Tables of Common Transform Pairs

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Engineers and students in communications and mathematics are confronted with transformations such as the z-Transform, the Fourier transform, or the Laplace transform. Often it is quite hard to quickly find the appropriate transform in a book or the Internet, much less to have a comprehensive overview of transformation pairs and corresponding properties.

In this document I compiled a handy collection of the most common transform pairs and properties of the

- \triangleright continuous-time frequency Fourier transform $(2\pi f)$,
- \triangleright continuous-time pulsation Fourier transform (ω) ,
- ▷ z-Transform,
- ▷ discrete-time Fourier transform DTFT, and
- \triangleright Laplace transform.

Please note that, before including a transformation pair in the table, I verified its correctness. Nevertheless, it is still possible that you may find errors or typos. I am very grateful to everyone dropping me a line and pointing out any concerns or typos.

Notation, Conventions, and Useful Formulas

| Imaginary unit | $j^2 = -1$ |
|--------------------------|--|
| Complex conjugate | $z = a + jb \longleftrightarrow z^* = a - jb$ |
| Real part | $\Re \mathfrak{e} \left\{ f(t) \right\} = \frac{1}{2} \left[f(t) + f^*(t) \right]$ |
| Imaginary part | $\mathfrak{Im}\left\{f(t)\right\} = \frac{1}{2j}\left[f(t) - f^*(t)\right]$ |
| Dirac delta/Unit impulse | $\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$ |
| Heaviside step/Unit step | $u[n] = \begin{cases} 1, & n \ge 0 \\ 0, & n < 0 \end{cases}$ |
| Sine/Cosine | $\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$ $\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$ |
| Sinc function | $\operatorname{sinc}(x) \equiv \frac{\sin(x)}{x}$ (unnormalized) |
| Rectangular function | $rect(\frac{t}{T}) = \begin{cases} 1 & \text{if } t \leqslant \frac{T}{2} \\ 0 & \text{if } t > \frac{T}{2} \end{cases}$ |
| Triangular function | triang $\left(\frac{t}{T}\right) = \operatorname{rect}\left(\frac{t}{T}\right) * \operatorname{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{ t }{T} & t \leqslant T \\ 0 & t > T \end{cases}$ |
| Convolution | continuous-time: $(f * g)(t) = \int_{-\infty}^{+\infty} f(\tau) g^*(t - \tau) d\tau$ |
| | discrete-time: $(u * v)[n] = \sum_{m=-\infty}^{\infty} u[m] v^*[n-m]$ |
| Parseval theorem | general statement: $\int_{-\infty}^{+\infty} f(t)g^*(t)dt = \int_{-\infty}^{+\infty} F(f)G^*(f)df$ |
| | continuous-time: $\int_{-\infty}^{+\infty} f(t) ^2 dt = \int_{-\infty}^{+\infty} F(f) ^2 df$ |
| | discrete-time: $\sum_{n=-\infty}^{+\infty} x[n] ^2 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(e^{j\omega}) ^2 d\omega$ |
| Geometric series | $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \qquad \sum_{k=0}^{n} x^k = \frac{1-x^{n+1}}{1-x}$ |
| | in general: $\sum_{k=m}^{n} x^k = \frac{x^m - x^{n+1}}{1 - x}$ |

Table of Continuous-time Frequency Fourier Transform Pairs

| $f(t) = \mathcal{F}^{-1} \left\{ F(f) \right\} = \int_{-\infty}^{+\infty} f(t) e^{j2\pi f t} df$ | $\overset{\mathcal{F}}{\longleftrightarrow}$ | $F(f) = \mathcal{F}\left\{f(t)\right\} = \int_{-\infty}^{+\infty} f(t)e^{-j2\pi ft}dt$ |
|---|--|---|
| transform $f(t)$ time reversal $f(-t)$ complex conjugation $f^*(t)$ reversed conjugation $f^*(-t)$ | $ \begin{array}{c} \stackrel{\mathcal{F}}{\longleftrightarrow} \\ \stackrel{\mathcal{F}}{\longleftrightarrow} \\ \stackrel{\mathcal{F}}{\longleftrightarrow} \\ \stackrel{\mathcal{F}}{\longleftrightarrow} \end{array} $ | $F(f)$ $F(-f)$ frequency reversal $F^*(-f)$ reversed conjugation $F^*(f)$ complex conjugation |
| $f(t) \text{ is purely real}$ $f(t) \text{ is purely imaginary}$ $\text{even/symmetry} \qquad f(t) = f^*(-t)$ $\text{odd/antisymmetry} \qquad f(t) = -f^*(-t)$ | $ \begin{array}{c} $ | $F(f) = F^*(-f)$ even/symmetry $F(f) = -F^*(-f)$ odd/antisymmetry $F(f)$ is purely real $F(f)$ is purely imaginary |
| time shifting $f(t-t_0)$ $f(t)e^{j2\pi f_0t}$ time scaling $f(af)$ $\frac{1}{ a }f\left(\frac{f}{a}\right)$ | $ \begin{array}{c} \stackrel{\mathcal{F}}{\longleftrightarrow} \\ \stackrel{\mathcal{F}}{\longleftrightarrow} \\ \stackrel{\mathcal{F}}{\longleftrightarrow} \\ \stackrel{\mathcal{F}}{\longleftrightarrow} \end{array} $ | $F(f)e^{-j2\pi ft_0}$ $F(f-f_0)$ frequency shifting $\frac{1}{ a }F\left(\frac{f}{a}\right)$ $F(af)$ frequency scaling |
| linearity $af(t) + bg(t)$ time multiplication $f(t)g(t)$ frequency convolution $f(t) * g(t)$ | $ \begin{array}{c} \stackrel{\mathcal{F}}{\longleftrightarrow} \\ \stackrel{\mathcal{F}}{\longleftrightarrow} \\ \stackrel{\mathcal{F}}{\longleftrightarrow} \end{array} $ | aF(f) + bG(t) F(f) * G(f) frequency convolution F(f)G(f) frequency multiplication |
| delta function $\delta(t)$ shifted delta function $\delta(t-t_0)$ 1 $e^{j2\pi f_0 t}$ | $ \begin{array}{c} \stackrel{\mathcal{F}}{\longleftrightarrow} \\ \stackrel{\mathcal{F}}{\longleftrightarrow} \\ \stackrel{\mathcal{F}}{\longleftrightarrow} \end{array} $ | 1 $e^{-j2\pi ft_0}$ $\delta(f)$ delta function $\delta(f-f_0)$ shifted delta function |
| two-sided exponential decay $e^{-a t } a>0$ $e^{-\pi t^2}$ $e^{j\pi t^2}$ | $ \begin{array}{c} $ | $ \frac{\frac{2a}{a^2+4\pi^2f^2}}{e^{-\pi f^2}} $ $ e^{j\pi(\frac{1}{4}-f^2)} $ |
| $\begin{array}{lll} \text{sine} & & \sin{(2\pi f_0 t + \phi)} \\ \text{cosine} & & \cos{(2\pi f_0 t + \phi)} \\ \text{sine modulation} & & f(t)\sin{(2\pi f_0 t)} \\ \text{cosine modulation} & & f(t)\cos{(2\pi f_0 t)} \\ \text{squared sine} & & \sin^2{(t)} \\ \text{squared cosine} & & \cos^2{(t)} \end{array}$ | $ \begin{array}{c} \stackrel{\mathcal{F}}{\longleftrightarrow}\\ \stackrel{\mathcal{F}}{\longleftrightarrow}\\ \stackrel{\mathcal{F}}{\longleftrightarrow}\\ \stackrel{\mathcal{F}}{\longleftrightarrow}\\ \stackrel{\mathcal{F}}{\longleftrightarrow}\\ \stackrel{\mathcal{F}}{\longleftrightarrow}\\ \stackrel{\mathcal{F}}{\longleftrightarrow}\\ $ | $ \frac{j}{2} \left[e^{-j\phi} \delta \left(f + f_0 \right) - e^{j\phi} \delta \left(f - f_0 \right) \right] \frac{1}{2} \left[e^{-j\phi} \delta \left(f + f_0 \right) + e^{j\phi} \delta \left(f - f_0 \right) \right] \frac{j}{2} \left[F \left(f + f_0 \right) - F \left(f - f_0 \right) \right] \frac{1}{2} \left[F \left(f + f_0 \right) + F \left(f - f_0 \right) \right] \frac{1}{4} \left[2\delta(f) - \delta \left(f - \frac{1}{\pi} \right) - \delta \left(f + \frac{1}{\pi} \right) \right] \frac{1}{4} \left[2\delta(f) + \delta \left(f - \frac{1}{\pi} \right) + \delta \left(f + \frac{1}{\pi} \right) \right] $ |
| rectangular $\operatorname{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant \frac{T}{2} \\ 0 & t > \frac{T}{2} \end{cases}$ triangular $\operatorname{triang}\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{ t }{T} & t \leqslant T \\ 0 & t > T \end{cases}$ | $\stackrel{\mathcal{F}}{\Longleftrightarrow}$ $\stackrel{\mathcal{F}}{\Longleftrightarrow}$ \mathcal{F} | $T \operatorname{sinc} T f$ $T \operatorname{sinc}^2 T f$ |
| $\begin{array}{llllllllllllllllllllllllllllllllllll$ | $\stackrel{\stackrel{r}{\longleftrightarrow}}{\longleftrightarrow}$ $\stackrel{\mathcal{F}}{\longleftrightarrow}$ $\stackrel{\mathcal{F}}{\longleftrightarrow}$ | $\frac{1}{j2\pi f} + \delta(f)$ $\frac{1}{j\pi f}$ $\frac{1}{B} \operatorname{rect}\left(\frac{f}{B}\right) = \frac{1}{B} 1_{\left[-\frac{B}{2}, +\frac{B}{2}\right]}(f)$ $\frac{1}{B} \operatorname{triang}\left(\frac{f}{B}\right)$ |
| n -th time derivative $\frac{d^n}{dt^n}f(t)$ n -th frequency derivative $t^nf(t)$ $\frac{1}{1+t^2}$ | $ \begin{array}{c} \stackrel{\mathcal{F}}{\longleftrightarrow} \\ \stackrel{\mathcal{F}}{\longleftrightarrow} \\ \stackrel{\mathcal{F}}{\longleftrightarrow} \end{array} $ | $(j2\pi f)^n F(f)$ $\frac{1}{(-j2\pi)^n} \frac{d^n}{df^n} F(f)$ $\pi e^{-2\pi f }$ |

