

## BÀI TẬP CHƯƠNG 3 \_ PHẦN 2

### I. Biểu diễn Fourier của chuỗi tuần hoàn (DTFS – Discrete time fourier serier)/Không tuần hoàn (Discrete time fourier transform)

#### Bài 1:

$$(a) \ x[n] = \cos(0,1\pi n) = \frac{1}{2} e^{j\frac{\pi}{10}n} + \frac{1}{2} e^{-j\frac{\pi}{10}n}, N_0 = 20, \Omega = \frac{\pi}{10}$$

$$x[n] = \sum_{k=0}^{N_0} X[k] e^{jk\Omega n}$$

$$\Rightarrow X[k] = \begin{cases} \frac{1}{2}, & k = \pm 1 \\ 0, & \text{otherwise} \end{cases}$$

$$(b) \ x[n] = \sin(0,1\pi n) = \frac{1}{2j} e^{j\frac{\pi}{10}n} - \frac{1}{2j} e^{-j\frac{\pi}{10}n}, N_0 = 20, \Omega = \frac{\pi}{10}$$

$$x[n] = \sum_{k=0}^{N_0} X[k] e^{jk\Omega n}$$

$$\Rightarrow X[k] = \begin{cases} \frac{1}{2j}, & k = 1 \\ -\frac{1}{2j}, & k = -1 \\ 0, & \text{otherwise} \end{cases}$$

$$(c) \ x[n] = 2\cos(1,6\pi n) + \sin(2,4\pi n), N_0 = 5, \Omega = \frac{2\pi}{5}$$

$$x[n] = e^{j4\frac{2\pi}{5}n} + e^{-j4\frac{2\pi}{5}n} + \frac{1}{2j} e^{j6\frac{2\pi}{5}n} - \frac{1}{2j} e^{-j6\frac{2\pi}{5}n} = \sum_{k=0}^{N_0} X[k] e^{jk\Omega n}$$

$$\Rightarrow X[k] = \begin{cases} 1, & k = \pm 4 \\ \frac{1}{2j}, & k = 6 \\ -\frac{1}{2j}, & k = -6 \\ 0, & \text{otherwise} \end{cases}$$

#### Bài 2:

#### Bài 3:

$$(a) \ x[n] = \left(\frac{1}{3}\right)^n u[n+2]$$

$$x[n] = \left(\frac{1}{3}\right)^n u[n+2] = 9 \cdot \left(\frac{1}{3}\right)^{n+2} u[n+2]$$

$$\left(\frac{1}{3}\right)^n u[n] \rightarrow \frac{1}{1 - \frac{1}{3}e^{-j\Omega}}$$

$$s[n+2] \rightarrow e^{j2\Omega} S(e^{j\Omega})$$

$$\Rightarrow X(e^{j\Omega}) = \frac{9e^{j2\Omega}}{1 - \frac{1}{3}e^{-j\Omega}}$$

$$(b) x[n] = (n-2)(u[n+4] - u[n-5])$$

$$u[n+4] - u[n-5] \rightarrow \frac{\sin\left(\frac{9\Omega}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)}$$

$$ns[n] \rightarrow j \frac{d}{d\Omega} S(e^{j\Omega})$$

$$\Rightarrow X(e^{j\Omega}) = j \frac{d}{d\Omega} \frac{\sin\left(\frac{9\Omega}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)} - 2 \frac{\sin\left(\frac{9\Omega}{2}\right)}{\sin\left(\frac{\Omega}{2}\right)}$$

$$(c) x[n] = \cos\left(\frac{\pi}{4}n\right) \left(\frac{1}{2}\right)^n u[n-2]$$

$$x[n] = \cos\left(\frac{\pi}{4}n\right) \left(\frac{1}{2}\right)^n u[n-2] = \cos\left(\frac{\pi}{4}n\right) \cdot \frac{1}{4} \cdot \left(\frac{1}{2}\right)^n u[n-2]$$

$$A[n] = \left(\frac{1}{2}\right)^n u[n] \rightarrow A(e^{j\Omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\Omega}}$$

