

Continuous-Time Fourier Transform (CTFT) & Discrete-Time Fourier Transform (DTFT)

TRAN Hoang Tung

Information and Communication Technology (ICT) Department
University of Science and Technology of Hanoi (USTH)

October 13 , 2015

1 Lesson Objectives

2 Continuous-Time Fourier Transform (CTFT)

3 Discrete-Time Fourier Transform (DTFT)

4 Homework

5 Mini-Test

Lesson Objectives

Signals &
Systems

TRAN
Hoang Tung

Objectives

CTFT

DTFT

Homework

Mini-Test

At the end of this lesson, you should be able to:

Lesson Objectives

Signals &
Systems

TRAN
Hoang Tung

Objectives

CTFT

DTFT

Homework

Mini-Test

At the end of this lesson, you should be able to:

- 1 convert signals from time-domain to frequency-domain

Lesson Objectives

Signals &
Systems

TRAN
Hoang Tung

Objectives

CTFT

DTFT

Homework

Mini-Test

At the end of this lesson, you should be able to:

- 1 convert signals from time-domain to frequency-domain
- 2 analyse systems using Fourier Transform

1 Lesson Objectives

2 Continuous-Time Fourier Transform (CTFT)

- Definition of CTFT
- CTFT for Signals
- Fourier Transform and Fourier Series
- CTFT for Systems

3 Discrete-Time Fourier Transform (DTFT)

4 Homework

5 Mini-Test

Continuous-Time Fourier Transform (CTFT)

Signals &
Systems

TRAN
Hoang Tung

Objectives

CTFT

Definition of
CTFT

CTFT for
Signals

Fourier Series

CTFT for
Systems

DTFT

Homework

Mini-Test

Definition

Time to Frequency:

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

Frequency to Time:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega$$

$$e^{-2(t-1)} \cdot u(t-1)$$

1 Lesson Objectives

2 Continuous-Time Fourier Transform (CTFT)

■ Definition of CTFT

■ CTFT for Signals

■ Fourier Transform and Fourier Series

■ CTFT for Systems

3 Discrete-Time Fourier Transform (DTFT)

4 Homework

5 Mini-Test

$\delta(t)$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

Signals &
Systems

TRAN
Hoang Tung

$$\delta(t) = \begin{cases} \neq 0 & \text{if } t = 0 \\ 0 & \text{else} \end{cases}$$

$$\int_{-\infty}^{+\infty} \delta(t) dt = 1$$

Objectives

CTFT

Definition of
CTFT

CTFT for
Signals

Fourier Series

CTFT for
Systems

DTFT

Homework

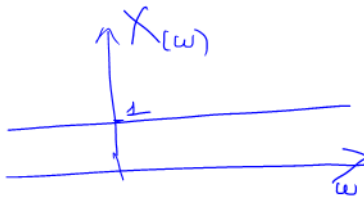
Mini-Test

Because $\delta(t) = 0$ everywhere except $t=0$

$$X(\omega) = \int_{-\infty}^{+\infty} \delta(t) \cdot e^{-j\omega t} dt$$

$$= \int_{-\infty}^{+\infty} \delta(t) \cdot \underbrace{e^{-j\omega(t=0)}}_1 dt$$

$$= \int_{-\infty}^{+\infty} \delta(t) dt = 1$$



$$\delta(t - 1)$$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

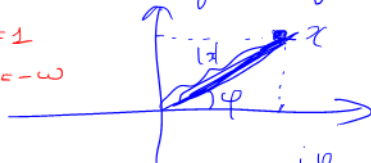
$$\underline{X(\omega) = e^{-j\omega} = 1 \cdot e^{j(-\omega)}}$$

↓

$$|X(\omega)| \cdot e^{j\angle X(\omega)}$$

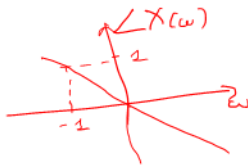
$$\Rightarrow \begin{cases} |X(\omega)| = 1 \\ \angle X(\omega) = -\omega \end{cases}$$

