

TÍN HIỆU VÀ HỆ THỐNG

Chương 3: Biểu diễn hệ thống tuyến tính bất biến trong miền tần số

Phần 3: BIỂU DIỄN FOURIER - TỔNG HỢP

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BẢNG TỔNG HỢP

Time Domain	Periodic (t, n)	Non periodic (t, n)	
Continuous (t)	Fourier Series $x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_0 t}$ $X[k] = \frac{1}{T} \int_0^T x(t)e^{-jk\omega_0 t} dt$ $x(t)$ has period T $\omega_0 = \frac{2\pi}{T}$	Fourier Transform $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)e^{j\omega t} d\omega$ $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$	Non periodic (k, ω)
Discrete (n)	Discrete-Time Fourier Series $x[n] = \sum_{k=0}^{N-1} X[k]e^{jk\Omega_0 n}$ $X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n]e^{-jk\Omega_0 n}$ $x[n]$ and $X[k]$ have period N $\Omega_0 = \frac{2\pi}{N}$	Discrete-Time Fourier Transform $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n} d\Omega$ $X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$ $X(e^{j\Omega})$ has period 2π	Periodic (k, Ω)
	Discrete (k)	Continuous (ω, Ω)	Frequency Domain

TÍNH ĐỔI LẦN GIỮA MIỀN THỜI GIAN VÀ MIỀN TẦN SỐ

<i>Time-Domain Property</i>	<i>Frequency-Domain Property</i>
continuous	nonperiodic
discrete	periodic
periodic	discrete
nonperiodic	continuous



TUYẾN TÍNH

$$z(t) = ax(t) + by(t)$$

$$z(t) = ax(t) + by(t)$$

$$z[n] = ax[n] + by[n]$$

$$z[n] = ax[n] + by[n]$$

$$\longleftrightarrow^{FT}$$

$$\longleftrightarrow^{FS; \omega_0}$$

$$\longleftrightarrow^{DTFT}$$

$$\longleftrightarrow^{DTFS; \Omega_0}$$

$$Z(j\omega) = aX(j\omega) + bY(j\omega)$$

$$Z[k] = aX[k] + bY[k]$$

$$Z(e^{j\Omega}) = aX(e^{j\Omega}) + bY(e^{j\Omega})$$

$$Z[k] = aX[k] + bY[k]$$

ĐỐI XỨNG

<i>Representation</i>	<i>Real-Valued Time Signals</i>	<i>Imaginary-Valued Time Signals</i>
FT	$X^*(j\omega) = X(-j\omega)$	$X^*(j\omega) = -X(-j\omega)$
FS	$X^*[k] = X[-k]$	$X^*[k] = -X[-k]$
DTFT	$X^*(e^{j\Omega}) = X(e^{-j\Omega})$	$X^*(e^{j\Omega}) = -X(e^{-j\Omega})$
DTFS	$X^*[k] = X[-k]$	$X^*[k] = -X[-k]$

NHÂN CHẬP

$$x(t) * z(t) \xleftrightarrow{FT} X(j\omega)Z(j\omega)$$

$$x(t) \circledast z(t) \xleftrightarrow{FS; \omega_0} TX[k]Z[k]$$

$$x[n] * z[n] \xleftrightarrow{DTFT} X(e^{j\Omega})Z(e^{j\Omega})$$

$$x[n] \circledast z[n] \xleftrightarrow{DTFS; \Omega_0} NX[k]Z[k]$$

NHÂN THƯỜNG

$$y(t) = g(t)x(t) \xleftrightarrow{FT} Y(j\omega) = \frac{1}{2\pi} G(j\omega) * X(j\omega)$$

$$y(t) = g(t)x(t) \xleftrightarrow{FT} Y(j\omega) = \sum_{k=-\infty}^{\infty} X[k]G(j(\omega - k\omega_o)).$$

VI PHÂN/TÍCH PHÂN

$$\frac{d}{dt}x(t) \xleftrightarrow{FT} j\omega X(j\omega)$$

$$\frac{d}{dt}x(t) \xleftrightarrow{FS; \omega_0} jk\omega_0 X[k]$$

$$-jtx(t) \xleftrightarrow{FT} \frac{d}{d\omega} X(j\omega)$$

$$-jnx[n] \xleftrightarrow{DTFT} \frac{d}{d\Omega} X(e^{j\Omega})$$

$$\int_{-\infty}^t x(\tau) d\tau \xleftrightarrow{FT} \frac{1}{j\omega} X(j\omega) + \pi X(j0)\delta(\omega)$$

DỊCH THỜI GIAN

$$x(t - t_o) \xleftrightarrow{FT} e^{-j\omega t_o} X(j\omega)$$

$$x(t - t_o) \xleftrightarrow{FS; \omega_o} e^{-jk\omega_o t_o} X[k]$$

$$x[n - n_o] \xleftrightarrow{DTFT} e^{-j\Omega n_o} X(e^{j\Omega})$$

$$x[n - n_o] \xleftrightarrow{DTFS; \Omega_o} e^{-jk\Omega_o n_o} X[k]$$

DỊCH TẦN SỐ

$$e^{j\gamma t}x(t) \xleftrightarrow{FT} X(j(\omega - \gamma))$$

$$e^{jk_0\omega_0 t}x(t) \xleftrightarrow{FS; \omega_0} X[k - k_0]$$

$$e^{j\Gamma n}x[n] \xleftrightarrow{DTFT} X(e^{j(\Omega - \Gamma)})$$

$$e^{jk_0\Omega_0 n}x[n] \xleftrightarrow{DTFS; \Omega_0} X[k - k_0]$$

PHÉP NÉN/GIÃN

$$z(t) = x(at) \xleftrightarrow{FT} (1/|a|)X(j\omega/a)$$

$$z(t) = x(at) \xleftrightarrow{FS; a\omega_o} Z[k] = X[k], \quad a > 0$$

QUAN HỆ PARSEVAL (PHỔ CÔNG SUẤT)

<i>Representation</i>	<i>Parseval Relation</i>
FT	$\int_{-\infty}^{\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) ^2 d\omega$
FS	$\frac{1}{T} \int_0^T x(t) ^2 dt = \sum_{k=-\infty}^{\infty} X[k] ^2$
DTFT	$\sum_{n=-\infty}^{\infty} x[n] ^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\Omega}) ^2 d\Omega$
DTFS	$\frac{1}{N} \sum_{n=0}^{N-1} x[n] ^2 = \sum_{k=0}^{N-1} X[k] ^2$