Data Structures in Python

1. **Hash Table**
2. Collision Resolution
3. Double Hashing & Rehashing
4. HashMap Coding

Agenda & Readings

- Agenda
  - Hashing
  - Hash Table
  - Hash Function
- Reference:
  - Problem Solving with Algorithms and Data Structures
  - Chapter 5 - Hashing

Overview

- Hashing or Hash Table Data Structure:

- Data structures so far

Array of size n	unsorted list	sorted array	Trees BST – average AVL – worst	Heap, Priority Queue	**Hashing**
`insert`	`find+O(1)`	`O(n)`	`O(log n)`	`O(log n)`	
`find`	`O(n)`	`O(log n)`	`O(log n)`	`O(log n)`	
`remove`	`find+O(1)`	`O(n)`	`O(log n)`	`O(log n)`	

Overview

- Hashing or Hash Table Data Structure:
supports insertion, deletion and search in average case constant time **O(1).**

- Data structures so far

Array of size n	unsorted list	sorted array	Trees BST – average AVL – worst	Heap, Priority Queue	**Hashing**
`insert`	`find+O(1)`	`O(n)`	`O(log n)`	`O(log n)`	*O(1)*
`find`	`O(n)`	`O(log n)`	`O(log n)`	`O(log n)`	*O(1)*
`remove`	`find+O(1)`	`O(n)`	`O(log n)`	`O(log n)`	*O(1)*

Overview

- Hashing or Hash Table Data Structure:
  supports insertion, deletion and search in average case constant time **O(1).**

- **Hash table**
  - It is data structure that stores **key-value pairs**.
  - The key is sent to a **hash function** that performs arithmetic operations on it.
  - The result is called **hash value** that is the **index of the key-value pa**ir in the **hash table**.

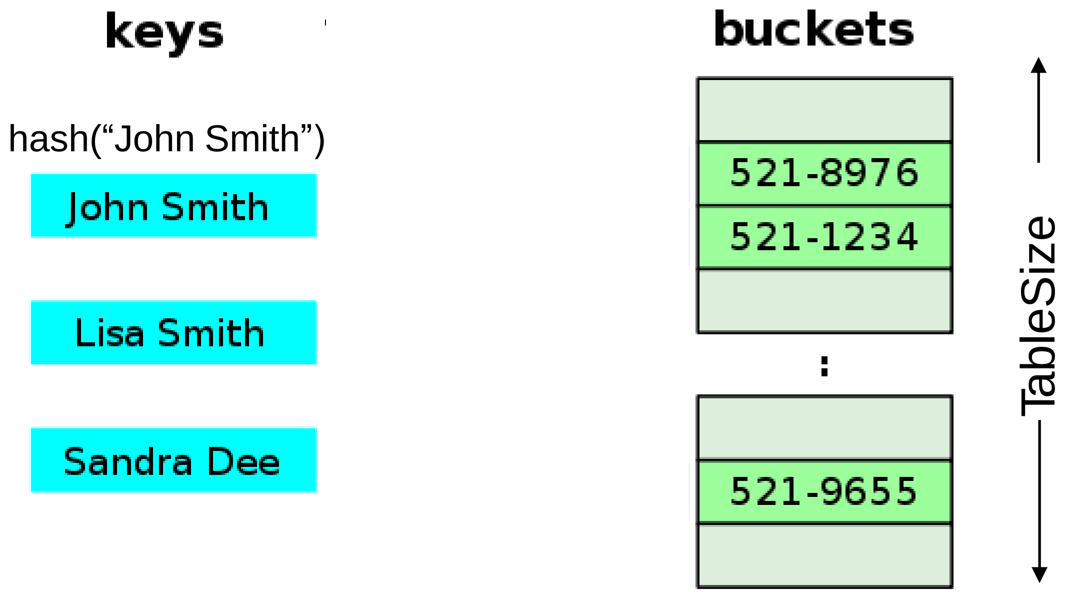
- **Hash function**
  - hash(key) → integer value
  - hash("string key") → integer value

