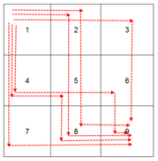
Count Paths



Write a program to count all the possible paths from top-left to bottom-right of a `m` X `n` matrix with the constraints that from each cell you can move only right, down, or diagonal (right and down).

Counting the number of paths can be trivially solved with recursive backtracking. Here is one example of how, for a 2D array `matrix` (with a call of `numberOfPaths(0, 0)`):

```
//Returns # of possible paths to reach cell at row m, column n from top-most, left-most cell
int numberOfPaths(int m, int n)
{
	//Base case, there is only one possible path if at end cell
	if (m == matrix.length - 1 || n == matrix[0].length - 1)
		return 1;

	return numberOfPaths(m+1, n) + numberOfPaths(m, n+1) + numberOfPaths(m+1, n+1);
}
```

Output, for a 3x3 matrix: `13`

However, the time complexity of above recursive solution is exponential. There are many overlapping sub-problems. The recursion tree would be similar to the recursion tree for the Longest Common Subsequence problem.

This problem has both properties (overlapping sub-problems and optimal sub-structure) of a dynamic programming problem. Like other typical DP problems, recomputations of same sub-problems can be avoided by constructing a temporary matrix and solving in "bottom up" manner.

Recursion, as you should know by now, solves problems in a "top down" fashion, breaking the problem into smaller and smaller sub-problems until the base case is reached, allowing the more complicated problems to then be solved in turn.

DP avoids re-computation by solving from the bottom up, *starting* with the most trivially easy problem and building *next* solution(s) from previous solution(s), until done.

Try to establish the base case(s) and recurrence relation on your own first; if you get stuck, copy the invisible text below, paste it into another document and change the color: