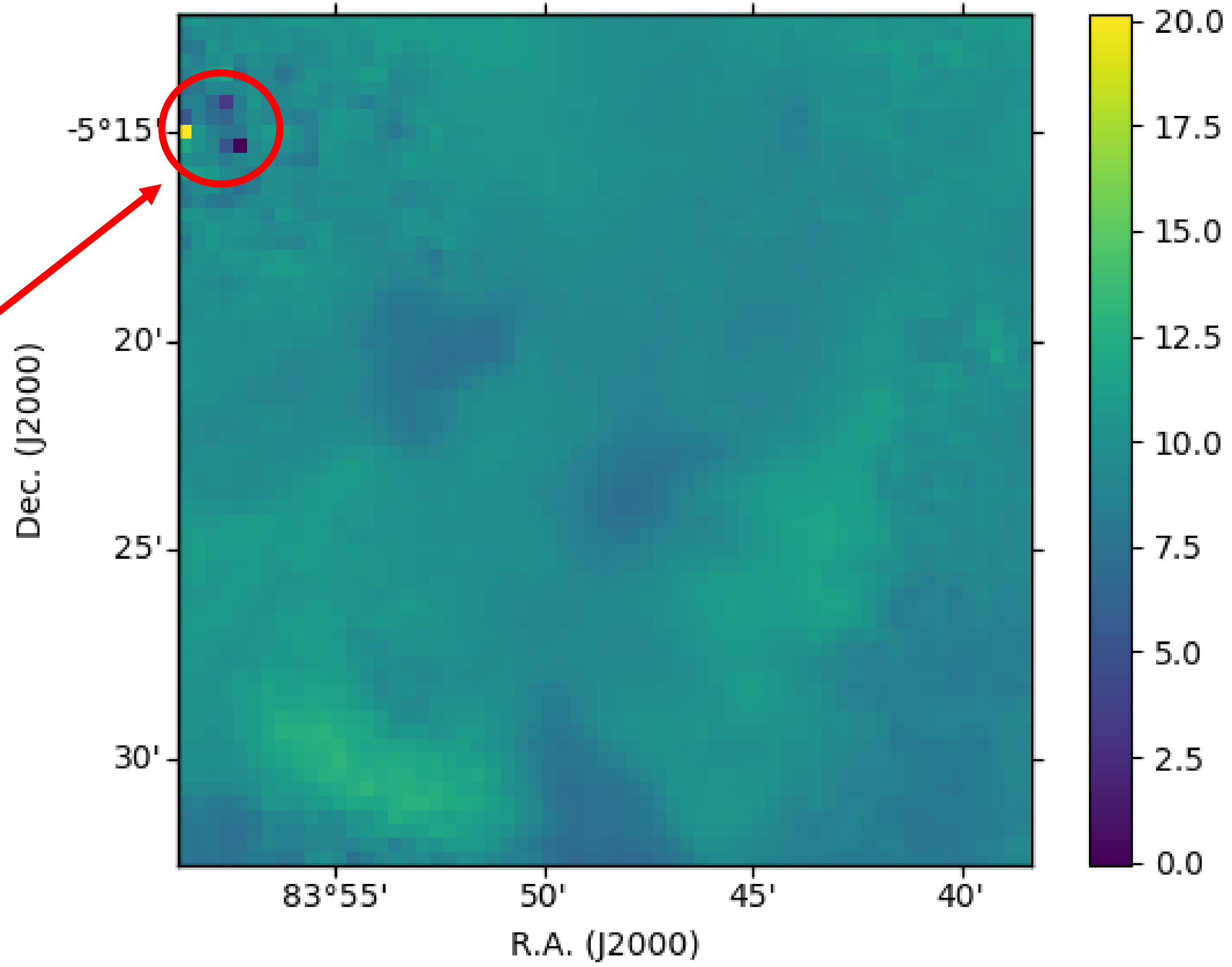
Single Dish Radio Observation

Derive reliable moment 1 and moment 2 vales
from the spectral map data

Exercise: intensity weighted velocity map

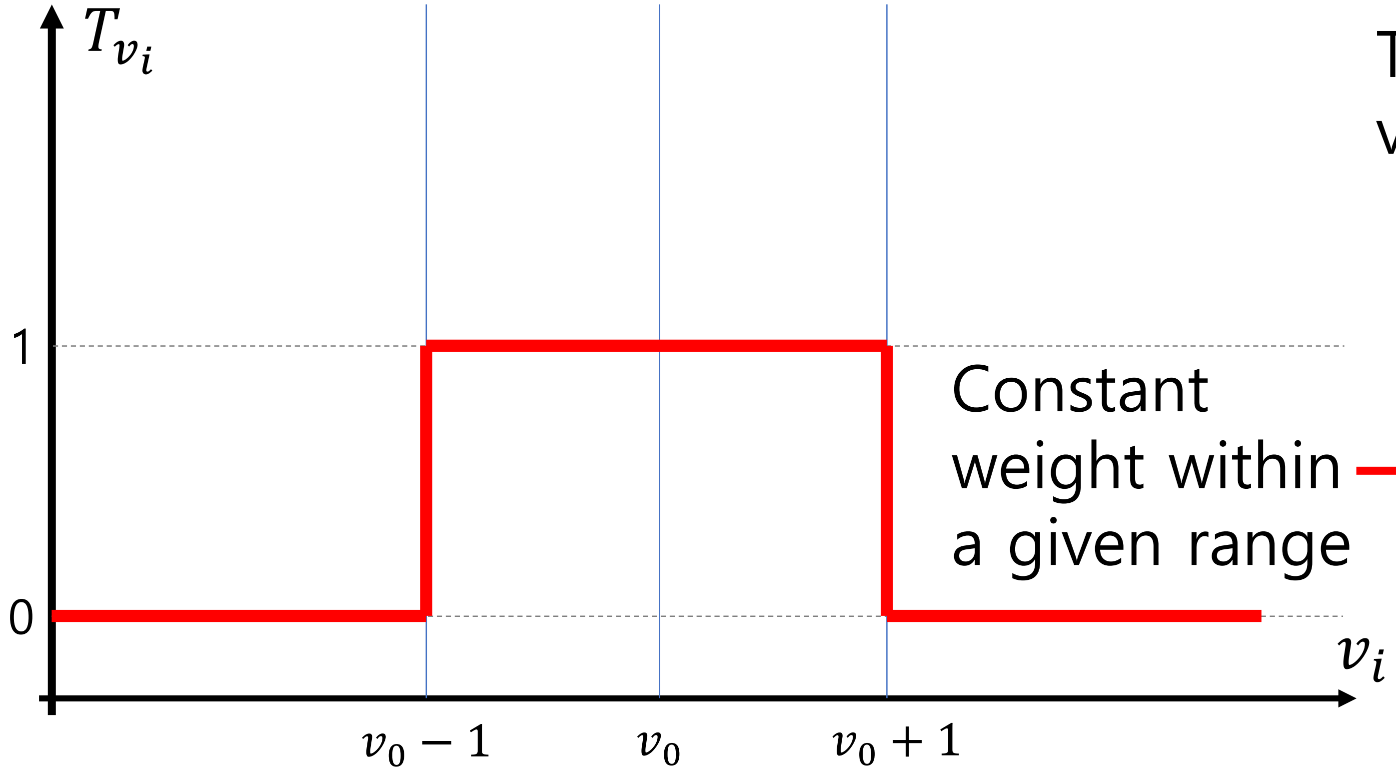
$$moment\ 1 = \frac{\sum_{v_1}^{v_2} T(v) \cdot v}{\sum_{v_1}^{v_2} T(v)}$$

Large (about 20 km s^{-1}) moment 1 value is not reliable.



