Consumption and Savings under Imperfect Perception of Expenditure Shocks[*]

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Abstract

I build a life cycle model of consumption and savings where households face exogenous expenditure shocks. households are heterogeneous by having different levels of perceptions of expenditure shocks. households who underestimate the expenditure shocks tend to spend more now and save less for the future. In a calibration exercise, I show that to match the dispersion of liquid wealth in data, it is crucial to have many households who underestimate the future expenditure shocks. The model exhibits the high marginal propensity consume comparable to empirical evidence even with the intertemporal elasticity of substitution below one. This high MPC is explained by the presence of large amount of hand-to-mouth households resulting from their underestimation of future expenditure shocks. The model also exhibits realistic level of liquid wealth over the life cycle.

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1 Introduction

[**P1. Research Question**] Why do households consume a large amount of additional spare money, and who spends the most? This question of magnitude and heterogeneity behind the marginal propensity to consume (MPC) is important when designing a policy, such as fiscal stimulus to boost economic activity. In practice, the magnitude can be important when the government attempts to gauge the size or duration of stimulus needed. The heterogeneity of consumption responses, related to who responds by a lot, can also be important when designing how the government should spread the stimulus across different groups. This paper investigates these two aspects of MPC by employing a model of heterogeneous agents with different degrees of behavioral biases.

[**P2. Liquidity Dispersion is Important for MPC**] A large piece of empirical evidence documents high MPC in various contexts. Moreover, much of the vast literature points to the strong association between low liquidity and large consumption responses (Souleles, 1999; Johnson et al., 2006; Parker et al., 2013; Baker, 2018; Baker et al., 2020; Fagereng et al., 2021; Aydin, 2021). Reflecting on the empirical evidence, seminal approaches of Carroll (1992, 1997), Kaplan and Violante (2014) and Carroll et al. (2017) all have hand-to-mouth households with low liquidity at their heart. Hence, in this liquidity view of generating large overall MPC, it is crucial to explain why and who are liquidity constrained.[1][2]

[**P3. Liquidity Dispersion is Severe than what Life Cycle Factors Imply**] The model needs to exhibit a realistic distribution of liquid assets, so the MPC in the model is high because of a real reason. How is the liquid asset distributed? Carroll et al. (2017) show that liquid wealth is distributed more unequally than the net worth. Then what kind of factors can explain this high dispersion of liquid wealth? Age can be an important factor driving a dispersion of liquid wealth since households across various stages of life would have different demands. However, I show that when we pool households by different age groups, the inequality of liquid wealth is still high. Thus, if a factor explaining the distribution of liquid wealth would be independent of the life cycle characteristics. I show that even in subgroups of people that are homogeneous in education groups, age, occupation, and income[3] dispersion of liquid wealth is very high. This result is robust when I restrict my analysis to highly liquid assets[4] which brings homogeneous returns. We need a model generating

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[1]This model's other detailed merits compared to the literature are explained in detail in the literature review section at the end of this introduction.

[2]Specifically, under the preferred calibration of this paper, Table 5 shows that the model matches the distribution of liquid wealth over income at different percentiles and age groups.

[3]In the literature, earnings heterogeneity (Castañeda et al., 2003), heterogeneity of asset returns (Hubmer et al., 2020), or entrepreneurship Quadrini (1999) are raised as important factors.

[4]The definitions of liquid and highly liquid assets are given in Section 2.

the dispersion of liquid wealth, even when income and asset returns are homogeneous in expectation.

[**P4. This paper**] I introduce a partial equilibrium model where households are only heterogeneous in the perceptions of future expenditure shocks, where income is homogeneous in expectation. Their consumption and savings of the households crucially depend on how much they under or overestimate their future expenditure shocks. Households that underestimate the expenditure shocks feel less need for precautionary savings, and they tend to consume more than other households that overestimate. This consumption gap creates a gap of savings between households that relatively underestimates and overestimates than others.[5] Over time, households that underestimate the expenditure shock consume more than they originally planned in the past, which depletes their liquid savings quickly. (**From P9: Heterogeneity of the MPC**) The channel leading to the dispersion of liquid wealth also drives the heterogeneity of the MPC. The households with low liquid wealth due to underestimating future expenditure shocks will be more likely to become hand-to-mouth where households consumes all their disposable income.

[**P5. Identification Strategy**] This paper, following Carroll et al. (2017), calibrates the model to match the share of liquid wealth at various percentiles while letting the model match the median liquid asset holdings over income. At the same time, it does not arbitrarily increase the overall MPC by introducing households with a low level of wealth in the economy. Targeting the level of median liquid wealth over income makes the overall level of liquid wealth in the model similar to the data. The result that consumption has a monotone relationship with the perception of expenditure shock is a crucial source of identification in this paper. If the data suggests that the share of wealth should be lower at the lower pentile, then the model will respond by introducing households that underestimate the future expenditure shocks, and the inequality of liquid wealth will be higher.

[**P6. Calibration result**] Calibration result implies that households perceive about a third of the true expenditure shock on average.[6] Also, there are a lot of relatively more optimistic households that underestimate future expenditure shocks with very low levels of liquid assets. Though not targeted directly, the calibration result also captures the level of liquid wealth at different percentiles over the life cycle.[7] *(Fix later) Applying the conventional definitions of 'hand-to-mouth' agents, which defines liquid asset balance over the income, this*

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[5]Specifically, if the distributions regarding future expenditure shock of Agent A first order-stochastically dominates distributions perceived by Agent B, then the consumption of Agent A will be lower than Agent B.

[6]I assume that the expenditure shocks are drawn from an exponential distribution with mean λ_t at each age t. In a loose sense, the calibration result implies that, on average, households perceive that the expenditure shock is drawn from an exponential distribution where the mean of $\lambda_t/3$.

[7]See Table 5 for this.

result implies that this model is also able to capture fraction of households that are hand-to-mouth over different stages of life. **Added:** Also, the calibration result implies that the majority of households do not exhibit consumption responses. Instead, a small group of hand-to-mouth households leads to a high overall MPC. This extensive margin of MPC is consistent with the pattern shown in Fuster et al. (2020).

[**P7. Why model survives the low IES**] Havránek (2015) conducts a meta-analysis on intertemporal elasticity of substitution (IES) and suggests mean IES based on micro evidence around 1/3. Though micro estimates tend to be below one (e.g., Best et al. (2020)), it is rare to use IES below one in the model. Aguiar et al. (2020), when using IES of 0.5, show that the consumption responses in the model Kaplan and Violante (2014) can be much smaller.[8] The IES controls the desire to smooth consumption throughout life. With low IES, households would want to smooth consumption more, and they tend to save more during the middle age to raise the consumption after retirement when the income is low. Using low IES in the model dampens forces towards the sensitive consumption response and is less likely to be hand-to-mouth since they eagerly hold liquid assets to balance the amount of consumption even across the near future under low IES. Even with the low IES suggested in the literature, my model can generate a sizeable amount of hand-to-mouth households by the mismatch between planned and actual consumption over the lifecycle.

[**Deleted. Heterogeneity of the MPC: liquidity and the extensive margin** (This paragraphs is divided into two parts and sent to P4 and P6)

[**P8. Policy implications: boost economic activity**] The policy implications of this paper depend on the two different goals of the government. When facing a recession, the government's goal can be boosting the economic activity to increase welfare. In this direction, the policy recommendation of this paper shares the same spirit with the previous literature focusing on the role of liquidity constraints: focusing on the low liquidity households would bring the maximal consumption response. As Carroll (1992) and Kaplan and Violante (2014) noted, households with low liquidity are more likely to be hand-to-mouth, exhibiting larger consumption response. This paper brings an additional channel. Since liquid wealth is a good proxy of whether agents are optimistic or not,[9] targeting the low-liquidity households would effectively cover the households that are more willing to consume more than the rational expectations benchmark under the same conditions.

[**P9. Policy implications: enhancing welfare**] From a long-term perspective, the government would want to enhancing the overall welfare in the economy. This paper the-

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[8]Compared to the MPC of 22.7 percent when using the annual discount factor of 0.93 and IES of 1.5, the annual discount factor of 0.94 and IES of 0.5 gives the MPC of 3.1 percent.

[9]See Figure 4 confirms this idea.

oretically shows that expected ex-post welfare, which the government wants to enhance, decreases as the agents' perception of future expenditure shock is farther from the actual. Hence, the policy recommendation crucially depends on the shape of the distribution of perception. Based on the calibrated distribution of perception with a large mass of households who underestimate the future expenditure shock, this paper recommends a policy that taxes the liquid wealth until the lower threshold and transfers the liquid wealth between a lower and upper threshold. This policy that is [1] self-fundable[10], [2] based on observables[11] and [3] not deceiving[12], the average consumption utility of the agents can be enhanced while raising additional funds from taxes.[13]

[**10-1. Macroeconomic literature review: Buffer-Stock Savings**] Literature Review of Macroeconomic Approaches: This paper is closely related to a literature strand that relates the liquidity constrained households with high MPC. Seminal works by Carroll (1992, 1997) show how adopting a borrowing constraint can make hand-to-mouth households exhibiting high MPC. However, as noted by Kaplan and Violante (2014), the fraction of households that become hand-to-mouth are not large enough to generate high overall MPC under the one-asset framework in a realistic setting.

[**P10-2. Macroeconomic literature review: KV**] The current workhorse model by Kaplan and Violante (2014) builds a model which separates saving for retirement and precautionary motives by having a liquid asset and illiquid asset requiring adjustment cost. Households have disproportionately small liquid assets compared to a large stock of illiquid savings due to higher returns on illiquid savings. As households have small liquid assets, they are more likely to be hand-to-mouth, which raises the overall MPC. However, as noted by Aguiar et al. (2020), the result is difficult to survive with the low IES, in line with the micro evidence, and some degree of preference heterogeneity is necessary to exhibit a realistic level of wealth and fraction of hand-to-mouth households.[14] This paper shares the same spirit of having households with a disproportionately small amount of liquid assets compared to illiquid assets. Agent in this paper can be time-inconsistent, and underestimating the future expenditure shock can lead to persistent low liquid wealth even with low IES.

[**P10-3. Macroeconomic literature review: CSTW**] Influential work by Carroll

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[10]As elaborated in Section 5, this policy does not require any additional funds, and no interpersonal transfers are required.

[11]This implies that the government cannot directly observe the 'optimism' and 'pessimism' of the agents.

[12]Agents correctly perceives what the government does.

[13]Table 7 illustrates the efficacy of this policy.

[14]Under the discount factor of 0.95, MPC of 13.2% when IES is 1.5 drops to 1.5% when IES is 0.5. This pattern is confirmed by the Table D.I of the appendix of Kaplan and Violante (2014) where rebate coefficient, measured using the method of Johnson et al. (2006), drops from 20% to 9% when IES decreases from 2 to 1.05.

et al. (2017) (henceforth, CSTW), equipped with discount factor heterogeneity, explains the high MPC driven by impatient households with a low level of wealth. A difference between CSTW and this paper is that CSTW mainly focuses on replicating the inequality of net worth, while this paper focuses on liquid wealth. However, CSTW shows that targeting liquid assets requires the model to use a wider dispersion of discount factors than the case of using the net worth due to even more severe inequality of liquid wealth.[15] Another difference is that, while CSTW uses an additional source of income dispersion by having households with different education levels, this paper does not have this fundamental heterogeneity in income: all households have the same income in expectation.[16] Lastly, this paper can generate a higher level of MPC comparable to the upper ranges of MPC reported in the literature.[17]

[**P11-1. Behavioral literature review:** (β, δ)] Literature Review of Behavioral Approaches: The quasi-hyperbolic discounting (Strotz, 1955; Pollak, 1968; Laibson, 1997) can also produce high marginal propensity to consume. Under this framework, Laibson et al. (2007) uses a two-asset life cycle model combined with quasi hyperbolic discounting and estimate the key parameters (β, δ)[18] to match the accumulation of wealth and credit card borrowing. The problem of using (β, δ) is that it is difficult to produce sizeable difference between sophisticated and naive agents, which mutes the role of perception regarding futures agents.[19] Also, the (β, δ) model generally does not guarantee a unique consumption solution unless the modeler uses the perception of β close to one, which implies the naive agent. In this paper, as mentioned earlier, there is a clear connection between the degree of perception and consumption unlike the quasi-hyperbolic discounting model.[20]

[**P11-2. Behavioral literature review: Lian**] Lian (2021) presents a model where

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[15]According to their work in Table 3, under the life cycle framework with IES of 1, targeting net worth requires that annual discount factors are distributed uniformly in $[0.86, 1.0]$ where 0.928 is the median. However, targeting liquid assets widens the domain of the discount factors to $[0.75, 1.0]$ where the median is 0.87 and far too small compared to the conventional use of discount factors. When IES is 2, Table 2 in the appendix of CSTW shows that the domain of discount factors widens even more to $[0.65, 1.07]$.

[16]This choice of having homogeneous households is motivated by the fact that education level or other demographic factors do not contribute much to the dispersion of liquid wealth.

[17]In CSTW, the benchmark case using net worth gives quarterly MPC of 5.8 percent. When using liquid wealth, the quarterly MPC can reach 16.3 percent.

[18]Under quasi-hyperbolic discounting framework, β implies the degree of present bias which discounts the utility from the future rewards compared to the reward now. δ is the long-run discount factor which measures how much the agent discount the utility of later rewards.

[19]When simulating the life cycle model under quasi-hyperbolic discounting, Angeletos et al. (2001) shows no significant difference between naive agents, who perceive future agents' present bias as $\beta = 1$, sophisticated agents, who perceive the future present bias correctly.

[20]In a stylized three-period problem, Salanie and Treich (2006) shows that the relationship between consumption and perception of β depends on the shape of the utility function. However, there is no guarantee that this result would hold in the longer horizon, and the existence or uniqueness of solutions can be difficult to characterize(Harris and Laibson, 2001, 2002).

mistakes in predicting future consumption create high MPC today, which can account for various behavioral biases. The key difference between Lian (2021) and this paper is how the direction of mistake affects consumption today. The model of Lian (2021) implies that bigger mistakes, in an absolute sense, in predicting future MPCs lead to larger MPC today. In this paper, whether the agent under or overestimate the future expenditure shock is important when determining the level of consumption. Another difference is that Lian (2021) emphasizes the MPC across all levels of wealth and is capable of explaining households with high consumption response and high liquid wealth, and this paper cannot explain this phenomenon. The mechanism of this paper crucially depends on the households maintaining disposable income around the borrowing constraint.

[**P11-3. Behavioral literature review: DE**] Bianchi et al. (2021) adopts insights of the psychology literature regarding selective memory recall and introduces a model where surprises can shape the future expectation in a biased way. Especially, a high-income shock today can generate optimistic beliefs regarding future liquidity and increase the marginal propensity to consume. The merit of Bianchi et al. (2021) is that they specify the learning behavior of agents, which is absent in this paper. However, it may not be easy to characterize the theoretical property of the model in a general life cycle setting, unlike this paper. Investigation of the role of diagnostic expectation in the life cycle setting to the dispersion of liquid wealth and high MPC can be interesting, but Bianchi et al. (2021) do not directly aim these objectives.[21]

[**P12. Structure of the paper**] The remaining of this paper proceeds as follows. In Section 2, I present two stylized facts. First, there is a severe distribution of liquid wealth even after controlling for demographic factors. Second, there is a large overall MPC, with the heterogeneity of consumption responses. Section 3 presents the model when it derives some theoretical properties. Section 4 takes the model to data, calibrates the distribution of perception, and examines the model's performance. Next, I present policy implications of this model in Section 5. Finally, section 6 concludes with possible extensions of the model.

