Chương 2

I. Xác định tín hiệu lối ra của hệ thống

Bài 1:

(a) $x[n] = 3\delta[n] - 2\delta[n-1]$

Tín hiệu lối ra: $y[n] = x[n] * h[n] = \{3\delta[n] - 2\delta[n-1]\} * h[n] = 3h[n] - 2h[n-1]$

$= 3\delta[n+1] - 7\delta[n] - 7\delta[n-2] + 5\delta[n-3] - 2\delta[n-4]$

(b) $x[n] = u[n+1] - u[n-3]$

$\rightarrow x[n] = \delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2]$

Tín hiệu lối ra: $y[n] = x[n] * h[n] = \{\delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2]\} * h[n]$

$= h[n+1] + h[n] + h[n-1] + h[n-2]$

$= 2\delta[n+2] + 4\delta[n+1] + 6\delta[n] + 5\delta[n-1] + 5\delta[n-2] + 2\delta[n-3] + \delta[n-5]$

(c) Từ hình ta có được tín hiệu đầu vào $x[n] = -\delta[n+2] + 2\delta[n] + 2\delta[n-3]$

Tín hiệu lối ra: $y[n] = x[n] * h[n] = \{-\delta[n+2] + 2\delta[n] + 2\delta[n-3]\} * h[n]$

$= -h[n+2] + 2h[n] + 2h[n-3]$

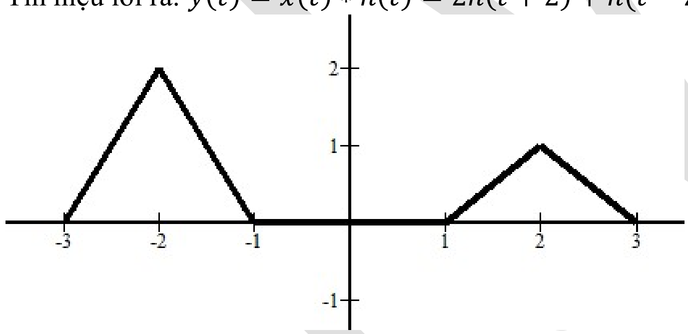
$= -\delta[n+3] - 3\delta[n+2] + 7\delta[n] + 3\delta[n-1] + 8\delta[n-3] + 4\delta[n-4]$

$-2\delta[n-5] + 2\delta[n-6]$

Bài 2:

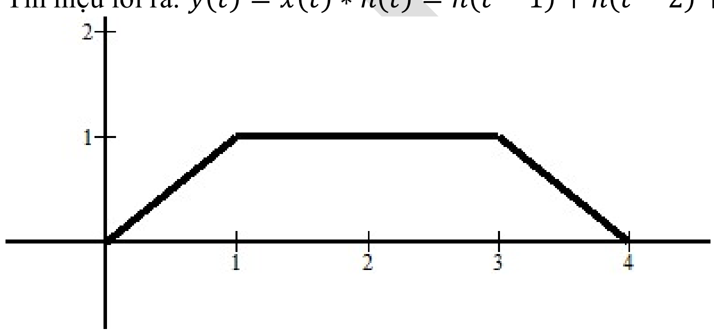
(a) $x(t) = 2\delta(t+2) + \delta(t-2)$

Tín hiệu lối ra: $y(t) = x(t) * h(t) = 2h(t+2) + h(t-2)$



(b) $x(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$

Tín hiệu lối ra: $y(t) = x(t) * h(t) = h(t-1) + h(t-2) + h(t-3)$



(c) $x(t) = \sum_{p=0}^{\infty}(-1)^p\delta(t-2p)$

Tín hiệu lối ra: $y(t) = \sum_{p=0}^{\infty}(-1)^p h(t-2p)$

II. Phương trình vi phân/sai phân

Bài 1:

(a) $5r + 10 = 0$
$r = -2$
$y^{(h)}(t) = ce^{-2t}$

(b) $r^2 + 6r + 8 = 0$
$r = -4, -2$
$y^{(h)}(t) = c_1 e^{-4t} + c_2 e^{-2t}$

(c) $r^2 + 4 = 0$
$r = \pm 2j$
$y^{(h)}(t) = c_1 e^{2j} \ + c_2 e^{-2jt}$

(d) $r^2 + 2r + 2 = 0$
$r = -1 \pm j$
$y^{(h)}(t) = c_1 e^{(-1-j)t} + c_2 e^{(-1+j)t}$

(e) $r^2 + 2r + 1 = 0$
$r = -1, -1$
$y^{(h)}(t) = c_1 e^{-t} + c_2 t e^{-t}$

Bài 2:

(a) $r - \alpha = 0 \to r = \alpha$
$y^{(h)}[n] = c_1 \alpha^n$

(b) $r^2 - \frac{1}{4} r - \frac{1}{8} = 0$
$$r = \frac{1}{2}, -\frac{1}{4}$$
$$y^{(h)}[n] = c_1 \left(\frac{1}{2}\right)^n + c_2 \left(-\frac{1}{4}\right)^n$$

(c) $r^2 + \frac{9}{16} = 0$
$$r = \pm j\frac{3}{4}$$
$$y^{(h)}[n] = c_1 \left(j\frac{3}{4}\right)^n + c_2 \left(-j\frac{3}{4}\right)^n$$

(d) $r^2 + r + \frac{1}{4} = 0$
$$r = -\frac{1}{2}, -\frac{1}{2}$$
$$y^{(h)}[n] = c_1 \left(-\frac{1}{2}\right)^n + c_2 n \left(-\frac{1}{2}\right)^n$$

Bài 3:

(a) $5\frac{d}{dt}y(t) + 10y(t) = 2x(t)$

(i) $x(t) = 2$

$$\begin{aligned} y^{(p)}(t) &= k \\ 10k &= 2(2) \\ k &= \frac{2}{5} \\ y^{(p)}(t) &= \frac{2}{5} \end{aligned}$$

(ii) $x(t) = e^{-t}$

$$\begin{aligned} y^{(p)}(t) &= ke^{-t} \\ -5ke^{-t} + 10ke^{-t} &= 2e^{-t} \\ k &= \frac{2}{5} \\ y^{(p)}(t) &= \frac{2}{5}e^{-t} \end{aligned}$$

(iii) $x(t) = \cos(3t)$

$$\begin{aligned} y^{(p)}(t) &= A\cos(3t) + B\sin(3t) \\ \frac{d}{dt}y^{(p)}(t) &= -3A\sin(3t) + 3B\cos(3t) \\ 5\left(-3A\sin(3t) + 3B\cos(3t)\right) + 10A\cos(3t) + 10B\sin(3t) &= 2\cos(3t) \\ -15A + 10B &= 0 \\ 10A + 15B &= 2 \end{aligned}$$

$$\begin{aligned} A &= \frac{4}{65} \\ B &= \frac{6}{65} \\ y^{(p)}(t) &= \frac{4}{65}\cos(3t) + \frac{6}{65}\sin(3t) \end{aligned}$$

