SE102:Multivariable Calculus

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Lecture 05
Exercises

Example

Find all extremals of $f(x, y) = xy - y + x - 2$ on the region $x^2 + y^2 \leq 2$.

Example

Find all extremals of $f(x, y) = -x^2 + 3xy - 2y^2$ on the region $2x^2 - 6xy + 5y^2 \leq 1$.

Example

Find the dimensions of the cube inscribed in the sphere of radius 2 whose surface area is maximum.

Example

Find the point on the surface $x^3 + y^2 + z = 2$ closest to the origin.

Example

Suppose $f(x, y)$ is a differentiable function defined on $\mathbf{R}^2$. If (x_0, y_0) is a critical point, explain why it is a saddle point if the Hessian $H_f(x_0, y_0)$ is negative regardless of the value of $f_{xx}(x_0, y_0)$.

Example

Let $\mathbf{v}$, $\mathbf{w}$, $\mathbf{u}$ be linearly independent 3-dimensional position vectors. Show that the volume of the parallelogram bounded by these vectors is $|(\mathbf{v} \times \mathbf{w}) \cdot \mathbf{u}|$.

Example

Discuss the difference on geometric configurations of three vectors $\mathbf{v}$, $\mathbf{w}$, $\mathbf{u}$ when the value of $(\mathbf{v} \times \mathbf{w}) \cdot \mathbf{u}$ is positive, negative, or zero.

Example

Prove or disprove: for any vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$,

1. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{w}) \cdot \mathbf{w}$.
2. $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$.

Example

Determine whether the limit exists:

$$\lim_{(x,y)\to(0,0)} \frac{xy + yx^2}{x^2 + y^2}$$

Example

Determine whether the limit exists: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2 y}{x^2 + 2y^2}$

Example

Determine whether the function is continuous at $(0, 0)$.

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

Example

Find all points where the directional derivative of $f(x, y) = x^2 + y^2 - 2x - 4y$ to the vector $\mathbf{u} = \frac{1}{2}(1, 1)$ is maximized.

Example

Given a fixed $c > 0$, show that the sum of three intercepts of any tangent plane to the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{c}$ is constant.

Example

Consider a small circle of radius b rolling inside the larger circle of radius a ($a > b$). Find the parametric equations of the trajectory of the point on the small circle.