Thonk: Math, Physics & Chemistry

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0 Preface

0.1 A Note to the Reader

0.1.1 Behind the Idea - A Short Story

This book is the result of a lot of contemplation. Psi25Omega was thinking about stuff he would like to learn and read about each day when he thought, "Why not take notes and make it a document? Why not 'LaTeX' it? It certainly would look good as well!". And then, he played with the different styles and options available for a few days and had a basic draft of the document. It was after this that he invited NSPKN6506 to the party after both of them found the project interesting to work on and helpful to people as well. And they have come this far, from struggling with multiple errors due to bad code (Yeah, they were not so good at LaTeX back then XD) and filling up the whole of January with content (Again, if you read it on par with the expectations; i.e. one thing a day) to the most recent draft you are gonna read. If you feel they did well, do try saying a "Hi" to them in school or on Discord or anywhere you meet them (You can discuss more about these things). They would love to clarify any doubts or clarifications from the book.

0.1.2 How you can start reading!

So, now you have finished the boring Introduction and have come to the part where you get some advice on how you can start reading the book. The thing is, there is no defined way in which you should read this. We recommend reading one "Box" a day as you can later search it up and gather more information going deeper into the topic. The content is a basic gist of the topic and tries kindle your interest to go to a browser and type in the topic on the search bar.

0.1.3 Final thoughts

Here are some final thoughts and reflections on the book. Please forgive us for any mistakes in the book and we'll try to correct them whenever you guys submit a pull request or we notice it. The book is of a higher level than standard Science and Math curriculum in India (9th and 10th grades). But don't worry, you can still understand most (95%) of the stuff (You can always use the internet to know about the things you don't understand). From another country's perspective, the book may be too trivial or tough, again depending on your curriculum's rigour. Overall, it's just another book that lists facts and theorems, but hopefully, in an inquisitive and interesting way.

0.2 Notations

0.2.1 The Box!

Here is an example of the typical ”box” we have used in the book. We also colour code the boxes to represent the different subjects the things cover;

Subject	Colour
Math	Yellow
Physics	Blue
Chemistry	Red

The Title (Sometimes involves Bad Puns)

This so-called ”box” usually contains the content and sometimes includes images, graphs or diagrams to make things interesting and clear. At times, we also have included footnotes[a] to be more reader-friendly.

[a] They are for clarifications that aren't really obvious but aren't significant at the same time.

0.2.2 Contributing

If you want to contribute to the project, feel free to fork our Github Repository and submit pull requests or get in touch with one of the authors.

0.2.3 Github

This is a link to our Github Repository where we host most of the Static files and LaTeX code.

Github, Thonk

1 January 2021

Lorem Ipsum Dolor Sit Amet.Lorem Ipsum Dolor Sit Amet.Lorem Ipsum Dolor Sit Amet.Lorem Ipsum Dolor Sit Amet.Lorem Ipsum Dolor Sit Amet.Lorem Ipsum Dolor Sit Amet.Lorem Ipsum Dolor Sit Amet.Lorem Ipsum Dolor Sit Amet.Lorem Ipsum Dolor Sit Amet.Lorem Ipsum Dolor Sit Amet. Lorem Ipsum Dolor Sit Amet.Lorem Ipsum Dolor Sit Amet.

1.1 A Simple Proof

Are there infinitely many prime numbers? If yes, how do we prove they exist? Here's a simple proof.
Assume we have only n prime numbers; $P_1, P_2, P_3, \dots P_n$.

$$\text{Let } N = P_1 \cdot P_2 \cdot P_3 \dots P_n + 1$$

N isn't divisible by any of the primes $P_1, P_2, P_3, \dots P_n$[a] which implies N's prime factorisation is $N \times 1$. N being prime contradicts our initial assumption.
Thus, there exist infinite primes.

[a]Prime numbers start from 2 and N is the LCM of all the prime numbers added to 1.

