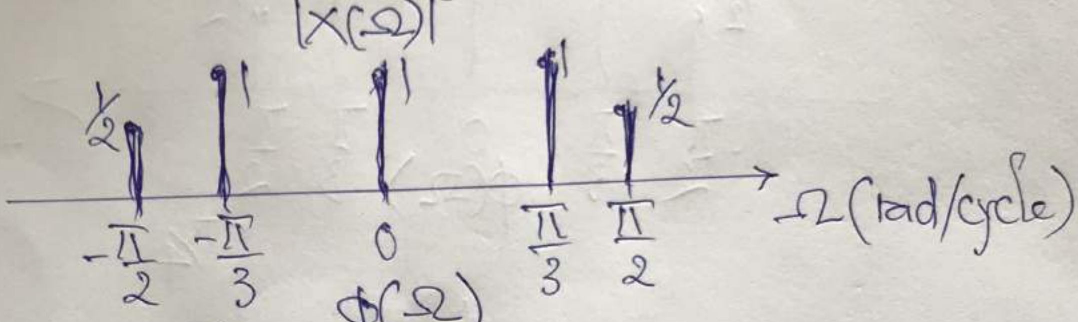
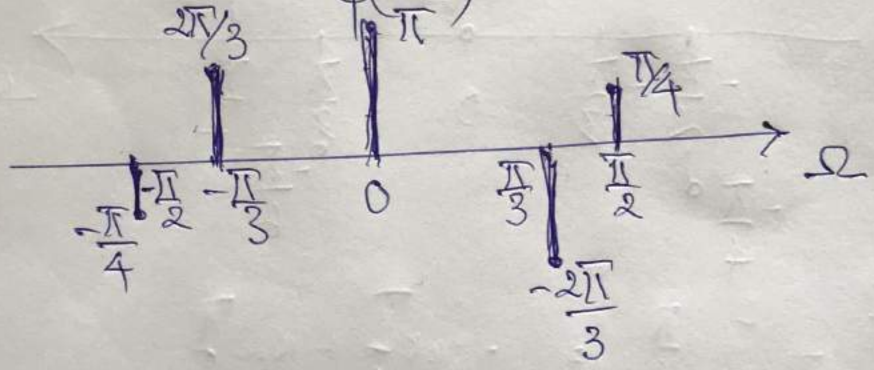
Câu 1. Phổ biên độ và phổ pha

a) $x[n] = \cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right) + 2\sin\left(\frac{\pi}{3}n - \frac{\pi}{6}\right) - 1$

$= \cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right) + 2\cos\left(\frac{\pi}{3}n - \frac{2\pi}{3}\right) + \cos(0n + \pi)$





