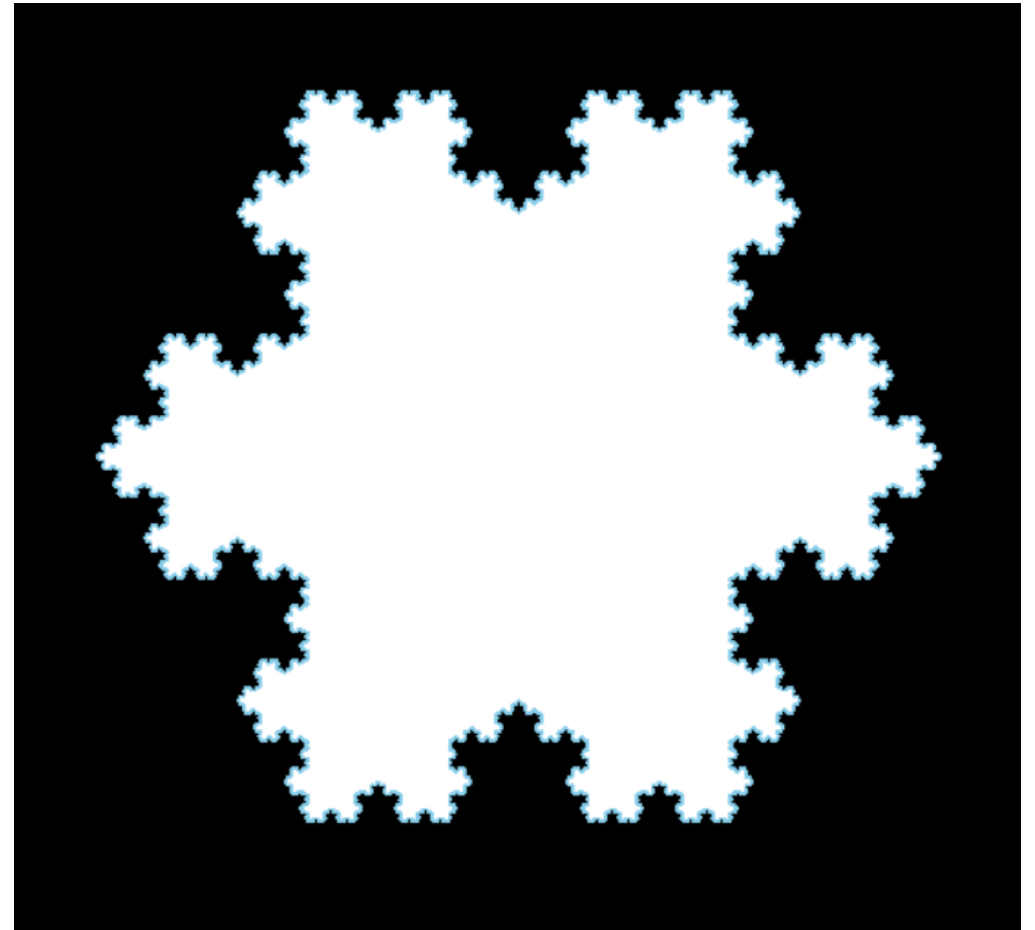


- The pattern:
 - Let us set the side length = $SIDE$.
 - Cut the $SIDE$ into 3 equal parts $SIDE/3$.
 - Replace the center part with 2 sides of length $SIDE/3$, such that it forms a spike.
 - Repeat the process for each of the 4 sides, until the length of each side is smaller than a given value.

Python Turtle - The Koch Snowflake using recursion

- The **algorithm** for drawing a Koch snowflake with n sides is:
 - for each side:
 - draw a Koch curve of the appropriate length and level
 - turn right $360.0/n$ degrees
- This is an example of a Koch snowflake made with three Koch curves.

```
for i in range(3):  
    koch(size, order)  
    right(120)
```