(b) $\frac{d^2}{dt^2}y(t) + 4y(t) = 3\frac{d}{dt}x(t)$

(i) $x(t) = t$

$$\begin{aligned} y^{(p)}(t) &= k_1 t + k_2 \\ 4k_1 t + 4k_2 &= 3 \\ k_1 &= 0 \\ k_2 &= \frac{3}{4} \\ y^{(p)}(t) &= \frac{3}{4} \end{aligned}$$

(ii) $x(t) = e^{-t}$

$$\begin{aligned} y^{(p)}(t) &= ke^{-t} \\ ke^{-t} + 4ke^{-t} &= -3e^{-t} \\ k &= -\frac{3}{5} \\ y^{(p)}(t) &= -\frac{3}{5}e^{-t} \end{aligned}$$

(iii) $x(t) = (\cos(t) + \sin(t))$

$$\begin{aligned} y^{(p)}(t) &= A\cos(t) + B\sin(t) \\ \frac{d}{dt}y^{(p)}(t) &= -A\sin(t) + B\cos(t) \\ \frac{d^2}{dt^2}y^{(p)}(t) &= -A\cos(t) - B\sin(t) \\ -A\cos(t) - B\sin(t) + 4A\cos(t) + 4B\sin(t) &= -3\sin(t) + 3\cos(t) \\ -A + 4A &= 3 \\ -B + 4B &= -3 \\ A &= 1 \\ B &= -1 \\ y^{(p)}(t) &= \cos(t) - \sin(t) \end{aligned}$$

(c) $\frac{d^2}{dt^2}y(t) + 2\frac{d}{dt}y(t) + y(t) = \frac{d}{dt}x(t)$

(i) $x(t) = e^{-3t}$

$$\begin{aligned} y^{(p)}(t) &= ke^{-3t} \\ 9ke^{-3t} - 6ke^{-3t} + ke^{-3t} &= -3e^{-3t} \\ k &= -\frac{3}{4} \\ y^{(p)}(t) &= -\frac{3}{4}e^{-3t} \end{aligned}$$

(ii) $x(t) = 2e^{-t}$

Since e^{-t} and te^{-t} are in the natural response, the particular soluction takes the form of

$$\begin{aligned} y^{(p)}(t) &= kt^2e^{-t} \\ \frac{d}{dt}y^{(p)}(t) &= 2kte^{-t} - kt^2e^{-t} \\ \frac{d^2}{dt^2}y^{(p)}(t) &= 2ke^{-t} - 4kte^{-t} + kt^2e^{-t} \\ -2e^{-t} &= 2ke^{-t} - 4kte^{-t} + kt^2e^{-t} + 2(2kte^{-t} - kt^2e^{-t}) + kt^2e^{-t} \\ k &= -1 \\ y^{(p)}(t) &= -t^2e^{-t} \end{aligned}$$

(iii) $x(t) = 2\sin(t)$

$$\begin{aligned}
y^{(p)}(t) &= A\cos(t) + B\sin(t) \\
\frac{d}{dt}y^{(p)}(t) &= -A\sin(t) + B\cos(t) \\
\frac{d^2}{dt^2}y^{(p)}(t) &= -A\cos(t) - B\sin(t) \\
-A\cos(t) - B\sin(t) - 2A\sin(t) + 2B\cos(t) + A\cos(t) + B\sin(t) &= 2\cos(t) \\
-A - 2B + A &= 2 \\
-B - 2A + B &= 0 \\
A &= 0 \\
B &= -1 \\
y^{(p)}(t) &= -\sin(t)
\end{aligned}$$

Bài 4:

(a) $y[n] - \frac{2}{5}y[n-1] = 2x[n]$

(i) $x[n] = 2u[n]$

$$\begin{aligned}
y^{(p)}[n] &= ku[n] \\
k - \frac{2}{5}k &= 4
\end{aligned}$$

$$\begin{aligned}
k &= \frac{20}{3} \\
y^{(p)}[n] &= \frac{20}{3}u[n]
\end{aligned}$$

(ii) $x[n] = -(\frac{1}{2})^n u[n]$

$$\begin{aligned}
y^{(p)}[n] &= k\left(\frac{1}{2}\right)^n u[n] \\
k\left(\frac{1}{2}\right)^n - \frac{2}{5}\left(\frac{1}{2}\right)^{n-1} k &= -2\left(\frac{1}{2}\right)^n \\
k &= -10 \\
y^{(p)}[n] &= -10\left(\frac{1}{2}\right)^n u[n]
\end{aligned}$$

(iii) $x[n] = \cos(\frac{\pi}{5}n)$

$$\begin{aligned}
y^{(p)}[n] &= A\cos(\frac{\pi}{5}n) + B\sin(\frac{\pi}{5}n) \\
2\cos(\frac{\pi}{5}n) &= A\cos(\frac{\pi}{5}n) + B\sin(\frac{\pi}{5}n) - \frac{2}{5}\left[A\cos(\frac{\pi}{5}(n-1)) + B\sin(\frac{\pi}{5}(n-1))\right) \\
&\quad \text{Using the trig identities} \\
\sin(\theta \pm \phi) &= \sin\theta\cos\phi \pm \cos\theta\sin\phi \\
\cos(\theta \pm \phi) &= \cos\theta\cos\phi \mp \sin\theta\sin\phi \\
y^{(p)}[n] &= 2.6381\cos(\frac{\pi}{5}n) + 0.9170\sin(\frac{\pi}{5}n)
\end{aligned}$$

(b) $y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + x[n-1]$

(i) $x[n] = nu[n]$

$$\begin{aligned}
y^{(p)}[n] &= k_1 nu[n] + k_2 u[n] \\
k_1 n + k_2 - \frac{1}{4}[k_1(n-1) + k_2] - \frac{1}{8}[k_1(n-2) + k_2] &= n + n - 1 \\
k_1 &= \frac{16}{5} \\
k_2 &= -\frac{104}{5} \\
y^{(p)}[n] &= \frac{16}{5} nu[n] - \frac{104}{5} u[n]
\end{aligned}$$

