Consumption and Savings under Imperfect Perception of Expenditure Shocks*

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Abstract

I build a life cycle model of consumption and savings where consumers face exogenous expenditure shocks. Consumers are heterogeneous by having different levels of perceptions of expenditure shocks. Agents who underestimate the expenditure shocks tend to spend more now and save less for the future. When calibrating the distribution of heterogeneous perceptions to match the distribution of liquid wealth, the model features many consumers who underestimate the expenditure shocks and hence have low liquid wealth. Based on the intertemporal elasticity of substitution below one, following the micro evidence, the calibrated model can still show the high marginal propensity to consume, which stems from the high dispersion of liquid wealth.

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1 Introduction

This paper studies a model of expenditure shocks that can explain the high marginal propensity to consume (MPC) by featuring enough liquidity-constrained households with the imperfect perception of future expenditure shocks. The MPC, which measures the amount consumers spend out of a windfall gain, is important when designing a policy, such as fiscal stimulus to boost economic activity. Not only is its overall magnitude crucial to gauge the size of stimulus needed, but also understanding the heterogeneity underlying the large MPC can help enhance the efficacy of such stimulus by focusing on the groups with larger consumption responses.

A large piece of empirical evidence documents the high MPC among households with low liquidity (Souleles, 1999; Johnson et al., 2006; Parker et al., 2013; Baker, 2018; Baker et al., 2020; Fagereng et al., 2021; Aydin, 2021). Reflecting on the empirical evidence, seminal works explaining high MPC has liquidity-constrained households at their hearts. Carroll (1992, 1997) show how adopting a borrowing constraint can make households that are hand-to-mouth and exhibit high MPC. Kaplan and Violante (2014) build a model where households have disproportionately small liquid assets compared to a large stock of illiquid savings and are effectively hand-to-mouth by separating savings for retirement from precautionary motives. Carroll et al. (2017), equipped with discount factor heterogeneity, explains the high MPC driven by impatient households with a low level of wealth.

The success of generating high MPC tends to rely on a key parameter, namely the degree of the intertemporal elasticity of substitution (IES). The IES controls the desire to smooth consumption throughout life. With low IES, households would want to smooth consumption more, and they tend to save more during the middle age to bring up the consumption after retirement when the income is low. Micro estimates suggest a third (Havránek, 2015) and they typically tend to be less than one (e.g. Best et al. (2020)). When using low IES in the model, this dampens forces towards the sensitive consumption response (Aguiar et al., 2020). Households are less likely to be hand-to-mouth since they eagerly hold liquid assets to balance the amount of consumption even across the near future under low IES. Unfortunately, most of the modeling success so far tends to rely on the high IES.

Moreover, the dispersion of liquid wealth associated with the high MPC is much unequal than prior literature on high MPC suggests. Using the Survey of Consumer Finances, I present a novel finding that severe inequality of liquid wealth exists within all stages of life. Hence, the low liquidity does not arise from a particular stage of life but rather due to a factor that persistently interferes throughout the life cycle.

This dispersion of liquid wealth cannot be rationalized by factors that drives the inequality of total wealth. For example, earnings heterogeneity (Castañeda et al., 2003), heterogeneity of asset returns (Hubmer et al., 2020), or entrepreneurship Quadrini (1999) can account for the inequality of total wealth. However, the degree of liquid wealth inequality among all households is not different from, the degree of inequality within subgroups of people with similar educational, occupational, and income characetristics. Separately focusing on the highly liquid assets, such as money market, checking and savings accounts, does not relieve the dispersion of wealth. Hence, this dispersion cannot be explained investment skill or luck. The dispersion reflects the choice, that is persistent in all stages of life.

To explain the high MPC arising from the realistic dispersion of liquid wealth independent of demographic factors, I introduce households with heterogeneous perceptions of future expenditure shocks. The consumption and savings of the households crucially depend on how much they under or overestimate the future expenditure shocks. Households that underestimate the expenditure shocks feel less need for precautionary savings, and they tend to spend more than other households that overestimate. Formally, if distributions regarding the future expenditure shocks agent A first-order stochastically dominate the distribution in agent B's mind, then consumption of agent A will be higher than agent B when they face the same level of current wealth. Dynamically, this creates a gap of wealth between households that relatively underestimates and overestimates than others. Especially, households that underestimate the expenditure shock spend more than they originally planned in the past, and their liquid savings get depleted quickly, and become hand-to-mouth.

The channel of generating high MPC by this paper is not completely separate from contributions of the previous literature. I adopt the two-asset environment à la Kaplan and Violante (2014). For theoretical tractability, I employ a streamlined version of Kaplan and Violante (2014) which allows the withdrawl of illiquid assets in a fixed retirement date, and households can save without any penalty as opposed to Kaplan and Violante (2014). Though there are subtle differences, the key mechanism generating is the same. Households have low liquid wealth by shifting all savings retirement to the illiquid wealth. The low liquidity is amplified among households who underestimate the future expenditure shocks.

I calibrate the model to gauge its power in explaining the dispersion of liquid wealth and high MPC. As the first step, I elicit the expenditure shocks from the data by estimating different expenditure categories' persistence and treating nonpersistent items as the expenditure shock. This classification incorporates the theoretical implication that the endogenous part of consumption is being smoothed over the life cycle and exhibits a positive serial correlation. However, the nonpersistent part has a weak serial correlation where short-run motives rather than long-run trends are more relevant, suggesting a deviation from the consumption smoothing. Furthermore, this classification leads to other striking differences between the nonpersistent expenditure shocks and the persistent endogenous part of consumption. First, the variance of the nonpersistent part is more than twice of the persistent part, while the size of the nonpersistent part is only a quarter of the persistent part. This large difference in the variance also suggests that the nonpersistent part of the expenditure deviates from the consumption smoothing principle. The heavily skewed nature of the expenditure shocks explains why households might underestimate the true level of shocks. For example, suppose an agent can make an unbiased estimation of the parameter ruling the exponential distribution. In that case, the median estimate of the mean from finitely many observations will always be less than the true mean. Hence, most households will have perceptions of the mean of the distribution that is less than the actual mean.

There are other aspects of data which suggests misprediction of future expenses. According to the Survey of Consumer Finances in 2019, among people who had loan experience in 2018, 37.3 percent were behind the loan payment schedule by two months or more. About 2.4 percent of the households rely on payday loans, which typically accompany high interest, and 38.1 percent answered that they had to confront emergency costs. In the same survey, 27.7 percent of subjects answered that their primary reason for saving is to prepare for the "rainy days," which is the second most picked reason for saving only after retirement purposes (34.1 percent). Also, 38.6 percent of working-age households have credit card loans where the median borrowing rate is 17 percent, and this high frequency of borrowing is difficult to be rationalized with such a high borrowing rate. As a related study, Berman et al. (2016) provides evidence that consumers tend to underweight the costs needed rather than income when predicting the spare money in the future. Howard et al. (2020) studies an expenditure prediction bias which is a tendency to underestimate future expenses. Fellowes and Willemin (2013) suggest that 25% of households withdraw 401(k) savings early, often to cover unexpected expenses.

Calibrating the distribution of perception required to mimic the share of wealth held at 20, 40, 60, and 80 percentiles and the median ratio of liquid over income, it turns out that households perceive about a third of the true expenditure shock. Hence, it features enough optimistic households with a low level of liquid assets by not realizing the precautionary saving they need to do the consumption smoothing properly. Regarding the dispersion of wealth, the calibrated model captures the share of wealth at different percentile and tracks the liquid wealth over the life cycle and its dispersion reasonably well.

This paper also makes several methodological contributions. First, it provides a way of identifying a distribution of heterogeneous households along with a continuous domain. To do this, I utilize two shape parameters of the beta distribution and a scale parameter that determines the size of the maximum value. Controlling three parameters lets us represent various types of distribution on a continuous domain by controlling the mean, skewness, and kurtosis. Potentially, this methodology may provide richer information than just analyzing a finite number of agents or agents along with a uniform distribution. Second, this paper provides a model with a monotone relationship between the agents' perception about the future and the consumption today, which is an important source of identification when we would like to take the model into the data.

The model in this paper calls for two types of policy recommendations. First, if the policy goal is to amplify consumption during a recession, it would be practical to target the households with a low perception of future expenditure shocks, which can be proxied by net worth. These households will typically be hand-to-mouth, which allows high consumption response even under the Ricardian equivalence. Second, if the policy aims to correct the misbalance of consumption during the lifetime, this cannot be done to optimistic households without taxing them. However, corrective policy to pessimistic households by providing additional liquidity is possible.

2 Stylized Facts

Fact 1: Severe Dispersion of Liquid Wealth Among Homogenous Groups/Assets Historically, the wealth inequality measured by the Gini coefficient has been persistently around 0.8. Compared to the income equality with the Gini coefficient around 0.5, there is much severe wealth inequality. In various directions, many models attempted to replicate this phenomenon. Important contributions highlights dispersion of labor efficiency (Castañeda et al., 2003), asset return heterogeneity (Hubmer et al., 2020), heterogeneity in discount factors (Krusell and Smith, 1998; Carroll et al., 2017; Hubmer et al., 2020), and entrepreneurship (Quadrini, 1999) to name a few.

Dispersion of liquid wealth is more severe than the net worth. Then, can factors that explain the dispersion of total wealth also apply to the dispersion of liquid wealth? To answer this question, if the dispersion of wealth measured among a homogeneous group of people/assets is significantly less than the dispersion among the overall population. Specifically, for each Survey of Consumer Finances survey, I only consider working-age households from 25 to 64 and drop the samples that earn under the annualized minimum wage. Also, I drop the samples that exhibit a negative amount of net worth. The definition of a *liquid asset* is the sum of money market, checking, savings and call accounts, directly held stocks, and bonds. To assess the inequality among households with similar characteristics, I make a subgroup of households by their respective education and income levels. Also, to control the effects of heterogeneous asset returns, I examine the inequality of *highly liquid asset* which comprises money market, checking, and savings accounts. Since the returns of items in the highly liquid asset are likely to be similar across households, their balances would represent households' desire to save and not differences in the investment skills or luck.

	All	Education			Income Tertiles		
		Graduate	Tertiary	Secondary	1st (highest)	2nd	3rd (lowest)
Liquid	0.89	0.87	0.86	0.86	0.85	0.80	0.90
Highly liquid	0.82	0.78	0.77	0.81	0.76	0.72	0.83

Table 1: Average Gini coefficients of liquid assets among working-age households by education and income levels: 1989-2019

Table 1 shows the average Gini coefficients in the Survey of Consumer Finances across all survey years between 1989-2019. Compared to the Gini coefficient of the net worth suggested in the literature, which is around 0.8, the Gini coefficient of the liquid wealth is over 0.8, which suggests severe wealth inequalty.[1] By grouping households by different education and income levels, the degree of inequality gets dampened but not by large. Especially, the Gini coefficient of the lowest income tertile is around 0.9, which suggests a high proportion of households with a near-zero level of wealth and producing a high Gini coefficient.

	Age groups			
	25-34	35-44	45-54	55-64
Liquid	0.80	0.86	0.89	0.90
Highly liquid	0.74	0.79	0.82	0.82

Table 2: Average Gini coefficients of liquid assets among working-age households by age groups: 1989-2019

The lifecycle model of consumption and savings implies that households accumulate large assets near the end of retirement, which would lead to the dispersion of wealth across different stages of life. Households with a near-zero level of wealth will be mostly young households since they face uphill income profiles and postpone saving. Households with low liquidity will vanish from the middle-age as they start accumulating wealth. Hence, if we group people by different age groups, inequality of wealth can be expected to go away. Surprisingly, grouping the households by different life stages does not contribute to the lower dispersion of wealth. As Table 2 suggests, there is a trend that the degree of dispersion increase as households reach the retirement age, but the Gini coefficients among households in the similar stages of life is roughly the same as the overall Gini coefficient. This implies that the households with low liquidity exist in all age groups, and life cycle properties are not the crucial factor leading to the severe dispersion of liquid wealth.

[1] This can be confirmed again by year-by-year calculation of the Gini coefficient in Table 8.

	Occupation group 1		Occupation group 2		
	Work for others	Self-employed	Managerial	Technical	Other
Liquid	0.87	0.89	0.87	0.88	0.81
Highly liquid	0.79	0.82	0.78	0.81	0.76

Table 3: Average Gini coefficients of liquid assets among working-age households by different occupation groups: 1989-2019

Note: 'Managerial' refers to manegerial and professional workers. 'Technical' refers to technical, sales and service workers.

Lastly, we check if different occupational characteristics can contribute to the dispersion of liquid wealth. Different occupational groups may have different occupational needs. For example, self-employed workers may have less predictable income which might increase desire for the precautionary savings. Also, households with higher income such as managerial and professional workers would have outliers that lead to greater dispersion of liquid wealth. Table 3 gauge inequality of liquid wealth by two classfications of occupation groups. In the first two columns, I contrast the group working for others, and the self-employed. Not only the the degree of inequality is similar, severe inequality exists in both groups. The remaining three columns also confirms that different types of occupations does not lead to different degree of liquid wealth inequality. Moreover, this inequality remains even when measured with highly liquid assets.

We checked if several sources that may explain the dispersion of total wealth can also explain the dispersion of liquid wealth. However, the exercise in this section shows that the high dispersion of liquid wealth persists within subgroups of people with similar education and income levels or stages of life. Hence, there is a force driving severe dispersion of life regardless of demographic or economic factors.

Fact 2: Large MPC and Low IES Most studies studying excess sensitivity of consumption agree that consumers' spending response to transitory income shock is large. For example, (Souleles, 1999) reports that consumers spend 0.344-0.64 out of the income tax refunds over a quarter. Agarwal and Qian (2014) measured that consumers spent 80 percent out of unanticipated fiscal stimulus in Singapore over ten months period. In response to social security tax reform, Parker (1999) found that households spent about 20 percent out of additional after-tax income over three-month periods on nondurable goods. In income tax rebate of 2001, Johnson et al. (2006) found that households spent 20 to 40 percent over a quarter on nondurable goods. In fiscal stimulus of 2008, Parker et al. (2013) found that households' total expenditure increased by 50 to 90 percent over a quarter. Parker (2017) finds strong spending responses among households that lack sophistication and financial planning and hints that persistent characteristics play an essential role. Using predictable payments from Alaska permanent fund, (Kueng, 2018) finds that households spent 20 percent for nondurable and services over a quarter.

Another piece of evidence points to heterogeneity of responses. Based on two stimuli of U.S. in years 2001 and 2008, Misra and Surico (2014) shows that around half of households do not show a significant and positive response to the stimulus. (Fuster et al., 2020) asked directly to survey respondents how much they would spend out of a windfall gain; around 70 percent of the respondents exhibited MPC of zero. Hence, the large MPC is driven by a small subset of households with a large response to a liquidity shock.

A large piece of evidence suggests a strong relationship between high consumption response and low liquidity, consistent with the buffer stock theory of saving (Souleles, 1999; Johnson et al., 2006; Parker et al., 2013; Baker, 2018; Baker et al., 2020; Fagereng et al., 2021; Aydin, 2021). However, there is another piece of evidence showing that persistent characteristics are more crucial (Parker, 2017). The view of this paper is that the persistent characteristics drive the low liquidity and thus bring the high consumption response as as Gelman et al. (2019), Gelman (2021), and Carroll et al. (2017).

In most models, explaining large MPCs require intertemporal elasticity of substitution (IES) around one or large. Smaller IES increases the desire to smooth out consumption over the life cycle, and households would exhibit small MPCs as noted by Aguiar et al. (2020). At the same time, a meta-analysis by Havránek (2015) reports that when accounting for selective reporting of the IES, the median estimate of IES is around 1/3. This paper tries to bridge these to contrasting facts by using low IES but still exhibiting high MPC overall.

Fact 3: Misprediction of Expenses Households generally save for two reasons. In the 2019 Survey of Consumer Finance, the most frequently chosen reason for saving was to prepare the retirement (34.1 percent) among working-age households. The following big reason for saving was to prepare for the 'rainy days,' chosen by 27.7 respondents.[2] The two main reasons for saving are consistent with implications of modern consumption savings models such as Carroll (1992). Households accumulate a large amount of wealth to prepare the retirement, and at the same time, some portion of the wealth is to prepare randomness of income and other temporary shocks.

However, households' portfolio choice in the Survey of Household Finances suggests a lack of preparation for the rainy days. Among working-age households, three percent of households even resort to payday loans which typically accompany high interest. As a most frequently chosen option, 38.3 percent of people answered that using a payday loan was

[2]The third reason to save was buy own house chosen by 5.8 percent.

because of an emergency. Furthermore, among subjects who currently borrow money, 38.9 percent answered that they had an experience being behind the payment by two months or more. These facts show that many households cannot avoid borrowing with high interest or paying penalties for being behind schedule, which brings additional costs. Thus, even though households save to prepare for the rainy days, the unpredicted expenses force households to use borrowing facilities that bring extra cost. Also, there is a prevalence of credit card borrowing where the median borrowing rate in the same survey is around 17 percent. Around 38.6 percent of households have credit card loans. The borrowing rate and the borrowing frequency are too high to be rationalized with the models without any behavioral frictions. [3] The pattern of borrowing over the life cycle is at odds with the implications of the life cycle

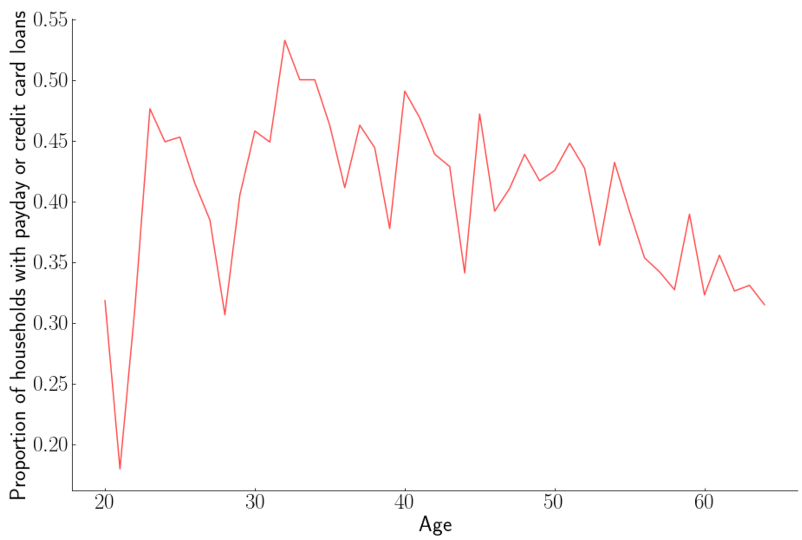

Figure 1: Proportion of having payday or credit card loans by age
Source: reproduced from the Survey of Consumer Finances at 2019

models without featuring a borrowing constraint. Such models would suggest that households would mainly borrow until the peak of the income profile and stop borrowing from then. However, Figure 1 does not show a sharp decrease in the frequency of borrowing near middle age, and the high frequency of borrowing also persists throughout the life cycle. Hence, contributions from other factors should persistently lead households to borrow throughout the life cycle.

As a related study, Berman et al. (2016) provides evidence that consumers tend to underweight the costs needed rather than income when predicting the spare money in the future.

[3] For example, the current workhorse model of consumption and savings by Kaplan and Violante (2014) targets 26 percent of borrowing rate with the nominal borrowing rate of 10 percent.

Howard et al. (2020) studies an expenditure prediction bias which is a tendency to underestimate future expenses. Fellowes and Willemin (2013) suggest that 25% of households withdraw 401(k) savings early, often to cover unexpected expenses.

3 Model

Households face consumption and savings problems for T number of periods. This paper takes a partial equilibrium approach where interest rates and income are assumed to be exogenous. There are two types of consumption. The first part of consumption is determined in the long-run perspective, and its trajectory is smooth over the life cycle. The second part of the consumption, which we call an expenditure shock, is not a choice variable where short-run motives are more important than consumption smoothing in the long run. Examples of expenditure shocks are vehicle repair costs or sudden medical expenses. Extra spending on items in this category does not bring additional utility. However, those are an unavoidable part of consumption, and we would not try to balance these types of expenditures throughout the life cycle. Typical examples of the long-run part of the consumption are food and utility expenditures.

I assume that the expenditure shock at period t is drawn from a random variable $\mathbf{\Gamma}_t$, which maps a set of events to nonnegative real numbers. Households have an imperfect perception of expenditure shocks, where the perception of $\mathbf{\Gamma}_t$, denoted as $\widetilde{\mathbf{\Gamma}}_t$, can be different from the actual $\mathbf{\Gamma}_t$. Similarly, variables with tilde, such as $\widetilde{x}$, represents the perception of a variable x. Without the time subscript, $\widetilde{\mathbf{\Gamma}} = \{\widetilde{\mathbf{\Gamma}}_1, \cdots, \widetilde{\mathbf{\Gamma}}_T\}$ represents sequence of perception of expenditure shocks. In the same way, $\mathbf{\Gamma}$ is the sequence of distribution of true expenditure shocks, $\mathbf{\Gamma} = \{\mathbf{\Gamma}_1, \cdots, \mathbf{\Gamma}_T\}$. Γ_t and $\widetilde{\Gamma}_t$ are particular realizations of random variables $\mathbf{\Gamma}_t$ and $\widetilde{\mathbf{\Gamma}}_t$.

Households retire at age T_r. During the working-age, households at time $t < T_r$ solve the following problem.

$$
\begin{aligned}
V_t^{\widetilde{\boldsymbol{\Gamma}}}(X_t, Z_t; \Gamma_t) &= \max_{C_t, S_t, A_t} u(C_t - \Gamma_t) + \delta \mathbb{E}_t \left[\widetilde{V}_{t+1}^{\widetilde{\boldsymbol{\Gamma}}}(X_{t+1}, Z_{t+1}; \widetilde{\Gamma}_{t+1}) \right] \quad \text{where} \qquad (1) \\
\widetilde{V}_{t'}^{\widetilde{\boldsymbol{\Gamma}}}(X_{t'}, Z_{t'}; \widetilde{\Gamma}_{t'}) &= \max_{C_{t'}, S_{t'}, A_{t'}} u(C_{t'} - \widetilde{\Gamma}_{t'}) + \delta \mathbb{E}_{t'} \left[\widetilde{V}_{t'+1}^{\widetilde{\boldsymbol{\Gamma}}}(X_{t'+1}, Z_{t'+1}; \widetilde{\Gamma}_{t'+1}) \right], \\
&\text{subject to} \\
Y_t + R_t^S S_{t-1} &= X_t \geq C_t + S_t + A_t, \\
Z_{t+1} &= R_{t+1}^A (A_t + Z_t), \\
C_t &\geq 0, A_t \geq 0 \text{ and } S_t \geq 0, \text{ and} \\
\widetilde{V}_{T_r-1}^{\widetilde{\boldsymbol{\Gamma}}}(X_{T_r-1}, Z_{T_r-1}; \widetilde{\Gamma}_{T_r-1}) &= \max_{C_{T_r-1}, S_{T_r-1}, A_{T_r-1}} u(C_{T_r-1} - \widetilde{\Gamma}_{T_r-1}) \\
&\quad + \delta \mathbb{E}_{T_r-1} \left[\widetilde{W}_{T_r}^{\widetilde{\boldsymbol{\Gamma}}}(Z_{T_r} + R_{T_r}^S S_{T_r-1}) \right].
\end{aligned}
$$

$V_t^{\widetilde{\Gamma}}(S_{t-1}, Z_{t-1}; \Gamma_t)$ is the current self's value function conditional on the savings of liquid asset S_{t-1} and illiquid asset Z_{t-1} from the previous period, and a realization of the expenditure shock Γ_t. Note that S_t is a stock variable, and A_t is a flow variable. The stock of illiquid asset at t is written as Z_t. The value function $V_t^{\widetilde{\Gamma}}(S_{t-1}, Z_{t-1}; \Gamma_t)$ also depends on the perception $\widetilde{\boldsymbol{\Gamma}}$, which determines the continuation value $\widetilde{V}_{t+1}^{\widetilde{\boldsymbol{\Gamma}}}(S_t, Z_t; \widetilde{\Gamma}_{t+1})$. The constraints $S_t \geq 0$ and $A_t \geq 0$ rule out borrowing. The wealth X_t available at time t is defined as the sum of current income Y_t and gross return of savings from liquid assets $R_t^S S_{t-1}$, where R_t^s is the gross interest rate on liquid assets.

After the retirement, households until retirement $t \in \{T_r, \cdots, T\}$ solve a following problem.

$$
\begin{aligned}
W_t^{\widetilde{\boldsymbol{\Gamma}}}(X_t; \Gamma_t) &= \max_{C_t, S_t} u(C_t - \Gamma_t) + \delta \mathbb{E}_t \left[\widetilde{W}_{t+1}^{\widetilde{\boldsymbol{\Gamma}}}(X_{t+1}; \widetilde{\Gamma}_{t+1}) \right] \quad \text{where} \qquad (2) \\
\widetilde{W}_{t'}^{\widetilde{\boldsymbol{\Gamma}}}(X_{t'}; \widetilde{\Gamma}_{t'}) &= \max_{C_{t'}, S_{t'}} u(C_{t'} - \widetilde{\Gamma}_{t'}) + \delta \mathbb{E}_{t'} \left[\widetilde{W}_{t'+1}^{\widetilde{\boldsymbol{\Gamma}}}(X_{t'+1}; \widetilde{\Gamma}_{t'+1}) \right], \\
&\text{subject to} \\
X_{T_r} &= Y_{T_r} + R_{T_r}^S S_{T_r-1} + R_{T_r}^A A_{T_r-1}, \\
Y_t + R_t S_{t-1} &= X_t \geq C_t + S_t \text{ if } t \neq T_r, \\
C_t &\geq 0, \text{ and } S_t \geq 0, \text{ and} \\
\widetilde{W}_T^{\widetilde{\boldsymbol{\Gamma}}}(X_T; \widetilde{\Gamma}_T) &= u(X_T - \widetilde{\Gamma}_T).
\end{aligned}
$$

After retirement, the problem becomes simple, and there are only two choice variables, C_t, and S_t. Also, at the period of retirement, T_r, agents can access the illiquid assets A_{T_r}. At the final period, agents consume all available wealth. I denote the value function after retirement using the letter W to distinguish value functions at two different regimes.

When I refer to optimal consumption and savings before retirement, they are functions solving (1) with inputs S_{t-1}, Z_{t-1}, and Γ_t based on the perception $\widetilde{\mathbf{\Gamma}}$). If there is no room for confusion, I will omit inputs of the functions to save space. Similarly, the optimal consumption and savings before retirement are functions solving (2) with inputs X_t, Γ_t, and $\mathbf{\Gamma}$. Following the literature, an agent is hand-to-mouth if the level of optimal consumption C_t equals available wealth X_t.

The domain of the utility function $u : \mathbb{R} \to \mathbb{R}$ consists of real numbers where $u' > 0$ and $u'' < 0$. Example of this kind of utility function is exponential utility function with constant absolute risk aversion. Inada condition cannot be incorporated in this model unless there is a guarantee that Γ_t is not too large. Alternatively, I can impose the Inada condition and assume that $\Gamma_t < Y_t$ and $\widetilde{\Gamma}_t < Y_t$ at every period. I also assume that R_t^A and R_t^S are deterministic, and $1/\delta > R_t^A > R_t^S$. This assumption removes the incentive to save liquid assets at the period just before the retirement $T_r - 1$, hence $S_{T_r} = 0$. At all periods before retirement $t < T_r$, the consumer saves illiquid and liquid assets at the same time only if

$$\mathbb{E}_t\left[R_t^S u'(C_{t+1})\right] = \delta^{T_r-t-1}\mathbb{E}_t\left[\left(\prod_{k=t+1}^{T_t} R_k^A\right) u'(C_{t+1})\right]$$

I call an agent who correctly perceives the future random variable, where the perception $\widetilde{\mathbf{\Gamma}}$ is identical to $\mathbf{\Gamma}$, as a *sophisticated* agent. Considering two different perceptions $\widetilde{\mathbf{\Gamma}}^1$ and $\widetilde{\mathbf{\Gamma}}^2$, I call that $\widetilde{\Gamma}^1$ is more *optimistic* (*pessimistic*) than $\widetilde{\Gamma}^2$ if cumulative density functions (CDF) in $\widetilde{\mathbf{\Gamma}}^2$ ($\widetilde{\mathbf{\Gamma}}^1$) first order stochastically dominates CDFs in $\widetilde{\mathbf{\Gamma}}^1$ ($\widetilde{\mathbf{\Gamma}}^2$). From now on, I denote $\widetilde{\mathbf{\Gamma}}_t^2 >_1 \widetilde{\mathbf{\Gamma}}_t^1$ if cumulative density function, CDF, of $\widetilde{\mathbf{\Gamma}}_t^2$ first order stochastically dominates CDF of $\widetilde{\mathbf{\Gamma}}_t^1$. Similarly, $\mathbf{\Gamma}^2 >_1 \mathbf{\Gamma}^1$ implies that $\mathbf{\Gamma}_t^2 >_1 \mathbf{\Gamma}_t^1$ for any t.

I investigate the role of $\widetilde{\mathbf{\Gamma}}$ in two parts. The first part investigates the relationship between the perception of the shock and the levels of consumption and savings. The second part focuses on the persistence of the endogenous part of savings $C_t - \Gamma_t$. To make the discussions easier, I focus on life after retirement, which is a one-asset model.

The first proposition argues that all else equal, the consumption of a relatively more optimistic agent is always higher than or equal to the agent with relatively more pessimistic beliefs.

Proposition 1. For every $t \geq T_r$, and $w_t \geq 0$, $C_t(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) \geq C_t(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^2)$ if $\widetilde{\mathbf{\Gamma}}^2 >_1 \widetilde{\mathbf{\Gamma}}^1$.

The intuition behind Proposition 1 is simple. The agent with more pessimistic beliefs perceives that she will face greater expenditure shock, which gives tighter liquidity constraints.

Hence, she still values the extra value of saving more than the optimistic agents.

The following result also shows a monotone relationship between the size of wealth and the perception of expenditure shocks. To see this, for $t \geq T_r$ and $k \leq T-t$ define recursively

$$X^i_{t+k}(X^i_{t+k-1}; \Gamma_{t+k}, \widetilde{\mathbf{\Gamma}}^i) = Y_{t+k} + R^S_{t+k}\left[X^i_{t+k-1} - C_{t+k-1}(X_{t+k-1}; \Gamma_{t+k-1}, \widetilde{\mathbf{\Gamma}}^i)\right].$$

Corollary 1. Fix $t \geq T_r$ and $X_t \geq 0$. Then for any $k \leq T-t$ and realization of $Y_t, \cdots, Y_{t+k}$ and $\Gamma_t, \cdots \Gamma_{t+k}$,

$$X^2_{t+k}(X^1_{t+k-1}; \Gamma_{t+k}, \widetilde{\mathbf{\Gamma}}^2) \geq X^1_{t+k}(X^1_{t+k-1}; \Gamma_{t+k}, \widetilde{\mathbf{\Gamma}}^1)$$

as long as $\widetilde{\mathbf{\Gamma}}^2 >_1 \widetilde{\mathbf{\Gamma}}^1$.

This result tells that the for any two agents where the perceptions are ordered as $\widetilde{\mathbf{\Gamma}}^2 <_1 \widetilde{\mathbf{\Gamma}}^1 <_1 \mathbf{\Gamma}$, the consumption profile over the life cycle of agent with $\widetilde{\mathbf{\Gamma}}^1$ would be more flatter. When starting with the same level of wealth X_t and facing same income and expenditure shocks, the wealth of the agent with more optimistic belief, $\widetilde{\mathbf{\Gamma}}^2$ would be depleted faster. Hence, at the end of the life cycle, the agent with pessimistic belief will have more left to consume. Based on this result, this paper uses the stock of liquid asset as a tool to elicit the distribution of relative optimism and pessimism.

Next we develop the dynamic properties of $C_t - \Gamma_t$ and C_t. To see this, let me explicitly write all the random variables as inputs of consumption this period, C_t, and the next period C_{t+1} as $C_t(X_t(Y_t), \Gamma_t)$ and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$. I omit $\widetilde{\mathbf{\Gamma}}$ because comparing perception of the expenditure is not important from now on. Total expenditure $C_t(X_t(Y_t), \Gamma_t)$ can be decomposed into two parts. The core part of consumption $C_t(X_t(Y_t), \Gamma_t) - \Gamma_t$ is what the agent tries to smooth in the lifetime. The rest, Γ_t is the expenditure shock. The key difference between the two is that, they have very different relationships with total consumption next period, $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$. Suppose that there is an increase in income Y_t or interest rate R^s_t. Then the available wealth at this period X_t will increase, and due to globally concave continuation value, consumption and savings at period t will strictly increase as long as the agent is not hand-to-mouth. This leads to increase in C_{t+1} as well since X_{t+1} increase. When there is an increase in the expenditure shock Γ_t, then the total expenditure C_t in this period will increase. However, the MPC cannot exceed one, so $C_t(X_t(Y_t), \Gamma_t) - \Gamma_t$ will decrease. At the same time, because of decrease in the available wealth next period, consumption next period will decrease in expectation. Hence, in any case, $C_t(X_t(Y_t), \Gamma_t) - \Gamma_t$ and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$ (and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1}) - \Gamma_{t+1}$ will exhibit positive correlation. However, the same mechanism will lead to negative correlation between Γ_t

and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$ (and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1}) - \Gamma_{t+1}$). Next proposition formally proves the intuition given above.

Proposition 2. Fix S_{t-1}. Then $\text{cov}(C_t - \Gamma_t, C_{t+1} - \Gamma_{t+1}|S_{t-1}) \geq \text{cov}(C_t, C_{t+1}|S_{t-1})$.

The result of 2 suggests a way of separating the endogenous part of consumption $C_t - \Gamma_t$ from the expenditure shock Γ_t. The categories of expenditure that exhibit significant serial correlation are likely to be smoothed over the life cycle. In contrast, the nonpersistent part of the expenditure would be motivated by short-run fluctuations in taste or urgent spending needs.

Proposition 3. For every $t < T_r$, $C_t(X_t, Z_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) \geq C_t(X_t, Z_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^2)$ if $\widetilde{\mathbf{\Gamma}}^2 >_1 \widetilde{\mathbf{\Gamma}}^1$.

However, the corollary 1 does not extend to the two-asset case. This is because even though household A is more pessimistic than household B, it does not necessarily imply that household A would save both liquid and illiquid assets more than B. For example, suppose imminent expenditure shock is larger than the expenditure shocks after retirement. In that case, the savings of illiquid assets of a pessimistic agent can be lower than the an optimistic agent. A numerical exercise in Figure 2 shows such a case. As the agent moves along the horizontal axis from left to right, the agent becomes more aware of the expenditure shock and becomes more pessimistic. The relatively optimistic households underestimate the need for precautionary saving in period two and mainly save using the illiquid assets, which brings a higher rate of return. However, the sophisticated agents who perfectly know the large expenditure shock mainly save using the liquid asset. The relationship of the consumption and savings with the perceptions in the two-asset case needs to be verified using a simulation, and I show in the later part of this section that the more pessimistic agents tend to accumulate more liquid illiquid assets than their optimistic counterparts.

Comparison with Alternative Modeling Strategy The key building blocks of this paper is from Carroll (1992, 1997). The stochastic income is likely to make the consumption function concave, and the presence of the borrowing constraint gives chance to generate low liquidity households that are hand-to-mouth.

Augemented on Carroll (1992, 1997), the two asset environment with an illiquid asset serves a similar role in Kaplan and Violante (2014). A difference is that Kaplan and Violante (2014) has a richer framework by allowing agents to withdraw their illiquid assets at any time as long as they pay the fixed cost. In this model, households are withdrawing illiquid assets possible only once in their lifetime. Hence, there is an infinite amount of fixed cost before the retirement age T_r in terms of Kaplan and Violante (2014). In other words, Kaplan and

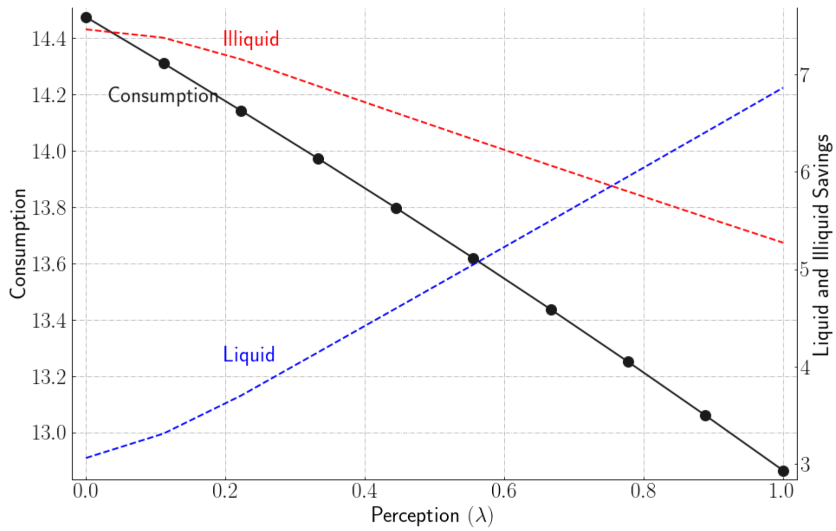

Figure 2: A case where pessimistic agent have lower illiquid savings than the optimistic agent
Notes: simulated by a three-period model with the following parameter values. Utility function is CRRA with $\sigma = 0.5$ (of $u(c) = c^{1-\sigma}/(1-\sigma)$. Income is $\{25, 10, 5\}$ for each period. Expenditure shocks of magnitudes 7 and 1 for periods two and three respectively occur by probability 0.5. The discount factor is 0.95, and the interest rates for liquid and illiquid assets are 0 and 2 percent, respectively. x-axis shows the perception of agents where a value λ in the x-axis denotes the percentage of the shock magnitude perceived by the agent.

Violante (2014) allows a symmetric adjustment costs, while this paper assumes an extreme asymmetric cost of adjustment. The rationale of allowing asymmetric cost is that saving in retirement account generally do not involve any costs. Also, letting the agents to withdraw the asset only once helps to build the theory since the presence of fixed can generate locally convex regions in the value function. Though there are some differences, the two-asset environment in this paper contributes to similar mechanism as the Kaplan and Violante (2014). Agents need less saving for the liquid assets by separating precautionary saving and saving for retirement, which increase chance of having more hand-to-mouth agents. Also, lower interest rate on liquid assets increases the MPC when the agents are not adjusting the illiquid assets.

As a related paper, Lian (2021) introduces a model where the mistakes in future consumption plan generates high MPC to the current self. There are two main differences between Lian (2021) and this paper. First, Lian (2021) derives a robust result regarding MPC, and able to explain high MPC among households with high liquidity. However, the main channel that this paper brings additional MPC is by generating enough low liquidity households.

In other words, Lian (2021) focuses shifting the gradient of the consumption function, this paper depends on deviation from rational expectation which pushes optimistic households to have lower liquidity and lead them to the binding borrowing constraint. Second, while all mistakes in the future lead to higher MPC in Lian (2021), this paper shows that only the households underestimates the shock in the future are likely to consume more and likely to have binding borrowing constraint. Hence, in this paper, the direction where the agents are 'wrong' is important.

This paper has direct connection to (β, δ) discounting model. Let us focus on stylized deterministic three-period setting with no liquidity constraints where $\delta = 1$. Two models can be represented as

$$\begin{aligned} U^{(\beta,\delta)}(C_1, C_2, C_3) =& u(C_1) + \beta\left[u(C_2) + u(C_3)\right], \text{ and} \\ U^{\Gamma,\widetilde{\Gamma}}(C_1, C_2, C_3) =& u(C_1 - \Gamma) + u(C_2 - \widetilde{\Gamma}) + u(C_3 - \widetilde{\Gamma}). \end{aligned}$$

Both models feature characteristics that look like a present bias. For example, the agents are indifferent between different future timings of receiving additional ε since $U^{(\beta,\delta)}(C, C + \varepsilon, C) = U^{(\beta,\delta)}(C, C, C+\varepsilon)$ and $U^{\Gamma,\widetilde{\Gamma}}(C, C+\varepsilon, C) = U^{\Gamma,\widetilde{\Gamma}}(C, C, C+\varepsilon)$. However, as long as $\beta < 1$ and $\Gamma > \widetilde{\Gamma}$, there is a preference reversal, such as $U^{(\beta,\delta)}(C + \varepsilon, C, C) > U^{(\beta,\delta)}(C, C + \varepsilon, C)$ and $U^{\Gamma,\widetilde{\Gamma}}(C+\varepsilon, C, C) > U^{\Gamma,\widetilde{\Gamma}}(C, C+\varepsilon, C)$. In this model, as Γ becomes stochastic, the agent might exhibit this present-biased behavior or not depending on the realized value Γ. Also, as the agent becomes more optimistic than before, there is a higher chance of exhibiting the present-biased behavior, corresponding to β being smaller in the (β, δ) model.

However, when analyzing optimizing agents, two models differ in two dimensions. Unless the agent is fully naive in the (β, δ) model, there can be discontinuities in the (β, δ) model since tension among future agents creates convex regions in the continuation value (Harris and Laibson, 2002). Moreover, the role of sophistication of β to consumption and savings is not clear. In a deterministic model, Salanie and Treich (2006) shows that the shape of the utility function determines the relationship between the sophistication and the level of consumption; hence the assumption of the utility function can influence the identification of β and $\widetilde{\beta}$ when calibrating in a life cycle model. This model does not feature tension among future agents, but this helps us build a monotone relationship between the perception of the expenditure shock and wealth, which is useful in the calibration process.

Miranda-Pinto et al. (2020) develops a model of consumption and savings with an expenditure shock and explains high consumption variance, the extensive margin of MPCs, and high MPC among low and high-income households. This model takes a parsimonious approach and assumes that expenditure shocks cannot be avoided. In terms of their model, the consumption threshold represents the expenditure shock, and the utility cost when consuming under the threshold tends to infinity. The benefit of taking this parsimonious approach is that it allows analyzing households with different levels of perceptions and accounts for wealth dispersion.

4 Calibration

In this section, we calibrate the distribution of perception to explain the dispersion of liquid assets. First, we check the model properties discussed in the previous section. Next, I explain the empirical target and the calibration strategy. Finally, the calibration result is presented in the last part of this section, and I also check if the models can generate enough MPC and liquid asset inequality.

