Thonk: Math, Physics & Chemistry

Contents

0 Preface

0.1   A Note to the Reader

0.1.1   Behind the Idea - A Short Story

This book is the result of a lot of contemplation. Psi25Omega was thinking about stuff he would like to learn and read about each day when he thought, "Why not take notes and make it a document? Why not 'LaTeX' it? It certainly would look good as well!". And then, he played with the different styles and options available for a few days and had a basic draft of the document. It was after this that he invited NSPKN6506 to the party after both of them found the project interesting to work on and helpful to people as well. And they have come this far, from struggling with multiple errors due to bad code (Yeah, they were not so good at LaTeX back then XD) and filling up the whole of January with content (Again, if you read it on par with the expectations; i.e. one thing a day) to the most recent draft you are gonna read. If you feel they did well, do say a "Hi" to them in school or on Discord or anywhere you meet them (You can certainly discuss more about these things). They would love to clarify any doubts on the book.

0.1.2   How you can start reading!

So, now you have finished the boring Introduction and have come to the part where you get some advice on how you can start reading the book. The thing is, there is no defined way in which you should read this. We recommend reading one "Box" a day as you can later search it up and gather more information going deeper into the topic. The content is a basic gist of the topic and tries kindle your interest to go to a browser and type in the topic on the search bar.

0.1.3   Final thoughts

Here are some final thoughts and reflections on the book. Please forgive us for any mistakes in the book and we'll try to correct them whenever you guys submit a pull request or we notice it. The book is of a higher level than standard Science and Math curriculum in India (9th and 10th grades). But don't worry, you can still understand most (95%) of the stuff (You can always use the internet to know about the things you don't understand). From another country's perspective, the book may be too trivial or tough, again depending on your curriculum's rigour. Overall, it's just another book that lists facts and theorems, but hopefully, in an inquisitive and interesting way.

0.2   Notations

0.2.1   The Box!

Here is an example of the typical "box" we have used in the book. We also colour code the boxes to represent the different subjects the things cover;

Subject	Colour
Math	Yellow
Physics	Blue
Chemistry	Red

The Title (Sometimes involves Bad Puns)

This so-called "box" usually contains the content and sometimes includes images, graphs or diagrams to make things interesting and clear. At times, we also have included footnotes[a] to be more reader-friendly. Also, all the illustrations given are made using LaTeX and by Psi25Omega. So, we aren't infringing any copyrights!!!

———————————————
[a]They are for clarifications that aren't really obvious but aren't significant at the same time.

0.3   Contributing

If you want to contribute to the project, feel free to fork our Github Repository and submit pull requests or get in touch with one of the authors.

0.3.1   Github

This is a link to our **Github** Repository where we host most of the Static files and LaTeX code.

1 January 2021

Mechanics, Position/Location Denote-rs, Notations, Conditions, Proofs, Prime Numbers, Particles, Constraints, Connections, Arrows, Diagrams, Chocolate, Conservation, Spaces, Additions, Multiplications, Conventions, Civil Engineering (Just Joking), Pulleys, Planes, Exam Centers, Geometry and What Not?

1.1 A Simple Proof

Are there infinitely many prime numbers? If yes, how do we prove they exist? Here's a simple proof.

Assume we have only n prime numbers; $P_1, P_2, P_3, \ldots P_n$.

$$\text{Let } N = P_1 \cdot P_2 \cdot P_3 \ldots P_n + 1$$

N isn't divisible by any of the primes $P_1, P_2, P_3, \ldots P_n{}^a$ which implies N's prime factorisation is $N \times 1$. N being prime contradicts our initial assumption.

Thus, there exist infinite primes.

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^{a}Prime numbers start from 2 and N is the LCM of all the prime numbers added to 1.

1.2 Are we down to the Least Hierarchical Particle?

Atoms are divisible, but are electrons, protons and neutrons?. A quark is an elementary particle and a fundamental constituent of matter. Quarks combine to form particles called hadrons. All commonly observable matter is composed of up quarks, down quarks and electrons. Due to a phenomenon called "Colour Confinement", quarks are never found existing individually, they can be found only composing hadrons, which include baryons (protons and neutrons) and mesons, or in quark–gluon plasma. They are the only elementary particles in the Standard Model of particle physics to experience all four fundamental interactions (electromagnetic force, gravitation, strong forces, and weak forces).

Quarks are of 6 types/flavours:

- Up
- Down
- Charm
- Strange
- Top
- Bottom

Up and down quarks have the lowest masses and such heavier quarks rapidly change into up and down quarks through the process of particle decay. Up and down quarks are generally stable and the most common in the universe, whereas strange, charm, bottom, and top quarks can only be produced in high energy collisions (such as those in particle accelerators like those found at CERN).

1.3   Quantum Numbers (Not a new System!?)

In chemistry and quantum physics, quantum numbers describe values of conserved quantities in the dynamics of a quantum system. An important aspect of quantum mechanics is the quantization of many observable quantities of interest. In particular, this leads to quantum numbers that take values in discrete sets of integers or half-integers[a]; although they approach infinity in some cases. Quantum numbers often describe specifically the energy levels of electrons in atoms, but other possibilities include angular momentum, spin, etc. An important family is flavour quantum numbers – internal quantum numbers which determine the type of a particle and its interactions with other particles through the fundamental forces. A system can have one or more quantum numbers; it is thus difficult to list all possible quantum numbers. Four quantum numbers can describe an electron in an atom completely:

- Principal quantum number (n)

- Azimuthal quantum number (l)

- Magnetic quantum number (m)

- Spin quantum number (s)

Also, the question of how many quantum numbers are needed to describe any given system has no universal answer; for each system, one must find the answer by performing a full analysis of the system. The most prominent system of nomenclature spawned from the molecular orbital theory of Friedrich Hund and Robert S. Mulliken, which incorporates Bohr energy levels as well as observations about electron spin. This model describes electrons using the above mentioned quantum numbers. It is also the common nomenclature in the classical description of nuclear particle states (e.g. protons and neutrons).

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[a]Numbers of the form $\frac{2n+1}{2}$

1.4   Exponentiation^{-1}

The logarithm function (denoted by log) is the inverse function of exponentiation. It denotes the exponent/power to which the 'base' has been raised to in a number. Here's an example;

$$\text{Let } x = a^b. \text{ This implies } \log_a(x) = b$$

The logarithm of x to the base a can be a whole number, decimal number, irrational number or even a complex number depending on its value. Often, log graph are used to make growth, forecast and even the number of cases during a pandemic. Logarithms involving complex numbers[a] (i) can be plotted on the real-complex plane.
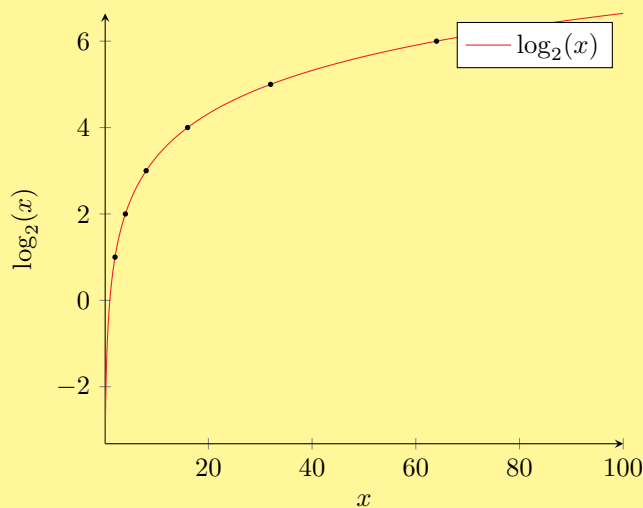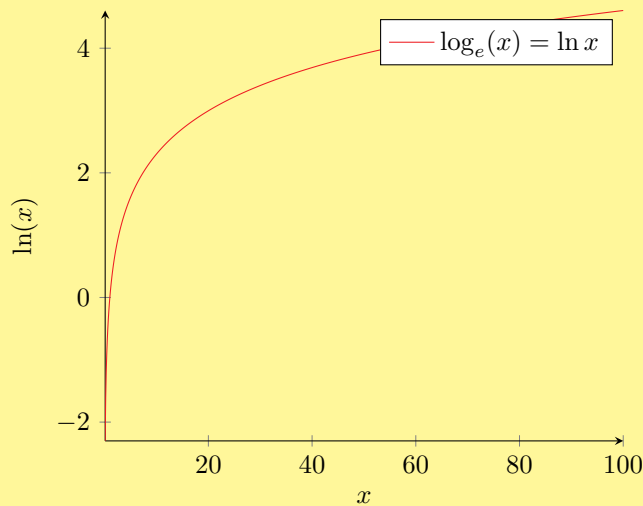
These are examples log-graphs to the base e^b and 2^c.