How the moment 1 value becomes unreliable?

$$moment\ 1 = \frac{\sum_{v_1}^{v_2} T(v) \cdot v}{\sum_{v_1}^{v_2} T(v)} = \frac{T_{v_1}v_1 + T_{v_2}v_2 + T_{v_3}v_3 + T_{v_4}v_4 + \cdots}{T_{v_1} + T_{v_2} + T_{v_3} + T_{v_4} + \cdots}$$



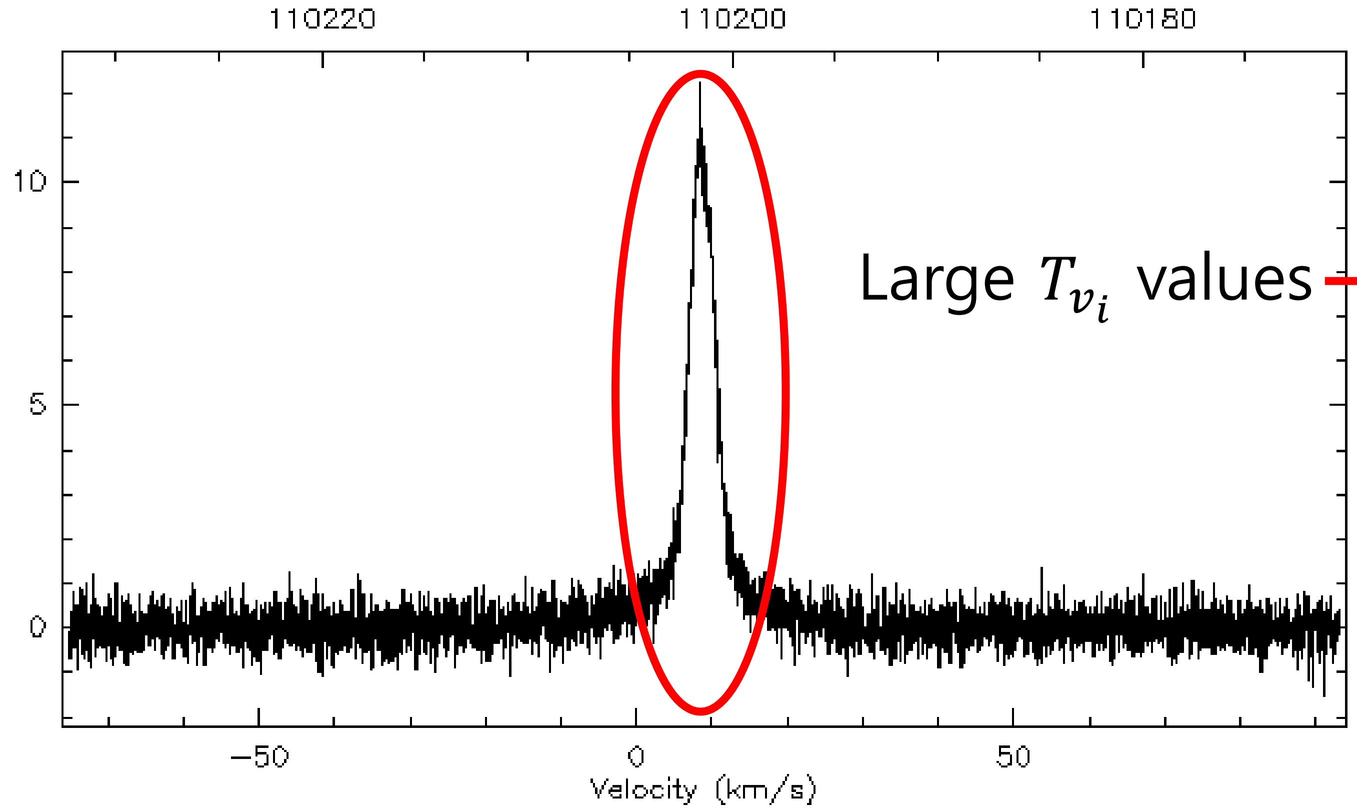
The moment 1 value is an average velocity weighted by the intensity.

Arithmetic mean value

$$\frac{T_{v_0-1} + T_{v_0} + T_{v_0+1}}{3}$$

How the moment 1 value becomes unreliable?

$$moment\ 1 = \frac{\sum_{v_1}^{v_2} T(v) \cdot v}{\sum_{v_1}^{v_2} T(v)} = \frac{T_{v_1}v_1 + T_{v_2}v_2 + T_{v_3}v_3 + T_{v_4}v_4 + \cdots}{T_{v_1} + T_{v_2} + T_{v_3} + T_{v_4} + \cdots}$$



