Name: *Anonymous*

Roll No : e.g., 190040001 | **Dept.**: e.g., CSE | **Sect.**: e.g., A4

IIT Bombay
CS 6001: GT&AMD
Quiz 2, 2022-23-I
Date: November 1, 2022

CS6001: Game Theory and Algorithmic Mechanism Design

Total: **10 × 3 = 30 marks,** *Duration:* **45 minutes, ATTEMPT ALL QUESTIONS**

Instructions:

1. This question paper and answersheet contains a total of 4 sheets of paper (8 pages, page 2 is blank). Please verify.
2. Write your name, roll number, department, section on **every side of every sheet** (except the blank sheet) of this booklet. Use only **black/blue ball-point pen**. The first 5 minutes of additional time is given exclusively for this activity.
3. Write final answers neatly with a pen **only in the given boxes**.
4. Use the rough sheets for scratch works / attempts to solution. **Write only the final solution (which may be a sequence of logical arguments) in a precise and succinct manner in the boxes provided**. Do not provide unnecessarily elaborate steps. The space within the boxes are sufficient for the correct and precise answers.
5. Submit your answerscripts to the teaching staff when you leave the exam hall or the time runs out (whichever is earlier). **Your exam will not be graded if you fail to return the paper**.
6. **This is a closed book, notes, internet exam. No communication device, e.g., cellphones, iPad, etc., is allowed**. Keep it switched off in your bag and keep the bag away from you. If anyone is found in possession of such devices during the exam, that answerscript may be disqualified for evaluation and DADAC may be invoked.

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Problem 1 (1 + 4 + 1 + 4 points). Suppose the following voting rule is used to choose the departmental general secretary of CSE, IIT Bombay. Every student of the department ranks the candidates from most preferred to least preferred (no ties), and submits this ranked list in a ballot box. Each candidate receives one point if she *beats* another candidate in a *pairwise election*. A pairwise election considers two candidates at a time, say a and b. In an election between only these two candidates with the current voting profile, if a wins then it gets a score of 1 and 0 otherwise (ties broken arbitrarily). Considering the pairwise election between all pairs, the candidate who thus amasses the greatest number of points wins the election. If two or more candidates are tied for first place in the number of points, the winner of the election is the candidate among them whose AADHAR number is smallest. Assume that there are *exactly* three candidates.

(a) Is this voting rule **monotone**? (Yes/No)

No

(b) Explain the answer above, i.e., if it is, then prove why, else provide a counterexample (with three candidates).

Consider candidates a, b, c. Three voters vote for them. Their preferences are as follows.

$a > b > c$
$b > c > a$
$c > a > b$

} *a wins*

According to the voting rule, each candidate gets a score of 1 since each beat one other candidate in pairwise election. Assuming a has the least AADHAR number, a wins. Now consider a similar profile, where only first voter's ballot has changed.

$a > c > b$
$b > c > a$
$c > a > b$

} *c wins*

Now, c beats both a and b in pairwise election -- its score is 2, and a = 1, b = 0. Hence c wins, but a's relative position did not change in these two profiles. This violates monotonicity.

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(c) Is this voting rule **manipulable**? (Yes/No)

Yes

(d) Explain the previous answer, i.e., if it is, then provide an example (with three candidates), else prove why.

Consider the same two profiles in part (b). Player 1 prefers a to c in the second profile, while the outcome is c. She can change her preference report to that of the first profile and get her favorite outcome a.

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Problem 2 (1 + 1 + 3 + 2 + 3 points). IIT Bombay plans to distribute a fixed amount of additional electricity to the students for their comfort during the summer. But the allocation will be different for every student and they will be charged according to their consumption. If x_i is the amount of electricity allocated to student i, then she pays x_i^2 for it. The comfort from electricity of the students is linear, but potentially with different slopes. It grows as $\theta_i x_i$, where θ_i is student i's private information. Therefore, the combined utility of student i from consuming x_i amount of electricity is

$$u_i(x, \theta_i) = \theta_i x_i - x_i^2.$$

(a) What is the type of preference each student has over her consumed electricity? Answer it in the appropriate terminology used in the class.

This preference is single-peaked over her consumed electricity. The utility of this student increases till $\theta_i/2$ and then drops.

(b) Is it possible to design a mechanism in this setting such that it reveals the private types of the students *truthfully*, and be *Pareto efficient* and *anonymous*? (Yes/No)

Yes.

(c) Explain your answer above, i.e., if it is possible, provide a mechanism that achieves it. If not, explain why it is not possible.

The setup is identical to that of task allocation. Hence the uniform rule that allocates electricity to all students satisfies the above three properties.

[A brief description of the uniform rule would be desirable but not necessary. However, if no mention of the uniform rule is made, then the mechanism should be completely explained.]

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(d) If you could find the mechanism, is the allocation returned by that mechanism also *envy-free*? (Yes/No). Hint: an envy-free allocation is one where every player weakly prefers her own allocated electricity than every other player's allocated electricity. Write 'NA' if your earlier answer was 'no such mechanisms exist'.

Yes.

(e) Explain your answer above, i.e., if the mechanism is envy-free, explain why. If not, why not?

In the uniform allocation rule, assuming that the total electricity is more than the sum of the student peaks (a complementary argument will work when it is less), the final allocation of electricity should be either an agent's peak or above it. For the students who did not get their peaks, their allocations are identical, and are smaller than that of the students who got their peaks. So, the peak-getting students don't envy them, since it is away from the peak. The students who did not get their peaks do not envy each other (since their allocations are identical) and not the folks who got the peaks, because it is on the other side of their own peaks from what they have been allocated (and by single-peakedness, that will be less preferred).

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Problem 3 (1 + (2 + 2 + 2) + (1 + 1 + 1) points). Let there be two indivisible objects, O_1 and O_2, which are to be allocated to the players P_1, P_2, and P_3. The constraint is that a player can get at most one of the objects. The values of the objects to the players are shown in the table below. Apply VCG mechanism and answer the questions below.

	P_1	P_2	P_3
O_1	105	80	110
O_2	150	130	90

(a) What is the allocation of the items to the players under this rule, i.e., who gets which item(s)?

$O_1 \to P_3$

$O_2 \to P_1$

This is the efficient allocation which is done in VCG

(b) Find the payment of P_1. Show/explain the steps of obtaining the final result.

payment of $P_1 = \sum_{j \neq 1} v_j(a^*_{-1}) - \sum_{j \neq 1} v_j(a^*) = 240 - 110 = 130$

when agent 1 is not present, the efficient allocation becomes $O_2 \to P_2, O_1 \to P_3$

which gives the sum value of 240.

(c) Find the payment of P_2. Show/explain the steps of obtaining the final result.

payment of P_2 *is zero, since under VCG, an agent who doesn't get an item pays nothing.*

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(d) Find the payment of P_3. Show/explain the steps of obtaining the final result.

payment of $P_3 = \sum_{j \neq 3} v_j(a^*_{-3}) - \sum_{j \neq 3} v_j(a^*) = 235 - 150 = 85$

when agent 3 is not present, the efficient allocation becomes $O_1 \to P_1, O_2 \to P_2$

which gives a sum value of 235.

(e) Find the utility of P_1. Show/explain the steps of obtaining the final result.

$u_1 = 150 - 130 = 20$

(f) Find the utility of P_2. Show/explain the steps of obtaining the final result.

$u_2 = 0$

(g) Find the utility of P_3. Show/explain the steps of obtaining the final result.

$u_3 = 110 - 85 = 25$

END OF QUESTION PAPER. GOOD LUCK!