Calibration of the Expenditure Shock (Needs to be cleaned) Separating the products in the household consumption basket by persistent and nonpersistent parts, we can find a stark difference between the two. For each product j, I measure the degree of persistence of expenditure using the following simple panel regression:

$$\begin{aligned}
\Delta C_{i,j,t} &= \rho_j \Delta C_{i,j,t-1} + \varepsilon_{i,j,t} + \text{Year FE}_t + \text{Month FE}_t + \varepsilon_{i,j,t}, \text{ where} \qquad (3)\\
\Delta C_{i,j,t} &= \gamma_j C_{i,j,t-1} + \text{Year FE}_t + \text{Month FE}_t + \xi_{i,j,t}.
\end{aligned}$$

To deal with endogeneity caused by adding lagged dependent variable, $C_{i,j,t-1}$, I employ the Anderson and Hsiao (1981) estimator by using a lag of level variable, $C_{i,j,t-1}$, as an instrument. I use a monthly panel since using quarterly data is too short of employing lagged instruments, and it fits the frequency used in the latter part of the paper. Year FE_t and Month FE_t stand for year and month fixed effects, respectively. In this simple regression, ρ_j captures the persistence of $C_{i,j,t}$. The first difference cancels out individual fixed effects. I use the Consumer Expenditure Survey from 1997 to 2013 using 44 product classifications by Kueng (2015) to estimate the persistence based on the model above.[45] I apply the sample selection criteria explained in the Appendix C.

Table 9 shows the estimated ρ_j in (3) for every 44 products. Twenty-one products are exhibit positive and significant persistence. Especially, alcohol, tobacco, food, and personal care expenditures are highly persistent. The remaining 23 products do not exhibit strong

[4] We can use detailed product classification, but many of the products in the lowest level share very similar characteristics, such as utility payments for gas and electricity. To reduce the size of the analysis, I use this 44 product classification that reduces this redundancy.

[5] I use the sample only from 1997-2013, where the classification can be applied.

persistence, such as education service, home management, insurance, and vehicle-related payments. First, denote the set of products that are persistent as S_p and nonpersistent as S_n. Then, we can construct variables $C^p_{i,t} = \sum_{j \in S_p} C_{i,j,t}$ and $C^n_{i,t} = \sum_{j \in S_t} C_{i,j,t}$ which aggregates expenditures over persistent and nonpersistent groups. Also, define the total expenditure of household i at time t as $C_{i,t}$. Next, to see the relationship between total expenditure and $C^p_{i,t}$ and $C^n_{i,t}$ by the following regression:

$$\Delta C_{i,t} = \alpha \Delta C^p_{i,t-1} + \beta \Delta C^n_{i,t-1} + \text{Year FE}_t + \text{Month FE}_t + \varepsilon_{i,t} \text{ where} \tag{4}$$

$$\begin{pmatrix} \Delta C^p_{i,t-1} \\ \Delta C^n_{i,t-1} \end{pmatrix} = (\eta^p, \eta^n) \begin{pmatrix} C^p_{i,t-2} \\ C^n_{i,t-2} \end{pmatrix} + \text{Year FE}_t + \text{Month FE}_t + \xi_{i,t}. \tag{5}$$

The estimated α and β are 0.033 and 0.006, respectively. Hence, increase in the nonpersistent part leads to almost no change in the total expenditure next period.[6] When changing the dependent variable in (4) to $C^p_{i,t-1}$, then the estimated α and β becomes 0.046 and -0.001, respectively.

Among all random shocks, if income and interest shocks were dominant, then positive income and interest rate shocks this period should affect positive change in consumption expenditure in this and the next period. Hence, $C^n_{i,t}$ was endogenous part of the consumption and savings problem, then we need to model a non-persistent shock that especially affects $C^n_{i,t}$, which dominates income and interest rate shocks. In this sense, this paper do not explicitly model randomness of preference parameters behind $C^n_{i,t}$ and treat $C^n_{i,t}$ as exogenous.

Next, I compare standard deviations of $C^n_{i,t}$ and $C^p_{i,t}$ in various angles. Over 12 months, I calculate standard deviation of expenditures for each person and take average of them over all households, that is $\text{sd}(C^m_{i,t}) = \sqrt{\sum_{t=1}^{12}(C^n_{i,t} - \bar{C}^n_i)/12}$ where $\bar{C}^n_i = \sum_{t=1}^{12} C_{i,t}/12$ for $m \in \{n, p\}$. The averages $\text{sd}(C^n_{i,t})$ and $\text{sd}(C^p_{i,t})$ over all households are 1,271 and 570, respectively. Though the nonpersistent part takes about a quarter of total expenditure, the standard deviation is more than twice as large compared to the persistent part. Moreover, averages of $\text{sd}(C^n_{i,t})/\bar{C}^n_i$ and $\text{sd}(C^p_{i,t})/\bar{C}^p_i$ over all households, which adjusts for sizes of expenditures, are 1.56 and 0.29 respectively. When adjusted for the size, we can see that the standard deviation of nonpersistent part is about 5 times larger than the persistent part.

All of the evidences so far can be summarizes as follows. (1) There is much more randomness in the nonpersistent part than the persistent part, and (2) the past value of nonpersistent part does not explain next period's expenditure of itself or the total expenditure, and (3) it decreases the persistent part of expenditure in the next period. Based on these facts, this paper assume that the nonpersistent part is exogenous since it violates the principles of

[6] Similar results are found when changing the lag of instruments variables in (5) from 3 to 4 instead of 2.

consumption smoothing compared to the persistent part.

The distribution of persistent and nonpersistent part has a stark difference. From now on, I use expenditures that are normalized by the permanent income as done in (**??**). Figure 3, shows the distribution of nonpersistent and persistent part of the expenditure. First, the distribution of nonpersistent part has a mode near zero, and the frequency of the histogram monotonically decrease when featuring higher values. It is heavily skewed to the right, and the shape can be well represented by an exponential distribution. On the other hand, the

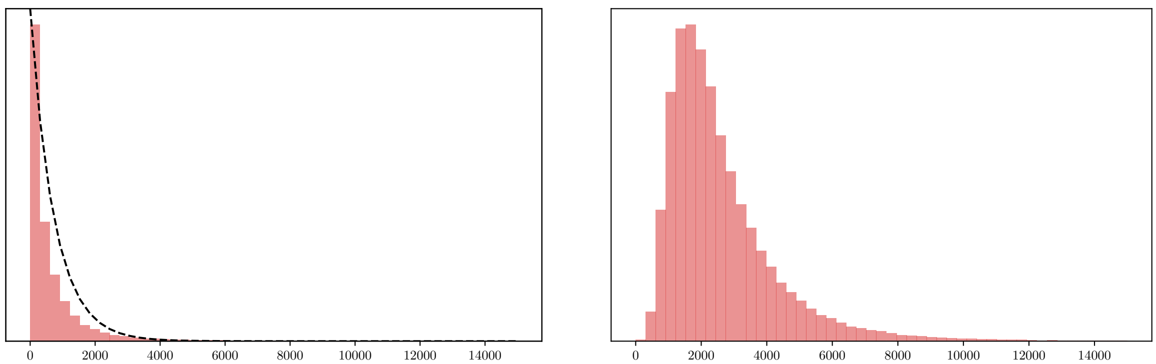

Figure 3: Distribution of normalized nonpersistent part (left) and the persistent part (right) of the expenditure

Note: black dotted line on the left-hand side figure is the probability density function of exponential distribution fitted to match data

nonpersistent part is centered around a positive value and exhibits a log-normal distribution. Thus, almost always, the persistent part is chosen as a positive value. However, nonpersistent are mostly zero in the consumption savings problem. Also, the chances of consuming a higher amount of nonpersistent components get smaller, and those intertemporal choices are uncorrelated.

Suppose that an agent fits the nonpersistent part distribution based on the exponential distribution with past observations. The efficient maximum likelihood estimator (MLE) for the parameter of exponential distribution would be the mean of the past observations. In a distribution, such as exponential distribution[7], the median is always less than the mean, which implies that the median person will observe a value that is less than the true mean. Her MLE estimate of the distribution will be underestimating the true parameter.[8]

The process of expenditure shock is also from the Consumer Expenditure Survey, based

[7]The result generalizes to the case where $f(c)$ is decreasing in c where $f(\cdot)$ is the probability density function.

[8]With exponential distribution, the result can be generalized as follows. Fix n, and $\bar{x} = (x_1 + \cdots + x_n)/n$ where $x_1, \cdots x_n$ are all drawn from the identical exponential distribution $\text{Exp}(\lambda)$ with parameter λ. Then median of $\bar{x}$ is less than λ. In the limit, $n \to \infty$, $\bar{x}$ tends to λ by the central limit theorem.

on the analysis of section 2. I apply fourth-order polynomial approximation with age to filter the trend of the mean level of expenditure shocks over the life-cycle. However, as we can see from Figure 4, it is still relatively flat compared to income, and it is small relative to income. To make the model solvable, I transform the utility function as $u(c) = (\max\{c, \underline{u}\})^{1-\sigma}/(1-\sigma)$, where $\underline{u} = 10^{-5}$ (1 cent). As we can see from Figure 4, since the expenditure shock has a small portion out of income, the lower bound $\underline{u}$ is rarely triggered, and varying this value does not affect the main results. All values of consumption and assets in the simulation represent 2019 dollars adjusted by the consumer price index.

Agent i will be endowed λ_i, representing the degree of perception of the expenditure shock. While the true distribution that draws the expenditure shock at time t is $\exp(\mu_t)$, the household i perceives that the expenditure shock will drawn from $\exp(\lambda_i \mu_t)$. Hence, $\lambda_i = 1$ implies that the agent i is sophisticated. If $\lambda < 1$ ($\lambda > 1$), then the agent is optimistic (pessimistic) compared to the sophisticated agents. I assume that the maximum level of λ_i is 1.5, enough to generate agents at the higher end of the distribution.

Income Process Following Carroll (1997), income process follows

$$
\begin{aligned}
Y_{i,a} &= P_{i,a}\Xi_a\varepsilon_{i,a} \qquad (6)\\
&\text{where}\\
P_{i,a} &= P_{i,a-1}\xi_{i,a}, \text{ and}\\
\Xi_a &= \beta_0 + \beta_1 a + \beta_2 a^2 + \beta_3 a^3 + \beta_4 a^4.
\end{aligned}
$$

Income of individual i at age a depends on permanent level of income $P_{i,a}$, age-specific term Ξ_a which shapes the overall income profile, and transitory shock $\varepsilon_{i,a}$. Shocks in permanent and transitory income, $\ln \varepsilon_{i,a}$ and $\ln \xi_{i,a}$, follows normal distribution with standard deviations 0.013 and 0.043 respectively, following Carroll et al. (2017)[9]. Age-specific term Ξ_a follows fourth-order polynomials of age, which is also standard. This paper estimates parameters determining Ξ_a by using the Consumer Expenditure Survey from year 1997 to 2013.

The choice of modeling permanent income as a geometric random walk follows Carroll (1997) which allows normalizing every variable by the permanent level of income. In the simulation, I also use the normalized variables for two reasons. First, normalizing by permanent income allows reducing the number of state variables. Second, the paper does not depend on income dispersion to explain liquid and illiquid wealth inequality. Hence, analyzing the normalized model allows controlling the influence of permanent income. Otherwise,

[9]This paper has monthly time frequency, so the standard deviations given in Carroll et al. (2017), which uses quarterly frequency, are divided by 3

it would be difficult to flesh out the contribution of permanent income towards the dispersion of wealth. The model lacks income, social security taxes, and unemployment (with unemployment benefits). Those features can make the analysis richer, but they all can potentially affect the dispersion of wealth and marginal propensity to consume. For example, adding unemployment would increase the desire for precautionary saving, but the marginal propensity to consume can be higher when the agent is unemployed. Social security tax or benefits are not in the model, but the income process used in this model follows realistic features such as low monthly income after retirement with a peak during the mid-age.

Discount Factor and Interest Rates Discount factor δ is 0.96 in annual terms, which is standard. The returns of liquid and illiquid assets are held constant. Following Kaplan and Violante (2014), the real return on liquid asset R^S is -0.7 percent, and illiquid asset is 2.84 percent respectively in annual terms. Hence $\delta < 1/R^A < 1/R^S$.

The Choice of IES I use the constant relative risk aversion utility function, $u(c) = c^{1-\sigma}/(1-\sigma)$. One of the most important parameters when modeling the life cycle consumption and savings problem is the intertemporal elasticity of substitution (IES), which is $1/\sigma$ in this paper. Empirically, when examining micro evidence, IES is suggested to be 1/3 (Havranek and Sokolova, 2020). In most applications, especially when explaining excess sensitivity of consumption or wealth dispersion, IES rarely goes lower 1. IES is crucial because it serves as one of the key components of the effective discount factor, $(\delta R)^{1/\sigma}$. With low values of $1/\sigma$, the effects from the discount factor become muted. Hence, to achieve excess sensitivity using the low value of IES plain vanilla consumption savings model require a lower value of discount factor δ to overcome low IES. However, if the IES is too high, it cannot generate an accumulation of wealth comparable to data (Aguiar et al., 2020). This paper uses 1/3 as the benchmark value of IES, as suggested in the empirics.

Agents enter the economy at age 25 and retire at age 65. Everyone dies at age 80 with no bequest. Adding bequest can help explain the increasing trend of wealth even after retirement, but I try to keep the model as simple as possible to rule out factors that can affect the paper's primary purpose: explaining the distribution of wealth and the MPC. The model has a monthly frequency, which results in 660 periods, with 480 periods before retirement.

Model Properties To investigate the fundamental property of the model, I simulate the model economy with 256,000 agents with λ_i that is evenly distributed over $[0, 2]$. Everyone starts with the same initial level of wealth, which is the median liquid and illiquid wealth of household heads aged 24-26 of the Survey of Consumer Finances.

Figure 4 tracks the median wealth, consumption, and MPC within every twenty groups of perceptions $[\lambda^j, \lambda^j + 0.1)$ where $\lambda^0 = 0$ and $\lambda^{20} = 1.9$. Then, I track the median wealth of households within each quintile of perceptions. As hinted by Propositions 1 and 3, Figure 4 shows that an agent with a higher level of λ_i accumulates a larger amount of liquid and illiquid assets throughout their life. Especially at age 65, the wealth of top group with $\lambda \in [1.9, 2.0)$ about is 6 times larger than the lowest group with $\lambda \in [0, 0.1)$.