1.2 Are we down to the least hierarchical Particle?

Atoms are divisible, but are electrons, protons and neutrons?. A quark is an elementary particle and a fundamental constituent of matter. Quarks combine to form particles called hadrons. All commonly observable matter is composed of up quarks, down quarks and electrons. Due to a phenomenon called "Colour Confinement", quarks are never found existing individually, they can be found only composing hadrons, which include baryons (protons and neutrons) and mesons, or in quark–gluon plasma. They are the only elementary particles in the Standard Model of particle physics to experience all four fundamental interactions (electromagnetic force, gravitation, strong forces, and weak forces).
Quarks are of 6 types/flavours:

- Up
- Down
- Charm
- Strange
- Top
- Bottom

Up and down quarks have the lowest masses and such heavier quarks rapidly change into up and down quarks through the process of particle decay. Up and down quarks are generally stable and the most common in the universe, whereas strange, charm, bottom, and top quarks can only be produced in high energy collisions (such as those in particle accelerators like those found at CERN).

1.3 Quantum Numbers (Not a new System!?)

In chemistry and quantum physics, quantum numbers describe values of conserved quantities in the dynamics of a quantum system. An important aspect of quantum mechanics is the quantization of many observable quantities of interest. In particular, this leads to quantum numbers that take values in discrete sets of integers or half-integers[a]; although they approach infinity in some cases. Quantum numbers often describe specifically the energy levels of electrons in atoms, but other possibilities include angular momentum, spin, etc. An important family is flavour quantum numbers – internal quantum numbers which determine the type of a particle and its interactions with other particles through the fundamental forces. A system can have one or more quantum numbers; it is thus difficult to list all possible quantum numbers.
Four quantum numbers can describe an electron in an atom completely:

- Principal quantum number (n)
- Azimuthal quantum number (l)
- Magnetic quantum number (m)
- Spin quantum number (s)

[a] Numbers of the form $\frac{2n+1}{2}$

1.4 Principal Quantum Number

In quantum mechanics, the principal quantum number (symbolized by n) is one of four quantum numbers assigned to each electron in an atom to describe that electron's state. Its values are natural numbers, making it a discrete variable.
Apart from the principal quantum number, the other quantum numbers for bound electrons are the azimuthal quantum number l, the magnetic quantum number m, and the spin quantum number s. As n increases, the electron also has higher energy and is, therefore, less tightly bound to the nucleus. For higher n the electron is farther from the nucleus. For each value of n there are n accepted l (azimuthal) values ranging from 0 to $n-1$ inclusively, hence higher-n electron states are numerous in definition. Accounting for two states of spin, each n-shell can accommodate up to $2n^2$ electrons (Given by Niels Bohr).
The principal quantum number was first created for use in the semi-classical Bohr model of the atom, distinguishing between different energy levels. With the development of modern quantum mechanics, the simple Bohr model was replaced with a more complex theory of atomic orbitals. However, the modern theory still requires the principal quantum number.

1.5 Azimuthal Quantum Number

The azimuthal quantum number is a quantum number for an atomic orbital that determines its orbital angular momentum as well as the shape of the orbital. The azimuthal quantum number is the second set of quantum numbers which describes the unique state of an electron (the others being the principal quantum number, the magnetic quantum number, and the spin quantum number). It is also known as the orbital angular momentum quantum number, orbital quantum number or second quantum number, and is symbolized as l.
Each of the different angular momentum states can take $2(2l + 1)$ electrons. This is because the third quantum number m (which can be thought of loosely as the quantized[a] projection of the angular momentum on the z-axis) runs from l to l in integer units, and so there are $2l + 1$ possible states. Each distinct n, l, m orbital can be occupied by two electrons with opposing spins (given by the quantum number $s = \pm\frac{1}{2}$), giving $2(2l + 1)$ electrons overall. Orbitals with higher l than given in the table are perfectly permissible, but these values cover all atoms so far discovered.

[a]Restricting the number of possible values of a quantity or states of a system so that certain variables can assume only certain magnitudes.

1.6 Magnetic Quantum Number

The magnetic quantum number (symbol m) is one of four quantum numbers in atomic physics. The magnetic quantum number distinguishes the orbitals available within a subshell, and is used to calculate the azimuthal component of the orientation of orbital in space. Electrons in a particular subshell (such as $s, p, d,$or$f$) are defined by values of l $(0, 1, 2,$ or $3)$. The value of m can range from $-l$ to $+l$, including zero. Thus the $s, p, d,$and$f$ subshells contain $1, 3, 5,$ and 7 orbitals each, with values of m within the ranges $0, \pm1, \pm2, \pm3$ respectively. Each of these orbitals can accommodate up to two electrons (with opposite spins), forming the basis of the periodic table.