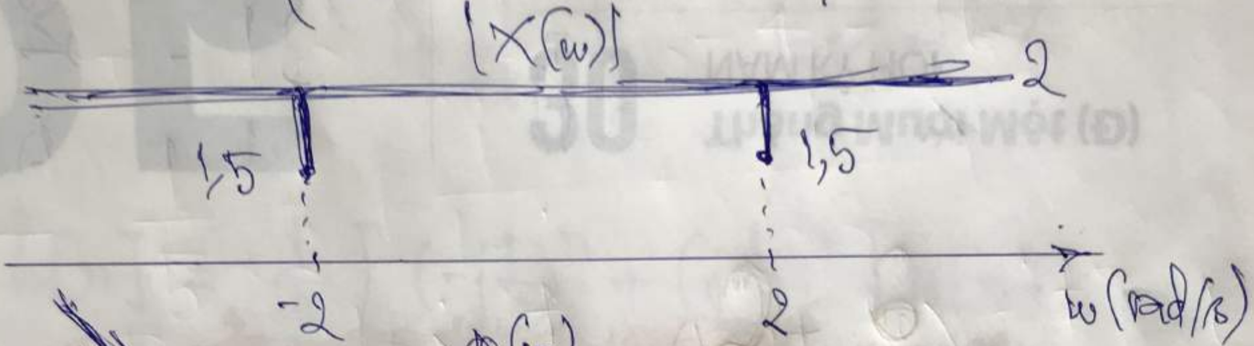
b) $x(t) = \cos(2t+1) + 2\delta(t-1)$

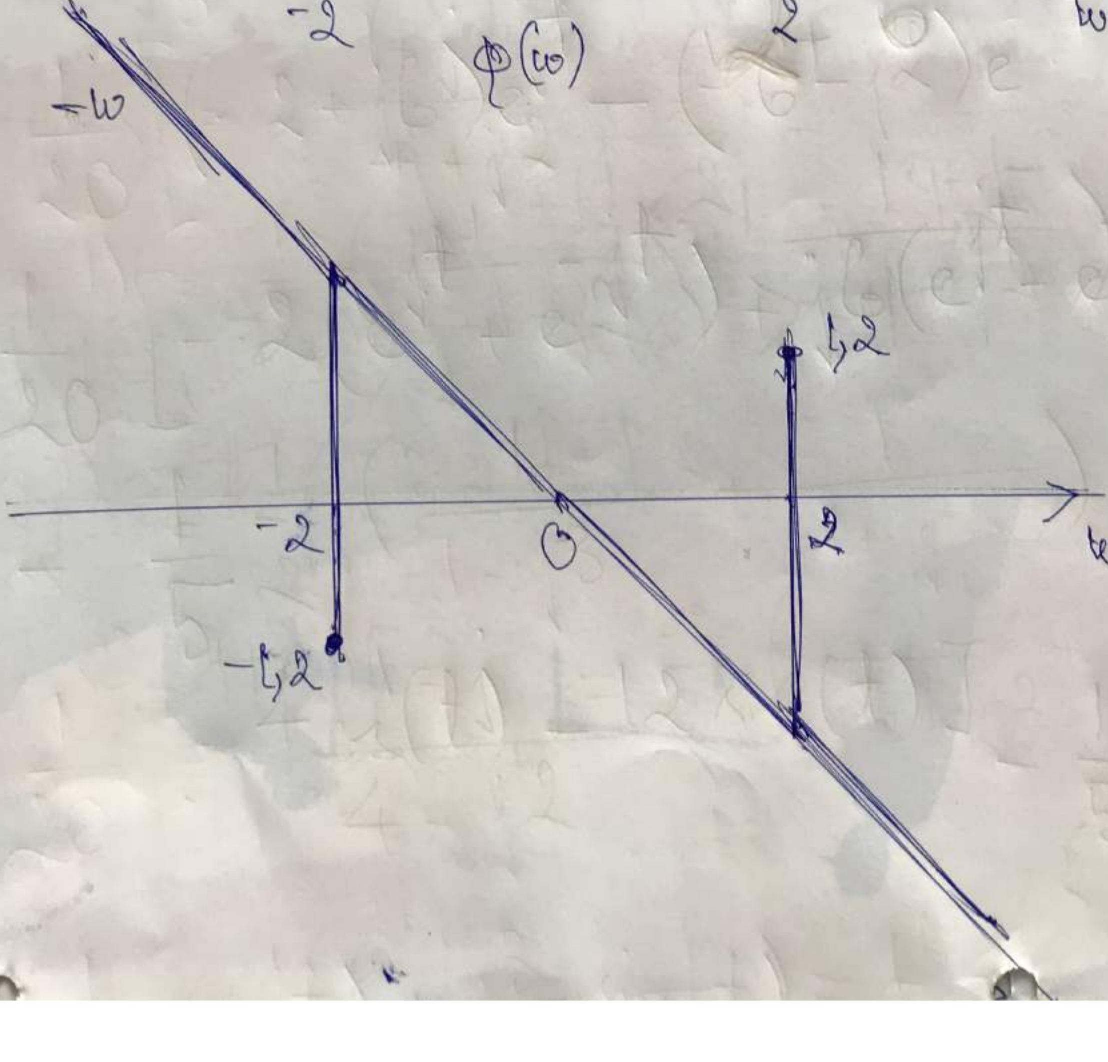
$$X(\omega) = \begin{cases} \frac{1}{2}e^{j} + 2e^{-j2} & \omega = 2 \\ \frac{1}{2}e^{-j} + 2e^{j2} & \omega = -2 \\ 2e^{-j\omega} & \text{còn lại} \end{cases}$$

$$= \begin{cases} -0{,}56 - j1{,}4 & \omega = 2 \\ -0{,}56 + j1{,}4 & \omega = -2 \\ 2e^{-j\omega} & \text{còn lại} \end{cases}$$

$$|X(\omega)| = \begin{cases} 1{,}5 & \omega = \pm 2 \\ 2 & \text{còn lại} \end{cases}$$

$$\phi(\omega) = \begin{cases} 1{,}2 & \omega = 2 \\ -1{,}2 & \omega = -2 \\ -\omega & \text{còn lại} \end{cases}$$