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[a] $\sqrt{-x}, \sqrt[3]{-x}, \ldots$

[b] Euler's constant; $e = 2.71$

[c] The points in the $\log_2(x)$ graph represent the points where the value of $\log_2(x)$ is an integer, i.e when x is a power of 2

1.5 Vectors!

Per definition, a vector is an element of a vector space (a set containing vectors). Essentially, a vector is any quantity represented taking into consideration both its direction and its magnitude. Vectors are represented by drawing a ray over them (preferably in their direction); i.e. a vectors as $\vec{a}$. A vector's output is decided by these 2 constituents;

- Magnitude; represented $||\vec{a}||$ or $|\vec{AB}|$ is the technically the length of the vector. Suppose

$$\vec{a} = m_1\hat{a_1} + m_2\hat{a_2} + m_3\hat{a_3} + \ldots m_n\hat{a_n}$$

  where $\hat{a_1}, \hat{a_2}, \hat{a_3}, \ldots \hat{a_n}$ represent the unit vectors in the n different axes/dimensions and $m_1, m_2, m_3, \ldots m_n$ are the respective coefficients, its magnitude is

$$\sqrt{m_1^2 + m_2^2 + m_3^2 + \ldots m_n^2}$$

- Direction; either positive or negative.

This is an example of a vector $(\vec{a})$ with co-ordinates x', y' and z' where $\vec{x}, \vec{y}$ and $\vec{z}$ are the unit vectors;
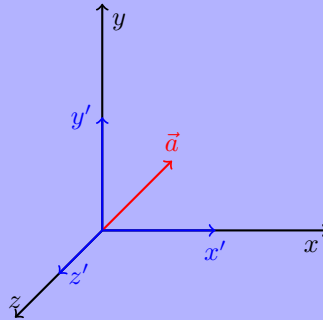
Vectors can be added to, subtracted from, multiplied or divided by another vector. Hence, $\vec{a}$ is actually the sum of three vectors x', y' and z'^a i.e

$$\vec{a} = \vec{x'} + \vec{y'} + \vec{z'}$$

There are a couple of laws and formulas that govern the arithmetic operations on vectors. Keep reading for those.

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[a]We call the vectors formed by the points x', y', z' the way they are named. But this isn't the case always. You will later come to know why its true in this case.
If you want to know right now, Hint: The vector's start is the origin $(0, 0, 0)$

1.6   Principal Quantum Number

In quantum mechanics, the principal quantum number (symbolized by n) is one of four quantum numbers assigned to each electron in an atom to describe that electron's state. Its values are natural numbers, making it a discrete variable.

Apart from the principal quantum number, the other quantum numbers for bound electrons are the azimuthal quantum number l, the magnetic quantum number m, and the spin quantum number s. As n increases, the electron also has higher energy and is, therefore, less tightly bound to the nucleus. For higher n the electron is farther from the nucleus. For each value of n there are n accepted l (azimuthal) values ranging from 0 to $n-1$ inclusively, hence higher-n electron states are numerous in definition. Accounting for two states of spin, each n-shell can accommodate up to $2n^2$ electrons (Given by Niels Bohr).

The principal quantum number was first created for use in the semi-classical Bohr model of the atom, distinguishing between different energy levels. With the development of modern quantum mechanics, the simple Bohr model was replaced with a more complex theory of atomic orbitals. However, the modern theory still requires the principal quantum number.

1.7 Sigma and Pi

Sigma and Pi are a part of the Greek alphabet used for notating different operations on a series of numbers which follow a specific pattern. $\sum$ represents the sum of the terms in series within the specified limits[a] while $\prod$ represents the product of the terms in the sequence.

- In $\sum$ notation, the part below (sub-scripted) the $\sum$ alphabet represents the start of the series and the part above (super-scripted) the $\sum$ represents the limit; i.e. the series's end. This is an example;

$$\sum_{n=0}^{25} n+1$$

   This symbolises the sum of $n+1$ for n from 0 to 25. $(1+2+3+4+\ldots 26)$

- In the $\prod$ notation, the part below (sub-scripted) the $\prod$ alphabet represents the start of the series and the part above (super-scripted) the $\prod$ represents the limit; i.e. the series's end. This is an example;

$$\prod_{n=0}^{25} n+1$$

   This symbolises the product of $n+1$ for n from 0 to 25. $(1 \times 2 \times 3 \times 4 \times \ldots 26)$

$\sum$ and $\prod$ are very useful in writing series operations as they are very easy to manipulate (and ergonomically, take up less space). For example, constants involved in the terms' definition can be put outside the Σ.

$$\text{In } \sum_{n=1}^{10} 5(n+1)$$

You can take 5 out and re-write it as

$$5 \sum_{n=1}^{10} n+1$$

since $5(2) + 5(3) + 5(4) + \ldots 5(11) = 5(2 + 3 + 4 + \ldots 11)$.
There are also a few formulas for the $\sum$ of different series. Some are given here.

$$\sum_{i=1}^{n} i = \frac{(n)(n+1)}{2}, \ \sum_{i=1}^{n} i^2 = \frac{(n)(n+1)(2n+1)}{6}$$

The former was devised by mathematician Gauss as a part of a punishment. There don't really exist any formulas for $\prod$ since it is very difficult to arrange stuff as you literally multiply a whole term by another.

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[a]Sometimes, the limit could just tend to infinity. We can also calculate the sums for these type of series if they converge; i.e. the terms decrease in value.

1.8   Arithmetic Operations on Vectors

The triangle rule of vector addition describes how we can add vectors;
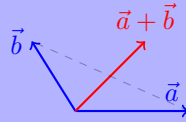
the vector from the tip of is resultant of the sum of $\vec{a}$ and $\vec{b}$.

- The magnitude of $\vec{a} + \vec{b}$ is

$$\sqrt{a^2 + 2ab\cos\theta + b^2}$$

  where a and b are the magnitudes of $\vec{a}$ and $\vec{b}$ respectively, and θ is the angle between $\vec{a}$ and $\vec{b}$

- If the angle made by $\vec{a} + \vec{b}$ with $\vec{a}$ is α, then

$$\tan\alpha = \frac{b\sin\theta}{a + b\cos\theta}$$

Coming to subtraction, it's the same but you take the negative value of the vector you subtract and add;
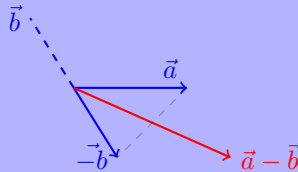
We can also multiply a vector by a scalar. If we have $\vec{a}$ with magnitude a and direction, multiplying it by a scalar m with magnitude 0.5 will give a new vector with magnitude $\frac{a}{2}$. Similarly, if we take 3 which is a scalar and multiply it by a $\vec{b}$, you get a new vector 3 times as long as $\vec{b}$. As a more physical example, the gravitational force on an object is a vector with its magnitude depending on a scalar; mass. If the mass of the object is doubled, the gravitational force is doubled as well.

$$\text{If } \vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$\text{then } |\vec{b}| = 2 \times |a| = 2 \cdot \sqrt{x^2 + y^2 + z^2}$$
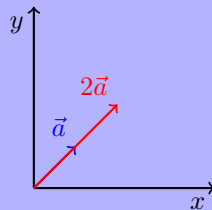
The same goes with division.

1.9   Azimuthal Quantum Number

The azimuthal quantum number is a quantum number for an atomic orbital that determines its orbital angular momentum as well as the shape of the orbital. The azimuthal quantum number is the second set of quantum numbers which describes the unique state of an electron (the others being the principal quantum number, the magnetic quantum number, and the spin quantum number). It is also known as the orbital angular momentum quantum number, orbital quantum number or second quantum number, and is symbolized as l.