2   Stylized Facts

Fact 1: Severe Dispersion of Liquid Wealth Among Homogenous Groups/Assets
Historically, the wealth inequality measured by the Gini coefficient has been persistently around 0.8. Compared to the income equality with the Gini coefficient around 0.5, there

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[21]Some empirical clarification regarding the relationship between key variables is important. For example, the result that high income may lead to high consumption implies higher income and consumption comovement, but this can be at odds with Miranda-Pinto et al. (2020).

is much severe wealth inequality. In various directions, many models attempted to replicate this phenomenon. Important contributions highlights dispersion of labor efficiency (Castañeda et al., 2003), asset return heterogeneity (Hubmer et al., 2020), heterogeneity in discount factors (Krusell and Smith, 1998; Carroll et al., 2017; Hubmer et al., 2020), and entrepreneurship (Quadrini, 1999) to name a few.

Dispersion of liquid wealth is more severe than the net worth. Then, can factors that explain the dispersion of total wealth also apply to the dispersion of liquid wealth? To answer this question, if the dispersion of wealth measured among a homogeneous group of people/assets is significantly less than the dispersion among the overall population. Specifically, for each Survey of Consumer Finances survey, I only consider working-age households from 25 to 64 and drop the samples that earn under the annualized minimum wage. Also, I drop the samples that exhibit a negative amount of net worth. The definition of a *liquid asset* is the sum of money market, checking, savings and call accounts, directly held stocks, and bonds. To assess the inequality among households with similar characteristics, I make a subgroup of households by their respective education and income levels. Also, to control the effects of heterogeneous asset returns, I examine the inequality of *highly liquid asset* which comprises money market, checking, and savings accounts. Since the returns of items in the highly liquid asset are likely to be similar across households, their balances would represent households' desire to save and not differences in the investment skills or luck.

	All	Education			Income Tertiles		
		Graduate	Tertiary	Secondary	1st (highest)	2nd	3rd (lowest)
Liquid	0.89	0.87	0.86	0.86	0.85	0.80	0.90
Highly liquid	0.82	0.78	0.77	0.81	0.76	0.72	0.83

Table 1: Average Gini coefficients of liquid assets among working-age households by education and income levels: 1989-2019

Table 1 shows the average Gini coefficients in the Survey of Consumer Finances across all survey years between 1989-2019. Compared to the Gini coefficient of the net worth suggested in the literature, which is around 0.8, the Gini coefficient of the liquid wealth is over 0.8, which suggests severe wealth inequalty.[22] By grouping households by different education and income levels, the degree of inequality gets dampened but not by large. Especially, the Gini coefficient of the lowest income tertile is around 0.9, which suggests a high proportion of households with a near-zero level of wealth and producing a high Gini coefficient.

The lifecycle model of consumption and savings implies that households accumulate large assets near the end of retirement, which would lead to the dispersion of wealth across different stages of life. Households with a near-zero level of wealth will be mostly young households

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[22]This can be confirmed again by year-by-year calculation of the Gini coefficient in Table 8.

| | Age groups | | | |
	25-34	35-44	45-54	55-64
Liquid	0.80	0.86	0.89	0.90
Highly liquid	0.74	0.79	0.82	0.82

Table 2: Average Gini coefficients of liquid assets among working-age households by age groups: 1989-2019

since they face uphill income profiles and postpone saving. Households with low liquidity will vanish from the middle-age as they start accumulating wealth. Hence, if we group people by different age groups, inequality of wealth can be expected to go away. Surprisingly, grouping the households by different life stages does not contribute to the lower dispersion of wealth. As Table 2 suggests, there is a trend that the degree of dispersion increase as households reach the retirement age, but the Gini coefficients among households in the similar stages of life is roughly the same as the overall Gini coefficient. This implies that the households with low liquidity exist in all age groups, and life cycle properties are not the crucial factor leading to the severe dispersion of liquid wealth.

| | Occupation group 1 | | Occupation group 2 | | |
	Work for others	Self-employed	Managerial	Technical	Other
Liquid	0.87	0.89	0.87	0.88	0.81
Highly liquid	0.79	0.82	0.78	0.81	0.76

Table 3: Average Gini coefficients of liquid assets among working-age households by different occupation groups: 1989-2019

Note: 'Managerial' refers to manegerial and professional workers. 'Technical' refers to technical, sales and service workers.

Lastly, we check if different occupational characteristics can contribute to the dispersion of liquid wealth. Different occupational groups may have different occupational needs. For example, self-employed workers may have less predictable income which might increase desire for the precautionary savings. Also, households with higher income such as managerial and professional workers would have outliers that lead to greater dispersion of liquid wealth. Table 3 gauge inequality of liquid wealth by two classficiations of occupation groups. In the first two columns, I contrast the group working for others, and the self-employed. Not only the the degree of inequality is similar, severe inequality exists in both groups. The remaining three columns also confirms that different types of occupations does not lead to different degree of liquid wealth inequality. Moreover, this inequality remains even when measured with highly liquid assets.

We checked if several sources that may explain the dispersion of total wealth can also explain the dispersion of liquid wealth. However, the exercise in this section shows that the

high dispersion of liquid wealth persists within subgroups of people with similar education and income levels or stages of life. Hence, there is a force driving severe dispersion of life regardless of demographic or economic factors.

Fact 2: Large MPC and Low IES Most studies studying excess sensitivity of consumption agree that consumers' spending response to transitory income shock is large. For example, (Souleles, 1999) reports that consumers spend 0.344-0.64 out of the income tax refunds over a quarter. Agarwal and Qian (2014) measured that consumers spent 80 percent out of unanticipated fiscal stimulus in Singapore over ten months period. In response to social security tax reform, Parker (1999) found that households spent about 20 percent out of additional after-tax income over three-month periods on nondurable goods. In income tax rebate of 2001, Johnson et al. (2006) found that households spent 20 to 40 percent over a quarter on nondurable goods. In fiscal stimulus of 2008, Parker et al. (2013) found that households' total expenditure increased by 50 to 90 percent over a quarter. Parker (2017) finds strong spending responses among households that lack sophistication and financial planning and hints that persistent characteristics play an essential role. Using predictable payments from Alaska permanent fund, (Kueng, 2018) finds that households spent 20 percent for nondurable and services over a quarter.

Another piece of evidence points to heterogeneity of responses. Based on two stimuli of U.S. in years 2001 and 2008, Misra and Surico (2014) shows that around half of households do not show a significant and positive response to the stimulus. (Fuster et al., 2020) asked directly to survey respondents how much they would spend out of a windfall gain; around 70 percent of the respondents exhibited MPC of zero. Hence, the large MPC is driven by a small subset of households with a large response to a liquidity shock.

A large piece of evidence suggests a strong relationship between high consumption response and low liquidity, consistent with the buffer stock theory of saving (Souleles, 1999; Johnson et al., 2006; Parker et al., 2013; Baker, 2018; Baker et al., 2020; Fagereng et al., 2021; Aydin, 2021). However, there is another piece of evidence showing that persistent characteristics are more crucial (Parker, 2017). The view of this paper is that the persistent characteristics drive the low liquidity and thus bring the high consumption response as as Gelman et al. (2019), Gelman (2021), and Carroll et al. (2017).

In most models, explaining large MPCs require intertemporal elasticity of substitution (IES) around one or large. Smaller IES increases the desire to smooth out consumption over the life cycle, and households would exhibit small MPCs as noted by Aguiar et al. (2020). At the same time, a meta-analysis by Havránek (2015) reports that when accounting for selective reporting of the IES, the median estimate of IES is around 1/3. This paper tries

to bridge these to contrasting facts by using low IES but still exhibiting high MPC overall.

Fact 3: Misprediction of Expenses Households generally save for two reasons. In the 2019 Survey of Consumer Finance, the most frequently chosen reason for saving was to prepare the retirement (34.1 percent) among working-age households. The following big reason for saving was to prepare for the 'rainy days,' chosen by 27.7 respondents.[23] The two main reasons for saving are consistent with implications of modern consumption savings models such as Carroll (1992). Households accumulate a large amount of wealth to prepare the retirement, and at the same time, some portion of the wealth is to prepare randomness of income and other temporary shocks.

However, households' portfolio choice in the Survey of Household Finances suggests a lack of preparation for the rainy days. Among working-age households, three percent of households even resort to payday loans which typically accompany high interest. As a most frequently chosen option, 38.3 percent of people answered that using a payday loan was because of an emergency. Furthermore, among subjects who currently borrow money, 38.9 percent answered that they had an experience being behind the payment by two months or more. These facts show that many households cannot avoid borrowing with high interest or paying penalties for being behind schedule, which brings additional costs. Thus, even though households save to prepare for the rainy days, the unpredicted expenses force households to use borrowing facilities that bring extra cost. Also, there is a prevalence of credit card borrowing where the median borrowing rate in the same survey is around 17 percent. Around 38.6 percent of households have credit card loans. The borrowing rate and the borrowing frequency are too high to be rationalized with the models without any behavioral frictions. [24] The pattern of borrowing over the life cycle is at odds with the implications of the life cycle models without featuring a borrowing constraint. Such models would suggest that households would mainly borrow until the peak of the income profile and stop borrowing from then. However, Figure 1 does not show a sharp decrease in the frequency of borrowing near middle age, and the high frequency of borrowing also persists throughout the life cycle. Hence, contributions from other factors should persistently lead households to borrow throughout the life cycle.

As a related study, Berman et al. (2016) provides evidence that consumers tend to underweight the costs needed rather than income when predicting the spare money in the future. Howard et al. (2020) studies an expenditure prediction bias which is a tendency to underestimate future expenses. Fellowes and Willemin (2013) suggest that 25% of households

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[23]The third reason to save was buy own house chosen by 5.8 percent.

[24]For example, the current workhorse model of consumption and savings by Kaplan and Violante (2014) targets 26 percent of borrowing rate with the nominal borrowing rate of 10 percent.
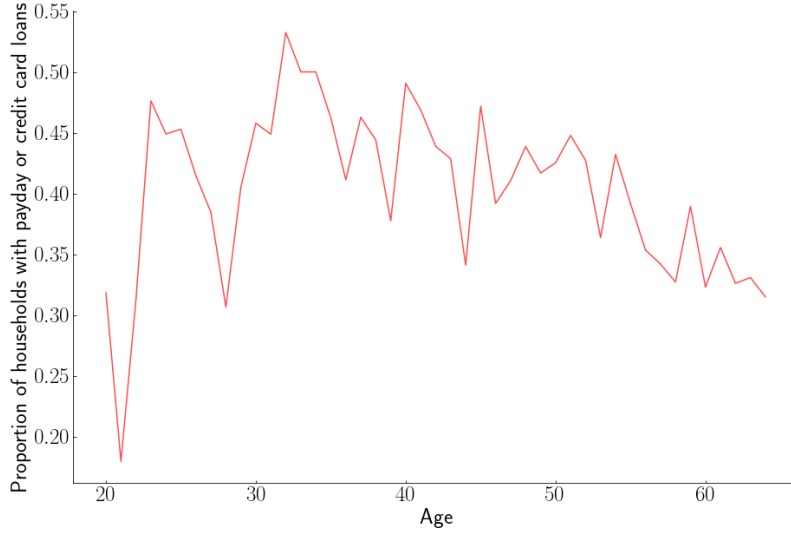
Figure 1: Proportion of having payday or credit card loans by age
Source: reproduced from the Survey of Consumer Finances at 2019

withdraw 401(k) savings early, often to cover unexpected expenses.

3   Model

Households face consumption and savings problems for T number of periods. This paper takes a partial equilibrium approach where interest rates and income are assumed to be exogenous. There are two types of consumption. The first part of consumption is determined in the long-run perspective, and its trajectory is smooth over the life cycle. The second part of the consumption, which we call an expenditure shock, is not a choice variable where short-run motives are more important than consumption smoothing in the long run. Examples of expenditure shocks are vehicle repair costs or sudden medical expenses. Extra spending on items in this category does not bring additional utility. However, those are an unavoidable part of consumption, and we would not try to balance these types of expenditures throughout the life cycle. Typical examples of the long-run part of the consumption are food and utility expenditures.

I assume that the expenditure shock at period t is drawn from a random variable $\boldsymbol{\Gamma}_t$, which maps a set of events to nonnegative real numbers. Households have an imperfect perception of expenditure shocks, where the perception of $\boldsymbol{\Gamma}_t$, denoted as $\widetilde{\boldsymbol{\Gamma}}_t$, can be different from the actual $\boldsymbol{\Gamma}_t$. Similarly, variables with tilde, such as $\widetilde{x}$, represents the perception of a

12

variable x. Without the time subscript, $\widetilde{\boldsymbol{\Gamma}} = \{\widetilde{\boldsymbol{\Gamma}}_1, \cdots, \widetilde{\boldsymbol{\Gamma}}_T\}$ represents sequence of perception of expenditure shocks. In the same way, $\boldsymbol{\Gamma}$ is the sequence of distribution of true expenditure shocks, $\boldsymbol{\Gamma} = \{\boldsymbol{\Gamma}_1, \cdots, \boldsymbol{\Gamma}_T\}$. Γ_t and $\widetilde{\Gamma}_t$ are particular realizations of random variables $\boldsymbol{\Gamma}_t$ and $\widetilde{\boldsymbol{\Gamma}}_t$.

Households retire at age T_r. During the working-age, households at time $t < T_r$ solve the following problem.

$$V_t^{\widetilde{\boldsymbol{\Gamma}}}(X_t, Z_t; \Gamma_t) = \max_{C_t, S_t, A_t} u(C_t - \Gamma_t) + \delta \mathbb{E}_t \left[\widetilde{V}_{t+1}^{\widetilde{\boldsymbol{\Gamma}}}(X_{t+1}, Z_{t+1}; \widetilde{\Gamma}_{t+1}) \right] \quad \text{where} \quad (1)$$

$$\widetilde{V}_{t'}^{\widetilde{\boldsymbol{\Gamma}}}(X_{t'}, Z_{t'}; \widetilde{\Gamma}_{t'}) = \max_{C_{t'}, S_{t'}, A_{t'}} u(C_{t'} - \widetilde{\Gamma}_{t'}) + \delta \mathbb{E}_{t'} \left[\widetilde{V}_{t'+1}^{\widetilde{\boldsymbol{\Gamma}}}(X_{t'+1}, Z_{t'+1}; \widetilde{\Gamma}_{t'+1}) \right],$$

$$\text{subject to}$$

$$Y_t + R_t^S S_{t-1} = X_t \geq C_t + S_t + A_t,$$

$$Z_{t+1} = R_{t+1}^A (A_t + Z_t),$$

$$C_t \geq 0, A_t \geq 0 \text{ and } S_t \geq 0, \text{ and}$$

$$\widetilde{V}_{T_r-1}^{\widetilde{\boldsymbol{\Gamma}}}(X_{T_r-1}, Z_{T_r-1}; \widetilde{\Gamma}_{T_r-1}) = \max_{C_{T_r-1}, S_{T_r-1}, A_{T_r-1}} u(C_{T_r-1} - \widetilde{\Gamma}_{T_r-1})$$

$$+ \delta \mathbb{E}_{T_r-1} \left[\widetilde{W}_{T_r}^{\widetilde{\boldsymbol{\Gamma}}}(Z_{T_r} + R_{T_r}^S S_{T_r-1}) \right].$$

$V_t^{\widetilde{\Gamma}}(S_{t-1}, Z_{t-1}; \Gamma_t)$ is the current self's value function conditional on the savings of liquid asset S_{t-1} and illiquid asset Z_{t-1} from the previous period, and a realization of the expenditure shock Γ_t. Note that S_t is a stock variable, and A_t is a flow variable. The stock of illiquid asset at t is written as Z_t. The value function $V_t^{\widetilde{\Gamma}}(S_{t-1}, Z_{t-1}; \Gamma_t)$ also depends on the perception $\widetilde{\boldsymbol{\Gamma}}$, which determines the continuation value $\widetilde{V}_{t+1}^{\widetilde{\Gamma}}(S_t, Z_t; \widetilde{\Gamma}_{t+1})$. The constraints $S_t \geq 0$ and $A_t \geq 0$ rule out borrowing. The wealth X_t available at time t is defined as the sum of current income Y_t and gross return of savings from liquid assets $R_t^S S_{t-1}$, where R_t^s is the gross interest rate on liquid assets.

