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Homework 4 Submission
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# CST 370
## Homework (Complexity Analysis)

1. Consider the following algorithm.

**ALGORITHM** *Mystery(n)*

//Input: A nonnegative integer n

S ← 0

**for** *i* ←1 **to** ***n*** **do**

S ← S + i*i

return S

a. What does this algorithm compute? = S(n) = $\sum_{i=1}^{n} i^2$

b. What is its basic operation? = **Multiplication,** (i * i)

c. How many times is the basic operation executed? = **n times**

d. What is the efficiency class of this algorithm? = **O(n)**

2. Consider the following algorithm.

**ALGORITHM** *Secret*(A[0…n-1])

//Input: An array A[O..n - 1] of *n* real numbers

*minval* ←A[O]; *maxval* ←A[O]

**for** i ←1to *n*- 1 **do**

if (A[i] < *minval)*

*minval* ←A[i]

if (*A[i]* > *maxval)*

*maxval* ←*A[i]*

**return** *maxval* - *minval*

a. What does this algorithm compute? = **The difference between the maximum and minimum values of the array's elements.**

b. What is its basic operation? = **The comparison operations,** A[i] < minval or A[i] > maxval

c. How many times is the basic operation executed? = **2(n-1) times**

d. What is the efficiency class of this algorithm? = **O(n)**

3. Compute the following sums.

a.1 + 3 + 5 + 7…. 999 = **250,000**

$1 + 2n = 999$
$n = 500$

$S(n) = ((1 + 999)500)/2$
$= 500{,}000/2$
$= 250{,}000$

b. 2 + 4 + 8 + 16 + . . . + 1024 = **2,046**

$1024 = 2 * 2^{n-1}$
$1024 = 4^{n-1}$
$512 = 2^{n-1}$
$2^9 = 2^{n-1}$
$9 = n - 1$
$n = 10$

$S(n) = 2(1-2^{10})/1 - 2$
$= 2(-1023) / -1$
$= -2{,}046 / -1$
$= 2{,}046$

4. Climbing stairs Problem: Find the number of different ways to climb an n-stair stair- case if each step is either one or two stairs.

**F(n) = F(n-1) + F(n-2)**
**for n>=3, F(1) = 1, F(2) = 2**

5. Solve the following recurrence relations.

a. $x(n) = x(n-1) + 5$ for $n > 1$, $x(1) = 0$ = **5(n-1)**

b. $x(n) = 3x(n-1)$ for $n > 1$, $x(1) = 4$ = **3^n-1 * 4**

c. $x(n) = x(n-1) + n$ for $n > 0$, $x(0) = 0$ **= n(n+1) / 2**

a. $= x(n) = x(n-1) + 5$
$x(n-1) = x(n-2) + 5$
$x(n) = x(n-2) + 5 + 5$
$= x(n) = x(n-2) + 5 * 2$
$x(n-2) = x(n-3) + 5 + 5$
$x(n) = x(n-3) + 5 + 5 + 5$
$= x(n) = x(n-3) + 5 * 3$

$= x(n-k) + 5 * k$
$= x(n-(n-1)) + 5 * (n-1)$
$= x(1) + 5(n-1)$
$= 0 + 5(n-1)$
$= 5(n-1)$

b. $= x(n) = 3x(n-1)$
$x(n-1) = 3x(n-2)$
$x(n) = 3(3x(n-2))$
$= x(n) = 3^2x(n-2)$
$x(n-2) = 3(3x(n-3))$
$x(n) = 3(3(3x(n-3)))$
$= x(n) = 3^3x(n-3)$

$= 3^k * x(n-k)$
$= 3^{n-1} * x(n-(n-1))$
$= 3^{n-1} * x(1)$
$= 3^{n-1} * 4$

c. $= x(n) = x(n-1) + n$
$x(n-1) = x(n-2) + n-1$
$x(n) = x(n-2) + n-1 + n$
$= x(n) = x(n-2) + 2n - 1$
$x(n-2) = x(n-3) + 2(n-1) - 1$
$x(n) = x(n-3) + 2n - 2 - 1 + n$
$= x(n) = x(n-3) + 3n -3$
$x(n-3) = x(n-4) + 3(n-1) - 3$
$x(n) = x(n-4) + 3n - 3 - 3 + n$
$= x(n) = x(n-4) + 4n - 6$
$x(n-4) = x(n-5) + 4(n-1) - 6$
$x(n) = x(n-5) + 4n - 4 - 6 - n$
$= x(n) = x(n-5) + 5n - 10$

$= x(n -k) + kn - n(n+1)/2$
$= x(n-(n-1) + n(n-1) - n(n+1)/2$
$= x(0) + n(n-1) - n(n+1)/2$
$= n(n+1)/2$