Hash Table

- Hash table is an array of fixed size elements

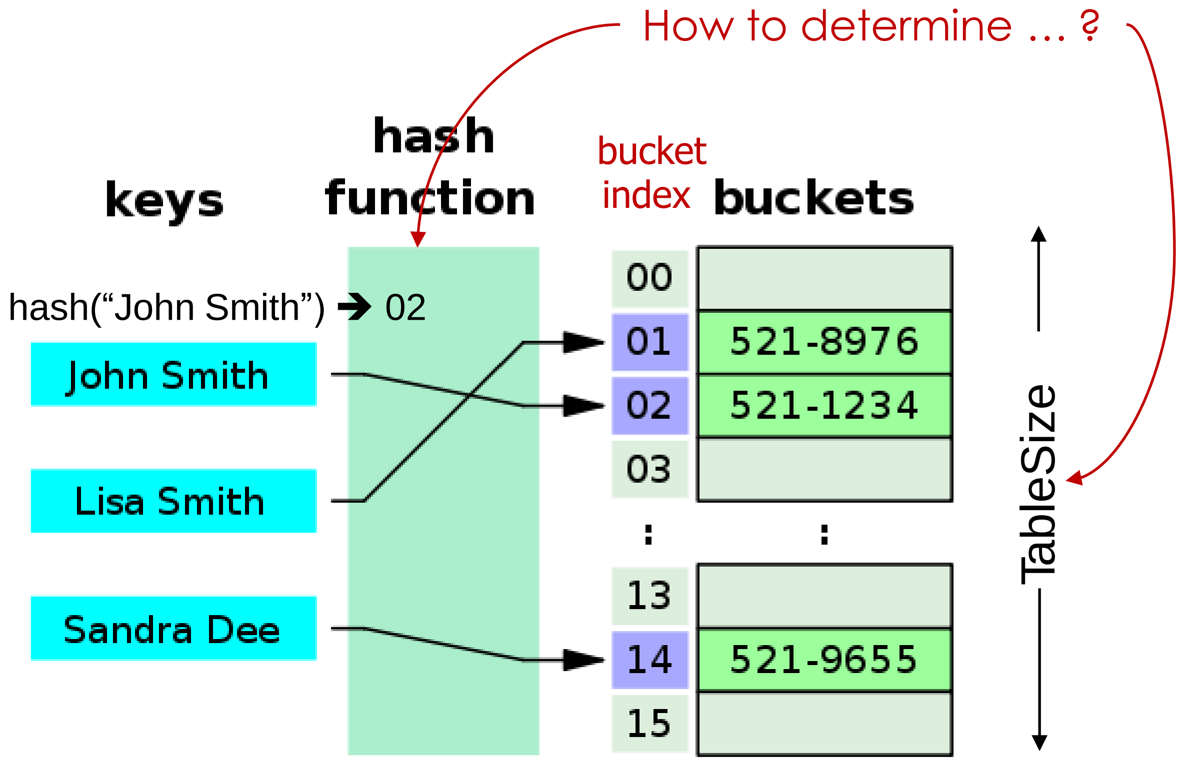
Let us suppose that there are one billion of names and numbers.
- Find, insert, and remove a number by a given name in **O(1)**.