$$B[n] = s[n-2] \rightarrow B(e^{j\Omega}) = e^{-j2\Omega} S(e^{j\Omega})$$

$$x[n] = \cos\left(\frac{\pi}{4}n\right) s[n-2] \rightarrow X(e^{j\Omega}) = \frac{1}{2} B(e^{j(\Omega - \frac{\pi}{4})}) + \frac{1}{2} B(e^{j(\Omega + \frac{\pi}{4})})$$

$$\Rightarrow X(e^{j\Omega}) = \frac{1}{8} \left[ \frac{e^{-2j(\Omega - \frac{\pi}{4})}}{1 - \frac{1}{2}e^{-j(\Omega - \frac{\pi}{4})}} + \frac{e^{-2j(\Omega + \frac{\pi}{4})}}{1 - \frac{1}{2}e^{-j(\Omega + \frac{\pi}{4})}} \right]$$

$$(d) x[n] = \left[ \frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n} \right] * \left[ \frac{\sin\left(\frac{\pi}{4}(n-8)\right)}{\pi(n-8)} \right]$$

$$s[n] = \frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n} \rightarrow S(e^{j\Omega}) = \begin{cases} 1, & |\Omega| \leq \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\Omega| \leq \pi \end{cases}$$

$$a[n] = s[n-8] \rightarrow A(e^{j\Omega}) = e^{-j8\Omega} S(e^{j\Omega})$$

$$\Rightarrow X(e^{j\Omega}) = A(e^{j\Omega}) S(e^{j\Omega}) = \begin{cases} e^{-j8\Omega}, & |\Omega| \leq \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\Omega| \leq \pi \end{cases}$$

$$(e) x[n] = \left[ \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} \right]^2 * \left[ \frac{\sin\left(\frac{\pi}{2}n\right)}{\pi n} \right]$$

$$a[n] = s[n]s[n] \rightarrow A(e^{j\Omega}) = \frac{1}{2\pi} S(e^{j\Omega}) * S(e^{j\Omega}) = \begin{cases} \frac{1}{2} - \frac{|\Omega|}{2}, & |\Omega| \leq \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < |\Omega| \leq \pi \end{cases}$$

$$\Rightarrow X(e^{j\Omega}) = \begin{cases} \frac{1}{2} - \frac{|\Omega|}{2}, & |\Omega| \leq \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < |\Omega| \leq \pi \end{cases}$$

#### Bài 4:

$$(a) X(e^{j\Omega}) = 2 \cos(2\Omega)$$

$$x[n] = \frac{1}{2\pi} \int_0^{2\pi} 2 \cos(2\Omega) e^{j\Omega n} d\Omega = \begin{cases} 1, & n = \pm 2 \\ 0, & \text{otherwise} \end{cases}$$

$$(b) X(e^{j\Omega}) = \begin{cases} e^{-j4\Omega}, & \frac{\pi}{2} < |\Omega| \leq \pi \\ 0, & \text{otherwise} \end{cases}$$

$$x[n] = \delta[n-4] - \frac{\sin\left(\frac{\pi(n-4)}{2}\right)}{\pi(n-4)}$$

#### Bài 5:

$$(a) Y[k] = X[k-5] + X[k+5] \rightarrow [e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}] x[n]$$

$$\Rightarrow y[n] = 2 \cos\left(\frac{\pi}{2}n\right) \frac{\sin\left(\frac{11\pi}{20}n\right)}{\sin\left(\frac{\pi}{20}n\right)}$$

$$(b) Y[k] = \cos\left(\frac{k\pi}{5}\right) X[k] \rightarrow \frac{1}{2} [x[n-2] + x[n+2]]$$

$$\Rightarrow y[n] = \frac{1}{2} \left[ \frac{\sin\left(\frac{11\pi}{20}(n-2)\right)}{\sin\left(\frac{\pi}{20}(n-2)\right)} + \frac{\sin\left(\frac{11\pi}{20}(n+2)\right)}{\sin\left(\frac{\pi}{20}(n+2)\right)} \right]$$

$$(c) Y[k] = X[k] \otimes X[k] \rightarrow (x[n])^2$$

$$\Rightarrow y[n] = \frac{\sin^2\left(\frac{11\pi}{20}n\right)}{\sin^2\left(\frac{\pi}{20}n\right)}$$

$$(d) Y[k] = \text{Re}\{X[k]\} \rightarrow \frac{x[n] + x[-n]}{2}$$

$$\Rightarrow y[n] = \frac{\sin\left(\frac{11\pi}{20}n\right)}{\sin\left(\frac{\pi}{20}n\right)}$$