Unlike liquid and illiquid wealth, different degrees of perception does not make remarkable differences in consumption except in the initial and terminal periods. Agents with correct perception exhibit a flat consumption profile until they reach retirement, which implies successful consumption smoothing under the low IES of 1/3. Since they are worried about the possibility of high expenditure shocks, they keep liquid assets for precautionary motives, which makes the consumption rise near the terminal period. The precautionary saving motive is higher among relatively more pessimistic households. Hence, agents with an extreme level of pessimism near $\lambda_i \simeq 2$ perceive that their initial liquid wealth is not enough and start accumulating liquid assets right away, making initial consumption lower than the other groups. Optimistic agents fail to underestimate the size of expenditure shock, which depletes liquid assets and high consumption at the beginning of life.

There are also large differences in the MPC over the life cycle by different levels of perceptions. First, the optimistic agents exhibit higher MPC than the pessimistic agents. Since optimistic agents occasionally meet binding borrowing constraints, their MPC is also larger than the pessimistic agents. The average monthly MPC of the most optimistic households, $\lambda_i \in [0, 0.1)$, is 22.4 while the middle group, $\lambda_i \in [0.9, 1.0)$, exhibits 2.2 percent, and the most pessimistic group, $\lambda_i \in [1.9, 2.0)$ exhibits 1.4 percent. Since the low liquidity drives the high MPC, there will be negligible differences among agents that accumulate enough wealth so that their borrowing constraint will be rarely binding. Hence, agents with perceptions $\lambda_i \in [0, 0.5)$ show high MPCs, while the others do not.

The trajectory of the MPC out of a transitory income shock follows the implication from the buffer-stock theory of savings (Carroll, 1992, 1997). The MPC is generally high at young ages since the current income is lower than the future income levels, which makes the borrowing constraint binding. As households accumulate assets at the middle age, MPC falls and becomes almost zero when the income is at its peak. However, consumption responses of optimistic agents versus rational expectations or pessimistic agents starkly differ near retirement. The amount of liquidity they have accumulated so far sets the consumption limit for all agents at retirement. However, optimistic agents who underestimate the need for precautionary savings may face larger expenditure shock than they expected and become

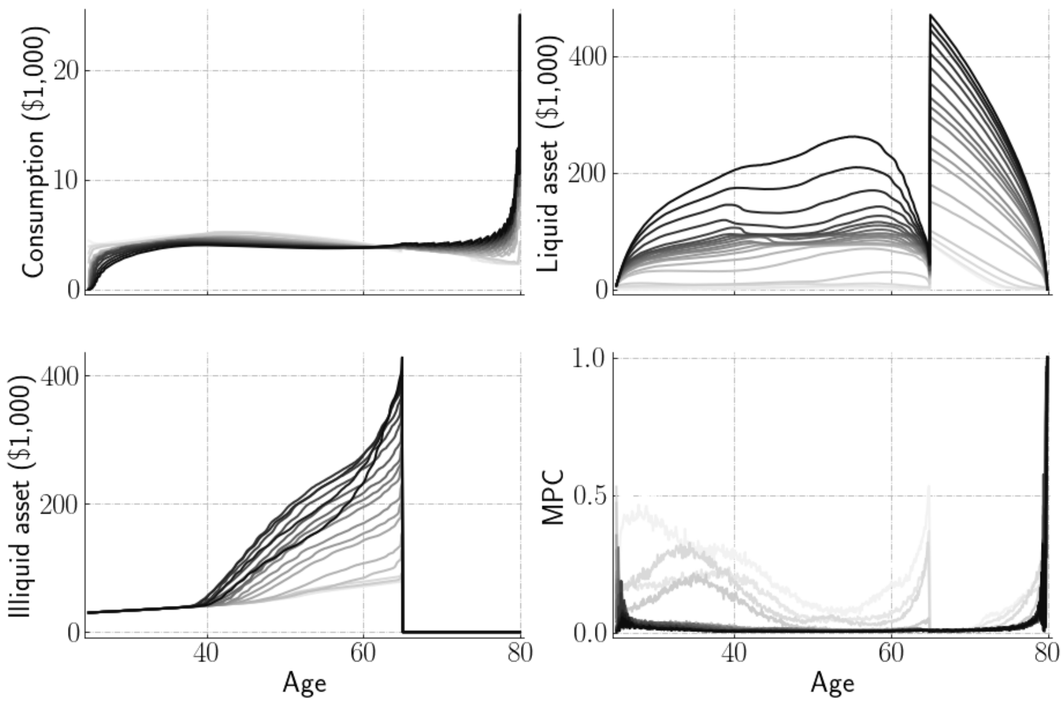

Figure 4: Trajectory of key variables by different levels of perception
Note: twenty groups of agents with perceptions $[\lambda^j, \lambda^j + 0.1)$ where $\lambda^1 = 0$ and $\lambda^{20} = 1.9$ are plotted where darker line indicates more pessimistic agents where λ^j is larger.

hand-to-mouth. On the other hand, agents who have accumulated enough assets do not exhibit a large consumption response from a transitory income shock. The mismatch between expected and actual liquidity available at retirement serves as a 'second terminal-period effect' and makes MPC rise as households retire.

Calibration Strategy and Empirical Target The key moment to match in the calibration exercise is the distribution of wealth. This paper adopts the strategy of Castañeda et al. (2003); Carroll et al. (2017) by targeting the share of liquid wealth held by 20, 40, 60, 80 percentiles of along with median liquid wealth among working-age households. There can be various reasons to hold illiquid assets, such as bequest motive and housing investment which are not modeled in this paper. Also, the dispersion of liquid wealth in the model can be affected by the introduction of large illiquid wealth at retirement; hence I only focus on the dispersion of liquid wealth before retirement. By introducing arbitrarily many optimistic households, the model has a chance to highlight households with very low liquidity and perfectly explain the share of wealth held at different percentiles. To prevent this issue, I let the model match the median liquid wealth over income similar to Carroll et al. (2017).

There have been various approaches to model the heterogeneity of consumers with different parameters. A commonly used approach is assuming a discrete number of agents with different utility or discounting parameters such as Krusell and Smith (1998). There is also an approach by Carroll et al. (2017) assuming uniform distribution and finding lower and upper bound of the parameter. In reality, the parameter of interest is likely to be drawn from a continuous distribution, and we cannot *a priori* guess the shape of the distribution. To employ a continuous and versatile distribution, I use a beta distribution that can have various shapes, nesting uniform distribution as a special case. Hence, the perception of agents λ_i is assumed to be drawn from a beta distribution. I fit three parameters where each has a distinct role. The lefthand side panel of Figure 5 shows the role of first two parameters, α and β. They control the shape of the beta distribution. The theoretical mean is $\alpha/(\alpha+\beta)$; hence, a larger value of α implies more agents with rational expectations in general. The size of both α and β controls the height of the peak. If $\alpha = \beta = 1$, then the distribution becomes a uniform distribution where the probability density function is flat. If α and β are high, the values drawn will be centered around a mean. To control the overall size of the values in the distribution, I also fit a scaling parameter γ representing the maximum value in the domain.

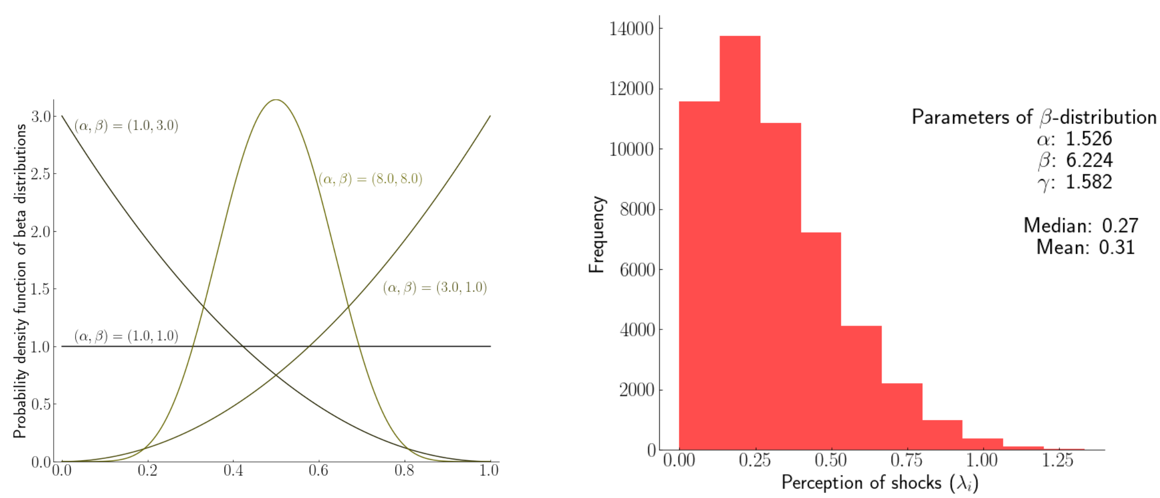

Figure 5: Examples of beta distribution (left) and the calibrated distribution (right)

Calibration Result The set of parameters to generate the moments in Table 4 is $(\alpha, \beta, \gamma) = (1.53, 6.22, 1.58)$. Calibrated parameters imply that the distribution of perception would be

	Share of wealth held at percentiles				Median liquid wealth over income
	20th	40th	60th	80th	
Data	<0.01	0.01	0.05	0.2	0.89
Model	<0.01	0.02	0.08	0.23	0.89

Table 4: Calibration result

looking as the righthand side panel of Figure 5. The average household would realize 27 percent of the true mean of the expenditure shocks. This low level of perception is necessary to make the model generate a low level of liquid wealth among working-age households. The maximum value of the distribution $\gamma = 1.58$ implies the existence of a small group of households that are more pessimistic than households with rational expectations.

Table 4 compares the moments in the data and moments in the simulated data where the choice of parameters are the ones that minimize the root mean squared error from the data. The model-generated moments are fairly close to the data. Especially, the median liquid wealth over income is well aligned with the data. Hence, the calibration result is not driven by arbitrarily generating many households with very low liquidity. However, there is some gap between the share of wealth at 60th and 80th percentile. Unlike the simulated model, households with an exceptionally high level of wealth near the top level of wealth exist, which is difficult to generate using the life-cycle model. As a result, Figure 6 shows that the model-generated Lorenz curve is slightly flatter than the data.

Table 5 compares the trend of liquid wealth over income at different percentiles of data and the simulated model. The numbers in the table are not the targets of the calibration exercise, but the simulated moments line up well with the actual data. Especially, it can explain that over 40 percent of the households in data do not have liquidity near their monthly income.

Next, I compute the marginal propensity to consume by giving \$800 to all households which are not taxed later. For household i at time t, I first compute the original level of consumption $c_t^i(w_t^i)$. Under the same environment, compute the counterfactual level of consumption using the rebate r, which is $\hat{c}_t^i(w_t^i + r)$. Then, the MPC of household i at time t can be calculated as $MPC_t^i = (\hat{c}_t^i(w_t^i + r) - c_t^i(w_t^i))/r$. In this setting, the model explains high overall MPC. The average monthly MPC among all households is 15.9 percent. In quarterly and annual terms, it can be translated to 40.5 and 87.4 percent following the conversion formula used by Carroll et al. (2017).[10] We can see that the extensive margin plays a big role from the lefthand side panel of Figure 7 since most of the households do not show a positive consumption response to the rebate. However, households with large

[10]The exact values for quarterly and annual MPC need to be examined by actually tracking the change in consumption over the quarter and year.

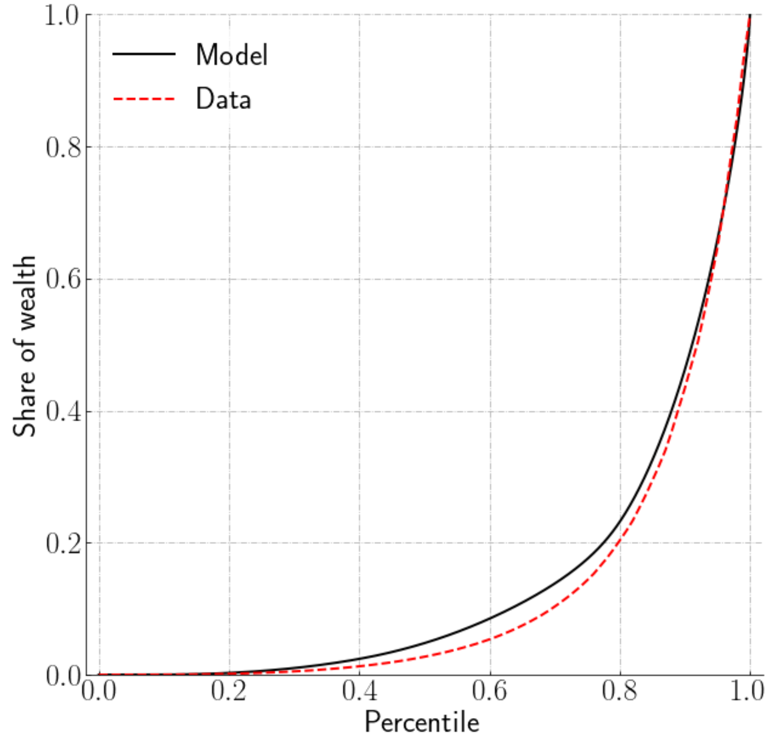

Figure 6: Lorenz Curve: Data versus Model

expenditure shocks would move to the region with a positive consumption response. This will be more effective among households with low levels of liquid wealth. The righthand side panel suggests extensive margin in the individual consumption response. When a household faces a large expenditure shock, then temporarily, they may exhibit high MPC. However, in most cases where they have negligible expenditure shocks, they behave as agents in the plain vanilla consumption and savings problem with a low MPC. The righthand side panel of Figure 7 shows the individual response of a sample agent who perceives 50 percent of the actual shock. Not only there is an extensive margin of consumption responses in the aggregate, but depending on the size of the expenditure shocks, each individual also exhibits infrequent but large consumption responses.

