

Consumption and Savings under Imperfect Perception of Expenditure Shocks*

Hyok Jung Kim[†]

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Abstract

In this paper, I build a life cycle model of consumption and savings where households face exogenous expenditure shocks. Households are heterogeneous as they have different levels of perceptions of expenditure shocks. Households who underestimate the expenditure shocks tend to spend more now and save less for the future. The calibration result matches the dispersion of liquid wealth in data, with many households underestimating the future expenditure shocks. The model exhibits high marginal propensity to consume as reported in the empirical evidence even with the intertemporal elasticity of substitution far below one. This high marginal propensity to consume is due to a large number of hand-to-mouth households resulting from their underestimation of future expenditure shocks. The model also exhibits a realistic level of liquid wealth over the life cycle. I also provide a policy recommendation that can enhance overall welfare.

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[†]Email: khjkim@ucdavis.edu

1 Introduction

Why do households consume a large quantity from additional income, and who spends the most? This question of magnitude and heterogeneity underlying the marginal propensity to consume (MPC) is essential to the design of a policy, like fiscal stimulus to boost economic activity. In practice, the magnitude is important when determining the size or duration of the fiscal stimulus. The heterogeneity is also important to identify the structure of the fiscal stimulus over diverse groups of people and induce the maximal consumption response. In this paper, I investigate these two aspects of MPC by employing a model of heterogeneous households with different degrees of behavioral biases.

A large piece of empirical evidence documents high MPCs in various contexts. Moreover, much of the vast literature indicates the strong association between low liquidity and large consumption responses (Souleles, 1999; Johnson et al., 2006; Parker et al., 2013; Baker, 2018; Baker et al., 2020; Fagereng et al., 2021; Aydin, 2021). Reflecting on the empirical evidence, influential approaches of Carroll (1992, 1997), Kaplan and Violante (2014), and Carroll et al. (2017) all have hand-to-mouth households with low liquidity at their heart. Hence, in this liquidity perspective of generating large overall MPC, it is crucial to explain why and who are liquidity constrained.

If a model depends on liquidity constraints to generate a high MPC, the model needs to exhibit a realistic distribution of liquid assets, and thus MPC in the model is high because of a real reason. How are liquid assets distributed? Carroll et al. (2017) demonstrates that liquid wealth is distributed more unequally than net worth. Then, what kind of factors can explain this high dispersion of liquid wealth? Age can be a crucial factor driving the dispersion of liquid wealth since households across various stages of life would have different demands. However, I demonstrate that when we pool households by different age groups, the inequality of liquid wealth is still high. Thus, a factor explaining the distribution of liquid wealth would be independent of age. I show that even in subgroups of homogeneous households in education groups, occupation, and income¹, the dispersion of liquid wealth is still extremely high. This result is robust when I restrict my analysis to highly liquid assets², that bring homogeneous returns.

There are several challenges in the current literature on high MPCs and the dispersion of liquid wealth. First, we need a model generating the dispersion of liquid wealth, even when income and asset returns are homogeneous in expectation. Carroll et al. (2017) successfully generate dispersion of wealth comparable to data with discount factor heterogeneity. How-

¹In the literature, earnings heterogeneity (Castañeda et al., 2003), heterogeneity of asset returns (Hubmer et al., 2020), or entrepreneurship (Quadrini, 1999) are mentioned as important factors.

²The definitions of liquid and highly liquid assets are provided in Section 2.

ever, they do not explicitly separate liquid and illiquid assets, and the dispersion of wealth depends on heterogeneity in education level, which drives additional dispersion of income. Second, it is rare for models to employ intertemporal elasticity of substitution (IES) significantly below one, as suggested by the meta-analysis by Havránek (2015) and other micro evidence—like that given in Best et al. (2020)—when explaining the high MPC. The lower IES strengthens households’ desire to smooth consumption, and the model will likely exhibit lower MPC, as noted by Aguiar et al. (2020).

In response to the challenge, I introduce a model that can explain the high MPC and dispersion of liquid wealth with a realistic choice of the IES. Households are heterogeneous in the perceptions of future expenditure shocks, whereas income is homogeneous in expectation. The consumption and savings of households are crucially dependent on how much they underestimate or overestimate their future expenditure shocks. Households that underestimate the expenditure shocks feel less need for precautionary saving and tend to consume more than other households that overestimate. This consumption gap creates a gap of savings between households that relatively underestimate and overestimate than others.³ Over time, households that underestimate the expenditure shock consume more than they originally planned in the past, which quickly depletes their liquid savings. The channel leading to the dispersion of liquid wealth also drives the heterogeneity of the MPC. The households with low liquid wealth due to the underestimation of future expenditure shocks will be more likely to become hand-to-mouth, where households consume all their disposable wealth. This paper also uses the mechanism of Kaplan and Violante (2014) where households hold a disproportionately small quantity of liquid assets compared to illiquid assets to explain a high MPC. However, the behavioral channel makes the mechanism robust to the low IES, as underestimating the future expenditure shock can lead to persistently low liquid wealth.

Following Carroll et al. (2017), this paper calibrates the model to match the share of liquid wealth at various percentiles, while letting the model match the median liquid asset holdings over income. Simultaneously, it does not arbitrarily increase the overall MPC by introducing households with a low level of wealth in the economy. Targeting the level of median liquid wealth over income makes the overall level of liquid wealth in the model similar to the data. The result that consumption has a monotonic relationship with the perception of expenditure shock is a crucial source of identification in this paper. If the data suggests that the share of wealth must be lower at the lower percentile, then the model will respond by introducing households that underestimate future expenditure shocks, and the

³Specifically, if distributions regarding future expenditure shock perceived by household A first-order stochastically dominates distributions perceived by household B , then the consumption of household A will be lower than that of household B .

inequality of liquid wealth will be higher.

The calibration result implies that households perceive approximately one-third of the true expenditure shock on average.⁴ In addition, there are a large number of relatively more optimistic households that underestimate future expenditure shocks with extremely low levels of liquid assets. Although not targeted directly, the calibration result also captures the level of liquid wealth at different percentiles over the life cycle. Furthermore, the calibration result implies that most households do not exhibit consumption responses. Instead, a small group of hand-to-mouth households leads to a high overall MPC. This extensive margin of MPC is consistent with the pattern presented in Fuster et al. (2020).

This paper provides two policy implications for two different goals of the government. Our first policy implication is regarding whom the government must target to boost economic activities during a recession: focusing on low-liquidity households would lead to an effective stimulus. As Carroll (1992) and Kaplan and Violante (2014) noted, households with low liquidity are more likely to be hand-to-mouth, thereby exhibiting a larger consumption response. This paper introduces an additional channel that supports this policy. Since liquid wealth is a good proxy of whether or not households are optimistic, targeting low-liquidity households would effectively cover the households that are more willing to consume. Our second policy is that if the government wants to enhance the overall ex-post welfare in the economy, it must introduce incentives to rectify the bias introduced by the misperception of future shocks. Based on the calibrated distribution of perception with a large mass of households that underestimate the future expenditure shock, this paper recommends a policy that taxes people with low liquid wealth and transfers back money when they have high liquid wealth. The average consumption utility of the households can be enhanced while raising additional funds from taxes. This is based on observables, as even though the government cannot observe optimism/pessimism, the levels of liquid wealth serve as an appropriate proxy.

Literature Review Carroll (1992, 1997) was among the first to adopt a borrowing constraint in a one-asset model to explain hand-to-mouth households exhibiting high MPC. Kaplan and Violante (2014) noted that the proportion of households that become hand-to-mouth is not sufficiently large to generate high overall MPC under the one-asset framework in a realistic setting. Kaplan and Violante (2014) introduce a new workhorse model with liquid and illiquid assets, which separates saving for retirement and precautionary motives. Aguiar et al. (2020) show that the original models cannot generate a realistic level of wealth and proportion of hand-to-mouth households. They introduce preference heterogeneity to

⁴I assume that the expenditure shocks are drawn from an exponential distribution, with mean λ_t at each age t . In a loose sense, the calibration result implies that, on average, households perceive that the expenditure shock is drawn from an exponential distribution, where the mean is $\lambda_t/3$.

remedy this. Aguiar et al. (2020) also note that these models still cannot generate high MPC with the low intertemporal elasticity of substitution (IES) estimated from the micro evidence.⁵

Carroll et al. (2017) introduce heterogeneity in the discount factor and education levels (that determine income) to explain the distribution of wealth and the high MPC. In their model, the households with low discount factors end up with low wealth, and consequently, generate a high MPC. There are three differences between this paper and Carroll et al. (2017). First, we consider liquid and illiquid assets separately, while Carroll et al. (2017) do not. Second, we consciously do not utilize income dispersion by education levels because education levels or other demographic factors do not contribute much to the dispersion of liquid wealth. Lastly, this paper can generate a higher level of MPC which is similar to the upper ranges of MPC reported in Havranek and Sokolova (2020).

The quasi-hyperbolic discounting (Strotz, 1955; Pollak, 1968; Laibson, 1997) can also produce high marginal propensity to consume. Under this framework, Laibson et al. (2007) used a two-asset life cycle model combined with quasi-hyperbolic discounting and estimate the key parameters (β, δ) ⁶ to match the accumulation of wealth and credit card borrowing. The problem of using (β, δ) is that it is difficult to produce a sizeable difference between sophisticated and naive agents, which mutes the role of perception regarding futures agents.⁷ Moreover, the (β, δ) model generally does not guarantee a unique consumption solution unless the modeler uses the perception of β close to one, which implies the naive agent. In this paper, as mentioned earlier, there is a clear connection between the degree of perception and consumption, unlike the quasi-hyperbolic discounting model.⁸

Lian (2021) presents a model where a larger absolute size of mistakes in predicting future consumption leads to a higher MPC today. In this paper, both the size and the direction of errors are crucial in determining the MPC. It is the households that underestimate the future expenditure shocks that will save less and are more likely to be hand-to-mouth in this

⁵Under the discount factor of 0.95, MPC of 13.2% when IES is 1.5 drops to 1.5% when IES is 0.5. This pattern is confirmed by the Table D.I of the appendix of Kaplan and Violante (2014) where rebate coefficient, measured using the method of Johnson et al. (2006), drops from 20% to 9% when IES decreases from 2 to 1.05.

⁶Under the quasi-hyperbolic discounting framework, β implies the degree of present bias that discounts the utility from the future rewards compared to the current reward. δ is the long-run discount factor which measures how much the agent discount the utility of later rewards.

⁷When simulating the life cycle model under quasi-hyperbolic discounting, Angeletos et al. (2001) shows no significant difference between naive agents, who perceive future agents' present bias as $\beta = 1$, and sophisticated agents, who perceive the future present bias correctly.

⁸In a stylized three-period problem, Salanie and Treich (2006) show that the relationship between consumption and perception of β depends on the shape of the utility function. However, there is no guarantee that this result would hold in the longer horizon, and the existence or uniqueness of solutions can be difficult to characterize (Harris and Laibson, 2001, 2002).

paper, which is a different implication from Lian (2021).

Bianchi et al. (2021) adopts insights of the psychology literature regarding selective memory recall and introduces a model where surprises can shape the future expectation in a biased manner. In particular, a high-income shock today can generate optimistic beliefs regarding future liquidity and increase the MPC. The merit of Bianchi et al. (2021) is that they specify the learning behavior of agents, which is absent in this paper. However, they do not apply their model in a general life-cycle setting, unlike this paper.

The remainder of this paper is organized in the following manner. In Section 2, I present two stylized facts. First, there is a severe distribution of liquid wealth even after controlling for demographic factors. Second, there is a large overall MPC, with the heterogeneity of consumption responses. Section 3 presents the model in which several theoretical properties are derived. Section 4 takes the model to data, calibrates the distribution of perception, and examines the model’s performance. Section 5 presents policy implications of this model. Finally, section 6 concludes with possible extensions.

2 Stylized Facts

2.1 Severe Dispersion of Liquid Wealth

Liquid wealth is a crucial part of households’ wealth and a major source of consumption. Ensuring that households have sufficient liquidity is important since households with too little liquid wealth may face a high borrowing cost in an emergency. Do households hold a sufficient amount of liquid wealth? To see the holdings of liquid wealth in data, I use the Survey of Consumer Finances from 1989 to 2019, one of the most comprehensive datasets regarding households’ portfolios. I focus on the working-age households between the ages 25 and 64 that earned above the minimum wage.⁹ I define the liquid asset as the sum of money market, checking, savings and call accounts, directly held stocks, and bonds.

Unfortunately, there is a large group of households with very low liquid wealth. The presence of households with low liquid assets drives the severe dispersion of liquid wealth. As seen from Table 1, the average Gini coefficient¹⁰ of the liquid asset in all survey waves

⁹I employ these sample selection criteria to make the data similar to the model environment in Section 3. I use households above the minimum wage, as the model of Section 3 has no unemployment. Households after retirement may have different timing of the withdrawal of illiquid assets, whereas the model assumes a fixed date.

¹⁰The Gini coefficient represents the degree of inequality where zero implies perfect equality, and one implies the opposite. To calculate the Gini coefficient using the sample weights given in the Consumer Survey of Finances, I use the methodology of Lerman and Yitzhaki (1989).

between 1989–2019 is 0.89.¹¹

Why is the inequality of liquid wealth so high? I first examine the demographic factors that are known to contribute to the high dispersion of net worth. Notable demographic factors are different education levels and the dispersion of income (Castañeda et al., 2003), asset return heterogeneity (Hubmer et al., 2020), and entrepreneurship (Quadrini, 1999). To test if a certain demographic factor can explain the dispersion of liquid wealth, I check if the Gini coefficient among subgroups of households classified using the demographic factor diminishes. The demographic factors used in this paper are education levels, income, occupation, and age. To isolate the dispersion of wealth from the asset return heterogeneity, I also present the results using *highly liquid asset* that comprises money market, checking, and savings accounts. Since the returns of items in the highly liquid assets are likely to be similar across households, their balances would represent households’ desire to save and not the differences in the investment skills or luck.

	All	Education			Income Tertiles		
		Graduate	Tertiary	Secondary	1st (highest)	2nd	3rd (lowest)
Liquid	0.89	0.87	0.86	0.86	0.85	0.80	0.90
Highly liquid	0.82	0.78	0.77	0.81	0.76	0.72	0.83

Table 1: Average Gini coefficients of liquid assets among working-age households by education and income levels: 1989–2019

Table 1 indicates that there is high inequality of liquid assets within subgroups of households that have similar education levels or income. Compared to the Gini coefficient of all households, the Gini coefficients within subgroups tend to be smaller, but the differences are small. In particular, the Gini coefficient of the lowest income tertile is approximately 0.9, which suggests a high proportion of households with a near-zero level of wealth and producing a high Gini coefficient. The Gini coefficient using the highly liquid asset is smaller than the case using liquid assets. Hence, assets that are not in highly liquid assets, such as stocks and bonds bring additional dispersion to the distribution of the highly liquid assets. However, we can still see a high degree of inequality across all demographic subgroups when using highly liquid assets.

The life cycle model of consumption and savings implies that households accumulate large assets near the end of retirement, which would lead to the dispersion of wealth across different stages of life. Households with a near-zero level of wealth will be mostly young households, since they face uphill income profiles and postpone saving. Households with low

¹¹In the same setting, the average Gini coefficient of the net worth is 0.74, which already indicates a high dispersion of wealth. The Gini coefficient of liquid wealth is consistently higher than the net worth in various situations this section considers.