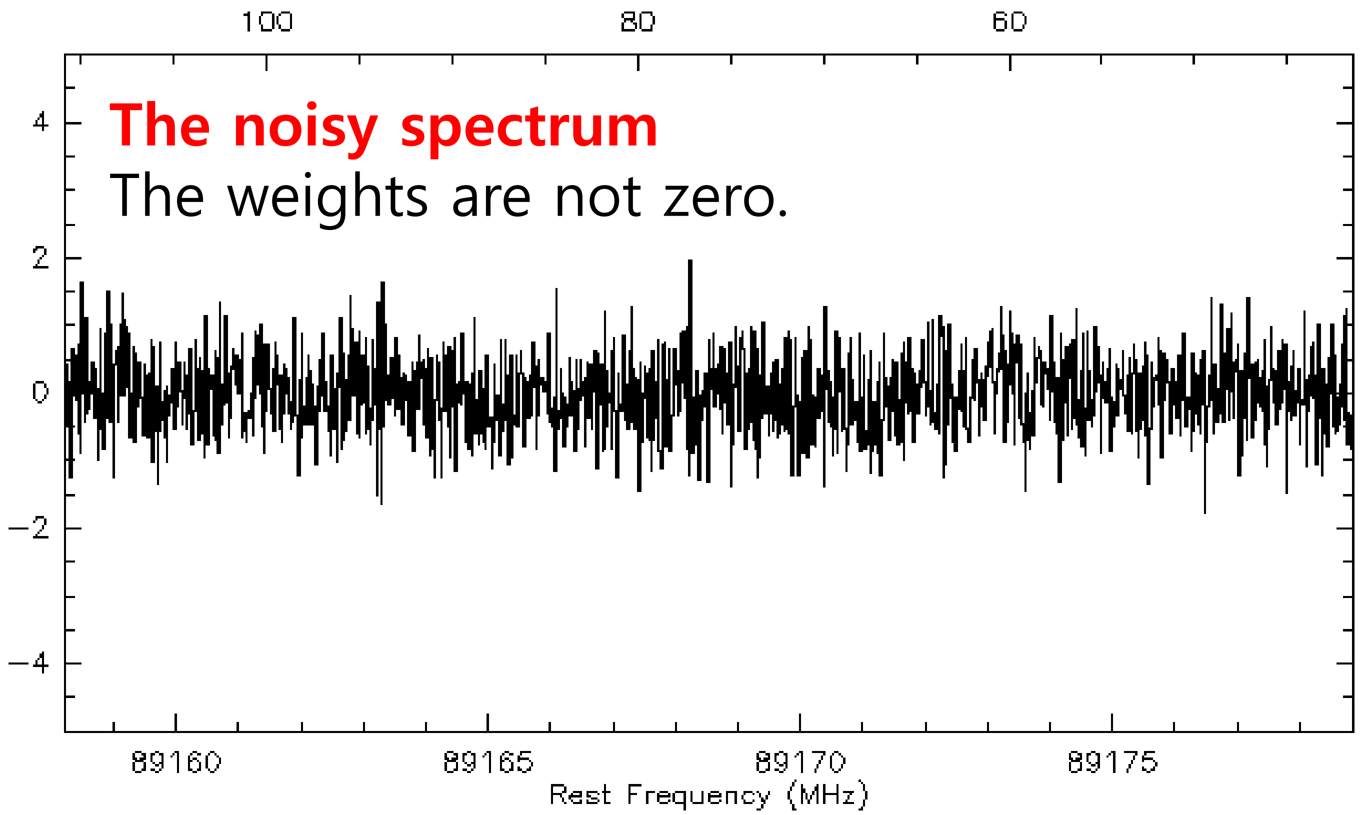
Large T_{v_i} values → Large $weight$ for v_i

∴ the moment 1 value would be near the velocity that has the maximum intensity (T_{peak})

The moment 1 value is an average velocity weighted by the intensity.

How the moment 1 value becomes unreliable?

$$moment\ 1 = \frac{\sum T(v) \cdot v}{\sum T(v)} = \frac{T_{v_1} v_1 + T_{v_2} v_2 + T_{v_3} v_3 + T_{v_4} v_4 + \cdots}{T_{v_1} + T_{v_2} + T_{v_3} + T_{v_4} + \cdots}$$



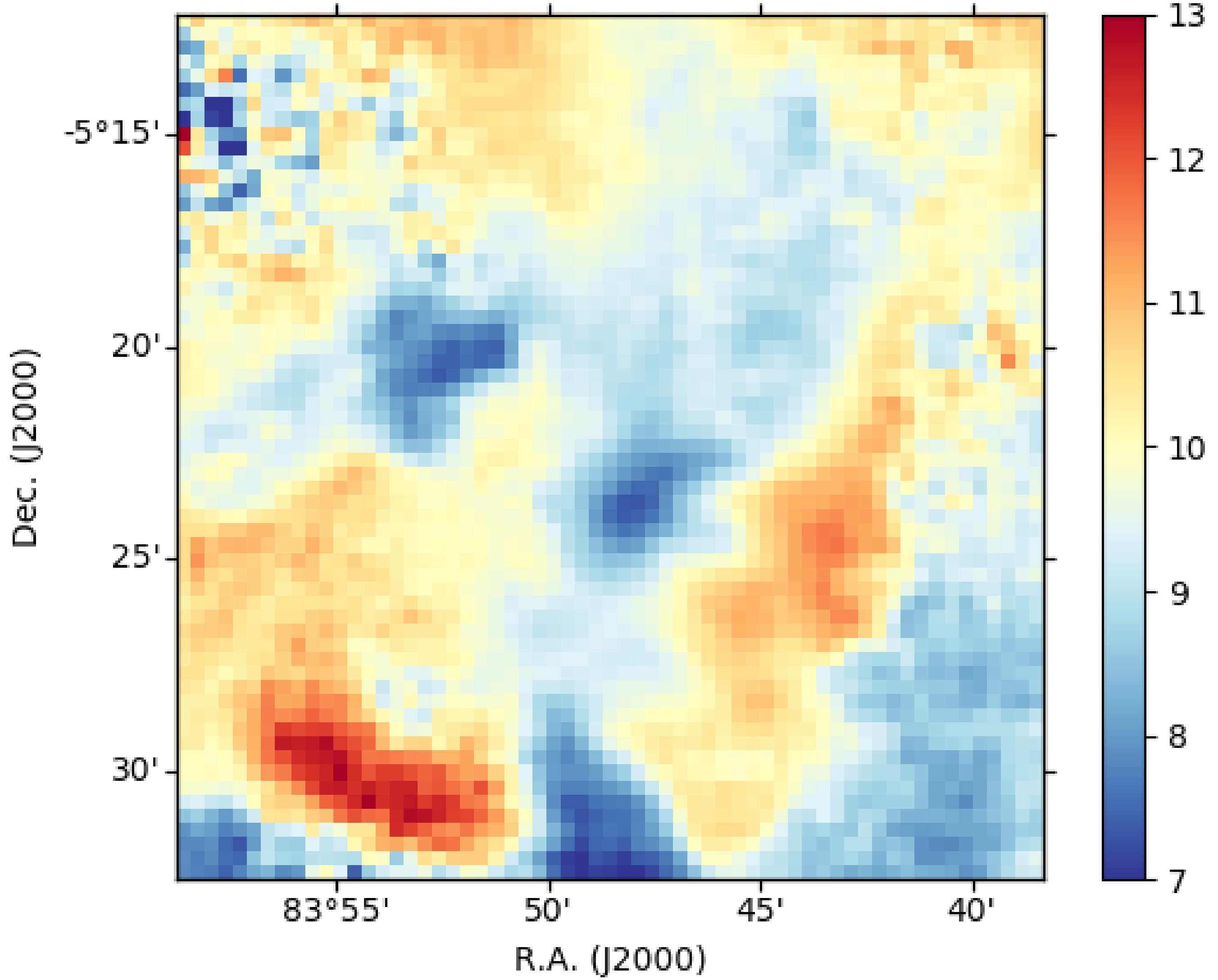
The moment 1 value is an average velocity weighted by the intensity.

Both $\sum T(v) \cdot v$ and $\sum T(v)$ can be non-zero values.

Very small $\sum T(v) \cdot v$ can be amplified by very small $\sum T(v)$.

What is the problem? How can we solve?