Each of the different angular momentum states can take $2(2l + 1)$ electrons. This is because the third quantum number m (which can be thought of loosely as the quantized[a] projection of the angular momentum on the z-axis) runs from l to l in integer units, and so there are $2l + 1$ possible states. Each distinct n, l, m orbital can be occupied by two electrons with opposing spins (given by the quantum number $s = \pm\frac{1}{2}$), giving $2(2l + 1)$ electrons overall. Orbitals with higher l than given in the table are perfectly permissible, but these values cover all atoms so far discovered.

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[a]Restricting the number of possible values of a quantity or states of a system so that certain variables can assume only certain magnitudes.

1.10   Scaled up and Tightened!

A vector space (also called a linear space) is a set of vectors, which may be added together and multiplied ("scaled") by numbers (scalars). Scalars are often taken to be real numbers, but there are also vector spaces with scalar multiplication by complex numbers, rational numbers, or generally any field. The operations of vector addition and scalar multiplication must satisfy certain requirements, called vector axioms (listed below). To specify that the scalars are real or complex numbers, the terms real vector space and complex vector space are often used.

Certain sets of Euclidean vectors are common examples of a vector space. They represent physical quantities such as forces, where any two forces (of the same type) can be added to yield a third, and the multiplication of a force vector by a real multiplier is another force vector. In the same way (but in a more geometric sense), vectors representing displacements in the plane or three-dimensional space also form vector spaces. Vectors in vector spaces do not necessarily have to be arrow-like objects: vectors are regarded as abstract mathematical objects with particular properties, which in some cases can be visualized as arrows.

For example, the $x - y - x$ plane is a vector space used commonly.

Vector spaces are the subject of linear algebra and are well characterized by their dimension, which, roughly speaking, specifies the number of independent directions in the space. Infinite-dimensional vector spaces arise naturally in mathematical analysis as function spaces, whose vectors are functions. These vector spaces are generally endowed with some additional structure such as a topology, which allows the consideration of issues of proximity and continuity.

1.11   Have a good Perspective!

In Physics, a frame of reference (or reference frame) consists of an abstract coordinate system[a] and the set of physical reference points[b] that uniquely fixes, locates and orients the coordinate system and standardizes measurements within the frame. For n dimensions, $n+1$ reference points are sufficient to fully define a its status. Using rectangular/Cartesian coordinates, a reference frame may be defined with a reference point at the origin and a reference point at one unit distance along each of the n coordinate axes[c]. In Einsteinian relativity[d], reference frames are used to specify the relationship between a moving observer and the phenomenon or phenomena under observation. In this context, the phrase often becomes "observational frame of reference" or "observational reference frame", which implies that the observer is at rest with respect to the frame, although not necessarily located at the origin. A relativistic reference frame includes the coordinate of time.
Inertial and Non-Inertial Frames of Reference

- Inertial Frame: An Inertial frame of reference is a reference frame moving with a constant velocity with respect to an observatory frame[e]; i.e not accelerating. An example is a ground frame as it doesn't have any acceleration with respect to the Earth. Newton's Laws which we observe everyday hold true only in Inertial Reference Frames. This is due to the fact that inertial frames are not accelerating and hence; don't experience an external force.

- Non Inertial Frame: A Non Inertial frame of reference is a reference frame that is accelerated with respect to an observatory frame. Newton's laws will not hold true in these frames as there is an external force. The frame of the moon is an example as it is accelerated with respect to the Earth. But if we want to make Newton's law hold here we need to take some mysterious/imaginary forces known as Pseudo Forces[f] to account for the force experience due to the acceleration.

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[a]Basically, a system to plot points.

[b]Points from where observations are made

[c]The axes representing the n dimensions

[d]A field in physics dedicated to the study of relative motion between particles and specifically, the Special Theory of Relativity.

[e]The frame with respect to which we take the relative motion of our reference frame.

[f]Here, Pseudo literally means imaginary or false.

1.12   Magnetic Quantum Number

The magnetic quantum number (symbol m) is one of four quantum numbers in atomic physics. The magnetic quantum number distinguishes the orbitals available within a subshell, and is used to calculate the azimuthal component of the orientation of orbital in space. Electrons in a particular subshell (such as s, p, d, or f) are defined by values of l $(0, 1, 2,$ or $3)$. The value of m can range from $-l$ to $+l$, including zero. Thus the s, p, d, and f subshells contain $1, 3, 5$, and 7 orbitals each, with values of m within the ranges $0, \pm 1, \pm 2, \pm 3$ respectively. Each of these orbitals can accommodate up to two electrons (with opposite spins), forming the basis of the periodic table.

1.13 Sets; Colloquially, Groups

In mathematics, a set is a well-defined collection of distinct elements[a] or members. Sets' elements can be anything; literally! They can define points or co ordinates, names, letters, geometrical shapes, equations, lines, constants, variables or even other sets. We have a few basic operations involving sets that resemble the four arithmetic operations.

Set Notation:

A set is defined by enclosing its elements within two parentheses (open and close) in the form "{Element 1, Element 2, Element 3, ... }" where the elements are separated by commas.

We can also include other sets inside a subset in these two ways "$\{\{S_{e_1}^1, S_{e_2}^1, \dots\}, \{S_{e_1}^2, S_{e_2}^2, \dots\}, \dots\}$" or "$\{S^1, S^2, \dots\}$" where $S^1, S^2, \dots$ represent the different sets and $e_!, e_2, \dots$ represent the different elements in each of the sets.

Subsets and Supersets:

A subset is a collection of elements as well, but all of its elements are part of a bigger or equal set[b]. For example we define $S = \{1, 2, 3, 4, 5, 6\}$, and $SS = \{1, 2, 3, 5\}$. We say SS is a subset of S since all of its elements are a part of S. This is conveyed mathematically in the form;

$$SS \subset S$$

where $\subset$ is the symbol for "subset".

A superset is the exact inverse and we can say S is a superset of SS. It's represented as;

$$S \supset SS$$

Continuing with the same example, if $SS = \{1, 2, 3, 4, 5, 6\}$, we say S ans SS are equal; i.e they precisely have the same elements. This is conveyed mathematically in the form;

$$SS = S$$

Unions, Intersections and Cardinality:

The union of two sets are the elements that are present in at least one of the sets. For example the union of $A = (1, 2, 3)$ and $B = (0, 2, 4)$ is $(0, 1, 2, 3, 4)$. This can be written as $A \cup B = \{0, 1, 2, 3, 4\}$.

The intersection of two sets are the elements that are present in both sets. The intersection of A and B is (2). This can be written as $A \cap B = \{2\}$.

The cardinality of a set is the number of elements in it. For example, the cardinality of both A and B is 3.

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[a]Elements are the distinct or same values that can be numbers, letters, words, codes, etc.

[b]Here, bigger and equal refer to the number of elements in the sets.

1.14   Dots and Crosses

In mathematics, the dot product or scalar product is an algebraic operation that takes two equal-length sequences of numbers (usually coordinate vectors), and returns a single number. Dot Product can be given in many ways (Just kidding there are just 3).

- $a \cdot b = \sum_{i=1}^{n} a_i \cdot b_i \hat{i}$

- $a \cdot b = a_x \cdot b_x + a_y \cdot b_y + a_z \cdot b_z$

- $a \cdot b = |a| \cdot |b| \cdot \cos \theta$

Now coming to Cross Products, in mathematics, the cross product or vector product (occasionally directed area product, to emphasize its geometric significance) is a binary operation on two vectors in a three-dimensional space. The cross product is anti-commutative and is distributive over addition. This is given as

$$A \cdot B = ||A|| \cdot ||B|| \ \sin \theta \cdot \hat{n}$$

Where A,B are Vectors and n is vector perpendicular to plane of A and B and $|A|$ and $|B|$ are the lengths of the vectors (magnitude).
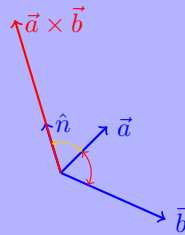
Here, the yellow angle is 90° and the red angle is θ.