1.7 Spin Quantum Number

In atomic physics, the spin quantum number is a quantum number that describes the intrinsic angular momentum (or spin angular momentum, or simply spin) of a given particle. The spin quantum number is designated by the letter s, and is the fourth of a set of quantum numbers (the principal quantum number, the azimuthal quantum number, the magnetic quantum number, and the spin quantum number), which completely describe the quantum state of an electron. this can be written as:-

$$||s|| = \sqrt{s(s+1)}h$$

where,

- s is the Spin Vector
- $||s||$ is the norm of the Spin Vector
- h is the Planck constant

1.8 A Salty Bond

The Ionic bond is a type of a chemical bond that is a result of the attraction between oppositely charged particles in ionic compounds like NaCl[a]. Ions are atoms (or a group of atoms) having a net charge. Atoms that gain electrons to become stable are called anions while those that lose electrons for the same are called cations. This transfer of electrons is known as electrovalence. Ionic bonds are mostly formed between metals and non-metals. In simpler words, an ionic bond is a result of the transfer of electrons from a metal (cation) to a non-metal (anion) in order for both atoms to attain stability.

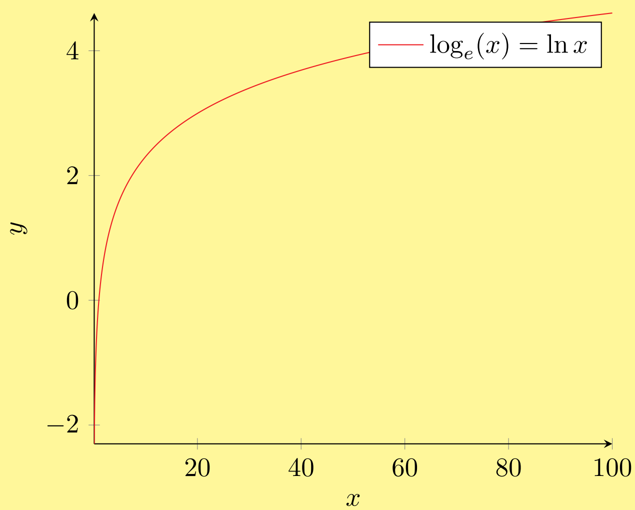
[a] More examples; KCl, $CaCl_2$

1.9 Exponentiation^{-1}

The logarithm function (denoted by log) is the inverse function of exponentiation. It denotes the exponent/power to which the 'base' has been raised to in a number. Here's an example;

$$\text{Let } x = a^b\text{. This implies} \log_a(x) = b$$

The logarithm of x to the base a can be a whole number, decimal number, irrational number or even a complex number depending on its value. Often, log graph are used to make growth, forecast and even the number of cases during a pandemic. Logarithms involving complex numbers[a] (i) can be plotted on the real-complex plane.



This is an example of a log-graph to the base e[b].

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[a] $\sqrt{-x}$, $\sqrt[3]{-x}, \ldots$

[b] Euler's constant; $e = 2.71$

1.10 Heisenberg's Life Questioning Inequality

In quantum mechanics[a], the uncertainty principle (also known as Heisenberg's uncertainty principle) is a variety of mathematical inequalities asserting a fundamental limit to the accuracy with which the values for certain pairs of physical quantities of a particle, such as position (x) and momentum (p) can be predicted from initial or known conditions. Heisenberg, in simple words stated that if you know velocity/momentum of the particle, you can't find it's position and vice versa. The equation stated in favour;

$$\Delta x \times \Delta P \geq \frac{h}{4\pi}$$

where $h = 6.626 \times 10^{-}34$ (Planck's Constant).
The more precisely the position of some particle is determined, the less precisely its momentum/velocity can be predicted from the known initial conditions.