Python Turtle - The Koch Snowflake using recursion

- Code:

```
from turtle import *

def koch(a, order):
    if order > 0:
        for t in [60, -120, 60, 0]:
            koch(a/3, order - 1)
            left(t)
    else:
        forward(a)
```

```
reset()

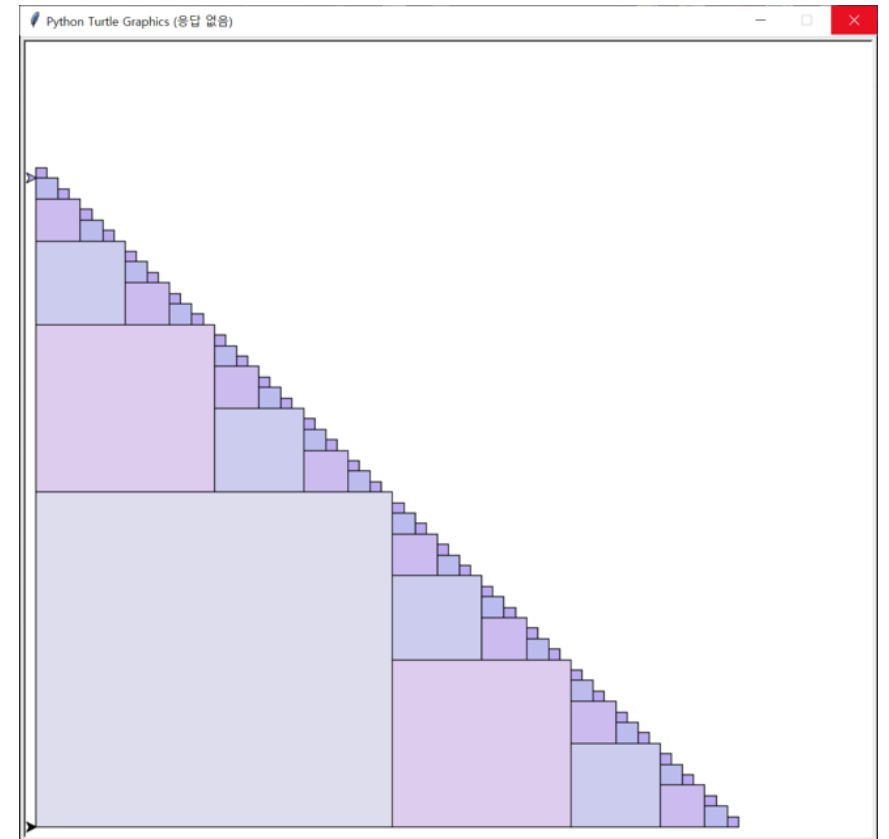
# Choose colors and size
color("sky blue", "white")
bgcolor("black")
size = 400
order = 7

# Ensure snowflake is centered
penup()
backward(size/1.732)
left(30)
pendown()

begin_fill() # Three Koch curves
for i in range(3):
    koch(size, order)
    right(120)
end_fill()
update()      # Make the last parts appear
```

Exercise: Stacking boxes

- Draw a stack of boxes using the recursion and turtle.
 - These boxes consists of a large box in the lower left, a smaller box just above it, and a smaller box to the right of it.



Exercise: Stacking boxes

- For this exercise, only two functions shown below will do the job.

```
from turtle import *

def draw_box(x, y, side, color):
    """x, y - the origin of the box to draw at lower left corner
       side - the length of the side of the box
       color - fill color for the box """

    # your code here

def stack_boxes(x, y, side, degree):
    """x, y - the origin of the box to draw at lower left corner
       side - the length of the side of the box
       degree - number of boxes to stack side or top.
               assume the max degree is limited to 8. """
    colormap = ["#unused", "#BBAAEE", "#BBBBEE", "#CCBBEE", \
               "#CCCCEE", "#DDCC EE", "#DDDDEE", "#EEDDEE", "#EEEEEE"]

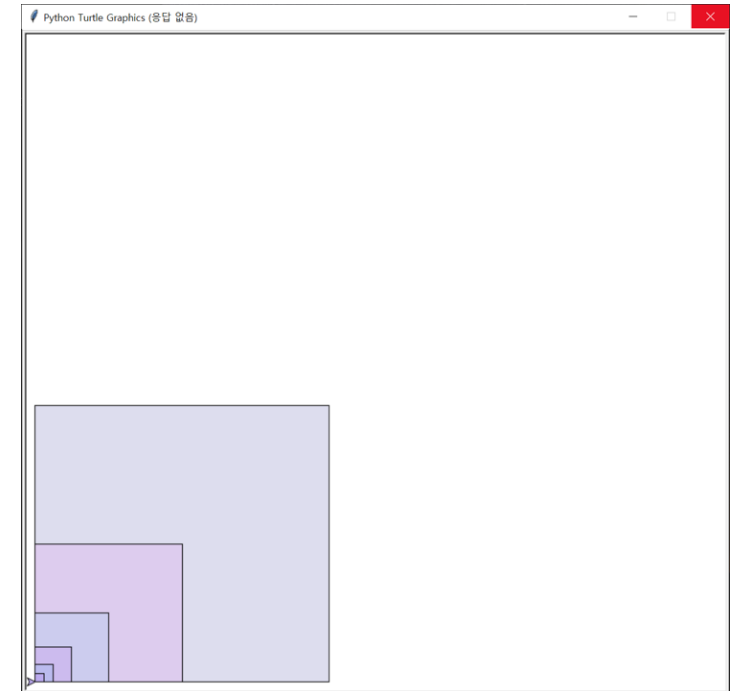
    # your code here

    return
```

```
if __name__ == '__main__':
    Turtle()
    setworldcoordinates(0, 0, 600, 600)
    stack_boxes(0, 0, 256, 6)
```

