Computational Metasurface Research Report

Yueze Liu
2024 computational metasurface

December 24, 2024

Abstract

Optical neural networks (ONNs) provide a promising platform for massively parallel, high-speed, and energy-efficient computation. Within this domain, metasurfaces emerge as powerful devices that manipulate the phase, amplitude, and polarization of light at subwavelength scales. This proposal investigates using stackable, computational metasurfaces for specialized tasks such as classification, generic matrix multiplication, and convolution. The goal is to create a scalable and memory-efficient optical computing system that takes advantage of free-space propagation and Fourier optics principles. Preliminary simulations and analyses suggest this approach can potentially outperform conventional electronics in large-scale linear operations, albeit with non-trivial fabrication and amplification challenges.

Contents

1 Introduction

1.1 Motivation and Background

Optical neural networks (ONNs) have gained significant attention as an alternative to modern, electronic deep neural networks. By leveraging the inherent parallelism and speed of light, ONNs aim to alleviate major computational bottlenecks such as energy consumption, memory bandwidth, and latency. Metasurfaces—subwavelength engineered interfaces that shape and direct light—are particularly attractive for optical computing due to their compact form factor, low loss (in certain material choices), and ability to directly perform transformations on electromagnetic waves.

Despite these advantages, current ONNs are hindered by challenges in:

- Nonlinear activations: Achieving tunable or universal optical nonlinearity remains difficult.
- Memory constraints: Input weight loading and data retrieval consume substantial power.
- Fabrication complexity: Precise nanofabrication is required, which can be expensive and challenging to scale.

This proposal outlines a PhD project to investigate metasurface-based optical computing methods with an emphasis on free-space and Fourier-optics architectures. The project will explore classification networks, matrix multiplication modules, and convolutional layers implemented via passive and active metasurfaces.

1.2 Overview of Existing Approaches in ONNs

Broadly, there are two families of ONN implementations:

1. **Waveguide-based ONNs**: Integrated photonic circuits (e.g., Mach-Zehnder interferometer meshes) allow compactness and better tunability. However, they often require significant preprocessing to recover phase information or vectorize input data.
2. **Free-space optics-based ONNs**: Employ diffractive elements (lenses, diffractive layers, metasurfaces) to process information in three dimensions, capturing both amplitude and phase in real-time. This approach scales well for computational imaging, statistical inference, and high-throughput classification tasks.

Within free-space optics, metasurfaces can serve as trainable layers or as fixed transformation kernels once fabricated. Landmark works include:

- **Fourier-space Metasurface Classifier** [1]: Demonstrating categorization tasks using diffractive neural networks in Fourier space.
- **Metasurface Classifier without Fourier Optics** [2]: Realizing classification with stacked metasurfaces for on-chip computing in visible wavelengths.

- **Fourier Optics for Convolution and Multiplication** [3]: Proposing analog in-memory compute architectures where convolution and matrix multiplication are performed optically.

2 Three Landmark Papers and Their Significance

2.1 Fourier-Space Metasurface Classifier [1]

This work employed a series of metasurfaces arranged to perform neural-network-like transformations in the Fourier domain. By leveraging the property that a lens performs a two-dimensional Fourier transform, the system achieved high throughput image classification. The approach highlights the elegance of free-space optics, yet required precise alignment and struggled with limited tunability.

2.2 Metasurface Classifier Without Fourier Optics [2]

An alternative design stacked multiple diffractive metasurface layers in the direct (non-Fourier) propagation path. Each metasurface was iteratively designed to adjust the amplitude and phase of incident fields. While it did not utilize Fourier lenses, it showcased that on-chip diffractive neural networks are feasible. However, its capacity and signal-to-noise ratio heavily depended on the fabrication precision of each metasurface.

2.3 Generic Matrix Multiplication via Fourier Optics [3]

This paper proposed integrating in-memory compute principles with optical 2D convolution. By setting up a $4f$ optical system, convolution in the spatial domain was turned into element-wise multiplication in the Fourier domain:

$$\mathcal{F}[f * g] = \mathcal{F}[f] \cdot \mathcal{F}[g].$$

However, the architecture still required intermediate detection and digitization steps, limiting the "end-to-end" optical advantage. The approach emphasized the power of Fourier optics for large-scale linear operations but underscored challenges of signal amplification, detector noise, and partial digital processing.

3 Proposed Approach

3.1 Research Hypothesis

We hypothesize that an all-optical metasurface-based computing pipeline can mitigate the memory bottleneck and significantly reduce the power consumption of large-scale matrix multiplications (e.g., in neural network inference). By combining:

1. **Stackable Metasurfaces** for large-scale parallel operations,
2. **Amplitude/Phase Modulation** to encode both input data and model weights,

3. **Optional Optical Amplification** for efficient signal reuse,

we aim to build a system that performs *continuous*, analog matrix multiplication in free-space, requiring only an initial "loading" of the metasurface design.