1.15 Spin Quantum Number

In atomic physics, the spin quantum number is a quantum number that describes the intrinsic angular momentum (or spin angular momentum, or simply spin) of a given particle. The spin quantum number is designated by the letter s, and is the fourth of a set of quantum numbers (the principal quantum number, the azimuthal quantum number, the magnetic quantum number, and the spin quantum number), which completely describe the quantum state of an electron. this can be written as:-

$$||s|| = \sqrt{s(s+1)}h$$

where,

- s is the Spin Vector

- $||s||$ is the norm of the Spin Vector

- h is the Planck constant

1.16   Prime Number Theorem

The prime number theorem describes the asymptotic[a] distribution of prime numbers among positive integers. It formalizes the intuitive idea that primes become less common as they become larger. The prime number theorem addresses this by precisely quantifying the rate at their frequency decreases. The first breakthrough was the $\pi(n)$ function, which calculates the probability that a random integer less than or equal to n is prime. It's defined as;

$$\pi(n) \sim \frac{1}{\ln(n)}$$

where $\ln(n)$ is the natural logarithm[b] of n.

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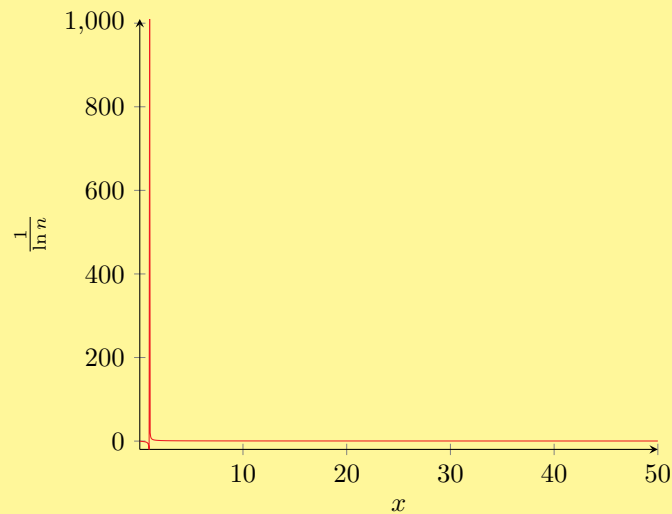
[a]Asymptotic here means approximate in mathematical terms.
[b]$\log_e(n)$

1.17   Free Body Diagrams

A Free Body Diagram consists of a diagrammatic representation of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it. It is a graphical illustration used to visualize the applied forces, moments, and resulting reactions on a body in a given condition (Sometimes also drawn to scale asymptotically). The body may consist of multiple internal members (a system of bodies), or be a single body. A series of free bodies and other diagrams may be necessary to solve complex problems. The reason why FBDs (Free Body Diagrams) are used widely is that they make problem solving easy. It is very useful in situations where the net force acting on the body is 0; i.e. the forces acting in the opposite directions can be equated to each other. Usually, arrows are drawn in the direction of the force. We can also take the components of the forces to make work easy. In FBDs, it is not only force that is represented, but we can also represent any acceleration the system is experiencing or the velocity with which it is moving. Usually internal forces are not represented, but it is no rule to not! Here are a few examples of FBDs:

A cube of mass m placed on a table:
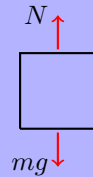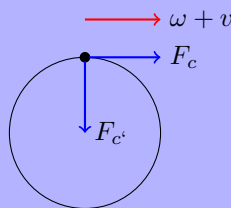
A cube of mass m with an acceleration a experiencing a frictional force F:
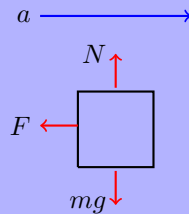
A point on the top of a roller moving with velocity v and angular velocity ω:

Try thinking through why there is no normal force or weight on the object and also, if you are curious about what F_c and $F_{c'}$ are, they are the centrifugal and centripetal forces. Find which one is which. XD

1.18   What are Subshells and Orbitals?

Subshells:
Each shell is composed of one or more subshells, which are themselves composed of atomic orbitals. For example, the first (K) shell has one subshell, called $1s$; the second (L) shell has two subshells, called $2s$ and $2p$; the third shell has $3s$, $3p$, and $3d$; the fourth shell has $4s$, $4p$, $4d$ and $4f$; the fifth shell has $5s$, $5p$, $5d$, and $5f$ and can theoretically hold more in the $5g$ subshell that is not occupied in the ground-state[a] electron configuration of any known element. Each subshell is constrained to hold $4l + 2$ electrons at most, namely:

- Each s subshell holds at most 2 electrons

- Each p subshell holds at most 6 electrons

- Each d subshell holds at most 10 electrons

- Each f subshell holds at most 14 electrons

- Each g subshell holds at most 18 electrons

Atomic Orbitals:
In atomic theory and quantum mechanics, an atomic orbital is a mathematical function describing the location and wave-like behavior of an electron in an atom.This function can be used to calculate the probability of finding any electron of an atom in any specific region around the atom's nucleus. The term atomic orbital may also refer to the physical region or space where the electron can be calculated to be present, as predicted by the particular mathematical form of the orbital. Orbitals have been given names, which are usually given in the form:
X orbital^y; where X is the energy level corresponding to the principal quantum number n; type is a lower-case letter denoting the shape or subshell of the orbital, corresponding to the angular quantum number l; and y is the number of electrons in that orbital.
For example, the orbital $1s^2$ (pronounced as the individual numbers and letters: "'one' 's' 'two'") has two electrons and is the lowest energy level ($n = 1$) and has an angular quantum number of $l = 0$, denoted by s.

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[a]The lowest energy state; i.e when the electron has very less energy to release or absorb.

1.19   Modular Arithmetic

Modular Arithmetic is an operation (used extensively in Mathematical Olympiads and Number Theory) to compute the remainder when a number is divided by another. For example, say 5 is the remainder when a is divided by b. We can represent it the following way:

$$a \equiv 5 \pmod{b}$$

where $\pmod{b}$ represents a taken modulo b and 5 is $a \pmod 5$.
Modulos can also be thought of this way.

$$14 \equiv 2 \pmod 3$$

On adding 3 to the remainder;

$$14 \equiv 5 \pmod 3$$

But this is true too (since 14 leaves a remainder of 5 on being divided by 9). Are we arriving upon a paradox? No, not at all. This actually leads us to our next point.
A property of congruence systems is that;

$$\text{If } a \equiv b \pmod n \implies \text{(it implies) } a \equiv b + n \pmod n$$

Modular Arithmetic systems also have the following properties;

- If $a \equiv b \pmod n$ and $c \equiv d \pmod n$; $ab \equiv cd \pmod n$

- If $a \equiv b \pmod n$ and $c \equiv d \pmod n$; $a + b \equiv c + d \pmod n$

The same apply for division and subtraction respectively.

1.20   Conservation of Energy

Conservation of energy, is a principle in physics according to which the energy of interacting bodies or particles in a closed system remains constant. The first kind of energy to be recognized was kinetic energy, or energy of motion. In certain particle collisions, called elastic, the sum of the kinetic energy of the particles before collision is equal to the sum of the kinetic energy of the particles after collision. The notion of energy was progressively widened to include other forms. The kinetic energy lost by a body slowing down as it travels upward against the force of gravity was regarded as being converted into potential energy, or stored energy, which in turn is converted back into kinetic energy as the body speeds up during its return to Earth. For example, when a pendulum swings upward, kinetic energy is converted to potential energy. When the pendulum stops briefly at the top of its swing, the kinetic energy is zero, and all the energy of the system is in potential energy. When the pendulum swings back down, the potential energy is converted back into kinetic energy. This is given as:-

$$K_1 + U_1 = K_2 + U_2$$

where K_1 and U_1 are the initial kinetic energy and potential energy respectively, while K_2 and U_2 are the final kinetic and potential energies.

1.21   Block Building Principle

The Aufbau principle, from the German "Aufbauprinzip" (building-up principle), also called the Aufbau rule, states that in the ground state of an atom or ion, electrons fill atomic orbitals of the lowest available energy levels before occupying higher levels. For example, the $1s$ subshell is filled before the $2s$ subshell is occupied. In this way, the electrons of an atom or ion form the most stable electron configuration possible. An example is the configuration $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^3$ for the phosphorus atom, meaning that the $1s$ subshell has 2 electrons, and so on.

