SE328:Topology

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Theorem

Every regular space with a countable basis is normal.

Theorem

Every metrizable space is normal.

Theorem

Every compact Hausdorff space is normal.

Example

If J is uncountable, then $\mathbf{R}^J$ is not normal.

Theorem (Urysohn Lemma)

Let X be a normal space and A, B be disjoint closed subsets of X. Then there is a continuous map

$$f : X \to [a, b]$$

such that $f(x) = a$ for all $x \in A$ and $f(x) = b$ for all $x \in B$.

Theorem (Urysohn metrization theorem)

Every regular space with a countable basis is metrizable.

Theorem

Let X be a T1-space and $\{f_\alpha : X \to \mathbf{R}\}$ be a family of continuous maps such that for each $x_0 \in X$ and each neighborhood U of x_0, there is f_α which is positive at x_0 and vanishes outside U. Then the function $F : X \to \mathbf{R}^J$ defined by

$$F(x) = (f_\alpha(x))_{\alpha \in J}$$

is an imbedding of X.

Definition

A T1-space X is called **completely regular** if for each point $x_0 \in X$ and each neighborhood U of x_0, there is a continuous map $f : X \to [0,1]$ such that $f(x_0) = 1$ and $f(X \setminus U) = 0$.

Lemma

A space X is completely regular if and only if it is homeomorphic to a subspace of $[0,1]^J$ for some J.

Definition

An m-**manifold** is a Hausdorff space X with a countable basis such that each point $x \in X$ has a neighborhood U homeomorphic to an open subset of $\mathbf{R}^m$.

Definition

The **support** of ϕ is the closure of the set $\phi^{-1}(\mathbf{R} \setminus \{0\})$.

Definition

Let $\{U_1, \cdots, U_n\}$ be a finite indexed open covering of the space X. A family of maps $\phi_i : X \to [0,1]$ for $i = 1, \cdots, n$ is called a **partition of unity dominated by** $\{U_i\}$ if it satisfies:

1. $\text{supp}\phi_i \subset U_i$ for $i = 1, \cdots, n$,
2. $\sum_{i=1}^{n} \phi_i(x) = 1$

Theorem

Let $\{U_1, \cdots, U_n\}$ be a finite open covering of a normal space X. Then there exists a partition of unity dominated by $\{U_i\}$.

Lemma

If X is a compact m-manifold, then X can be imbedded in $\mathbf{R}^N$ for some positive N.