Contents

	Acknowledgments	page xiv	
	Notations	XV	
	Introduction	xxiii	
1	The game of chess	1	
	1.1 Schematic description of the game	1	
	1.2 Analysis and results	2	
	1.3 Remarks	7	
	1.4 Exercises	7	
2	Utility theory	9	
	2.1 Preference relations and their representation	9	
	2.2 Preference relations over uncertain outcomes: the model	12	
	2.3 The axioms of utility theory	14	
	2.4 The characterization theorem for utility functions	19	
	2.5 Utility functions and affine transformations	22	
	2.6 Infinite outcome set	23	
	2.7 Attitude towards risk	23	
	2.8 Subjective probability	26	
	2.9 Discussion	27	
	2.10 Remarks	31	
	2.11 Exercises	31	
3	Extensive-form games	39	
	3.1 An example	40	
	3.2 Graphs and trees	41	
	3.3 Game trees	42	
	3.4 Chomp: David Gale's game	47	
	3.5 Games with chance moves	49	
	3.6 Games with imperfect information	52	
	3.7 Exercises	57	

4 Sti	rategic-form games	75
4.1	Examples and definition of strategic-form games	76
4.2	The relationship between the extensive form and the	
	strategic form	82
4.3	Strategic-form games: solution concepts	84
4.4		85
4.5	Domination	85
4.6	Second-price auctions	91
4.7	The order of elimination of dominated strategies	95
4.8	Stability: Nash equilibrium	95
4.9	Properties of the Nash equilibrium	100
4.1	0 Security: the maxmin concept	102
4.1	1 The effect of elimination of dominated strategies	106
4.1	2 Two-player zero-sum games	110
4.1	3 Games with perfect information	118
4.1	4 Games on the unit square	121
4.1	5 Remarks	128
4.1	6 Exercises	128
5 Mi	xed strategies	144
 J 1911		
5.1	2 2	145
5.2		152
5.3	1	166
5.4	C	170
5.5	, ,	172
5.6	1 7 6	176
5.7	Imperfect information: the value of information	180
5.8	Evolutionarily stable strategies	186
	Remarks	194
5.1	0 Exercises	194
6 Be	havior strategies and Kuhn's Theorem	219
6.1	Behavior strategies	221
6.2	E	226
6.3		235
6.4		238
6.5		243
6.6		244
0.0	—	

Equilibrium refinements	251
7.1 Subgame perfect equilibrium	252
•	260
	262
	271
• •	284
	284
Correlated equilibria	300
8.1 Examples	301
8.2 Definition and properties of correlated equilibrium	305
8.3 Remarks	313
8.4 Exercises	313
Games with incomplete information and common priors	319
9.1 The Aumann model of incomplete information and the concept	
of knowledge	322
9.2 The Aumann model of incomplete information with beliefs	334
9.3 An infinite set of states of the world	344
9.4 The Harsanyi model of games with incomplete	
information	345
9.5 Incomplete information as a possible interpretation of	
mixed strategies	361
9.6 The common prior assumption: inconsistent beliefs	365
9.7 Remarks	367
9.8 Exercises	368
Comes with incomplete information; the general model	296
	386
	386
•	391
<u>.</u>	394
	400
	407
1	415
	423
10.8 Exercises	423
The universal belief space	440
11.1 Belief hierarchies	442
11.2 Types	450
	7.1 Subgame perfect equilibrium 7.2 Rationality, backward induction, and forward induction 7.3 Perfect equilibrium 7.4 Sequential equilibrium 7.5 Remarks 7.6 Exercises Correlated equilibria 8.1 Examples 8.2 Definition and properties of correlated equilibrium 8.3 Remarks 8.4 Exercises Games with incomplete information and common priors 9.1 The Aumann model of incomplete information and the concept of knowledge 9.2 The Aumann model of incomplete information with beliefs 9.3 An infinite set of states of the world 9.4 The Harsanyi model of games with incomplete information 9.5 Incomplete information as a possible interpretation of mixed strategies 9.6 The common prior assumption: inconsistent beliefs 9.7 Remarks 9.8 Exercises Games with incomplete information: the general model 10.1 Belief spaces 10.2 Belief and knowledge 10.3 Examples of belief spaces 10.4 Belief subspaces 10.5 Games with incomplete information 10.6 The concept of consistency 10.7 Remarks 10.8 Exercises The universal belief space