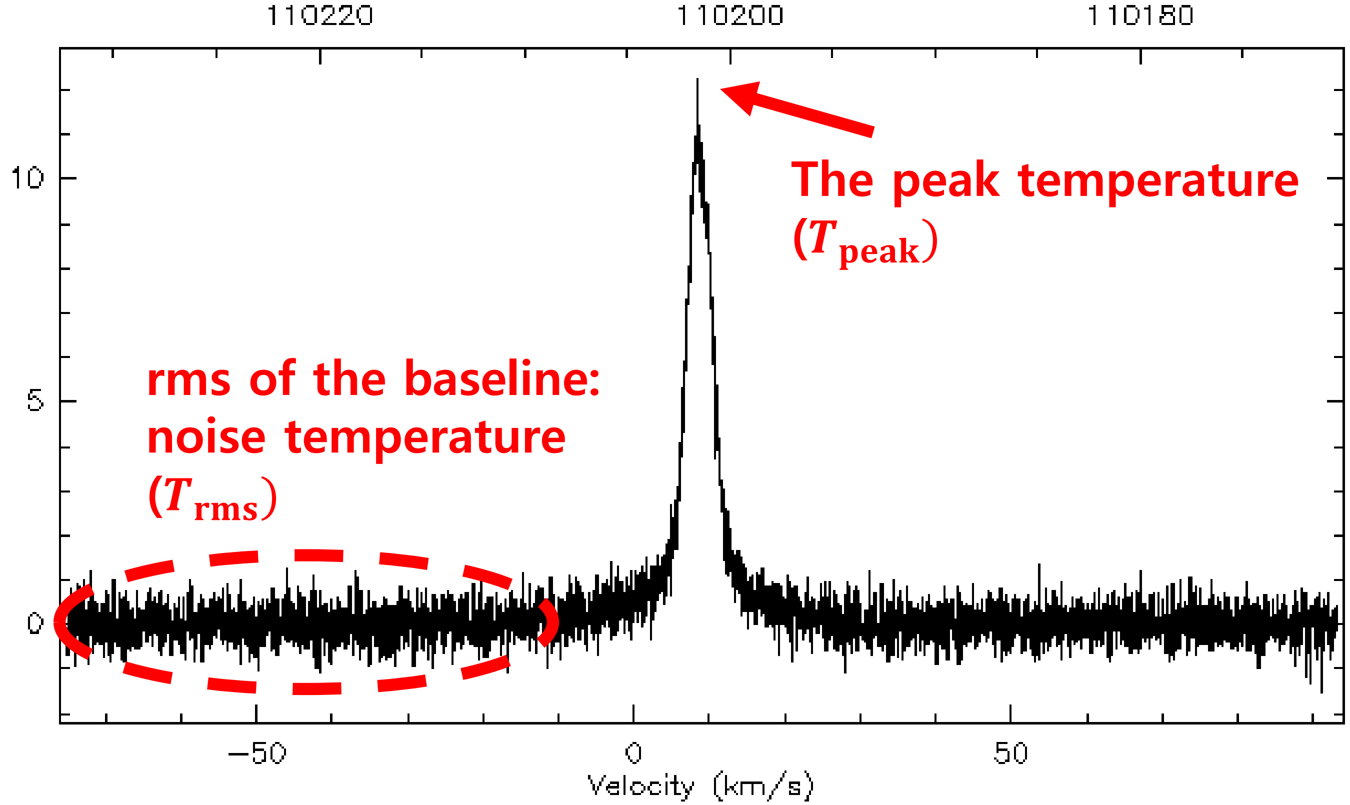
- Vmin=7
- Vmax=13
- Cmap = cm.RdYlBu



Signal to noise ratio (SNR)

- One of the important parameter in observations.
- $SNR = \frac{T_{\mathrm{peak}}}{T_{\mathrm{rms}}}$

^{13}CO J=1-0 toward Orion KL with a high SNR



Signal to noise ratio (SNR)

hs-yun@hsyun-Vostro-470: ~/Desktop/TRAO_spectra

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```
import numpy as np
from astropy.io import fits
from astropy.wcs import WCS
import matplotlib.pyplot as plt
import matplotlib.gridspec as gridspec
import matplotlib.cm as cm
import copy