	Age groups			
	25–34	35–44	45–54	55–64
Liquid	0.80	0.86	0.89	0.90
Highly liquid	0.74	0.79	0.82	0.82

Table 2: Average Gini coefficients of liquid assets among working-age households by age groups: 1989–2019

liquidity will vanish from the middle-age as they begin accumulating wealth. Hence, if we group people by different age groups, wealth inequality is expected to go away. Surprisingly, grouping the households by different life stages does not contribute to lowering the dispersion of liquid wealth. As Table 2 suggests, there is a trend that the degree of dispersion increase as households reach the retirement age, but the Gini coefficients among households in similar stages of life are approximately the same as the overall Gini coefficient. This implies that households with low liquidity exist in all age groups, and life cycle properties are not the crucial factors that lead to the severe dispersion of liquid wealth.

	Occupation group 1		Occupation group 2		
	Work for others	Self-employed	Managerial	Technical	Other
Liquid	0.87	0.89	0.87	0.88	0.81
Highly liquid	0.79	0.82	0.78	0.81	0.76

Table 3: Average Gini coefficients of liquid assets among working-age households by different occupation groups: 1989–2019

Note: “Managerial” refers to managerial and professional workers. “Technical” refers to technical, sales, and service workers.

Lastly, we check if different occupational characteristics can contribute to the dispersion of liquid wealth. Different occupational groups may have different occupational needs. For example, self-employed workers may have a less predictable income, which might increase the desire for precautionary savings. Moreover, households with higher income, such as managerial and professional workers, would have outliers that lead to greater liquid wealth dispersion. Table 3 gauge inequality of liquid wealth by two classifications of occupation groups. In the first two columns, I contrast the group working for others and the self-employed. Not only is the degree of inequality similar, but severe inequality also exists in both groups. The remaining three columns provide an alternative classification of occupation groups by their roles. This alternative method for classifying the occupation confirms that different occupations do not lead to different degrees of liquid wealth inequality. Moreover, this inequality remains even when measured with highly liquid assets.

We checked if potential demographic factors that explain the dispersion of total wealth can also explain the dispersion of liquid wealth. However, the analysis in this section shows

that the high dispersion of liquid wealth persists within subgroups of people with similar education and income levels or stages of life. Hence, there is a force driving severe dispersion of life regardless of demographic or economic factors.

2.2 Large and Heterogeneous MPC

Numerous empirical studies report excess sensitivity of consumption: consumers' MPC of a windfall gain is large. For example, Souleles (1999) reports that consumers spend 34.4–64 percent of income tax refunds over a quarter. Agarwal and Qian (2014) measured that consumers spent 80 percent of unanticipated fiscal stimulus in Singapore over a 10-month period. In response to the social security tax reform, Parker (1999) found that households spent approximately 20 percent of additional after-tax income over three-month periods on nondurable goods. In the income tax rebate of 2001, Johnson et al. (2006) found that households spent 20 percent to 40 percent of the tax rebate over a quarter on nondurable goods. In fiscal stimulus of 2008, Parker et al. (2013) found that households' spent 50 percent to 90 percent of the stimulus payments over a quarter. Using predictable payments from the Alaska permanent fund, (Kueng, 2018) finds that households spend 20 percent for nondurable and services over a quarter. The magnitude of the MPC may differ, but they are usually over twenty percent when measuring the increase in consumption over a quarter. Havranek and Sokolova (2020) collect estimates of MPC in both micro and macro literature. In their data, when restricting the sample with micro-level evidence and MPC measured over a quarter, the median MPC is approximately 22.7 percent, whereas the mean MPC is 30.2 percent.

Heterogeneity of Consumption Responses Households exhibit heterogeneous consumption responses. Three notable patterns of the heterogeneous consumption responses are [1] association of low liquidity and high consumption responses, [2] extensive margin of consumption response, and [3] the role of sophistication and financial planning.

First, a large piece of evidence suggests a strong relationship between high consumption response and low liquidity, consistent with the buffer stock theory of saving (Souleles, 1999; Johnson et al., 2006; Parker et al., 2013; Baker, 2018; Baker et al., 2020; Fagereng et al., 2021; Aydin, 2021). In addition, there is another piece of evidence that indicates the importance of persistent characteristics (Parker, 2017). The stance of this paper is that the persistent characteristics drive low liquidity and thus, bring the high consumption response, as Gelman et al. (2019), Gelman (2021), and Carroll et al. (2017).

Second, there is evidence of an extensive margin of consumption responses. Not all households exhibit a large consumption response, and the high overall MPC is led by a small group

of households in the economy. For example, based on two stimuli from U.S. in years 2001 and 2008, Misra and Surico (2014) show that approximately half of the households do not show a significant and positive response to the stimulus. (Fuster et al., 2020) asked directly to survey respondents how much they would spend out of a windfall gain; approximately 70 percent of the respondents exhibited MPC of zero.

Third, households’ attitudes and perceptions regarding the future are important. Parker (2017) finds strong spending responses among households that lack sophistication and financial planning and hints that persistent characteristics play an essential role. Motivated by this finding, this paper adopts persistent behavioral friction, which can lead to different consumption responses.

2.3 Misprediction of Expenditures

Households generally save for two reasons. In the 2019 Survey of Consumer Finance, the most frequently selected reason for saving was to prepare for retirement (34.1 percent) among working-age households. The following big reason for saving was to prepare for the ‘rainy days,’ which was selected by 27.7 percent of the respondents.¹² The two main reasons for saving are consistent with implications of modern consumption savings models, such as Carroll (1992). Households accumulate a large amount of wealth to prepare for retirement, and simultaneously, some portion of the wealth is left to deal with the income fluctuations and other random events, such as liquidity or expenditure shocks.

However, households’ portfolio choice in the Survey of Household Finances suggests a lack of preparation for rainy days. Among working-age households, three percent of households even resort to payday loans, which typically accompany high interest. As a most frequently selected option, 38.3 percent of people answered that using a payday loan was because of an emergency. Furthermore, among subjects who currently borrow money, 38.9 percent answered that they had an experience being behind the payment by two months or more. These facts show that many households cannot avoid borrowing with high interest or paying penalties for being behind schedule, which brings additional costs. Thus, even though households save to prepare for the rainy days, the unpredicted expenses force households to use borrowing facilities that bring extra cost. Also, there is a prevalence of credit card borrowing where the median borrowing rate in the same survey is around 17 percent. Approximately 38.6 percent of households have credit card loans. The borrowing rate and the borrowing frequency are too high to be rationalized with the models without any behavioral frictions.¹³ The pattern of borrowing over the life cycle is at odds with the

¹²The third reason to save was to buy own house, selected by 5.8 percent of the sample.

¹³For example, the current workhorse model of consumption and savings by Kaplan and Violante (2014)

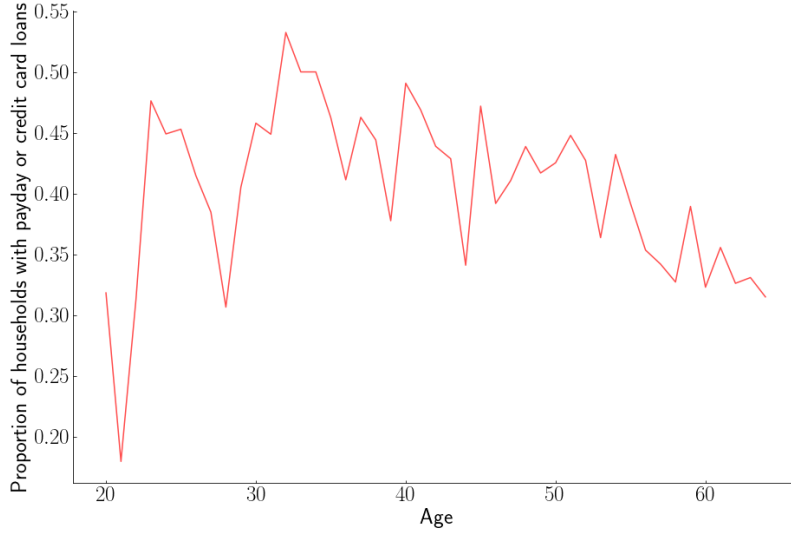


Figure 1: Proportion of having payday or credit card loans by age
Source: Reproduced from the Survey of Consumer Finances at 2019

implications of the life cycle models without featuring a borrowing constraint. Such models would suggest that households would mainly borrow until the peak of the income profile and stop borrowing from then. However, Figure 1 does not indicate a sharp decrease in the frequency of borrowing near the middle age, and the high frequency of borrowing also persists throughout the life cycle. Hence, contributions from other factors must persistently lead households to borrow throughout the life cycle.

As a related study, Berman et al. (2016) provides evidence that consumers tend to underweight the costs rather than income when predicting the future spare money. Howard et al. (2020) studies an expenditure prediction bias which is a tendency to underestimate future expenses. Fellowes and Willemin (2013) suggest that 25% of households withdraw 401(k) savings early, often to cover unexpected expenses.

3 Model

Households face consumption and savings problems for T number of periods. Interest rates and income are exogenous. There are two types of consumption. The first part of consumption is determined in the long-run perspective, and its trajectory is smooth over the life cycle. The second part of the consumption, which we call an expenditure shock, is not

targets 26 percent of the households to borrow, under the nominal borrowing rate of 10 percent.

a choice variable where short-run motives are more important than consumption smoothing in the long run. Examples of expenditure shocks are vehicle repair costs or sudden medical expenses. Extra spending on items in this category does not bring additional utility. However, such expenss are an unavoidable part of consumption, and determine the minimum level of consumption at each period.

The expenditure shock at period t is drawn from a random variable $\mathbf{\Gamma}_t$, which maps a set of events to nonnegative real numbers. Households have an imperfect perception of expenditure shocks, where the perception of $\mathbf{\Gamma}_t$, denoted as $\tilde{\mathbf{\Gamma}}_t$, can be different from the actual $\mathbf{\Gamma}_t$. Similarly, variables with tilde, such as \tilde{x} , represents the perception of a variable x . Without the time subscript, $\tilde{\mathbf{\Gamma}} = \{\tilde{\mathbf{\Gamma}}_1, \dots, \tilde{\mathbf{\Gamma}}_T\}$ represents the sequence of perception of expenditure shocks. In the same way, $\mathbf{\Gamma}$ is the sequence of distribution of true expenditure shocks, $\mathbf{\Gamma} = \{\mathbf{\Gamma}_1, \dots, \mathbf{\Gamma}_T\}$. Γ_t and $\tilde{\Gamma}_t$ are particular realizations of random variables $\mathbf{\Gamma}_t$ and $\tilde{\mathbf{\Gamma}}_t$.

Households retire at age T_r . During the working-age, households at time $t < T_r$ solve the following problems:

$$\begin{aligned}
V_t^{\tilde{\mathbf{\Gamma}}}(X_t, Z_t; \Gamma_t) &= \max_{C_t, S_t, A_t} u(C_t - \Gamma_t) + \delta \mathbb{E}_t \left[\tilde{V}_{t+1}^{\tilde{\mathbf{\Gamma}}}(X_{t+1}, Z_{t+1}; \tilde{\Gamma}_{t+1}) \right], \quad \text{where} \quad (1) \\
\tilde{V}_{t'}^{\tilde{\mathbf{\Gamma}}}(X_{t'}, Z_{t'}; \tilde{\Gamma}_{t'}) &= \max_{C_{t'}, S_{t'}, A_{t'}} u(C_{t'} - \tilde{\Gamma}_{t'}) + \delta \mathbb{E}_{t'} \left[\tilde{V}_{t'+1}^{\tilde{\mathbf{\Gamma}}}(X_{t'+1}, Z_{t'+1}; \tilde{\Gamma}_{t'+1}) \right], \\
&\text{subject to} \\
Y_t + R_t^S S_{t-1} &= X_t \geq C_t + S_t + A_t, \\
Z_{t+1} &= R_{t+1}^A (A_t + Z_t), \\
C_t \geq 0, A_t \geq 0 \text{ and } S_t \geq 0, &\text{ and} \\
\tilde{V}_{T_r-1}^{\tilde{\mathbf{\Gamma}}}(X_{T_r-1}, Z_{T_r-1}; \tilde{\Gamma}_{T_r-1}) &= \max_{C_{T_r-1}, S_{T_r-1}, A_{T_r-1}} u(C_{T_r-1} - \tilde{\Gamma}_{T_r-1}) \\
&\quad + \delta \mathbb{E}_{T_r-1} \left[\tilde{W}_{T_r}^{\tilde{\mathbf{\Gamma}}}(Z_{T_r} + R_{T_r}^S S_{T_r-1}) \right].
\end{aligned}$$

$V_t^{\tilde{\mathbf{\Gamma}}}(S_{t-1}, Z_{t-1}; \Gamma_t)$ is the current self's value function conditional on the savings of liquid asset S_{t-1} and illiquid asset Z_{t-1} from the previous period, and a realization of the expenditure shock Γ_t . Note that S_t is a stock variable, and A_t is a flow variable. The stock of illiquid asset at t is written as Z_t . The value function $V_t^{\tilde{\mathbf{\Gamma}}}(S_{t-1}, Z_{t-1}; \Gamma_t)$ also depends on the perception $\tilde{\mathbf{\Gamma}}$, which determines the continuation value $\tilde{V}_{t+1}^{\tilde{\mathbf{\Gamma}}}(S_t, Z_t; \tilde{\Gamma}_{t+1})$. The disposable wealth X_t at time t is defined as the sum of current income Y_t and gross return of savings from liquid assets $R_t^S S_{t-1}$, where R_t^S is the gross interest rate on liquid assets.

The constraints $S_t \geq 0$ and $A_t \geq 0$ eliminate borrowing. In particular, The constraint on the flow of illiquid asset $A_t \geq 0$ ensures that the withdrawal of illiquid assets before retire-

ment is impossible. Since there are no costs of saving A_t , this model imposes an extreme case of asymmetric adjustment cost. An alternative means of imposing illiquidity of an asset can be requiring an adjustment cost, as in Kaplan and Violante (2014). In particular, Kaplan and Violante (2014) assumes symmetric adjustment cost for illiquid assets. In reality, the withdrawal of illiquid assets should be possible, but paying back a mortgage or saving in a retirement account does not require any adjustment cost. Hence, the adjustment costs of illiquid assets in the real world will be somewhere in between the extreme asymmetric cost of this model and the symmetric adjustment cost of Kaplan and Violante (2014). The assumption that this model makes is for theoretical tractability. In particular with adjustment costs, the continuation values can be locally convex due to the non-convexity of choice sets. To acquire a clear relationship between the degrees of perception, and consumption, I do not impose adjustments that can add unnecessary difficulty in the theoretical analysis.

After retirement, $t \in \{T_r, \dots, T\}$, households solve the following problems:

$$\begin{aligned}
W_t^{\tilde{\Gamma}}(X_t; \Gamma_t) &= \max_{C_t, S_t} u(C_t - \Gamma_t) + \delta \mathbb{E}_t \left[\widetilde{W}_{t+1}^{\tilde{\Gamma}}(X_{t+1}; \tilde{\Gamma}_{t+1}) \right], \quad \text{where} \\
\widetilde{W}_{t'}^{\tilde{\Gamma}}(X_{t'}; \tilde{\Gamma}_{t'}) &= \max_{C_{t'}, S_{t'}} u(C_{t'} - \tilde{\Gamma}_{t'}) + \delta \mathbb{E}_{t'} \left[\widetilde{W}_{t'+1}^{\tilde{\Gamma}}(X_{t'+1}; \tilde{\Gamma}_{t'+1}) \right], \\
&\text{subject to} \\
X_{T_r} &= Y_{T_r} + R_{T_r}^S S_{T_r-1} + R_{T_r}^A Z_{T_r-1}, \\
Y_t + R_t S_{t-1} &= X_t \geq C_t + S_t \text{ if } t \neq T_r, \\
C_t &\geq 0, \text{ and } S_t \geq 0, \text{ and} \\
\widetilde{W}_T^{\tilde{\Gamma}}(X_T; \tilde{\Gamma}_T) &= u(X_T - \tilde{\Gamma}_T).
\end{aligned} \tag{2}$$

After retirement, the problem becomes simple with only two choice variables, C_t , and S_t . Also, at the period of retirement, T_r , households can access the illiquid assets Z_{T_r} , they accumulated thus far. In the final period, households consume all available wealth. I denote the value function after retirement using the letter W instead of V to distinguish value functions in two different regimes.

