SE328:Topology

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Week 14

Definition

A retraction of X onto a subset $A \subset X$ is a continuous map $r : X \to A$ such that $r|_A$ is the identity map. If such map r exists, we say that A is a retract of X.

Lemma

If A is a retract of X, then the homomorphism of fundamental groups induced by inclusion $j : A \to X$ is injective.

Theorem

There is no retraction of $\mathbf{B}^2$ *onto* $\mathbf{S}^2$.

Lemma

Let $h : \mathbf{S}^1 \to X$ be a continuous map. Then the followings are equivalent:

1. h is nulhomotopic.
2. h extends to a continuous map $h : \mathbf{B}^2 \to X$.
3. h_* is the trivial homomorphism of fundamental groups.

Corollary

The inclusion map $j : \mathbf{S}^1 \to \mathbf{R}^2 - \{0\}$ is not nulhomotopic. The identity map $i : \mathbf{S}^1 \to \mathbf{S}^1$ is not nulhomotopic.

Theorem

Given a nonvanishing vector field on $\mathbf{B}^2$*, there exists a point of* $\mathbf{S}^1$ *where the vector field points directly inward and a point of* $\mathbf{S}^1$ *where it points directly outward.*

Theorem (Brouwer fixed-point theorem)

If $f : \mathbf{B}^2 \to \mathbf{B}^2$ is continuous, then there exists a point $x \in \mathbf{B}^2$ such that $f(x) = x$.

Corollary

Let A be a 3×3 matrix of positive real numbers. Then A has a positive real eigenvalue.

Theorem

There is an $\varepsilon > 0$ such that for every open covering $\mathcal{A}$ of T by sets of diameter less than ε, some point of T belongs to at least three elements of $\mathcal{A}$.

Theorem

Any polynomial has at least one complex root.

Definition

A map $h : \mathbf{S}^n \to \mathbf{S}^m$ is said to be antipodal-preserving if $h(-x) = -h(x)$ for all $x \in \mathbf{S}^n$.

Theorem

If $h : \mathbf{S}^1 \to \mathbf{S}^1$ is continuous and antipodal-preserving, then h is not nulhomotopic.

Theorem

There is no continuous antipodal-preserving map $g : \mathbf{S}^2 \to \mathbf{S}^1$.

Theorem (Borsuk-Ulam)

Given a continuous map $f : \mathbf{S}^2 \to \mathbf{R}^2$, *there is a point* $x \in \mathbf{S}^2$ *such that* $f(x) = f(-x)$.

Theorem

Given two bounded polygonal regions in $\mathbf{R}^2$, there exists a line in $\mathbf{R}^2$ that bisects each of them.