Table 6 compares the MPCs across different modeling strategy. Under rational expectations, the MPC without the use of expenditure shock is only 1.9 percent. Using the two-asset model under rational expectations, the MPC becomes 3.2 percent, almost twice the one-asset model. Unlike Kaplan and Violante (2014), separation of precautionary savings and savings for retirement does not have a dramatic effect on the MPC. This is because when using

		Liquid wealth over income			
		20th	40th	60th	80th
25-34	Data	0.12	0.44	1.06	2.67
	Model	0.02	0.35	1.14	2.6
35-44	Data	0.15	0.51	1.18	3.14
	Model	0.14	0.36	1.17	2.6
45-54	Data	0.20	0.61	1.48	4.29
	Model	0.32	0.67	1.38	4.35
55-64	Data	0.28	0.86	2.31	7.23
	Model	0.16	0.57	1.48	8.03
Overall	Data	0.21	0.68	1.74	5.40
	Model	0.14	0.48	1.26	4.22

Table 5: Liquid wealth over income at 20, 40, 60, and 80th percentiles

Note: numbers in parenthesis are from the simulated model, and others are from the data

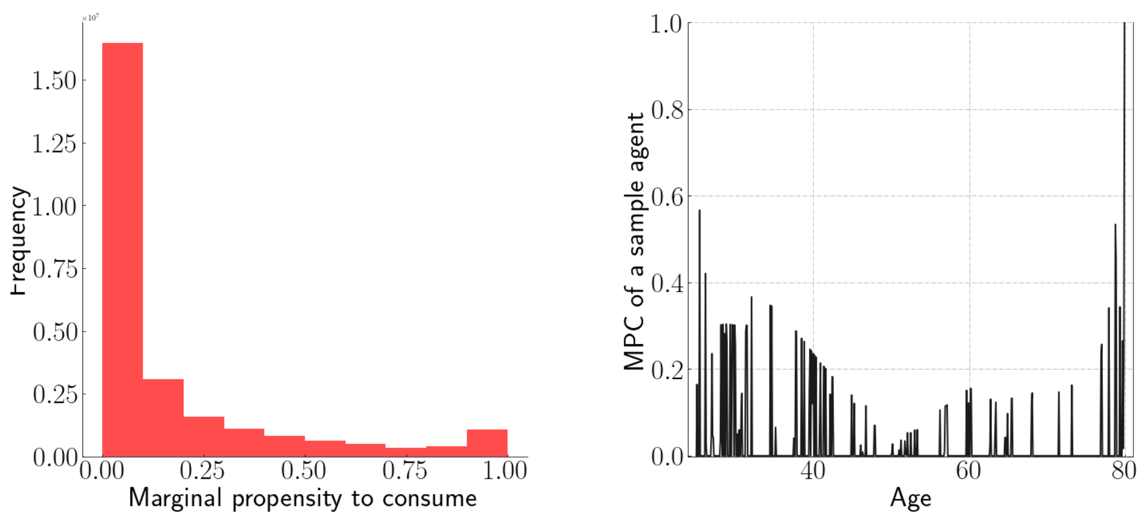

Figure 7: Distribution of MPC (left) and sample individual consumption response (right)

Number of Assets	Expectations	Use expenditure shocks	MPC
One	Rational expectation	No	1.9
Two	Rational expectation	No	3.2
One	Rational expectation	Yes	1.4
One	Fully optimistic	Yes	13.3
Two	Rational expectation	Yes	2.3
Two	Fully optimistic	Yes	23.9

Table 6: Comparison of MPCs across different models

the small IES, the desire to smooth consumption is strong, and the consumption profile will be flat as shown in Figure 4. Hence, households save enough liquid assets even with a low-interest rate on liquid assets, making the borrowing constraint rarely binding.

The rational expectation agent exhibits a lesser consumption response and an MPC of 1.4 percent under the one-asset framework by introducing the expenditure shocks. By introducing additional shock, the increased need for precautionary saving makes households save more. Also, there is a clear distinction between agents with rational expectations and fully optimistic beliefs. The fully optimistic agent under the one-asset framework exhibits an MPC of 13.3 percent. When households are optimistic, their expected wealth accumulation does not meet the actual wealth since they face larger expenditure shocks than anticipated before.

The two-asset framework amplifies the channel of generating higher MPC. For rational expectation agents and fully optimistic agents, the MPC is about twice the MPC under the one-asset framework. Though the large IES dampens the mechanism generating high MPC in the two-asset frame under rational expectations, introducing the agents with behavioral frictions makes lower liquid assets among optimistic agents, and separating the savings for retirement exacerbates the low liquidity where the liquid assets only serve the role for precautionary saving.

5 Policy Implications

In this section, we investigate the role of perception $\widetilde{\mathbf{\Gamma}}$ in consumer welfare. For simplicity, I consider the one asset model which features life after retirement. I define the *welfare function* $W_t^*(X_t;\Gamma_t,\widetilde{\mathbf{\Gamma}})$ in a paternalistic view, which captures expected *realized* lifetime utility based on the perception $\widetilde{\mathbf{\Gamma}}$ as

$$
\begin{gathered}
W_t^*(X_t;\Gamma_t,\widetilde{\mathbf{\Gamma}}) = u(C_t(X_t;\Gamma_t,\widetilde{\mathbf{\Gamma}}) - \Gamma_t) + \delta\mathbb{E}\left[W_{t+1}^*(X_{t+1};\Gamma_{t+1},\widetilde{\mathbf{\Gamma}})\right] \\
\text{where} \\
C_t(X_t;\Gamma_t,\widetilde{\mathbf{\Gamma}}) = \arg\max_{c_t} u(C_t - \Gamma_t) + \delta\mathbb{E}_t\left[\delta\widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t+1};\widetilde{\Gamma}_{t+1})\right], \text{ and} \\
C_t \leq X_t = Y_t + R_t^s(X_{t-1} - C_{t-1}).
\end{gathered}
$$

The following proposition establishes a basic result: there is monotonicity of the welfare function between two pessimistic (or optimistic) agents.

Proposition 4. For any $t \geq T_r$, if $\mathbf{\Gamma} >_1 \widetilde{\mathbf{\Gamma}}^1 >_1 \widetilde{\mathbf{\Gamma}}^2$, then $W_t^*(X_t;\Gamma_t,\widetilde{\mathbf{\Gamma}}^1) \geq W_t^*(X_t;\Gamma_t\widetilde{\mathbf{\Gamma}}^2)$. Also, $\mathbf{\Gamma} <_1 \widetilde{\mathbf{\Gamma}}^1 <_1 \widetilde{\mathbf{\Gamma}}^2$, implies that $W_t^*(X_t;\Gamma_t,\widetilde{\mathbf{\Gamma}}^1) \geq W_t^*(X_t;\Gamma_t\widetilde{\mathbf{\Gamma}}^2)$.

The message of Proposition 4 is simple. If we have two optimistic or pessimistic agents. The agent that is closest to the sophisticated case, $\mathbf{\Gamma}$ will have larger lifetime expected welfare. For example, for a pessimistic agent, the agent suffers from welfare loss by his overconsumption. Though he optimally chooses his level of expenditure based on his belief $\widetilde{\mathbf{\Gamma}}$, this is not optimal in the sense of welfare, and the consumption will not be smooth enough. Moreover, this welfare loss gets accumulated over the life cycle. As the agent becomes more and more pessimistic, the welfare loss will become larger as well.

This result shows that when choosing priorities of who to 'correct,' the government should first target the agents with abnormally low or high amounts of savings for a paternalistic government. For these agents, the expected increase in welfare will be higher by government intervention.

Ricardian Equivalence Sometimes, the government wants to boost economic activity rather than enhance welfare by maintaining a stable consumption path. In this case, Proposition 1 remind that the agents with most optimistic beliefs will likely show greater consumption response. However, in the real world setting, the fiscal policy can be constrained by methods of financing fiscal stimulus. In an extreme case where agents realize the increase in future tax burden, the fiscal policy would have no effect at all, which is the famous Ricardian equivalence theorem (Barro, 1974). Unfortunately, in this model, when the agents are not hand-to-mouth, fiscal transfer funded by the tax next period will not affect agents' decisions. However, since the marginal continuation value of the optimistic agent is always lower than the pessimistic agent, we can expect that the optimistic agent will likely have more binding liquidity constraints. For these agents, the fiscal policy can still have a net effect even when agents perceive that it will be taxed later.

An Example of Ex-Post Welfare Improving Policy What kind of policy can improve the welfare of households with behavioral frictions? This question can be important in reality since if the government can correct the behavior of households with simple policy tools, it can prevent households from facing high borrowing costs when they face large expenditure shocks.

To answer this question, I impose three criteria that a policy should satisfy. First, a policy should not depend on external funds, and preferably, the implementation should be possible without inter-personal transfers. Without this condition, there can be obvious welfare improving policy of spreading money to households. Also, if the government can make inter-personal transfers, it would be crucial to assign the utility weights to different households and discuss what type of households we should prioritize. Second, the policy instruments should follow observable variables. Hence, the government cannot design a policy conditional on the agents' optimism and pessimism, which would also be impossible in the real world situation. Third, the government implements dynamically consistent policies, and the households should form correct beliefs about them. Without this condition, the degree of freedom for the government is large, and the government will eventually lose credibility.

I suggest the following policy penalizing low liquidity and giving households enough buffer to prevent expenditure from depleting their liquid wealth. The government imposes a proportional tax τ_l up to the liquid wealth of $W_{l,t}$ at time t. For wealth that exceeds $W_{l,t}$ and less than $W_{r,t}$, the government gives a tax rebate proportional to τ_r. Hence, the tax $T_{i,t}(X_{i,t})$ that households i with the level of liquid wealth $X_{i,t}$ pay at time t would be

$$T_{i,t}(X_{i,t}) = \min\{W_{l,t}, X_{i,t}\}\tau_l - \min\{\max\{X_{i,t} - W_{l,t}, 0\}, W_{r,t} - W_{l,t}\}\tau_r, \tag{7}$$

which is also the tax revenue for the government. In simple terms, the government penalizes households with wealth less than $W_{l,t}$ but induces optimistic households to accumulate the liquid wealth at least $W_{l,t}$. However, such policy can make households with the rational expectation to undersave and keep the level of wealth between $W_{l,t}$ and $W_{r,t}$, so there is no guarantee for overall welfare improvement. However, if we keep $W_{r,t}$ small enough, households that already accumulate large liquid wealth will not be affected by this policy. At the same time, $W_{r,t}$ should be large enough to make low liquidity households be prepared for the underestimated expenditure shocks.

If $W_{r,t} = 2W_{l,t}$ and $t_l = t_r$, then all of the transfers $(X_{i,t} - W_{l,t})t_r$ can be covered by the taxes $X_{i,t}t_l$ collected earlier, so this policy does not need any interpersonal transfers. Hence, if households correctly form beliefs about (7), then such policy will satisfy all three criteria mentioned above. I simulate the economy with the above setting under various t_l and $W_{l,t}$ based on the calibrated distribution of perception in Figure 5. I impose that $W_{l,t} = \bar{w}\Xi_t$ where Ξ_t is the age specific trend of income appeared in (6). Rather than fixing $W_{l,t}$ and $W_{r,t}$, by making $W_{l,t}$ to be proportional to the trend of income, I can make the intended minimum savings $W_{l,t}$ to vary with income so optimistic households will be naturally inclined to save larger amount of liquid wealth during the middle age when there income peaks.

Table 7 shows the result of the calibration exercise. I have chosen four different tax (or transfer) rates, which are 1, 2, 3, and 4 percent in an annualized term. In some of the parameters, the average utility is higher than the benchmark case without any policy. Especially when $T_{l,t}/\Xi_t = 2$ and $T_{r,t}/\Xi_t = 4$, the average utility is higher than the benchmark case. Hence, inducing agents to save the liquid wealth of twice their income trend can be desirable. At the same time, the government can raise tax revenue by increasing the τ_l and

$T_{l,t}/\Xi_t$	$T_{r,t}/\Xi_t$	τ_l	τ_r	Avg. utility	Avg. revenue
1	2	0.01	0.01	-191.6	0.06
		0.02	0.02	-191.7	0.07
		0.03	0.03	-192.1	0.1
		0.04	0.04	-192.9	0.14
2	4	0.01	0.01	-189.7	0.13
		0.02	0.02	-188.8	0.23
		0.03	0.03	-188.8	0.29
		0.04	0.04	-189.5	0.53
No policy benchmark				-191.8	0

Table 7: Policy Simulation Based on the Calibrated Model

Note: 'Avg. utility' refers to mean level of utility which is defined as $\frac{1}{I\times T}\sum_{i\in I}\sum_{t\in T}u(C_{i,t}-\Gamma_{i,t})$. where I is the number of the simulated households and T is the total number of periods. 'Avg. revenue' refers to the average revenue that the government raises which is defined as $\frac{1}{I\times T}\sum_{i\in I}\sum_{t\in T}T_{i,t}(X_{i,t})$. The unit of revenue is \$1,000. τ_l and τ_r in the table are annualized, actual value τ_l^{actual} used in the simulation is $\tau^{*,\text{actual}}=1-(1+\tau^*)^{1/12}$ for $\tau^*=\{\tau_l,\tau_r\}$.

τ_r as well.

6 Conclusion

Using a model of heterogeneous agents with different perceptions of expenditure shocks, this paper generated high overall MPC by matching the severe dispersion of liquid wealth found in the data. The calibration shows that most households have a low perception of future expenditure shock, which leads them to have a very low level of liquidity and exhibit high MPC. I conclude by discussing several extensions of this paper.

First, the model in this paper assumed a fixed retirement date, with no early withdrawal of illiquid assets. Alternatively, we can impose adjustment costs as Kaplan and Violante (2014). Introducing adjustment costs can further incorporate real-world behavior such as early withdrawal of retirement accounts. However, adjustment costs may break the concavity of the continuation value in the two-asset problem and hence disrupt the differentiability of the value function, which can impede obtaining theoretical results in Proposition 3. Nevertheless, the benefits of quantitative exercise incorporating adjustment costs certainly exist and desire attention.

Second, this paper depended on the parsimonious model of expenditure shocks which was assumed to be exogenous. Though nonpersistent short-run temptations mainly drive products classified as expenditure shocks, they cannot be entirely outside the life cycle consumption and savings plan. This assumption can be relieved by modeling the expenditure system where households have an imperfect perception of preferences. However, to rationalize the high variance of the nonpersistent part and its stark difference distribution with the persistent part, it would be necessary to incorporate large and temporary preference shocks to endogenize the expenditure shock introduced in this paper.