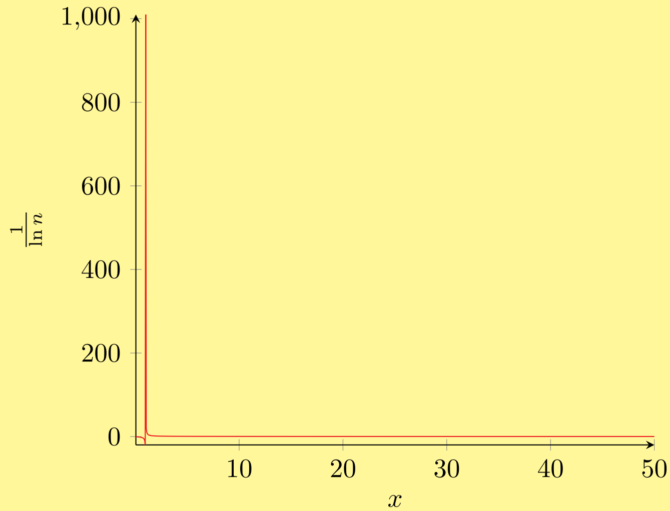
[a]Branch of Physics dedicated to observing the physical properties at the subatomic scale.

1.11 Prime Number Theorem

The prime number theorem describes the asymptotic[a] distribution of prime numbers among positive integers. It formalizes the intuitive idea that primes become less common as they become larger. The prime number theorem addresses this by precisely quantifying the rate at their frequency decreases. The first breakthrough was the $\pi(n)$ function, which calculates the probability that a random integer less than or equal to n is prime. It's defined as;

$$\pi(n) \sim \frac{1}{\ln(n)}$$

where $\ln(n)$ is the natural logarithm[b] of n.



[a]Asymptotic here means approximate in mathematical terms.
[b]$\log_e(n)$

1.12 Catalan's Conjecture

The Catalan's conjecture, conjectured by the mathematician Eugène Charles Catalan in 1844 and proven in 2002 by Preda Mihăilescu. It states that there exists only one solution to the equation

$$x^a - y^b = 1$$

where $x = 3, a = 2, y = 2, b = 3$ for $a, b > 1$ and $x, y > 0$.

1.13 Angular Momentum of an Electron

The angular momentum (L) of an electron in the n^{th} orbit is given by

$$L = \frac{nh}{2\pi}$$

where h is the Planck's constant.

1.14 Gibbs Free Energy

In thermodynamics, the Gibbs Free Energy (G) (named after Josiah Willard Gibbs) is a thermodynamic potential that calculates the maximum reversible work performed by a thermodynamic system at a constant temperature (T) and pressure (P). It is given by

$$\Delta G = \Delta H - T\Delta S$$

where S represents its Entropy, i.e. the measure of randomness. S.I unit - Joules

1.15 Modular Arithmetic

Modular Arithmetic is an operation (used extensively in Mathematical Olympiads and Number Theory) to compute the remainder when a number is divided by another. For example, say 5 is the remainder when a is divided by b. We can represent it the following way:

$$a \equiv 5 \pmod{b}$$

where $\pmod{b}$ represents a taken modulo b and 5 is $a \pmod{5}$.
Modulos can also be thought of this way. We know that $14 \equiv 2 \pmod{3}$. On adding 3 to the remainder, we get $14 \equiv 5 \pmod{3}$. But this is true too (since 14 leaves a remainder of 5 on being divided by 9). Are we arriving upon a paradox? No, not at all. This actually leads us to our next point.
A property of congruence systems is that;
if $a \equiv b \pmod{n}$ it implies $(\implies)$ $a \equiv b + n \pmod{n}$.
Modular Arithmetic systems also have the following properties;

- If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$; $ab \equiv cd \pmod{n}$
- If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$; $a + b \equiv c + d \pmod{n}$

The same apply for subtraction and division respectively.

1.16 Quadratic Residues (Not to be residued!)

An interesting part of modular arithmetic congruence systems is quadratic residues which represent the remainder when a perfect square is divided by some integer n.
For example, one can observe that 0 and 1 are the quadratic residues when any perfect square is taken modulo 4; i.e. $\pmod 4$.
This has a very simple proof. We know that any integer x is either $0, 1, 2$ or $3 \pmod 4$. From the multiplicative property,

$$x \equiv 0, 1, 2, 3 \pmod 4$$
$$\implies x \times x \equiv 0 \times 0, 1 \times 1, 2 \times 2, 3 \times 3 \pmod 4$$
$$\implies x \equiv 0, 1, 4, 9 \pmod 4$$
$$\text{On writing 4 and 9 as } 4 + 0 \text{ and } 2 \times 4 + 1,$$
$$x \equiv 0, 1 \pmod 4$$