Table of Continuous-time Pulsation Fourier Transform Pairs

| time reversal $x(-t)$ $\xrightarrow{F_{so}}$ $X(-\omega)$ frequency reversal complex conjugation $x'(t)$ $\xrightarrow{F_{so}}$ $X^*(-\omega)$ reversed conjugation $x^*(-t)$ $\xrightarrow{F_{so}}$ $X^*(-\omega)$ complex conjugation $x^*(-t)$ $\xrightarrow{F_{so}}$ $X^*(\omega)$ complex conjugation $x^*(t)$ is purely read $x(t)$ is purely imaginary $x(t) = x^*(-t)$ $\xrightarrow{F_{so}}$ $X(f) = X^*(-\omega)$ odd/antisymmetry $x(t) = x^*(-t)$ $\xrightarrow{F_{so}}$ $X(\omega)$ is purely imaginary $x(t) = x^*(-t)$ $\xrightarrow{F_{so}}$ $x(\omega)$ $\xrightarrow{F_{so}}$ $x(\omega)$ is purely imaginary $x(t) = x^*(-t)$ $\xrightarrow{F_{so}}$ $x(\omega)$ $\xrightarrow{F_{so}}$ $x(\omega)$ is purely imaginary $x(t) = x^*(-t)$ $\xrightarrow{F_{so}}$ $x(\omega)$ $\xrightarrow{F_{so}}$ $x(\omega)$ is purely imaginary $x(t) = x^*(-t)$ $\xrightarrow{F_{so}}$ $x(\omega)$ $\xrightarrow{F_{so}}$ $x(\omega)$ is purely imaginary $x(t) = x^*(-t)$ $\xrightarrow{F_{so}}$ $x(\omega)$ $\xrightarrow{F_{so}}$ $x(\omega)$ is purely imaginary $x(\omega)$ in $x(\omega)$ | $x(t) = \mathcal{F}_{\omega}^{-1} \{X(\omega)\} = \int_{-\infty}^{+\infty} x(t)e^{j\omega t}d\omega$ | $\xleftarrow{\mathcal{F}_{\omega}}$ | $X(\omega) = \mathcal{F}_{\omega} \{x(t)\} = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$ |
|--|---|--|--|
| complex conjugation reversed conjugation $x^*(t)$ $\stackrel{F_{\infty}}{\Sigma_{\infty}} X^*(-\omega)$ reversed conjugation $x(t)$ is purely real $x(t)$ is purely imaginary $x(t)$ is purely imaginary $x(t)$ is purely imaginary $x(t)$ is purely imaginary $x(t)$ $x(t)$ is purely imaginary $x(t)$ | transform $x(t)$ | $\leftarrow \xrightarrow{\mathcal{F}_{\omega}}$ | $X(\omega)$ |
| $x(t) \text{ is purely real} \qquad x(t) \text{ is purely real} \qquad x(t) \text{ is purely imaginary} \qquad x(t) = x^*(-t) \qquad x(t) =$ | time reversal $x(-t)$ | $\stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow}$ | $X(-\omega)$ frequency reversa |
| $x(t) \text{ is purely real} \qquad x(t) \text{ is purely real} \qquad x(t) \text{ is purely imaginary} \qquad x(t) = x^*(-t) \qquad x(t) =$ | complex conjugation $x^*(t)$ | $\xleftarrow{\mathcal{F}_{\omega}}$ | $X^*(-\omega)$ reversed conjugation |
| $x(t) \text{ is purely imaginary} \\ x(t) = x^*(-t) \\ x(t) = x^*(-t) \\ x(t) = -x^*(-t) \\$ | reversed conjugation $x^*(-t)$ | | $X^*(\omega)$ complex conjugation |
| $x(t) \text{ is purely imaginary} \qquad x(t) = x^*(-t) \qquad x(t) \text{ is purely real odd/antisymmetry} \qquad x(t) = x^*(-t) \qquad x(t) \Rightarrow x(t) \Rightarrow$ | x(t) is purely real | $\leftarrow \xrightarrow{\mathcal{F}_{\omega}}$ | $X(f) = X^*(-\omega)$ even/symmetry |
| even/symmetry $x(t) = x^*(-t)$ $\stackrel{\mathcal{F}}{\longleftarrow} X(\omega)$ is purely real odd/antisymmetry $x(t) = -x^*(-t)$ $\stackrel{\mathcal{F}}{\longleftarrow} X(\omega)$ is purely imaginary time shifting $x(t) = -x^*(-t)$ $\stackrel{\mathcal{F}}{\longleftarrow} X(\omega)$ is purely imaginary time scaling $x(t) = -x^*(-t)$ $\stackrel{\mathcal{F}}{\longleftarrow} X(\omega) = -y\omega t_0$ frequency shifting $x(af) = \frac{1}{ a }x(\frac{a}{a})$ $\frac{1}{ a }x(\frac{a}{a})$ $\frac{1}{ a }x(\frac{a}{a})$ $\frac{1}{ a }x(\frac{a}{a})$ $\frac{1}{ a }x(\frac{a}{a})$ frequency scaling $\frac{1}{ a }x(t) + bx_2(t)$ $\frac{\mathcal{F}}{\longleftarrow} X(a\omega)$ frequency scaling it ime multiplication $x_1(t) + bx_2(t)$ $\frac{\mathcal{F}}{\longleftarrow} X_1(\omega) + bx_2(\omega)$ frequency convolution $x_1(t) * x_2(t)$ $\frac{\mathcal{F}}{\longleftarrow} X_1(\omega) \times x_2(\omega)$ frequency convolution frequency convolution $x_1(t) * x_2(t)$ $\frac{\mathcal{F}}{\longleftarrow} x_1(\omega) \times x_2(\omega)$ frequency multiplication $x_1(t) * x_2(t)$ $\frac{\mathcal{F}}{\longleftarrow} x_1(\omega) \times x_2(\omega)$ frequency multiplication $x_1(t) * x_2(t)$ $\frac{\mathcal{F}}{\longleftarrow} x_1(\omega) \times x_2(\omega)$ frequency multiplication $x_1(t) * x_2(t)$ $\frac{\mathcal{F}}{\longleftarrow} x_1(\omega) \times x_2(\omega)$ frequency multiplication $x_1(t) * x_2(t)$ $\frac{\mathcal{F}}{\longleftarrow} x_1(\omega) \times x_2(\omega)$ frequency multiplication $x_1(t) * x_2(t)$ $\frac{\mathcal{F}}{\longleftarrow} x_1(\omega) \times x_2(\omega)$ frequency convolution $x_1(t) * x_2(t)$ $\frac{\mathcal{F}}{\longleftarrow} x_1(\omega) \times x_2(\omega)$ frequency convolution $x_1(t) * x_2(t)$ $\frac{\mathcal{F}}{\longleftarrow} x_1(\omega) \times x_2(\omega)$ frequency convolution $x_1(t) * x_2(t) \times x_2(\omega)$ $\frac{\mathcal{F}}{\longleftarrow} x_1(\omega) \times x_2(\omega)$ frequency convolution $x_1(t) * x_2(t) \times x_2(\omega)$ $\frac{\mathcal{F}}{\longleftarrow} x_1(\omega) \times x_2(\omega)$ frequency convolution $x_1(t) \times x_2(t) \times x_2(\omega)$ $\frac{\mathcal{F}}{\longleftarrow} x_1(\omega) \times x_2(\omega)$ frequency convolution $x_1(t) \times x_2(t) \times x_2(\omega)$ $\frac{\mathcal{F}}{\longleftarrow} x_1(\omega) \times x_2(\omega)$ frequency convolution $x_1(t) \times x_2(t) \times x_2(\omega)$ $\frac{\mathcal{F}}{\longleftarrow} x_2(\omega) \times x_2(\omega)$ frequency convolution $x_1(t) \times x_2(t) \times x_2(\omega)$ frequency convolution $x_1(t) \times x_2(\omega)$ frequency convolution $x_1(t) \times x_2(\omega)$ frequency convolution $x_1(t) \times x_2(\omega)$ frequency convolution $x_1(t$ | | $\leftarrow \xrightarrow{\mathcal{F}_{\omega}}$ | |
| $ \begin{array}{c} \operatorname{odd/antisymmetry} & x(t) = -x^*(-t) & \xrightarrow{\mathcal{F}_{\omega}} & X(\omega) \text{ is purely imaginary} \\ \\ \operatorname{time shifting} & x(t-t_0) & \xrightarrow{\mathcal{F}_{\omega}} & X(\omega)e^{-j\omega t_0} \\ & x(t)e^{j\omega t_0t} & \xrightarrow{\mathcal{F}_{\omega}} & X(\omega-\omega_0) & \text{frequency shifting} \\ & x(af) & \xrightarrow{\mathcal{F}_{\omega}} & 1_{ \alpha }X\left(\frac{\omega}{a}\right) \\ & \frac{1}{ \alpha }x\left(\frac{1}{a}\right) & \xrightarrow{\mathcal{F}_{\omega}} & X(\omega\omega) & \text{frequency scaling} \\ \\ \operatorname{linearity} & ax_1(t) + bx_2(t) & \xrightarrow{\mathcal{F}_{\omega}} & aX_1(\omega) + bX_2(\omega) \\ \operatorname{time multiplication} & x_1(t)x_2(t) & \xrightarrow{\mathcal{F}_{\omega}} & \frac{1}{2\pi}X_1(\omega) * X_2(\omega) & \text{frequency convolution} \\ \operatorname{frequency convolution} & x_1(t)*x_2(t) & \xrightarrow{\mathcal{F}_{\omega}} & \frac{1}{2\pi}X_1(\omega) * X_2(\omega) & \text{frequency convolution} \\ \operatorname{frequency convolution} & x_1(t)*x_2(t) & \xrightarrow{\mathcal{F}_{\omega}} & \frac{1}{2\pi}X_1(\omega) * X_2(\omega) & \text{frequency convolution} \\ \operatorname{frequency convolution} & x_1(t)*x_2(t) & \xrightarrow{\mathcal{F}_{\omega}} & \frac{1}{2\pi}X_1(\omega) * X_2(\omega) & \text{frequency convolution} \\ \operatorname{frequency convolution} & x_1(t) * x_2(t) & \xrightarrow{\mathcal{F}_{\omega}} & \frac{1}{2\pi}X_1(\omega) * X_2(\omega) & \text{frequency convolution} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & \frac{1}{2\pi}X_1(\omega) * X_2(\omega) & \text{frequency convolution} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & \frac{1}{2\pi}X_1(\omega) * X_2(\omega) & \text{frequency convolution} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & \frac{1}{2\pi}X_1(\omega) * X_2(\omega) & \text{frequency convolution} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & \frac{1}{2\pi}X_1(\omega) * X_2(\omega) & \text{frequency convolution} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & 2\pi\delta(\omega) & \text{delta function} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & 2\pi\delta(\omega) & \text{delta function} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & 2\pi\delta(\omega) & \text{delta function} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & 2\pi\delta(\omega) & \text{delta function} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & 2\pi\delta(\omega) & \text{delta function} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & 2\pi\delta(\omega) & \text{delta function} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & 2\pi\delta(\omega) & \text{delta function} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & 2\pi\delta(\omega) & \text{delta function} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & 2\pi\delta(\omega) & \text{delta function} \\ \operatorname{delta function} & \delta(t) & \xrightarrow{\mathcal{F}_{\omega}} & 2\pi\delta(\omega) & \text{delta function} \\ delta$ | | | (*) |