Third, learning of expenditure shocks was not allowed. This paper isolated the chance for learning based on the calibrated distribution of the expenditure shocks. Since the distribution of the expenditure shock is extremely skewed, median households will underestimate the mean and mainly observe the shocks with low values. Also, there are many categories of consumption that would make it difficult to learn all the aspects of the expenditure shocks. Therefore, I limited the model's scope and the degree of freedom by focusing on the pure role of optimism and pessimism. However, some extreme events can shape the expectation of households, and such learning behaviors can also lead to an interesting source of heterogeneity.

A Proofs

Proof of Proposition 1

Proof. Write the cumulative density functions of $\widetilde{\mathbf{\Gamma}}_t^1$ and $\widetilde{\mathbf{\Gamma}}_t^2$ as $F_t^1 : \mathbb{R}_+ \to [0,1]$ and $F_t^2 : \mathbb{R}_+ \to [0,1]$, respectively. I prove using backward induction. At the final period, consumers spend all available wealth, hence $C_T(X_T, \Gamma_T; \widetilde{\mathbf{\Gamma}}) = X_T$ whatever Γ_T and $\widetilde{\mathbf{\Gamma}}$ is. Since $F_t^1 >_1 F_t^2$, it follows that

$$\mathbb{E}_{T-1}\left[\frac{\partial W_T^{\widetilde{\mathbf{\Gamma}}^1}(X_T;\Gamma_T)}{\partial X_T}\right] = \int u'(X_T - \Gamma_T)dF_T^1(\Gamma_T) > \int u'(X_T - \Gamma_T)dF_T^2(\Gamma_T)$$
$$= \mathbb{E}_{T-1}\left[\frac{\partial W_T^{\widetilde{\mathbf{\Gamma}}^2}(X_T;\Gamma_T)}{\partial X_T}\right].$$

It is obvious that $\mathbb{E}_{T-1}\left[W_T^{\widetilde{\mathbf{\Gamma}}^1}(X_T;\Gamma_T)\right]$ is concave to X_T. Now we proceed to general periods. Let the following properties [T1] and [T2] that hold at period $t+1$:

$$[\text{T1}] \quad 0 < \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^2}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}}\right] < \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^1}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}}\right], \text{ and}$$
$$[\text{T2}] \quad \mathbb{E}_t\left[\frac{\partial^2 W_{t+1}^{\widetilde{\mathbf{\Gamma}}^2}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}^2}\right] < 0 \text{ and } \mathbb{E}_t\left[\frac{\partial^2 W_{t+1}^{\widetilde{\mathbf{\Gamma}}^2}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}^2}\right] < 0.$$

Then, I show that properties [T1] and [T2] also hold at period t, and moreover, $C_t(X_T;\Gamma_t,\widetilde{\mathbf{\Gamma}}^1) \leq C_t(X_T;\Gamma_t,\widetilde{\mathbf{\Gamma}}^2)$ where the inequality is strict when the consumer saves a positive amount.

For any $t \geq T_r$, fix a shock Γ_t. By [T1] and [T2], if the agent is hand-to-mouth under $\widetilde{\mathbf{\Gamma}}^1$, then the agent is also hand-to-mouth under $\widetilde{\mathbf{\Gamma}}^2$ since $u'(C_t - \Gamma_t) > \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^1}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}}\right] > \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^2}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}}\right]$ for any $C_t \in (0, X_t]$. Hence, in this case, $C_t(X_T;\Gamma_t,\widetilde{\mathbf{\Gamma}}^1) = C_t(X_T;\Gamma_t,\widetilde{\mathbf{\Gamma}}^2) = X_t$ by [T2]. If the agent saves under $\widetilde{\mathbf{\Gamma}}^1$ where $C_t(X_T;\Gamma_t,\widetilde{\mathbf{\Gamma}}^1) < X_T$, then

$$u'(C_t(X_T;\Gamma_t,\widetilde{\mathbf{\Gamma}}^1)) = \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^1}(w_{t+1};\widetilde{\Gamma}_{t+1})}{\partial w_{t+1}}\right] > \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^2}(w_{t+1};\widetilde{\Gamma}_{t+1})}{\partial w_{t+1}}\right]$$

by [T1]. Then we have $C_t(X_T;\Gamma_t,\widetilde{\mathbf{\Gamma}}^1) < C_t(X_T;\Gamma_t,\widetilde{\mathbf{\Gamma}}^2)$. Whether or not the agent is hand-to-mouth under $\widetilde{\mathbf{\Gamma}}^1$, this leads to

$$\frac{\partial \widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}^1}(X_t;\widetilde{\Gamma}_t)}{\partial X_t} = u'(C_t(X_T;\widetilde{\Gamma}_t,\widetilde{\mathbf{\Gamma}}^1) - \widetilde{\Gamma}_t) \geq \text{`}u'(C_t(X_T;\widetilde{\Gamma}_t,\widetilde{\mathbf{\Gamma}}^2) - \widetilde{\Gamma}_t) = \frac{\partial \widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}^2}(X_t;\widetilde{\Gamma}_t)}{\partial X_t}.$$

Using the fact that $F^1 >_1 F^2$, we can conclude that [T1] holds at time t. [T2] is obvious because consumption is an increasing function of wealth. $\square$

Proof of Proposition 2

Proof. When fixing S_{t-1}, the consumption function in terms of random variables Y_t, Y_{t+1}, Γ_t and Γ_{t+1} can be written as $C_t(X_t(Y_t), \Gamma_t)$ and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)$ for periods t and $t+1$.

We need to show that $\text{cov}(\Gamma_t, C_{t+1}X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)) < 0$. Define $\bar{\Gamma}_t = \mathbb{E}[\Gamma_t]$. Fix Y_t, Y_{t+1}, and Γ_{t+1} Then,

$$\left[\Gamma_t - \bar{\Gamma}_t\right]\left[C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t) - C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \bar{\Gamma}_t)\right] \leq 0.$$

Taking expectation with respect to Γ_t conditional on Y_t, Y_{t+1}, and Γ_{t+1} gives,

$$\mathbb{E}\left[\left[\Gamma_t - \bar{\Gamma}_t\right] C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)\middle| Y_t, Y_{t+1}, \Gamma_{t+1}\right] \leq 0.$$

Now taking expectation over Y_t, Y_{t+1}, and Γ_{t+1} gives,

$$\mathbb{E}\left[\Gamma_t C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)\right] - \bar{\Gamma}_t \mathbb{E}\left[C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)\right] \leq 0$$

This shows that $\text{cov}(C_t - \Gamma_t, C_{t+1}) \geq \text{cov}(C_t, C_{t+1})$. $\square$

Lemma 1. Suppose that $u(c)$ is strictly concave, and $F(s,a)$ is concave. For a convex set $B(\bar{x})$, a problem

$$V(\bar{x}, \bar{A}) = \max_{s,a} u(\bar{x} - s - a) + F(s, Z)$$
$$\text{s.t. } s + a < \bar{x}, Z = \bar{A} + a, s \geq 0 \text{ and } a \geq 0.$$

[1] only allows a unique solution for $c = \bar{x} - s - a$, [2] value function $V(\bar{x}, \bar{A})$ is concave, and [3] if $u(\cdot)$ and $F(s,a)$ are concave and differentiable, then $V(\bar{x})$ is also differentiable at $\bar{x}$ and A.

1. I first show the uniqueness of c. Suppose that another triplet c', s' and a' where $c \neq c'$ exists. Then for a $\lambda \in (0,1)$

$$u(\lambda c + (1-\lambda)c') + F(\lambda s + (1-\lambda)s', \lambda Z + (1-\lambda)Z') > u(c) + F(s, Z)$$

which is a contradiction.

[2] For concavity of V, let $S' = (s', a')$ and $c' = \bar{x}' - s' - a'$ to denote the solution that maximize the objective function when given $\bar{x}'$ and $\bar{A}'$. Similarly, let $S'' = (s'', a'')$ and $c'' = \bar{x}'' - s'' - a''$ be the maximizers when given $\bar{x}''$ and $\bar{A}''$.

For any $\lambda \in [0, 1]$, $\lambda S' + (1 - \lambda)S''$ is feasible when given $\bar{x}^* = \lambda\bar{x}' + (1 - \lambda)\bar{x}''$ and $\bar{A}^* = \lambda\bar{A}' + (1 - \lambda)\bar{A}''$. Define $S^* = \lambda S' + (1 - \lambda)S''$ and $c^* = \lambda c' + (1 - \lambda)c''$. Then,

$$\begin{aligned} V(\bar{x}^*, \bar{A}^*) &\geq u(c^*) + F(s^*, Z^*) \\ &\geq \lambda\left[u(c') + F(s', Z')\right] + (1 - \lambda)\left[u(c'') + F(s'', Z'')\right] \\ &> \lambda V(\bar{x}', \bar{A}') + (1 - \lambda)V(\bar{x}'', \bar{A}''). \end{aligned}$$

[3] For differentiabilty, Lemma Benveniste and Scheinkman (1979) shows that if $W(x, A|s, a) = u(x - s - a) + F(s, a + A)$ is a concave function on a convex set B, then a concave function $V(x, A)$ where $V(x^*, A^*) = W(x^*, A^*)$ and $V(x, A) \geq W(x, A)$ for all other $(x, A) \in B$, then V is differentiable at (x^*, A^*). □

Proof of Proposition 3

Proof. I prove using backward induction. In each step, I show that [T1] following two inequalities

$$\frac{\partial\widetilde{V}^1_{t+1}(X_{t+1}, Z_{t+1})}{\partial X_{t+1}} > \frac{\partial\widetilde{V}^2_{t+1}(X_{t+1}, Z_{t+1})}{\partial X_{t+1}}, \text{ and } \frac{\partial\widetilde{V}^1_{t+1}(X_{t+1}, Z_{t+1})}{\partial Z_{t+1}} > \frac{\partial\widetilde{V}^2_{t+1}(X_{t+1}, Z_{t+1})}{\partial Z_{t+1}}$$

hold, and [T2] $\widetilde{V}^i_{t+1}(X_t, Z_t)$ is a concave function.

Starting from period $T_r - 1$, the agent i solves

$$\max_{C,S,A} u(C - \Gamma_{T_r-1}) + \delta\mathbb{E}_{T_r-1}\left[\widetilde{W}^i_{T_r}(R^S_{T_r}S + R^A_{T_r}(A + Z_{T_r-1})\right] \tag{8}$$

$$\text{subject to } C + S + A \leq X_{T_r-1}, C \geq 0, S \geq 0, \text{ and } A \geq 0. \tag{9}$$

Trivially, $S = 0$ since it is an inferior asset compared to A with $R^A_{T_r} > R^S_{T_r}$. Hence, this is effectively an one asset case. We can apply same steps as in Proposition 1 and claim that $C^2_{T_r-1}(X_{T_r-1}, Z_{T_r-1}) \geq C^1_{T_r-1}(X_{T_r-1}, Z_{T_r-1})$. Also,

$$\begin{aligned} \frac{\partial\widetilde{V}^i_{T_r-1}}{\partial Z_{T_r-1}} &= u'(C^i_{T_r-1} - \widetilde{\Gamma}^i_t)\frac{\partial C^i_{T_r-1}}{\partial Z_{T_r-1}} + \delta\mathbb{E}_{T_r-1}\left[\widetilde{W}^i_{T_r}{}'(X_{T_r})R^A_{T_r}\left[1 + \frac{\partial A_{T_r-1}}{\partial Z_{T_r-1}}\right]\right] \\ &= \delta\mathbb{E}_{T_r-1}\left[\widetilde{W}^i_{T_r}{}'(X_{T_r})R^A_{T_r}\right] \end{aligned}$$

whether or not the constraint on $C^i_{T_r}$ is binding. If agents 1 and 2 are both hand-to-mouth, then the wealth at period T_r is the same and this gives

$$\frac{\partial \widetilde{V}^2_{T_r-1}}{\partial Z_{T_r-1}} = \delta \mathbb{E}_{T_r-1}\left[\widetilde{W}^{2}_{T_r}{}'(X_{T_r})R^A_{T_r}\right] < \delta \mathbb{E}_{T_r-1}\left[\widetilde{W}^{1}_{T_r}{}'(X_{T_r})R^A_{T_r}\right] < \frac{\partial \widetilde{V}^1_{T_r-1}}{\partial Z_{T_r-1}}.$$

If the agent 2 is hand-to-mouth but agent 1 saves, then

$$\frac{\partial \widetilde{V}^2_{T_r-1}}{\partial Z_{T_r-1}} = \delta \mathbb{E}_{T_r-1}\left[\widetilde{W}^{i}_{T_r}{}'(X_{T_r})R^A_{T_r}\right] < u'(C^2_{T_r-1} - \widetilde{\Gamma}^2_{T_r-1}) < u'(C^1_{T_r-1} - \widetilde{\Gamma}^1_{T_r-1}) = \frac{\partial \widetilde{V}^1_{T_r-1}}{\partial Z_{T_r-1}}.$$

If both agents save, then

$$\frac{\partial \widetilde{V}^2_{T_r-1}}{\partial Z_{T_r-1}} = u'(C^2_{T_r-1} - \widetilde{\Gamma}^2_{T_r-1}) < u'(C^1_{T_r-1} - \widetilde{\Gamma}^1_{T_r-1}) = \frac{\partial \widetilde{V}^1_{T_r-1}}{\partial Z_{T_r-1}}.$$

Similar argument can show us that $\frac{\partial \widetilde{V}^2_{T_r-1}}{\partial X_{T_r-1}} < \frac{\partial \widetilde{V}^1_{T_r-1}}{\partial X_{T_r-1}}$ as we have done in Proposition 1 but I do not repeat it here. Also, $\widetilde{V}^i_{T_r-1}(X_{T_r-1}, Z_{T_r} - 1)$ is concave by Lemma 1.

Now we go to any arbitrary period $t < T_r - 1$ given that [T1] and [T2] holds at period $t+1$. Let us denote $\widetilde{F}^i_t(S^i_t, A^i_t) = \mathbb{E}_t\left[\widetilde{V}^i_{t+1}(X^i_{t+1}(S^i_t), Z^i_{t+1}(A^i_t))\right]$. Nothe that the objective function $u(X_t - S^i_t - A^i_t) + \widetilde{F}_t(S^i_t, A^i_t)$ is differentiable and concave to S^i_t and A^i_t with linear constraints by [T1] and [T2], hence Kuhn-Tucker optimality conditions are necessary and sufficient.