The optimal consumption and savings before retirement at t are functions solving (1) with inputs when X_t , Z_t , and Γ_t are based on the perception $\tilde{\Gamma}$. If there is no room for confusion, I will omit inputs of the functions to save space. Similarly, the optimal consumption and savings before retirement are functions that solve (2) with inputs X_t , Γ_t , and $\tilde{\Gamma}$. Following the literature, a household is hand-to-mouth if the level of optimal consumption C_t equals available wealth X_t , that is $C_t(X_t, Z_t, \Gamma_t; \tilde{\Gamma}) = X_t$.

The domain of the utility function $u : \mathbb{R} \rightarrow \mathbb{R}$ consists of real numbers, where $u' > 0$ and $u'' < 0$. An example of this kind of utility function is the exponential utility function with

constant absolute risk aversion. The Inada condition cannot be incorporated in this model unless there is a guarantee that Γ_t is not too large. Alternatively, I can impose the Inada condition and assume that $\Gamma_t < Y_t$ and $\tilde{\Gamma}_t < Y_t$ in every period. I also assume that R_t^A and R_t^S are deterministic, and $1/\delta > R_t^A > R_t^S$. This assumption eliminates the incentive to save liquid assets in the period just before retirement $T_r - 1$; hence $S_{T_r} = 0$.

Optimism and Pessimism I call a household that correctly perceives the future random variable, where the perception $\tilde{\Gamma}$ is identical to Γ , a *sophisticated* household. Considering two different perceptions $\tilde{\Gamma}^1$ and $\tilde{\Gamma}^2$, I consider that $\tilde{\Gamma}^1$ is more *optimistic* (*pessimistic*) than $\tilde{\Gamma}^2$ if cumulative density functions (CDF) in $\tilde{\Gamma}^2$ ($\tilde{\Gamma}^1$) first-order stochastically dominates CDFs in $\tilde{\Gamma}^1$ ($\tilde{\Gamma}^2$). From now on, I denote $\tilde{\Gamma}_t^2 >_1 \tilde{\Gamma}_t^1$ if CDF, of $\tilde{\Gamma}_t^2$ first-order stochastically dominates CDF of $\tilde{\Gamma}_t^1$. Similarly, $\Gamma^2 >_1 \Gamma^1$ implies that $\Gamma_t^2 >_1 \Gamma_t^1$ for any t .

Relationship between Consumption and Perception How is consumption related to the perception of future expenditure shocks? I establish the relationship between the level of consumption, and the degree of optimism and pessimism. I first investigate the behavior of households after the retirement. In the first proposition, I show that when two households face the same disposable wealth X_t and the expenditure shock Γ_t , the household that are more optimistic will spend more.

Proposition 1. For every $t \geq T_r$, and $w_t \geq 0$, $C_t(X_t; \Gamma_t, \tilde{\Gamma}^1) \geq C_t(X_t; \Gamma_t, \tilde{\Gamma}^2)$ if $\tilde{\Gamma}^2 >_1 \tilde{\Gamma}^1$.

The intuition underlying Proposition 1 is simple. The household with more pessimistic beliefs perceives that it will face a larger expenditure shock in the future. The presence of large expenditure shocks in the future decrease $C_t - \Gamma_t$ at any future period t . The marginal value of saving will also be higher, since the pessimistic households will perceive that the long-run part of the consumption will be lower. However, optimistic households do not value savings as much as pessimistic households. Note that Proposition 1 does not specify where Γ_t is drawn from. As long as the perceptions of households can be ordered, the comparative statics of consumption is possible under the same X_t and Γ_t .

If consumption at any point, under the same condition, is always higher among optimistic households, can we also claim a similar argument for assets? To answer this question, consider the following scenario. Fix a sequence of income Y_t, Y_{t+1}, \dots, Y_T , interest rates $R_t^S, R_{t+1}^S, \dots, R_T^S$, and expenditure shocks $\Gamma_t, \Gamma_{t+1}, \dots, \Gamma_T$. Moreover, for household i , and when $t \geq T_r$ and $k \leq T - t$, I define the endogenous disposable income of i at time $t + k$ recursively as,

$$X_{t+k}^i(X_{t+k-1}^i; \Gamma_{t+k}, \tilde{\Gamma}^i) = Y_{t+k} + R_{t+k}^S \left[X_{t+k-1}^i - C_{t+k-1}(X_{t+k-1}^i; \Gamma_{t+k-1}, \tilde{\Gamma}^i) \right],$$

which depends on the endogenous choices of consumption, $C_t, C_{t+1}, \dots, C_{t+k-1}$.

In this manner, $X_{t_k}^i(\cdot; \cdot, \tilde{\Gamma}^i)$ tracks the accumulation of assets of household i over the life cycle. Since we fixed the exogenous factors, such as income, interest rates, and expenditure shocks, the comparison between $X_{t_k}^i(\cdot; \cdot, \tilde{\Gamma}^i)$ and $X_{t_k}^i(\cdot; \cdot, \tilde{\Gamma}^j)$ let us focus on how the different degree of perception can lead to a different path of asset accumulation.

Corollary 1. Fix $t \geq T_r$ and $X_t \geq 0$. Then, for any $k \leq T - t$ and realization of Y_t, \dots, Y_{t+k} and $\Gamma_t, \dots, \Gamma_{t+k}$,

$$X_{t+k}^2(X_{t+k-1}^1; \Gamma_{t+k}, \tilde{\Gamma}^2) \geq X_{t+k}^1(X_{t+k-1}^1; \Gamma_{t+k}, \tilde{\Gamma}^1)$$

as long as $\tilde{\Gamma}^2 >_1 \tilde{\Gamma}^1$.

This result reveals that for any two households where the perceptions are ordered as $\tilde{\Gamma}^2 <_1 \tilde{\Gamma}^1 <_1 \Gamma$, the consumption profile of the household with $\tilde{\Gamma}^1$ would be more flatter than the other household with $\tilde{\Gamma}^2$. Moreover, when beginning with the same level of wealth X_t and facing the same income and expenditure shocks, the wealth of the household with more optimistic belief, $\tilde{\Gamma}^2$ would be depleted faster. Hence, at the end of the life cycle, the household with pessimistic beliefs will have more left to consume. Based on this result, this paper uses the stock of liquid assets as a tool to elicit the distribution of relative optimism and pessimism.

We obtain a similar result as Proposition 1 after the retirement, which is a two-asset model. The intuition underlying Proposition 2 is the same as that for Proposition 1.

Proposition 2. For every $t < T_r$, $C_t(X_t, Z_t; \Gamma_t, \tilde{\Gamma}^1) \geq C_t(X_t, Z_t; \Gamma_t, \tilde{\Gamma}^2)$ if $\tilde{\Gamma}^2 >_1 \tilde{\Gamma}^1$.

However, Corollary 1 does not extend to the two-asset case. This is because even though household A is more pessimistic than household B , it does not necessarily imply that household A would save both liquid and illiquid assets more than B . For example, suppose imminent expenditure shock is larger than the expenditure shocks after retirement. In such a case, the savings of illiquid assets of a pessimistic household can be lower than that of the optimistic household. A numerical exercise in Figure 2 shows such a case. As the household moves along the horizontal axis from left to right, the household becomes more aware of the expenditure shock and becomes more pessimistic. The relatively optimistic households underestimate the need for precautionary saving in period two and mainly save using the illiquid assets, which brings a higher rate of return. However, the sophisticated households that perfectly know the large expenditure shock mainly save using the liquid asset. The relationship of the consumption and savings with the perceptions in the two-asset case needs to

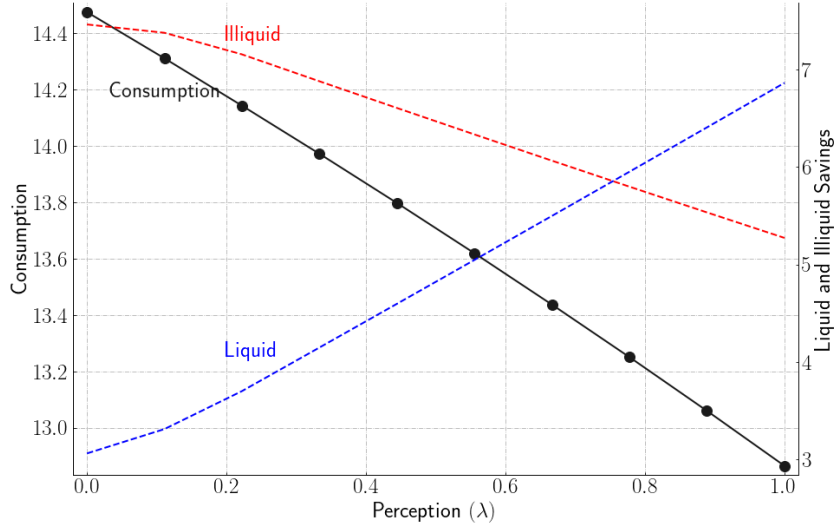


Figure 2: A case where pessimistic household have lower illiquid savings than an optimistic household

Notes: The figure presents the simulation result of a three-period model with the following parameter values. Utility function is CRRA with $\sigma = 0.5$ (of $u(c) = c^{1-\sigma}/(1-\sigma)$). Income is $\{25, 10, 5\}$ for each period. Expenditure shocks of magnitudes 7 and 1 for periods two and three, respectively, occur with probability 0.5. The discount factor is 0.95, and the interest rates for liquid and illiquid assets are 0 and 2 percent, respectively. The x -axis shows the perception of households, where a value λ on the x -axis denotes the percentage of the shock magnitude perceived by the household.

be verified using a simulation, and I show in Section 4 that the more pessimistic households tend to accumulate more liquid illiquid assets than their optimistic counterparts.

Fluctuations of C_t and $C_t - \Gamma_t$ Next, we develop the dynamic properties of $C_t - \Gamma_t$ and C_t . To see this, let I explicitly write all the random variables as inputs of consumption in this period, C_t , and the next period C_{t+1} as $C_t(X_t(Y_t), \Gamma_t)$ and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$. I omit $\tilde{\Gamma}$ because comparing the perception of the expenditure is not important here. The total expenditure $C_t(X_t(Y_t), \Gamma_t)$ can be decomposed into two parts. The core part of consumption $C_t(X_t(Y_t), \Gamma_t) - \Gamma_t$ is what the household attempts to smooth in the lifetime. The rest, Γ_t , is the expenditure shock. The key difference between the two is that, they have very different relationships with total consumption next period, $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$. Suppose that there is an increase in income Y_t or interest rate R_t^s . Then, the available wealth at this period X_t will increase and due to the globally concave continuation value, consumption and savings at period t will strictly increase as long as the household is not hand-to-mouth.

This leads to increase in C_{t+1} as well since X_{t+1} increase. When there is an increase in the expenditure shock Γ_t , then the total expenditure C_t in this period will increase. However, the MPC cannot exceed one, so $C_t(X_t(Y_t), \Gamma_t) - \Gamma_t$ will decrease. Simultaneously, because of decrease in the available wealth next period, consumption next period will decrease in expectation. Hence, in any case, $C_t(X_t(Y_t), \Gamma_t) - \Gamma_t$ and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$ (and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1}) - \Gamma_{t+1}$) will exhibit a positive correlation. However, the same mechanism will lead to a negative correlation between Γ_t and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$ (and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1}) - \Gamma_{t+1}$). The next proposition formally proves the intuition given above.

Proposition 3. Fix S_{t-1} . Then $\text{cov}(C_t - \Gamma_t, C_{t+1} - \Gamma_{t+1} | S_{t-1}) \geq \text{cov}(C_t, C_{t+1} | S_{t-1})$.

The result of 3 suggests a method of separating the endogenous part of consumption $C_t - \Gamma_t$ from the expenditure shock, Γ_t . The categories of expenditure that exhibit significant serial correlation are likely to be smoothed over the life cycle. In contrast, the nonpersistent part of the expenditure would be motivated by short-run fluctuations in taste or urgent spending needs.

4 Calibration

In this section, we calibrate key elements of the model using data. First, we check model properties after calibrating the basic parameters, income process, and expenditure shocks. Second, we check the model's performance after calibrating the distribution of perception to match the dispersion of liquid wealth.

Income Process Following Carroll (1997), the income process follows

$$Y_{i,a} = P_{i,a} \Xi_a \varepsilon_{i,a} \quad (3)$$

where

$$P_{i,a} = P_{i,a-1} \xi_{i,a}, \text{ and}$$

$$\Xi_a = \beta_0 + \beta_1 a + \beta_2 a^2 + \beta_3 a^3 + \beta_4 a^4.$$

The income of individual i at age a depends on the permanent level of income $P_{i,a}$, age-specific term Ξ_a —which shapes the overall income profile—and transitory shock $\varepsilon_{i,a}$. Shocks in permanent and transitory income, $\ln \varepsilon_{i,a}$ and $\ln \xi_{i,a}$, follows normal distribution with standard deviations 0.013 and 0.043 respectively, following Carroll et al. (2017)¹⁴. The age-specific

¹⁴This paper has monthly time frequency, so the standard deviations given in Carroll et al. (2017), which uses quarterly frequency, are divided by three.

term Ξ_a follows fourth-order polynomials of age, which is also standard. This paper estimates parameters determining Ξ_a by using the Consumer Expenditure Survey from 1997 to 2013. The choice of modeling permanent income as a geometric random walk enables the normalization of every variable by the permanent level of income, which reduces the computational complexity.

The model of this paper lacks a few realistic features. First, I do not have unemployment in the model. Since occupational characteristics do not explain the dispersion of liquid wealth, I build a model with a single type of household that always earns labor income. This abstraction ensures that the dispersion of liquid wealth is not a result of the additional dispersion of income from unemployment. Second, there are no income or social security taxes. Introducing taxes can be useful to conduct realistic policy experiments, but this model has single representative households with homogeneous income in expectation. I assume that every household has similar trajectory of expected income over the life cycle, which controls the contribution of income dispersion towards the dispersion of liquid wealth.

Calibration of the Expenditure Shock Separating the products in the household consumption basket into persistent and nonpersistent parts, we can find a stark difference between the two. For each product j , I measure the degree of persistence of expenditure using the following simple panel regression:

$$\begin{aligned}\Delta C_{i,j,t} &= \rho_j \Delta C_{i,j,t-1} + \varepsilon_{i,j,t} + \text{Year FE}_t + \text{Month FE}_t + \varepsilon_{i,j,t}, \text{ where} \\ \Delta C_{i,j,t-1} &= \gamma_j C_{i,j,t-2} + \text{Year FE}_{t-1} + \text{Month FE}_{t-1} + \xi_{i,j,t-1}.\end{aligned}\tag{4}$$

To deal with endogeneity caused by adding lagged dependent variable, $C_{i,j,t-1}$, I employ the Anderson and Hsiao (1981) estimator by using a lag of level variable, $C_{i,j,t-2}$, as an instrument. I use a monthly panel since quarterly data only allows four data points per household, which is not long enough to estimate (4). Moreover, monthly frequency matches well with the time-frequency used in the model. Year FE_{*t*} and Month FE_{*t*} stand for year and month fixed effects, respectively. In this simple regression, ρ_j captures the persistence of $C_{i,j,t}$. The first difference cancels out individual fixed effects. I use the Consumer Expenditure Survey from 1997 to 2013 using 44 product classifications by Kueng (2015) to estimate the persistence based on the model above.¹⁵¹⁶

¹⁵We can use a more detailed product classification, but if the category of products is too narrow, there can be redundancy when estimating the persistence. For example, I use utility payments as a category of consumption instead of treating gas and electricity separately, which might share similar characteristics. Using a broad category of products can also help us to reduce the size of the analysis.

