SE328:Topology

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Definition

For two metric space X, Y, a function $f : X \to Y$ is said to be **uniformly continuous** if for any $\varepsilon > 0$, there exists $\delta > 0$ such that for any $x_1, x_2 \in X$,

$$d(x_1, x_2) < \delta \text{ implies } d(f(x_1), f(x_2)) < \varepsilon$$

Theorem (Uniform continuity theorem)

Let $f : X \to Y$ be a continuous function between metric spaces. If X is compact, then f is uniformly continuous.

Definition

Let x_n be sequence in X and x_{n_i} be a subsequence. If every sequence in X has a convergent subsequence, then X is said to be **sequentially compact**. Meanwhile, X is said to be **limit point compact** if every infinite subset of X has a limit point.

Proposition

If X is a metric space, then the followings are equivalent:

1. *X is compact,*
2. *X is sequenctially compact,*
3. *X is limit point compact.*