We deal with following four cases.

- Case 1: both agents 1 and 2 are hand-to-mouth.

  In this case, there is nothing to prove since the consumption of two agents are identical. Also, [T1] holds trivially.

- Case 2: only agent 2 is hand-to-mouth.

  Also in this case, the assumption already implies that [T1] holds.

- Case 3: only agent 1 is hand-to-mouth.

  I show the impossibility of this case. Assume that the agent 1 is only saving illiquid asset A^1_t. The assumption implies that

  $$u'(C^2_t - \Gamma_t) = \frac{\partial \widetilde{F}^2_t(S^2_t, A^2_t)}{\partial S^2_t} \leq \frac{\partial \widetilde{F}^1_t(S^2_t, A^2_t)}{\partial S^2_t}.$$

  Hence, by Inada condition on $u(\cdot)$ and continuity and concavity of $\widetilde{F}_t(\cdot,\cdot)$, there should exist some levels of $\Delta > 0$ and $\Delta' > 0$ such that

$$u'(C_t^2 - \Delta - \Delta' - \Gamma_t) = \frac{\partial \widetilde{F}_t^1(S_t^2 + \Delta, A_t^2 + \Delta')}{\partial S_t^2}.$$

However, this cannot be true because $C_t^2 - \Delta - \Delta'$, $S_t^2 + \Delta$ and A_t^2 is also a solution for agent 1's problem and we only allow for unique optimal consumption by Lemma 1. Similar arguments can be done when the agent 1 is only saving liquid asset S_t^1.

- Case 4: all agents are not hand-to-mouth.

  The argument is the same as case 3.

Hence, the consumption is always higher for the agent 2. Concavity and differentiability of the value function are guaranteed by Lemma 1. Then $F_{t-1}^i(S_{t-1}^i, A_{t-1}^i) = \mathbb{E}_t\left[V_{t+1}^i(X_{t+1}^i, Z_{t+1}^i)\right]$ is also concave since $F_{t-1}^i(S_{t-1}^i, A_{t-1}^i)$ is sum of a concave functions.

Finally, we need to show that [T1] holds at periods at t. For the first inequality, $C_t^2(X_t, Z_t; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^2) \geq C_t^1(X_t, Z_t; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^1)$ implies that

$$\begin{aligned}
& u'\left(C_t^1(X_t, Z_t; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^1)) - \widetilde{\Gamma}_t\right) \geq u'\left(C_t^2(X_t, Z_t; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^2)) - \widetilde{\Gamma}_t\right) \\
& \Rightarrow \mathbb{E}_{t-1}\left[\frac{\partial \widetilde{V}_t^1}{\partial X_t}\right] \geq \mathbb{E}_{t-1}\left[\frac{\partial \widetilde{V}_t^2}{\partial X_t}\right],
\end{aligned}$$

where the last line follows from the fact that $\widetilde{\Gamma}_t^1 >_1 \widetilde{\Gamma}_t^2$. □

Proof of Proposition 4

Proof. I only show the proof for the case where $\mathbf{\Gamma} >_1 \widetilde{\mathbf{\Gamma}}^1 >_1 \widetilde{\mathbf{\Gamma}}^2$. The proof for the other case is similar to this case. Assume that following holds for continuation value at $t+1$:

$$\begin{aligned}
[a] \quad & \mathbb{E}_t\left[W_{t+1}^*(X_{t+1}; \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^1)\right] \geq \mathbb{E}_t\left[W_{t+1}^*(X_{t+1}; \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^2)\right] \\
[b] \quad & \mathbb{E}_t\left[W_{t+1}^*(Y_{t+1} + R_{t+1}^S(\bar{S} + \Delta); \Gamma_t, \widetilde{\mathbf{\Gamma}}) - W_{t+1}^*(Y_{t+1} + R_{t+1}^S\bar{S}; \Gamma_t, \widetilde{\mathbf{\Gamma}})\right] \\
& \geq \mathbb{E}_t\left[\widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(Y_{t+1} + R_{t+1}^S(\bar{S} + \Delta); \widetilde{\Gamma}_t) - \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(Y_{t+1} + R_{t+1}^S\bar{S}; \widetilde{\Gamma}_t)\right] \text{ for all } \Delta \geq 0.
\end{aligned}$$

I show that using [a] and [b], Fix $X_t > 0$ and denote consumption under $\widetilde{\mathbf{\Gamma}}^1$ as C^1. Then Consumption under $\widetilde{\mathbf{\Gamma}}^2$ can be written as $C^1 + \Delta$ for some $\Delta \geq 0$. Then we are comparing

$$U^1 = u(C^1) + \delta \mathbb{E}_t\left[W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^1)\right] \text{ and} \tag{10}$$

$$U^2 = u(C^1 + \Delta) + \delta \mathbb{E}_t\left[W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1 - \Delta); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^1)\right]. \tag{11}$$

Then,

$$
\begin{aligned}
u(C^1+\Delta)-u(C^1) &\le \delta\mathbb{E}_t\big[\widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^1}(Y_{t+1}+R_{t+1}^S(X_t-C^1);\widetilde{\Gamma}_{t+1}) \\
&\qquad -\widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}^1}(Y_{t+1}+R_{t+1}^S(X_t-C^1-\Delta);\widetilde{\Gamma}_{t+1})\big] \\
&\le \delta\mathbb{E}_t\big[W_{t+1}^*(Y_{t+1}+R_{t+1}^S(X_t-C^1);\Gamma_{t+1},\widetilde{\mathbf{\Gamma}}^1) \\
&\qquad -W_{t+1}^*(Y_{t+1}+R_{t+1}^S(X_t-C^1-\Delta);\Gamma_{t+1},\widetilde{\mathbf{\Gamma}}^1)\big] \\
&\le \delta\mathbb{E}_t\big[W_{t+1}^*(Y_{t+1}+R_{t+1}^S(X_t-C^1);\Gamma_{t+1},\widetilde{\mathbf{\Gamma}}^1) \\
&\qquad -W_{t+1}^*(Y_{t+1}+R_{t+1}^S(X_t-C^1-\Delta);\Gamma_{t+1},\widetilde{\mathbf{\Gamma}}^2)\big].
\end{aligned}
$$

Rearranging the last row shows that $U^1 \ge U^2$. Not I show that [a] and [b] holds with the continuation value at t. [a] is obvious, which is just taking expectation on the above inequality. To show [b], note that $F(x) >_1 G(x)$ between two CDFs $F(x)$ amd $G(x)$ implies that

$$
\int v(x)dF(x) \ge \int u(x)dG(x)
$$

if $v(x) \ge u(x)$ for all x and $u(x)$ is increasing in x. Using this strategy, we need to show that

$$
\begin{aligned}
&W_t^*(Y_t+R_t^S(X_t-C_t+\Delta);\Gamma_t,\widetilde{\mathbf{\Gamma}})-W_t^*(Y_t+R_t^S\bar{S};\Gamma_t,\widetilde{\mathbf{\Gamma}}) \\
&\ge \widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}}(Y_t+R_t^S(\bar{S}+\Delta);\Gamma_t)-\widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}}(Y_t+R_t^S\bar{S};\Gamma_t)
\end{aligned}
$$

for all Δ and Γ_t. Note that this reduces back to [b] at $t+1$. Last thing to show is that $\widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}}(Y_t+R_t^S(\bar{S}+\Delta);\Gamma_t)-\widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}}(Y_t+R_t^S\bar{S};\Gamma_t)$ is increasing in Γ_t. Denote $X'_{t+1}=Y_t+R_t^S(\bar{S}+\Delta)$ and $X_t=Y_t+R_t^S\bar{S}$. Note that (if the agent is not hand-to-mouth)

$$
\begin{aligned}
\frac{\partial W_t^{\widetilde{\mathbf{\Gamma}}}(X_t;\Gamma_t)}{\partial\Gamma_t} &= u'(C_t-\Gamma_t)\left[\frac{\partial C_t}{\partial\Gamma_t}-1\right]-\delta R_{t+1}^S\mathbb{E}_t\left[\frac{\partial W_{t+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t+1};\widetilde{\Gamma}_{t+1})}{\partial X_{t+1}}\right]\frac{\partial C_t}{\partial\Gamma_t} \\
&= -u'(C_t-\Gamma_t) < 0.
\end{aligned}
$$

If the agent is hand-to-mouth, then we have the same result immediately. Then we immediately have the result that $\widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}}(Y_t+R_t^S(\bar{S}+\Delta);\Gamma_t)-\widetilde{W}_t^{\widetilde{\mathbf{\Gamma}}}(Y_t+R_t^S\bar{S};\Gamma_t)$ is increasing in Γ_t. Last thing to show is that [a] and [b] holds when constructing the continuation value at the terminal period, but this is obvious. □

B Other Tables and Figures

		89'	92'	95'	98'	01'	04'	07'	10'	13'	16'	19'	Avg.
All	NW	0.71	0.71	0.70	0.72	0.74	0.74	0.74	0.77	0.77	0.79	0.78	0.74
	LIQ	0.88	0.87	0.89	0.89	0.90	0.89	0.90	0.91	0.90	0.91	0.89	0.89
	Illiquid	0.68	0.68	0.66	0.68	0.70	0.70	0.70	0.72	0.73	0.76	0.76	0.71
Graduate	NW	0.70	0.68	0.68	0.68	0.72	0.71	0.70	0.73	0.73	0.75	0.75	0.71
	LIQ	0.86	0.86	0.86	0.86	0.89	0.87	0.88	0.89	0.88	0.87	0.87	0.87
	Illiquid	0.66	0.66	0.64	0.64	0.68	0.66	0.67	0.67	0.70	0.73	0.73	0.68
Tertiary education	NW	0.69	0.67	0.67	0.69	0.70	0.68	0.71	0.70	0.72	0.76	0.72	0.70
	LIQ	0.85	0.83	0.86	0.87	0.87	0.86	0.87	0.85	0.85	0.89	0.85	0.86
	Illiquid	0.64	0.64	0.65	0.66	0.66	0.65	0.68	0.67	0.68	0.72	0.69	0.67
Secondary education	NW	0.67	0.67	0.64	0.65	0.67	0.69	0.66	0.71	0.73	0.74	0.75	0.69
	LIQ	0.85	0.82	0.87	0.83	0.83	0.85	0.85	0.88	0.90	0.88	0.89	0.86
	Illiquid	0.66	0.64	0.60	0.61	0.63	0.64	0.64	0.67	0.69	0.72	0.73	0.66
1st income tertile	NW	0.64	0.65	0.64	0.66	0.66	0.65	0.66	0.68	0.67	0.71	0.70	0.67
	LIQ	0.85	0.84	0.86	0.86	0.87	0.85	0.86	0.86	0.84	0.86	0.83	0.85
	Illiquid	0.61	0.62	0.61	0.62	0.63	0.62	0.63	0.64	0.64	0.67	0.68	0.63
2nd income tertile	NW	0.58	0.60	0.58	0.58	0.62	0.60	0.58	0.60	0.67	0.63	0.64	0.61
	LIQ	0.76	0.80	0.82	0.79	0.81	0.78	0.78	0.77	0.85	0.81	0.82	0.80
	Illiquid	0.53	0.53	0.50	0.49	0.52	0.51	0.49	0.51	0.57	0.54	0.55	0.52
3rd income tertile	NW	0.72	0.72	0.70	0.69	0.74	0.75	0.71	0.75	0.78	0.76	0.74	0.73
	LIQ	0.88	0.86	0.87	0.86	0.90	0.93	0.91	0.92	0.93	0.92	0.89	0.90
	Illiquid	0.71	0.69	0.64	0.66	0.67	0.67	0.67	0.69	0.70	0.71	0.71	0.68

Table 8: Gini index of the United States: 1989-2019

Notes: Illiquid assets refers to total assets minus the liquid assets. Net worth is the total assets minus debt. 'Avg.' refers to the average of all survey years for each row.

	ρ	p-value	Expenditure shock
Adult care	0.338	0.008	
Alcohol away from home	0.190	<0.001	
Alcohol at home	1.000	<0.001	
Child care	0.118	0.026	O
Clothes	-0.015	0.009	O
Clothing services	0.205	<0.001	
Domestic services	0.257	0.136	O
Education durables	0.034	0.063	O
Education services	0.003	0.731	O
Entertainment durables	-0.003	0.588	O
Entertainment services	0.054	0.005	
Fees and charges	0.023	0.215	O
Food away from home	0.064	<0.001	
Food at home	0.883	<0.001	
Furniture rental	1.000	<0.001	
Gasoline expenses	0.447	<0.001	
Health care durable	0.010	0.397	O
Health insurance	0.242	<0.001	
Health care service	0.026	<0.001	
Other household expenditures	0.066	0.002	
Home insurance	0.002	0.933	O
Home management	-0.042	0.004	O
Home maintenance and repairs	0.036	<0.001	
Home-related equipment and supplies	0.093	0.093	O
Household furnishings and equipment	0.013	0.282	O
Household textiles and linens	0.041	0.298	O
Jewelry	0.005	0.657	O
Life insurance	-0.084	0.041	O
Occupational expenses	0.115	<0.001	
Parking expenses	~0.001	0.941	O
Public transportation	0.012	0.078	O
Personal care products	-0.036	0.217	O
Telephone services	0.246	<0.001	
Personal care services	0.999	<0.001	
Reading material	1.000	<0.001	
Rent	0.063	<0.001	
Rented vehicles	0.048	<0.001	
Tobacco	1.000	<0.001	
Utilities	0.056	<0.001	
Vehicle	0.002	0.172	O
Other vehicle-related durables	0.021	0.277	O
Vehicle insurance	-0.050	<0.001	O
Vehicle service	-0.005	0.430	O
Water and other public services	-0.142	<0.001	O

Table 9: Classification Expenditure Shocks

C Sample Selection when using the Consumer Expenditure Survey

The following households are dropped. (1) Households with top-coded income. (2) Household head lives in a student housing (dormitory). (3) Household head age is not reported. (4) Family size is not reported. (5) Age is less than 25 which is the initial age of the simulation. (6) Age exceeds 80 which is the terminal age of the simulation. (7) Family size has changed during the survey quarters. (8) Nondurable goods expenditure is at bottom 1 percent of the sample or the food consumption is zero. (9) Households do not have full twelve month survey responses.

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