After the retirement, households until retirement $t \in \{T_r, \cdots, T\}$ solve a following prob-

lem.

$$W_t^{\widetilde{\mathbf{\Gamma}}}(X_t; \Gamma_t) = \max_{C_t, S_t} u(C_t - \Gamma_t) + \delta \mathbb{E}_t \left[\widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t+1}; \widetilde{\Gamma}_{t+1}) \right] \quad \text{where} \tag{2}$$

$$\widetilde{W}_{t'}^{\widetilde{\mathbf{\Gamma}}}(X_{t'}; \widetilde{\Gamma}_{t'}) = \max_{C_{t'}, S_{t'}} u(C_{t'} - \widetilde{\Gamma}_{t'}) + \delta \mathbb{E}_{t'} \left[\widetilde{W}_{t'+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t'+1}; \widetilde{\Gamma}_{t'+1}) \right],$$

$$\text{subject to}$$

$$X_{T_r} = Y_{T_r} + R_{T_r}^S S_{T_r-1} + R_{T_r}^A A_{T_r-1},$$

$$Y_t + R_t S_{t-1} = X_t \geq C_t + S_t \text{ if } t \neq T_r,$$

$$C_t \geq 0, \text{ and } S_t \geq 0, \text{ and}$$

$$\widetilde{W}_T^{\widetilde{\mathbf{\Gamma}}}(X_T; \widetilde{\Gamma}_T) = u(X_T - \widetilde{\Gamma}_T).$$

After retirement, the problem becomes simple, and there are only two choice variables, C_t, and S_t. Also, at the period of retirement, T_r, agents can access the illiquid assets A_{T_r}. At the final period, agents consume all available wealth. I denote the value function after retirement using the letter W to distinguish value functions at two different regimes.

When I refer to optimal consumption and savings before retirement, they are functions solving (1) with inputs S_{t-1}, Z_{t-1}, and Γ_t based on the perception $\widetilde{\mathbf{\Gamma}}$). If there is no room for confusion, I will omit inputs of the functions to save space. Similarly, the optimal consumption and savings before retirement are functions solving (2) with inputs X_t, Γ_t, and $\mathbf{\Gamma}$. Following the literature, an agent is hand-to-mouth if the level of optimal consumption C_t equals available wealth X_t.

The domain of the utility function $u : \mathbb{R} \to \mathbb{R}$ consists of real numbers where $u' > 0$ and $u'' < 0$. Example of this kind of utility function is exponential utility function with constant absolute risk aversion. Inada condition cannot be incorporated in this model unless there is a guarantee that Γ_t is not too large. Alternatively, I can impose the Inada condition and assume that $\Gamma_t < Y_t$ and $\widetilde{\Gamma}_t < Y_t$ at every period. I also assume that R_t^A and R_t^S are deterministic, and $1/\delta > R_t^A > R_t^S$. This assumption removes the incentive to save liquid assets at the period just before the retirement $T_r - 1$, hence $S_{T_r} = 0$. At all periods before retirement $t < T_r$, the consumer saves illiquid and liquid assets at the same time only if

$$\mathbb{E}_t \left[R_t^S u'(C_{t+1}) \right] = \delta^{T_r-t-1} \mathbb{E}_t \left[\left(\prod_{k=t+1}^{T_t} R_k^A \right) u'(C_{t+1}) \right]$$

I call an agent who correctly perceives the future random variable, where the perception $\widetilde{\mathbf{\Gamma}}$ is identical to $\mathbf{\Gamma}$, as a *sophisticated* agent. Considering two different perceptions $\widetilde{\mathbf{\Gamma}}^1$ and $\widetilde{\mathbf{\Gamma}}^2$, I call that $\widetilde{\Gamma}^1$ is more *optimistic* (*pessimistic*) than $\widetilde{\Gamma}^2$ if cumulative density functions (CDF)

in $\widetilde{\mathbf{\Gamma}}^2$ ($\widetilde{\mathbf{\Gamma}}^1$) first order stochastically dominates CDFs in $\widetilde{\mathbf{\Gamma}}^1$ ($\widetilde{\mathbf{\Gamma}}^2$). From now on, I denote $\widetilde{\mathbf{\Gamma}}_t^2 >_1 \widetilde{\mathbf{\Gamma}}_t^1$ if cumulative density function, CDF, of $\widetilde{\mathbf{\Gamma}}_t^2$ first order stochastically dominates CDF of $\widetilde{\mathbf{\Gamma}}_t^1$. Similarly, $\mathbf{\Gamma}^2 >_1 \mathbf{\Gamma}^1$ implies that $\mathbf{\Gamma}_t^2 >_1 \mathbf{\Gamma}_t^1$ for any t.

I investigate the role of $\widetilde{\mathbf{\Gamma}}$ in two parts. The first part investigates the relationship between the perception of the shock and the levels of consumption and savings. The second part focuses on the persistence of the endogenous part of savings $C_t - \Gamma_t$. To make the discussions easier, I focus on life after retirement, which is a one-asset model.

The first proposition argues that all else equal, the consumption of a relatively more optimistic agent is always higher than or equal to the agent with relatively more pessimistic beliefs.

Proposition 1. For every $t \geq T_r$, and $w_t \geq 0$, $C_t(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) \geq C_t(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^2)$ if $\widetilde{\mathbf{\Gamma}}^2 >_1 \widetilde{\mathbf{\Gamma}}^1$.

The intuition behind Proposition 1 is simple. The agent with more pessimistic beliefs perceives that she will face greater expenditure shock, which gives tighter liquidity constraints. Hence, she still values the extra value of saving more than the optimistic agents.

The following result also shows a monotone relationship between the size of wealth and the perception of expenditure shocks. To see this, for $t \geq T_r$ and $k \leq T - t$ define recursively

$$X_{t+k}^i(X_{t+k-1}^i; \Gamma_{t+k}, \widetilde{\mathbf{\Gamma}}^i) = Y_{t+k} + R_{t+k}^S \left[X_{t+k-1}^i - C_{t+k-1}(X_{t+k-1}; \Gamma_{t+k-1}, \widetilde{\mathbf{\Gamma}}^i) \right].$$

Corollary 1. Fix $t \geq T_r$ and $X_t \geq 0$. Then for any $k \leq T - t$ and realization of $Y_t, \cdots, Y_{t+k}$ and $\Gamma_t, \cdots \Gamma_{t+k}$,

$$X_{t+k}^2(X_{t+k-1}^1; \Gamma_{t+k}, \widetilde{\mathbf{\Gamma}}^2) \geq X_{t+k}^1(X_{t+k-1}^1; \Gamma_{t+k}, \widetilde{\mathbf{\Gamma}}^1)$$

as long as $\widetilde{\mathbf{\Gamma}}^2 >_1 \widetilde{\mathbf{\Gamma}}^1$.

This result tells that the for any two agents where the perceptions are ordered as $\widetilde{\mathbf{\Gamma}}^2 <_1 \widetilde{\mathbf{\Gamma}}^1 <_1 \mathbf{\Gamma}$, the consumption profile over the life cycle of agent with $\widetilde{\mathbf{\Gamma}}^1$ would be more flatter. When starting with the same level of wealth X_t and facing same income and expenditure shocks, the wealth of the agent with more optimistic belief, $\widetilde{\mathbf{\Gamma}}^2$ would be depleted faster. Hence, at the end of the life cycle, the agent with pessimistic belief will have more left to consume. Based on this result, this paper uses the stock of liquid asset as a tool to elicit the distribution of relative optimism and pessimism.

Next we develop the dynamic properties of $C_t - \Gamma_t$ and C_t. To see this, let me explicitly write all the random variables as inputs of consumption this period, C_t, and the next period C_{t+1} as $C_t(X_t(Y_t), \Gamma_t)$ and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$. I omit $\widetilde{\mathbf{\Gamma}}$ because comparing perception of the expenditure is not important from now on. Total expenditure $C_t(X_t(Y_t), \Gamma_t)$ can

be decomposed into two parts. The core part of consumption $C_t(X_t(Y_t), \Gamma_t) - \Gamma_t$ is what the agent tries to smooth in the lifetime. The rest, Γ_t is the expenditure shock. The key difference between the two is that, they have very different relationships with total consumption next period, $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$. Suppose that there is an increase in income Y_t or interest rate R_t^s. Then the available wealth at this period X_t will increase, and due to globally concave continuation value, consumption and savings at period t will strictly increase as long as the agent is not hand-to-mouth. This leads to increase in C_{t+1} as well since X_{t+1} increase. When there is an increase in the expenditure shock Γ_t, then the total expenditure C_t in this period will increase. However, the MPC cannot exceed one, so $C_t(X_t(Y_t), \Gamma_t) - \Gamma_t$ will decrease. At the same time, because of decrease in the available wealth next period, consumption next period will decrease in expectation. Hence, in any case, $C_t(X_t(Y_t), \Gamma_t) - \Gamma_t$ and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$ (and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1}) - \Gamma_{t+1}$ will exhibit positive correlation. However, the same mechanism will lead to negative correlation between Γ_t and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1})$ (and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}, \Gamma_t), \Gamma_{t+1}) - \Gamma_{t+1}$). Next proposition formally proves the intuition given above.

Proposition 2. Fix S_{t-1}. Then $\text{cov}(C_t - \Gamma_t, C_{t+1} - \Gamma_{t+1} | S_{t-1}) \geq \text{cov}(C_t, C_{t+1} | S_{t-1})$.

The result of 2 suggests a way of separating the endogenous part of consumption $C_t - \Gamma_t$ from the expenditure shock Γ_t. The categories of expenditure that exhibit significant serial correlation are likely to be smoothed over the life cycle. In contrast, the nonpersistent part of the expenditure would be motivated by short-run fluctuations in taste or urgent spending needs.

Proposition 3. For every $t < T_r$, $C_t(X_t, Z_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) \geq C_t(X_t, Z_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^2)$ if $\widetilde{\mathbf{\Gamma}}^2 >_1 \widetilde{\mathbf{\Gamma}}^1$.

However, the corollary 1 does not extend to the two-asset case. This is because even though household A is more pessimistic than household B, it does not necessarily imply that household A would save both liquid and illiquid assets more than B. For example, suppose imminent expenditure shock is larger than the expenditure shocks after retirement. In that case, the savings of illiquid assets of a pessimistic agent can be lower than the an optimistic agent. A numerical exercise in Figure 2 shows such a case. As the agent moves along the horizontal axis from left to right, the agent becomes more aware of the expenditure shock and becomes more pessimistic. The relatively optimistic households underestimate the need for precautionary saving in period two and mainly save using the illiquid assets, which brings a higher rate of return. However, the sophisticated agents who perfectly know the large expenditure shock mainly save using the liquid asset. The relationship of the consumption and savings with the perceptions in the two-asset case needs to be verified
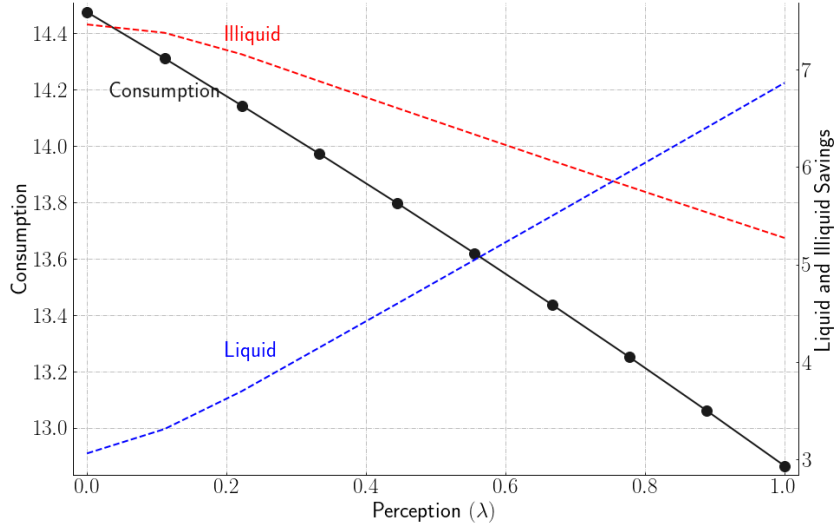
Figure 2: A case where pessimistic agent have lower illiquid savings than the optimistic agent
Notes: simulated by a three-period model with the following parameter values. Utility function is CRRA with $\sigma = 0.5$ (of $u(c) = c^{1-\sigma}/(1-\sigma)$). Income is $\{25, 10, 5\}$ for each period. Expenditure shocks of magnitudes 7 and 1 for periods two and three respectively occur by probability 0.5. The discount factor is 0.95, and the interest rates for liquid and illiquid assets are 0 and 2 percent, respectively. x-axis shows the perception of agents where a value λ in the x-axis denotes the percentage of the shock magnitude perceived by the agent.

using a simulation, and I show in the later part of this section that the more pessimistic agents tend to accumulate more liquid illiquid assets than their optimistic counterparts.

Comparison with Alternative Modeling Strategy The key building blocks of this paper is from Carroll (1992, 1997). The stochastic income is likely to make the consumption function concave, and the presence of the borrowing constraint gives chance to generate low liquidity households that are hand-to-mouth.

Augemented on Carroll (1992, 1997), the two asset environment with an illiquid asset serves a similar role in Kaplan and Violante (2014). A difference is that Kaplan and Violante (2014) has a richer framework by allowing agents to withdraw their illiquid assets at any time as long as they pay the fixed cost. In this model, households are withdrawing illiquid assets possible only once in their lifetime. Hence, there is an infinite amount of fixed cost before the retirement age T_r in terms of Kaplan and Violante (2014). In other words, Kaplan and Violante (2014) allows a symmetric adjustment costs, while this paper assumes an extreme asymmetric cost of adjustment. The rationale of allowing asymmetric cost is that saving in

retirement account generally do not involve any costs. Also, letting the agents to withdraw the asset only once helps to build the theory since the presence of fixed can generate locally convex regions in the value function. Though there are some differences, the two-asset environment in this paper contributes to similar mechanism as the Kaplan and Violante (2014). Agents need less saving for the liquid assets by separating precautionary saving and saving for retirement, which increase chance of having more hand-to-mouth agents. Also, lower interest rate on liquid assets increases the MPC when the agents are not adjusting the illiquid assets.