¹⁶I use the sample only from 1997-2013, where the classification can be applied.

The following households are excluded from the analysis, consistent with the sample selection criteria used in the literature. First, I exclude households living in college housing. Second, households with top-coded income, missing age and family size, and not having full 12-month survey responses, are excluded. Third, I select households between the ages of 25 and 81 years to be consistent with the model. Fourth, households must not have any variation in the family size to isolate the effects from the change in family composition. Fifth, I exclude the households with zero food expenditure from the sample.

Table 9 shows the estimated ρ_j in (4) for 44 products. Twenty-one products exhibit positive and significant persistence. In particular, alcohol, tobacco, food, and personal care expenditures are highly persistent. The remaining 23 products do not exhibit strong persistence, such as education service, home management, insurance, and vehicle-related payments. First, denote the set of products that are persistent as S_p and nonpersistent as S_n . Then, we construct variables $C_{i,t}^p = \sum_{j \in S_p} C_{i,j,t}$ and $C_{i,t}^n = \sum_{j \in S_n} C_{i,j,t}$, which aggregates expenditures over persistent and nonpersistent groups. Moreover, we define the total expenditure of household i at time t as $C_{i,t}$.

The persistence of aggregate variables Next, we check the distinctive properties of $C_{i,t}^p$ and $C_{i,t}^n$. Following Proposition 3, I constructed $C_{i,t}^p$ to include products that have a high serial correlation. However, this does not guarantee that $C_{i,t}^p$ as a whole would also have higher persistence than the total expenditure $C_{i,t}$. Using the regression in (4), I measure the persistence of aggregate variables $C_{i,t}^p$ and $C_{i,t}^n$ and their estimated persistence are 0.041 and 0.008, respectively. We can conclude that the high persistence of individual products are well transferred to the aggregate variable.

Another manner of observing the persistence is to examine the relationship between the total expenditure, and its components $C_{i,t}^p$ and $C_{i,t}^n$ by the following regression:

$$\Delta C_{i,t} = \alpha \Delta C_{i,t-1}^p + \beta \Delta C_{i,t-1}^n + \text{Year FE}_t + \text{Month FE}_t + \varepsilon_{i,t}, \text{ where} \quad (5)$$

$$\begin{pmatrix} \Delta C_{i,t-1}^p \\ \Delta C_{i,t-1}^n \end{pmatrix} = (\eta^p, \eta^n) \begin{pmatrix} C_{i,t-2}^p \\ C_{i,t-2}^n \end{pmatrix} + \text{Year FE}_t + \text{Month FE}_t + \xi_{i,t}. \quad (6)$$

The estimated α and β are 0.033 and 0.006, respectively. Hence, an increase in the non-persistent part leads to almost no change in the total expenditure next period.¹⁷ When changing the dependent variable in (5) to $C_{i,t-1}^p$, the estimated α and β becomes 0.046 and -0.001, respectively. Although $C_{i,t}^n$ represents items that are individually nonpersistent, they are also largely unrelated to the total expenditure $C_{i,t}$ next period. In this sense, I model

¹⁷Similar results are found when changing the lag of instruments variables in (6) to 3 and 4 instead of 2.

$C_{i,t}^n$ as an exogenous part of the expenditure, which does not exhibit any autocorrelation and has a weak relationship with the total expenditure over time.

Variances of Persistent and Nonpersistent Expenditures If a product is a part of the consumption and savings problem in a long horizon, the consumption growth rate will be steady. In particular, in a short period, there will be small variations to the expenditure if income is the major source of the random shocks. Note that $C_{i,t}^n$ had nonpersistent components of the expenditure, but this does not imply that those components will also have high variance.¹⁸ I show that $C_{i,t}^n$ actually has a larger variance than $C_{i,t}^p$, even though its size is much smaller, which indicates that $C_{i,t}^n$ as a whole violates consumption smoothing.

For each household i , over 12 months, I calculate the standard deviation of $C_{i,t}^n$ and $C_{i,t}^p$. In other words, I calculate $\text{sd}(C_{i,t}^m) = \sqrt{\sum_{t=1}^{12} (C_{i,t}^m - \bar{C}_i^m)^2 / 12}$ where $\bar{C}_i^m = \sum_{t=1}^{12} C_{i,t}^m / 12$ for $m \in \{n, p\}$. The averages of $\text{sd}(C_{i,t}^n)$ and $\text{sd}(C_{i,t}^p)$ over all households are 1,271 and 570, respectively. Although the nonpersistent part constitutes approximately a quarter of the total expenditure, the standard deviation is twice as large as the persistent part. Moreover, the averages of $\text{sd}(C_{i,t}^n) / \bar{C}_i^n$ and $\text{sd}(C_{i,t}^p) / \bar{C}_i^p$ for overall households, which adjusts magnitudes of expenditures, are 1.56 and 0.29, respectively. When adjusted for the size, it is evident that the standard deviation of the nonpersistent part is approximately five times larger than the persistent part.

Distributions of Persistent and Nonpersistent Expenditures There is a stark difference between distributions of persistent and nonpersistent expenditures. Figure 3 depicts the distribution of the nonpersistent and persistent expenditures. First, the distribution of nonpersistent expenditure $C_{i,t}^n$ has a mode near zero, and the frequency decreases for higher expenditure levels. Moreover, it is heavily skewed to the right, and resembles the exponential distribution. On the other hand, the persistent expenditure $C_{i,t}^p$ is centered around a positive value and has a shape similar to the log-normal distribution. Thus, almost always, the persistent part is consumed strictly positive values.

Two different distributions represent the fundamental difference between persistent and nonpersistent expenditures. The persistent expenditure is a crucial part of the consumption and savings problem, and the consumer must spend a certain quantity. However, households do not spend any of the nonpersistent components in many cases, which cannot be rationalized with a utility function that satisfies the Inada condition. To make the nonpersistent

¹⁸Suppose that a random variable x_t follows $x_t = \rho x_{t-1} + e_t$. Other things being equal, while keeping the variance of e_t constant, an increase in $\rho \geq 0$ will make the unconditional variance of x_t increase as well. This property implies that the method of creating $C_{i,t}^n$ by choosing persistent component is not necessarily related to its low variance.

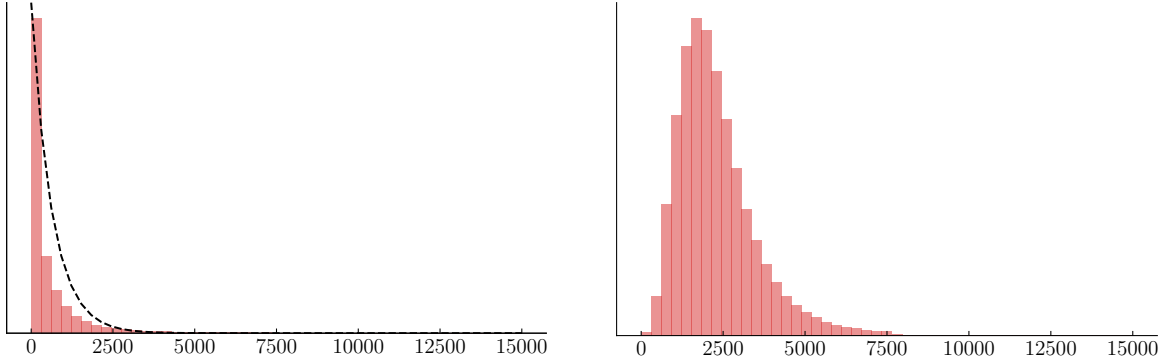


Figure 3: Distribution of the nonpersistent (left) and the persistent (right) expenditures
Note: The black dotted line on the left-hand side figure fits the probability density function of exponential distribution.

component an endogenous part of the consumption and savings problem, we need a large preference shock and a utility function that violates the Inada condition.

Feeding the Expenditure Shocks to the Model I calibrate the expenditure shocks in following steps. First, I estimate the individual persistent income P_i using the model in (3).¹⁹ Then, I make a normalized expenditure shock, by $c_{i,t}^n = C_{i,t}^n / P_i$. Based on the observation from Figure 3, I assume that the expenditure shocks follow an exponential distribution where the mean represents all information about the distribution. Of course, other distributions can represent the expenditure shocks better, but I avoid the use of distributions that may require complicated combinations of parameters.

Households may have different levels of expenditure shocks over the life cycle. To account for this, I estimate the trend of $\mu_a = \sum_{i=1}^{I(a)} c_{i,a}^n / I(a)$, where $I(a)$ is the number of households at age a . Specifically, I use the fourth-order polynomial of ages to filter the trend of μ_a . However, as evident from Figure 4, it is still relatively flat compared to income. In addition, the size of the expenditure shock is small, approximately a fifth of the income, which implies that arbitrarily large expenditure shocks do not drive the results of this paper.

Endowing the Degree of Perception The household i is endowed with λ_i , representing the degree of perception of the expenditure shock. While the true distribution that draws the expenditure shock at age a is $\exp(\mu_a)$, the household i perceives that the expenditure shock will be drawn from $\exp(\lambda_i \mu_a)$. Hence, $\lambda_i = 1$ implies that the household i is sophisticated.

¹⁹The Consumer Expenditure Survey asks for the income twice at most. For each household i , I use the mean income Y_i over two different periods, which can help to isolate the transitory component. Then, I measure the persistent component of household i as $P_i = Y_i / \Xi_{i,a}$ based on the age a .

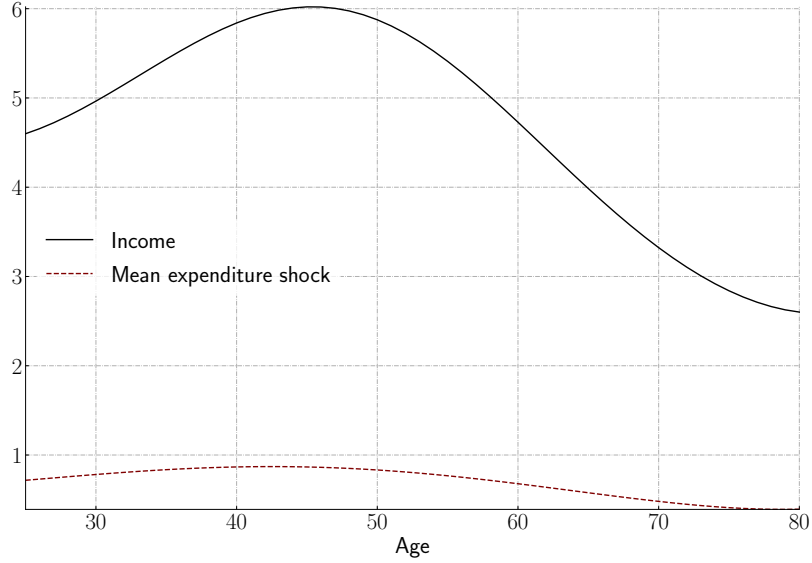


Figure 4: Trend of Income and Calibrated Expenditure Shocks

Note: The unit of the y -axis is \$1,000 dollars.

If $\lambda < 1$ ($\lambda > 1$), then the household is optimistic (pessimistic) relative to the sophisticated households.

Why Households May Persistently Underestimate Expenditure Shocks Suppose that a household fits the nonpersistent part distribution based on the exponential distribution with past observations. Then, the efficient maximum likelihood estimator (MLE) for the parameter of exponential distribution would be the mean of the past observations. In a distribution, like an exponential distribution²⁰, the median is always less than the mean, which implies that the median household will observe a value that is less than the true mean. Thus, the MLE estimate based on the limited observations will be underestimating the true parameter.²¹

Discount Factor and Interest Rates The discount factor δ is 0.96 in annual terms, which is standard. The returns of liquid and illiquid assets are held constant. Following Kaplan and Violante (2014), the real returns on liquid and illiquid assets are -1.5 percent

²⁰The result generalizes to the case where $f(c)$ is decreasing in c where $f(\cdot)$ is the probability density function.

²¹With exponential distribution, the result can be generalized in the following manner. Fix n , and $\bar{x} = (x_1 + \dots + x_n)/n$ where x_1, \dots, x_n are all drawn from the identical exponential distribution $\text{Exp}(\lambda)$ with parameter λ . Then, the median of \bar{x} is less than λ . In the limit, $n \rightarrow \infty$, \bar{x} tends to λ by the central limit theorem.

and 2.3 percent, respectively, in annual terms.

The Choice of IES I use the constant relative risk aversion utility function, $u(c) = c^{1-\sigma}/(1-\sigma)$. One of the most important parameters when modeling the life cycle consumption and savings problem is the IES, which is $1/\sigma$ in this paper. Empirically, when examining micro evidence, IES is suggested to be $1/3$ (Havranek and Sokolova, 2020), and I follow this recommendation.

Other Features of the Model Households enter the economy at age 25 and retire at age 65. All households die at age 80, with no bequest. Adding bequest can help explain the increasing trend of wealth even after retirement, but I attempt to keep the model as simple as possible to rule out other factors that can affect dispersion of liquid wealth. The model has a monthly frequency, which results in 660 periods, with 480 periods before retirement. To make the model solvable, I transform the utility function as $u(c) = (\max\{c, \underline{u}\})^{1-\sigma}/(1-\sigma)$, where $\underline{u} = 10^{-5}$ (1 cent).

Calculation of the MPC This paper calculates the MPC by giving \$800 that are not subsequently taxed to all households. For household i at time t , I first compute the original level of consumption $c_t^i(w_t^i)$. Fixing the level of disposable income and illiquid assets, to create the same environment, I compute the counterfactual level of consumption using the rebate r , which is $\hat{c}_t^i(w_t^i + r)$. Then, the MPC of household i at time t can be calculated as $MPC_t^i = (\hat{c}_t^i(w_t^i + r) - c_t^i(w_t^i))/r$. This method of calculating MPC is similar to that of Carroll et al. (2017), Aguiar et al. (2020), and Fuster et al. (2020) who measure the pure increase in consumption by winning a lottery, which is effectively calculating the slope of the consumption function. The size of the stimulus is larger than that in some of previous literature such as Kaplan and Violante (2014), Aguiar et al. (2020), and Fuster et al. (2020), who give \$500 to households. The choice of giving \$800 is to give households a quantity approximately between \$500 (2001 tax rebate) and \$1,200 (the individual payments in CARES Act in one of the stimulus and relief packages for COVID-19). Choosing a lower quantity of rebate will induce higher consumption responses due to the concavity of the consumption function Carroll and Kimball (1996), and the existence of the liquidity constraint.

An alternative method to measure a consumption response that resembles the actual fiscal stimulus can be embedding the belief that the stimulus has to be funded by taxes in the future. After the government spreads the rebate, it can collect taxes k periods later, and households that are not liquidity constrained for $k - 1$ periods will not react to the stimulus by Ricardian equivalence, as shown in Barro (1974). The policy experiment of Kaplan and

Violante (2014) reproducing the 2001 tax rebate is an example of measuring consumption response in this manner where taxes were collected 10 years after the rebate was given. This paper abstracts from realistic components of tax rebates and does not specifically aim to replicate a fiscal stimulus. In this paper, the measurement of the MPC focuses on purely representing the slope of the consumption function.

Model Properties To investigate the consumption and savings over the life cycle under different perceptions of expenditure shocks, I simulate the model economy with 256,000 households with λ_i that is evenly distributed over $[0, 2]$. Every household begins with the same initial level of wealth, which is the median liquid and illiquid wealth of household heads aged 24–26 of the Survey of Consumer Finances.