Electron behavior is elaborated by other principles of atomic physics, such as Hund's rule and the Pauli exclusion principle. Hund's rule asserts that if multiple orbitals of the same energy are available, electrons will occupy different orbitals singly before any are occupied doubly. If double occupation does occur, the Pauli exclusion principle requires that electrons that occupy the same orbital must have different spins. That is $\pm\frac{1}{2}$.
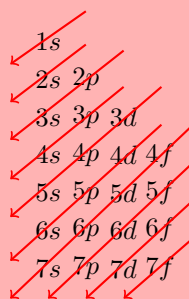
A diagrammatic representation of the order to fill electron subshells.

1.22   Quadratic Residues (Not to be residued!)

An interesting part of modular arithmetic congruence systems is quadratic residues which represent the remainder when a perfect square is divided by some integer n.

For example, one can observe that 0 and 1 are the quadratic residues when any perfect square is taken modulo 4; i.e. (mod 4).

This has a very simple proof. We know that any integer x is either $0, 1, 2$ or 3 (mod 4). From the multiplicative property,

$$x \equiv 0, 1, 2, 3 \pmod 4$$
$$\implies x \times x \equiv 0 \times 0, 1 \times 1, 2 \times 2, 3 \times 3 \pmod 4$$
$$\implies x \equiv 0, 1, 4, 9 \pmod 4$$
On writing 4 and 9 as $4 + 0$ and $2 \times 4 + 1$,
$$x \equiv 0, 1 \pmod 4$$

The following table lists quadratic residues when taken (mod n);

n	$x^2 \pmod n$
1	0
2	$0, 1$
3	$0, 1$
4	$0, 1$
5	$0, 1, 4$
6	$0, 1, 3, 4$
7	$0, 1, 2, 4$
8	$0, 1, 4$

Quadratic residues are extensively used to compute Polynomials and i quadratic equations.

Some Problems!

- Can the sum of 2 squares leave a reminder of 3 when divided by 4?

- Can the difference of 2 squares be of the form $2(2k + 1)$?

1.23   Conservative and Non Conservative

In Potential Energy and Conservation of Energy, any transition between kinetic and potential energy conserved the total energy of the system. This was path independent, meaning that we can start and stop at any two points in the problem, and the total energy of the system—kinetic plus potential at these points are equal to each other. This is characteristic of a conservative force. We dealt with conservative forces in the preceding section, such as the gravitational force and spring force. When comparing the motion of the football The total energy of the system never changes, even though the gravitational potential energy of the football increases, as the ball rises relative to ground and falls back to the initial gravitational potential energy when the football player catches the ball. Non-conservative forces are dissipative forces such as friction or air resistance. These forces take energy away from the system as the system progresses, energy that you can't get back. These forces are path dependent; therefore it matters where the object starts and stops.

1.24   Hund's Choco Orbital Rule

According to Hund's rule:
Before the double occupation of any orbital, every orbital in the sub level is singly occupied. For the maximization of total spin, all electrons in a single occupancy orbital have the same spin. An electron will not pair with another electron in a half-filled orbital as it has the ability to fill all its orbitals with similar energy. Many unpaired electrons are present in atoms which are at the ground state. If two electrons come in contact they would show the same behaviour as two magnets do. The electrons first try to get as far away from each other as possible before they have to pair up.
The electrons enter an empty orbital before pairing up. The electrons repel each other as they are negatively charged. The electrons do not share orbitals to reduce repulsion.
When we consider the second rule, the spins of unpaired electrons in singly occupied orbitals are the same. The initial electrons spin in the sub-level decides what the spin of the other electrons would be. For instance, a carbon atom's electron configuration would be $1s^2\ 2s^2\ 2p^2$. The same orbital will be occupied by the two 2s electrons although different orbitals will be occupied by the two $2p$ electrons in reference to Hund's rule.

1.25   Triangle Centers

There are 4 centers (among the many that are insignificant to us) that are present in a triangle. They are the Centroid, Orthocentre, Circumcenter and Incenter.

Centroid: The centroid is the intersecting point of all the 3 medians of a triangle. It is also the center of mass of a regular (in terms of mass, not equilateral) triangle with a plane surface. Thus, if you place a triangle's centroid on a small point and try to balance it, it will most likely work if your graphing skills are dope like **Psi25Omega** (Just Kidding) and you ensure no natural factors affect the triangle.

Orthocenter: The orthocentre is the meeting point of all altitudes. It essentially isn't the center of mass but has some special properties that are too overpowered to be discussed here.

Circumcenter: The circumcenter of a triangle is the center of a circle passing through the three vertices of a triangle (the circumscribed circle). It is also the intersection of the perpendicular bisectors of the sides of the triangle.

Incenter: The incenter is the center of the inscribed circle (the circle tangent to the three sides and drawn inside the triangle). It is also incidentally, the intersetion of the angle bisectors of the 3 angles in the triangle.

The centers are usually denoted by these alphabtets;

Centroid - G, Orthocenter - H, Circumcenter - O, Incenter - I.

Some Special Cases:[a]

- In the case of a right triangle, the circumcenter is the vertex opposite to the hypotenuse.

- In an equilateral triangle, all these converge and are basically the same point.

Just this one illustration;

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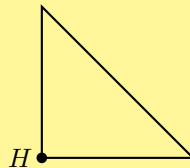
[a]My (Psi25Omega's) LaTeX skills aren't so pro, so I can't draw an illustration for every center.

1.26   Thonk about This!

When it is given that a specific pulley is mass less then the tensions on both the sides of that pulley are equal.

If you encounter with a situation as shown in the below picture and if the pulley is mass-less, then the the tension on both sides are equal; i.e.
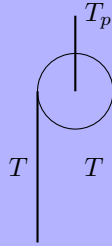
$$2T_p = T + T = 2T$$

There are many such conditions and situations under which you can equate tension in different places in a pulley-system. Pulleys are also called the Atwood's Machine. There's also the concept of "slack" to calculate the magnitude of acceleration that bodies in a pulley system when they are released - left to free fall. But there's a nice formula to calculate the force and the acceleration for a 2-body system.
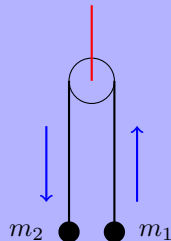
Let's say the acceleration of both the blocks is a (it is gonna be the same except for the direction).

Then the tension in the string T is given by

$$T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

while the acceleration a^a is given by

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

---

[a]Here, consider $m_2 > m_1$ and the direction of acceleration of block with mass m_1 as positive. (Also, try deriving these on your own with an FBD)

1.27 Pauli's "Excluded" Principle

The Pauli's exclusion principle is the quantum mechanical principle which states that two or more identical fermions[a] cannot occupy the same quantum state within a quantum system simultaneously. This principle was formulated by Austrian physicist Wolfgang Pauli in 1925 for electrons, and later extended to all fermions with his spin–statistics theorem of 1940. In the case of electrons in atoms, it can be stated as follows: it is impossible for two electrons of a poly-electron atom to have the same values of the four quantum numbers: n, the principal quantum number, l, the azimuthal quantum number, m, the magnetic quantum number, and s, the spin quantum number. For example, if two electrons reside in the same orbital, then their n, l, and m values are the same, therefore their s must be different, and thus the electrons must have opposite half-integer spin projections of $\frac{1}{2}$ and $-\frac{1}{2}$.

---

[a]Particles with half-integer spin

1.28   GCD and 2 Proofs

The GCD (Greatest Common Divisor) of 2 or more numbers is the largest integer that divides all of the numbers. There are a few properties GCD has and we can exploit them to prove lemmas among others.
For 2 numbers a and b;

$$\gcd(a, b) = \gcd(a - b, b)$$

This is also heavily used in the result of the Euclidean Division Algorithm.
Proof:
Let's assume

$$\gcd(a, b) = k$$

where k is any integer and let

$$a = ka' \text{ and } b = kb'$$

From this,

$$a - b = k(a' - b')$$

Notice that this is also divisible by k.
This proves the statement and wait, we have a problem!