(ii) $x[n] = (\frac{1}{8})^n u[n]$

$$\begin{aligned}
y^{(p)}[n] &= k\left(\frac{1}{8}\right)^n u[n] \\
k\left(\frac{1}{8}\right)^n - \frac{1}{4}\left(\frac{1}{8}\right)^{n-1} k - \frac{1}{8}\left(\frac{1}{8}\right)^{n-2} k &= \left(\frac{1}{8}\right)^n + \left(\frac{1}{8}\right)^{n-1} \\
k &= -1 \\
y^{(p)}[n] &= -1\left(\frac{1}{8}\right)^n u[n]
\end{aligned}$$

(iii) $x[n] = e^{j\frac{\pi}{4}n} u[n]$

$$\begin{aligned}
y^{(p)}[n] &= ke^{j\frac{\pi}{4}n} u[n] \\
ke^{j\frac{\pi}{4}n} - \frac{1}{4}ke^{j\frac{\pi}{4}(n-1)} - \frac{1}{8}ke^{j\frac{\pi}{4}(n-2)} &= e^{j\frac{\pi}{4}n} + e^{j\frac{\pi}{4}(n-1)} \\
k &= \frac{1 + e^{-j\frac{\pi}{4}}}{1 - \frac{1}{4}e^{-j\frac{\pi}{4}} - \frac{1}{8}ke^{-j\frac{\pi}{2}}}
\end{aligned}$$

(iv) $x[n] = (\frac{1}{2})^n u[n]$

Since $\left(\frac{1}{2}\right)^n u[n]$ is in the natural response, the particular solution takes the form of:

$$\begin{aligned}
y^{(p)}[n] &= kn\left(\frac{1}{2}\right)^n u[n] \\
kn\left(\frac{1}{2}\right)^n - k\frac{1}{4}(n-1)\left(\frac{1}{2}\right)^{n-1} - k\frac{1}{8}(n-2)\left(\frac{1}{2}\right)^{n-2} &= \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n-1} \\
k &= 2 \\
y^{(p)}[n][n] &= 2n\left(\frac{1}{2}\right)^n u[n]
\end{aligned}$$

(c) $y[n] + y[n-1] + \frac{1}{2}y[n-2] = x[n] + 2x[n-1]$

(i) $x[n] = u[n]$

$$\begin{aligned} y^{(p)}[n][n] &= ku[n] \\ k + k + \frac{1}{2}k &= 2 + 2 \\ k &= \frac{8}{5} \\ y^{(p)}[n] &= \frac{8}{5}u[n] \end{aligned}$$

(ii) $x[n] = (\frac{-1}{2})^n u[n]$

$$\begin{aligned} y^{(p)}[n] &= k\left(-\frac{1}{2}\right)^n u[n] \\ k\left(-\frac{1}{2}\right)^n + k\left(-\frac{1}{2}\right)^{n-1} + \frac{1}{2}\left(-\frac{1}{2}\right)^{n-2} k &= \left(-\frac{1}{2}\right)^n + 2\left(-\frac{1}{2}\right)^{n-1} \\ k &= -3 \\ y^{(p)}[n] &= -3\left(-\frac{1}{2}\right)^n u[n] \end{aligned}$$

Bài 4:

(a) $\frac{d}{dt}y(t) + 10y(t) = 2x(t), \quad y(0^-) = 1, x(t) = u(t)$

$$\begin{aligned} t \geq 0 & \quad \text{natural: characteristic equation} \\ r + 10 &= 0 \\ r &= -10 \\ y^{(n)}(t) &= ce^{-10t} \\ & \quad \text{particular} \\ y^{(p)}(t) &= ku(t) = \frac{1}{5}u(t) \\ y(t) &= \frac{1}{5} + ce^{-10t} \\ y(0^-) = 1 &= \frac{1}{5} + c \\ c &= \frac{4}{5} \\ y(t) &= \frac{1}{5}\left[1 + 4e^{-10t}\right] u(t) \end{aligned}$$

(b) $\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 4y(t) = \frac{d}{dt}x(t), \quad y(0^-) = 0, \left.\frac{d}{dt}y(t)\right|_{t=0^-} = 1, x(t) = \sin(t)u(t)$

$$
\begin{aligned}
t \geq 0 & \qquad \text{natural: characteristic equation} \\
r^2 + 5r + 4 &= 0 \\
r &= -4, \; -1 \\
y^{(n)}(t) &= c_1 e^{-4t} + c_2 e^{-t} \\
& \qquad \text{particular} \\
y^{(p)}(t) &= A\sin(t) + B\cos(t) \\
&= \frac{5}{34}\sin(t) + \frac{3}{34}\cos(t) \\
y(t) &= \frac{5}{34}\sin(t) + \frac{3}{34}\cos(t) + c_1 e^{-4t} + c_2 e^{-t} \\
y(0^-) = 0 &= \frac{3}{34} + c_1 + c_2 \\
\left.\frac{d}{dt}y(0)\right|_{t=0^-} = 1 &= \frac{5}{34} - 4c_1 - c_2 \\
c_1 &= -\frac{13}{51} \\
c_2 &= \frac{1}{6} \\
y(t) &= \frac{5}{34}\sin(t) + \frac{3}{34}\cos(t) - \frac{13}{51}e^{-4t} + \frac{1}{6}e^{-t}
\end{aligned}
$$

(c) $\frac{d^2}{dt^2}y(t) + 6\frac{d}{dt}y(t) + 8y(t) = 2x(t), \quad y(0^-) = -1, \left.\frac{d}{dt}y(t)\right|_{t=0^-} = 1, x(t) = e^{-t}u(t)$

$$
\begin{aligned}
t \geq 0 & \qquad \text{natural: characteristic equation} \\
r^2 + 6r + 8 &= 0 \\
r &= -4, \; -2 \\
y^{(n)}(t) &= c_1 e^{-2t} + c_2 e^{-4t} \\
& \qquad \text{particular} \\
y^{(p)}(t) &= ke^{-t}u(t) \\
&= \frac{2}{3}e^{-t}u(t) \\
y(t) &= \frac{2}{3}e^{-t}u(t) + c_1 e^{-2t} + c_2 e^{-4t} \\
y(0^-) = -1 &= \frac{2}{3} + c_1 + c_2 \\
\left.\frac{d}{dt}y(0)\right|_{t=0^-} = 1 &= -\frac{2}{3} - 2c_1 - 4c_2 \\
c_1 &= -\frac{5}{2} \\
c_2 &= \frac{5}{6} \\
y(t) &= \frac{2}{3}e^{-t}u(t) - \frac{5}{2}e^{-2t} + \frac{5}{6}e^{-4t}
\end{aligned}
$$