def what_do_you_want_to_see(velo,spec):
    moment1_tmp = np.sum(spec*velo)
    sum_I = np.sum(spec)
    moment1_val = moment1_tmp/sum_I
    return moment1_val
```

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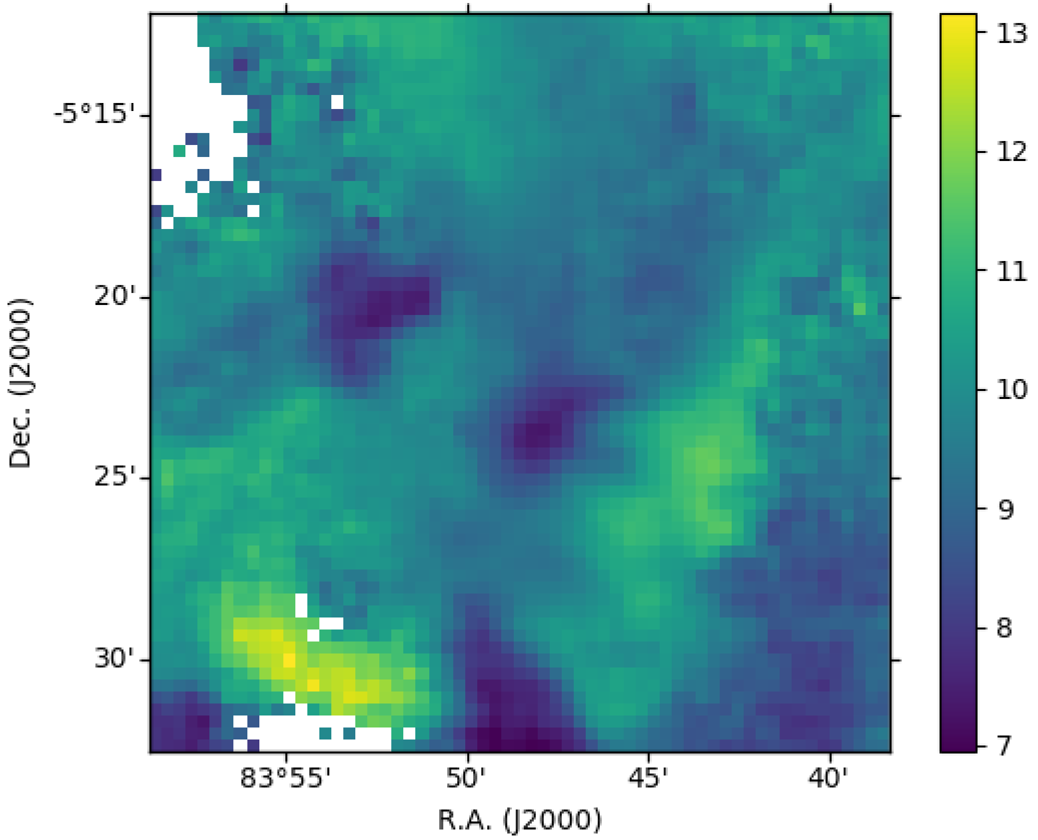
- Add a selection rule.

Return $\begin{cases} moment\ 1 & if\ T_{peak} > 5 \cdot T_{rms} \\ np.nan & if\ T_{peak} < 5 \cdot T_{rms} \end{cases}$



Velocity dispersion map (moment 2)

- Intensity weighted velocity difference

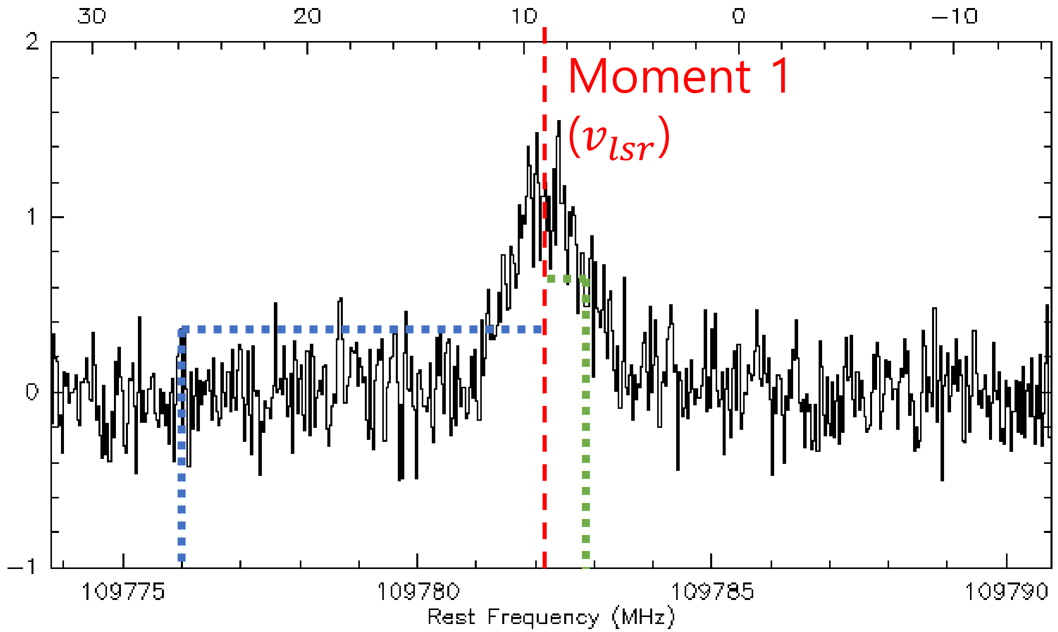
$$moment\ 2 = \sqrt{\frac{\sum T(v) \cdot (v - moment\ 1)^2}{\sum T(v)}}$$