Figure 5 tracks the median wealth, consumption, and MPC within every 20 groups of perceptions $[\lambda^j, \lambda^j + 0.1)$ where $\lambda^0 = 0$ and $\lambda^{20} = 1.9$. Then, I track the median wealth of households within each quintile of perceptions. As hinted by Propositions 1 and 2, Figure 5 indicates that a household with a higher level of λ_i accumulates a larger quantity of liquid and illiquid assets throughout their life. In particular at the age of 65, the wealth of the top group with $\lambda \in [1.9, 2.0)$ is approximately six times larger than the lowest group with $\lambda \in [0, 0.1)$.

Unlike liquid and illiquid wealth, different degrees of perception do not make remarkable differences in consumption except in the initial and terminal periods. Households with correct perception exhibit a flat consumption profile until they reach retirement, which implies successful consumption smoothing under the low IES of $1/3$. Since they are worried about the possibility of high expenditure shocks, they retain liquid assets for precautionary motives, which makes the consumption rise near the terminal period. The precautionary saving motive is higher among relatively more pessimistic households. Hence, households with an extreme level of pessimism near $\lambda_i \simeq 2$ perceive that their initial liquid wealth is not enough and begin accumulating liquid assets right away, thereby making the initial consumption lower than that of the other groups. Optimistic households underestimate the size of the expenditure shock, which depletes liquid assets and high consumption at the beginning of life.

There are also large differences in the MPC over the life cycle by different levels of perceptions. First, the optimistic households exhibit higher MPC than the pessimistic households. Since optimistic households occasionally meet binding borrowing constraints, their MPC is also larger than that of pessimistic households. The average monthly MPC of the most optimistic households, $\lambda_i \in [0, 0.1)$, is 22.4, while the middle group, $\lambda_i \in [0.9, 1.0)$, exhibits 2.2 percent and the most pessimistic group, $\lambda_i \in [1.9, 2.0)$ exhibits 1.4 percent. Since the low liquidity drives the high MPCs, there will be negligible differences among households

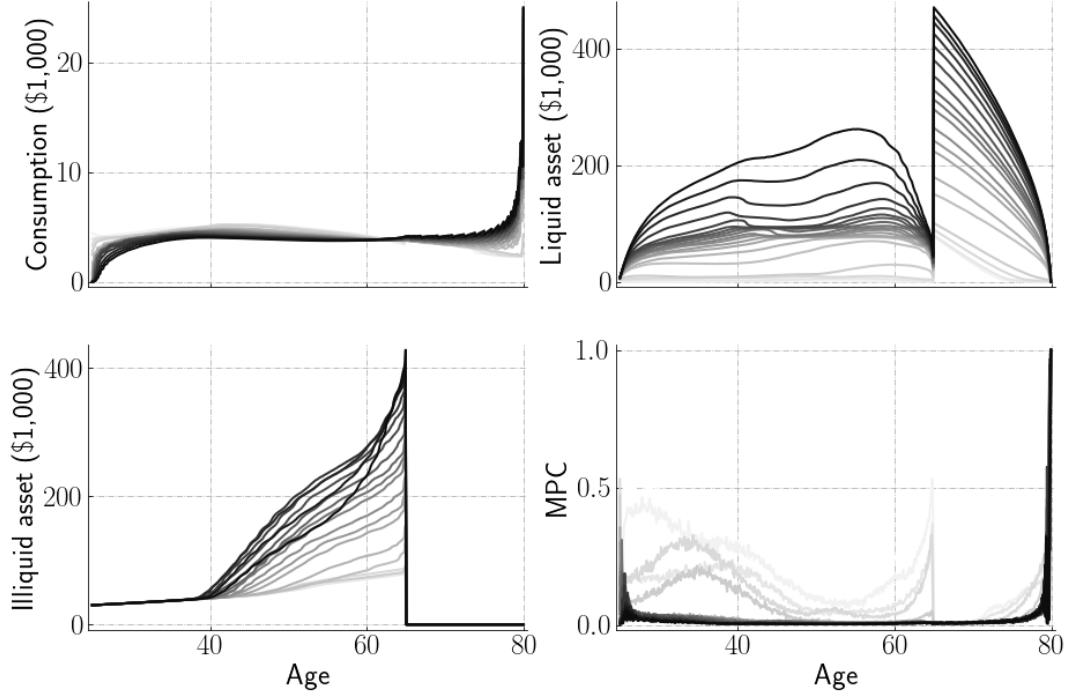


Figure 5: Trajectory of key variables by different levels of perception

Note: Twenty groups of households with perceptions $[\lambda^j, \lambda^j + 0.1)$, where $\lambda^1 = 0$ and $\lambda^{20} = 1.9$ are plotted and the darker line indicates more pessimistic households where λ^j is larger.

that accumulate sufficient wealth so that their borrowing constraint will be rarely binding. By this reason, households with perceptions $\lambda_i \in [0, 0.5)$ show high MPCs, while the others do not.

The trajectory of the MPC out of a transitory income shock follows the implication from the buffer-stock theory of savings (Carroll, 1992, 1997). The MPC is generally high at young ages since the current income is lower than the future income levels, which makes the borrowing constraint binding. As households accumulate assets at the middle age, MPC falls and becomes almost zero when the income is at its peak. However, consumption responses of optimistic households versus rational expectations or pessimistic households differ starkly close to retirement. The quantity of liquidity they have accumulated thus far sets the consumption limit for all households at retirement. However, optimistic households that underestimate the need for precautionary savings may face larger expenditure shock than they expected and become hand-to-mouth. On the other hand, households that have accumulated sufficient assets do not exhibit a large consumption response from a transitory

income shock. The mismatch between expected and actual liquidity available at retirement serves as a “second terminal-period effect” and makes MPC rise as households retire.

Calibration Strategy and Empirical Target The key moment to match in the calibration is the distribution of wealth. This paper adopts the strategy of Castañeda et al. (2003); Carroll et al. (2017) by targeting the share of liquid wealth held by 20th, 40th, 60th, 80th percentiles along with median liquid wealth among working-age households. There can be various reasons to hold illiquid assets, such as bequest motive and housing investment, which are not modeled in this paper. Moreover, the dispersion of liquid wealth in the model can be affected by the introduction of large illiquid wealth at retirement. By this reasons, I only focus on the dispersion of liquid wealth before retirement. By arbitrarily introducing numerous optimistic households, the model has a chance to highlight households with very low liquidity and perfectly explain the share of wealth held at different percentiles. To prevent this issue, I let the model match the median liquid wealth over income, similar to Carroll et al. (2017).

There have been various approaches to model the heterogeneity of consumers with different parameters. A commonly used approach is assuming a discrete number of households with different utility or discounting parameters such as Krusell and Smith (1998). There is also an approach by Carroll et al. (2017) that assumes uniform distribution and computes the lower and upper bounds of the parameter. In reality, the parameter of interest is likely to be drawn from a continuous distribution, and we cannot *a priori* guess the shape of the distribution. To employ a continuous and versatile distribution, I use a beta distribution that can have various shapes, nesting uniform distribution as a special case. Hence, the perception of households λ_i is assumed to be drawn from a beta distribution. I fit three parameters where each has a distinct role. The lefthand side panel of Figure 6 illustrates the role of first two parameters, α and β . They control the shape of the beta distribution. The theoretical mean is $\alpha/(\alpha + \beta)$; hence, a larger value of α implies more households with rational expectations in general. The size of both α and β controls the height of the peak. If $\alpha = \beta = 1$, then the distribution becomes a uniform distribution where the probability density function is flat. If α and β are high, the values drawn will be centered around a mean. To control the overall size of the values in the distribution, I also fit a scaling parameter γ representing the maximum value in the domain.

Calibration Result The set of parameters to generate the moments in Table 4 is $(\alpha, \beta, \gamma) = (1.53, 6.22, 1.58)$. Calibrated parameters imply that the distribution of perception would look like what is depicted in the righthand side of Figure 6. Based on this result, the average

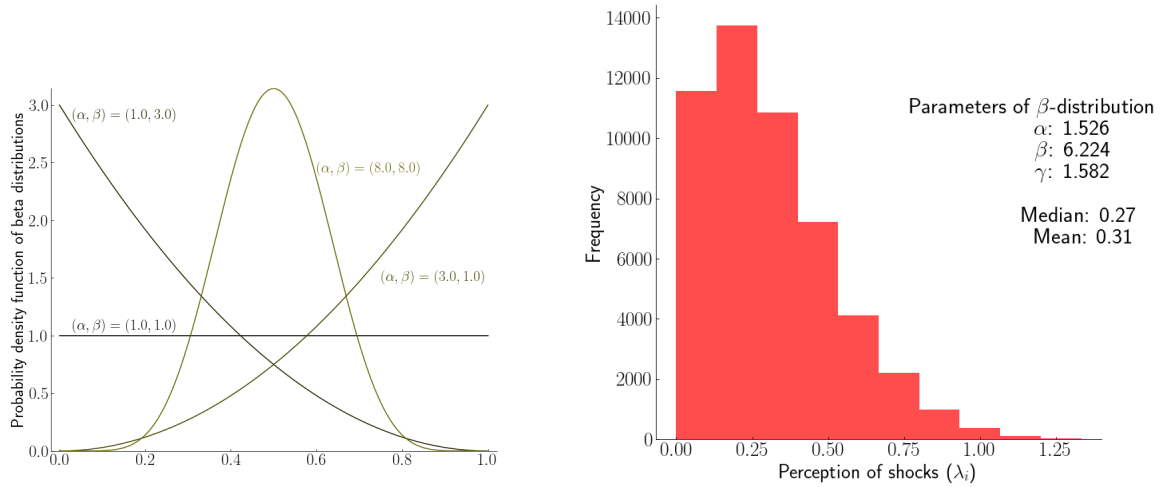


Figure 6: Examples of beta distribution (left) and the calibrated distribution (right)

	Share of wealth held at percentiles				Median liquid wealth over income
	20th	40th	60th	80th	
Data	<0.01	0.01	0.05	0.2	0.89
Model	<0.01	0.02	0.08	0.23	0.89

Table 4: Calibration result

household would realize 27 percent of the true mean of the expenditure shocks. This low level of perception is necessary to make the model generate a low level of liquid wealth among working-age households. The maximum value of the distribution $\gamma = 1.58$ implies the existence of a small group of households that are more pessimistic than households with rational expectations.

Comparison of key moments between the model and data Table 4 compares the moments of actual data versus simulated data from the calibrated model. The model-generated moments are fairly close to the data. In particular, the median liquid wealth over income is well aligned with the data. Controlling the median liquid wealth ensures that the calibration result is not driven by arbitrarily generating many households with very low liquidity. However, there is some gap between the share of wealth at the 60th and 80th percentile. Unlike the simulated model, households with an exceptionally high level of wealth near the top level of wealth exist, which is difficult to generate using the life-cycle model. As a result, Figure 7 displays that the model-generated Lorenz curve is slightly flatter than the data.

Table 5 compares the trend of liquid wealth over income at different percentiles of data

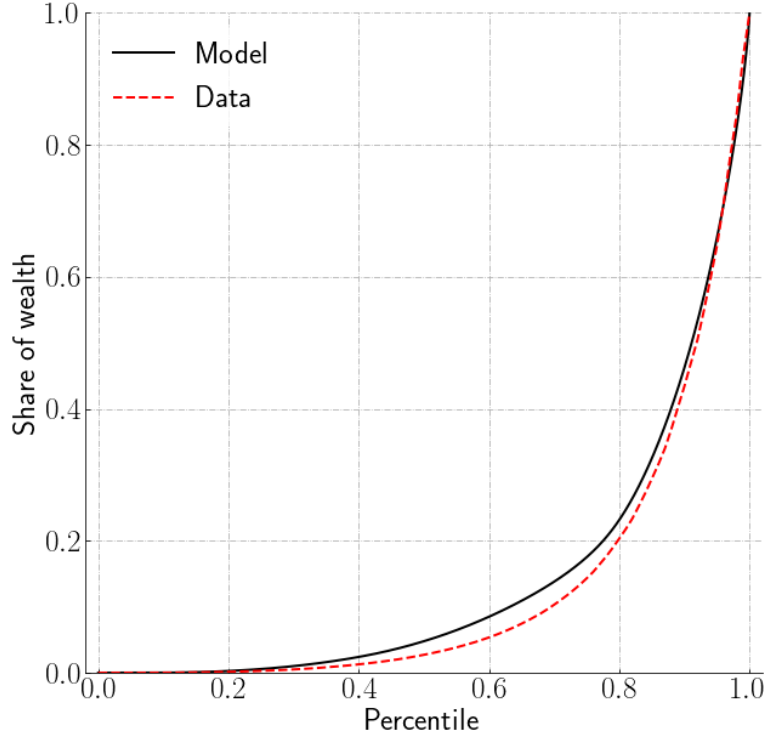


Figure 7: Lorenz Curve: Data versus Model

and the simulated model. The numbers in the table are not the targets of the calibration, but the simulated moments line up well with the actual data. In particular, it can explain that over 40 percent of the households in data do not have liquidity near their monthly income.

MPC in the calibrated model The model exhibits high overall MPC. The average monthly MPC among all households is 15.9 percent. In quarterly and annual terms, it can be translated to 40.5 and 87.4 percent following the conversion formula used by Carroll et al. (2017).²²

It is evident from the lefthand side panel of Figure 8 that extensive margin plays a big role since most of the households do not show a positive consumption response to the rebate. However, households with large expenditure shocks would move to the region with a positive consumption response. This will be more effective among households with low levels of liquid

²²The exact values for quarterly and annual MPC need to be examined by actually tracking the change in consumption over the quarter and year. I convert the monthly MPC of x to the MPC over k months, by $1 - (1 - x)^k$.

		Liquid wealth over income			
		20th	40th	60th	80th
25–34	Data	0.12	0.44	1.06	2.67
	Model	0.02	0.35	1.14	2.6
35–44	Data	0.15	0.51	1.18	3.14
	Model	0.14	0.36	1.17	2.6
45–54	Data	0.20	0.61	1.48	4.29
	Model	0.32	0.67	1.38	4.35
55–64	Data	0.28	0.86	2.31	7.23
	Model	0.16	0.57	1.48	8.03
Overall	Data	0.21	0.68	1.74	5.40
	Model	0.14	0.48	1.26	4.22

Table 5: Liquid wealth over income at 20th, 40th, 60th, and 80th percentiles
Note: Numbers in parenthesis are from the simulated model, and others are from the data

wealth. The righthand side panel suggests extensive margin in the individual consumption response. When a household faces a large expenditure shock, then they may exhibit a high MPC temporarily. However, in most cases where they face negligible expenditure shocks, they exhibit a low MPC to cope with the desire to smooth consumption.

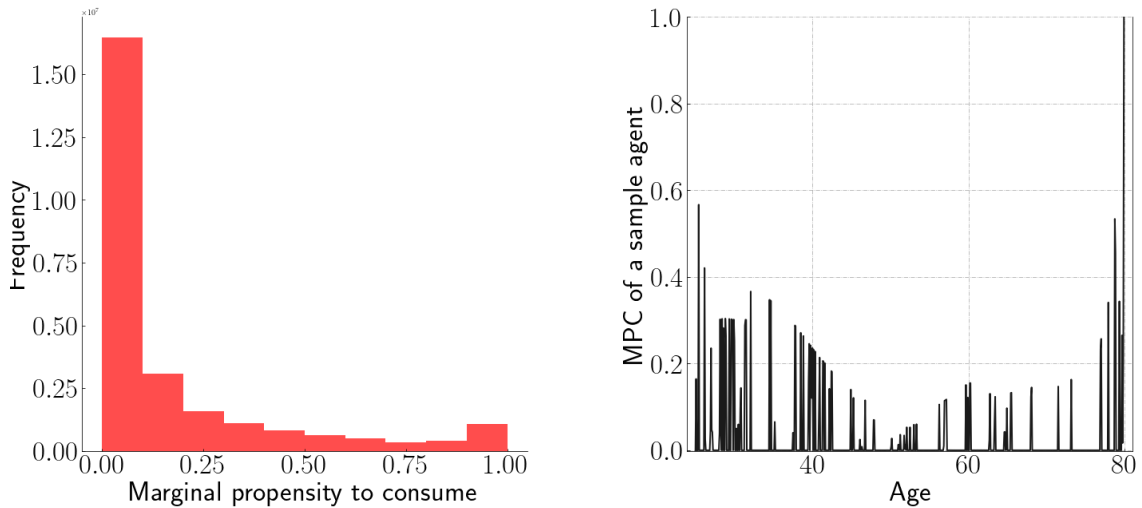


Figure 8: Distribution of MPC (left) and sample individual consumption response (right)

The righthand side panel of Figure 8 shows the individual response of a sample household that perceives 50 percent of the actual shock. Not only is there an extensive margin of consumption responses in the aggregate, but depending on the size of the expenditure shocks, each individual also exhibits infrequent but large consumption responses.

