Bài 1

a)

$$x(t) = \cos\frac{\pi}{3}t + \sin\frac{\pi}{4}t = x_1(t) + x_2(t)$$

where $x_1(t) = \cos(\pi/3)t = \cos\omega_1 t$ is periodic with $T_1 = 2\pi/\omega_1 = 6$ and $x_2(t) = \sin(\pi/4)t = \sin\omega_2 t$ is periodic with $T_2 = 2\pi/\omega_2 = 8$. Since $T_1/T_2 = \frac{6}{8} = \frac{3}{4}$ is a rational number, $x(t)$ is periodic with fundamental period $T_0 = 4T_1 = 3T_2 = 24$.

b)

$$x(t) = \cos t + \sin\sqrt{2}\,t = x_1(t) + x_2(t)$$

where $x_1(t) = \cos t = \cos\omega_1 t$ is periodic with $T_1 = 2\pi/\omega_1 = 2\pi$ and $x_2(t) = \sin\sqrt{2}\,t = \sin\omega_2 t$ is periodic with $T_2 = 2\pi/\omega_2 = \sqrt{2}\,\pi$. Since $T_1/T_2 = \sqrt{2}$ is an irrational number, $x(t)$ is nonperiodic.

c)

$$x(t) = e^{j[(\pi/2)t - 1]} = e^{-j}e^{j(\pi/2)t} = e^{-j}e^{j\omega_0 t} \longrightarrow \omega_0 = \frac{\pi}{2}$$

$x(t)$ is periodic with fundamental period $T_0 = 2\pi/\omega_0 = 4$.

d)

$$x[n] = \cos\frac{\pi}{3}n + \sin\frac{\pi}{4}n = x_1[n] + x_2[n]$$

where

$$x_1[n] = \cos\frac{\pi}{3}n = \cos\Omega_1 n \longrightarrow \Omega_1 = \frac{\pi}{3}$$

$$x_2[n] = \sin\frac{\pi}{4}n = \cos\Omega_2 n \longrightarrow \Omega_2 = \frac{\pi}{4}$$

Since $\Omega_1/2\pi = \frac{1}{6}$ (= rational number), $x_1[n]$ is periodic with fundamental period $N_1 = 6$, and since $\Omega_2/2\pi = \frac{1}{8}$ (= rational number), $x_2[n]$ is periodic with fundamental period $N_2 = 8$. Thus, from the result of Prob. 1.15, $x[n]$ is periodic and its fundamental period is given by the least common multiple of 6 and 8, that is, $N_0 = 24$.

Bài 2

a)

The sinusoidal signal $x(t)$ is periodic with $T_0 = 2\pi/\omega_0$. Then by the result from Prob. 1.18, the average power of $x(t)$ is

$$P = \frac{1}{T_0}\int_0^{T_0}[x(t)]^2\,dt = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} A^2\cos^2(\omega_0 t + \theta)\,dt$$

$$= \frac{A^2\omega_0}{2\pi}\int_0^{2\pi/\omega_0}\frac{1}{2}[1 + \cos(2\omega_0 t + 2\theta)]\,dt = \frac{A^2}{2} < \infty$$

Thus, $x(t)$ is a power signal. Note that periodic signals are, in general, power signals.

b)

$$E = \lim_{T \to \infty} \int_{-T/2}^{T/2} |x(t)|^2 dt = \lim_{T \to \infty} \int_0^{T/2} t^2 \, dt = \lim_{T \to \infty} \frac{(T/2)^3}{3} = \infty$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \lim_{T \to \infty} \frac{1}{T} \int_0^{T/2} t^2 \, dt = \lim_{T \to \infty} \frac{1}{T} \frac{(T/2)^3}{3} = \infty$$

Thus, $x(t)$ is neither an energy signal nor a power signal.