Complex plane



$$x = |x| e^{j\varphi}$$

$$\angle x = \varphi$$



$$\delta(t - 1)$$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

Time-Shift Property

if $x(t) \longleftrightarrow X(\omega)$ then:

$$x(t - t_0) \longleftrightarrow e^{-j\omega t_0} X(\omega)$$

Rectangular Signal

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

Signals &
Systems

TRAN
Hoang Tung

Objectives

CTFT

Definition of
CTFT

CTFT for
Signals

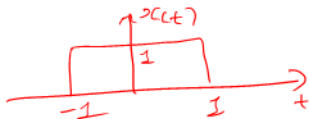
Fourier Series

CTFT for
Systems

DTFT

Homework

Mini-Test



$$x(t) = \begin{cases} 1 & -1 \leq t \leq 1 \\ 0 & \text{else} \end{cases}$$

$$\Rightarrow X(\omega) = \int_{-1}^1 e^{-j\omega t} dt$$

$$\text{sinc}(t) = \frac{\sin(t)}{t}$$

$$= \frac{e^{-j\omega t}}{-j\omega} \Big|_{-1}^1 = \frac{1}{-j\omega} (e^{-j\omega} - e^{j\omega})$$

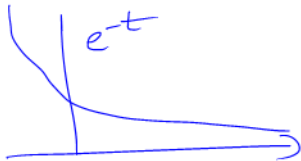
$$e^{j\omega} = \cos \omega + j \sin \omega$$

$$= 2 \frac{\sin \omega}{\omega}$$

$$e^{-t}u(t)$$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt$$

$$\begin{aligned} X(\omega) &= \int_{-\infty}^{+\infty} e^{-t} \cdot e^{-j\omega t} dt \\ &= \int_0^{+\infty} e^{-(1+j\omega)t} dt \end{aligned}$$



$$\begin{aligned} &= \left. \frac{e^{-(1+j\omega)t}}{-(1+j\omega)} \right|_0^{+\infty} \\ &= \frac{e^{-t} \cdot e^{-j\omega t}}{-(1+j\omega)} \Big|_{t=\infty} - \frac{1}{-(1+j\omega)} \\ &= \frac{1}{1+j\omega} \end{aligned}$$

$$e^{-t}u(t)$$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt$$

Formular

$$x(t) = e^{-at}u(t) \longleftrightarrow X(\omega) = \frac{1}{a + j\omega}$$

1 Lesson Objectives

2 Continuous-Time Fourier Transform (CTFT)

- Definition of CTFT
- CTFT for Signals
- **Fourier Transform and Fourier Series**
- CTFT for Systems

3 Discrete-Time Fourier Transform (DTFT)

4 Homework

5 Mini-Test

Fourier Transform for Periodic Signals

Signals &
Systems

TRAN
Hoang Tung

Objectives

CTFT

Definition of
CTFT

CTFT for
Signals

Fourier Series

CTFT for
Systems

DTFT

Homework

Mini-Test

Until Now

- Fourier **Series** only exists with **periodic** signals.
- Fourier **Transform** is for **aperiodic** signals.

Fourier Transform for Periodic Signals

Signals &
Systems

TRAN
Hoang Tung

Objectives

CTFT

Definition of
CTFT

CTFT for
Signals

Fourier Series

CTFT for
Systems

DTFT

Homework

Mini-Test

Until Now

- Fourier **Series** only exists with **periodic** signals.
- Fourier **Transform** is for **aperiodic** signals.

From Now on

- Fourier **Series** only exists with **periodic** signals.
- Fourier **Transform** is for **all** signals.

[Consult section 4.2 for more information.]

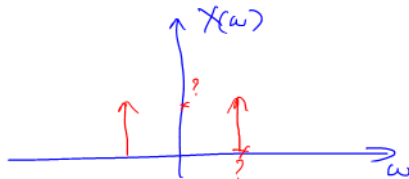
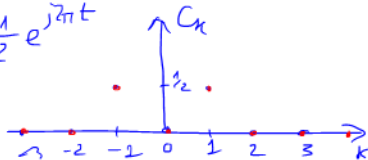
Fourier Transform of $\cos(2\pi t)$

Signals &
Systems

TRAN
Hoang Tung

$$\cos(2\pi t) = \frac{e^{j2\pi t} + e^{-j2\pi t}}{2} \quad \begin{cases} C_{-1} = \frac{1}{2} \\ C_1 = \frac{1}{2} \end{cases}$$

$$= \frac{1}{2} \cdot e^{-j2\pi t} + \frac{1}{2} e^{j2\pi t}$$



Objectives

CTFT

Definition of
CTFT

CTFT for
Signals

Fourier Series

CTFT for
Systems

DTFT

Homework

Mini-Test

1 Lesson Objectives

2 Continuous-Time Fourier Transform (CTFT)

- Definition of CTFT
- CTFT for Signals
- Fourier Transform and Fourier Series
- CTFT for Systems