	MPC under different IES					
	1/3	0.5	1/1.5	1	1.5	2
No expenditure shocks with rational expectations						
One-asset	2.1	2.5	2.6	2.6	2.4	2.1
Two-asset	3.1	5.6	7.8	10.7	15.0	19.8
With expenditure shocks						
Two-asset						
Rational expctations	2.7	2.6	3.1	7.8	8.9	9.9
Fully optimistic	25.2	30.5	33.5	37.2	40.7	43.2
One-asset						
Rational expctations	2.0	2.1	2.1	2.3	2.2	1.9
Fully optimistic	17.7	20.7	21.7	22.0	21.1	19.1

Table 6: Comparison of MPCs across different models

Note: IES of 1/3 is the baseline case with annual discount factor of 0.96. The other cases with IES of α uses discount factor of $(0.96 \times R)^{\alpha/3}/R$ in annual terms. $R = 1.017$, which is the annual gross interest rate of the one-asset model in Kaplan and Violante (2014).

MPCs in other benchmark models Table 6 compares the MPCs across different modeling strategies and various values of IES. Following the calibration strategy thus far, I assume that $\delta = 0.96$ when IES is 1/3. For all other values of IES, I convert the annual discount factor that makes the consumption choice approximately equal over different choices of IES.²³ The reason for applying this conversion is to compare MPCs across different values of IES while maintaining the models to generate similar levels of wealth as in Kaplan and Violante (2014). An alternative approach would be recalibrating the discount factor each time for different values of IES, and different types of models, as in Kaplan and Violante (2014).²⁴

A clear pattern is that, generally, low IES leads to lower MPC. Under the baseline case with IES of 1/3, the MPC without expenditure shock under the one-asset framework and rational expectations is only 2.1 percent. Under the two-asset model under rational expectations, the MPC becomes 3.1 percent, which is larger than the one-asset case, but the difference is negligible. The separation of liquid and illiquid assets does not have a dramatic effect on the MPC. This result is because the desire to smooth consumption is strong when using the small IES, and to maintain a flat consumption profile, as shown in Figure 5, households must accumulate both liquid and illiquid assets. In this case, the borrowing

²³For two different pairs of (δ_1, σ_1) and (δ_2, σ_2) and common interest rate R , the consumption choice will be the same if $(\delta_1 R)^{1/\sigma_1} = (\delta_2 R)^{1/\sigma_2}$. Hence, to make the comparison between (δ_1, σ_1) and (δ_2, σ_2) fair, we can convert the δ_2 to satisfy $(\delta_1 R)^{\sigma_2/\sigma_1}/R$. In a stochastic model, this is not exact, and one needs to simulate the model by matching a particular statistic. However, the approximation appears to be accurate since MPCs of the one-asset model in Table 6 are stable across different values of IES.

²⁴I use the same discount factor across different types of models if the IES is the same. This is to compare the role of different assumptions, such as introducing two assets, introducing expenditure shocks, and changing expectations.

constraints will rarely be binding, even with the low-interest rate on liquid assets. Moreover, the choice of low IES makes households insensitive to interest rate differences in liquid and illiquid assets.

However, the use of high IES can lead to higher MPC. Even among households with rational expectations, the MPC under the two-asset framework is approximately 20 percent with IES of two. Although there is a difference in the modeling strategy where Kaplan and Violante (2014) uses symmetric adjustment cost, and I impose an asymmetric adjustment cost, the amplification of consumption responses by separating two different assets are present in both approaches, particularly when using the high IES.

Behavioral bias can let the model overcome having smaller consumption responses induced by small IES. Under the one-asset framework, fully optimistic households exhibit an MPC of 17.7 percent, significantly higher than the rational expectations benchmark. Using the two-asset model, there is an additional increase in the MPC, where MPC now becomes 25.2 percent. Even under the low IES, behavioral frictions lead to lower liquid assets among optimistic households. Separating the savings for retirement exacerbates the low accumulation of liquid assets, which serves as the only tool for precautionary saving.

We see a clear pattern between models with and without expenditure shocks when focusing only on households with rational expectations. In all cases, the models with expenditure shocks produce smaller MPCs than those without expenditure shocks for both one- and two-asset cases. This pattern can be attributed to the fact that households may require additional savings when adding an extra shock to the model for precautionary purposes. Analysis thus far hints that the presence of expenditure shock alone is not a factor that drives high MPCs. The behavioral bias, where households have a limited perception of expenditure shock, leads to high MPCs.

5 Policy Implications

In this section, we investigate the role of perception $\tilde{\Gamma}$ in consumer welfare, and provides policy recommendations.

Optimism and Welfare For simplicity, I consider the one-asset model which features life after retirement. I define the *welfare function* $W_t^*(X_t; \Gamma_t, \tilde{\Gamma})$ in a paternalistic view, which

captures expected lifetime utility based on perception $\tilde{\Gamma}$ as

$$W_t^*(X_t; \Gamma_t, \tilde{\Gamma}) = u(C_t(X_t; \Gamma_t, \tilde{\Gamma}) - \Gamma_t) + \delta \mathbb{E} \left[W_{t+1}^*(X_{t+1}; \Gamma_{t+1}, \tilde{\Gamma}) \right],$$

where

$$C_t(X_t; \Gamma_t, \tilde{\Gamma}) = \arg \max_{c_t} u(C_t - \Gamma_t) + \delta \mathbb{E}_t \left[\tilde{W}_{t+1}^{\tilde{\Gamma}}(X_{t+1}; \tilde{\Gamma}_{t+1}) \right], \text{ and}$$

$$C_t \leq X_t = Y_t + R_t^s(X_{t-1} - C_{t-1}).$$

Both actual and perceived distributions of the expenditure shock are important to determine the welfare. When calculating the current utility $u(C_t - \Gamma_t)$, Γ_t is drawn from the actual distribution of the expenditure shock. On the other hand, the household makes a decision based on $\tilde{\Gamma}$ to gauge continuation value $\tilde{W}_{t+1}^{\tilde{\Gamma}}$. An alternative and stronger means of defining the welfare would be using *ex-post* realizations of the expenditure shocks. However, there can be a possibility that the random draws of Γ_t can be close to the values drawn from $\tilde{\Gamma}_t$, and there is no guarantee that the households with correct perception will be better off. Defining the welfare in *ex-ante* perspective provides a chance to establish a monotonic relationship between the welfare function and the degree of perception, as indicated in the following proposition:

Proposition 4. For any $t \geq T_r$, if $\Gamma >_1 \tilde{\Gamma}^1 >_1 \tilde{\Gamma}^2$, then $W_t^*(X_t; \Gamma_t, \tilde{\Gamma}^1) \geq W_t^*(X_t; \Gamma_t, \tilde{\Gamma}^2)$. Also, $\Gamma <_1 \tilde{\Gamma}^1 <_1 \tilde{\Gamma}^2$, implies that $W_t^*(X_t; \Gamma_t, \tilde{\Gamma}^1) \geq W_t^*(X_t; \Gamma_t, \tilde{\Gamma}^2)$.

Proposition 4 demonstrates that if we have two different households comparable with their relative optimism and pessimism, the household that is closest to the sophisticated case, Γ , will have higher welfare.

The mechanism leading to this result is simple. An optimistic household will suffer welfare loss by the overconsumption now; when a large expenditure shock hits, the household will not have enough resources to spend money where the marginal utility will be greater than the current period. Since the loss of welfare accumulates over time, the fact that a household with the perception that is farther away from the reality is worse off holds for any period. A similar explanation applies to pessimistic households.

This result reveals that when choosing priorities of correction, the government should first target the households with abnormally low or high degrees of perception. For these households, the expected increase in welfare will be higher by government intervention. Then, it is important to design a policy based on the degree of perception elicited by the real data. In the following account, I suggest a policy to enhance the overall welfare by correcting the low liquid savings of optimistic households.

An Example of an Ex-Post Welfare Improving Policy What kind of policy can improve the welfare of households with behavioral frictions? This question can be important in reality since if the government can correct the behavior of households with simple policy tools, it can prevent households from facing high borrowing costs when they face large expenditure shocks.

To answer this question, I impose three criteria that a policy must satisfy. First, a policy should not depend on external funds and, preferably, the implementation should be possible without inter-personal transfers. Without this condition, there can be obvious welfare improving policy of spreading money to households. Moreover, if the government can make inter-personal transfers, it would be crucial to assign the utility weights to different households and discuss what type of households we should prioritize. Second, the policy instruments must follow observable variables. Hence, the government cannot design a policy conditional on the households' optimism and pessimism, which would also be impossible in the real world situation. Third, the government implements dynamically consistent policies, and the households should form correct beliefs about them. Without this condition, the degree of freedom for the government is large, and the government will eventually lose credibility.

I suggest the following policy to penalize low liquidity and giving households sufficient buffer to prevent expenditure from depleting their liquid wealth. The government imposes a proportional tax τ_l up to the liquid wealth of $W_{l,t}$ at time t . For wealth that exceeds $W_{l,t}$ and less than $W_{r,t}$, the government gives a tax rebate proportional to τ_r . Hence, the tax $T_{i,t}(X_{i,t})$ that households i with the level of liquid wealth $X_{i,t}$ pay at time t would be

$$T_{i,t}(X_{i,t}) = \min\{W_{l,t}, X_{i,t}\}\tau_l - \min\{\max\{X_{i,t} - W_{l,t}, 0\}, W_{r,t} - W_{l,t}\}\tau_r, \quad (7)$$

which is also the tax revenue for the government. In simple terms, the government penalizes households with wealth less than $W_{l,t}$ which induces optimistic households to accumulate liquid wealth, at least $W_{l,t}$. However, such policy can make households with the rational expectation to undersave and keep the level of wealth between $W_{l,t}$ and $W_{r,t}$, so there is no guarantee for overall welfare improvement. However, if we keep $W_{r,t}$ sufficiently small, households that already accumulate large liquid wealth will not be affected by this policy. Simultaneously, $W_{r,t}$ must be sufficiently large to make low liquidity households prepare for the underestimated expenditure shocks.

If $W_{r,t} = 2W_{l,t}$ and $t_l = t_r$, then all the transfers $(X_{i,t} - W_{l,t})t_r$ can be covered by the taxes $X_{i,t}t_l$ collected earlier, so this policy does not need any interpersonal transfers. Hence, if households correctly form beliefs regarding (7), then such a policy will satisfy all three

criteria mentioned above. I simulate the economy with the above setting under various t_l and $W_{l,t}$ based on the calibrated distribution of perception in Figure 6. I impose that $W_{l,t} = \bar{w}\Xi_t$, where Ξ_t is the age specific trend of income in (3). Rather than fixing $W_{l,t}$ and $W_{r,t}$, by making $W_{l,t}$ proportional to the trend of income, I can make the intended minimum savings $W_{l,t}$ vary with income so optimistic households will be naturally inclined to save a larger quantity of liquid wealth during the middle age when the income peaks.

$T_{l,t}/\Xi_t$	$T_{r,t}/\Xi_t$	τ_l	τ_r	Avg. utility	Avg. revenue
1	2	0.01	0.01	-191.6	0.06
		0.02	0.02	-191.7	0.07
		0.03	0.03	-192.1	0.1
		0.04	0.04	-192.9	0.14
2	4	0.01	0.01	-189.7	0.13
		0.02	0.02	-188.8	0.23
		0.03	0.03	-188.8	0.29
		0.04	0.04	-189.5	0.53
No policy benchmark				-191.8	0

Table 7: Policy Simulation Based on the Calibrated Model

Note: ‘Avg. utility’ refers to mean level of utility, which is defined as $\frac{1}{I \times T} \sum_{i \in I} \sum_{t \in T} u(C_{i,t} - \Gamma_{i,t})$, where I is the number of the simulated households and T is the total number of periods. ‘Avg. revenue’ refers to the average revenue that the government raises, which is defined as $\frac{1}{I \times T} \sum_{i \in I} \sum_{t \in T} T_{i,t}(X_{i,t})$. The unit of revenue is \$1,000. τ_l and τ_r in the table are annualized, actual value τ_l^{actual} used in the simulation is $\tau^{*,\text{actual}} = 1 - (1 + \tau^*)^{1/12}$ for $\tau^* = \{\tau_l, \tau_r\}$.

Table 7 presents the result of the calibration exercise. I selected four different tax (or transfer) rates, which are 1%, 2%, 3%, and 4% in annual terms. In some of the parameters, the average utility is higher than the benchmark case without any policy. In particular, when $T_{l,t}/\Xi_t = 2$ and $T_{r,t}/\Xi_t = 4$, the average utility is higher than the benchmark case. Hence, inducing households to save the liquid wealth of twice their income trend can be desirable. Simultaneously, the government can raise tax revenue by increasing the τ_l and τ_r as well.

Policy to Boost Economic Activity There can be cases where government would want to boost economic activity rather than enhance welfare when facing an economic recession. In this case, Propositions 1 and 2 reminds us that the households with the more optimistic beliefs will likely show greater consumption responses. As in the previous welfare improving policy, the quantity of liquid assets is a good proxy for the degree of perception in this model. To achieve a greater consumption response from the stimulus, the government must spread the rebates to low liquid wealth households. First, they will spend more due to the concavity of the consumption function Carroll and Kimball (1996). Second, this paper allows

an additional boost to the consumption response, as low liquidity households that are more likely to be optimistic will spend more by underestimating expenditure shocks.

In a real-world setting, the fact that fiscal stimulus has to be taxed later can limit the effectiveness of a fiscal policy. In a case where households realize the increase in future tax burden and are not hand-to-mouth, the fiscal policy would have no effect at all, which is the famous Ricardian equivalence theorem (Barro, 1974). This wisdom also applies to this model, and the strength of the fiscal policy will depend on the size of hand-to-mouth households. Moreover, in this realistic setting, the policy recommendation is the same, and the government must target households with low liquid wealth. The tax incidence will have a lesser effect on this group of households because households must be more likely to be optimistic will have a higher likelihood of retaining their hand-to-mouth status.

6 Conclusion

Using a model of heterogeneous agents with different perceptions of expenditure shocks, this paper generated high overall MPC by matching the severe dispersion of liquid wealth found in the data. The calibration reveals that most households have a low perception of future expenditure shock, which leads them to have a very low level of liquidity and exhibit high MPCs. I conclude by discussing several extensions of this paper.

First, the model in this paper assumed a fixed retirement date with no early withdrawal of illiquid assets. Alternatively, we can impose adjustment costs like Kaplan and Violante (2014). This paper had to rely on a fixed retirement date to build a two-asset model with clean theoretical predictions. However, introducing adjustment costs can allow real-world behaviors such as early withdrawal of retirement accounts and lumpy adjustments of illiquid assets.