Time Complexity



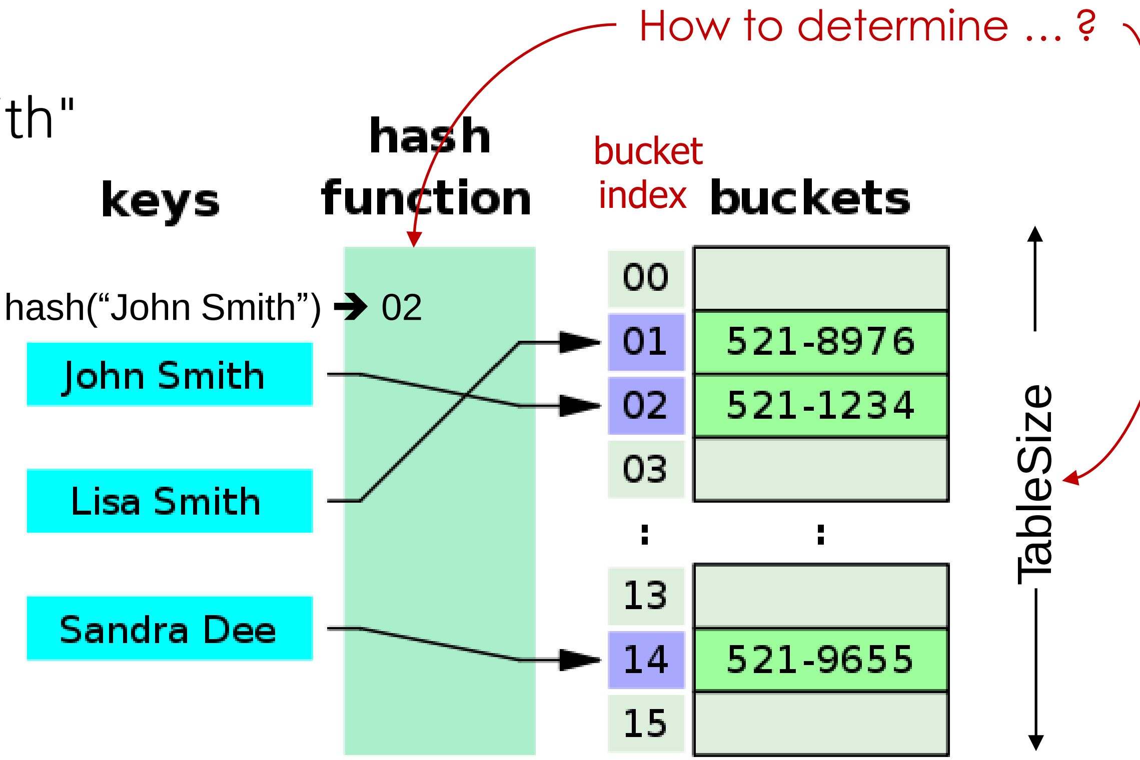
Hash Table

- Hash table is an array of fixed size elements
- Array elements indexed by a key mapped to a bucket index[0 .. TableSize-1]
- Mapping from key to index using hash(), hash function
  - e.g., hash("John Smith") → 02