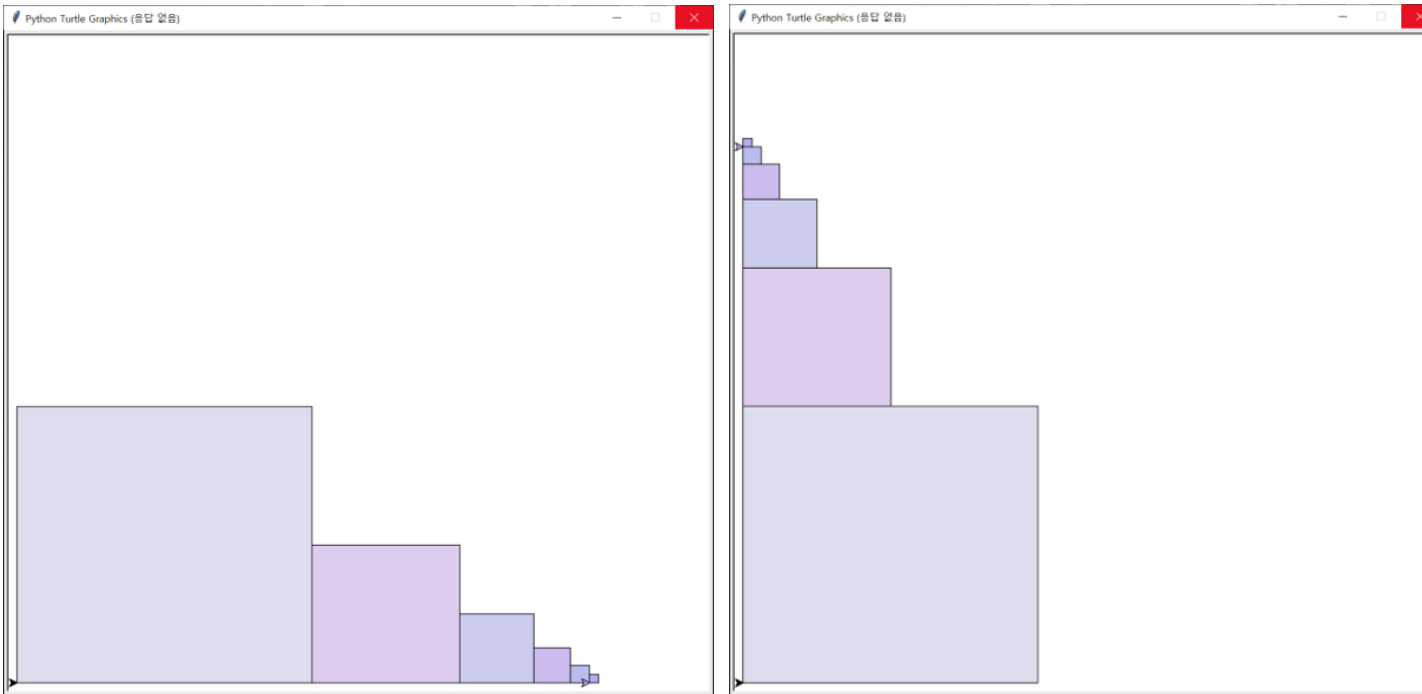
Exercise: Stacking boxes

- Step 1: Draw boxes at the origin in two ways.
 - **Iterative method:** In `stack_boxes()`, invoke `draw_boxes()` repeatedly such that all boxes are drawn at the origin.
 - **Recursive method:** In `stack_boxes()`, invoke `draw_boxes()` once, but `stack_boxes()` invoked recursively makes `draw_boxes()` called many times such that all boxes are drawn at the origin.
- The bottom left corner of the window is set to the origin(0, 0).
- The degree determines the number of boxes to draw.
- The size of boxes are set to a power of 2.
- The maximum degree is limited to 8 for easy coloring.
- `draw_box()` starts and finishes its drawing at the origin given.
- `fillcolor()` is set by `colormap[degree]`.