| time shifting $x(t-t_0) \qquad \stackrel{\mathcal{F}_{\omega}}{\underset{x(t) \in \mathbb{P}^{\omega}}{\sum}} \qquad X(\omega)e^{-j\omega t_0} \qquad x(t)e^{-j\omega t_0} \qquad x(t)e^{-j\omega t_0} \qquad X(\omega-\omega_0) \qquad \text{frequency shifting time scaling} \qquad x(af) \qquad \stackrel{\mathcal{F}_{\omega}}{\underset{ a }{\sum}} \qquad \frac{1}{ a }X\left(\frac{a}{a}\right) \qquad \qquad x(af) \qquad \stackrel{\mathcal{F}_{\omega}}{\underset{ a }{\sum}} \qquad \frac{1}{ a }X\left(\frac{a}{a}\right) \qquad \qquad \text{frequency scaling} \qquad \qquad x(af) \qquad \stackrel{\mathcal{F}_{\omega}}{\underset{ a }{\sum}} \qquad \frac{1}{ a }X\left(\frac{a}{a}\right) \qquad \qquad \text{frequency scaling} \qquad \qquad x(af) \qquad \stackrel{\mathcal{F}_{\omega}}{\underset{ a }{\sum}} \qquad \frac{1}{ a }X\left(\frac{a}{a}\right) \qquad \qquad \text{frequency scaling} \qquad \qquad x(af) \qquad \stackrel{\mathcal{F}_{\omega}}{\underset{ a }{\sum}} \qquad \frac{1}{ a }X\left(\frac{a}{a}\right) \qquad \qquad \text{frequency scaling} \qquad \qquad x(af) \qquad \stackrel{\mathcal{F}_{\omega}}{\underset{ a }{\sum}} \qquad \frac{1}{ a }X\left(\frac{a}{a}\right) \qquad \qquad \text{frequency scaling} \qquad \qquad x(af) \qquad \stackrel{\mathcal{F}_{\omega}}{\underset{ a }{\sum}} \qquad \frac{1}{ a }X\left(\frac{a}{a}\right) \qquad \qquad \text{frequency scaling} \qquad \qquad x(af) \qquad \stackrel{\mathcal{F}_{\omega}}{\underset{ a }{\sum}} \qquad \frac{1}{ a }X\left(\frac{a}{a}\right) \qquad \qquad x(af) \qquad \qquad x(af) \qquad \stackrel{\mathcal{F}_{\omega}}{\underset{ a }{\sum}} \qquad x(a\omega) \qquad \qquad \text{frequency scaling} \qquad \qquad x(af) \qquad \qquad $ | | | • |
| $x(t)e^{j\omega_0t} \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad X(\omega-\omega_0) \qquad \text{frequency shifting} \\ x(af) \qquad \frac{\mathcal{F}_{\omega}}{ a } \times \frac{1}{ a } X\left(\frac{\omega}{a}\right) \qquad \text{frequency scaling} \\ \frac{1}{ a }x\left(\frac{t}{a}\right) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \frac{1}{ a }X\left(\frac{\omega}{a}\right) \qquad \text{frequency scaling} \\ \frac{1}{ a }x\left(\frac{t}{a}\right) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad X(a\omega) \qquad \text{frequency scaling} \\ \text{Ilinearity} \qquad ax_1(t) + bx_2(t) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad 2x_1(\omega) + bX_2(\omega) \qquad \text{frequency convolution} \\ x_1(t) * x_2(t) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \frac{1}{2\pi}X_1(\omega) * X_2(\omega) \qquad \text{frequency convolution} \\ \text{delta function} \qquad \delta(t) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad 2\pi\delta(\omega) \qquad \text{frequency multiplication} \\ \text{delta function} \qquad \delta(t-t_0) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad 2\pi\delta(\omega) \qquad \text{delta function} \\ \text{shifted delta function} \qquad \delta(t-t_0) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad 2\pi\delta(\omega-\omega_0) \qquad \text{shifted delta function} \\ \text{e}^{j\omega_0 t} \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad 2\pi\delta(\omega-\omega_0) \qquad \text{shifted delta function} \\ \text{evenomential decay} \qquad e^{-a t } \qquad 3 < 0 \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad 2\pi\delta(\omega-\omega_0) \qquad \text{shifted delta function} \\ \text{exponential decay} \qquad e^{-at}u(t) \Re\{a\} > 0 \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \frac{a^2+\omega^2}{a^2+\omega^2} \qquad 2\pi^2 \\ \text{exponential decay} \qquad e^{-at}u(t) \Re\{a\} > 0 \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \frac{1}{a^2+\omega^2} \qquad 2\pi^2 \\ \text{sine} \qquad \sin(\omega_0 t + \phi) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}^2} \qquad \mathcal{F}_{\omega} \qquad \sqrt{2\pi}e^{-\frac{2^2\omega^2}{2^2}} \qquad 2\pi^2 \\ \text{sine} \qquad \sin(\omega_0 t + \phi) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}^2} \qquad \mathcal{F}_{\omega} \qquad \pi \left[e^{-j\phi}\delta(\omega+\omega_0) - e^{j\phi}\delta(\omega-\omega_0)\right] \\ \text{sine mudulation} \qquad x(t) \sin(\omega_0 t) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \pi \left[e^{-j\phi}\delta(\omega+\omega_0) - e^{j\phi}\delta(\omega-\omega_0)\right] \\ \text{sine mudulation} \qquad x(t) \cos(\omega_0 t) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \pi \left[e^{-j\phi}\delta(\omega+\omega_0) - e^{j\phi}\delta(\omega-\omega_0)\right] \\ \text{sine mudulation} \qquad x(t) \sin(\omega_0 t) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \pi \left[e^{-j\phi}\delta(\omega+\omega_0) - e^{j\phi}\delta(\omega-\omega_0)\right] \\ \text{supposed sine} \qquad \sin^2(\omega_0 t) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \pi^2\left[2\delta(\omega + \omega_0) - k(\omega+\omega_0)\right] \\ \text{supposed sine} \qquad \sin^2(\omega_0 t) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \pi^2\left[2\delta(\omega + \omega_0) - k(\omega+\omega_0)\right] \\ \text{supposed sine} \qquad \cos^2(\omega_0 t) \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \pi^2\left[2\delta(\omega + \omega_0) - k(\omega+\omega_0) - k(\omega+\omega_0)\right] \\ \text{supposed sine} \qquad \sin^2(t) = \begin{cases} 1 & t \leq T \\ 0 & t > T \end{cases} \qquad T \sin^2(\frac{\omega^T}{2$ | | | |
| time scaling $ \begin{array}{c} x\left(af\right) & \xrightarrow{\mathcal{F}_{\omega}} & \frac{1}{ a }X\left(\frac{\omega}{a}\right) \\ \frac{1}{ a }x\left(\frac{f}{a}\right) & \xrightarrow{\mathcal{F}_{\omega}} & X(a\omega) \end{array} \text{frequency scaling} \\ \\ \frac{1}{ a }x\left(\frac{f}{a}\right) & \xrightarrow{\mathcal{F}_{\omega}} & X(a\omega) \end{array} \text{frequency scaling} \\ \\ \frac{1}{ a }x\left(\frac{f}{a}\right) & \xrightarrow{\mathcal{F}_{\omega}} & X(a\omega) \end{array} \text{frequency scaling} \\ \\ \frac{1}{ a }x\left(\frac{f}{a}\right) & \xrightarrow{\mathcal{F}_{\omega}} & X(a\omega) \end{array} \text{frequency scaling} \\ \\ \frac{1}{ a }x\left(\frac{f}{a}\right) & \xrightarrow{\mathcal{F}_{\omega}} & X(a\omega) \end{array} \text{frequency scaling} \\ \\ \frac{1}{ a }x\left(\frac{f}{a}\right) & \xrightarrow{\mathcal{F}_{\omega}} & X(a\omega) \end{array} \text{frequency scaling} \\ \\ \frac{1}{ a }x\left(\frac{f}{a}\right) & \xrightarrow{\mathcal{F}_{\omega}} & X(a\omega) \end{array} \text{frequency convolution} \\ \\ x_1(t)x_2(t) & \xrightarrow{\mathcal{F}_{\omega}} & X_1(\omega) + bX_2(\omega) \\ \\ x_1(\omega)x_2(\omega) & \text{frequency convolution} \\ \\ \\ x_1(\omega)x_2(\omega) & \xrightarrow{\mathcal{F}_{\omega}} & \frac{1}{2\pi}X_1(\omega) \times X_2(\omega) \end{array} \text{frequency convolution} \\ \\ \text{delta function} \\ \\ \text{shifted delta function} \\ \\ \text{shifted delta function} \\ \\ \\ \text{shifted delta function} \\ \\ \\ \text{shifted delta function} \\ \text{shifted delta function} \\ \text{shifted delta function} \\ \text{shifted delta function} \\ \\ \text{shifted delta function} $ | · • • • • • • • • • • • • • • • • • • • | | |
| $\frac{1}{ a } \chi \left(\frac{f}{a}\right) \qquad \xrightarrow{\mathcal{F}_{\omega}} \qquad \lambda(a\omega) \qquad \text{frequency scaling}$ linearity $ax_1(t) + bx_2(t) \qquad \xrightarrow{\mathcal{F}_{\omega}} \qquad aX_1(\omega) + bX_2(\omega)$ time multiplication $x_1(t) x_2(t) \qquad \xrightarrow{\mathcal{F}_{\omega}} \qquad \frac{1}{2\pi} X_1(\omega) * X_2(\omega) \qquad \text{frequency convolution}$ $x_1(t) * x_2(t) \qquad \xrightarrow{\mathcal{F}_{\omega}} \qquad \frac{1}{2\pi} X_1(\omega) * X_2(\omega) \qquad \text{frequency convolution}$ $x_1(t) * x_2(t) \qquad \xrightarrow{\mathcal{F}_{\omega}} \qquad \frac{1}{2\pi} X_1(\omega) X_2(\omega) \qquad \text{frequency multiplication}$ delta function $\delta(t) \qquad \xrightarrow{\mathcal{F}_{\omega}} \qquad 2\pi \delta(\omega) \qquad \text{delta function}$ shifted delta function $e^{j\omega_0 t} \qquad \xrightarrow{\mathcal{F}_{\omega}} \qquad 2\pi \delta(\omega) \qquad \text{delta function}$ two-sided exponential decay $e^{-a t } \qquad a > 0 \qquad \xrightarrow{\mathcal{F}_{\omega}} \qquad 2\pi \delta(\omega) \qquad \text{delta function}$ two-sided exponential decay $e^{-a t } \qquad a > 0 \qquad \xrightarrow{\mathcal{F}_{\omega}} \qquad 2\pi \delta(\omega) \qquad \text{delta function}$ two-sided exponential decay $e^{-at} u(t) \qquad \Re\{a\} > 0 \qquad \xrightarrow{\mathcal{F}_{\omega}} \qquad \frac{2a}{a^2 + a^2} \qquad$ | * * | | |