#### Bài 6:

$$(a) Y(e^{j\Omega}) = e^{-j4\Omega} X(e^{j\Omega}) \rightarrow x[n-4]$$

$$\Rightarrow y[n] = x[n-4] = (n-4) \left(\frac{3}{4}\right)^{|n-4|}$$

$$(b) Y(e^{j\Omega}) = \text{Re}\{X(e^{j\Omega})\} \rightarrow x_e[n]$$

Do  $x[n]$  là số thực và là hàm lẻ

$$\Rightarrow y[n] = 0$$

$$(c) Y(e^{j\Omega}) = \frac{d}{d\Omega} X(e^{j\Omega}) \rightarrow -jnx[n]$$

$$\Rightarrow y[n] = -jn^2 \left(\frac{3}{4}\right)^{|n|}$$

$$(d) Y(e^{j\Omega}) = X(e^{j\Omega}) \otimes X\left(e^{j\left(\Omega - \frac{\pi}{2}\right)}\right) \rightarrow 2\pi \cdot x[n] \cdot e^{j\frac{\pi}{2}n} x[n]$$

$$\Rightarrow y[n] = 2\pi \left(\frac{3}{4}\right)^{2|n|} e^{j\frac{\pi}{2}n}$$

$$(e) Y(e^{j\Omega}) = \frac{d}{d\Omega} X(e^{j2\Omega})$$

$$\begin{aligned} \frac{d}{d\Omega} S(e^{j\Omega}) &\rightarrow -jn^2 \left(\frac{3}{4}\right)^{2|n|} \\ A(e^{j\Omega}) &\rightarrow \begin{cases} a[n], & n \text{ chẵn} \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

$$\Rightarrow y[n] = \begin{cases} -jn^2 \left(\frac{3}{4}\right)^{2|n|}, & n \text{ chẵn} \\ 0, & \text{otherwise} \end{cases}$$

$$(f) Y(e^{j\Omega}) = X(e^{j\Omega}) + X(e^{-j\Omega}) \rightarrow x[n] + x[-n]$$

$$\Rightarrow y[n] = n\left(\frac{3}{4}\right)^{|n|} - n\left(\frac{3}{4}\right)^{|-n|} = 0$$

## II. Đáp ứng tần số

### Bài 1:

$$(a) y[n] = \frac{1}{2}(x[n] + x[n-1])$$

$$\rightarrow Y(e^{j\Omega}) = \frac{1}{2}(1 + e^{-j\Omega})X(e^{j\Omega})$$

$$H(e^{j\Omega}) = \frac{1}{2} \frac{1}{1 + e^{-j\Omega}}$$

$$(b) y[n] = \frac{1}{2}(x[n] - x[n-1])$$

$$\rightarrow Y(e^{j\Omega}) = \frac{1}{2}(1 - e^{-j\Omega})X(e^{j\Omega})$$

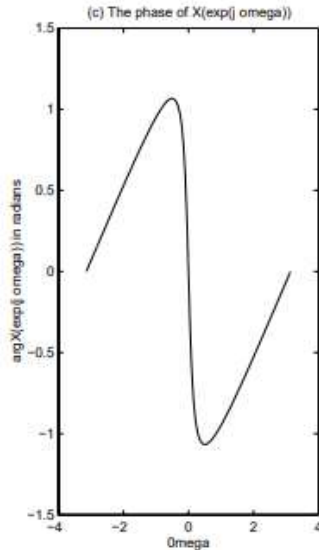
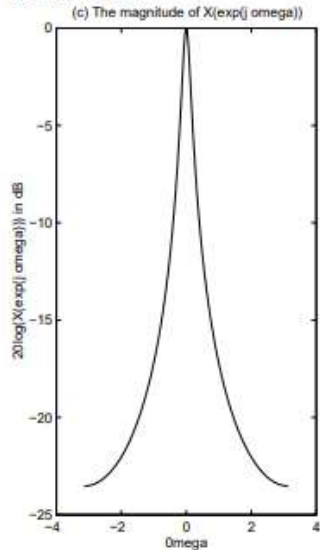
$$H(e^{j\Omega}) = \frac{1}{2} \frac{1}{1 - e^{-j\Omega}}$$

### Bài 2:

$$(a) h[n] = \frac{1}{8} \left(\frac{7}{8}\right)^n u[n]$$

$$H(e^{j\Omega}) = \frac{1}{8 - 7e^{-j\Omega}}$$

Low Pass filter.