	11.3	Definition of the universal belief space	453
	11.4	Remarks	456
	11.5	Exercises	456
12	Auct	ions	461
	12.1	Notation	464
	12.2	Common auction methods	464
	12.3	Definition of a sealed-bid auction with private values	465
	12.4	Equilibrium	468
	12.5	The symmetric model with independent private values	471
	12.6	The Envelope Theorem	484
	12.7	Risk aversion	488
	12.8	Mechanism design	492
	12.9	Individually rational mechanisms	500
	12.10	Finding the optimal mechanism	501
	12.11	Remarks	508
	12.12	Exercises	509
13	Repe	ated games	519
	13.1	The model	520
	13.2	Examples	521
	13.3	The <i>T</i> -stage repeated game	524
	13.4	Characterization of the set of equilibrium payoffs of the <i>T</i> -stage repeated	
		game	530
	13.5	Infinitely repeated games	537
	13.6	The discounted game	542
	13.7	Uniform equilibrium	546
	13.8	Discussion	554
		Remarks	555
		Exercises	555
14	Repe	ated games with vector payoffs	569
	14.1	Notation	570
	14.2	The model	572
	14.3	Examples	573
	14.4	Connections between approachable and excludable sets	574
	14.5	A geometric condition for the approachability of a set	576
	14.6	Characterizations of convex approachable sets	585
	14.7	Application 1: Repeated games with incomplete information	590
	14.8	Application 2: Challenge the expert	600
	14.9	Discussion	606
		Remarks	607
		Evaraisas	608

15	Bargaining games	622
	15.1 Notation	625
	15.2 The model	625
	15.3 Properties of the Nash solution	626
	15.4 Existence and uniqueness of the Nash solution	630
	15.5 Another characterization of the Nash solution	635
	15.6 The minimality of the properties of the Nash solution	639
	15.7 Critiques of the properties of the Nash solution	641
	15.8 Monotonicity properties	643
	15.9 Bargaining games with more than two players	650
	15.10 Remarks	653
	15.11 Exercises	653
16	Coalitional games with transferable utility	659
	16.1 Examples	661
	16.2 Strategic equivalence	668
	16.3 A game as a vector in a Euclidean space	670
	16.4 Special families of games	671
	16.5 Solution concepts	672
	16.6 Geometric representation of the set of imputations	676
	16.7 Remarks	678
	16.8 Exercises	678
17	The core	686
	17.1 Definition of the core	687
	17.2 Balanced collections of coalitions	691
	17.3 The Bondareva–Shapley Theorem	695
	17.4 Market games	702
	17.5 Additive games	712
	17.6 The consistency property of the core	715
	17.7 Convex games	717
	17.8 Spanning tree games	721
	17.9 Flow games	724
	17.10 The core for general coalitional structures	732
	17.11 Remarks	735
	17.12 Exercises	735
18	The Shapley value	748
	18.1 The Shapley properties	749
	18.2 Solutions satisfying some of the Shapley properties	751
	18.3 The definition and characterization of the Shapley value	754
	18.4 Examples	758

Contents

xii

		18.5 An alternative characterization of the Shapley value	760
		18.6 Application: the Shapley–Shubik power index	763
		18.7 Convex games	767
		18.8 The consistency of the Shapley value	768
		18.9 Remarks	774
		18.10 Exercises	774
		2010 2000	,,,
	19	The bargaining set	782
		19.1 Definition of the bargaining set	784
		19.2 The bargaining set in two-player games	788
		19.3 The bargaining set in three-player games	788
		19.4 The bargaining set in convex games	794
		19.5 Discussion	797
		19.6 Remarks	798
		19.7 Exercises	798
_	20	The nucleolus	801
		20.1 Definition of the nucleolus	802
		20.2 Nonemptiness and uniqueness of the nucleolus	805
		20.3 Properties of the nucleolus	809
		20.4 Computing the nucleolus	815
		20.5 Characterizing the prenucleolus	816
		20.6 The consistency of the nucleolus	823
		20.7 Weighted majority games	825
		20.8 The bankruptcy problem	831
		20.9 Discussion	842
		20.10 Remarks	843
		20.11 Exercises	844
_	21	Social choice	853
		21.1 Social welfare functions	856
		21.2 Social choice functions	864
		21.3 Non-manipulability	871
		21.4 Discussion	873
		21.5 Remarks	874
		21.6 Exercises	874
	22	Stable matching	884
-		22.1 The model	886
		22.2 Existence of stable matching: the men's courtship algorithm	888
		22.3 The women's courtship algorithm	890

xiii

Contents

	22.4 Comparing matchings	892
	22.5 Extensions	898
	22.6 Remarks	905
	22.7 Exercises	905
23	Appendices	916
	23.1 Fixed point theorems	916
	23.2 The Separating Hyperplane Theorem	943
	23.3 Linear programming	945
	23.4 Remarks	950
	23.5 Exercises	950
	References	958
	Index	968