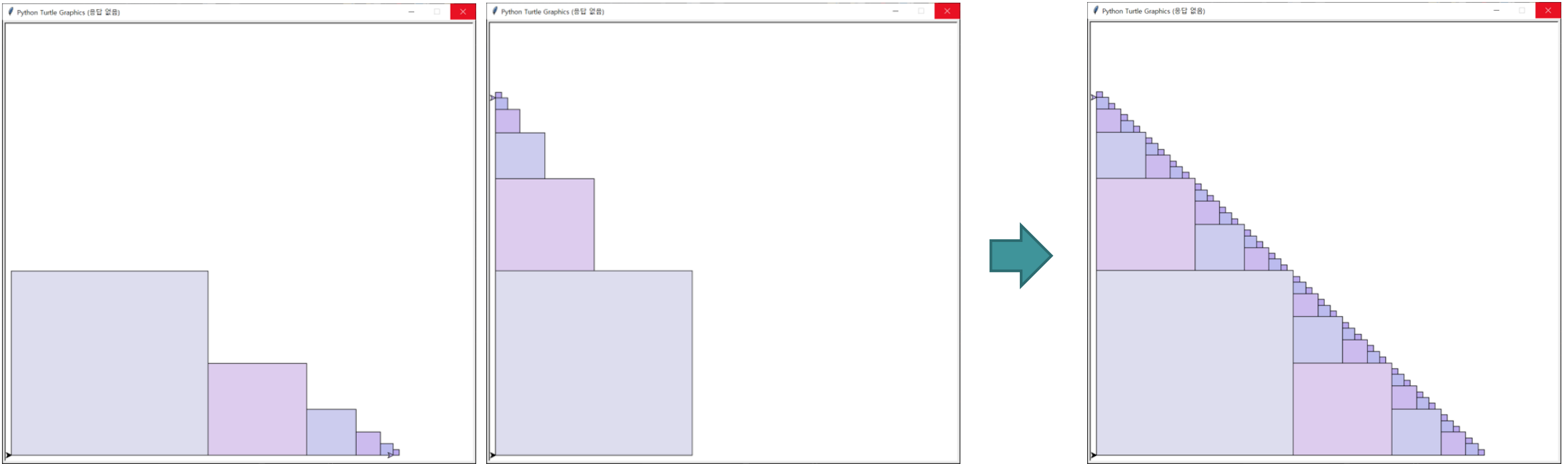
Exercise: Stacking boxes

- Step 2: Draw a stack of boxes at the bottom first and at the top next, recursively.
 - It is a matter of setting a new origin of the boxes to draw.



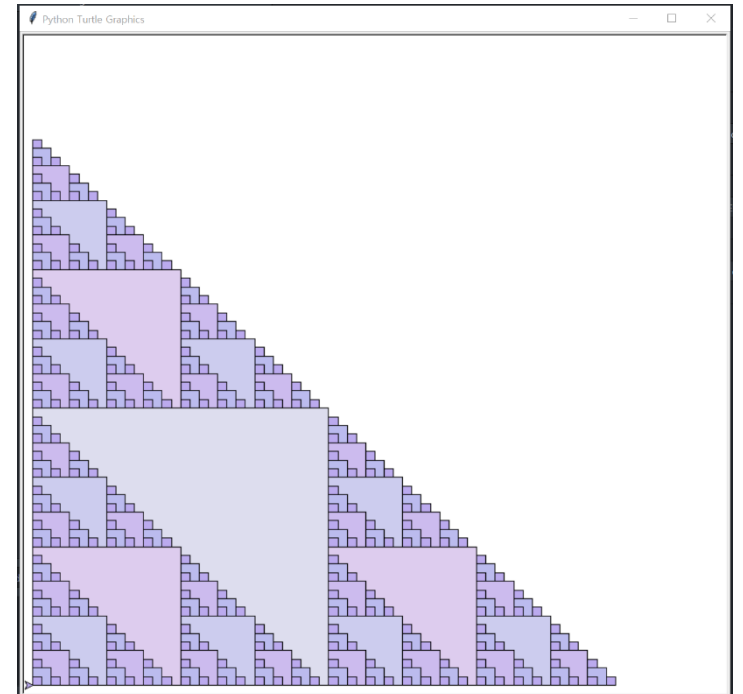
Exercise: Stacking boxes

- Step 3: Invoke `stack_boxes()` twice such that boxes, one to the right and one for at the top, can be recursively drawn, by setting the new origin.
 - Stop if the degree is less than 1.