Second, this paper depended on the parsimonious model of expenditure shocks, which was assumed to be exogenous. In reality, all the categories of consumption can depend on the level of income and wealth. This assumption can be relieved by modeling the expenditure system where households have an imperfect perception of preferences. However, to rationalize the high variance of the nonpersistent part and its stark difference distribution with the persistent part, it would be necessary to incorporate large and temporary preference shocks to make expenditure shocks endogenous.

Third, the learning of expenditure shocks was not allowed. This paper isolated the learning based on the calibrated distribution of the expenditure shocks. Since the distribution of the expenditure shock is extremely skewed, median households will underestimate the mean and mainly observe a negligible magnitude of shocks. Also, there are many categories

of consumption that would make it difficult to learn all the aspects of the expenditure shocks. Therefore, I limited the model's scope and the degree of freedom by focusing on the pure role of optimism and pessimism. However, some extreme events can shape the expectation of households, and such learning behaviors can also lead to an interesting source of heterogeneity and consumption dynamics.

A Proofs

Proof of Proposition 1

Proof. Write the cumulative density functions of $\tilde{\Gamma}_t^1$ and $\tilde{\Gamma}_t^2$ as $F_t^1 : \mathbb{R}_+ \rightarrow [0, 1]$ and $F_t^2 : \mathbb{R}_+ \rightarrow [0, 1]$, respectively. I prove the proposition using backward induction. In the final period, consumers spend all available wealth, hence $C_T(X_T, \Gamma_T; \tilde{\Gamma}) = X_T$ irrespective of what Γ_T and $\tilde{\Gamma}$ is. Since $F_t^1 >_1 F_t^2$, it follows that

$$\begin{aligned} \mathbb{E}_{T-1} \left[\frac{\partial W_T^{\tilde{\Gamma}^1}(X_T; \Gamma_T)}{\partial X_T} \right] &= \int u'(X_T - \Gamma_T) dF_T^1(\Gamma_T) > \int u'(X_T - \Gamma_T) dF_T^2(\Gamma_T) \\ &= \mathbb{E}_{T-1} \left[\frac{\partial W_T^{\tilde{\Gamma}^2}(X_T; \Gamma_T)}{\partial X_T} \right]. \end{aligned}$$

It is obvious that $\mathbb{E}_{T-1} [W_T^{\tilde{\Gamma}^1}(X_T; \Gamma_T)]$ is concave to X_T . Now, we proceed to general periods. Let the following properties [T1] and [T2] hold at period $t + 1$:

$$\begin{aligned} \text{[T1]} \quad 0 &< \mathbb{E}_t \left[\frac{\partial \tilde{W}_{t+1}^{\tilde{\Gamma}^2}(X_{t+1}; \tilde{\Gamma}_{t+1})}{\partial X_{t+1}} \right] < \mathbb{E}_t \left[\frac{\partial \tilde{W}_{t+1}^{\tilde{\Gamma}^1}(X_{t+1}; \tilde{\Gamma}_{t+1})}{\partial X_{t+1}} \right], \text{ and} \\ \text{[T2]} \quad \mathbb{E}_t \left[\frac{\partial^2 \tilde{W}_{t+1}^{\tilde{\Gamma}^2}(X_{t+1}; \tilde{\Gamma}_{t+1})}{\partial X_{t+1}^2} \right] &< 0 \text{ and } \mathbb{E}_t \left[\frac{\partial^2 \tilde{W}_{t+1}^{\tilde{\Gamma}^1}(X_{t+1}; \tilde{\Gamma}_{t+1})}{\partial X_{t+1}^2} \right] < 0. \end{aligned}$$

Then, I show that properties [T1] and [T2] also hold at period t ; moreover, $C_t(X_T; \Gamma_t, \tilde{\Gamma}^1) \leq C_t(X_T; \Gamma_t, \tilde{\Gamma}^2)$ where the inequality is strict when the consumer saves a positive amount.

For any $t \geq T_r$, fix a shock Γ_t . By [T1] and [T2], if the agent is hand-to-mouth under $\tilde{\Gamma}^1$, then the agent is also hand-to-mouth under $\tilde{\Gamma}^2$ since $u'(C_t - \Gamma_t) > \mathbb{E}_t \left[\frac{\partial \tilde{W}_{t+1}^{\tilde{\Gamma}^1}(X_{t+1}; \tilde{\Gamma}_{t+1})}{\partial X_{t+1}} \right] > \mathbb{E}_t \left[\frac{\partial \tilde{W}_{t+1}^{\tilde{\Gamma}^2}(X_{t+1}; \tilde{\Gamma}_{t+1})}{\partial X_{t+1}} \right]$ for any $C_t \in (0, X_t]$. Hence, in this case, $C_t(X_T; \Gamma_t, \tilde{\Gamma}^1) = C_t(X_T; \Gamma_t, \tilde{\Gamma}^2) = X_t$ by [T2]. If the agent saves under $\tilde{\Gamma}^1$, where $C_t(X_T; \Gamma_t, \tilde{\Gamma}^1) < X_t$, then

$$u'(C_t(X_T; \Gamma_t, \tilde{\Gamma}^1)) = \mathbb{E}_t \left[\frac{\partial \tilde{W}_{t+1}^{\tilde{\Gamma}^1}(w_{t+1}; \tilde{\Gamma}_{t+1})}{\partial w_{t+1}} \right] > \mathbb{E}_t \left[\frac{\partial \tilde{W}_{t+1}^{\tilde{\Gamma}^2}(w_{t+1}; \tilde{\Gamma}_{t+1})}{\partial w_{t+1}} \right]$$

by [T1]. This leads to $C_t(X_T; \Gamma_t, \tilde{\Gamma}^1) < C_t(X_T; \Gamma_t, \tilde{\Gamma}^2)$. Irrespective of whether the agent is hand-to-mouth under $\tilde{\Gamma}^1$, this leads to

$$\frac{\partial \tilde{W}_t^{\tilde{\Gamma}^1}(X_t; \tilde{\Gamma}_t)}{\partial X_t} = u'(C_t(X_T; \tilde{\Gamma}_t, \tilde{\Gamma}^1) - \tilde{\Gamma}_t) \geq u'(C_t(X_T; \tilde{\Gamma}_t, \tilde{\Gamma}^2) - \tilde{\Gamma}_t) = \frac{\partial \tilde{W}_t^{\tilde{\Gamma}^2}(X_t; \tilde{\Gamma}_t)}{\partial X_t}.$$

Using the fact that $F^1 >_1 F^2$, we can conclude that [T1] holds at time t . [T2] is obvious because consumption is an increasing function of wealth. \square

Proof of Proposition 3

Proof. When fixing S_{t-1} , the consumption function in terms of random variables Y_t , Y_{t+1} , Γ_t and Γ_{t+1} can be written as $C_t(X_t(Y_t), \Gamma_t)$ and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)$ for periods t and $t + 1$.

We need to show that $\text{cov}(\Gamma_t, C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)) < 0$. Define $\bar{\Gamma}_t = \mathbb{E}[\Gamma_t]$. Fix Y_t, Y_{t+1} , and Γ_{t+1} . Then,

$$[\Gamma_t - \bar{\Gamma}_t] [C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t) - C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \bar{\Gamma}_t)] \leq 0.$$

Taking expectation with respect to Γ_t conditional on Y_t, Y_{t+1} , and Γ_{t+1} yields,

$$\mathbb{E} [[\Gamma_t - \bar{\Gamma}_t] C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t) | Y_t, Y_{t+1}, \Gamma_{t+1}] \leq 0.$$

Now taking expectation over Y_t, Y_{t+1} , and Γ_{t+1} yields,

$$\mathbb{E} [\Gamma_t C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)] - \bar{\Gamma}_t \mathbb{E} [C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)] \leq 0.$$

This shows that $\text{cov}(C_t - \Gamma_t, C_{t+1}) \geq \text{cov}(C_t, C_{t+1})$. \square

Lemma 1. Suppose that $u(c)$ is strictly concave, and $F(s, a)$ is concave. For a convex set $B(\bar{x})$,

$$\begin{aligned} V(\bar{x}, \bar{A}) &= \max_{s, a} u(\bar{x} - s - a) + F(s, Z) \\ \text{s.t. } & s + a < \bar{x}, Z = \bar{A} + a, s \geq 0 \text{ and } a \geq 0. \end{aligned}$$

- only allows a unique solution for $c = \bar{x} - s - a$,
- value function $V(\bar{x}, \bar{A})$ is concave, and
- if $u(\cdot)$ and $F(s, a)$ are concave and differentiable, then $V(\bar{x})$ is also differentiable at \bar{x} and \bar{A} .

1. I first show the uniqueness of c . Suppose that another triplet c' , s' , and a' where $c \neq c'$ exists. Then, for a $\lambda \in (0, 1)$,

$$u(\lambda c + (1 - \lambda)c') + F(\lambda s + (1 - \lambda)s', \lambda Z + (1 - \lambda)Z') > u(c) + F(s, Z)$$

is a contradiction.

For concavity of V , let $S' = (s', a')$ and $c' = \bar{x}' - s' - a'$ denote the solution that maximizes the objective function, given \bar{x}' and \bar{A}' . Similarly, let $S'' = (s'', a'')$ and $c'' = \bar{x}'' - s'' - a''$ be the maximizers given \bar{x}'' and \bar{A}'' .

For any $\lambda \in [0, 1]$, $\lambda S' + (1 - \lambda)S''$ is feasible when given $\bar{x}^* = \lambda \bar{x}' + (1 - \lambda)\bar{x}''$ and $\bar{A}^* = \lambda \bar{A}' + (1 - \lambda)\bar{A}''$. Define $S^* = \lambda S' + (1 - \lambda)S''$ and $c^* = \lambda c' + (1 - \lambda)c''$. Then,

$$\begin{aligned} V(\bar{x}^*, \bar{A}^*) &\geq u(c^*) + F(s^*, Z^*) \\ &\geq \lambda [u(c') + F(s', Z')] + (1 - \lambda) [u(c'') + F(s'', Z'')] \\ &> \lambda V(\bar{x}', \bar{A}') + (1 - \lambda)V(\bar{x}'', \bar{A}''). \end{aligned}$$

For differentiability, Lemma Benveniste and Scheinkman (1979) show that if $W(x, A|s, a) = u(x - s - a) + F(s, a + A)$ is a concave function on a convex set B , then a concave function $V(x, A)$ where $V(x^*, A^*) = W(x^*, A^*)$ and $V(x, A) \geq W(x, A)$ for all other $(x, A) \in B$, then V is differentiable at (x^*, A^*) . \square

Proof of Proposition 2

Proof. I provide the proof using backward induction. In each step, I show that [T1] following two inequalities

$$\frac{\partial \tilde{V}_{t+1}^1(X_{t+1}, Z_{t+1})}{\partial X_{t+1}} > \frac{\partial \tilde{V}_{t+1}^2(X_{t+1}, Z_{t+1})}{\partial X_{t+1}}, \text{ and } \frac{\partial \tilde{V}_{t+1}^1(X_{t+1}, Z_{t+1})}{\partial Z_{t+1}} > \frac{\partial \tilde{V}_{t+1}^2(X_{t+1}, Z_{t+1})}{\partial Z_{t+1}}$$

hold, and [T2] $\tilde{V}_{t+1}^i(X_t, Z_t)$ is a concave function.

Starting from period $T_r - 1$, agent i solves

$$\max_{C, S, A} u(C - \Gamma_{T_r-1}) + \delta \mathbb{E}_{T_r-1} \left[\tilde{W}_{T_r}^i(R_{T_r}^S S + R_{T_r}^A (A + Z_{T_r-1})) \right] \quad (8)$$

$$\text{subject to } C + S + A \leq X_{T_r-1}, C \geq 0, S \geq 0, \text{ and } A \geq 0. \quad (9)$$

Trivially, $S = 0$ since it is an inferior asset compared to A with $R_{T_r}^A > R_{T_r}^S$. Hence, this is effectively a one-asset case. We can apply the same steps as those in Proposition 1 and claim that $C_{T_r-1}^2(X_{T_r-1}, Z_{T_r-1}) \geq C_{T_r-1}^1(X_{T_r-1}, Z_{T_r-1})$. Moreover,

$$\begin{aligned} \frac{\partial \tilde{V}_{T_r-1}^i}{\partial Z_{T_r-1}} &= u'(C_{T_r-1}^i - \tilde{\Gamma}_t^i) \frac{\partial C_{T_r-1}^i}{\partial Z_{T_r-1}} + \delta \mathbb{E}_{T_r-1} \left[\tilde{W}_{T_r}^i{}'(X_{T_r}) R_{T_r}^A \left[1 + \frac{\partial A_{T_r-1}}{\partial Z_{T_r-1}} \right] \right] \\ &= \delta \mathbb{E}_{T_r-1} \left[\tilde{W}_{T_r}^i{}'(X_{T_r}) R_{T_r}^A \right] \end{aligned}$$

whether or not the constraint on $C_{T_r}^i$ is binding. If agents 1 and 2 are both hand-to-mouth, then the wealth at period T_r is the same; this yields

$$\frac{\partial \tilde{V}_{T_r-1}^2}{\partial Z_{T_r-1}} = \delta \mathbb{E}_{T_r-1} \left[\tilde{W}_{T_r}^2'(X_{T_r}) R_{T_r}^A \right] < \delta \mathbb{E}_{T_r-1} \left[\tilde{W}_{T_r}^1'(X_{T_r}) R_{T_r}^A \right] < \frac{\partial \tilde{V}_{T_r-1}^1}{\partial Z_{T_r-1}}.$$

If the agent 2 is hand-to-mouth but agent 1 saves, then

$$\frac{\partial \tilde{V}_{T_r-1}^2}{\partial Z_{T_r-1}} = \delta \mathbb{E}_{T_r-1} \left[\tilde{W}_{T_r}^2'(X_{T_r}) R_{T_r}^A \right] < u'(C_{T_r-1}^2 - \tilde{\Gamma}_{T_r-1}^2) < u'(C_{T_r-1}^1 - \tilde{\Gamma}_{T_r-1}^1) = \frac{\partial \tilde{V}_{T_r-1}^1}{\partial Z_{T_r-1}}.$$

If both agents save, then

$$\frac{\partial \tilde{V}_{T_r-1}^2}{\partial Z_{T_r-1}} = u'(C_{T_r-1}^2 - \tilde{\Gamma}_{T_r-1}^2) < u'(C_{T_r-1}^1 - \tilde{\Gamma}_{T_r-1}^1) = \frac{\partial \tilde{V}_{T_r-1}^1}{\partial Z_{T_r-1}}.$$

A similar argument can show us that $\frac{\partial \tilde{V}_{T_r-1}^2}{\partial X_{T_r-1}} < \frac{\partial \tilde{V}_{T_r-1}^1}{\partial X_{T_r-1}}$ as we have done in Proposition 1, but I do not repeat it here. Moreover, $\tilde{V}_{T_r-1}^i(X_{T_r-1}, Z_{T_r-1})$ is concave by Lemma 1.

Now, we go to any arbitrary period $t < T_r - 1$ given that [T1] and [T2] hold at period $t + 1$. Let us denote $\tilde{F}_t^i(S_t^i, A_t^i) = \mathbb{E}_t \left[\tilde{V}_{t+1}^i(X_{t+1}^i(S_t^i), Z_{t+1}^i(A_t^i)) \right]$. Note that the objective function $u(X_t - S_t^i - A_t^i) + \tilde{F}_t^i(S_t^i, A_t^i)$ is differentiable and concave to S_t^i and A_t^i with linear constraints by [T1] and [T2], hence, Kuhn-Tucker optimality conditions are necessary and sufficient.

We deal with the following four cases:

- Case 1: Both agents 1 and 2 are hand-to-mouth.

In this case, there is nothing to prove since the consumption of two agents is identical. Moreover, [T1] holds trivially.

- Case 2: Only agent 2 is hand-to-mouth.

Also in this case, the assumption already implies that [T1] holds.