| linearity $ax_1(t) + bx_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} aX_1(\omega) + bX_2(\omega)$ time multiplication $x_1(t)x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency convolution $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency convolution frequency convolution $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency multiplication $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency multiplication $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency multiplication $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency multiplication $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency multiplication $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency convolution $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency convolution $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency convolution $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency convolution $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency convolution $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency multiplication $x_1(t) * x_2(t)$ $\stackrel{\mathcal{F}_{\omega}}{\longleftarrow} \frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ delta function $x_1(t) * x_2(\omega) * x_2(\omega)$ $\xrightarrow{x_1(t)} \frac{1}{2\pi}X_1(\omega) * x_2(\omega)$ delta function $x_1(t) * x_2(\omega) * x_2(\omega)$ $\xrightarrow{x_1(t)} \frac{1}{2\pi}X_1(\omega) * x_2(\omega)$ sine $x_1(t) * x_2(t) * x_2(\omega) * x_2(\omega)$ $\xrightarrow{x_1(t)} \frac{1}{2\pi}X_1(\omega) * x_2(\omega)$ shifted delta function $x_1(t) * x_2(\omega) * x_2(\omega)$ $\xrightarrow{x_1(t)} \frac{1}{2\pi}X_1(\omega) * x_2(\omega)$ shifted delta function $x_1(t) * x_2(\omega) * x_2(\omega)$ $\xrightarrow{x_1(t)} \frac{1}{2\pi}X_1(\omega) * x_2(\omega)$ shifted delta function $x_1(t) * x_2(\omega) * x_2(\omega)$ $\xrightarrow{x_1(t)} \frac{1}{2\pi}X_1(\omega) * x_2(\omega)$ shifted delta function $x_1(t) * x_2(\omega) * x_2(\omega)$ $\xrightarrow{x_1(t)} \frac{1}{2\pi}X_1(\omega) * x_2(\omega)$ shifted delta function $x_1(t) * x_2(\omega) * x_2(\omega)$ shifted delta function $x_1(t) * x$ | time scaling $x(af)$ | $\stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow}$ | $\frac{1}{ a }X\left(\frac{\omega}{a}\right)$ |
| time multiplication $x_1(t)x_2(t) \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \frac{1}{2\pi}X_1(\omega)*X_2(\omega) \text{frequency convolution}$ $x_1(t)*x_2(t) \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad X_1(\omega)X_2(\omega) \text{frequency multiplication}$ $\frac{\delta(t)}{\delta(t-t_0)} \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \frac{1}{2\pi\delta(\omega)} \qquad \frac{1}{\delta(\omega)} \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \frac{1}{\delta(\omega)} \qquad \frac{\mathcal{F}_{\omega}}{\delta(\omega)} \qquad \frac{2a}{a^2+\omega^2} \qquad \frac{1}{a+j\omega} \qquad \frac{1}{a+j\omega} \qquad \frac{1}{\delta(\omega)} \qquad \frac{1}{\delta(\omega)} \qquad \frac{\mathcal{F}_{\omega}}{\delta(\omega)} \qquad \frac{1}{\delta(\omega)} \qquad \frac{1}{\delta(\omega$ | $\frac{1}{ a }x\left(rac{f}{a} ight)$ | $\stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow}$ | $X(a\omega)$ frequency scaling |
| time multiplication $x_1(t)x_2(t) \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \frac{1}{2\pi}X_1(\omega)*X_2(\omega) \text{frequency convolution}$ $x_1(t)*x_2(t) \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad X_1(\omega)X_2(\omega) \text{frequency multiplication}$ $\frac{\delta(t)}{\delta(t-t_0)} \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \frac{1}{2\pi\delta(\omega)} \qquad \frac{1}{\delta(\omega)} \qquad \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \qquad \frac{1}{\delta(\omega)} \qquad \frac{\mathcal{F}_{\omega}}{\delta(\omega)} \qquad \frac{2a}{a^2+\omega^2} \qquad \frac{1}{a+j\omega} \qquad \frac{1}{a+j\omega} \qquad \frac{1}{\delta(\omega)} \qquad \frac{1}{\delta(\omega)} \qquad \frac{\mathcal{F}_{\omega}}{\delta(\omega)} \qquad \frac{1}{\delta(\omega)} \qquad \frac{1}{\delta(\omega$ | linearity $ax_1(t) + bx_2(t)$ | $\leftarrow \xrightarrow{\mathcal{F}_{\omega}}$ | $aX_1(\omega) + bX_2(\omega)$ |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | | \longleftrightarrow | $\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$ frequency convolution |
| $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | | $\stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow}$ | 211 |
| $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | dolto function S(t) | \mathcal{F}_{ω} | 1 |
| $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | ` ' | | |
| $e^{j\omega_0 t} \stackrel{F_{\omega}}{\longleftarrow} \qquad 2\pi\delta(\omega-\omega_0) \text{shifted delta function}$ two-sided exponential decay $e^{-a t } a > 0 \stackrel{F_{\omega}}{\longleftarrow} \qquad \frac{2a}{a^2+\omega^2}$ exponential decay $e^{-at}u(t) \Re\{a\} > 0 \stackrel{F_{\omega}}{\longleftarrow} \qquad \frac{1}{a-j\omega}$ reversed exponential decay $e^{-at}u(-t) \Re\{a\} > 0 \stackrel{F_{\omega}}{\longleftarrow} \qquad \frac{1}{a-j\omega}$ $e^{\frac{t^2}{2\sigma^2}} \stackrel{F_{\omega}}{\longleftarrow} \qquad \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$ sine $\sin(\omega_0 t + \phi) \stackrel{F_{\omega}}{\longleftarrow} \qquad \frac{1}{a-j\omega} \frac{1}{a-j\omega}$ of $e^{\frac{t^2}{2\sigma^2}} \stackrel{F_{\omega}}{\longleftarrow} \qquad \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$ sine $\sin(\omega_0 t + \phi) \stackrel{F_{\omega}}{\longleftarrow} \qquad \frac{1}{a-j\omega} \frac{1}{a-j\omega} \frac{1}{a-j\omega}$ of $e^{-at}u(-t) \Re\{a\} > 0 \stackrel{F_{\omega}}{\longleftarrow} \qquad \frac{1}{a-j\omega} \frac{1}{a-j\omega}$ of $e^{\frac{t^2}{2\sigma^2}} \stackrel{F_{\omega}}{\longleftarrow} \qquad \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$ sine $\cos(\omega_0 t + \phi) \stackrel{F_{\omega}}{\longleftarrow} \qquad \frac{1}{a-j\omega} $ | | | |
| two-sided exponential decay $e^{-a t }$ $a>0$ $\stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons}$ $\frac{2a}{a^2+\omega^2}$ exponential decay $e^{-at}u(t)$ $\Re\{a\}>0$ $\stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons}$ $\frac{1}{a-j\omega}$ reversed exponential decay $e^{-at}u(-t)$ $\Re\{a\}>0$ $\stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons}$ $\frac{1}{a-j\omega}$ $\frac{1}{$ | _ | | * / |
| exponential decay $e^{-at}u(t)$ $\Re\{a\} > 0$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\frac{1}{a+j\omega}$ reversed exponential decay $e^{-at}u(-t)$ $\Re\{a\} > 0$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\frac{1}{a-j\omega}$ $\frac{t^2}{2\sigma^2}$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$ $\frac{1}{2}$ sine $\sin(\omega_0 t + \phi)$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\int j\pi \left[e^{-j\phi}\delta\left(\omega + \omega_0\right) - e^{j\phi}\delta\left(\omega - \omega_0\right)\right]$ cosine $\cos(\omega_0 t + \phi)$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\pi \left[e^{-j\phi}\delta\left(\omega + \omega_0\right) - e^{j\phi}\delta\left(\omega - \omega_0\right)\right]$ sine modulation $x(t)\sin(\omega_0 t)$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\frac{1}{2}\left[X\left(\omega + \omega_0\right) - X\left(\omega - \omega_0\right)\right]$ cosine modulation $x(t)\cos(\omega_0 t)$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\frac{1}{2}\left[X\left(\omega + \omega_0\right) + X\left(\omega - \omega_0\right)\right]$ squared sine $\sin^2(\omega_0 t)$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\pi^2\left[2\delta(f) - \delta\left(\omega - \omega_0\right) - \delta\left(\omega + \omega_0\right)\right]$ squared cosine $\cos^2(\omega_0 t)$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\pi^2\left[2\delta(\omega) + \delta\left(\omega - \omega_0\right) + \delta\left(\omega + \omega_0\right)\right]$ rectangular $\cot\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant \frac{T}{2} \\ 0 & t > \frac{T}{2} \end{cases}$ $\xrightarrow{\mathcal{F}_{\omega}}$ $T \operatorname{sinc}\left(\frac{\omega T}{2}\right)$ triangular $\cot\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant T \\ 0 & t > T \end{cases}$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\pi\delta(f) + \frac{1}{j\omega}$ signum $\operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases}$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\frac{1}{T} \operatorname{rect}\left(\frac{\omega T}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f)$ squared sinc $\operatorname{sinc}^2(Tt)$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\frac{1}{T} \operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ | <i>e</i> 1∞0° | | $2\pi\delta(\omega-\omega_0)$ shifted delta function |
| reversed exponential decay $e^{-at}u(-t)$ $\Re\{a\} > 0$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\frac{1}{a-j\omega}$ $\frac{1}{a-j\omega}$ $\frac{t^2}{2\sigma^2}$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$ sine $\sin(\omega_0t+\phi)$ $\xrightarrow{\mathcal{F}_{\omega}}$ $\int_{\mathcal{F}_{\omega}} \int_{\mathcal{F}_{\omega}} \int_{$ | · | | |