(d) $\frac{d^2}{dt^2}y(t) + y(t) = 3\frac{d}{dt}x(t), \quad y(0^-) = -1, \frac{d}{dt}y(t)\big|_{t=0^-} = 1, x(t) = 2te^{-t}u(t)$

$$\begin{aligned}
t \geq 0 & \quad \text{natural: characteristic equation} \\
r^2 + 1 &= 0 \\
r &= \pm j \\
y^{(n)}(t) &= A\cos(t) + B\sin(t) \\
& \quad \text{particular} \\
y^{(p)}(t) &= kte^{-t}u(t) \\
\frac{d^2}{dt^2}y^{(p)}(t) &= -2ke^{-t} + kte^{-t} \\
-2ke^{-t} + kte^{-t} + kte^{-t} &= 3[2e^{-t} - 2te^{-t}] \\
k &= -3 \\
y^{(p)}(t) &= -3te^{-t}u(t) \\
y(t) &= -3te^{-t}u(t) + A\cos(t) + B\sin(t) \\
y(0^-) &= -1 = 0 + A + 0 \\
\frac{d}{dt}y(t)\Big|_{t=0^-} &= 1 = -3 + 0 + B \\
y(t) &= -3te^{-t}u(t) - \cos(t) + 4\sin(t)
\end{aligned}$$

Bài 5:

(a) $y[n] - \frac{1}{2}y[n-1] = 2x[n], \quad y[-1] = 3, x[n] = (\frac{-1}{2})^n u[n]$

$$\begin{aligned}
n \geq 0 & \quad \text{natural: characteristic equation} \\
r - \frac{1}{2} &= 0 \\
y^{(n)}[n] &= c\left(\frac{1}{2}\right)^n \\
& \quad \text{particular} \\
y^{(p)}[n] &= k\left(-\frac{1}{2}\right)^n u[n] \\
k\left(-\frac{1}{2}\right)^n - \frac{1}{2}k\left(-\frac{1}{2}\right)^{n-1} &= 2\left(-\frac{1}{2}\right)^n \\
k &= 1 \\
y^{(p)}[n] &= \left(-\frac{1}{2}\right)^n u[n]
\end{aligned}$$

$$
\begin{aligned}
y[n] &= \frac{1}{2}y[n-1] + 2x[n] \\
y[0] &= \frac{1}{2}3 + 2 = \frac{7}{2} \\
y[n] &= \left(-\frac{1}{2}\right)^n u[n] + c\left(\frac{1}{2}\right)^n u[n] \\
\frac{7}{2} &= 1 + c \\
c &= \frac{5}{2} \\
y[n] &= \left(-\frac{1}{2}\right)^n u[n] + \frac{5}{2}\left(\frac{1}{2}\right)^n u[n]
\end{aligned}
$$

(b) $y[n] - \frac{1}{9}y[n-2] = x[n-1], y[-1] = 1, y[-2] = 0, x[n] = u[n]$

$$
\begin{aligned}
n \geq 0 & \quad \text{natural: characteristic equation} \\
r^2 - \frac{1}{9} &= 0 \\
r &= \pm\frac{1}{3} \\
y^{(n)}[n] &= c_1\left(\frac{1}{3}\right)^n + c_2\left(-\frac{1}{3}\right)^n \\
& \quad \text{particular} \\
y^{(p)}[n] &= ku[n] \\
k - \frac{1}{9}k &= 1 \\
k &= \frac{9}{8} \\
y^{(p)}[n] &= \frac{9}{8}u[n]
\end{aligned}
$$

$$
\begin{aligned}
y[n] &= \frac{9}{8}u[n] + c_1\left(\frac{1}{3}\right)^n + c_2\left(-\frac{1}{3}\right)^n \\
& \quad \text{Translate initial conditions} \\
y[n] &= \frac{1}{9}y[n-2] + x[n-1] \\
y[0] &= \frac{1}{9}0 + 0 = 0 \\
y[1] &= \frac{1}{9}1 + 1 = \frac{10}{9} \\
0 &= \frac{9}{8} + c_1 + c_2 \\
\frac{10}{9} &= \frac{9}{8} + \frac{1}{3}c_1 - \frac{1}{3}c_2 \\
y[n] &= \frac{9}{8}u[n] - \frac{7}{12}\left(\frac{1}{3}\right)^n - \frac{13}{24}\left(-\frac{1}{3}\right)^n
\end{aligned}
$$

$$
\begin{aligned}
y^{(p)}[n] &= \frac{32}{3}u[n] \\
y[n] &= \frac{32}{3}u[n] + c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{4}\right)^n \\
& \text{Translate initial conditions} \\
y[n] &= \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + 2x[n] \\
y[0] &= \frac{3}{4}1 - \frac{1}{8}(-1) + 2(2) = \frac{39}{8} \\
y[1] &= \frac{3}{4}\left(\frac{39}{8}\right) - \frac{1}{8}1 + 2(2) = \frac{241}{32} \\
\frac{39}{8} &= \frac{32}{3} + c_1 + c_2 \\
\frac{241}{32} &= \frac{32}{3} + \frac{1}{2}c_1 + \frac{1}{4}c_2 \\
y[n] &= \frac{32}{3}u[n] - \frac{27}{4}\left(\frac{1}{2}\right)^n + \frac{23}{24}\left(\frac{1}{4}\right)^n
\end{aligned}
$$

(c) $y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + x[n-1]$, $\quad y[-1] = 4, y[-2] = -2, x[n] = (-1)^n u[n]$

$$
\begin{aligned}
n \geq 0 & \text{natural: characteristic equation} \\
r^2 + \frac{1}{4}r - \frac{1}{8} &= 0 \\
r &= -\frac{1}{4}, \frac{1}{2} \\
y^{(n)}[n] &= c_1\left(\frac{1}{2}\right)^n + c_2\left(-\frac{1}{4}\right)^n \\
& \text{particular} \\
y^{(p)}[n] &= k(-1)^n u[n] \\
\text{for } n \geq 1 & \\
k(-1)^n + k\frac{1}{4}(-1)^{n-1} - k\frac{1}{8}(-1)^{n-2} &= (-1)^n + (-1)^{n-1} = 0 \\
k &= 0 \\
y^{(p)}[n] &= 0 \\
y[n] &= c_1\left(\frac{1}{2}\right)^n + c_2\left(-\frac{1}{4}\right)^n \\
& \text{Translate initial conditions} \\
y[n] &= -\frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + x[n] + x[n-1] \\
y[0] &= \frac{1}{4}4 + \frac{1}{8}(-2) + 1 + 0 = -\frac{1}{4} \\
y[1] &= -\frac{1}{4}\left(-\frac{1}{4}\right) + \frac{1}{8}4 + -1 + 1 = \frac{9}{16} \\
-\frac{1}{4} &= c_1 + c_2 \\
\frac{9}{16} &= \frac{1}{2}c_1 - \frac{1}{4}c_2 \\
y[n] &= \frac{2}{3}\left(\frac{1}{2}\right)^n - \frac{11}{12}\left(-\frac{1}{4}\right)^n
\end{aligned}
$$