3 Discrete-Time Fourier Transform (DTFT)

4 Homework

5 Mini-Test

Convolution & Fourier

$$y(t) = x(t) * h(t) \longleftrightarrow Y(\omega) = X(\omega)H(\omega)$$

Objectives

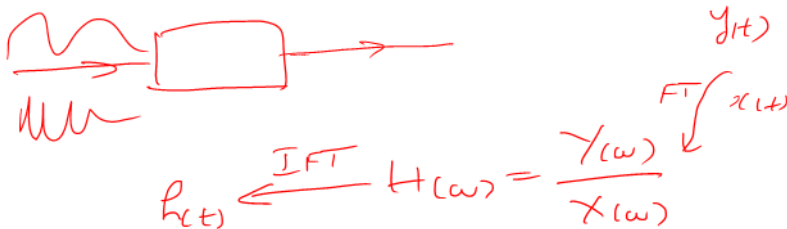
CTFT

Definition of
CTFT
CTFT for
Signals
Fourier Series
CTFT for
Systems

DTFT

Homework

Mini-Test



Convolution & Fourier

$$y(t) = x(t) * h(t) \longleftrightarrow Y(\omega) = X(\omega)H(\omega)$$

Differentiation Property

if $x(t) \longleftrightarrow X(\omega)$ then:

$$\frac{dx(t)}{dt} \longleftrightarrow j\omega X(\omega)$$

Example

Signals &
Systems

TRAN
Hoang Tung

Objectives

CTFT

Definition of
CTFT

CTFT for
Signals

Fourier Series

CTFT for
Systems

DTFT

Homework

Mini-Test

Given a LTI system by the differential equation:

$$\frac{d^2 y(t)}{dt^2} + 6 \frac{dy(t)}{dt} + 8y(t) = x(t)$$

Find the impulse of this system

Guide: $x(t) \rightarrow X(\omega)$
 $y(t) \rightarrow Y(\omega)$
 $(\dots) Y(\omega) = X(\omega)$

$$\Rightarrow H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{(\dots)}$$

↓
 $h(t)$

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = x(t)$$

Signals & Systems

TRAN
Hoang Tung

Objectives

CTFT

Definition of
CTFT

CTFT for
Signals

Fourier Series

**CTFT for
Systems**

DTFT

Homework

Mini-Test

1 Lesson Objectives

2 Continuous-Time Fourier Transform (CTFT)

3 Discrete-Time Fourier Transform (DTFT)

- Definition of DTFT
- DTFT for Signals

4 Homework

5 Mini-Test

Discrete-Time Fourier Transform (DTFT)

Signals &
Systems

TRAN
Hoang Tung

Objectives

CTFT

DTFT

Definition of
DTFT

DTFT for
Signals

Homework

Mini-Test

Definition

Time to Frequency:

$$X(\omega) = \sum_{n=-\infty}^{+\infty} x(n)e^{-j\omega n}$$

Frequency to Time:

$$x(n) = \frac{1}{2\pi} \int_{2\pi} X(\omega)e^{j\omega n} d\omega$$

1 Lesson Objectives

2 Continuous-Time Fourier Transform (CTFT)

3 Discrete-Time Fourier Transform (DTFT)

■ Definition of DTFT

■ DTFT for Signals

4 Homework

5 Mini-Test

$$X(\omega) = \sum x(n)e^{-j\omega n}$$

- 1 Lesson Objectives
- 2 Continuous-Time Fourier Transform (CTFT)
- 3 Discrete-Time Fourier Transform (DTFT)
- 4 Homework**
- 5 Mini-Test

CTFT - Chapter 4

4.1, 4.2, 4.3, 4.7, 4.19, 4.21, 4.22

DTFT - Chapter 5

5.1, 5.2, 5.6, 5.19, 5.20, 5.22, 5.29

- 1 Lesson Objectives
- 2 Continuous-Time Fourier Transform (CTFT)
- 3 Discrete-Time Fourier Transform (DTFT)
- 4 Homework
- 5 Mini-Test**

$$e^{-2(t-1)} u(t-1)$$