- The moment 2 value is the same as the standard deviation (σ) if the spectrum is a single gaussian function.

Velocity dispersion map (moment 2)

- Intensity weighted velocity difference

$$moment\ 2 = \sqrt{\frac{\sum T(v) \cdot (v - moment\ 1)^2}{\sum T(v)}}$$



1. Large $T(v)$ with v that is close to v_{lsr}

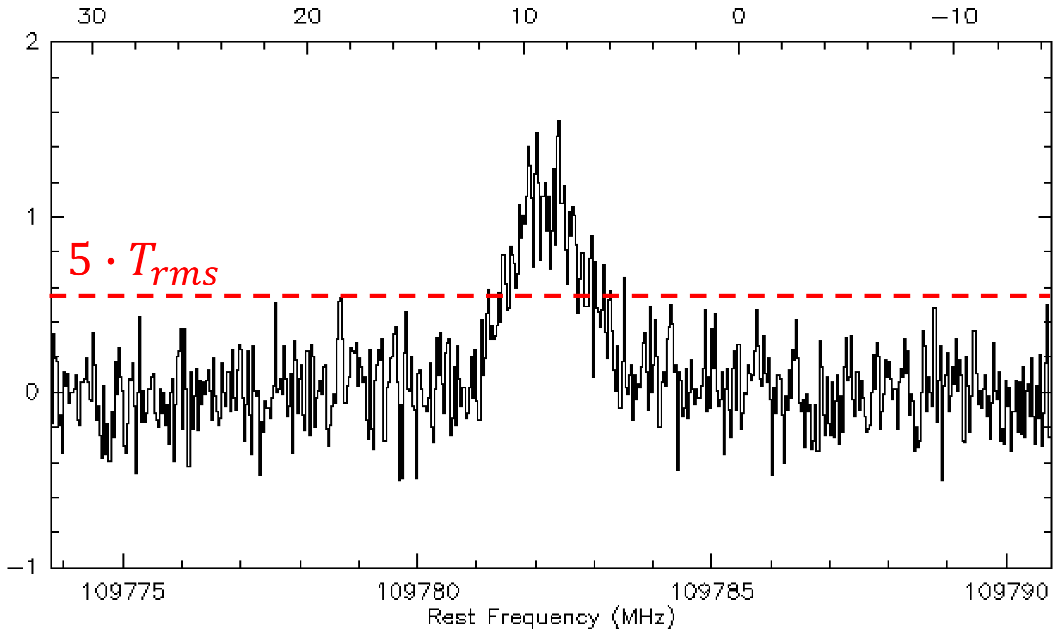
2. Smalll $T(v)$ with v that is far from v_{lsr}

What is the problem? How can we solve them?

Velocity dispersion map (moment 2)

- Intensity weighted velocity difference

$$moment\ 2 = \sqrt{\frac{\sum T(v) \cdot (v - moment\ 1)^2}{\sum T(v)}}$$



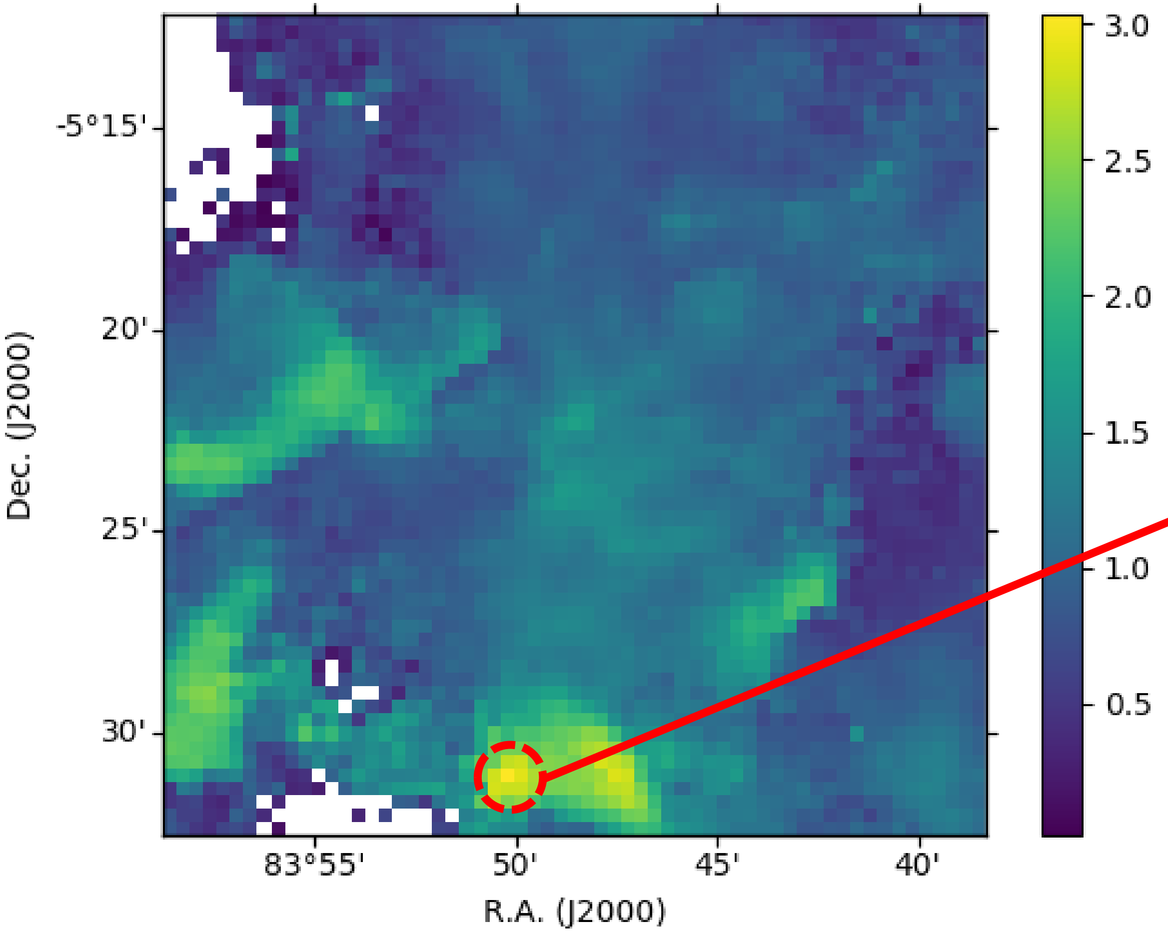
Let us consider the signal over $5 \cdot T_{rms}$.