Bài 3:

By Eq. (2.6)

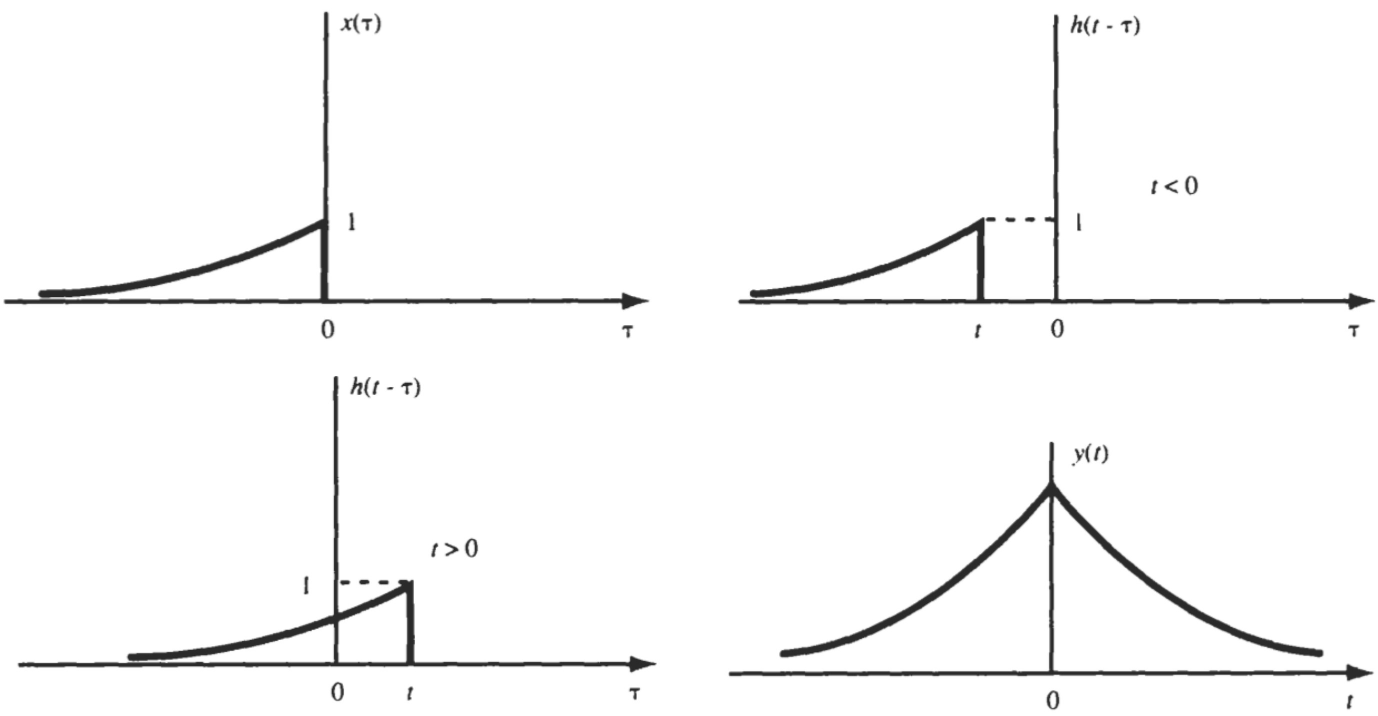
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) \, d\tau$$

Functions $x(\tau)$ and $h(t-\tau)$ are shown in Fig. 2-5(a) for $t < 0$ and $t > 0$. From Fig. 2-5(a) we see that for $t < 0$, $x(\tau)$ and $h(t - \tau)$ overlap from $\tau = -\infty$ to $\tau = t$, while for $t > 0$, they overlap from $\tau = -\infty$ to $\tau = 0$. Hence, for $t < 0$, we have

$$y(t) = \int_{-\infty}^{t} e^{\alpha\tau} e^{-\alpha(t-\tau)} \, d\tau = e^{-\alpha t} \int_{-\infty}^{t} e^{2\alpha\tau} \, d\tau = \frac{1}{2\alpha} e^{\alpha t} \tag{2.66a}$$

For $t > 0$, we have

$$y(t) = \int_{-\infty}^{0} e^{\alpha\tau} e^{-\alpha(t-\tau)} \, d\tau = e^{-\alpha t} \int_{-\infty}^{0} e^{2\alpha\tau} \, d\tau = \frac{1}{2\alpha} e^{-\alpha t} \tag{2.66b}$$



Combining Eqs. (2.66a) and (2.66b), we can write $y(t)$ as

$$y(t) = \frac{1}{2\alpha} e^{-\alpha|t|} \qquad \alpha > 0 \tag{2.67}$$

which is shown in Fig. 2-5(b).

Bài 4: https://www.youtube.com/watch?v=rzV1iWmqtX0
Bài 5: https://www.youtube.com/watch?v=C_kwjJLepLg