As a related paper, Lian (2021) introduces a model where the mistakes in future consumption plan generates high MPC to the current self. There are two main differences between Lian (2021) and this paper. First, Lian (2021) derives a robust result regarding MPC, and able to explain high MPC among households with high liquidity. However, the main channel that this paper brings additional MPC is by generating enough low liquidity households. In other words, Lian (2021) focuses shifting the gradient of the consumption function, this paper depends on deviation from rational expectation which pushes optimistic households to have lower liquidity and lead them to the binding borrowing constraint. Second, while all mistakes in the future lead to higher MPC in Lian (2021), this paper shows that only the households underestimates the shock in the future are likely to consume more and likely to have binding borrowing constraint. Hence, in this paper, the direction where the agents are 'wrong' is important.

This paper has direct connection to (β, δ) discounting model. Let us focus on stylized deterministic three-period setting with no liquidity constraints where $\delta = 1$. Two models can be represented as

$$
\begin{aligned}
U^{(\beta,\delta)}(C_1, C_2, C_3) &= u(C_1) + \beta\left[u(C_2) + u(C_3)\right], \text{ and}\\
U^{\Gamma,\widetilde{\Gamma}}(C_1, C_2, C_3) &= u(C_1 - \Gamma) + u(C_2 - \widetilde{\Gamma}) + u(C_3 - \widetilde{\Gamma}).
\end{aligned}
$$

Both models feature characteristics that look like a present bias. For example, the agents are indifferent between different future timings of receiving additional ε since $U^{(\beta,\delta)}(C, C + \varepsilon, C) = U^{(\beta,\delta)}(C, C, C+\varepsilon)$ and $U^{\Gamma,\widetilde{\Gamma}}(C, C+\varepsilon, C) = U^{\Gamma,\widetilde{\Gamma}}(C, C, C+\varepsilon)$. However, as long as $\beta < 1$ and $\Gamma > \widetilde{\Gamma}$, there is a preference reversal, such as $U^{(\beta,\delta)}(C + \varepsilon, C, C) > U^{(\beta,\delta)}(C, C + \varepsilon, C)$ and $U^{\Gamma,\widetilde{\Gamma}}(C+\varepsilon, C, C) > U^{\Gamma,\widetilde{\Gamma}}(C, C+\varepsilon, C)$. In this model, as Γ becomes stochastic, the agent might exhibit this present-biased behavior or not depending on the realized value Γ. Also, as the agent becomes more optimistic than before, there is a higher chance of exhibiting the present-biased behavior, corresponding to β being smaller in the (β, δ) model.

However, when analyzing optimizing agents, two models differ in two dimensions. Unless

the agent is fully naive in the (β, δ) model, there can be discontinuities in the (β, δ) model since tension among future agents creates convex regions in the continuation value (Harris and Laibson, 2002). Moreover, the role of sophistication of β to consumption and savings is not clear. In a deterministic model, Salanie and Treich (2006) shows that the shape of the utility function determines the relationship between the sophistication and the level of consumption; hence the assumption of the utility function can influence the identification of β and $\widetilde{\beta}$ when calibrating in a life cycle model. This model does not feature tension among future agents, but this helps us build a monotone relationship between the perception of the expenditure shock and wealth, which is useful in the calibration process.

Miranda-Pinto et al. (2020) develops a model of consumption and savings with an expenditure shock and explains high consumption variance, the extensive margin of MPCs, and high MPC among low and high-income households. This model takes a parsimonious approach and assumes that expenditure shocks cannot be avoided. In terms of their model, the consumption threshold represents the expenditure shock, and the utility cost when consuming under the threshold tends to infinity. The benefit of taking this parsimonious approach is that it allows analyzing households with different levels of perceptions and accounts for wealth dispersion.

4 Calibration

In this section, we calibrate the distribution of perception to explain the dispersion of liquid assets. First, we check the model properties discussed in the previous section. Next, I explain the empirical target and the calibration strategy. Finally, the calibration result is presented in the last part of this section, and I also check if the models can generate enough MPC and liquid asset inequality.

Calibration of the Expenditure Shock (Needs to be cleaned) Separating the products in the household consumption basket by persistent and nonpersistent parts, we can find a stark difference between the two. For each product j, I measure the degree of persistence of expenditure using the following simple panel regression:

$$\Delta C_{i,j,t} = \rho_j \Delta C_{i,j,t-1} + \varepsilon_{i,j,t} + \text{Year FE}_t + \text{Month FE}_t + \varepsilon_{i,j,t}, \text{ where} \tag{3}$$
$$\Delta C_{i,j,t} = \gamma_j C_{i,j,t-1} + \text{Year FE}_t + \text{Month FE}_t + \xi_{i,j,t}.$$

To deal with endogeneity caused by adding lagged dependent variable, $C_{i,j,t-1}$, I employ the Anderson and Hsiao (1981) estimator by using a lag of level variable, $C_{i,j,t-1}$, as an

instrument. I use a monthly panel since using quarterly data is too short of employing lagged instruments, and it fits the frequency used in the latter part of the paper. Year FE_t and Month FE_t stand for year and month fixed effects, respectively. In this simple regression, ρ_j captures the persistence of $C_{i,j,t}$. The first difference cancels out individual fixed effects. I use the Consumer Expenditure Survey from 1997 to 2013 using 44 product classifications by Kueng (2015) to estimate the persistence based on the model above.[25][26] I apply the sample selection criteria explained in the Appendix C.

Table 9 shows the estimated ρ_j in (3) for every 44 products. Twenty-one products are exhibit positive and significant persistence. Especially, alcohol, tobacco, food, and personal care expenditures are highly persistent. The remaining 23 products do not exhibit strong persistence, such as education service, home management, insurance, and vehicle-related payments. First, denote the set of products that are persistent as S_p and nonpersistent as S_n. Then, we can construct variables $C_{i,t}^p = \sum_{j \in S_p} C_{i,j,t}$ and $C_{i,t}^n = \sum_{j \in S_t} C_{i,j,t}$ which aggregates expenditures over persistent and nonpersistent groups. Also, define the total expenditure of household i at time t as $C_{i,t}$. Next, to see the relationship between total expenditure and $C_{i,t}^p$ and $C_{i,t}^n$ by the following regression:

$$\Delta C_{i,t} = \alpha \Delta C_{i,t-1}^p + \beta \Delta C_{i,t-1}^n + \text{Year FE}_t + \text{Month FE}_t + \varepsilon_{i,t} \text{ where} \qquad (4)$$

$$\begin{pmatrix} \Delta C_{i,t-1}^p \\ \Delta C_{i,t-1}^n \end{pmatrix} = (\eta^p, \eta^n) \begin{pmatrix} C_{i,t-2}^p \\ C_{i,t-2}^n \end{pmatrix} + \text{Year FE}_t + \text{Month FE}_t + \xi_{i,t}. \qquad (5)$$

The estimated α and β are 0.033 and 0.006, respectively. Hence, increase in the nonpersistent part leads to almost no change in the total expenditure next period.[27] When changing the dependent variable in (4) to $C_{i,t-1}^p$, then the estimated α and β becomes 0.046 and -0.001, respectively.

Among all random shocks, if income and interest shocks were dominant, then positive income and interest rate shocks this period should affect positive change in consumption expenditure in this and the next period. Hence, $C_{i,t}^n$ was endogenous part of the consumption and savings problem, then we need to model a non-persistent shock that especially affects $C_{i,t}^n$, which dominates income and interest rate shocks. In this sense, this paper do not explicitly model randomness of preference parameters behind $C_{i,t}^n$ and treat $C_{i,t}^n$ as exogenous.

Next, I compare standard deviations of $C_{i,t}^n$ and $C_{i,t}^p$ in various angles. Over 12 months, I

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[25]We can use detailed product classification, but many of the products in the lowest level share very similar characteristics, such as utility payments for gas and electricity. To reduce the size of the analysis, I use this 44 product classification that reduces this redundancy.

[26]I use the sample only from 1997-2013, where the classification can be applied.

[27]Similar results are found when changing the lag of instruments variables in (5) from 3 to 4 instead of 2.

calculate standard deviation of expenditures for each person and take average of them over all households, that is $\mathrm{sd}(C_{i,t}^m) = \sqrt{\sum_{t=1}^{12}(C_{i,t}^n - \bar{C}_i^n)/12}$ where $\bar{C}_i^n = \sum_{t=1}^{12} C_{i,t}/12$ for $m \in \{n, p\}$. The averages $\mathrm{sd}(C_{i,t}^n)$ and $\mathrm{sd}(C_{i,t}^p)$ over all households are 1,271 and 570, respectively. Though the nonpersistent part takes about a quarter of total expenditure, the standard deviation is more than twice as large compared to the persistent part. Moreover, averages of $\mathrm{sd}(C_{i,t}^n)/\bar{C}_i^n$ and $\mathrm{sd}(C_{i,t}^p)/\bar{C}_i^p$ over all households, which adjusts for sizes of expenditures, are 1.56 and 0.29 respectively. When adjusted for the size, we can see that the standard deviation of nonpersistent part is about 5 times larger than the persistent part.

All of the evidences so far can be summarizes as follows. (1) There is much more randomness in the nonpersistent part than the persistent part, and (2) the past value of nonpersistent part does not explain next period's expenditure of itself or the total expenditure, and (3) it decreases the persistent part of expenditure in the next period. Based on these facts, this paper assume that the nonpersistent part is exogenous since it violates the principles of consumption smoothing compared to the persistent part.

The distribution of persistent and nonpersistent part has a stark difference. From now on, I use expenditures that are normalized by the permanent income as done in (**??**). Figure 3, shows the distribution of nonpersistent and persistent part of the expenditure. First, the distribution of nonpersistent part has a mode near zero, and the frequency of the histogram monotonically decrease when featuring higher values. It is heavily skewed to the right, and the shape can be well represented by an exponential distribution. On the other hand, the
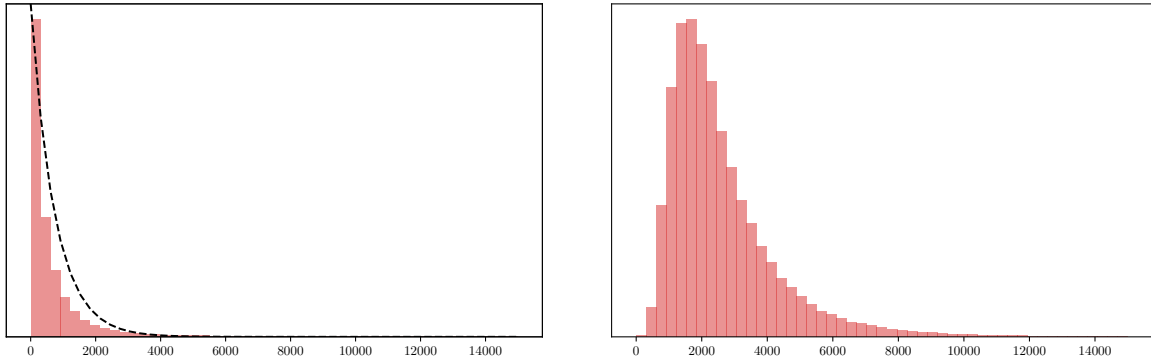
Figure 3: Distribution of normalized nonpersistent part (left) and the persistent part (right) of the expenditure

Note: black dotted line on the left-hand side figure is the probability density function of exponential distribution fitted to match data

nonpersistent part is centered around a positive value and exhibits a log-normal distribution. Thus, almost always, the persistent part is chosen as a positive value. However, nonpersistent are mostly zero in the consumption savings problem. Also, the chances of consuming a

higher amount of nonpersistent components get smaller, and those intertemporal choices are uncorrelated.

Suppose that an agent fits the nonpersistent part distribution based on the exponential distribution with past observations. The efficient maximum likelihood estimator (MLE) for the parameter of exponential distribution would be the mean of the past observations. In a distribution, such as exponential distribution[28], the median is always less than the mean, which implies that the median person will observe a value that is less than the true mean. Her MLE estimate of the distribution will be underestimating the true parameter.[29]

The process of expenditure shock is also from the Consumer Expenditure Survey, based on the analysis of section 2. I apply fourth-order polynomial approximation with age to filter the trend of the mean level of expenditure shocks over the life-cycle. However, as we can see from Figure 4, it is still relatively flat compared to income, and it is small relative to income. To make the model solvable, I transform the utility function as $u(c) = (\max\{c, \underline{u}\})^{1-\sigma}/(1-\sigma)$, where $\underline{u} = 10^{-5}$ (1 cent). As we can see from Figure 4, since the expenditure shock has a small portion out of income, the lower bound $\underline{u}$ is rarely triggered, and varying this value does not affect the main results. All values of consumption and assets in the simulation represent 2019 dollars adjusted by the consumer price index.

Agent i will be endowed λ_i, representing the degree of perception of the expenditure shock. While the true distribution that draws the expenditure shock at time t is $\exp(\mu_t)$, the household i perceives that the expenditure shock will drawn from $\exp(\lambda_i \mu_t)$. Hence, $\lambda_i = 1$ implies that the agent i is sophisticated. If $\lambda < 1$ ($\lambda > 1$), then the agent is optimistic (pessimistic) compared to the sophisticated agents. I assume that the maximum level of λ_i is 1.5, enough to generate agents at the higher end of the distribution.

Income Process   Following Carroll (1997), income process follows

$$Y_{i,a} = P_{i,a}\Xi_a\varepsilon_{i,a} \tag{6}$$

$$\text{where}$$

$$P_{i,a} = P_{i,a-1}\xi_{i,a}, \text{ and}$$

$$\Xi_a = \beta_0 + \beta_1 a + \beta_2 a^2 + \beta_3 a^3 + \beta_4 a^4.$$

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[28]The result generalizes to the case where $f(c)$ is decreasing in c where $f(\cdot)$ is the probability density function.

[29]With exponential distribution, the result can be generalized as follows. Fix n, and $\bar{x} = (x_1 + \cdots + x_n)/n$ where $x_1, \cdots x_n$ are all drawn from the identical exponential distribution $\text{Exp}(\lambda)$ with parameter λ. Then median of $\bar{x}$ is less than λ. In the limit, $n \to \infty$, $\bar{x}$ tends to λ by the central limit theorem.

Income of individual i at age a depends on permanent level of income $P_{i,a}$, age-specific term Ξ_a which shapes the overall income profile, and transitory shock $\varepsilon_{i,a}$. Shocks in permanent and transitory income, $\ln \varepsilon_{i,a}$ and $\ln \xi_{i,a}$, follows normal distribution with standard deviations 0.013 and 0.043 respectively, following Carroll et al. (2017)[30]. Age-specific term Ξ_a follows fourth-order polynomials of age, which is also standard. This paper estimates parameters determining Ξ_a by using the Consumer Expenditure Survey from year 1997 to 2013.

The choice of modeling permanent income as a geometric random walk follows Carroll (1997) which allows normalizing every variable by the permanent level of income. In the simulation, I also use the normalized variables for two reasons. First, normalizing by permanent income allows reducing the number of state variables. Second, the paper does not depend on income dispersion to explain liquid and illiquid wealth inequality. Hence, analyzing the normalized model allows controlling the influence of permanent income. Otherwise, it would be difficult to flesh out the contribution of permanent income towards the dispersion of wealth. The model lacks income, social security taxes, and unemployment (with unemployment benefits). Those features can make the analysis richer, but they all can potentially affect the dispersion of wealth and marginal propensity to consume. For example, adding unemployment would increase the desire for precautionary saving, but the marginal propensity to consume can be higher when the agent is unemployed. Social security tax or benefits are not in the model, but the income process used in this model follows realistic features such as low monthly income after retirement with a peak during the mid-age.