Câu 2 $y[n] + \frac{1}{4} y[n-2] = x[n]$

a) $H(z) = \dfrac{1}{1+\frac{1}{4}z^{-2}}$

Hệ thống nhân quả có 2 trị cực $\pm j\frac{1}{2}$ nằm bên trong đường tròn đơn vị $\longrightarrow$ ổn định

b) $H(z) = \dfrac{1}{2}\left(\dfrac{1}{1+j\frac{1}{2}z^{-1}} + \dfrac{1}{1-j\frac{1}{2}z^{-1}}\right)$

$\Rightarrow h[n] = \dfrac{1}{2}\left[\left(-j\frac{1}{2}\right)^n + \left(j\frac{1}{2}\right)^n\right]u[n]$

$= \begin{cases} 2^{-n}u[n] & n \text{ chẵn} \\ 0 & n \text{ lẻ} \end{cases}$

$H(\Omega) = \dfrac{1}{1+\frac{1}{4}e^{-j2\Omega}}$ (vì hệ thống ổn định)

c) $y[-1] = 1$, $y[-2] = 0$

$$y_0[n] = c_1\left(-j\frac{1}{2}\right)^n + c_2\left(j\frac{1}{2}\right)^n$$

$$y_0[-1] = c_1\left(-\frac{1}{j}2\right) + c_2\left(\frac{1}{j}2\right)$$

$$= j2c_1 - j2c_2 = 1$$

$$y_0[-2] = \frac{1}{j^2}4(c_1 + c_2) = 0$$

$$\Rightarrow \begin{cases} c_1 = -j\frac{1}{4} \\ c_2 = j\frac{1}{4} \end{cases}$$

$n \geq 0$:

$$y_0[n] = \left[-j\frac{1}{4}\left(-j\frac{1}{2}\right)^n + j\frac{1}{4}\left(j\frac{1}{2}\right)^n\right]u[n]$$

$$= \begin{cases} 0 & n \text{ chẵn} \\ -2^{-n-1}u[n] & n \text{ lẻ} \end{cases}$$

d) $x[n] = u[n-1] - u[n-4]$

$= \delta[n-1] + \delta[n-2] + \delta[n-3]$

$\rightarrow y[n] = x[n] * h[n]$

$= h[n-1] + h[n-2] + h[n-3]$

+ $n \leq 0$: $y[n] = 0$

+ $n = 1$: $y[1] = h[0] = -1$

+ $n = 2$: $y[2] = h[1] + h[0] = -1$

+ $n = 3$: $y[3] = h[2] + h[1] + h[0]$

$= -\frac{1}{4} - 1 = -1{,}25$

+ n chẵn > 3 :

$y[n] = h[n-2] = -2^{-n+2}$

+ n lẻ > 3 :

$y[n] = h[n-1] + h[n-3]$

$= -2^{-n+1} - 2^{-n+3} = -10 \times 2^{-n}$

Câu 3 $h(t) = e^{-t}\cos(2t)u(t)$

a) $\int_{-\infty}^{+\infty} |h(t)|dt = \int_{-\infty}^{+\infty} |e^{-t}\cos(2t)u(t)|dt$

$< \int_{0}^{+\infty} e^{-t}dt = 1 \Rightarrow$ ổn định

b) $h(t) = \frac{1}{2}\left[e^{(j2-1)t} + e^{(-j2-1)t}\right]u(t)$

$\Rightarrow H(s) = \frac{1}{2}\left(\frac{1}{s+1-j2} + \frac{1}{s+1+j2}\right)$

ROC$(H(s))$: $Re(s) > -1$

Vì hệ thống ổn định :