IMO 1959 P1: Prove that the fraction $\frac{21n+4}{14n+3}$ is irreducible (cannot be simplified further) for every natural number n.[a]

A special relation between the LCM and GCD of 2 numbers exists;

$$\text{lcm}(a, b) \cdot \gcd(a, b) = ab$$

The proof involves some prime factorization of the gcd combined with some manipulation.

---

[a]Here, we essentially need to prove $\gcd(21n + 4, 14n + 3) = 1$

1.29   Pulleys, MA and Planes!

Pulleys A pulley is a wheel on an axle or shaft that is designed to support movement and change of direction of a taut cable or belt, or transfer of power between the shaft and cable or belt. In the case of a pulley supported by a frame or shell that does not transfer power to a shaft, but is used to guide the cable or exert a force, the supporting shell is called a block, and the pulley may be called a sheave.

A pulley may have a groove or grooves between flanges around its circumference to locate the cable or belt. The drive element of a pulley system can be a rope, cable, belt, or chain.

It can be of three types

- **Fixed**:A fixed pulley has an axle mounted in bearings attached to a supporting structure. A fixed pulley changes the direction of the force on a rope or belt that moves along its circumference. Mechanical advantage is gained by combining a fixed pulley with a movable pulley or another fixed pulley of a different diameter.

- **Movable**:A movable pulley has an axle in a movable block. A single movable pulley is supported by two parts of the same rope and has a mechanical advantage of two.

- **Compound**:A combination of fixed and movable pulleys forms a block and tackle. A block and tackle can have several pulleys mounted on the fixed and moving axles, further increasing the mechanical advantage.

Inclined Planes An inclined plane, also known as a ramp, is a flat supporting surface tilted at an angle, with one end higher than the other, used as an aid for raising or lowering a load. Due to conservation of energy, the same amount of mechanical energy (work) is required to lift a given object by a given vertical distance, disregarding losses from friction, but the inclined plane allows the same work to be done with a smaller force exerted over a greater distance. The angle of friction, also sometimes called the angle of repose, is the maximum angle at which a load can rest motionless on an inclined plane due to friction, without sliding down. This angle is equal to the arctangent of the coefficient of static friction s between the surfaces.

The mechanical advantage of an inclined plane depends on its slope, meaning its gradient or steepness. The smaller the slope, the larger the mechanical advantage, and the smaller the force needed to raise a given weight. A plane's slope s is equal to the difference in height between its two ends, or "rise", divided by its horizontal length, or "run".This can be also said as:

$$\theta = \tan^- 1 \frac{Rise}{Run}$$

Mechanical Advantage The mechanical advantage MA of a simple machine is defined as the ratio of the output force exerted on the load to the input force applied. For the inclined plane the output load force is just the gravitational force of the load object on the plane, its weight F_w. The input force is the force F_i exerted on the object, parallel to the plane, to move it up the plane. The mechanical advantage is

$$MA = \frac{F_w}{F_i}$$

1.30   A Salty Bond

The Ionic bond is a type of a chemical bond that is a result of the attraction between oppositely charged particles in ionic compounds like NaCl[a]. Ions are atoms (or a group of atoms) having a net charge. Atoms that gain electrons to become stable are called anions while those that lose electrons for the same are called cations. This transfer of electrons is known as electrovalence. Ionic bonds are mostly formed between metals and non-metals. In simpler words, an ionic bond is a result of the transfer of electrons from a metal (cation) to a non-metal (anion) in order for both atoms to attain stability.

It is important to recognize that clean ionic bonding — in which one atom or molecule completely transfers an electron to another — cannot exist: all ionic compounds have some degree of covalent bonding (where electron pairs are shared), or electron sharing. Thus, the term "ionic bonding" is given when the ionic character is greater than the covalent character.

Ionic compounds conduct electricity when molten or in dissolved in a solvent, typically not when solid. Ionic compounds generally have a high melting point because the bond is ironically very strong, though held by just electron clouds and the attraction. Depending on the charge of the ions they consist of, higher charges mean stronger cohesive forces and a higher melting point. They also tend to be soluble in water; and the stronger the cohesive forces, the lower the solubility.

Here is an example of an ionic bond between Na[b] and Cl[c].
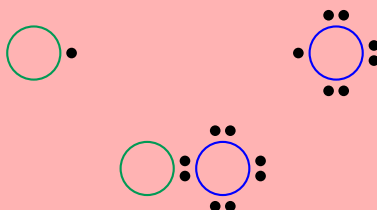
Here, the black dots represent the electrons.

---

[a]More examples; KCl, CaCl$_2$

[b]The green one

[c]The blue one

1.31   Constraint Relation

Constraint Relation is a beautiful and interesting concept which helps in solving questions related to pulleys and strings. It can be used to solve even the most complicated problems. Once understood it will be a very useful tool in solving problems in dynamics.

Constraint relation says that the sum of products of all tensions in strings and velocities of respective blocks connected to the strings is equal to 0. In other words it says that the total power by tension is zero. Mathematically it is represented by :

$$\sum T \cdot \vec{v} = 0$$

If the velocity vector is constant then differentiating the above equation with respect to time we get another relation. If the velocity vector is constant then the sum of products of all tensions in strings and accelerations of respective blocks connected to the strings is equal to 00. It is mathematically represented by :

$$\sum T \cdot \vec{a} = 0$$

Note: Constraint relation works only when the strings are inextensible and taut.

A Clarification

We skipped Math on the 31st and interchanged it with Physics because we felt it would be a sin to move Constraint Relation to February as it the month has a completely different outlook and context.[a]

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[a]OK! Considering it a sin is an exaggeration but yeah, moving it would cause unneeded confusion and we needed to break the arbitrariness of the schedule to do so.

2 Feb

2.1 Diophantine Equations

In mathematics, a Diophantine equation is a polynomial equation, usually involving two or more unknowns, such that the only solutions of interest are the integer ones (an integer solution is such that all the unknowns take integer values). In Diophantine equations, the variable can take any degree be it 1 or 100. But we are usually interested in finding the integer solutions as they have many real world applications.

Diophantine problems have fewer equations than unknowns[a] and involve finding integers that solve simultaneously all equations. As such systems of equations define algebraic curves, algebraic surfaces, or, more generally, algebraic sets, their study is a part of algebraic geometry that is called Diophantine geometry.

The Diophantine equation $a^2 + b^2 = c^2$ is an example of such an equation. There are infinite solutions to it when $a, b, c \in$ integers[b]

Chinese Remainder Theorem

The Chinese remainder theorem describes an important class of linear Diophantine systems of equations: let $n_1, n_2, \ldots n_k$ be k pairwise co-prime integers greater than 1, $a_1, a_2, \ldots a_k$ be k arbitrary[c] integers, and N be the product $n_1, n_2, \ldots, n_k = n_1 \cdot n_2 \cdot n_3 \ldots n_k$.

The Chinese remainder theorem asserts that the following linear Diophantine system has exactly one solution $(x, x_1, \ldots x_k)$ such that $0 \leq x \leq N$, and that the other solutions are obtained by adding to x a multiple of N:

$$x = a_1 + n_1\, x_1$$

$$\vdots$$

$$x = a_k + n_k\, x_k$$

This can be easily proved with modular arithmetic. So, we unleash an author's best weapon.

"The rest of the exercise is left to the reader as it is trivial."

---

[a]Usually, you require n distinct (Here, distinct refers to not reversing another existing equation or a derivation from it) equations to solve for n variables.

[b]$\in$ means "in". Here, we use it to say a, b, c are a part of the set of integers, i.e. $\mathbb{Z}$.

[c]Without any condition on them

2.2   Bernoulli's Principle

For steady, incompressible, nonviscous and irrotational flow, the quantity p + $\frac{1}{2}\rho g$ +$\rho g y$ is conserved i.e

$$p + \frac{1}{2}\rho g + \rho g y$$

where p is the pressure of the fluid ρ is the density of the fluid, y is height or depth and g is acceleration First derived (1738) by the Swiss mathematician Daniel Bernoulli, the theorem states, in effect, that the total mechanical energy of the flowing fluid, comprising the energy associated with fluid pressure, the gravitational potential energy of elevation, and the kinetic energy of fluid motion, remains constant. Bernoulli's theorem is the principle of energy conservation for ideal fluids in steady, or streamline, flow and is the basis for many engineering applications. Bernoulli's theorem implies, therefore, that if the fluid flows horizontally so that no change in gravitational potential energy occurs, then a decrease in fluid pressure is associated with an increase in fluid velocity. If the fluid is flowing through a horizontal pipe of varying cross-sectional area, for example, the fluid speeds up in constricted areas so that the pressure the fluid exerts is least where the cross section is smallest. This phenomenon is sometimes called the Venturi effect, after the Italian scientist G.B. Venturi (1746–1822), who first noted the effects of constricted channels on fluid flow.