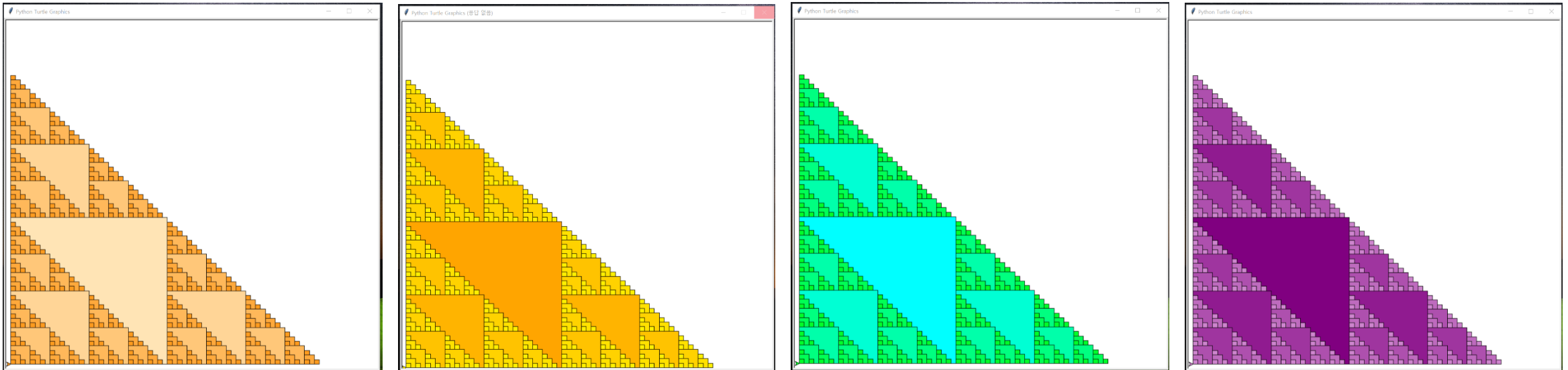
Exercise: Stacking boxes

- Step 4: Pile up more boxes in boxes.
 - Pile up more boxes inside boxes by invoking one more recursive call.



Exercise: Stacking boxes

- Step 5: Coloring boxes.
 - We want to replace the colormap which is hard-coded using a fixed (magic) number.
 - Before invoking `stack_boxes()`, make a colormap based on degree using `color_fader()`. `color_fader()` returns a color between two colors mixed (0 to 1 ratio).
 - Pass this new colormap as an additional argument of `stack_boxes()`.



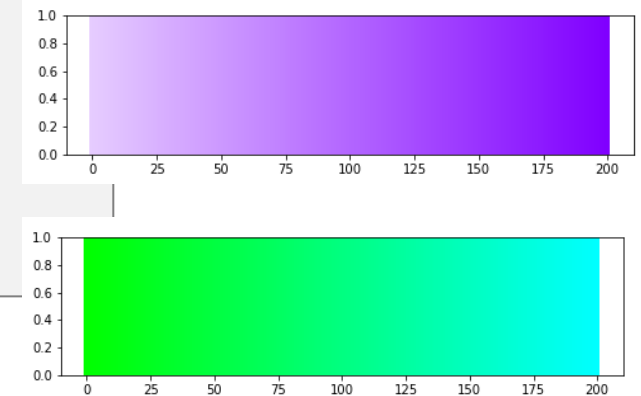
Exercise: Stacking boxes

- Step 5: Coloring boxes.

```
import matplotlib as mpl
import matplotlib.pyplot as plt
import numpy

def color_fader(c1,c2,mix=0):
    #fade (linear interpolate) from color c1 (at mix=0) to c2 (mix=1)
    c1=numpy.array(mpl.colors.to_rgb(c1))
    c2=numpy.array(mpl.colors.to_rgb(c2))
    return mpl.colors.to_hex((1-mix)*c1 + mix*c2)

if __name__ == '__main__':
    c1 = '#e6ccff'      #light purple      c1 = 'lime'
    c2 = '#8000ff'      #dark purple       c2 = 'cyan'
    n=200
    fig, ax = plt.subplots(figsize=(8, 2))
    for x in range(n+1):
        ax.axvline(x, color=color_fader(c1,c2,x/n), linewidth=4)
    plt.show()
```



Summary

- We visually experienced how the recursion works through recursive graphics.
- All good recursion must come to an end. Sooner or later method must NOT call itself recursively. It must have the base case(s).
- Recursion is a powerful tool!