$H(\omega) = H(s)\Big|_{s=j\omega} = \frac{1}{2}\left(\frac{1}{j\omega+1-j2} + \frac{1}{j\omega+1+j2}\right)$

c) $x(t) = 2\sin(t) - 1$

$$= \frac{1}{j}e^{jt} - \frac{1}{j}e^{-jt} - e^{j0t}$$

$$\rightarrow y(t) = \frac{1}{j}H(1)e^{+jt} - \frac{1}{j}H(-1)e^{-jt} - H(0)$$

$$H(\omega=1) = \frac{1}{2}\left(\frac{1}{j+1-j2} + \frac{1}{j+1+j2}\right)$$

$$= \frac{1}{2}\left(\frac{1}{1-j} + \frac{1}{1+3j}\right)$$

$$= \frac{1+j}{4+j2}$$

$$H(\omega=-1) = \frac{1}{2}\left(\frac{1}{-j+1-j2} + \frac{1}{-j+1+j2}\right)$$

$$= \frac{1}{2}\left(\frac{1}{1-j3} + \frac{1}{1+j}\right)$$

$$= \frac{1-j}{4-j2}$$

$$H(\omega=0) = \frac{1}{2}\left(\frac{1}{1-j2} + \frac{1}{1+j2}\right) = \frac{1}{5}$$

$$\rightarrow \quad y(t) = \frac{1+j}{-2+j4}e^{jt} - \frac{1-j}{2+j4}e^{-jt} - \frac{1}{5}$$

$$= \frac{(1+j)(2+j4)}{-4-16}e^{jt} - \frac{(1-j)(-2+j4)}{-4-16}e^{-jt} - \frac{1}{5}$$

$$= -\frac{1}{20}\left[(-2+j6)e^{jt} - (2+j6)e^{-jt}\right] - \frac{1}{5}$$

$$= -\frac{1}{20}\left[-2(e^{jt}+e^{-jt}) + j6(e^{jt}-e^{-jt})\right] - \frac{1}{5}$$

$$= -\frac{1}{20}\left[-4\cos(t) - 12\sin(t)\right] - \frac{1}{5}$$

$$= \frac{1}{5}\cos(t) + \frac{3}{5}\sin(t) - \frac{1}{5}$$

d) $x(t) = u(t-1)$

$$\rightarrow X(s) = \frac{e^{-s}}{s}$$

$$Y(s) = H(s)X(s)$$

$$= \frac{1}{2}\left(\frac{1}{s+1-j2} + \frac{1}{s+1+j2}\right)\frac{e^{-s}}{s}$$

$$= \frac{1}{2}\left[\frac{1}{s(s+1-j2)} + \frac{1}{s(s+1+j2)}\right]e^{-s}$$

$$= \frac{1}{2}\left[\frac{1}{1-j2}\left(\frac{1}{s} - \frac{1}{s+1-j2}\right)\right.$$

$$\left. + \frac{1}{1+j2}\left(\frac{1}{s} - \frac{1}{s+1+j2}\right)\right]e^{-s}$$

$$= \left(\frac{1}{5}\cdot\frac{1}{s} - \frac{1}{2-j4}\cdot\frac{1}{s+1-j2} - \frac{1}{2+j4}\cdot\frac{1}{s+1+j2}\right)e^{-s}$$

$$= Y_1(s)\,e^{-s}$$

$$y_1(t) = \frac{1}{5}u(t) - \frac{1}{2-j4}e^{-t+j2t}u(t) - \frac{1}{2+j4}e^{-t-j2t}u(t)$$

$$= u(t)\left[\frac{1}{5} - e^{-t}\frac{e^{j2t}(1+j2) + e^{-j2t}(1-j2)}{2\times 5}\right]$$

$$= \frac{1}{5}\left[1 - \frac{1}{2}e^{-t}\left(e^{j2t} + e^{-j2t} + j2e^{j2t} - j2e^{-j2t}\right)\right] \times u(t)$$

$$= \frac{1}{5}\left[1 - e^{-t}\cos(2t) + e^{-t}\sin(2t)\right]u(t)$$

$$Y(s) = Y_1(s)e^{-s} \Longrightarrow y(t) = y_1(t-1)$$

$$\rightarrow y(t) = \frac{1}{5}\left[1 - e^{-t+1}\left(\cos(2t-2) + \sin(2t-2)\right)\right] \times u(t-1)$$