2.3   Thermal Stuff! (Hot)

The Zeroth Law of Thermodynamics states that if two systems are in thermodynamic equilibrium with a third system, the two original systems are in thermal equilibrium with each other. Basically, if system A is in thermal equilibrium with system C and system B is also in thermal equilibrium with system C, system A and system B are in thermal equilibrium with each other.

2.4   Pell's Equation

Pell's equation, also called the Pell–Fermat equation, is any Diophantine equation of the form

$$x^2 - ny^2 = 1$$

where n is a given positive non-square integer. It has infinitely many solutions (proved by Joseph Lagrange) in x and y and forms a hyperbola when plotted in Cartesian co-ordinates.

We can also find its solutions through a recursive algorithm where;

If (x_0, y_0) is a solution to $x^2 - dy^2 = N$ and (u_n, v_n) is a solution to $u^2 - dv^2 = 1$ then (x_n, y_n) such that $x_n + y_n\sqrt{d} = (x_0 + y_0\sqrt{d})$ is a solution to $x^2 - dy^2 = N$.

This is the graph for $n = 2$:
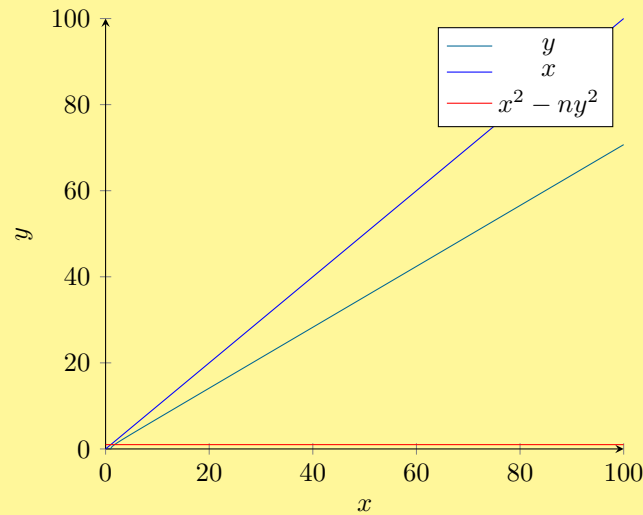
2.5   Toricelli's Theorm

Torricelli's theorem, also called Torricelli's law, Torricelli's principle, or Torricelli's equation, statement that the speed, v, of a liquid flowing under the force of gravity out of an opening in a tank is proportional jointly to the square root of the vertical distance, h, between the liquid surface and the centre of the opening and to the square root of twice the acceleration caused by gravity, $2g$, or simply $v = \sqrt{2gh}$.
The speed of a portion of water flowing through an opening in a tank a given distance, h, below the water surface is the same as the speed that would be attained by a drop of water falling freely under the force of gravity alone (that is, neglecting effects of air) through the same distance, h. The speed of efflux is independent of the direction of flow; at the point of the opening the speed is given by this equation, whether the opening is directed upward, downward, or horizontally.

2.6   The Basic Principle of the Thermal World

The first law of thermodynamics, also known as Law of Conservation of Energy, states that energy can neither be created nor destroyed; energy can only be transferred or changed from one form to another. For example, turning on a light would seem to produce energy; however, it is electrical energy that is converted.

$$\Delta U = Q + W$$

Where U is change in energy and Q is heat added and W is work done by system This law says that there are two kinds of processes, heat and work, that can lead to a change in the internal energy of a system. Since both heat and work can be measured and quantified, this is the same as saying that any change in the energy of a system must result in a corresponding change in the energy of the surroundings outside the system. In other words, energy cannot be created or destroyed. If heat flows into a system or the surroundings do work on it, the internal energy increases and the sign of q and w are positive. Conversely, heat flow out of the system or work done by the system (on the surroundings) will be at the expense of the internal energy, and q and w will therefore be negative.

2.7 Nooks and Corners

In mathematics, a parametric equation defines a group of quantities as functions of one or more independent variables called parameters. Parametric equations are commonly used to express the coordinates of the points that make up a geometric object such as a curve or surface (which are generally hard to express using linear equations and other such simpler ones), in which case the equations are collectively called a parametric representation or parameterization of the object.

Examples are parametric equations $a = \cos x$ and $b = \sin x$ are the parameterization of a unit circle with x as the parameter such that $(a, b) = (\cos x, \sin x)$ are the co-ordinates.
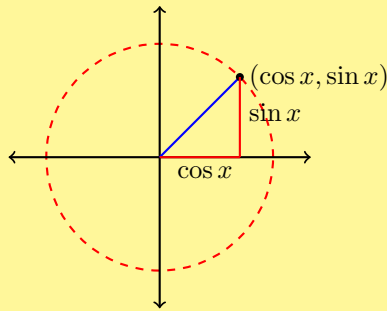
Explicit equations

More generally, any curve given by an explicit equation

$$y = f(x)$$

Parabola

The simplest equation for a parabola is

$$y = x^2$$

Circle

Any circle can be parameterized by the following equation where r is the radius of the circle.

$$x^2 + y^2 = r^2$$

Ellipse

An ellipse in canonical position (center at origin, major axis along the X-axis) with semi-axes a and b can be represented parametrically as

$$x = a \cos t$$
$$y = b \sin t$$

Parametric Equations are very useful in expressing geometric shapes using algebraic terms.

2.8   "Bulky" Modulus

When we increase the pressure on a material by an amount Δp ,it's density will increase.The fractional change in Volume will be $\frac{\Delta V}{V}$,which is negative if the volume decreases. The ratio between these quantities is called Bulk Modulus (B)

$$B = -\frac{P}{\Delta V/V}$$

The Minus $(-)$ sign has been inserted in this definition to make B positive,because Δp and v have opposite signs.

2.9   Enthal/Ropy

Enthalpy: When a process occurs at constant pressure, the heat evolved (either released or absorbed) is equal to the change in enthalpy.Enthalpy(H) is the sum of the internal energy (U) and Product of Pressure and Volume.

$$H = U + PV$$

Under constant Pressure

$$\Delta H = \Delta U + \Delta PV$$

Under constant pressure and temperature

$$\Delta H = \Delta U + P\Delta V$$

Entropy: Entropy, the measure of a system's thermal energy per unit temperature that is unavailable for doing useful work. Because work is obtained from ordered molecular motion, the amount of entropy is also a measure of the molecular disorder, or randomness, of a system. The concept of entropy provides deep insight into the direction of spontaneous change for many everyday phenomena. Its introduction by the German physicist Rudolf Clausius in 1850 is a highlight of 19th-century physics.

$$S = k_b ln(\Omega)$$

Where S is Entropy, k_b is Boltzmann's constant and Ω is amount of microscopic observations.

2.10   Venturi's effect

The Venturi effect is the reduction in fluid pressure that results when a fluid flows through a constricted section (or choke) of a pipe. The Venturi effect is named after its discoverer, Giovanni Battista Venturi.In fluid dynamics, an incompressible fluid's velocity must increase as it passes through a constriction in accord with the principle of mass continuity, while its static pressure must decrease in accord with the principle of conservation of mechanical energy (Bernoulli's principle). Thus, any gain in kinetic energy a fluid may attain by its increased velocity through a constriction is balanced by a drop in pressure.

By measuring pressure, the flow rate can be determined, as in various flow measurement devices such as Venturi meters, Venturi nozzles and orifice plates.