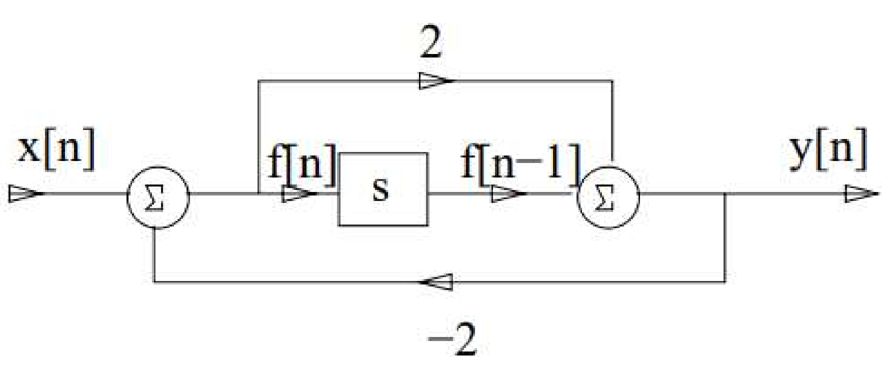
(d) $y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n], \quad y[-1] = 1, y[-2] = -1, x[n] = 2u[n]$

$$
\begin{aligned}
n \geq 0 & \quad \text{natural: characteristic equation} \\
r^2 - \frac{3}{4}r + \frac{1}{8} &= 0 \\
r &= \frac{1}{4}, \frac{1}{2} \\
y^{(n)}[n] &= c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{4}\right)^n \\
& \quad \text{particular} \\
y^{(p)}[n] &= ku[n] \\
k - k\frac{3}{4} + k\frac{1}{8} &= 4 \\
k &= \frac{32}{3} \\
y^{(p)}[n] &= \frac{32}{3}u[n] \\
y[n] &= \frac{32}{3}u[n] + c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{4}\right)^n \\
& \quad \text{Translate initial conditions} \\
y[n] &= \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + 2x[n] \\
y[0] &= \frac{3}{4}1 - \frac{1}{8}(-1) + 2(2) = \frac{39}{8} \\
y[1] &= \frac{3}{4}\left(\frac{39}{8}\right) - \frac{1}{8}1 + 2(2) = \frac{241}{32} \\
\frac{39}{8} &= \frac{32}{3} + c_1 + c_2 \\
\frac{241}{32} &= \frac{32}{3} + \frac{1}{2}c_1 + \frac{1}{4}c_2 \\
y[n] &= \frac{32}{3}u[n] - \frac{27}{4}\left(\frac{1}{2}\right)^n + \frac{23}{24}\left(\frac{1}{4}\right)^n
\end{aligned}
$$

III. Sơ đồ khối hệ thống

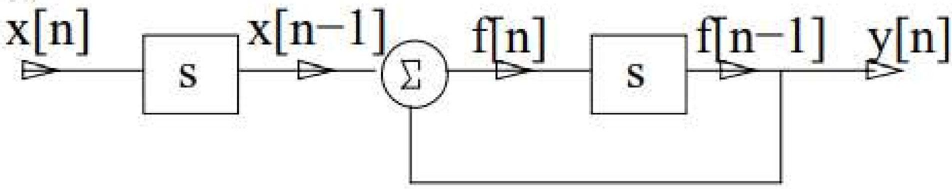
Bài 1:

(a)



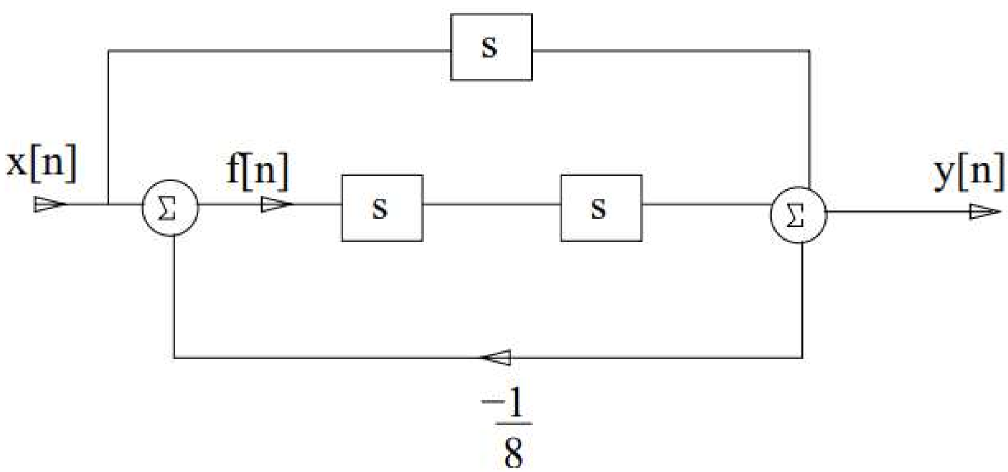
$$
\begin{aligned}
f[n] &= -2y[n] + x[n] \\
y[n] &= f[n-1] + 2f[n] \\
&= -2y[n-1] + x[n-1] - 4y[n] + 2x[n] \\
5y[n] + 2y[n-1] &= x[n-1] + 2x[n]
\end{aligned}
$$

(b)



$$
\begin{aligned}
f[n] &= y[n] + x[n-1] \\
y[n] &= f[n-1] \\
&= y[n-1] + x[n-2]
\end{aligned}
$$

c)



$$
\begin{aligned}
f[n] &= x[n] - \frac{1}{8}y[n] \\
y[n] &= x[n-1] + f[n-2] \\
y[n] + \frac{1}{8}y[n-2] &= x[n-1] + x[n-2]
\end{aligned}
$$

Bài 2:

(a) $y(t) = x^{(1)}(t) + 2y^{(1)}(t)$

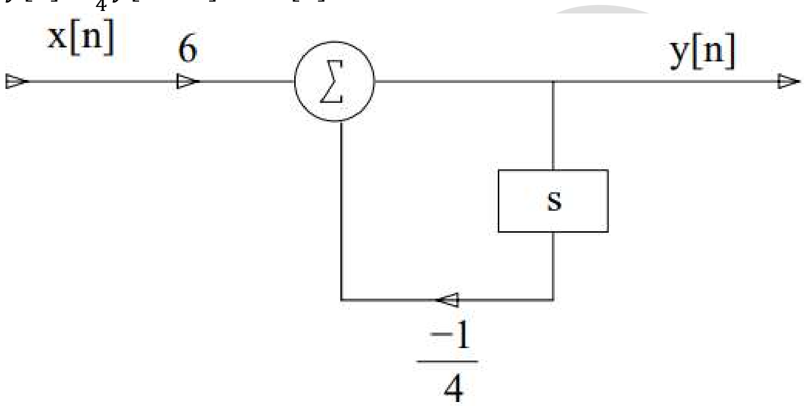
⇨ $\frac{d}{dt}y(t) - 2y(t) = x(t)$

(b) $y(t) = x^{(1)}(t) + 2y^{(1)}(t) - y^{(2)}(t)$

⇨ $\frac{d^2}{dt^2}y(t) - 2\frac{d}{dt}y(t) + y(t) = \frac{d}{dt}x(t)$

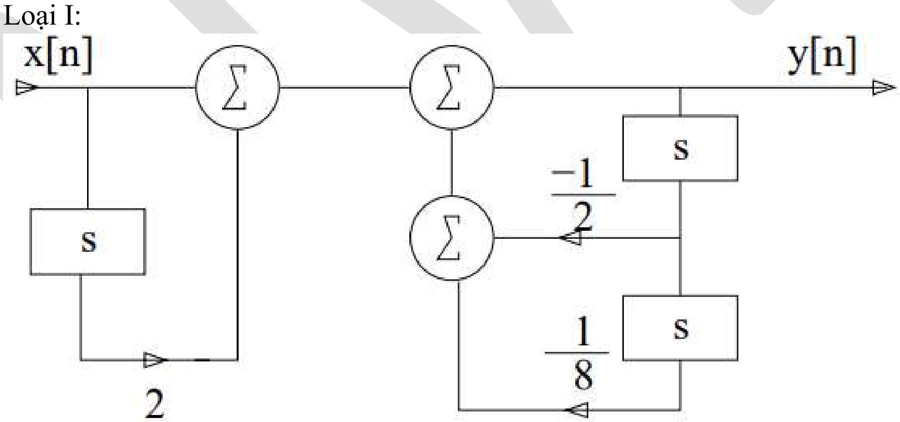
Bài 3:

(a) $y[n] - \frac{1}{4}y[n-1] = 6x[n]$

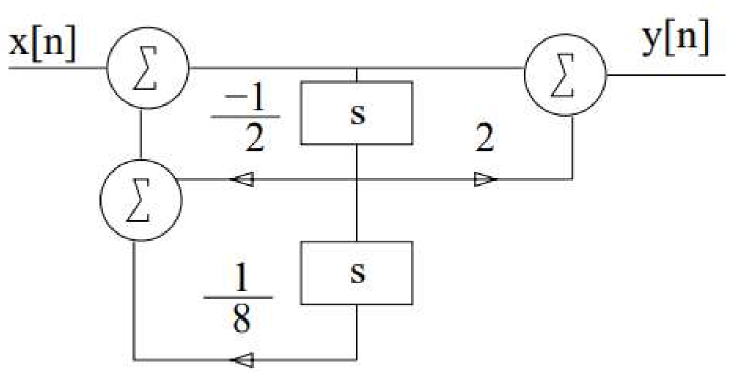


(b) $y[n] + \frac{1}{2}y[n-1] - \frac{1}{8}y[n-2] = x[n] + 2x[n-1]$

(i) Loại I:

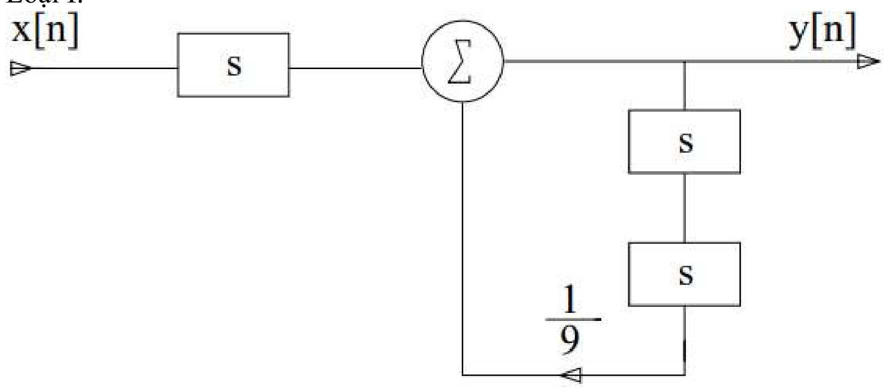


(ii) Loại II:

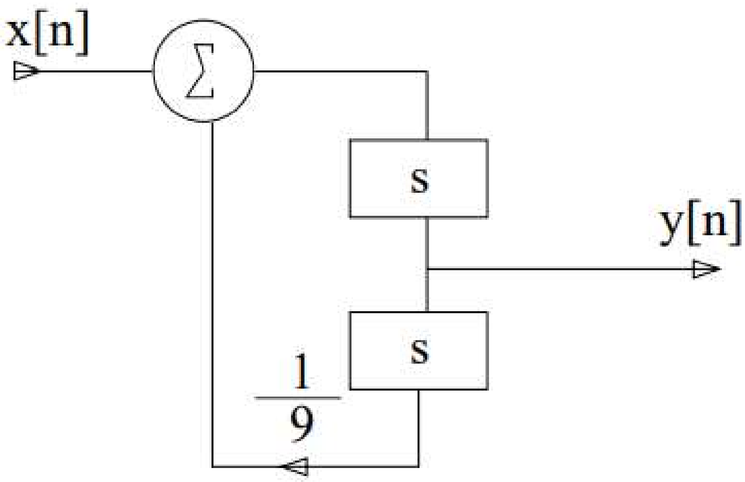


(c) $y[n]-\frac{1}{9}y[n-2]=x[n-1]$

(i) Loại I:

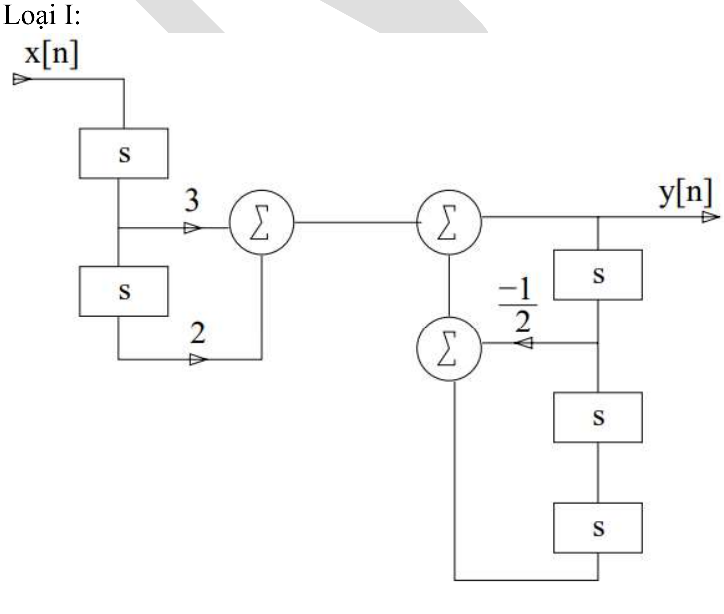


(ii) Loại II:

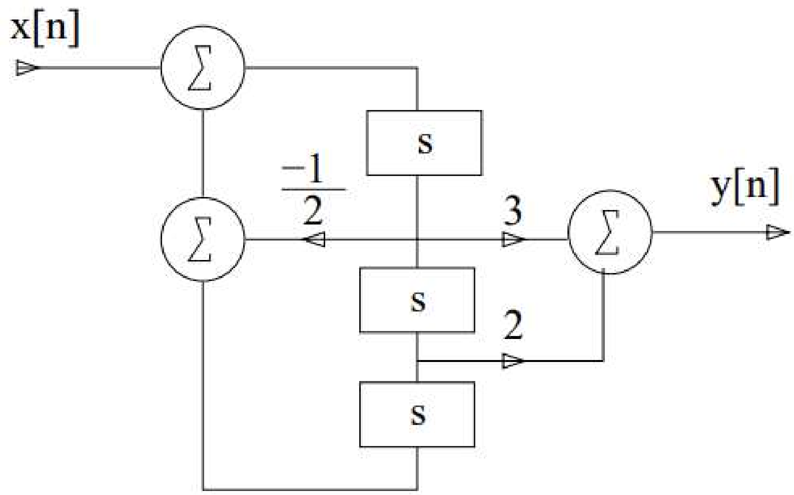


(d) $y[n]+\frac{1}{2}y[n-1]-y[n-3]=3x[n-1]+2x[n-2]$

(i) Loại I:



(ii) Loại II:

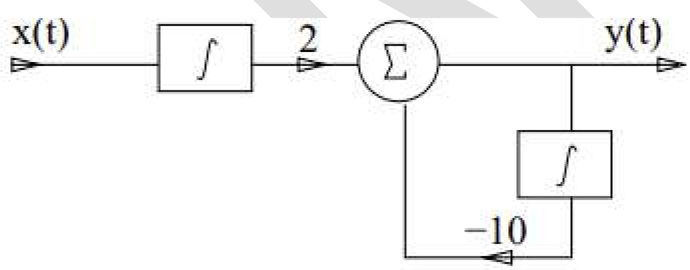


Bài 4:

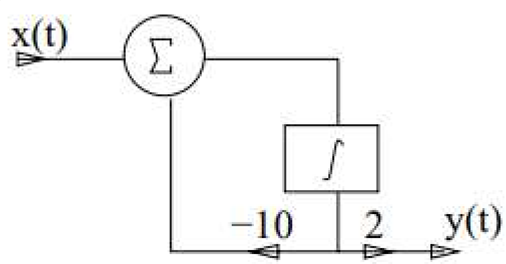
(a) $\frac{d}{dt}y(t) + 10y(t) = 2x(t)$

$$\begin{aligned} y(t) + 10y^{(1)}(t) &= 2x^{(1)}(t) \\ y(t) &= 2x^{(1)}(t) - 10y^{(1)}(t) \end{aligned}$$

Loại I



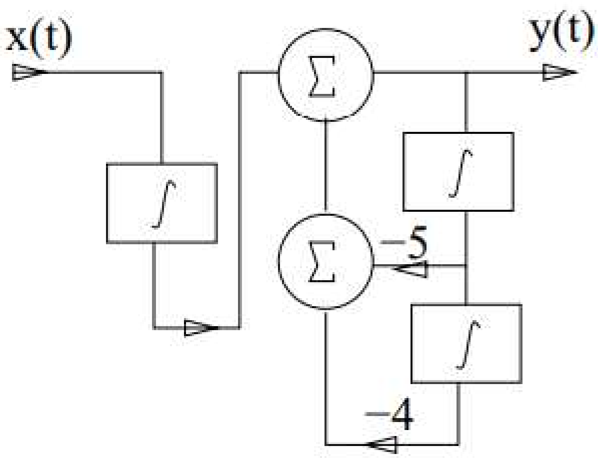
Loại II



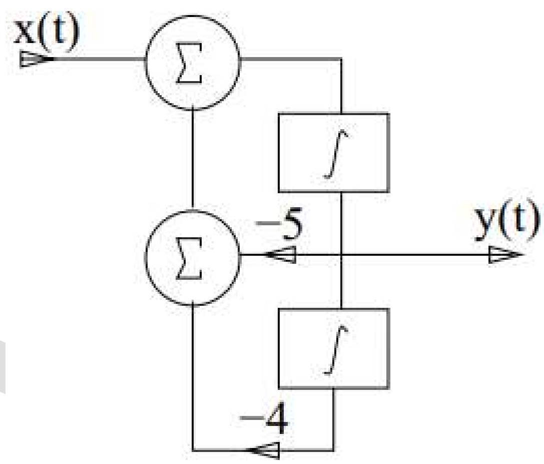
(b) $\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 4y(t) = \frac{d}{dt}x(t)$

$$\begin{aligned} y(t) + 5y^{(1)}(t) + 4y^{(2)}(t) &= x^{(1)}(t) \\ y(t) &= x^{(1)}(t) - 5y^{(1)}(t) - 4y^{(2)}(t) \end{aligned}$$