- Case 3: Only agent 1 is hand-to-mouth.

I show the impossibility of this case. Assume that agent 1 is only saving illiquid asset A_t^1 . The assumption implies that

$$u'(C_t^2 - \Gamma_t) = \frac{\partial \tilde{F}_t^2(S_t^2, A_t^2)}{\partial S_t^2} \leq \frac{\partial \tilde{F}_t^1(S_t^2, A_t^2)}{\partial S_t^2}.$$

Hence, by the Inada condition on $u(\cdot)$ and continuity and concavity of $\tilde{F}_t(\cdot, \cdot)$, there must exist certain levels of $\Delta > 0$ and $\Delta' > 0$ such that

$$u'(C_t^2 - \Delta - \Delta' - \Gamma_t) = \frac{\partial \tilde{F}_t^1(S_t^2 + \Delta, A_t^2 + \Delta')}{\partial S_t^2}.$$

However, this cannot be true because $C_t^2 - \Delta - \Delta'$, $S_t^2 + \Delta$, and A_t^2 is also a solution for agent 1's problem and we only allow for unique optimal consumption by Lemma 1. Similar arguments can be made when agent 1 is only saving liquid asset S_t^1 .

- Case 4: All agents are not hand-to-mouth.

The argument is the same as that for case 3.

Hence, the consumption is always higher for agent 2. Concavity and differentiability of the value function are guaranteed by Lemma 1. Then, $F_{t-1}^i(S_{t-1}^i, A_{t-1}^i) = \mathbb{E}_t [V_{t+1}^i(X_{t+1}^i, Z_{t+1}^i)]$ is also concave since $F_{t-1}^i(S_{t-1}^i, A_{t-1}^i)$ is the sum of concave functions.

Finally, we need to show that [T1] holds at period t . For the first inequality, $C_t^2(X_t, Z_t; \tilde{\Gamma}_t, \tilde{\Gamma}^2) \geq C_t^1(X_t, Z_t; \tilde{\Gamma}_t, \tilde{\Gamma}^1)$ implies that

$$\begin{aligned} u'(C_t^1(X_t, Z_t; \tilde{\Gamma}_t, \tilde{\Gamma}^1) - \tilde{\Gamma}_t) &\geq u'(C_t^2(X_t, Z_t; \tilde{\Gamma}_t, \tilde{\Gamma}^2) - \tilde{\Gamma}_t) \\ \Rightarrow \mathbb{E}_{t-1} \left[\frac{\partial \tilde{V}_t^1}{\partial X_t} \right] &\geq \mathbb{E}_{t-1} \left[\frac{\partial \tilde{V}_t^2}{\partial X_t} \right], \end{aligned}$$

where the last line follows from the fact that $\tilde{\Gamma}_t^1 >_1 \tilde{\Gamma}_t^2$. □

Proof of Proposition 4

Proof. I only show the proof for the case where $\mathbf{\Gamma} >_1 \tilde{\mathbf{\Gamma}}^1 >_1 \tilde{\mathbf{\Gamma}}^2$. The proof for the other case is similar to that for this case. Assume that following holds for continuation value at $t + 1$:

$$\begin{aligned} [a] \quad \mathbb{E}_t [W_{t+1}^*(X_{t+1}; \Gamma_{t+1}, \tilde{\mathbf{\Gamma}}^1)] &\geq \mathbb{E}_t [W_{t+1}^*(X_{t+1}; \Gamma_{t+1}, \tilde{\mathbf{\Gamma}}^2)] \\ [b] \quad \mathbb{E}_t [W_{t+1}^*(Y_{t+1} + R_{t+1}^S(\bar{S} + \Delta); \Gamma_{t+1}, \tilde{\mathbf{\Gamma}}) - W_{t+1}^*(Y_{t+1} + R_{t+1}^S \bar{S}; \Gamma_{t+1}, \tilde{\mathbf{\Gamma}})] \\ &\geq \mathbb{E}_t [\tilde{W}_{t+1}^{\tilde{\mathbf{\Gamma}}} (Y_{t+1} + R_{t+1}^S(\bar{S} + \Delta); \tilde{\Gamma}_{t+1}) - \tilde{W}_{t+1}^{\tilde{\mathbf{\Gamma}}} (Y_{t+1} + R_{t+1}^S \bar{S}; \tilde{\Gamma}_{t+1})] \text{ for all } \Delta \geq 0. \end{aligned}$$

As the first step, I show that using [a] and [b], we have $W_t^*(X_t; \Gamma_t, \tilde{\mathbf{\Gamma}}^1) \geq W_t^*(X_t; \Gamma_t, \tilde{\mathbf{\Gamma}}^2)$. Fix $X_t > 0$ and denote consumption under $\tilde{\mathbf{\Gamma}}^1$ as C^1 . Then, consumption under $\tilde{\mathbf{\Gamma}}^2$ can be

written as $C^1 + \Delta_C$ for some $\Delta_C \geq 0$. Then we will compare the following two terms:

$$U^1 = u(C^1) + \delta \mathbb{E}_t \left[W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1); \Gamma_{t+1}, \tilde{\Gamma}^1) \right], \text{ and} \quad (10)$$

$$U^2 = u(C^1 + \Delta_C) + \delta \mathbb{E}_t \left[W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1 - \Delta_C); \Gamma_{t+1}, \tilde{\Gamma}^1) \right]. \quad (11)$$

Then, we can deduce

$$\begin{aligned} u(C^1 + \Delta_C) - u(C^1) &\leq \delta \mathbb{E}_t \left[\widetilde{W}_{t+1}^{\tilde{\Gamma}^1}(Y_{t+1} + R_{t+1}^S(X_t - C^1); \tilde{\Gamma}_{t+1}) \right. \\ &\quad \left. - \widetilde{W}_{t+1}^{\tilde{\Gamma}^1}(Y_{t+1} + R_{t+1}^S(X_t - C^1 - \Delta_C); \tilde{\Gamma}_{t+1}) \right] \\ &\leq \delta \mathbb{E}_t \left[W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1); \Gamma_{t+1}, \tilde{\Gamma}^1) \right. \\ &\quad \left. - W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1 - \Delta_C); \Gamma_{t+1}, \tilde{\Gamma}^1) \right] \\ &\leq \delta \mathbb{E}_t \left[W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1); \Gamma_{t+1}, \tilde{\Gamma}^1) \right. \\ &\quad \left. - W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1 - \Delta_C); \Gamma_{t+1}, \tilde{\Gamma}^2) \right]. \end{aligned}$$

Rearranging the last row shows that $U^1 \geq U^2$.

Now, to continue the backward induction, I show that [a] and [b] holds with the continuation value at t . [a] is obvious, which is just taking the expectation on the above inequality with the same random variable.

To show [b], note that $F(x) >_1 G(x)$ between two CDFs $F(x)$ and $G(x)$ implies that

$$\int v(x) dF(x) \geq \int w(x) dG(x)$$

if $v(x) \geq w(x)$ for all x and if $w(x)$ is increasing in x . Using this strategy, we need to show that

$$\begin{aligned} v(\Gamma_t) &= W_t^*(Y_t + R_t^S(\bar{S} + \Delta); \Gamma_t, \tilde{\Gamma}) - W_t^*(Y_t + R_t^S \bar{S}; \Gamma_t, \tilde{\Gamma}) \\ &\geq \widetilde{W}_t^{\tilde{\Gamma}}(Y_t + R_t^S(\bar{S} + \Delta); \Gamma_t) - \widetilde{W}_t^{\tilde{\Gamma}}(Y_t + R_t^S \bar{S}; \Gamma_t) = w(\Gamma_t) \end{aligned}$$

for all Δ and Γ_t , and the terms on the right-hand side, $w(\Gamma_t)$ is increasing in Γ . To show the former, I denote the $C_t(\Delta)$ as the level of consumption under available wealth $Y_t + R_t^S(\bar{S} + \Delta)$, and $X_{t+1}(\Delta)$ to denote $X_{t+1}(\Delta) = Y_{t+1} + R_{t+1}^S(Y_t + R_t^S(\bar{S} + \Delta) - C_t(\Delta))$. Note that $C_t(\Delta)$

and $X_{t+1}(\Delta)$ are increasing functions of Δ . We can rearrange $v(\Gamma_t) - w(\Gamma_t)$ as,

$$\begin{aligned}
v(\Gamma_t) - w(\Gamma_t) &= u(C_t(\Delta)) + \delta \mathbb{E}_t \left[W_{t+1}^*(X_{t+1}(\Delta); \Gamma_{t+1}, \tilde{\Gamma}) \right] \\
&\quad - u(C_t(0)) - \delta \mathbb{E}_t \left[W_{t+1}^*(X_{t+1}(0); \Gamma_{t+1}, \tilde{\Gamma}) \right] \\
&\quad - u(C_t(\Delta)) - \delta \mathbb{E}_t \left[\widetilde{W}_{t+1}^{\tilde{\Gamma}}(X_{t+1}(\Delta); \tilde{\Gamma}_{t+1}) \right] \\
&\quad + u(C_t(0)) + \delta \mathbb{E}_t \left[\widetilde{W}_{t+1}^{\tilde{\Gamma}}(X_{t+1}(0); \tilde{\Gamma}_{t+1}) \right] \\
&= \delta \mathbb{E}_t \left[W_{t+1}^*(X_{t+1}(\Delta); \Gamma_{t+1}, \tilde{\Gamma}) \right] - \delta \mathbb{E}_t \left[W_{t+1}^*(X_{t+1}(0); \Gamma_{t+1}, \tilde{\Gamma}) \right] \\
&\quad - \left[\delta \mathbb{E}_t \left[\widetilde{W}_{t+1}^{\tilde{\Gamma}}(X_{t+1}(0); \tilde{\Gamma}_{t+1}) \right] - \delta \mathbb{E}_t \left[\widetilde{W}_{t+1}^{\tilde{\Gamma}}(X_{t+1}(\Delta); \tilde{\Gamma}_{t+1}) \right] \right].
\end{aligned}$$

By the fact that [b] holds at period $t + 1$, this term is positive.

Now, we turn to the fact that $w(\Gamma_t)$ is an increasing function. Using the differentiability of $w(\Gamma_t)$ ²⁵, it is sufficient to show that $\partial \widetilde{W}_t^{\tilde{\Gamma}}(X_t; \Gamma_t) / \partial X_t = u'(C_t(X_t; \Gamma_t) - \Gamma_t)$ is increasing in Γ_t . For any $\Delta_\Gamma > 0$, $C_t(X_t; \Gamma_t + \Delta_\Gamma) - C_t(X_t; \Gamma_t) < \Delta_\Gamma$, hence $w(\Gamma_t)$ is increasing in Γ_t by the concavity of $u(\cdot)$. \square

²⁵As in Propositions 1 and 2, differentiability is guaranteed by Lemma 1.

B Other Tables and Figures

		89'	92'	95'	98'	01'	04'	07'	10'	13'	16'	19'	Avg.
All	NW	0.71	0.71	0.70	0.72	0.74	0.74	0.74	0.77	0.77	0.79	0.78	0.74
	LIQ	0.88	0.87	0.89	0.89	0.90	0.89	0.90	0.91	0.90	0.91	0.89	0.89
	Illiquid	0.68	0.68	0.66	0.68	0.70	0.70	0.70	0.72	0.73	0.76	0.76	0.71
Graduate	NW	0.70	0.68	0.68	0.68	0.72	0.71	0.70	0.73	0.73	0.75	0.75	0.71
	LIQ	0.86	0.86	0.86	0.86	0.89	0.87	0.88	0.89	0.88	0.87	0.87	0.87
	Illiquid	0.66	0.66	0.64	0.64	0.68	0.66	0.67	0.67	0.70	0.73	0.73	0.68
Tertiary education	NW	0.69	0.67	0.67	0.69	0.70	0.68	0.71	0.70	0.72	0.76	0.72	0.70
	LIQ	0.85	0.83	0.86	0.87	0.87	0.86	0.87	0.85	0.85	0.89	0.85	0.86
	Illiquid	0.64	0.64	0.65	0.66	0.66	0.65	0.68	0.67	0.68	0.72	0.69	0.67
Secondary education	NW	0.67	0.67	0.64	0.65	0.67	0.69	0.66	0.71	0.73	0.74	0.75	0.69
	LIQ	0.85	0.82	0.87	0.83	0.83	0.85	0.85	0.88	0.90	0.88	0.89	0.86
	Illiquid	0.66	0.64	0.60	0.61	0.63	0.64	0.64	0.67	0.69	0.72	0.73	0.66
First income tertile	NW	0.64	0.65	0.64	0.66	0.66	0.65	0.66	0.68	0.67	0.71	0.70	0.67
	LIQ	0.85	0.84	0.86	0.86	0.87	0.85	0.86	0.86	0.84	0.86	0.83	0.85
	Illiquid	0.61	0.62	0.61	0.62	0.63	0.62	0.63	0.64	0.64	0.67	0.68	0.63
Second income tertile	NW	0.58	0.60	0.58	0.58	0.62	0.60	0.58	0.60	0.67	0.63	0.64	0.61
	LIQ	0.76	0.80	0.82	0.79	0.81	0.78	0.78	0.77	0.85	0.81	0.82	0.80
	Illiquid	0.53	0.53	0.50	0.49	0.52	0.51	0.49	0.51	0.57	0.54	0.55	0.52
Third income tertile	NW	0.72	0.72	0.70	0.69	0.74	0.75	0.71	0.75	0.78	0.76	0.74	0.73
	LIQ	0.88	0.86	0.87	0.86	0.90	0.93	0.91	0.92	0.93	0.92	0.89	0.90
	Illiquid	0.71	0.69	0.64	0.66	0.67	0.67	0.67	0.69	0.70	0.71	0.71	0.68

Table 8: Gini index of the United States: 1989–2019

Notes: Illiquid assets refers to total assets minus the liquid assets. Net worth is the total assets minus debt. “Avg.” refers to the average of all survey years for each row.

	ρ	p -value	Expenditure shock
Adult care	0.338	0.008	
Alcohol away from home	0.190	<0.001	
Alcohol at home	1.000	<0.001	
Child care	0.118	0.026	O
Clothes	-0.015	0.009	O
Clothing services	0.205	<0.001	
Domestic services	0.257	0.136	O
Education durables	0.034	0.063	O
Education services	0.003	0.731	O
Entertainment durables	-0.003	0.588	O
Entertainment services	0.054	0.005	
Fees and charges	0.023	0.215	O
Food away from home	0.064	<0.001	
Food at home	0.883	<0.001	
Furniture rental	1.000	<0.001	
Gasoline expenses	0.447	<0.001	
Health care durable	0.010	0.397	O
Health insurance	0.242	<0.001	
Health care service	0.026	<0.001	
Other household expenditures	0.066	0.002	
Home insurance	0.002	0.933	O
Home management	-0.042	0.004	O
Home maintenance and repairs	0.036	<0.001	
Home-related equipment and supplies	0.093	0.093	O
Household furnishings and equipment	0.013	0.282	O
Household textiles and linens	0.041	0.298	O
Jewelry	0.005	0.657	O
Life insurance	-0.084	0.041	O
Occupational expenses	0.115	<0.001	
Parking expenses	~0.001	0.941	O
Public transportation	0.012	0.078	O
Personal care products	-0.036	0.217	O
Telephone services	0.246	<0.001	
Personal care services	0.999	<0.001	
Reading material	1.000	<0.001	
Rent	0.063	<0.001	
Rented vehicles	0.048	<0.001	
Tobacco	1.000	<0.001	
Utilities	0.056	<0.001	
Vehicle	0.002	0.172	O
Other vehicle-related durables	0.021	0.277	O
Vehicle insurance	-0.050	<0.001	O
Vehicle service	-0.005	0.430	O
Water and other public services	-0.142	<0.001	O

Table 9: Classification of Expenditure Shocks

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