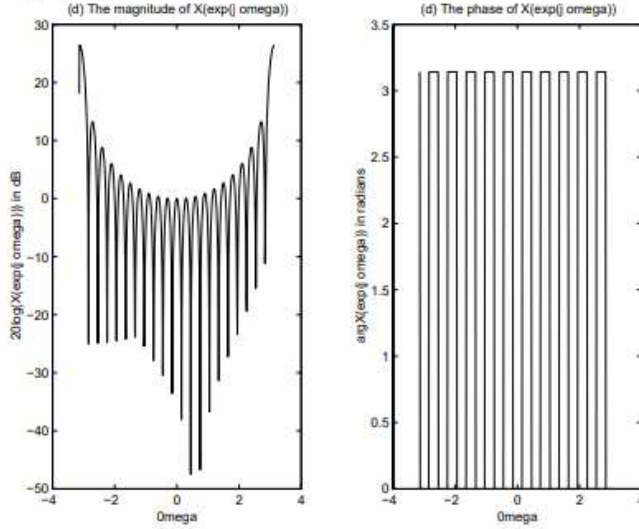


$$(b) h[n] = \begin{cases} (-1)^n, & |n| \leq 10 \\ 0, & \text{otherwise} \end{cases}$$

$$h[n] = \begin{cases} e^{j\pi n}, & |n| \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

$$H(e^{j\Omega}) = \frac{\sin\left(\frac{21}{2}(\Omega - \pi)\right)}{\sin\left(\frac{1}{2}(\Omega - \pi)\right)}$$

High pass filter.



**Bài 3:**

$$(a) \quad x[n] = \left(\frac{1}{2}\right)^n u[n], \quad y[n] = \frac{1}{4}\left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[n]$$

$$X(e^{j\Omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\Omega}}$$

$$Y(e^{j\Omega}) = \frac{1}{4} \frac{1}{1 - \frac{1}{2}e^{-j\Omega}} + \frac{1}{1 - \frac{1}{4}e^{-j\Omega}}$$

$$H(e^{j\Omega}) = \frac{1}{4} + \frac{1 - \frac{1}{2}e^{-j\Omega}}{1 - \frac{1}{4}e^{-j\Omega}}$$

$$\Rightarrow h[n] = \frac{1}{4}\delta[n] + \left(\frac{1}{4}\right)^n u[n] - \frac{1}{2}\left(\frac{1}{4}\right)^{n-1} u[n-1]$$

$$(b) \quad x[n] = \left(\frac{1}{4}\right)^n u[n], \quad y[n] = \left(\frac{1}{4}\right)^n u[n] - \left(\frac{1}{4}\right)^{n-1} u[n-1]$$

$$X(e^{j\Omega}) = \frac{1}{1 - \frac{1}{4}e^{-j\Omega}}$$

$$Y(e^{j\Omega}) = \frac{1}{1 - \frac{1}{4}e^{-j\Omega}} - \frac{e^{-j\Omega}}{1 - \frac{1}{4}e^{-j\Omega}}$$

$$H(e^{j\Omega}) = 1 + e^{-j\Omega}$$

$$\Rightarrow h[n] = \delta[n] - \delta[n-1]$$

**Bài 4:**

$$\begin{aligned}
\text{(a)} \quad y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] &= 3x[n] - \frac{3}{4}x[n-1] \\
&\rightarrow \left(1 - \frac{1}{4}e^{-j\Omega} - \frac{1}{8}e^{-j2\Omega}\right)Y(e^{j\Omega}) = \left(3 - \frac{3}{4}e^{-j\Omega}\right)X(e^{j\Omega}) \\
&\rightarrow H(e^{j\Omega}) = \frac{3 - \frac{3}{4}e^{-j\Omega}}{1 - \frac{1}{4}e^{-j\Omega} - \frac{1}{8}e^{-j2\Omega}} = \frac{1}{1 - \frac{1}{2}e^{-j\Omega}} + \frac{2}{1 - \frac{1}{4}e^{-j\Omega}}
\end{aligned}$$