The following table lists quadratic residues when taken $\pmod n$;

n	$x^2 \pmod n$
1	0
2	$0, 1$
3	$0, 1$
4	$0, 1$
5	$0, 1, 4$
6	$0, 1, 3, 4$
7	$0, 1, 2, 4$
8	$0, 1, 4$

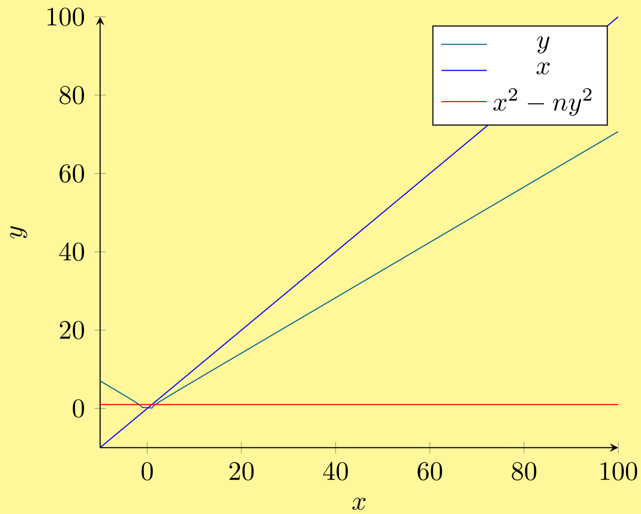
1.17 Pell's Equation

Pell's equation, also called the Pell–Fermat equation, is any Diophantine equation of the form

$$x^2 - ny^2 = 1$$

where n is a given positive non-square integer. It has infinitely many solutions (proved by Joseph Lagrange) in x and y and forms a hyperbola when plotted in Cartesian co-ordinates.
We can also find its solutions through a recursive algorithm where;
If (x_0, y_0) is a solution to $x^2 - dy^2 = N$ and (u_n, v_n) is a solution to $u^2 - dv^2 = 1$ then (x_n, y_n) such that $x_n + y_n\sqrt{d} = (x_0 + y_0\sqrt{d})$ is a solution to $x^2 - dy^2 = N$.

This is the graph for $n = 2$:



1.18 Pauli's "Excluded" Principle

The Pauli's exclusion principle is the quantum mechanical principle which states that two or more identical fermions[a] cannot occupy the same quantum state within a quantum system simultaneously. This principle was formulated by Austrian physicist Wolfgang Pauli in 1925 for electrons, and later extended to all fermions with his spin–statistics theorem of 1940. In the case of electrons in atoms, it can be stated as follows: it is impossible for two electrons of a poly-electron atom to have the same values of the four quantum numbers: n, the principal quantum number, l, the azimuthal quantum number, m, the magnetic quantum number, and s, the spin quantum number. For example, if two electrons reside in the same orbital, then their n, l, and m values are the same, therefore their s must be different, and thus the electrons must have opposite half-integer spin projections of $\frac{1}{2}$ and $-\frac{1}{2}$.

[a]Particles with half-integer spin

1.19 Half life Of Atoms

Half-life represented by $t_{\frac{1}{2}}$ is the time required for a quantity to reduce to half of its initial value. It's used in nuclear physics to describe how quickly unstable atoms undergo radioactive decay or how long stable atoms survive. Half-life is mainly defined in terms of probability; i.e. "Half-life is the time required for exactly half of the entities to decay on average". In other words, the probability of a radioactive atom decaying within its half-life is 50Exponential decay can be described by any of the following three equivalent formulas:

$$N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$N(t) = N_0 e^{-\frac{t}{\tau}}$$

$$N(t) = N_0 e^{-\lambda t}$$

where;

- N_0 is the initial quantity of the substance that will decay,
- $N(t)$ is the quantity that still remains and has not yet decayed after a time t,
- $t_{\frac{1}{2}}$ is the half-life of the decaying quantity, τ is a positive number called the mean lifetime of the decaying quantity,
- λ is a positive number called the decay constant of the decaying quantity.

The three parameters $t_{\frac{1}{2}}$, τ, and λ are all directly related in the following way:

$$t_{\frac{1}{2}} = \frac{\ln(2)}{\lambda} = \tau \ln(2)$$