Discount Factor and Interest Rates   Discount factor δ is 0.96 in annual terms, which is standard. The returns of liquid and illiquid assets are held constant. Following Kaplan and Violante (2014), the real return on liquid asset R^S is -0.7 percent, and illiquid asset is 2.84 percent respectively in annual terms. Hence $\delta < 1/R^A < 1/R^S$.

The Choice of IES   I use the constant relative risk aversion utility function, $u(c) = c^{1-\sigma}/(1-\sigma)$. One of the most important parameters when modeling the life cycle consumption and savings problem is the intertemporal elasticity of substitution (IES), which is $1/\sigma$ in this paper. Empirically, when examining micro evidence, IES is suggested to be 1/3 (Havranek and Sokolova, 2020). In most applications, especially when explaining excess sensitivity of consumption or wealth dispersion, IES rarely goes lower 1. IES is crucial because it serves as one of the key components of the effective discount factor, $(\delta R)^{1/\sigma}$. With low values of $1/\sigma$, the effects from the discount factor become muted. Hence, to achieve

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[30]This paper has monthly time frequency, so the standard deviations given in Carroll et al. (2017), which uses quarterly frequency, are divided by 3

excess sensitivity using the low value of IES plain vanilla consumption savings model require a lower value of discount factor δ to overcome low IES. However, if the IES is too high, it cannot generate an accumulation of wealth comparable to data (Aguiar et al., 2020). This paper uses $1/3$ as the benchmark value of IES, as suggested in the empirics.

Agents enter the economy at age 25 and retire at age 65. Everyone dies at age 80 with no bequest. Adding bequest can help explain the increasing trend of wealth even after retirement, but I try to keep the model as simple as possible to rule out factors that can affect the paper's primary purpose: explaining the distribution of wealth and the MPC. The model has a monthly frequency, which results in 660 periods, with 480 periods before retirement.

Model Properties   To investigate the fundamental property of the model, I simulate the model economy with 256,000 agents with λ_i that is evenly distributed over $[0, 2]$. Everyone starts with the same initial level of wealth, which is the median liquid and illiquid wealth of household heads aged 24-26 of the Survey of Consumer Finances.

Figure 4 tracks the median wealth, consumption, and MPC within every twenty groups of perceptions $[\lambda^j, \lambda^j + 0.1)$ where $\lambda^0 = 0$ and $\lambda^{20} = 1.9$. Then, I track the median wealth of households within each quintile of perceptions. As hinted by Propositions 1 and 3, Figure 4 shows that an agent with a higher level of λ_i accumulates a larger amount of liquid and illiquid assets throughout their life. Especially at age 65, the wealth of top group with $\lambda \in [1.9, 2.0)$ about is 6 times larger than the lowest group with $\lambda \in [0, 0.1)$.

Unlike liquid and illiquid wealth, different degrees of perception does not make remarkable differences in consumption except in the initial and terminal periods. Agents with correct perception exhibit a flat consumption profile until they reach retirement, which implies successful consumption smoothing under the low IES of $1/3$. Since they are worried about the possibility of high expenditure shocks, they keep liquid assets for precautionary motives, which makes the consumption rise near the terminal period. The precautionary saving motive is higher among relatively more pessimistic households. Hence, agents with an extreme level of pessimism near $\lambda_i \simeq 2$ perceive that their initial liquid wealth is not enough and start accumulating liquid assets right away, making initial consumption lower than the other groups. Optimistic agents fail to underestimate the size of expenditure shock, which depletes liquid assets and high consumption at the beginning of life.

There are also large differences in the MPC over the life cycle by different levels of perceptions. First, the optimistic agents exhibit higher MPC than the pessimistic agents. Since optimistic agents occasionally meet binding borrowing constraints, their MPC is also larger than the pessimistic agents. The average monthly MPC of the most optimistic households,
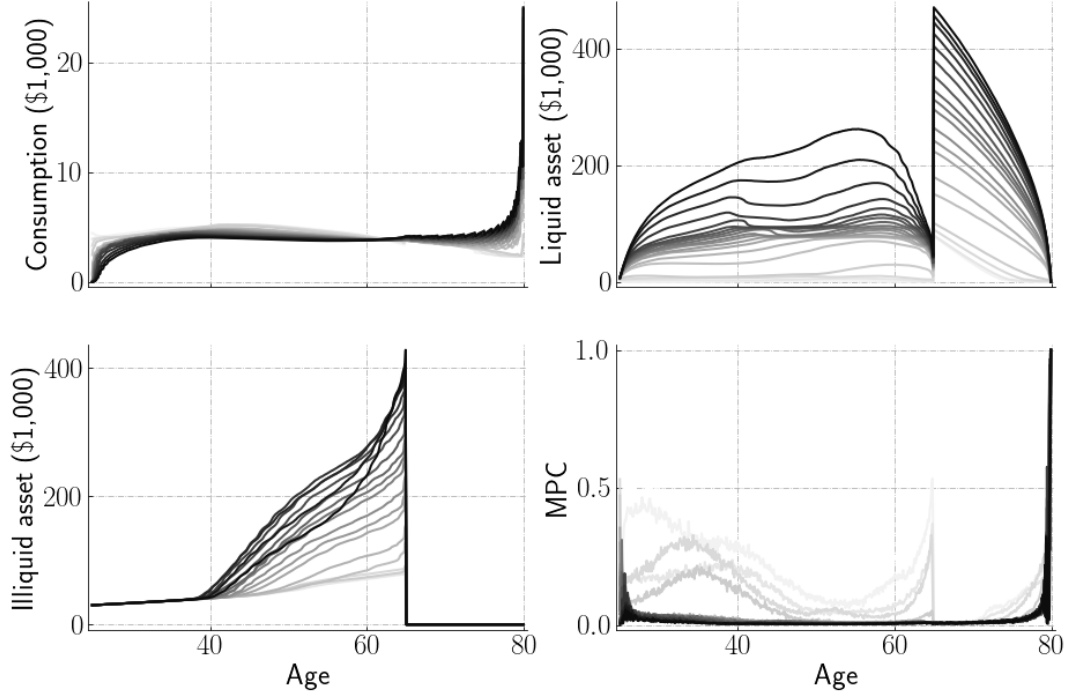
Figure 4: Trajectory of key variables by different levels of perception
Note: twenty groups of agents with perceptions $[\lambda^j, \lambda^j + 0.1)$ where $\lambda^1 = 0$ and $\lambda^{20} = 1.9$ are plotted where darker line indicates more pessimistic agents where λ^j is larger.

$\lambda_i \in [0, 0.1)$, is 22.4 while the middle group, $\lambda_i \in [0.9, 1.0)$, exhibits 2.2 percent, and the most pessimistic group, $\lambda_i \in [1.9, 2.0)$ exhibits 1.4 percent. Since the low liquidity drives the high MPC, there will be negligible differences among agents that accumulate enough wealth so that their borrowing constraint will be rarely binding. Hence, agents with perceptions $\lambda_i \in [0, 0.5)$ show high MPCs, while the others do not.

The trajectory of the MPC out of a transitory income shock follows the implication from the buffer-stock theory of savings (Carroll, 1992, 1997). The MPC is generally high at young ages since the current income is lower than the future income levels, which makes the borrowing constraint binding. As households accumulate assets at the middle age, MPC falls and becomes almost zero when the income is at its peak. However, consumption responses of optimistic agents versus rational expectations or pessimistic agents starkly differ near retirement. The amount of liquidity they have accumulated so far sets the consumption limit for all agents at retirement. However, optimistic agents who underestimate the need for precautionary savings may face larger expenditure shock than they expected and become

hand-to-mouth. On the other hand, agents who have accumulated enough assets do not exhibit a large consumption response from a transitory income shock. The mismatch between expected and actual liquidity available at retirement serves as a 'second terminal-period effect' and makes MPC rise as households retire.

Calibration Strategy and Empirical Target The key moment to match in the calibration exercise is the distribution of wealth. This paper adopts the strategy of Castañeda et al. (2003); Carroll et al. (2017) by targeting the share of liquid wealth held by 20, 40, 60, 80 percentiles of along with median liquid wealth among working-age households. There can be various reasons to hold illiquid assets, such as bequest motive and housing investment which are not modeled in this paper. Also, the dispersion of liquid wealth in the model can be affected by the introduction of large illiquid wealth at retirement; hence I only focus on the dispersion of liquid wealth before retirement. By introducing arbitrarily many optimistic households, the model has a chance to highlight households with very low liquidity and perfectly explain the share of wealth held at different percentiles. To prevent this issue, I let the model match the median liquid wealth over income similar to Carroll et al. (2017).

There have been various approaches to model the heterogeneity of consumers with different parameters. A commonly used approach is assuming a discrete number of agents with different utility or discounting parameters such as Krusell and Smith (1998). There is also an approach by Carroll et al. (2017) assuming uniform distribution and finding lower and upper bound of the parameter. In reality, the parameter of interest is likely to be drawn from a continuous distribution, and we cannot *a priori* guess the shape of the distribution. To employ a continuous and versatile distribution, I use a beta distribution that can have various shapes, nesting uniform distribution as a special case. Hence, the perception of agents λ_i is assumed to be drawn from a beta distribution. I fit three parameters where each has a distinct role. The lefthand side panel of Figure 5 shows the role of first two parameters, α and β. They control the shape of the beta distribution. The theoretical mean is $\alpha/(\alpha + \beta)$; hence, a larger value of α implies more agents with rational expectations in general. The size of both α and β controls the height of the peak. If $\alpha = \beta = 1$, then the distribution becomes a uniform distribution where the probability density function is flat. If α and β are high, the values drawn will be centered around a mean. To control the overall size of the values in the distribution, I also fit a scaling parameter γ representing the maximum value in the domain.

Calibration Result The set of parameters to generate the moments in Table 4 is $(\alpha, \beta, \gamma) = (1.53, 6.22, 1.58)$. Calibrated parameters imply that the distribution of perception would be
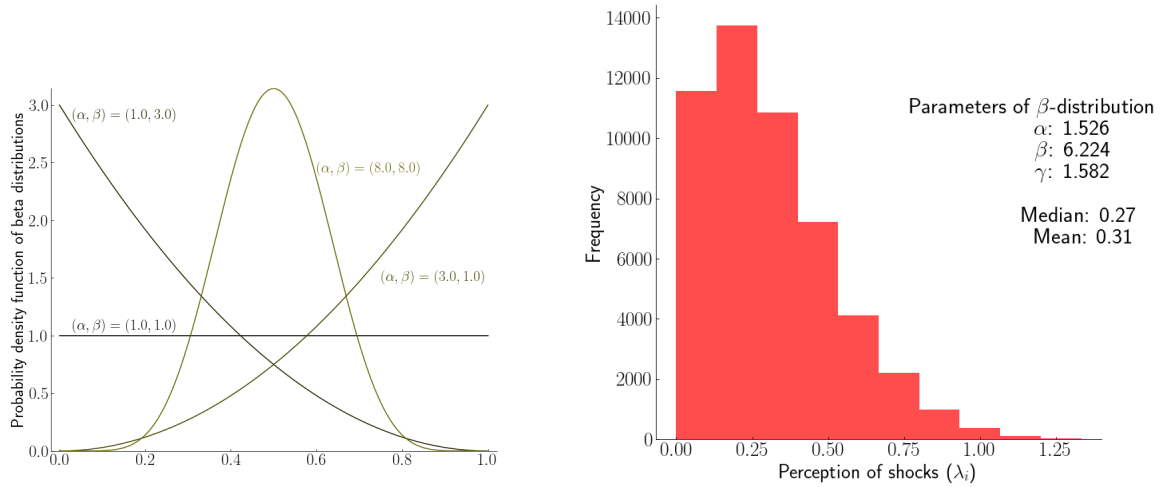
Figure 5: Examples of beta distribution (left) and the calibrated distribution (right)

| | Share of wealth held at percentiles | | | | Median liquid wealth |
	20th	40th	60th	80th	over income
Data	<0.01	0.01	0.05	0.2	0.89
Model	<0.01	0.02	0.08	0.23	0.89

Table 4: Calibration result

looking as the righthand side panel of Figure 5. The average household would realize 27 percent of the true mean of the expenditure shocks. This low level of perception is necessary to make the model generate a low level of liquid wealth among working-age households. The maximum value of the distribution $\gamma = 1.58$ implies the existence of a small group of households that are more pessimistic than households with rational expectations.

Table 4 compares the moments in the data and moments in the simulated data where the choice of parameters are the ones that minimize the root mean squared error from the data. The model-generated moments are fairly close to the data. Especially, the median liquid wealth over income is well aligned with the data. Hence, the calibration result is not driven by arbitrarily generating many households with very low liquidity. However, there is some gap between the share of wealth at 60th and 80th percentile. Unlike the simulated model, households with an exceptionally high level of wealth near the top level of wealth exist, which is difficult to generate using the life-cycle model. As a result, Figure 6 shows that the model-generated Lorenz curve is slightly flatter than the data.

Table 5 compares the trend of liquid wealth over income at different percentiles of data and the simulated model. The numbers in the table are not the targets of the calibration
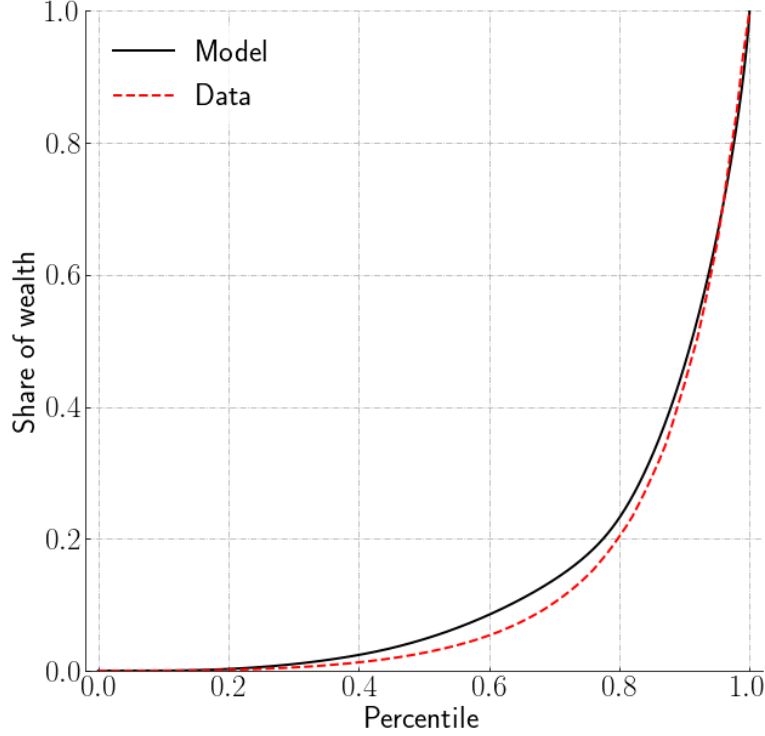
Figure 6: Lorenz Curve: Data versus Model

exercise, but the simulated moments line up well with the actual data. Especially, it can explain that over 40 percent of the households in data do not have liquidity near their monthly income.