| $\frac{t^2}{e^{\frac{1}{2}\sigma^2}} \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$ sine $\sin(\omega_0t + \phi) \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} j\pi \left[e^{-j\phi}\delta\left(\omega + \omega_0\right) - e^{j\phi}\delta\left(\omega - \omega_0\right)\right]$ cosine $\cos(\omega_0t + \phi) \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} \pi \left[e^{-j\phi}\delta\left(\omega + \omega_0\right) - e^{j\phi}\delta\left(\omega - \omega_0\right)\right]$ sine modulation $x(t)\sin(\omega_0t) \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} \frac{j}{2}\left[X\left(\omega + \omega_0\right) - X\left(\omega - \omega_0\right)\right]$ cosine modulation $x(t)\cos(\omega_0t) \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} \frac{1}{2}\left[X\left(\omega + \omega_0\right) - X\left(\omega - \omega_0\right)\right]$ squared sine $\sin^2(\omega_0t) \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} \pi^2\left[2\delta(f) - \delta\left(\omega - \omega_0\right) - \delta\left(\omega + \omega_0\right)\right]$ squared cosine $\cos^2(\omega_0t) \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} \pi^2\left[2\delta(\omega) + \delta\left(\omega - \omega_0\right) + \delta\left(\omega + \omega_0\right)\right]$ rectangular $\cot\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant \frac{T}{2} \\ 0 & t > \frac{T}{2} \end{cases} \qquad \mathcal{F}_{\omega} \qquad T \operatorname{sinc}\left(\frac{\omega T}{2}\right)$ triangular $\tan\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{ t }{T} & t \leqslant T \\ 0 & t > T \end{cases} \qquad \mathcal{F}_{\omega} \qquad T \operatorname{sinc}^2\left(\frac{\omega T}{2}\right)$ step $u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \qquad \pi\delta(f) + \frac{1}{j\omega}$ signum $\sin\left(Tt\right) \qquad \frac{\mathcal{F}_{\omega}}{T} \qquad \frac{1}{T} \operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f)$ sinc $\sin\left(Tt\right) \qquad \frac{\mathcal{F}_{\omega}}{T} \qquad \frac{1}{T} \operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ | exponential decay $e^{-at}u(t)$ $\Re\{a\} > 0$ | | $\frac{1}{a+j\omega}$ |
| $e^{\frac{t^2\sigma^2}{2\sigma^2}} \stackrel{\mathcal{F}_{\omega}}{\Longleftrightarrow} \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$ sine $\sin(\omega_0t + \phi) \stackrel{\mathcal{F}_{\omega}}{\Longleftrightarrow} j\pi \left[e^{-j\phi}\delta\left(\omega + \omega_0\right) - e^{j\phi}\delta\left(\omega - \omega_0\right)\right]$ cosine $\cos(\omega_0t + \phi) \stackrel{\mathcal{F}_{\omega}}{\Longleftrightarrow} \pi \left[e^{-j\phi}\delta\left(\omega + \omega_0\right) - e^{j\phi}\delta\left(\omega - \omega_0\right)\right]$ sine modulation $x(t)\sin(\omega_0t) \stackrel{\mathcal{F}_{\omega}}{\Longleftrightarrow} \pi \left[e^{-j\phi}\delta\left(\omega + \omega_0\right) - e^{j\phi}\delta\left(\omega - \omega_0\right)\right]$ cosine modulation $x(t)\cos(\omega_0t) \stackrel{\mathcal{F}_{\omega}}{\Longleftrightarrow} \frac{j}{2}\left[X\left(\omega + \omega_0\right) - X\left(\omega - \omega_0\right)\right]$ squared sine $\sin^2(\omega_0t) \stackrel{\mathcal{F}_{\omega}}{\Longleftrightarrow} \pi^2\left[2\delta(f) - \delta\left(\omega - \omega_0\right) - \delta\left(\omega + \omega_0\right)\right]$ squared cosine $\cos^2(\omega_0t) \stackrel{\mathcal{F}_{\omega}}{\Longleftrightarrow} \pi^2\left[2\delta(\omega) + \delta\left(\omega - \omega_0\right) + \delta\left(\omega + \omega_0\right)\right]$ rectangular $\cot\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant \frac{T}{2} \\ 0 & t > \frac{T}{2} \end{cases} \qquad \mathcal{F}_{\omega} \qquad T \operatorname{sinc}\left(\frac{\omega T}{2}\right)$ triangular $\tan\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{ t }{T} & t \leqslant T \\ 0 & t > T \end{cases} \qquad \mathcal{F}_{\omega} \qquad T \operatorname{sinc}\left(\frac{\omega T}{2}\right)$ step $u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \qquad \mathcal{F}_{\omega} \qquad \pi\delta(f) + \frac{1}{j\omega}$ signum $\operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases} \qquad \mathcal{F}_{\omega} \qquad \frac{T}{T} \operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f)$ sinc $\operatorname{sinc}(Tt) \qquad \mathcal{F}_{\omega} \qquad \frac{1}{T} \operatorname{rect}\left(\frac{\omega}{2\pi T}\right)$ | reversed exponential decay $e^{-at}u(-t)$ $\Re\{a\} > 0$ | $\stackrel{\mathcal{F}_{\omega}}{\Longleftrightarrow}$ | $\frac{1}{a-i\omega}$ |
| cosine $\cos(\omega_{0}t + \phi) \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \Rightarrow \pi \left[e^{-j\phi} \delta \left(\omega + \omega_{0} \right) + e^{j\phi} \delta \left(\omega - \omega_{0} \right) \right]$ sine modulation $x(t) \sin(\omega_{0}t) \iff \frac{j}{2} \left[X \left(\omega + \omega_{0} \right) - X \left(\omega - \omega_{0} \right) \right]$ cosine modulation $x(t) \cos(\omega_{0}t) \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \Rightarrow \frac{1}{2} \left[X \left(\omega + \omega_{0} \right) - X \left(\omega - \omega_{0} \right) \right]$ squared sine $\sin^{2}(\omega_{0}t) \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \Rightarrow \pi^{2} \left[2\delta(f) - \delta \left(\omega - \omega_{0} \right) - \delta \left(\omega + \omega_{0} \right) \right]$ rectangular $\cos^{2}(\omega_{0}t) \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \Rightarrow \pi^{2} \left[2\delta(\omega) + \delta \left(\omega - \omega_{0} \right) - \delta \left(\omega + \omega_{0} \right) \right]$ triangular $\cot\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant \frac{T}{2} \\ 0 & t > \frac{T}{2} \end{cases} \iff T \operatorname{sinc}\left(\frac{\omega T}{2}\right)$ triangular $\cot\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant T \\ 0 & t > T \end{cases} \Rightarrow T \operatorname{sinc}^{2}\left(\frac{\omega T}{2}\right)$ step $u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \Rightarrow \pi\delta(f) + \frac{1}{j\omega}$ signum $\operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases} \Rightarrow \frac{\mathcal{F}_{\omega}}{J} \Rightarrow \frac{1}{T} \operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f)$ squared sinc $\operatorname{sinc}^{2}(Tt) \iff \frac{\mathcal{F}_{\omega}}{J} \Rightarrow \frac{1}{T} \operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ | $e^{rac{t^2}{2\sigma^2}}$ | $\stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow}$ | $\sigma\sqrt{2\pi}e^{-rac{\sigma^2\omega^2}{2}}$ |
| cosine $\cos(\omega_{0}t + \phi) \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \Rightarrow \pi \left[e^{-j\phi} \delta \left(\omega + \omega_{0} \right) + e^{j\phi} \delta \left(\omega - \omega_{0} \right) \right]$ sine modulation $x(t) \sin(\omega_{0}t) \iff \frac{j}{2} \left[X \left(\omega + \omega_{0} \right) - X \left(\omega - \omega_{0} \right) \right]$ cosine modulation $x(t) \cos(\omega_{0}t) \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \Rightarrow \frac{1}{2} \left[X \left(\omega + \omega_{0} \right) - X \left(\omega - \omega_{0} \right) \right]$ squared sine $\sin^{2}(\omega_{0}t) \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \Rightarrow \pi^{2} \left[2\delta(f) - \delta \left(\omega - \omega_{0} \right) - \delta \left(\omega + \omega_{0} \right) \right]$ rectangular $\cos^{2}(\omega_{0}t) \iff \frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}} \Rightarrow \pi^{2} \left[2\delta(\omega) + \delta \left(\omega - \omega_{0} \right) - \delta \left(\omega + \omega_{0} \right) \right]$ triangular $\cot\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant \frac{T}{2} \\ 0 & t > \frac{T}{2} \end{cases} \iff T \operatorname{sinc}\left(\frac{\omega T}{2}\right)$ triangular $\cot\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant T \\ 0 & t > T \end{cases} \Rightarrow T \operatorname{sinc}^{2}\left(\frac{\omega T}{2}\right)$ step $u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \Rightarrow \pi\delta(f) + \frac{1}{j\omega}$ signum $\operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases} \Rightarrow \frac{\mathcal{F}_{\omega}}{J} \Rightarrow \frac{1}{T} \operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f)$ squared sinc $\operatorname{sinc}^{2}(Tt) \iff \frac{\mathcal{F}_{\omega}}{J} \Rightarrow \frac{1}{T} \operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ | sine $\sin(\omega_0 t + \phi)$ | $\leftarrow \xrightarrow{\mathcal{F}_{\omega}}$ | $i\pi \left[e^{-j\phi}\delta\left(\omega+\omega_{0}\right)-e^{j\phi}\delta\left(\omega-\omega_{0}\right)\right]$ |
| sine modulation $x(t)\sin\left(\omega_{0}t\right) \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} \frac{j}{2}\left[X\left(\omega+\omega_{0}\right)-X\left(\omega-\omega_{0}\right)\right]$ cosine modulation $x(t)\cos\left(\omega_{0}t\right) \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} \frac{j}{2}\left[X\left(\omega+\omega_{0}\right)-X\left(\omega-\omega_{0}\right)\right]$ squared sine $\sin^{2}\left(\omega_{0}t\right) \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} \pi^{2}\left[2\delta(f)-\delta\left(\omega-\omega_{0}\right)-\delta\left(\omega+\omega_{0}\right)\right]$ squared cosine $\cos^{2}\left(\omega_{0}t\right) \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} \pi^{2}\left[2\delta(\omega)+\delta\left(\omega-\omega_{0}\right)+\delta\left(\omega+\omega_{0}\right)\right]$ rectangular $\cot\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant \frac{T}{2} \\ 0 & t > \frac{T}{2} \end{cases} \qquad \mathcal{F}_{\omega} \qquad T \operatorname{sinc}\left(\frac{\omega T}{2}\right)$ triangular $\tan\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{ t }{T} & t \leqslant T \\ 0 & t > T \end{cases} \qquad \mathcal{F}_{\omega} \qquad T \operatorname{sinc}^{2}\left(\frac{\omega T}{2}\right)$ step $u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \qquad \pi\delta(f) + \frac{1}{j\omega}$ signum $\operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases} \qquad \frac{\mathcal{F}_{\omega}}{J} \qquad \frac{2}{J\omega}$ sinc $\operatorname{sinc}(Tt) \qquad \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} \qquad \frac{1}{T} \operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f)$ squared sinc $\operatorname{sinc}^{2}(Tt) \qquad \stackrel{\mathcal{F}_{\omega}}{\rightleftharpoons} \qquad \frac{1}{T} \operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ | | | , , , , , |
| cosine modulation $x(t)\cos(\omega_0 t) \iff \frac{\mathcal{F}_{\omega}}{\frac{1}{2}} \left[X(\omega + \omega_0) + X(\omega - \omega_0)\right]$ squared sine $\sin^2(\omega_0 t) \iff \pi^2 \left[2\delta(f) - \delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right]$ squared cosine $\cos^2(\omega_0 t) \iff \pi^2 \left[2\delta(\omega) + \delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]$ rectangular $\cot\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant \frac{T}{2} \\ 0 & t > \frac{T}{2} \end{cases} \iff T \operatorname{sinc}\left(\frac{\omega T}{2}\right)$ triangular $\tan\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{ t }{T} & t \leqslant T \\ 0 & t > T \end{cases} \iff T \operatorname{sinc}^2\left(\frac{\omega T}{2}\right)$ step $u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \iff \pi\delta(f) + \frac{1}{j\omega}$ signum $\operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases} \iff \frac{\mathcal{F}_{\omega}}{T} \operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f)$ squared sinc $\operatorname{sinc}(Tt) \iff \frac{\mathcal{F}_{\omega}}{T} \operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ | | | |
| $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | | | 2 |
| | | | 21 () 0/1 |
| rectangular $ \operatorname{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant \frac{T}{2} \\ 0 & t > \frac{T}{2} \end{cases} \qquad \overset{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad T \operatorname{sinc}\left(\frac{\omega T}{2}\right) $ triangular $ \operatorname{triang}\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{ t }{T} & t \leqslant T \\ 0 & t > T \end{cases} \qquad T \operatorname{sinc}^2\left(\frac{\omega T}{2}\right) $ step $ u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \qquad \overset{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \pi \delta(f) + \frac{1}{j\omega} $ signum $ \operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases} \qquad \overset{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \frac{2}{j\omega} $ sinc $ \operatorname{sinc}(Tt) \qquad \overset{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \frac{1}{T} \operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f) $ squared sinc $ \operatorname{sinc}^2(Tt) \qquad \overset{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \frac{1}{T} \operatorname{triang}\left(\frac{\omega}{2\pi T}\right) $ | · · · | | |
| $u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \qquad \stackrel{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \pi \delta(f) + \frac{1}{j\omega}$ signum $\operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases} \qquad \stackrel{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \frac{2}{j\omega}$ sinc $\operatorname{sinc}(Tt) \qquad \stackrel{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \frac{1}{T}\operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f)$ squared sinc $\operatorname{sinc}^2(Tt) \qquad \stackrel{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \frac{1}{T}\operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ | | | |
| $u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \qquad \stackrel{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \pi \delta(f) + \frac{1}{j\omega}$ signum $\operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases} \qquad \stackrel{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \frac{2}{j\omega}$ sinc $\operatorname{sinc}(Tt) \qquad \stackrel{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \frac{1}{T}\operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f)$ squared sinc $\operatorname{sinc}^2(Tt) \qquad \stackrel{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \frac{1}{T}\operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ | rectangular $\operatorname{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leqslant \frac{\tau}{2} \\ 0 & t > \frac{T}{2} \end{cases}$ | $\stackrel{\mathcal{F}_{\omega}}{\Longleftrightarrow}$ | $T\operatorname{sinc}\left(\frac{\omega T}{2}\right)$ |
| $u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \qquad \stackrel{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \pi \delta(f) + \frac{1}{j\omega}$ signum $\operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases} \qquad \stackrel{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \frac{2}{j\omega}$ sinc $\operatorname{sinc}(Tt) \qquad \stackrel{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \frac{1}{T}\operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f)$ squared sinc $\operatorname{sinc}^2(Tt) \qquad \stackrel{\mathcal{F}_{\omega}}{\Longrightarrow} \qquad \frac{1}{T}\operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ | triangular $ \operatorname{triang}\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{ t }{T} & t \leq T \\ 0 & t > T \end{cases} $ | $\stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow}$ | $T\operatorname{sinc}^2\left(\frac{\omega T}{2}\right)$ |
| $\operatorname{signum} \qquad \operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 & \stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow} & \frac{2}{j\omega} \\ -1 & t < 0 & \stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow} & \frac{1}{T}\operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f) \end{cases}$ $\operatorname{sinc}(Tt) \qquad \stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow} \qquad \frac{1}{T}\operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ $\operatorname{squared sinc} \qquad \operatorname{sinc}^2(Tt) \qquad \stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow} \qquad \frac{1}{T}\operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ | step $u(t) = 1_{[0,+\infty]}(t) = \begin{cases} 1 & t \ge 0 \end{cases}$ | $\stackrel{\mathcal{F}_{\omega}}{\Longleftrightarrow}$ | $\pi\delta(f) + \frac{1}{\dot{\beta}}$ |
| sinc $\sin(Tt) \iff \frac{\mathcal{F}_{\omega}}{T} \operatorname{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T}1_{[-\pi T, +\pi T]}(f)$ squared sinc $\sin^{2}(Tt) \iff \frac{1}{T}\operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ <i>n</i> -th time derivative $\frac{d^{n}}{dt^{n}}f(t) \iff \frac{\mathcal{F}_{\omega}}{dt^{n}} \qquad (j\omega)^{n}X(\omega)$ <i>n</i> -th frequency derivative $t^{n}f(t) \iff j^{n}\frac{d^{n}}{df^{n}}X(\omega)$ time inverse. | signum $\operatorname{sgn}(t) = \begin{cases} 1 & t \geqslant 0 \\ -1 & t < 0 \end{cases}$ | $\stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow}$ | $rac{2}{j\omega}$ |
| squared sinc $\sin^2(Tt) \iff \frac{\mathcal{F}_{\omega}}{T} \operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ <i>n</i> -th time derivative $\frac{d^n}{dt^n} f(t) \iff \frac{\mathcal{F}_{\omega}}{t^n} \iff (j\omega)^n X(\omega)$ <i>n</i> -th frequency derivative $t^n f(t) \iff j^n \frac{d^n}{dt^n} X(\omega)$ time inverse | sinc $\operatorname{sinc}(Tt)$ | $\stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow}$ | $\frac{1}{T} \operatorname{rect} \left(\frac{\omega}{2\pi T} \right) = \frac{1}{T} \mathbb{1}_{[-\pi T. + \pi T]}(f)$ |
| $\frac{d^n}{dt^n}f(t) \iff \frac{f_\omega}{dt^n}X(\omega)$ n -th frequency derivative $t^nf(t) \iff f(t) \iff f(t)$ $f(t) \iff f(t) \iff f(t)$ $f(t) \iff f(t) \iff f(t)$ | squared sinc $\operatorname{sinc}^{2}\left(Tt\right)$ | $\xleftarrow{\mathcal{F}_{\omega}}$ | $\frac{1}{T}\operatorname{triang}\left(\frac{\omega}{2\pi T}\right)$ |
| n -th frequency derivative $t^n f(t) \stackrel{\mathcal{F}_{\omega}}{\longleftrightarrow} j^n \frac{d^n}{df^n} X(\omega)$ | n -th time derivative $\frac{d^n}{dx^n} f(t)$ | $\leftarrow \xrightarrow{\mathcal{F}_{\omega}}$ | $(j\omega)^n X(\omega)$ |
| time inverse $\frac{1}{F_{\omega}}$ $\frac{\mathcal{F}_{\omega}}{\mathcal{F}_{\omega}}$ | n -th frequency derivative $t^n f(t)$ | $\leftarrow \xrightarrow{\mathcal{F}_{\omega}}$ | $i^n \frac{d^n}{d^n} X(\omega)$ |
| | time inverse $\frac{1}{4}$ | \mathcal{F}_{ω} | $-i\pi \operatorname{sgn}(\omega)$ |