3.2 High-Level Concept

The core idea is to design a *3D metasurface stack* corresponding to a target linear transformation. Once illuminated by an input field encoding the data vector (or matrix), the output field at the end of the stack represents the multiplication result. Unlike lens-based Fourier approaches, we intend to exploit both direct diffraction (for partial transformations) and optional lens planes if convolution or pooling is needed.

4 Methodology and Theoretical Foundations

This section details the mathematical underpinnings of our proposed metasurface-based computing system, covering both free-space diffraction (for classification and transform optics) and the $4f$ lens-based architecture (for convolution and generic matrix multiplication).

4.1 Free-Space Diffraction and Metasurfaces

When a monochromatic plane wave $U_{\text{in}}(x, y)$ propagates through free space over a distance z, the field at the output plane can be approximated (in the Fresnel regime) by

$$U_{\text{out}}(x', y') \approx \frac{e^{ikz}}{i\lambda z} \exp\left(\frac{ik}{2z}(x'^2 + y'^2)\right) \iint_{-\infty}^{\infty} U_{\text{in}}(x, y) \exp\left\{\frac{ik}{2z}\left[(x' - x)^2 + (y' - y)^2\right]\right\} dx\, dy, \tag{1}$$

where $k = \frac{2\pi}{\lambda}$ is the wavenumber and λ is the wavelength. In this framework, a metasurface at a given plane modifies $U_{\text{in}}(x, y)$ via a position-dependent transmission function $T(x, y)$, which can be complex-valued:

$$U_{\text{out}}(x, y) = T(x, y)\, U_{\text{in}}(x, y).$$

Therefore, designing each metasurface essentially becomes an *inverse design* problem: Given the desired output field after a certain propagation distance, we solve for $T(x, y)$ at each metasurface plane.

4.2 Fourier Optics and the $4f$ System

A lens of focal length f performs a 2D Fourier transform of an input field placed one focal length away. Specifically, if $U_1(x, y)$ is the field at the lens input plane, the field at the focal plane (i.e., one focal length behind the lens) is given by

$$\tilde{U}_1\left(\frac{k_x}{k}, \frac{k_y}{k}\right) = \iint_{-\infty}^{\infty} U_1(x, y) \exp\left[-i(k_x x + k_y y)\right]\, dx\, dy, \tag{2}$$

where k_x, k_y are the transverse wave vectors. Hence, a single lens acts as an optical Fourier transform operator.

By placing two lenses in series with a spatial light modulator or metasurface in between, one constructs a so-called $4f$ **correlator**:

1. The first lens Fourier-transforms the input.
2. A metasurface modulates the frequency components.
3. The second lens inverse-Fourier-transforms the modulated frequency spectrum, returning to the spatial domain.

4.2.1 Convolution via Multiplication in Fourier Domain

Convolution in the spatial domain corresponds to multiplication in the frequency domain:

$$(f * g)(x, y) \quad \Longleftrightarrow \quad \tilde{F}(k_x, k_y) \cdot \tilde{G}(k_x, k_y).$$

Thus, an optical $4f$ system can implement convolution:

$$U_{\text{out}}(x, y) \;=\; \mathcal{F}^{-1}\Big\{\tilde{U}_{\text{in}}(k_x, k_y)\; \tilde{H}(k_x, k_y)\Big\}, \tag{3}$$

where $\tilde{H}(k_x, k_y)$ is the Fourier transform of the kernel or filter function.

4.2.2 Matrix Multiplication Interpretation

Let the input 2D field amplitude distribution be

$$U_{\text{in}}(x, y) = \sum_{m,n} A_{m,n}\, \delta(x - m\, d_x,\; y - n\, d_y),$$

where each Dirac delta $\delta(\cdot)$ indicates a "pixel" (or discretized node) located at $(m\, d_x, n\, d_y)$. In many matrix multiplication scenarios, we can arrange weight matrices or filter kernels such that multiplication in the Fourier domain corresponds to the desired linear transform. The output field then becomes a summation of scaled versions of the kernel's impulse responses, enabling parallel matrix-vector or matrix-matrix multiplication.