Next, I compute the marginal propensity to consume by giving \$800 to all households which are not taxed later. For household i at time t, I first compute the original level of consumption $c_t^i(w_t^i)$. Under the same environment, compute the counterfactual level of consumption using the rebate r, which is $\hat{c}_t^i(w_t^i + r)$. Then, the MPC of household i at time t can be calculated as $MPC_t^i = (\hat{c}_t^i(w_t^i + r) - c_t^i(w_t^i))/r$. In this setting, the model explains high overall MPC. The average monthly MPC among all households is 15.9 percent. In quarterly and annual terms, it can be translated to 40.5 and 87.4 percent following the conversion formula used by Carroll et al. (2017).[31] We can see that the extensive margin plays a big role from the lefthand side panel of Figure 7 since most of the households do not show a positive consumption response to the rebate. However, households with large

---

[31]The exact values for quarterly and annual MPC need to be examined by actually tracking the change in consumption over the quarter and year.

		Liquid wealth over income			
		20th	40th	60th	80th
25-34	Data	0.12	0.44	1.06	2.67
	Model	0.02	0.35	1.14	2.6
35-44	Data	0.15	0.51	1.18	3.14
	Model	0.14	0.36	1.17	2.6
45-54	Data	0.20	0.61	1.48	4.29
	Model	0.32	0.67	1.38	4.35
55-64	Data	0.28	0.86	2.31	7.23
	Model	0.16	0.57	1.48	8.03
Overall	Data	0.21	0.68	1.74	5.40
	Model	0.14	0.48	1.26	4.22

Table 5: Liquid wealth over income at 20, 40, 60, and 80th percentiles

Note: numbers in parenthesis are from the simulated model, and others are from the data

expenditure shocks would move to the region with a positive consumption response. This will be more effective among households with low levels of liquid wealth. The righthand side panel suggests extensive margin in the individual consumption response. When a household faces a large expenditure shock, then temporarily, they may exhibit high MPC. However, in most cases where they have negligible expenditure shocks, they behave as agents in the plain vanilla consumption and savings problem with a low MPC. The righthand side panel
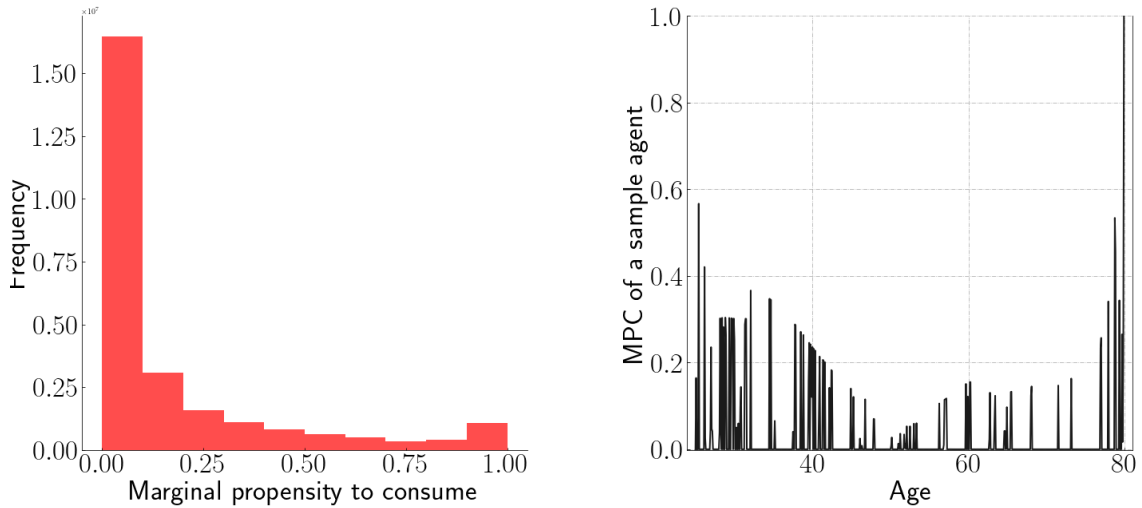

Figure 7: Distribution of MPC (left) and sample individual consumption response (right)

of Figure 7 shows the individual response of a sample agent who perceives 50 percent of the actual shock. Not only there is an extensive margin of consumption responses in the

aggregate, but depending on the size of the expenditure shocks, each individual also exhibits infrequent but large consumption responses.

Number of Assets	Expectations	Use expenditure shocks	MPC
One	Rational expectation	No	1.9
Two	Rational expectation	No	3.2
One	Rational expectation	Yes	1.4
One	Fully optimistic	Yes	13.3
Two	Rational expectation	Yes	2.3
Two	Fully optimistic	Yes	23.9

Table 6: Comparison of MPCs across different models

Table 6 compares the MPCs across different modeling strategy. Under rational expectations, the MPC without the use of expenditure shock is only 1.9 percent. Using the two-asset model under rational expectations, the MPC becomes 3.2 percent, almost twice the one-asset model. Unlike Kaplan and Violante (2014), separation of precautionary savings and savings for retirement does not have a dramatic effect on the MPC. This is because when using the small IES, the desire to smooth consumption is strong, and the consumption profile will be flat as shown in Figure 4. Hence, households save enough liquid assets even with a low-interest rate on liquid assets, making the borrowing constraint rarely binding.

The rational expectation agent exhibits a lesser consumption response and an MPC of 1.4 percent under the one-asset framework by introducing the expenditure shocks. By introducing additional shock, the increased need for precautionary saving makes households save more. Also, there is a clear distinction between agents with rational expectations and fully optimistic beliefs. The fully optimistic agent under the one-asset framework exhibits an MPC of 13.3 percent. When households are optimistic, their expected wealth accumulation does not meet the actual wealth since they face larger expenditure shocks than anticipated before.

The two-asset framework amplifies the channel of generating higher MPC. For rational expectation agents and fully optimistic agents, the MPC is about twice the MPC under the one-asset framework. Though the large IES dampens the mechanism generating high MPC in the two-asset frame under rational expectations, introducing the agents with behavioral frictions makes lower liquid assets among optimistic agents, and separating the savings for retirement exacerbates the low liquidity where the liquid assets only serve the role for precautionary saving.

5 Policy Implications

In this section, we investigate the role of perception $\widetilde{\mathbf{\Gamma}}$ in consumer welfare. For simplicity, I consider the one asset model which features life after retirement. I define the *welfare function* $W_t^*(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}})$ in a paternalistic view, which captures expected *realized* lifetime utility based on the perception $\widetilde{\mathbf{\Gamma}}$ as

$$W_t^*(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}) = u(C_t(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}) - \Gamma_t) + \delta \mathbb{E}\left[W_{t+1}^*(X_{t+1}; \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}})\right]$$

where

$$C_t(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}) = \arg\max_{c_t} u(C_t - \Gamma_t) + \delta \mathbb{E}_t \left[\delta \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(X_{t+1}; \widetilde{\Gamma}_{t+1})\right], \text{ and}$$

$$C_t \leq X_t = Y_t + R_t^s(X_{t-1} - C_{t-1}).$$

The following proposition establishes a basic result: there is monotonicity of the welfare function between two pessimistic (or optimistic) agents.

Proposition 4. For any $t \geq T_r$, if $\mathbf{\Gamma} >_1 \widetilde{\mathbf{\Gamma}}^1 >_1 \widetilde{\mathbf{\Gamma}}^2$, then $W_t^*(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) \geq W_t^*(X_t; \Gamma_t \widetilde{\mathbf{\Gamma}}^2)$. Also, $\mathbf{\Gamma} <_1 \widetilde{\mathbf{\Gamma}}^1 <_1 \widetilde{\mathbf{\Gamma}}^2$, implies that $W_t^*(X_t; \Gamma_t, \widetilde{\mathbf{\Gamma}}^1) \geq W_t^*(X_t; \Gamma_t \widetilde{\mathbf{\Gamma}}^2)$.

The message of Proposition 4 is simple. If we have two optimistic or pessimistic agents. The agent that is closest to the sophisticated case, $\mathbf{\Gamma}$ will have larger lifetime expected welfare. For example, for a pessimistic agent, the agent suffers from welfare loss by his overconsumption. Though he optimally chooses his level of expenditure based on his belief $\widetilde{\mathbf{\Gamma}}$, this is not optimal in the sense of welfare, and the consumption will not be smooth enough. Moreover, this welfare loss gets accumulated over the life cycle. As the agent becomes more and more pessimistic, the welfare loss will become larger as well.

This result shows that when choosing priorities of who to 'correct,' the government should first target the agents with abnormally low or high amounts of savings for a paternalistic government. For these agents, the expected increase in welfare will be higher by government intervention.

Ricardian Equivalence Sometimes, the government wants to boost economic activity rather than enhance welfare by maintaining a stable consumption path. In this case, Proposition 1 remind that the agents with most optimistic beliefs will likely show greater consumption response. However, in the real world setting, the fiscal policy can be constrained by methods of financing fiscal stimulus. In an extreme case where agents realize the increase in future tax burden, the fiscal policy would have no effect at all, which is the famous Ricardian equivalence theorem (Barro, 1974). Unfortunately, in this model, when the agents are not

hand-to-mouth, fiscal transfer funded by the tax next period will not affect agents' decisions. However, since the marginal continuation value of the optimistic agent is always lower than the pessimistic agent, we can expect that the optimistic agent will likely have more binding liquidity constraints. For these agents, the fiscal policy can still have a net effect even when agents perceive that it will be taxed later.

An Example of Ex-Post Welfare Improving Policy   What kind of policy can improve the welfare of households with behavioral frictions? This question can be important in reality since if the government can correct the behavior of households with simple policy tools, it can prevent households from facing high borrowing costs when they face large expenditure shocks.

To answer this question, I impose three criteria that a policy should satisfy. First, a policy should not depend on external funds, and preferably, the implementation should be possible without inter-personal transfers. Without this condition, there can be obvious welfare improving policy of spreading money to households. Also, if the government can make inter-personal transfers, it would be crucial to assign the utility weights to different households and discuss what type of households we should prioritize. Second, the policy instruments should follow observable variables. Hence, the government cannot design a policy conditional on the agents' optimism and pessimism, which would also be impossible in the real world situation. Third, the government implements dynamically consistent policies, and the households should form correct beliefs about them. Without this condition, the degree of freedom for the government is large, and the government will eventually lose credibility.

I suggest the following policy penalizing low liquidity and giving households enough buffer to prevent expenditure from depleting their liquid wealth. The government imposes a proportional tax τ_l up to the liquid wealth of $W_{l,t}$ at time t. For wealth that exceeds $W_{l,t}$ and less than $W_{r,t}$, the government gives a tax rebate proportional to τ_r. Hence, the tax $T_{i,t}(X_{i,t})$ that households i with the level of liquid wealth $X_{i,t}$ pay at time t would be

$$T_{i,t}(X_{i,t}) = \min\{W_{l,t}, X_{i,t}\}\tau_l - \min\{\max\{X_{i,t} - W_{l,t}, 0\}, W_{r,t} - W_{l,t}\}\tau_r, \qquad (7)$$

which is also the tax revenue for the government. In simple terms, the government penalizes households with wealth less than $W_{l,t}$ but induces optimistic households to accumulate the liquid wealth at least $W_{l,t}$. However, such policy can make households with the rational expectation to undersave and keep the level of wealth between $W_{l,t}$ and $W_{r,t}$, so there is no guarantee for overall welfare improvement. However, if we keep $W_{r,t}$ small enough, households that already accumulate large liquid wealth will not be affected by this policy. At the

same time, $W_{r,t}$ should be large enough to make low liquidity households be prepared for the underestimated expenditure shocks.

If $W_{r,t} = 2W_{l,t}$ and $t_l = t_r$, then all of the transfers $(X_{i,t} - W_{l,t})t_r$ can be covered by the taxes $X_{i,t}t_l$ collected earlier, so this policy does not need any interpersonal transfers. Hence, if households correctly form beliefs about (7), then such policy will satisfy all three criteria mentioned above. I simulate the economy with the above setting under various t_l and $W_{l,t}$ based on the calibrated distribution of perception in Figure 5. I impose that $W_{l,t} = \bar{w}\Xi_t$ where Ξ_t is the age specific trend of income appeared in (6). Rather than fixing $W_{l,t}$ and $W_{r,t}$, by making $W_{l,t}$ to be proportional to the trend of income, I can make the intended minimum savings $W_{l,t}$ to vary with income so optimistic households will be naturally inclined to save larger amount of liquid wealth during the middle age when there income peaks.

$T_{l,t}/\Xi_t$	$T_{r,t}/\Xi_t$	τ_l	τ_r	Avg. utility	Avg. revenue
		0.01	0.01	-191.6	0.06
1	2	0.02	0.02	-191.7	0.07
		0.03	0.03	-192.1	0.1
		0.04	0.04	-192.9	0.14
		0.01	0.01	-189.7	0.13
2	4	0.02	0.02	-188.8	0.23
		0.03	0.03	-188.8	0.29
		0.04	0.04	-189.5	0.53
No policy benchmark				-191.8	0

Table 7: Policy Simulation Based on the Calibrated Model

Note: 'Avg. utility' refers to mean level of utility which is defined as $\frac{1}{I \times T}\sum_{i \in I}\sum_{t \in T} u(C_{i,t} - \Gamma_{i,t})$. where I is the number of the simulated households and T is the total number of periods. 'Avg. revenue' refers to the average revenue that the government raises which is defined as $\frac{1}{I \times T}\sum_{i \in I}\sum_{t \in T} T_{i,t}(X_{i,t})$. The unit of revenue is \$1,000. τ_l and τ_r in the table are annualized, actual value τ_l^{actual} used in the simulation is $\tau^{*,\text{actual}} = 1 - (1 + \tau^*)^{1/12}$ for $\tau^* = \{\tau_l, \tau_r\}$.

Table 7 shows the result of the calibration exercise. I have chosen four different tax (or transfer) rates, which are 1, 2, 3, and 4 percent in an annualized term. In some of the parameters, the average utility is higher than the benchmark case without any policy. Especially when $T_{l,t}/\Xi_t = 2$ and $T_{r,t}/\Xi_t = 4$, the average utility is higher than the benchmark case. Hence, inducing agents to save the liquid wealth of twice their income trend can be desirable. At the same time, the government can raise tax revenue by increasing the τ_l and τ_r as well.

6   Conclusion

Using a model of heterogeneous agents with different perceptions of expenditure shocks, this paper generated high overall MPC by matching the severe dispersion of liquid wealth found in the data. The calibration shows that most households have a low perception of future expenditure shock, which leads them to have a very low level of liquidity and exhibit high MPC. I conclude by discussing several extensions of this paper.

First, the model in this paper assumed a fixed retirement date, with no early withdrawal of illiquid assets. Alternatively, we can impose adjustment costs as Kaplan and Violante (2014). Introducing adjustment costs can further incorporate real-world behavior such as early withdrawal of retirement accounts. However, adjustment costs may break the concavity of the continuation value in the two-asset problem and hence disrupt the differentiability of the value function, which can impede obtaining theoretical results in Proposition 3. Nevertheless, the benefits of quantitative exercise incorporating adjustment costs certainly exist and desire attention.

Second, this paper depended on the parsimonious model of expenditure shocks which was assumed to be exogenous. Though nonpersistent short-run temptations mainly drive products classified as expenditure shocks, they cannot be entirely outside the life cycle consumption and savings plan. This assumption can be relieved by modeling the expenditure system where households have an imperfect perception of preferences. However, to rationalize the high variance of the nonpersistent part and its stark difference distribution with the persistent part, it would be necessary to incorporate large and temporary preference shocks to endogenize the expenditure shock introduced in this paper.

