Stochastic Processes
Assignment 4, due 2022-02-18

Note: Start each problem on a **new page**. (Multiple pages for a single problem are fine if needed.)

Problem 1. Is it possible for a branching process to be reversible? What can be said about ξ in that case (The number of children of each individual is an independent copy of ξ.)

Problem 2. Find the probability generating function $G(s) = \mathbb{E}s^X$ for the following distributions:
(a) $X = \mathrm{Bernoulli}(p)$: here $P(X = 1) = p$ and $P(X = 0) = 1 - p$.
(b) $X = \mathrm{Bin}(n, p)$.
(c) $X = \mathrm{Poi}(\lambda)$.
(d) $X = \mathrm{Geom}(p)$, so $P(X = n) = p(1 - p)^{n-1}$ for $n = 1, 2, \dots$.

Problem 3. Find the probability generating function $G(s) = \mathbb{E}s^X$ for the following distributions:
(a) $X = A + B$ where $A = \mathrm{Bin}(n, p)$ and $B = \mathrm{Geom}(p)$ are independent.
(b) Let $N = \mathrm{Poi}(\lambda)$ and $Y_1, Y_2, \dots$ be $\mathrm{Geom}(p)$ for some p, λ and all are independent. Let $X = Y_1 + \dots + Y_N$.
(c) Let $N = \mathrm{Geom}(q)$ and $Y_1, Y_2, \dots$ be $\mathrm{Geom}(p)$ for some p, q and all are independent. Let $X = Y_1 + \dots + Y_N$.

Problem 4. Consider a branching process with offspring distribution ξ with

$$P(\xi = 0) = \alpha \qquad P(\xi = 1) = \beta \qquad P(\xi = 2) = 0 \qquad P(\xi = 3) = 1 - \alpha - \beta.$$

(a) If $\alpha = \beta = 1/3$, find the probability the process becomes extinct.
(b) If we start with 4 individuals instead of 1, find the probability the process becomes extinct.
(c) If α, β are such that $\mathbb{E}(\xi) = 2$, what α and β minimize the probability of extinction (starting with 1 individual)? What α, β maximize that probability?

Problem 5. Find transition probabilities on $S = \{0, 1, 2, \dots\}$ so that the stationary distribution is $\mathrm{Poi}(\lambda)$ and the only transitions that can hove non-zero probability are $P_{n,n-1}, P_{n,n+1}$, and $P_{n,n}$ for all n.

Extra practice problems Do not hand these in. (Feel free to ask for hints is stuck.)
Ross, chapter 4: problems 64,66,71,72,73,74.

Read ahead We will start Chapter 5 next week.

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