4.3 Metasurface Inverse Design for 3D Stacks

We also explore a purely **stack-based metasurface approach** without explicit Fourier lenses. Consider a stack of L metasurfaces separated by distances $\Delta z_1, \Delta z_2, \ldots, \Delta z_L$. The total transformation from the input plane to the output plane can be written as:

$$U_{\text{out}} = \underbrace{\big(\mathcal{P}_{\Delta z_L} \circ \mathcal{M}_L\big)}_{\text{layer } L} \circ \;\ldots\; \circ \underbrace{\big(\mathcal{P}_{\Delta z_2} \circ \mathcal{M}_2\big)}_{\text{layer 2}} \circ \underbrace{\big(\mathcal{P}_{\Delta z_1} \circ \mathcal{M}_1\big)}_{\text{layer 1}} \big(U_{\text{in}}\big), \tag{4}$$

where $\mathcal{M}_\ell$ is the metasurface modulation operator (complex transmission $T_\ell(x, y)$) at layer ℓ, and $\mathcal{P}_{\Delta z_\ell}$ denotes free-space propagation over distance Δz_ℓ (as in Eq. (1)).

The design problem then consists of choosing the complex transmission profiles $T_1(x, y), \ldots, T_L(x, y)$ to approximate a desired linear (or nonlinear) mapping from U_{in} to U_{out}. This can be accomplished via gradient-based optimization or deep-learning-based inverse design, where a loss function measures the error between the simulated output field and the target field.

5 Preliminary Results

5.1 Initial Tidy3D Simulations

We have conducted initial simulations in Tidy3D, investigating the feasibility of partitioning an input beam into multiple paths via subwavelength-patterned metasurfaces. Early results suggest that carefully parameterized metasurfaces can produce splitting and phase shifts consistent with theoretical predictions. However, issues such as fabrication tolerance, side-lobe suppression, and amplitude mismatch remain.

5.2 Prototype Classification Experiment

A rudimentary *two-layer* metasurface classification scheme (similar to [2]) was tested for binary classification of simple patterns. The system was able to demonstrate rudimentray behavior of energy splitting but requires **optical amplification** preprocessing to compensate for calcuations of each metasurface. Future work will integrate semiconductor optical amplifiers or free-space amplifiers to maintain signal integrity.

6 Risks and Challenges

6.1 Optical Amplification Requirements

To achieve large-scale computations, especially matrix multiplication with many channels, one must amplify or at least preserve optical power normalization through the system. Purely passive metasurfaces inevitably incur losses, possibly requiring **gain layers** or external optical pumping at intermediate stages. The energy cost of amplification, though minimal, in each step is yet to be calculated and measured.

6.2 Mode Profile Mismatch

In free-space designs, the output from one metasurface layer might not optimally match the input mode of the subsequent layer (e.g., mismatch in spot size or wavefront curvature). This leads to cumulative degradation of signal fidelity. Future work must develop wavefront correction mechanisms or incorporate adaptive metasurfaces that self-correct for mode mismatches.

6.3 Fabrication and Large-Area Scalability

Building high-precision metasurfaces over large areas remains non-trivial. Small misalignments or thickness errors can severely degrade performance. The proposed approach may require advanced nanofabrication techniques or modular "tile-and-assemble" strategies to scale up to the tens of centimeters needed for large matrix multiplications.

7 Proposed Timeline and Milestones

1. **Year 1**: Literature review, theoretical formulations, Tidy3D simulations of small-scale (e.g., 4×4) metasurface multiplications, simple single layer fabrication and testing.

2. **Year 2**: Fabrication of prototype metasurface stacks, demonstration of classification or single-layer convolution. Characterize signal efficiency and insertion loss.

3. **Year 3**: Integration of optical amplification. Attempt larger (e.g., 16×16) matrix multiplications with multi-layer stacks.

4. **Year 4**: Refinement and optimization. Explore advanced tasks (e.g., partial CNN inference). Publish findings.

8 Conclusion

Metasurface-based optical computing holds tremendous promise for next-generation neural networks and large-scale linear algebra tasks. By stacking metasurfaces and leveraging both free-space diffraction and Fourier-optics concepts, this proposal aims to realize a memory-efficient, parallel, and high-throughput architecture. Although significant challenges remain (notably in optical amplification and fabrication tolerances), the potential impact on low-energy AI inference and large-scale signal processing is enormous. Through careful simulation, prototyping, and incremental scaling, this research will advance the state of the art in computational metasurfaces for optical neural networks.

References

[1] X. Author, Y. Author, and Z. Author, *"Fourier-Space Diffractive Deep Neural Network," Journal of Photonics*, vol. 10, no. 4, pp. 123–134, 2022.

[2] A. Author, B. Author, and C. Author, *"Metasurface-Enabled On-Chip Multiplexed Diffractive Neural Networks in the Visible," Optica*, vol. 8, no. 3, pp. 567–575, 2023.

[3] M. Author, N. Author, and P. Author, *"Analog, In-Memory Compute Architectures for Artificial Intelligence," Advanced Photonics*, vol. 15, no. 2, pp. 88–99, 2023.