Referring to the adjacent diagram, using Bernoulli's equation in the special case of steady, incompressible, inviscid flows (such as the flow of water or other liquid, or low speed flow of gas) along a streamline, the theoretical pressure drop at the constriction is given by:

$$p_1 + p_2 = \frac{\rho}{2}(v_2^2 - v_1^2)$$

where ρ, is the density of the fluid, v_1 is the (slower) fluid velocity where the pipe is wider, v_2 is the (faster) fluid velocity where the pipe is narrower

Choked Flow The limiting case of the Venturi effect is when a fluid reaches the state of choked flow, where the fluid velocity approaches the local speed of sound. When a fluid system is in a state of choked flow, a further decrease in the downstream pressure environment will not lead to an increase in velocity, unless the fluid is compressed.

The mass flow rate for a compressible fluid will increase with increased upstream pressure, which will increase the density of the fluid through the constriction (though the velocity will remain constant). This is the principle of operation of a de Laval nozzle. Increasing source temperature will also increase the local sonic velocity, thus allowing for increased mass flow rate but only if the nozzle area is also increased to compensate for the resulting decrease in density.

Expansion of section The Bernoulli equation is invertible, and pressure should rise when a fluid slows down. Nevertheless, if there is an expansion of the tube section, turbulence will appear and the theorem will not hold. In all experimental Venturi tubes, the pressure in the entrance is compared to the pressure in the middle section; the output section is never compared with them.

2.11   Heisenberg's Life Questioning Inequality

In quantum mechanics[a], the uncertainty principle (also known as Heisenberg's uncertainty principle) is a variety of mathematical inequalities asserting a fundamental limit to the accuracy with which the values for certain pairs of physical quantities of a particle, such as position (x) and momentum (p) can be predicted from initial or known conditions. Heisenberg, in simple words stated that if you know velocity/momentum of the particle, you can't find it's position and vice versa. The equation stated in favour;

$$\Delta x \times \Delta P \geq \frac{h}{4\pi}$$

where $h = 6.626 \times 10^-34$ (Planck's Constant).
The more precisely the position of some particle is determined, the less precisely its momentum/velocity can be predicted from the known initial conditions.

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[a]Branch of Physics dedicated to observing the physical properties at the subatomic scale.

2.12   Catalan's Conjecture

The Catalan's conjecture, conjectured by the mathematician Eugène Charles Catalan in 1844 and proven in 2002 by Preda Mihăilescu. It states that there exists only one solution to the equation

$$x^a - y^b = 1$$

where $x = 3, a = 2, y = 2, b = 3$ for $a, b > 1$ and $x, y > 0$.

2.13   Magnus's effect

The Magnus effect is an observable phenomenon that is commonly associated with a spinning object moving through air or another fluid. The path of the spinning object is deflected in a manner that is not present when the object is not spinning. The deflection can be explained by the difference in pressure of the fluid on opposite sides of the spinning object. The Magnus Effect depends on the speed of rotation.

The most readily observable case of the Magnus effect is when a spinning sphere (or cylinder) curves away from the arc it would follow if it were not spinning. It is often used by football players, baseball pitchers and cricket bowlers. Consequently, the phenomenon is important in the study of the physics of many ball sports. It is also an important factor in the study of the effects of spinning on guided missiles—and has some engineering uses, for instance in the design of rotor ships and Flettner aeroplanes.

Topspin in ball games is defined as spin about a horizontal axis perpendicular to the direction of travel that moves the top surface of the ball in the direction of travel. Under the Magnus effect, topspin produces a downward swerve of a moving ball, greater than would be produced by gravity alone. Backspin produces an upwards force that prolongs the flight of a moving ball

2.14   Entropy rules!

The second law of thermodynamics says that the entropy of any isolated system always increases. Isolated systems spontaneously evolve towards thermal equilibrium—the state of maximum entropy of the system. More simply put: the entropy of the universe (the ultimate isolated system) only increases and never decreases.[a]

A simple way to think of the second law of thermodynamics is that a room, if not cleaned and tidied, will invariably become more messy and disorderly with time – regardless of how careful one is to keep it clean. When the room is cleaned, its entropy decreases, but the effort to clean it has resulted in an increase in entropy outside the room that exceeds the entropy lost.

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[a]RIP Universe. It is becoming more and more random with time.

2.15   Zero is weird

The third law of thermodynamics states that the entropy of a system approaches a constant value as the temperature approaches absolute zero. The entropy of a system at absolute zero is typically zero, and in all cases is determined only by the number of different ground states it has. Specifically, the entropy of a pure crystalline substance (perfect order) at absolute zero temperature is zero. This statement holds true if the perfect crystal has only one state with minimum energy.

2.16   Angular Momentum of an Electron

The angular momentum (L) of an electron in the nth orbit is given by

$$L = \frac{nh}{2\pi}$$

where h is the Planck's constant.

2.17   Gibbs Free Energy

In thermodynamics, the Gibbs Free Energy (G) (named after Josiah Willard Gibbs) is a thermodynamic potential that calculates the maximum reversible work performed by a thermodynamic system at a constant temperature (T) and pressure (P). It is given by

$$\Delta G = \Delta H - T\Delta S$$

where S represents its Entropy, i.e. the measure of randomness. S.I unit - Joules

2.18   Meme Time!

In chemical thermodynamics, the fugacity of a real gas is an effective partial pressure which replaces the mechanical partial pressure in an accurate computation of the chemical equilibrium constant. It is equal to the pressure of an ideal gas which has the same temperature and molar Gibbs free energy as the real gas. Fugacities are determined experimentally or estimated from various models such as a Van der Waals gas that are closer to reality than an ideal gas. The real gas pressure and fugacity are related through the dimensionless fugacity coefficient. ϕ

$$\phi = \frac{f}{P}$$

Where ϕ is fugacity coeffiecient, f is small pressure change and P is large pressure change

2.19 Inequalities

In mathematics, an inequality is a relation which makes a non-equal comparison between two numbers or other mathematical expressions. It is used most often to compare two numbers on the number line by their size. There are several different notations used to represent different kinds of inequalities:

- The notation $a < b$ means that a is less than b.

- The notation $a > b$ means that a is greater than b.

In either case, a is not equal to b. These relations are known as strict inequalities, meaning that a is strictly less than or strictly greater than b. Equivalence is excluded.In contrast to strict inequalities, there are two types of inequality relations that are not strict:

- The notation $a \leq b$ means that a is less than or equal to b (or, equivalently, at most b, or not greater than b).

- The notation $a \geq b$ means that a is greater than or equal to b (or, equivalently, at least b, or not less than b).

There are many inequalities between means. For example, for any positive numbers $a_1, a_2, \ldots a_n$ we have $H \leq G \leq A \leq Q$, where

- $H = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \ldots \frac{1}{a_n}}$

- $G = \sqrt[n]{a_1 \cdot a_2 \cdot a_3 \cdot \ldots a_n}$

- $A = \frac{a_1 + a_2 + a_3 \ldots \ldots a_n}{n}$

- $Q = \sqrt{\frac{a_1^2 + a_2^2 + a_3^2 \ldots a_n^2}{n}}$

A Problem:
Prove $a^2 + b^2 \geq 2ab$ for all integers a, b.[a]

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[a]This is actually the first step/example in the road to prove $G \leq A$.

2.20   Half Life of Atoms

Half-life represented by $t_{\frac{1}{2}}$ is the time required for a quantity to reduce to half of its initial value. It's used in nuclear physics to describe how quickly unstable atoms undergo radioactive decay or how long stable atoms survive. Half-life is mainly defined in terms of probability; i.e. "Half-life is the time required for exactly half of the entities to decay on average". In other words, the probability of a radioactive atom decaying within its half-life is 50Exponential decay can be described by any of the following three equivalent formulas:

$$N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$N(t) = N_0 e^{-\frac{t}{\tau}}$$

$$N(t) = N_0 e^{-\lambda t}$$

where;

- N_0 is the initial quantity of the substance that will decay,

- $N(t)$ is the quantity that still remains and has not yet decayed after a time t,

- $t_{\frac{1}{2}}$ is the half-life of the decaying quantity, τ is a positive number called the mean lifetime of the decaying quantity,

- λ is a positive number called the decay constant of the decaying quantity.

The three parameters $t_{\frac{1}{2}}$, τ, and λ are all directly related in the following way:

$$t_{\frac{1}{2}} = \frac{\ln(2)}{\lambda} = \tau \ln(2)$$