TÍN HIỆU VÀ HỆ THỐNG

Chương 3: Biểu diễn hệ thống tuyến tính bất biến trong miền tần số

Phần 3: BIỂU DIỄN FOURIER - TỔNG HỢP

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BẢNG TỔNG HỢP

Time Domain	Periodic (t, n)	Non periodic (t, n)	
Continuous (t)	**Fourier Series** $x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_o t}$ $X[k] = \frac{1}{T}\int_0^T x(t) e^{-jk\omega_o t}\,dt$ $x(t)$ has period T $\omega_o = \frac{2\pi}{T}$	**Fourier Transform** $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{j\omega t}\,d\omega$ $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\,dt$	Nonperiodic (k, ω)
Discrete (n)	**Discrete-Time Fourier Series** $x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\Omega_o n}$ $X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-jk\Omega_o n}$ $x[n]$ and $X[k]$ have period N $\Omega_o = \frac{2\pi}{N}$	**Discrete-Time Fourier Transform** $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega}) e^{j\Omega n}\,d\Omega$ $X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$ $X(e^{j\Omega})$ has period 2π	Periodic (k, Ω)
	Discrete (k)	Continuous (ω, Ω)	Frequency Domain

TÍNH ĐỔI LẪN GIỮA MIỀN THỜI GIAN VÀ MIỀN TẦN SỐ

Time-Domain Property	*Frequency-Domain Property*
continuous	nonperiodic
discrete	periodic
periodic	discrete
nonperiodic	continuous

TUYẾN TÍNH

$$z(t) = ax(t) + by(t) \xleftrightarrow{FT} Z(j\omega) = aX(j\omega) + bY(j\omega)$$

$$z(t) = ax(t) + by(t) \xleftrightarrow{FS;\omega_o} Z[k] = aX[k] + bY[k]$$

$$z[n] = ax[n] + by[n] \xleftrightarrow{DTFT} Z(e^{j\Omega}) = aX(e^{j\Omega}) + bY(e^{j\Omega})$$

$$z[n] = ax[n] + by[n] \xleftrightarrow{DTFS;\Omega_o} Z[k] = aX[k] + bY[k]$$

ĐỐI XỨNG

Representation	*Real-Valued Time Signals*	*Imaginary-Valued Time Signals*
FT	$X^*(j\omega) = X(-j\omega)$	$X^*(j\omega) = -X(-j\omega)$
FS	$X^*[k] = X[-k]$	$X^*[k] = -X[-k]$
DTFT	$X^*(e^{j\Omega}) = X(e^{-j\Omega})$	$X^*(e^{j\Omega}) = -X(e^{-j\Omega})$
DTFS	$X^*[k] = X[-k]$	$X^*[k] = -X[-k]$

NHÂN CHẬP

$$x(t) * z(t) \xleftrightarrow{FT} X(j\omega)Z(j\omega)$$

$$x(t) \circledast z(t) \xleftrightarrow{FS;\omega_o} TX[k]Z[k]$$

$$x[n] * z[n] \xleftrightarrow{DTFT} X(e^{j\Omega})Z(e^{j\Omega})$$

$$x[n] \circledast z[n] \xleftrightarrow{DTFS;\Omega_o} NX[k]Z[k]$$

NHÂN THƯỜNG

$$y(t) = g(t)x(t) \xleftrightarrow{FT} Y(j\omega) = \frac{1}{2\pi} G(j\omega) * X(j\omega)$$

$$\boxed{y(t) = g(t)x(t) \xleftrightarrow{FT} Y(j\omega) = \sum_{k=-\infty}^{\infty} X[k]G(j(\omega - k\omega_o)).}$$

VI PHÂN/TÍCH PHÂN

$$\frac{d}{dt}x(t) \xleftrightarrow{FT} j\omega X(j\omega)$$

$$\frac{d}{dt}x(t) \xleftrightarrow{FS;\omega_o} jk\omega_o X[k]$$

$$-jtx(t) \xleftrightarrow{FT} \frac{d}{d\omega}X(j\omega)$$

$$-jnx[n] \xleftrightarrow{DTFT} \frac{d}{d\Omega}X(e^{j\Omega})$$

$$\int_{-\infty}^{t} x(\tau)\,d\tau \xleftrightarrow{FT} \frac{1}{j\omega}X(j\omega) + \pi X(j0)\delta(\omega)$$

DỊCH THỜI GIAN

$$x(t - t_o) \xleftrightarrow{FT} e^{-j\omega t_o} X(j\omega)$$

$$x(t - t_o) \xleftrightarrow{FS;\omega_o} e^{-jk\omega_o t_o} X[k]$$

$$x[n - n_o] \xleftrightarrow{DTFT} e^{-j\Omega n_o} X(e^{j\Omega})$$

$$x[n - n_o] \xleftrightarrow{DTFS;\Omega_o} e^{-jk\Omega_o n_o} X[k]$$

DỊCH TẦN SỐ

$$e^{j\gamma t}x(t) \xleftrightarrow{FT} X(j(\omega - \gamma))$$

$$e^{jk_o\omega_o t}x(t) \xleftrightarrow{FS;\omega_o} X[k - k_o]$$

$$e^{j\Gamma n}x[n] \xleftrightarrow{DTFT} X(e^{j(\Omega-\Gamma)})$$

$$e^{jk_o\Omega_o n}x[n] \xleftrightarrow{DTFS;\Omega_o} X[k - k_o]$$

PHÉP NÉN/GIÃN

$$z(t) = x(at) \xleftrightarrow{FT} (1/|a|)X(j\omega/a)$$

$$z(t) = x(at) \xleftrightarrow{FS;\, a\omega_o} Z[k] = X[k], \qquad a > 0$$

QUAN HỆ PARSEVAL (PHỔ CÔNG SUẤT)

Representation	*Parseval Relation*
FT	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 \, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(j\omega)\rvert^2 \, d\omega$
FS	$\frac{1}{T}\int_0^T \lvert x(t)\rvert^2 \, dt = \sum_{k=-\infty}^{\infty} \lvert X[k]\rvert^2$
DTFT	$\sum_{n=-\infty}^{\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \lvert X(e^{j\Omega})\rvert^2 \, d\Omega$
DTFS	$\frac{1}{N}\sum_{n=0}^{N-1} \lvert x[n]\rvert^2 = \sum_{k=0}^{N-1} \lvert X[k]\rvert^2$