$$\Leftrightarrow h[n] = \left[\left(\frac{1}{2}\right)^n + 2\left(\frac{1}{4}\right)^n\right]u[n]$$

$$\begin{aligned}
\text{(b)} \quad y[n] + \frac{1}{2}y[n-1] &= x[n] - 2x[n-1] \\
&\rightarrow \left(1 + \frac{1}{2}e^{-j\Omega}\right)Y(e^{j\Omega}) = (1 - 2e^{-j\Omega})X(e^{j\Omega}) \\
&\rightarrow H(e^{j\Omega}) = \frac{1 - 2e^{-j\Omega}}{1 + \frac{1}{2}e^{-j\Omega}}
\end{aligned}$$

$$\Leftrightarrow h[n] = \left(-\frac{1}{2}\right)^n u[n] - 2\left(-\frac{1}{2}\right)^{n-1} u[n-1]$$

**Bài 5:**

$$\text{(a)} \quad h[n] = \alpha^n u[n], \quad |\alpha| < 1$$

$$\begin{aligned}
H(e^{j\Omega}) &= \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \frac{1}{1 - \alpha e^{-j\Omega}} \\
&\rightarrow Y(e^{j\Omega})(1 - \alpha e^{-j\Omega}) = X(e^{j\Omega})
\end{aligned}$$

$$\Leftrightarrow y[n] - \alpha y[n-1] = x[n]$$

$$\text{(b)} \quad h[n] = \delta[n] + 2\left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{2}\right)^n u[n]$$

$$\begin{aligned}
H(e^{j\Omega}) &= \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} = 1 + \frac{2}{1 - \frac{1}{2}e^{-j\Omega}} \frac{1}{1 + \frac{1}{2}e^{-j\Omega}} = \frac{4 + \frac{1}{2}e^{-j\Omega} - \frac{1}{4}e^{-j\Omega}}{1 - \frac{1}{4}e^{-j2\Omega}} \\
&\rightarrow Y(e^{j\Omega})\left(1 - \frac{1}{4}e^{-j2\Omega}\right) = X(e^{j\Omega})\left(4 + \frac{1}{2}e^{-j\Omega} - \frac{1}{4}e^{-j\Omega}\right)
\end{aligned}$$

$$\Leftrightarrow y[n] - \frac{1}{4}y[n-2] = 4x[n] + \frac{1}{2}x[n-1] - \frac{1}{4}x[n-2]$$

**Bài 6:**

$$\begin{aligned}
\text{(a)} \quad H(e^{j\Omega}) &= \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \frac{1+e^{-j\Omega}}{e^{-j2\Omega}+3} \\
&\rightarrow Y(e^{j\Omega})(e^{-j2\Omega}+3) = X(e^{j\Omega})(1+e^{-j\Omega})
\end{aligned}$$

$$\Leftrightarrow 3y[n] + y[n-2] = x[n] + x[n-1]$$

$$\begin{aligned}
\text{(b)} \quad H(e^{j\Omega}) &= \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} = 1 + \frac{e^{-j\Omega}}{\left(1 - \frac{1}{2}e^{-j\Omega}\right)\left(1 + \frac{1}{4}e^{-j\Omega}\right)} = \frac{1 + \frac{3}{4}e^{-j\Omega} - \frac{1}{8}e^{-j2\Omega}}{1 - \frac{1}{4}e^{-j\Omega} - \frac{1}{8}e^{-j2\Omega}} \\
&\rightarrow Y(e^{j\Omega})\left(1 - \frac{1}{4}e^{-j\Omega} - \frac{1}{8}e^{-j2\Omega}\right) = X(e^{j\Omega})\left(1 + \frac{3}{4}e^{-j\Omega} - \frac{1}{8}e^{-j2\Omega}\right)
\end{aligned}$$

$$\Leftrightarrow y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + \frac{3}{4}x[n-1] - \frac{1}{8}x[n-2]$$