Loại I



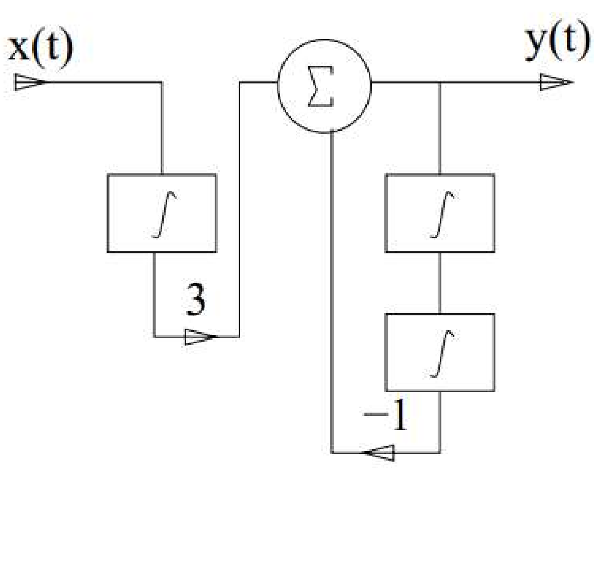
Loại II



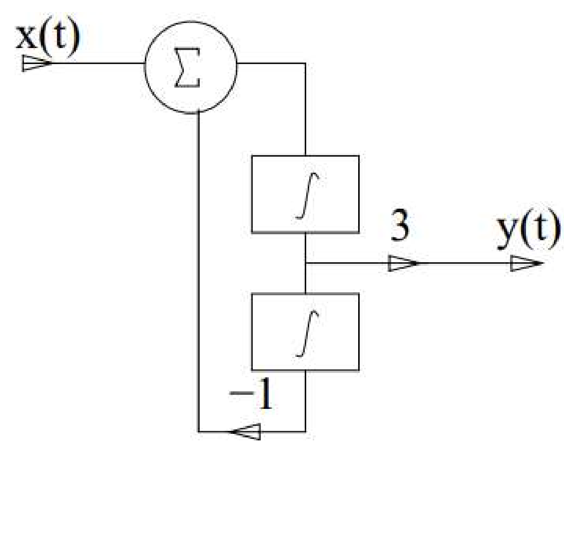
(c) $\frac{d^2}{dt^2}y(t) + y(t) = 3\frac{d}{dt}x(t)$

$$\begin{aligned} y(t) + y^{(2)}(t) &= 3x^{(1)}(t) \\ y(t) &= 3x^{(1)}(t) - y^{(2)}(t) \end{aligned}$$

Loại I



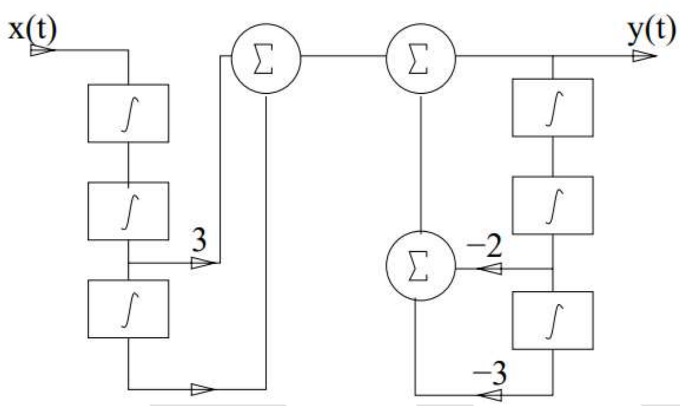
Loại II



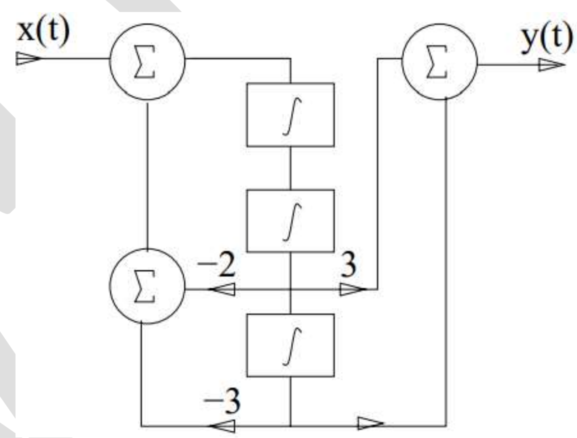
(d) $\frac{d^3}{dt^3}y(t) + 2\frac{d}{dt}y(t) + 3y(t) = x(t) + 3\frac{d}{dt}x(t)$

$$\begin{aligned} y(t) + 2y^{(2)}(t) + 3y^{(3)}(t) &= x^{(3)}(t) + 3x^{(2)}(t) \\ y(t) &= x^{(3)}(t) + 3x^{(2)}(t) - 2y^{(2)}(t) - 3y^{(3)}(t) \end{aligned}$$

Loại I



Loại II



IV. Phân loại hệ thống

Bài 1:

Tính nhân quả

Một hệ TTBB là nhân quả nếu và chỉ nếu đáp ứng xung của nó nhân quả: $h(t) = 0$ với $\forall t < 0$.

Tính ổn định

Một hệ TTBB là ổn định nếu và chỉ nếu đáp ứng xung của nó thoả mãn: $\int_{-\infty}^{\infty} |h(t)|dt < \infty$.

Hệ thống có nhớ/không nhớ

Một hệ TTBB là không nhớ (hệ thống tĩnh) nếu và chỉ nếu đáp ứng xung của nó thoả mãn: $h(t) = 0$ với $\forall t \neq 0$.
Ngược lại gọi là hệ thống là có nhớ (hệ thống động).

(a) $h(t) = \cos(\pi t)$
   (i) Có nhớ
   (ii) Phi nhân quả
   (iii) Không ổn định
(b) $h(t) = e^{-2t}u(t-1)$
   (i) Có nhớ
   (ii) Nhân quả
   (iii) Ổn định
(c) $h(t) = u(t-1)$
   (i) Có nhớ
   (ii) Phi nhân quả
   (iii) Không ổn định
(d) $h(t) = 3\delta(t)$
   (i) Không nhớ
   (ii) Nhân quả
   (iii) Ổn định
(e) $h(t) = \cos(\pi t)\, u(t)$
   (i) Có nhớ
   (ii) Nhân quả
   (iii) Không ổn dịnh
(f) $h[n] = (-1)^n u[n]$
   (i) Có nhớ
   (ii) Phi nhân quả
   (iii) Không ổn định
(g) $h[n] = \left(\frac{1}{2}\right)^{|n|}$
   (i) Có nhớ
   (ii) Phi nhân quả
   (iii) Không ổn định
(h) $h[n] = \cos\left(\frac{\pi}{8}n\right)\{u[n] - u[n-10]\}$
   (i) Có nhớ
   (ii) Nhân quả
   (iii) Ổn định
(i) $h[n] = 2u[n] - 2u[n-5]$
   (i) Có nhớ
   (ii) Nhân quả
   (iii) Ổn định
(j) $h[n] = \sin\left(\frac{\pi}{2}n\right)$
   (i) Có nhớ
   (ii) Phi nhân quả
   (iii) Không ổn định
(k) $h[n] = \sum_{p=-1}^{\infty} \delta[n-2p]$
   (i) Có nhớ
   (ii) Phi nhân quả
   (iii) Không ổn định