Third, learning of expenditure shocks was not allowed. This paper isolated the chance for learning based on the calibrated distribution of the expenditure shocks. Since the distribution of the expenditure shock is extremely skewed, median households will underestimate the mean and mainly observe the shocks with low values. Also, there are many categories of consumption that would make it difficult to learn all the aspects of the expenditure shocks. Therefore, I limited the model's scope and the degree of freedom by focusing on the pure role of optimism and pessimism. However, some extreme events can shape the expectation of households, and such learning behaviors can also lead to an interesting source of heterogeneity.

A   Proofs

Proof of Proposition 1

Proof. Write the cumulative density functions of $\widetilde{\boldsymbol{\Gamma}}_t^1$ and $\widetilde{\boldsymbol{\Gamma}}_t^2$ as $F_t^1 : \mathbb{R}_+ \to [0,1]$ and $F_t^2 : \mathbb{R}_+ \to [0,1]$, respectively. I prove using backward induction. At the final period, consumers spend all available wealth, hence $C_T(X_T, \Gamma_T; \widetilde{\boldsymbol{\Gamma}}) = X_T$ whatever Γ_T and $\widetilde{\boldsymbol{\Gamma}}$ is. Since $F_t^1 >_1 F_t^2$, it follows that

$$
\mathbb{E}_{T-1}\left[\frac{\partial W_T^{\widetilde{\boldsymbol{\Gamma}}^1}(X_T; \Gamma_T)}{\partial X_T} \right] = \int u'(X_T - \Gamma_T) dF_T^1(\Gamma_T) > \int u'(X_T - \Gamma_T) dF_T^2(\Gamma_T)
$$

$$
= \mathbb{E}_{T-1}\left[\frac{\partial W_T^{\widetilde{\boldsymbol{\Gamma}}^2}(X_T; \Gamma_T)}{\partial X_T} \right].
$$

It is obvious that $\mathbb{E}_{T-1}\left[W_T^{\widetilde{\boldsymbol{\Gamma}}^1}(X_T; \Gamma_T) \right]$ is concave to X_T. Now we proceed to general periods. Let the following properties [T1] and [T2] that hold at period $t+1$:

$$
[\text{T1}] \quad 0 < \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\boldsymbol{\Gamma}}^2}(X_{t+1}; \widetilde{\Gamma}_{t+1})}{\partial X_{t+1}} \right] < \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\boldsymbol{\Gamma}}^1}(X_{t+1}; \widetilde{\Gamma}_{t+1})}{\partial X_{t+1}} \right], \text{ and}
$$

$$
[\text{T2}] \quad \mathbb{E}_t\left[\frac{\partial^2 W_{t+1}^{\widetilde{\boldsymbol{\Gamma}}^2}(X_{t+1}; \widetilde{\Gamma}_{t+1})}{\partial X_{t+1}^2} \right] < 0 \text{ and } \mathbb{E}_t\left[\frac{\partial^2 W_{t+1}^{\widetilde{\boldsymbol{\Gamma}}^2}(X_{t+1}; \widetilde{\Gamma}_{t+1})}{\partial X_{t+1}^2} \right] < 0.
$$

Then, I show that properties [T1] and [T2] also hold at period t, and moreover, $C_t(X_T; \Gamma_t, \widetilde{\boldsymbol{\Gamma}}^1) \leq C_t(X_T; \Gamma_t, \widetilde{\boldsymbol{\Gamma}}^2)$ where the inequality is strict when the consumer saves a positive amount.

For any $t \geq T_r$, fix a shock Γ_t. By [T1] and [T2], if the agent is hand-to-mouth under $\widetilde{\boldsymbol{\Gamma}}^1$, then the agent is also hand-to-mouth under $\widetilde{\boldsymbol{\Gamma}}^2$ since $u'(C_t - \Gamma_t) > \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\boldsymbol{\Gamma}}^1}(X_{t+1}; \widetilde{\Gamma}_{t+1})}{\partial X_{t+1}} \right] > \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\boldsymbol{\Gamma}}^2}(X_{t+1}; \widetilde{\Gamma}_{t+1})}{\partial X_{t+1}} \right]$ for any $C_t \in (0, X_t]$. Hence, in this case, $C_t(X_T; \Gamma_t, \widetilde{\boldsymbol{\Gamma}}^1) = C_t(X_T; \Gamma_t, \widetilde{\boldsymbol{\Gamma}}^2) = X_t$ by [T2]. If the agent saves under $\widetilde{\boldsymbol{\Gamma}}^1$ where $C_t(X_T; \Gamma_t, \widetilde{\boldsymbol{\Gamma}}^1) < X_T$, then

$$
u'(C_t(X_T; \Gamma_t, \widetilde{\boldsymbol{\Gamma}}^1)) = \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\boldsymbol{\Gamma}}^1}(w_{t+1}; \widetilde{\Gamma}_{t+1})}{\partial w_{t+1}} \right] > \mathbb{E}_t\left[\frac{\partial \widetilde{W}_{t+1}^{\widetilde{\boldsymbol{\Gamma}}^2}(w_{t+1}; \widetilde{\Gamma}_{t+1})}{\partial w_{t+1}} \right]
$$

by [T1]. Then we have $C_t(X_T; \Gamma_t, \widetilde{\boldsymbol{\Gamma}}^1) < C_t(X_T; \Gamma_t, \widetilde{\boldsymbol{\Gamma}}^2)$. Whether or not the agent is hand-to-mouth under $\widetilde{\boldsymbol{\Gamma}}^1$, this leads to

$$
\frac{\partial \widetilde{W}_t^{\widetilde{\boldsymbol{\Gamma}}^1}(X_t; \widetilde{\Gamma}_t)}{\partial X_t} = u'(C_t(X_T; \widetilde{\Gamma}_t, \widetilde{\boldsymbol{\Gamma}}^1) - \widetilde{\Gamma}_t) \geq {}^\backprime u'(C_t(X_T; \widetilde{\Gamma}_t, \widetilde{\boldsymbol{\Gamma}}^2) - \widetilde{\Gamma}_t) = \frac{\partial \widetilde{W}_t^{\widetilde{\boldsymbol{\Gamma}}^2}(X_t; \widetilde{\Gamma}_t)}{\partial X_t}.
$$

Using the fact that $F^1 >_1 F^2$, we can conclude that [T1] holds at time t. [T2] is obvious because consumption is an increasing function of wealth. □

Proof of Proposition 2

Proof. When fixing S_{t-1}, the consumption function in terms of random variables Y_t, Y_{t+1}, Γ_t and Γ_{t+1} can be written as $C_t(X_t(Y_t), \Gamma_t)$ and $C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)$ for periods t and $t+1$.

We need to show that $\text{cov}(\Gamma_t, C_{t+1} X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)) < 0$. Define $\bar{\Gamma}_t = \mathbb{E}[\Gamma_t]$. Fix Y_t, Y_{t+1}, and Γ_{t+1} Then,

$$\left[\Gamma_t - \bar{\Gamma}_t\right] \left[C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t) - C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \bar{\Gamma}_t)\right] \le 0.$$

Taking expectation with respect to Γ_t conditional on Y_t, Y_{t+1}, and Γ_{t+1} gives,

$$\mathbb{E}\left[\left[\Gamma_t - \bar{\Gamma}_t\right] C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t) \big| Y_t, Y_{t+1}, \Gamma_{t+1}\right] \le 0.$$

Now taking expectation over Y_t, Y_{t+1}, and Γ_{t+1} gives,

$$\mathbb{E}\left[\Gamma_t C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)\right] - \bar{\Gamma}_t \mathbb{E}\left[C_{t+1}(X_{t+1}(Y_t, Y_{t+1}), \Gamma_{t+1}, \Gamma_t)\right] \le 0$$

This shows that $\text{cov}(C_t - \Gamma_t, C_{t+1}) \ge \text{cov}(C_t, C_{t+1})$. □

Lemma 1. Suppose that $u(c)$ is strictly concave, and $F(s, a)$ is concave. For a convex set $B(\bar{x})$, a problem

$$V(\bar{x}, \bar{A}) = \max_{s,a} u(\bar{x} - s - a) + F(s, Z)$$

$$\text{s.t. } s + a < \bar{x}, Z = \bar{A} + a, s \ge 0 \text{ and } a \ge 0.$$

[1] only allows a unique solution for $c = \bar{x} - s - a$, [2] value function $V(\bar{x}, \bar{A})$ is concave, and [3] if $u(\cdot)$ and $F(s, a)$ are concave and differentiable, then $V(\bar{x})$ is also differentiable at $\bar{x}$ and A.

1. I first show the uniqueness of c. Suppose that another triplet c', s' and a' where $c \ne c'$ exists. Then for a $\lambda \in (0, 1)$

$$u(\lambda c + (1 - \lambda)c') + F(\lambda s + (1 - \lambda)s', \lambda Z + (1 - \lambda)Z') > u(c) + F(s, Z)$$

which is a contradiction.

[2] For concavity of V, let $S' = (s', a')$ and $c' = \bar{x}' - s' - a'$ to denote the solution that maximize the objective function when given $\bar{x}'$ and $\bar{A}'$. Similarly, let $S'' = (s'', a'')$ and $c'' = \bar{x}'' - s'' - a''$ be the maximizers when given $\bar{x}''$ and $\bar{A}''$.

For any $\lambda \in [0,1]$, $\lambda S' + (1-\lambda)S''$ is feasible when given $\bar{x}^* = \lambda \bar{x}' + (1-\lambda)\bar{x}''$ and $\bar{A}^* = \lambda \bar{A}' + (1-\lambda)\bar{A}''$. Define $S^* = \lambda S' + (1-\lambda)S''$ and $c^* = \lambda c' + (1-\lambda)c''$. Then,

$$
\begin{aligned}
V(\bar{x}^*, \bar{A}^*) &\geq u(c^*) + F(s^*, Z^*) \\
&\geq \lambda \left[u(c') + F(s', Z') \right] + (1-\lambda) \left[u(c'') + F(s'', Z'') \right] \\
&> \lambda V(\bar{x}', \bar{A}') + (1-\lambda) V(\bar{x}'', \bar{A}'').
\end{aligned}
$$

[3] For differentiabilty, Lemma Benveniste and Scheinkman (1979) shows that if $W(x, A|s, a) = u(x - s - a) + F(s, a + A)$ is a concave function on a convex set B, then a concave function $V(x, A)$ where $V(x^*, A^*) = W(x^*, A^*)$ and $V(x, A) \geq W(x, A)$ for all other $(x, A) \in B$, then V is differentiable at (x^*, A^*). $\qquad\square$

Proof of Proposition 3

Proof. I prove using backward induction. In each step, I show that [T1] following two inequalities

$$
\frac{\partial \widetilde{V}^1_{t+1}(X_{t+1}, Z_{t+1})}{\partial X_{t+1}} > \frac{\partial \widetilde{V}^2_{t+1}(X_{t+1}, Z_{t+1})}{\partial X_{t+1}}, \text{ and } \frac{\partial \widetilde{V}^1_{t+1}(X_{t+1}, Z_{t+1})}{\partial Z_{t+1}} > \frac{\partial \widetilde{V}^2_{t+1}(X_{t+1}, Z_{t+1})}{\partial Z_{t+1}}
$$

hold, and [T2] $\widetilde{V}^i_{t+1}(X_t, Z_t)$ is a concave function.

Starting from period $T_r - 1$, the agent i solves

$$
\max_{C,S,A} u(C - \Gamma_{T_r-1}) + \delta \mathbb{E}_{T_r-1} \left[\widetilde{W}^i_{T_r}(R^S_{T_r}S + R^A_{T_r}(A + Z_{T_r-1})) \right] \tag{8}
$$

$$
\text{subject to } C + S + A \leq X_{T_r-1}, C \geq 0, S \geq 0, \text{ and } A \geq 0. \tag{9}
$$

Trivially, $S = 0$ since it is an inferior asset compared to A with $R^A_{T_r} > R^S_{T_r}$. Hence, this is effectively an one asset case. We can apply same steps as in Proposition 1 and claim that $C^2_{T_r-1}(X_{T_r-1}, Z_{T_r-1}) \geq C^1_{T_r-1}(X_{T_r-1}, Z_{T_r-1})$. Also,

$$
\begin{aligned}
\frac{\partial \widetilde{V}^i_{T_r-1}}{\partial Z_{T_r-1}} &= u'(C^i_{T_r-1} - \widetilde{\Gamma}^i_t)\frac{\partial C^i_{T_r-1}}{\partial Z_{T_r-1}} + \delta \mathbb{E}_{T_r-1} \left[\widetilde{W}^i_{T_r}{}'(X_{T_r})R^A_{T_r} \left[1 + \frac{\partial A_{T_r-1}}{\partial Z_{T_r-1}} \right] \right] \\
&= \delta \mathbb{E}_{T_r-1} \left[\widetilde{W}^i_{T_r}{}'(X_{T_r})R^A_{T_r} \right]
\end{aligned}
$$

whether or not the constraint on $C^i_{T_r}$ is binding. If agents 1 and 2 are both hand-to-mouth,

then the wealth at period T_r is the same and this gives

$$\frac{\partial \widetilde{V}^2_{T_r-1}}{\partial Z_{T_r-1}} = \delta \mathbb{E}_{T_r-1}\left[\widetilde{W}^2_{T_r}{}'(X_{T_r})R^A_{T_r}\right] < \delta \mathbb{E}_{T_r-1}\left[\widetilde{W}^1_{T_r}{}'(X_{T_r})R^A_{T_r}\right] < \frac{\partial \widetilde{V}^1_{T_r-1}}{\partial Z_{T_r-1}}.$$

If the agent 2 is hand-to-mouth but agent 1 saves, then

$$\frac{\partial \widetilde{V}^2_{T_r-1}}{\partial Z_{T_r-1}} = \delta \mathbb{E}_{T_r-1}\left[\widetilde{W}^i_{T_r}{}'(X_{T_r})R^A_{T_r}\right] < u'(C^2_{T_r-1} - \widetilde{\Gamma}^2_{T_r-1}) < u'(C^1_{T_r-1} - \widetilde{\Gamma}^1_{T_r-1}) = \frac{\partial \widetilde{V}^1_{T_r-1}}{\partial Z_{T_r-1}}.$$

If both agents save, then

$$\frac{\partial \widetilde{V}^2_{T_r-1}}{\partial Z_{T_r-1}} = u'(C^2_{T_r-1} - \widetilde{\Gamma}^2_{T_r-1}) < u'(C^1_{T_r-1} - \widetilde{\Gamma}^1_{T_r-1}) = \frac{\partial \widetilde{V}^1_{T_r-1}}{\partial Z_{T_r-1}}.$$

Similar argument can show us that $\frac{\partial \widetilde{V}^2_{T_r-1}}{\partial X_{T_r-1}} < \frac{\partial \widetilde{V}^1_{T_r-1}}{\partial X_{T_r-1}}$ as we have done in Proposition 1 but I do not repeat it here. Also, $\widetilde{V}^i_{T_r-1}(X_{T_r-1}, Z_{T_r}-1)$ is concave by Lemma 1.