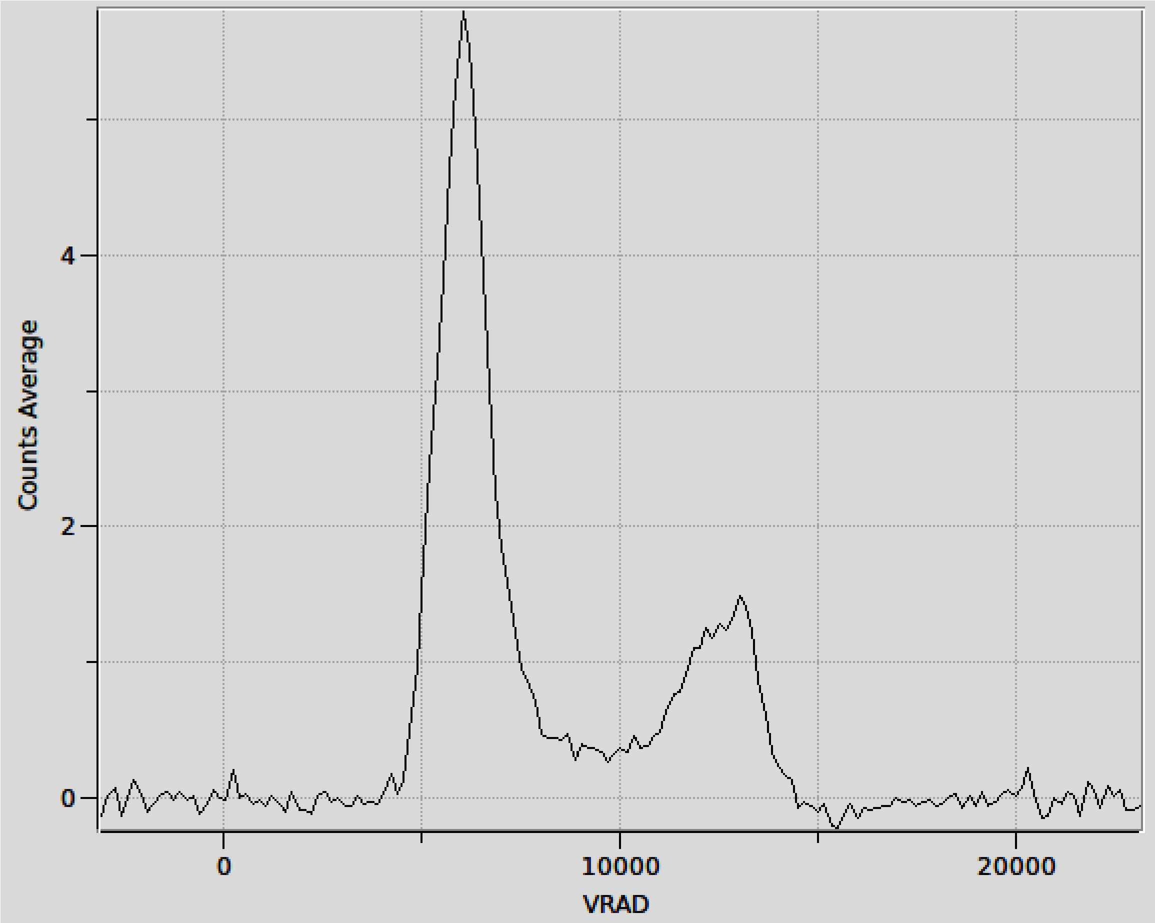
Calculate moment 1 value for the spectra, which has $T_{peak} > 5 \cdot T_{rms}$

Calculate moment 2 values for the signals, which has $T(v) > 5 \cdot T_{rms}$

Velocity dispersion map (moment 2)

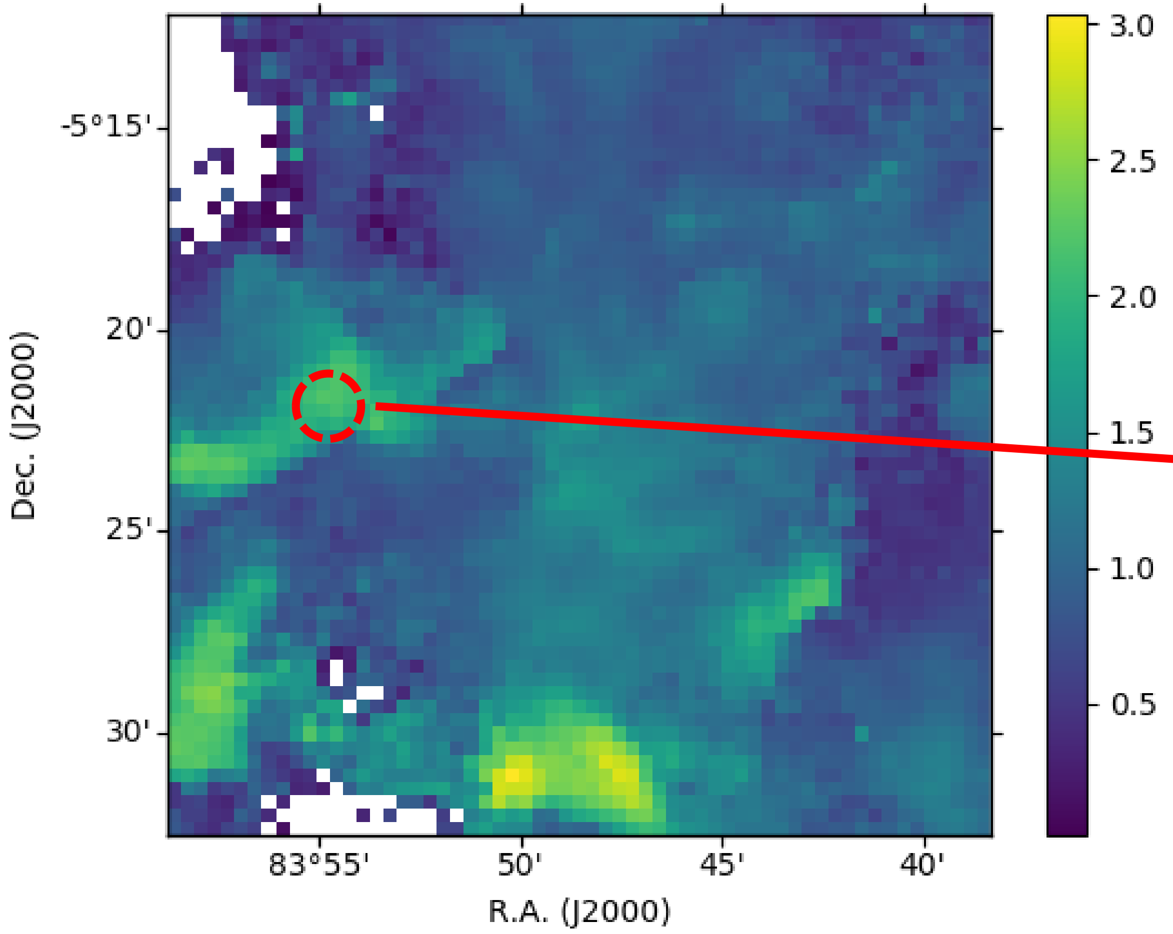
- Intensity weighted velocity difference

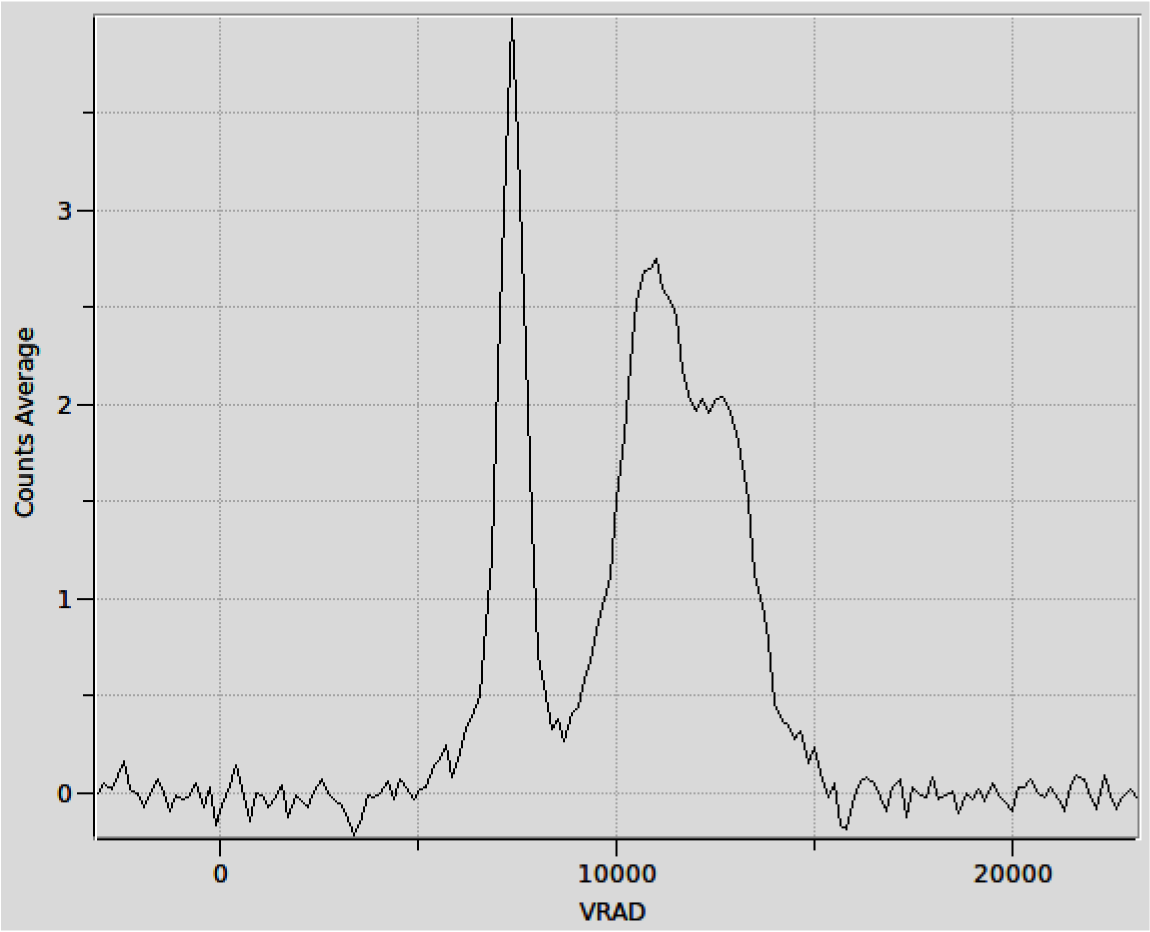




Velocity dispersion map (moment 2)

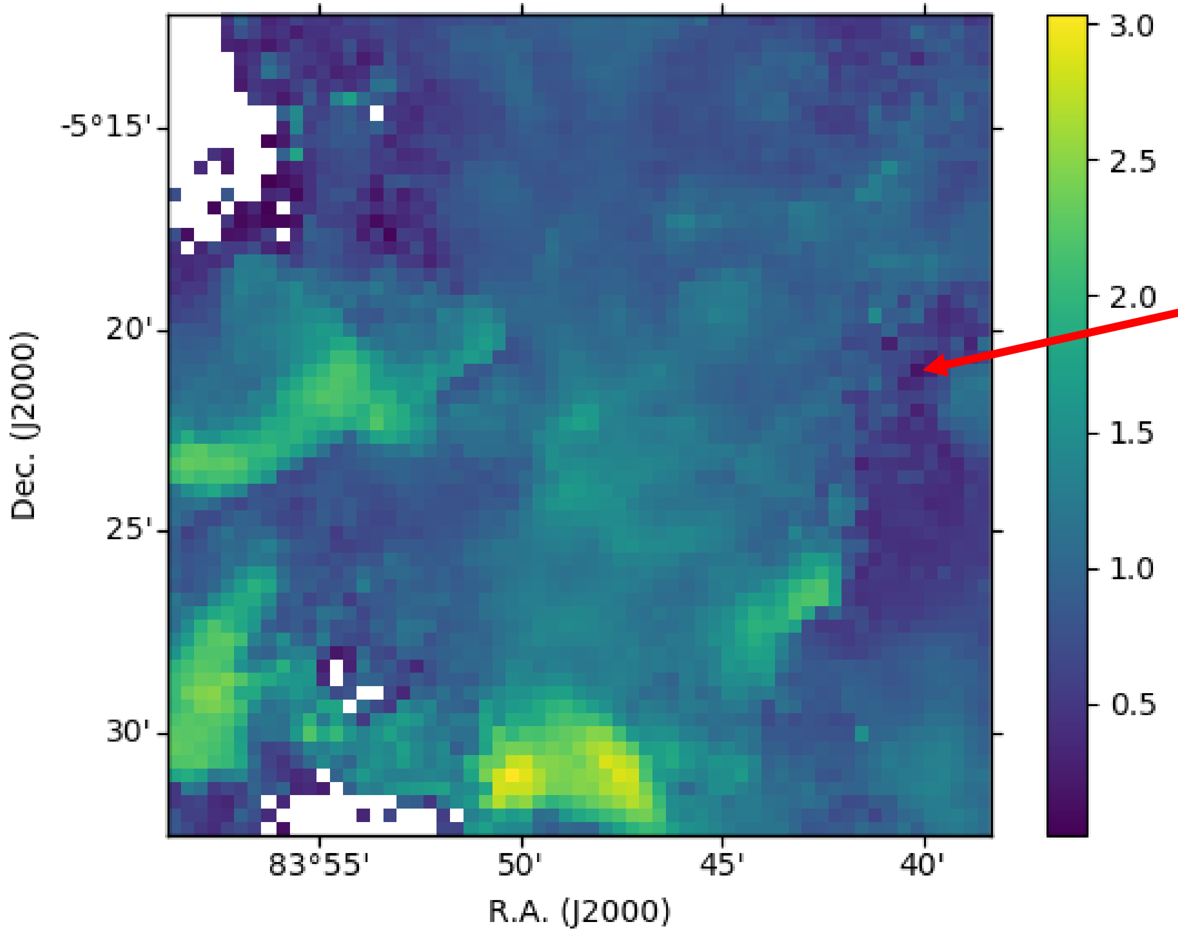
- Intensity weighted velocity difference





Velocity dispersion map (moment 2)

- Intensity weighted velocity difference



- There are several pixels that have very small moment 2 values (about 0.1 km s^{-1})