Table of z-Transform Pairs

| $x[n] = \mathcal{Z}^{-1} \left\{ X(z) \right\} = \frac{1}{2}$ | $\frac{1}{2\pi j} \oint X(z)z^{n-1}dz$ | $\stackrel{\mathcal{Z}}{\longleftrightarrow}$ | $X(z) = \mathcal{Z}\left\{x[n]\right\} = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$ | ROC |
|--|---|---|---|--|
| transform time reversal complex conjugation reversed conjugation | $x[n]$ $x[-n]$ $x^*[n]$ $x^*[-n]$ | $ \begin{array}{c} \stackrel{\mathcal{Z}}{\longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\longleftrightarrow} \end{array} $ | $X(z)$ $X(\frac{1}{z})$ $X^*(z^*)$ $X^*(\frac{1}{z^*})$ | R_x $\frac{1}{R_x}$ R_x $\frac{1}{R_x}$ |
| real part imaginary part | $\Re \{x[n]\}$ $\Im \{x[n]\}$ | $\overset{\mathcal{Z}}{\Longleftrightarrow}$ | $\frac{1}{2}[X(z) + X^*(z^*)]$ $\frac{1}{2j}[X(z) - X^*(z^*)]$ | R_x R_x |
| time shifting scaling in \mathcal{Z} downsampling by N | $x[n - n_0]$ $a^n x[n]$ $x[Nn], N \in \mathbb{N}_0$ | $ \begin{array}{c} \stackrel{\mathcal{Z}}{\longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\longleftrightarrow} \end{array} $ | $z^{-n_0}X(z)$ $X\left(\frac{z}{a}\right)$ $\frac{1}{N}\sum_{k=0}^{N-1}X\left(W_N^kz^{\frac{1}{N}}\right) W_N = e^{-\frac{j2\omega}{N}}$ | R_x $ a R_x$ R_x |
| linearity time multiplication frequency convolution | $ax_1[n] + bx_2[n]$ $x_1[n]x_2[n]$ $x_1[n] * x_2[n]$ | $ \begin{array}{c} \stackrel{\mathcal{Z}}{\longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\longleftrightarrow} \end{array} $ | $aX_1(z) + bX_2(z)$ $\frac{1}{2\pi j} \oint X_1(u)X_2\left(\frac{z}{u}\right)u^{-1}du$ $X_1(z)X_2(t)$ | $R_x \cap R_y$ $R_x \cap R_y$ $R_x \cap R_y$ |
| delta function shifted delta function | $\delta[n] \\ \delta[n-n_0]$ | $\overset{\mathcal{Z}}{\longleftrightarrow}$ | $1 \\ z^{-n_0}$ | $\forall z$ $\forall z$ |
| step | u[n] - u[-n-1] | $\stackrel{\mathcal{Z}}{\Longleftrightarrow}$ $\stackrel{\mathcal{Z}}{\Longleftrightarrow}$ | $\frac{z}{z-1}$ $\frac{z}{z-1}$ | z > 1 $ z < 1$ |
| ramp | $-u[-n-1]$ $nu[n]$ $n^{2}u[n]$ $-n^{2}u[-n-1]$ $n^{3}u[n]$ $-n^{3}u[-n-1]$ $(-1)^{n}$ | $\begin{array}{c} \stackrel{Z}{\longleftrightarrow} \\ \end{array}$ | $ \frac{z}{(z-1)^2} \\ \frac{z}{(z-1)^2} \\ \frac{z(z+1)}{(z-1)^3} \\ \frac{z(z+1)}{(z-1)^3} \\ \frac{z(z^2+4z+1)}{(z-1)^4} \\ \frac{z(z^2+4z+1)}{(z-1)^4} \\ \frac{z}{z+1} $ | z < 1 z > 1 z > 1 z < 1 z < 1 z < 1 |
| exponential | $a^{n}u[n]$ $-a^{n}u[-n-1]$ $a^{n-1}u[n-1]$ $na^{n}u[n]$ $n^{2}a^{n}u[n]$ $e^{-an}u[n]$ | $\begin{array}{c} \stackrel{\mathcal{Z}}{\Longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\Longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\Longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\Longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\Longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\Longleftrightarrow} \end{array}$ | $ \frac{z}{z-a} $ $ \frac{z}{z-a} $ $ \frac{1}{z-a} $ $ \frac{az}{(z-a)^2} $ $ az(z+a) $ $ (z-a)^3 $ $ \frac{z}{z-e^{-a}} $ | $ z > a $ $ z < a $ $ z > a $ $ z > a $ $ z > a $ $ z > a $ $ z > e^{-a} $ |
| exp. interval $\begin{cases} a^n \\ 0 \end{cases}$ | $n = 0, \dots, N - 1$ otherwise | $\stackrel{\mathcal{Z}}{\Longleftrightarrow}$ | $\frac{1-a^Nz^{-N}}{1-az^{-1}}$ | z > 0 |
| sine cosine | $\sin(\omega_0 n) u[n]$ $\cos(\omega_0 n) u[n]$ $a^n \sin(\omega_0 n) u[n]$ $a^n \cos(\omega_0 n) u[n]$ | $\begin{array}{c} \stackrel{\mathcal{Z}}{\longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\longleftrightarrow} \end{array}$ | $ \begin{array}{c} z \sin(\omega_0) \\ \hline z^2 - 2\cos(\omega_0)z + 1 \\ z(z - \cos(\omega_0)) \\ z^2 - 2\cos(\omega_0)z + 1 \\ za\sin(\omega_0) \\ z^2 - 2a\cos(\omega_0)z + a^2 \\ z(z - a\cos(\omega_0)) \\ z^2 - 2a\cos(\omega_0)z + a^2 \end{array} $ | $\begin{aligned} z &> 1 \\ z &> 1 \\ z &> 1 \\ z &> a \\ z &> a \end{aligned}$ |
| differentiation in \mathcal{Z} integration in \mathcal{Z} | $nx[n]$ $\frac{x[n]}{n}$ $\stackrel{i=1}{=} (n-i+1) a^m u[n]$ | $ \begin{array}{c} \stackrel{\mathcal{Z}}{\longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\longleftrightarrow} \\ \stackrel{\mathcal{Z}}{\longleftrightarrow} \end{array} $ | $-z\frac{dX(z)}{dz} - \int_0^z \frac{X(z)}{z} dz$ $-\frac{1}{2} \frac{Z(z)}{(z-a)^{m+1}} dz$ | R_x R_x |

Note:

$$\frac{z}{z-1} = \frac{1}{1-z^{-1}}$$

Table of Common Discrete Time Fourier Transform (DTFT) Pairs

| $x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(e^{j\omega}) e^{j\omega n} d\omega$ | $\stackrel{DTFT}{\Longleftrightarrow}$ | $X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$ |
|--|---|---|
| $ \begin{array}{ccc} \text{transform} & x[n] \\ \text{time reversal} & x[-n] \\ \text{complex conjugation} & x^*[n] \\ \text{reversed conjugation} & x^*[-n] \end{array} $ | $\begin{array}{c} DTFT \\ \longleftrightarrow \\ DTFT \\ \longleftrightarrow \\ DTFT \\ \longleftrightarrow \\ DTFT \\ \end{array}$ | $X(e^{j\omega})$ $X(e^{-j\omega})$ $X^*(e^{-j\omega})$ $X^*(e^{j\omega})$ |
| $x[n] \text{ is purely real} \\ x[n] \text{ is purely imaginary} \\ \text{even/symmetry} \qquad x[n] = x^*[-n] \\ \text{odd/antisymmetry} \qquad x[n] = -x^*[-n]$ | $ \begin{array}{c} DTFT \\ DTFT \\ DTFT \\ DTFT \\ DTFT \end{array} $ | $X(e^{j\omega})=X^*(e^{-j\omega})$ even/symmetry $X(e^{j\omega})=-X^*(e^{-j\omega})$ odd/antisymmetry $X(e^{j\omega})$ is purely real $X(e^{j\omega})$ is purely imaginary |
| time shifting $x[n-n_0] \ x[n]e^{j\omega_0 n}$ | $\overset{DTFT}{\Longleftrightarrow}$ $\overset{DTFT}{\longleftrightarrow}$ | $X(e^{j\omega})e^{-j\omega n_0}$ $X(e^{j(\omega-\omega_0)})$ frequency shifting |
| downsampling by N $x[Nn] N \in \mathbb{N}_0$ upsampling by N $\begin{cases} x\left[\frac{n}{N}\right] & n = kN \\ 0 & otherwise \end{cases}$ | $\stackrel{DTFT}{\longleftrightarrow}$ | $\frac{1}{N} \sum_{k=0}^{N-1} X(e^{j\frac{\omega - 2\pi k}{N}})$ $X(e^{jN\omega})$ |
| linearity $ax_1[n] + bx_2[n]$ time multiplication $x_1[n]x_2[n]$ | $\overset{DTFT}{\longleftrightarrow}$ | $aX_1(e^{j\omega}) + bX_2(e^{j\omega})$ $X_1(e^{j\omega}) * X_2(e^{j\omega}) = $ frequency convolution $\frac{1}{2\pi} \int_{-\pi}^{+\pi} X_1(e^{j(\omega-\sigma)}) X_2(e^{j\sigma}) d\sigma$ |
| frequency convolution $x_1[n] * x_2[n]$ | $\stackrel{DTFT}{\longleftrightarrow}$ | $X_1(e^{j\omega})X_2(e^{j\omega})$ frequency multiplication |
| delta function $\delta[n]$ shifted delta function $\delta[n-n_0]$ $\frac{1}{e^{j\omega_0 n}}$ | $ \begin{array}{c} DTFT \\ DTFT \\ DTFT \\ DTFT \\ DTFT \end{array} $ | $\begin{array}{ll} 1 \\ e^{-j\omega n_0} \\ \tilde{\delta}(\omega) & \text{delta function} \\ \tilde{\delta}(\omega-\omega_0) & \text{shifted delta function} \end{array}$ |
| sine $\sin(\omega_0 n + \phi)$ cosine $\cos(\omega_0 n + \phi)$ | $\overset{DTFT}{\longleftrightarrow}$ | $\frac{j}{2}[e^{-j\phi}\tilde{\delta}(\omega+\omega_0+2\pi k)-e^{+j\phi}\tilde{\delta}(\omega-\omega_0+2\pi k)]$ $\frac{1}{2}[e^{-j\phi}\tilde{\delta}(\omega+\omega_0+2\pi k)+e^{+j\phi}\tilde{\delta}(\omega-\omega_0+2\pi k)]$ |
| rectangular $\operatorname{rect}\left(\frac{n}{M}\right) = \begin{cases} 1 & n \leqslant M \\ 0 & \text{otherwise} \end{cases}$ | $\stackrel{DTFT}{\longleftrightarrow}$ | $\frac{\sin\left[\omega\left(M+\frac{1}{2}\right)\right]}{\sin(\omega/2)}$ |
| step $u[n]$ decaying step $a^n u[n]$ $(a < 1)$ special decaying step $(n+1)a^n u[n]$ $(a < 1)$ | $ \begin{array}{c} DTFT \\ DTFT \\ DTFT \end{array} $ | $\frac{\frac{1}{1-e^{-j\omega}} + \frac{1}{2}\tilde{\delta}(\omega)}{\frac{1}{1-ae^{-j\omega}}} \frac{1}{\left(1-ae^{-j\omega}\right)^2}$ |
| $\frac{\sin(\omega_c n)}{\pi n} = \frac{\omega_c}{\pi} \operatorname{sinc}(\omega_c n)$ | $\stackrel{DTFT}{\Longleftrightarrow}$ | $\tilde{\operatorname{rect}}\left(\frac{\omega}{\omega_c}\right) = \begin{cases} 1 & \omega < \omega_c \\ 0 & \omega_c < \omega < \pi \end{cases}$ |
| MA $\operatorname{rect}\left(\frac{n}{M} - \frac{1}{2}\right) = \begin{cases} 1 & 0 \leqslant n \leqslant M \\ 0 & \text{otherwise} \end{cases}$ | $\stackrel{DTFT}{\longleftrightarrow}$ | $\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)}e^{-j\omega M/2}$ |
| MA rect $\left(\frac{n}{M-1} - \frac{1}{2}\right) = \begin{cases} 1 & 0 \leqslant n \leqslant M-1 \\ 0 & \text{otherwise} \end{cases}$ | $\stackrel{DTFT}{\longleftrightarrow}$ | $\frac{\sin[\omega M/2]}{\sin(\omega/2)}e^{-j\omega(M-1)/2}$ |
| derivation $nx[n]$ difference $x[n] - x[n-1]$ $\frac{a^n \sin[\omega_0(n+1)]}{\sin \omega_0} u[n] a < 1$ | $ \begin{array}{c} DTFT \\ DTFT \\ DTFT \end{array} $ | $j\frac{d}{d\omega}X(e^{j\omega})$ $(1 - e^{-j\omega})X(e^{j\omega})$ $\frac{1}{1 - 2a\cos(\omega_0 e^{-j\omega}) + a^2 e^{-j2\omega}}$ |

Note:
$$\tilde{\delta}(\omega) = \sum_{k=-\infty}^{+\infty} \delta(\omega + 2\pi k) \qquad \qquad \text{rect}(\omega) = \sum_{k=-\infty}^{+\infty} \text{rect}(\omega + 2\pi k)$$
Parseval:
$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} |X(e^{j\omega})|^2 d\omega$$

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} |X(e^{j\omega})|^2 d\omega$$

Table of Laplace Transform Pairs

| $f(t) = \mathcal{L}^{-1} \{ F(s) \} = \frac{1}{2\pi j} \lim_{T \to T} f(s)$ | $\rightarrow \infty \int_{c-jT}^{c+jT} F(s) e^{st} ds$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $F(s) = \mathcal{L} \{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-st}dt$ |
|---|---|---|---|
| transform | f(t) | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | F(s) |
| complex conjugation | $f^*(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $F^*(s^*)$ |
| time shifting | $f(t-a)$ $t \geqslant a > 0$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $a^{-as}F(s)$ |
| | $e^{-at}f(t)$ | | F(s+a) frequency shifting |
| time scaling | f(at) | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{1}{ a }F(\frac{s}{a})$ |
| linearity | $af_1(t) + bf_2(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $aF_1(s) + bF_2(s)$ |
| time multiplication | $f_1(t)f_2(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $F_1(s) * F_2(s)$ frequency convolution |
| time convolution | $f_1(t) * f_2(t)$ | $\stackrel{\mathcal{L}}{\longleftrightarrow}$ | $F_1(s)F_2(s)$ frequency product |
| delta function | $\delta(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | 1 |
| shifted delta function | $\delta(t-a)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | e^{-as} exponential decay |
| unit step | u(t) | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{1}{s}$ |
| ramp | tu(t) | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{1}{s}$ $\frac{1}{s^{2}}$ $\frac{2}{s^{3}}$ |
| parabola | $t^2u(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{2}{s^3}$ |
| <i>n</i> -th power | t^n | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{n!}{s^{n+1}}$ |
| exponential decay | e^{-at} | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{1}{s+a}$ |
| two-sided exponential decay | $e^{-a t }$ | $\stackrel{\mathcal{L}}{\longleftrightarrow}$ | $\frac{2a}{a^2 - s^2}$ |
| | te^{-at} | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{1}{(s+a)^2}$ |
| | $(1-at)e^{-at}$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{s}{(s+a)^2}$ |
| exponential approach | $1 - e^{-at}$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{a}{s(s+a)}$ |
| sine | $\sin{(\omega t)}$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{\omega}{s^2 + \omega^2}$ |
| cosine | $\cos{(\omega t)}$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | |
| | ` ′ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{s}{s^2 + \omega^2}$ |
| hyperbolic sine | $\sinh (\omega t)$ | $\stackrel{\longleftarrow}{\longleftarrow}$ | $\frac{\omega}{s^2 - \omega^2}$ |
| hyperbolic cosine | $\cosh\left(\omega t\right)$ | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{s}{s^2 - \omega^2}$ |
| exponentially decaying sine | $e^{-at}\sin\left(\omega t\right)$ | $\stackrel{\sim}{\longleftrightarrow}$ | $\frac{\omega}{(s+a)^2+\omega^2}$ |
| exponentially decaying cosine | $e^{-at}\cos\left(\omega t\right)$ | $\stackrel{\mathcal{L}}{\longleftrightarrow}$ | $\frac{s+a}{(s+a)^2+\omega^2}$ |
| frequency differentiation | tf(t) | $\overset{\mathcal{L}}{\Longleftrightarrow}$ | -F'(s) |
| frequency n -th differentiation | $t^n f(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $(-1)^n F^{(n)}(s)$ |
| time differentiation | $f'(t) = \frac{d}{dt}f(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | sF(s) - f(0) |
| time 2nd differentiation | $f'(t) = \frac{d}{dt} f(t)$ $f''(t) = \frac{d^2}{dt^2} f(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $s^2F(s) - sf(0) - f'(0)$ |
| time n -th differentiation | $f^{(n)}(t) = \frac{d^n}{dt^n} f(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$ |
| time integration | $\int_{0}^{t} f(\tau)d\tau = (u * f)(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{1}{s}F(s)$ |
| frequency integration | $\int_{0}^{t} f(\tau)d\tau = (u * f)(t)$ $\frac{1}{t}f(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\int_{s}^{\infty} F(u) du$ |
| time inverse | $f^{-1}(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{F(s)-f^{-1}}{F(s)} = \frac{F(s)+f^{-1}(0)}{s^n} + \frac{f^{-2}(0)}{s^{n-1}} + \dots + \frac{f^{-n}(0)}{s}$ |
| time differentiation | $f^{-n}(t)$ | $\stackrel{\mathcal{L}}{\Longleftrightarrow}$ | $\frac{F(s)}{r} + \frac{f^{-1}(0)}{r} + \frac{f^{-2}(0)}{r-1} + \dots + \frac{f^{-n}(0)}{r-1}$ |