Now we go to any arbitrary period $t < T_r - 1$ given that [T1] and [T2] holds at period $t+1$. Let us denote $\widetilde{F}^i_t(S^i_t, A^i_t) = \mathbb{E}_t\left[\widetilde{V}^i_{t+1}(X^i_{t+1}(S^i_t), Z^i_{t+1}(A^i_t))\right]$. Nothe that the objective function $u(X_t - S^i_t - A^i_t) + \widetilde{F}_t(S^i_t, A^i_t)$ is differentiable and concave to S^i_t and A^i_t with linear constraints by [T1] and [T2], hence Kuhn-Tucker optimality conditions are necessary and sufficient.

We deal with following four cases.

- Case 1: both agents 1 and 2 are hand-to-mouth.

  In this case, there is nothing to prove since the consumption of two agents are identical. Also, [T1] holds trivially.

- Case 2: only agent 2 is hand-to-mouth.

  Also in this case, the assumption already implies that [T1] holds.

- Case 3: only agent 1 is hand-to-mouth.

  I show the impossibility of this case. Assume that the agent 1 is only saving illiquid asset A^1_t. The assumption implies that

  $$u'(C^2_t - \Gamma_t) = \frac{\partial \widetilde{F}^2_t(S^2_t, A^2_t)}{\partial S^2_t} \le \frac{\partial \widetilde{F}^1_t(S^2_t, A^2_t)}{\partial S^2_t}.$$

  Hence, by Inada condition on $u(\cdot)$ and continuity and concavity of $\widetilde{F}_t(\cdot, \cdot)$, there should

38

exist some levels of $\Delta > 0$ and $\Delta' > 0$ such that

$$u'(C_t^2 - \Delta - \Delta' - \Gamma_t) = \frac{\partial \widetilde{F}_t^1(S_t^2 + \Delta, A_t^2 + \Delta')}{\partial S_t^2}.$$

However, this cannot be true because $C_t^2 - \Delta - \Delta'$, $S_t^2 + \Delta$ and A_t^2 is also a solution for agent 1's problem and we only allow for unique optimal consumption by Lemma 1. Similar arguments can be done when the agent 1 is only saving liquid asset S_t^1.

- Case 4: all agents are not hand-to-mouth.

  The argument is the same as case 3.

Hence, the consumption is always higher for the agent 2. Concavity and differentiability of the value function are guaranteed by Lemma 1. Then $F_{t-1}^i(S_{t-1}^i, A_{t-1}^i) = \mathbb{E}_t\left[V_{t+1}^i(X_{t+1}^i, Z_{t+1}^i)\right]$ is also concave since $F_{t-1}^i(S_{t-1}^i, A_{t-1}^i)$ is sum of a concave functions.

Finally, we need to show that [T1] holds at periods at t. For the first inequality, $C_t^2(X_t, Z_t; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^2) \geq C_t^1(X_t, Z_t; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^1)$ implies that

$$u'\left(C_t^1(X_t, Z_t; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^1)) - \widetilde{\Gamma}_t\right) \geq u'\left(C_t^2(X_t, Z_t; \widetilde{\Gamma}_t, \widetilde{\mathbf{\Gamma}}^2)) - \widetilde{\Gamma}_t\right)$$

$$\Rightarrow \mathbb{E}_{t-1}\left[\frac{\partial \widetilde{V}_t^1}{\partial X_t}\right] \geq \mathbb{E}_{t-1}\left[\frac{\partial \widetilde{V}_t^2}{\partial X_t}\right],$$

where the last line follows from the fact that $\widetilde{\Gamma}_t^1 >_1 \widetilde{\Gamma}_t^2$. $\qquad \square$

Proof of Proposition 4

Proof. I only show the proof for the case where $\mathbf{\Gamma} >_1 \widetilde{\mathbf{\Gamma}}^1 >_1 \widetilde{\mathbf{\Gamma}}^2$. The proof for the other case is similar to this case. Assume that following holds for continuation value at $t+1$:

[a] $\quad \mathbb{E}_t\left[W_{t+1}^*(X_{t+1}; \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^1)\right] \geq \mathbb{E}_t\left[W_{t+1}^*(X_{t+1}; \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^2)\right]$

[b] $\quad \mathbb{E}_t\left[W_{t+1}^*(Y_{t+1} + R_{t+1}^S(\bar{S} + \Delta); \Gamma_t, \widetilde{\mathbf{\Gamma}}) - W_{t+1}^*(Y_{t+1} + R_{t+1}^S \bar{S}; \Gamma_t, \widetilde{\mathbf{\Gamma}})\right]$

$\quad\quad \geq \mathbb{E}_t\left[\widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(Y_{t+1} + R_{t+1}^S(\bar{S} + \Delta); \widetilde{\Gamma}_t) - \widetilde{W}_{t+1}^{\widetilde{\mathbf{\Gamma}}}(Y_{t+1} + R_{t+1}^S \bar{S}; \widetilde{\Gamma}_t)\right]$ for all $\Delta \geq 0$.

I show that using [a] and [b], Fix $X_t > 0$ and denote consumption under $\widetilde{\mathbf{\Gamma}}^1$ as C^1. Then Consumption under $\widetilde{\mathbf{\Gamma}}^2$ can be written as $C^1 + \Delta$ for some $\Delta \geq 0$. Then we are comparing

$$U^1 = u(C^1) + \delta\mathbb{E}_t\left[W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^1)\right] \text{ and} \tag{10}$$

$$U^2 = u(C^1 + \Delta) + \delta\mathbb{E}_t\left[W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1 - \Delta); \Gamma_{t+1}, \widetilde{\mathbf{\Gamma}}^1)\right]. \tag{11}$$

Then,

$$
\begin{aligned}
u(C^1 + \Delta) - u(C^1) \leq \delta\mathbb{E}_t\big[& \widetilde{W}_{t+1}^{\widetilde{\boldsymbol{\Gamma}}^1}(Y_{t+1} + R_{t+1}^S(X_t - C^1); \widetilde{\Gamma}_{t+1}) \\
& - \widetilde{W}_{t+1}^{\widetilde{\boldsymbol{\Gamma}}^1}(Y_{t+1} + R_{t+1}^S(X_t - C^1 - \Delta); \widetilde{\Gamma}_{t+1})\big] \\
\leq \delta\mathbb{E}_t\big[& W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1); \Gamma_{t+1}, \widetilde{\boldsymbol{\Gamma}}^1) \\
& - W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1 - \Delta); \Gamma_{t+1}, \widetilde{\boldsymbol{\Gamma}}^1)\big] \\
\leq \delta\mathbb{E}_t\big[& W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1); \Gamma_{t+1}, \widetilde{\boldsymbol{\Gamma}}^1) \\
& - W_{t+1}^*(Y_{t+1} + R_{t+1}^S(X_t - C^1 - \Delta); \Gamma_{t+1}, \widetilde{\boldsymbol{\Gamma}}^2)\big].
\end{aligned}
$$

Rearranging the last row shows that $U^1 \geq U^2$. Not I show that [a] and [b] holds with the continuation value at t. [a] is obvious, which is just taking expectation on the above inequality. To show [b], note that $F(x) >_1 G(x)$ between two CDFs $F(x)$ amd $G(x)$ implies that

$$
\int v(x)dF(x) \geq \int u(x)dG(x)
$$

if $v(x) \geq u(x)$ for all x and $u(x)$ is increasing in x. Using this strategy, we need to show that

$$
\begin{aligned}
W_t^*(Y_t + R_t^S(X_t - C_t + \Delta); \Gamma_t, \widetilde{\boldsymbol{\Gamma}}) &- W_t^*(Y_t + R_t^S \bar{S}; \Gamma_t, \widetilde{\boldsymbol{\Gamma}}) \\
&\geq \widetilde{W}_t^{\widetilde{\boldsymbol{\Gamma}}}(Y_t + R_t^S(\bar{S} + \Delta); \Gamma_t) - \widetilde{W}_t^{\widetilde{\boldsymbol{\Gamma}}}(Y_t + R_t^S \bar{S}; \Gamma_t)
\end{aligned}
$$

for all Δ and Γ_t. Note that this reduces back to [b] at $t+1$. Last thing to show is that $\widetilde{W}_t^{\widetilde{\boldsymbol{\Gamma}}}(Y_t + R_t^S(\bar{S} + \Delta); \Gamma_t) - \widetilde{W}_t^{\widetilde{\boldsymbol{\Gamma}}}(Y_t + R_t^S \bar{S}; \Gamma_t)$ is increasing in Γ_t. Denote $X'_{t+1} = Y_t + R_t^S(\bar{S} + \Delta)$ and $X_t = Y_t + R_t^S \bar{S}$. Note that (if the agent is not hand-to-mouth)

$$
\begin{aligned}
\frac{\partial W_t^{\widetilde{\boldsymbol{\Gamma}}}(X_t; \Gamma_t)}{\partial \Gamma_t} &= u'(C_t - \Gamma_t)\left[\frac{\partial C_t}{\partial \Gamma_t} - 1\right] - \delta R_{t+1}^S \mathbb{E}_t\left[\frac{\partial W_{t+1}^{\widetilde{\boldsymbol{\Gamma}}}(X_{t+1}; \widetilde{\Gamma}_{t+1})}{\partial X_{t+1}}\right]\frac{\partial C_t}{\partial \Gamma_t} \\
&= -u'(C_t - \Gamma_t) < 0.
\end{aligned}
$$

If the agent is hand-to-mouth, then we have the same result immediately. Then we immediately have the result that $\widetilde{W}_t^{\widetilde{\boldsymbol{\Gamma}}}(Y_t + R_t^S(\bar{S} + \Delta); \Gamma_t) - \widetilde{W}_t^{\widetilde{\boldsymbol{\Gamma}}}(Y_t + R_t^S \bar{S}; \Gamma_t)$ is increasing in Γ_t. Last thing to show is that [a] and [b] holds when constructing the continuation value at the terminal period, but this is obvious. $\qquad\square$

B   Other Tables and Figures

		89'	92'	95'	98'	01'	04'	07'	10'	13'	16'	19'	Avg.
	NW	0.71	0.71	0.70	0.72	0.74	0.74	0.74	0.77	0.77	0.79	0.78	0.74
All	LIQ	0.88	0.87	0.89	0.89	0.90	0.89	0.90	0.91	0.90	0.91	0.89	0.89
	Illiquid	0.68	0.68	0.66	0.68	0.70	0.70	0.70	0.72	0.73	0.76	0.76	0.71
Graduate	NW	0.70	0.68	0.68	0.68	0.72	0.71	0.70	0.73	0.73	0.75	0.75	0.71
	LIQ	0.86	0.86	0.86	0.86	0.89	0.87	0.88	0.89	0.88	0.87	0.87	0.87
	Illiquid	0.66	0.66	0.64	0.64	0.68	0.66	0.67	0.67	0.70	0.73	0.73	0.68
Tertiary	NW	0.69	0.67	0.67	0.69	0.70	0.68	0.71	0.70	0.72	0.76	0.72	0.70
education	LIQ	0.85	0.83	0.86	0.87	0.87	0.86	0.87	0.85	0.85	0.89	0.85	0.86
	Illiquid	0.64	0.64	0.65	0.66	0.66	0.65	0.68	0.67	0.68	0.72	0.69	0.67
Secondary	NW	0.67	0.67	0.64	0.65	0.67	0.69	0.66	0.71	0.73	0.74	0.75	0.69
education	LIQ	0.85	0.82	0.87	0.83	0.83	0.85	0.85	0.88	0.90	0.88	0.89	0.86
	Illiquid	0.66	0.64	0.60	0.61	0.63	0.64	0.64	0.67	0.69	0.72	0.73	0.66
1st	NW	0.64	0.65	0.64	0.66	0.66	0.65	0.66	0.68	0.67	0.71	0.70	0.67
income	LIQ	0.85	0.84	0.86	0.86	0.87	0.85	0.86	0.86	0.84	0.86	0.83	0.85
tertile	Illiquid	0.61	0.62	0.61	0.62	0.63	0.62	0.63	0.64	0.64	0.67	0.68	0.63
2nd	NW	0.58	0.60	0.58	0.58	0.62	0.60	0.58	0.60	0.67	0.63	0.64	0.61
income	LIQ	0.76	0.80	0.82	0.79	0.81	0.78	0.78	0.77	0.85	0.81	0.82	0.80
tertile	Illiquid	0.53	0.53	0.50	0.49	0.52	0.51	0.49	0.51	0.57	0.54	0.55	0.52
3rd	NW	0.72	0.72	0.70	0.69	0.74	0.75	0.71	0.75	0.78	0.76	0.74	0.73
income	LIQ	0.88	0.86	0.87	0.86	0.90	0.93	0.91	0.92	0.93	0.92	0.89	0.90
tertile	Illiquid	0.71	0.69	0.64	0.66	0.67	0.67	0.67	0.69	0.70	0.71	0.71	0.68

Table 8: Gini index of the United States: 1989-2019

Notes: Illiquid assets refers to total assets minus the liquid assets. Net worth is the total assets minus debt. 'Avg.' refers to the average of all survey years for each row.

	ρ	p-value	Expenditure shock
Adult care	0.338	0.008	
Alcohol away from home	0.190	<0.001	
Alcohol at home	1.000	<0.001	
Child care	0.118	0.026	O
Clothes	-0.015	0.009	O
Clothing services	0.205	<0.001	
Domestic services	0.257	0.136	O
Education durables	0.034	0.063	O
Education services	0.003	0.731	O
Entertainment durables	-0.003	0.588	O
Entertainment services	0.054	0.005	
Fees and charges	0.023	0.215	O
Food away from home	0.064	<0.001	
Food at home	0.883	<0.001	
Furniture rental	1.000	<0.001	
Gasoline expenses	0.447	<0.001	
Health care durable	0.010	0.397	O
Health insurance	0.242	<0.001	
Health care service	0.026	<0.001	
Other household expenditures	0.066	0.002	
Home insurance	0.002	0.933	O
Home management	-0.042	0.004	O
Home maintenance and repairs	0.036	<0.001	
Home-related equipment and supplies	0.093	0.093	O
Household furnishings and equipment	0.013	0.282	O
Household textiles and linens	0.041	0.298	O
Jewelry	0.005	0.657	O
Life insurance	-0.084	0.041	O
Occupational expenses	0.115	<0.001	
Parking expenses	$\sim$0.001	0.941	O
Public transportation	0.012	0.078	O
Personal care products	-0.036	0.217	O
Telephone services	0.246	<0.001	
Personal care services	0.999	<0.001	
Reading material	1.000	<0.001	
Rent	0.063	<0.001	
Rented vehicles	0.048	<0.001	
Tobacco	1.000	<0.001	
Utilities	0.056	<0.001	
Vehicle	0.002	0.172	O
Other vehicle-related durables	0.021	0.277	O
Vehicle insurance	-0.050	<0.001	O
Vehicle service	-0.005	0.430	O
Water and other public services	-0.142	<0.001	O

Table 9: Classification Expenditure Shocks

C Sample Selection when using the Consumer Expenditure Survey

The following households are dropped. (1) Households with top-coded income. (2) Household head lives in a student housing (dormitory). (3) Household head age is not reported. (4) Family size is not reported. (5) Age is less than 25 which is the initial age of the simulation. (6) Age exceeds 80 which is the terminal age of the simulation. (7) Family size has changed during the survey quarters. (8) Nondurable goods expenditure is at bottom 1 percent of the sample or the food consumption is zero. (9) Households do not have full twelve month survey responses.

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