Hash Table

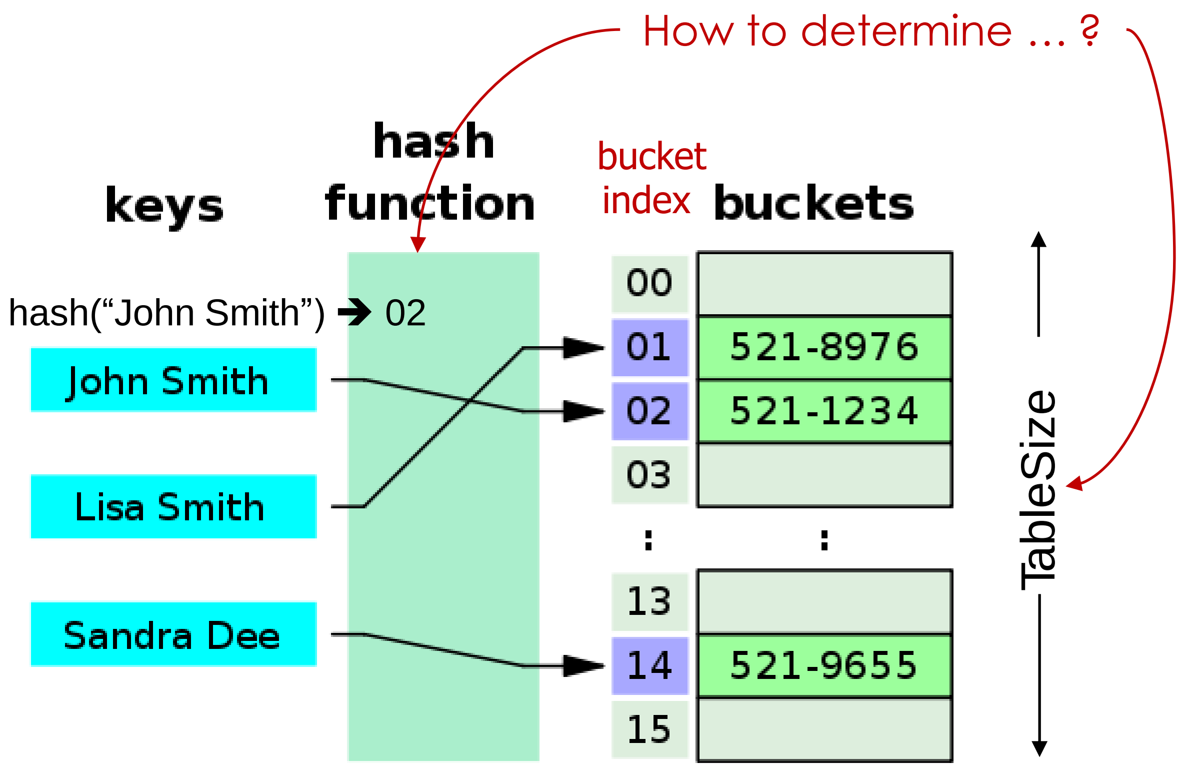
- insert
  - `HashTable[hash("John Smith")] = <"John Smith", 521-1234>`
- remove
  - `HashTable[hash("John Smith")] = None`
- find
  - `HashTable[hash("John Smith")]`
    returns the element hashed for "John Smith"



What happens
if hash("John Smith") == hash("Joe Blow")?
"Collision"

