

FINAL EXAMINATION
Course: **Signals and Systems**
Duration: 90 minutes

Part 1 (Multiple-choice questions): For problems in this part, you only have to give the letter of the correct answer (A/B/C/D). Explanations are not required.

Problem 1. Which one of the systems described by the following impulse responses is both causal and stable?

- A. $h(t) = \sin(3\pi t)[u(t+1) - u(t-1)]$
- B. $h(n) = (1/3)^n[u(n) - u(2n-1)]$
- C. $h(n) = -nu(n)$
- D. $h(t) = e^{2t}u(t/2)$

Answer: B (1 point)

Problem 2. Which one of the following systems is NOT a linear time-invariant system?

- A. $\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + x(t)$
- B. $y(n) + y(n-1) = 2^n x(n)$
- C. $\frac{d^2 y(t)}{dt^2} - \frac{dy(t)}{dt} = -x(t)$
- D. $y(n) - y(n-1) + 2y(n+1) = x(n-1)$

Answer: B (1 point)

Problem 3. Given a system described by the following transfer function:

$$X(s) = \frac{2s+1}{s^2+3s+2}$$

which one of the following statements about this system is NOT correct?

- A. This system can be both causal and stable.

- B. This system can not be both non-causal and stable.
- C. If this system is causal then its frequency response exists.
- D. If this system is non-causal then its frequency response exists.

Answer: D (1 point)

Problem 4. Which one of the following statements is NOT correct?

- A. A stable linear time-invariant system can not have a periodic impulse response.
- B. The frequency response of a stable linear time-invariant system is the Fourier transform of its impulse response.
- C. The frequency response of a stable discrete-time linear time-invariant system is discrete.
- D. The frequency response of a stable discrete-time linear time-invariant system is continuous.

Answer: C (1 point)

Part 2 (Exercises): For problems in this part, detailed explanations/derivations that lead to the answer must be provided.

Problem 5. Given a causal linear time-invariant system described by the following differential equation:

$$\frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 2 y(t) = x(t) - \frac{dx(t)}{dt}$$

- a) Determine the transfer function of the given system.
- b) Determine the impulse response of the given system.
- c) Determine the step response of the system.

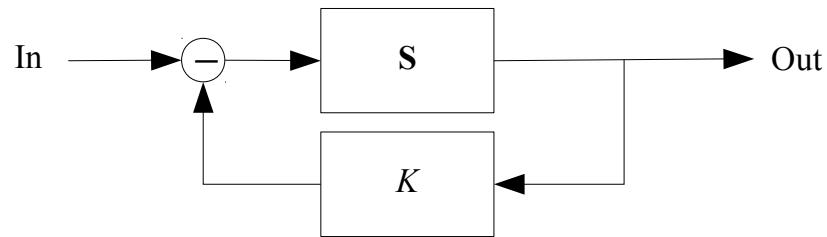
Answer:

$$a) \quad H(s) = \frac{1-s}{s^2+3s+2} \quad (1 \text{ point})$$

$$b) \quad h(t) = (2e^{-t} - 3e^{-2t})u(t) \quad (1 \text{ point})$$

$$c) \quad y(t) = \left(\frac{1}{2} - 2e^{-t} + \frac{3}{2}e^{-2t}\right)u(t) \quad (1 \text{ point})$$

Problem 6. Given a system **T** described by the following block diagram:



in which, **S** is a discrete-time causal linear time-invariant system described by the difference equation $y(n) + 2y(n-1) = x(n-1)$ and K is a real value.

- Determine the transfer function of **T**.
- Determine the frequency response of **T** (if it exists) when $K = 1$ and when $K = -2$.
- Determine the condition for K so that **T** is stable.

Answer:

a) $H(z) = \frac{K}{z + K + 2}$ (1 point)

b) $K = 0$: not exist, $K = -2$: $H(\Omega) = -2e^{-j\Omega}$ (1 point)

c) $-3 < K < -1$ (1 point)

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