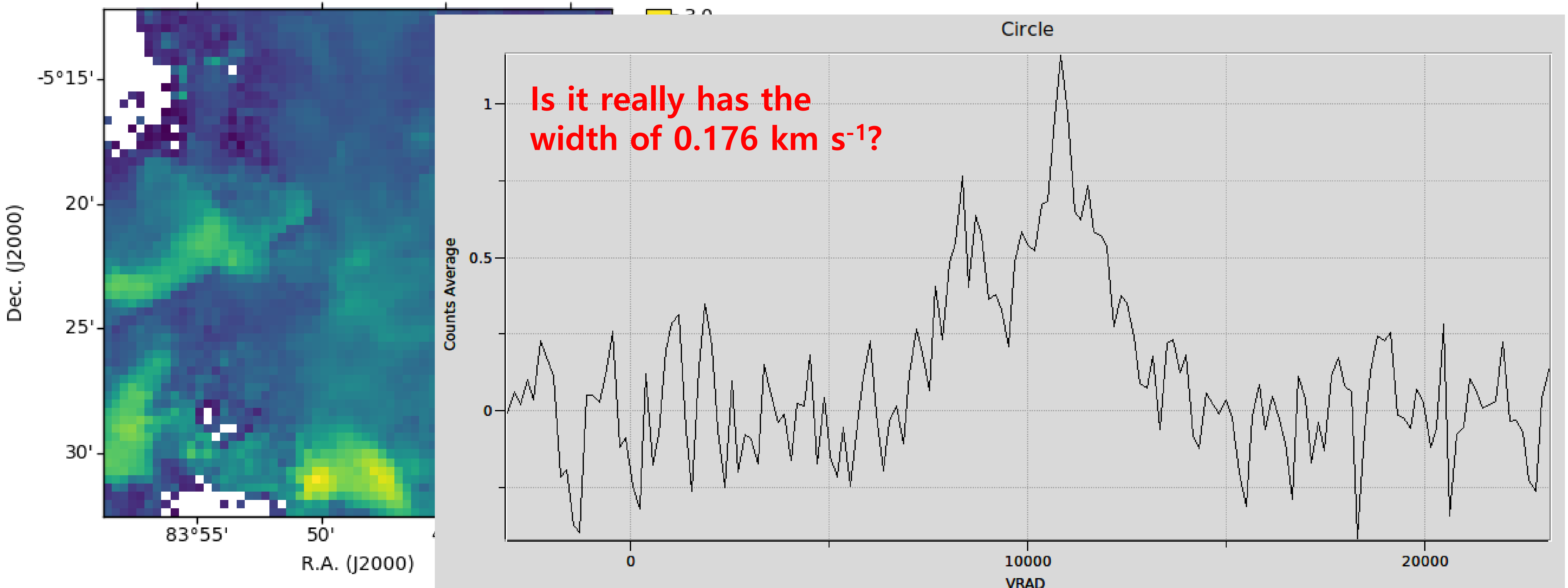
- Moment 2 = 0.176
- Thermal broadening width

$$\sigma_{th} = \sqrt{\frac{kT}{m}}$$

- Assuming m of 2u (the mass of H_2), $\sigma_{th} = 0.176\, km\, s^{-1}$ can be produced with $T = 7.35K$.

Velocity dispersion map (moment 2)

- Intensity weighted velocity difference



Positive bias by the clipping method

- 'Clipping' is one of the method that frequently used.
- However, at every reasonable clipping level the technique suffers from biases.
  - The positive bias
  - Weak extended emission can be clipped.
- The moment mask method

  (Dame, 2011, arxiv.org/abs/1101.1499)