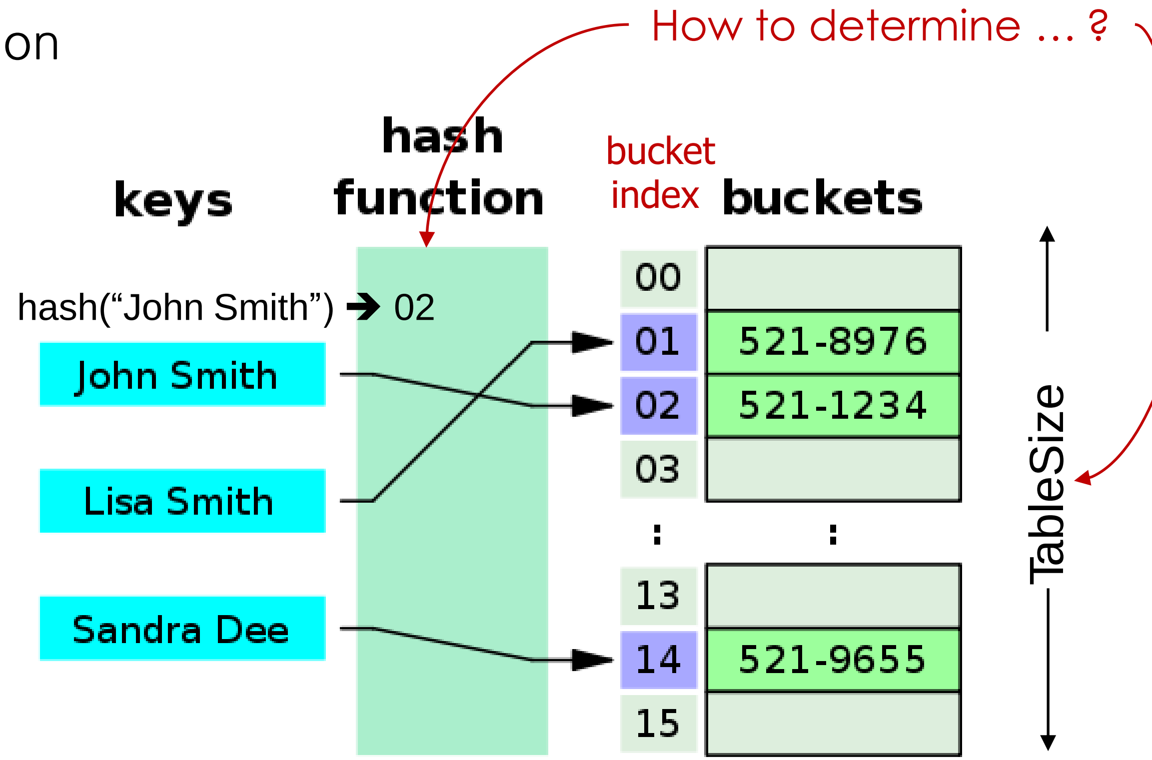
Hash Table

- Factors affecting Hash Table Design
  - Hash function
  - Table size - Usually fixed at the start
  - Collision handling schemes - Array or Linked List



Hash Function

- It maps an element's key into a valid hash table index
  - hash(key) → hash table index
- Note that this is (slightly) different from saying:
  - hash(string) → int
  - Because the key can be of any type
    - e.g., "hash(int) → int" is also a hash function



Hash Function Properties

- It maps an element's key into a valid hash table index
  - hash(key) → hash table index
- It maps key to integer
  - Constraint: Integer should be between **`[0, TableSize-1]`**
- A hash function can result in a many-to-one mapping (causing collision)
  - Collision occurs when hash function maps two or more keys to same array index
- Collisions **cannot** be avoided but its chances can be reduced using a "good" hash function

Hash Function - Effective use of table size

- Simple hash function (assume integer keys)
  - `hash(Key) = Key % TableSize`

- For random keys, `hash()` distributes keys evenly over table
  - What if `TableSize = 100` and keys are ALL multiples of 10?
  - Better if `TableSize` is a **prime number**

Hash Function Example: String Keys

- Using a very simple function to map strings to integers:
  - Add up character ASCII values (0-255) to produce integer keys
    - e.g., "abcd" = 97 + 98 + 99 + 100 = 394
    - hash("abcd") = 394 % TableSize
- Potential problems:
  - Anagrams will map to the same index
    - hash("abcd") = hash("dbac")
  - Small strings may not use all of table
    - strlen(s) * 255 < TableSize
  - Time proportional to length of the string

Hash Function Example: String Keys

- Another approach:
  - Treat first 3 characters of string as base-27 integer(26 letters plus space)
    - e.g., $Key = s[0] + (27^1 * s[1]) + (27^2 * s[2])$
    - Better than previous approach because …
- But, potential problems:
- Apple
  Apply
  Appointment
  Apricot

  → collision

Hash Function Example: String Keys

- Last approach:
  - Use all N characters of string as an N-digit and base-K number
  - Choose K to be prime number larger than number of different digits (characters)
    - i.e., K = 29, 31, 37
    - If L = Length of string S, then

$$hash(S) = \sum_{i=0}^{L-i} S[L-1-i] * 37^i \,\%\, TableSize \tag{1}$$

    - Use Horner's rule to compute hash(S).
    - Limit L for long strings

```
# a hash function for strings
hash(key, tablesize)
    code = 0
    for x in key:
        code = code * 37 + x
    code %= tablesize
    if code < 0: code += tablesize
    return code
```

  - Potential problems
    - Overflow
    - Larger runtime

Summary

- Using a hash table we can, on average (if table large enough and hash function suitable), insert, delete and search for items in constant time - **O(1).**
- The **hash function** is the mapping between an item and the slot where the item is stored.
- A **collision** occurs when an item is mapped to an occupied slot.
- A **perfect hash function** is able to map m items into a table of size m with no collisions. Perfect hash functions are hard to come by.
- Handling collisions systematically is required - **collision resolution.**