Name:	*Anonymous*					
Roll No : e.g., 190040001		**Dept.**: e.g., CSE		**Sect.**: e.g., A4		

IIT Bombay
CS 6001: GT&AMD
Endsem, 2022-23-I
Date: November 18, 2022

CS6001: Game Theory and Algorithmic Mechanism Design

Total: **10 $\times$ 4 = 40 marks,** *Duration:* **2 hours, ATTEMPT ALL QUESTIONS**

Instructions:

1. This question paper-cum-answersheet contains a total of 5 sheets of paper (10 pages, page 2 is blank). Please verify.
2. Write your name, roll number, department, section on **every side of every sheet** (except the blank sheet) of this booklet. Use only **black/blue ball-point pen**. The first 5 minutes of additional time is given exclusively for this activity.
3. Write final answers neatly with a pen **only in the given boxes**.
4. Use the rough sheets for scratch works / attempts to solution. **Write only the final solution (which may be a sequence of logical arguments) in a precise and succinct manner in the boxes provided**. Do not provide unnecessarily elaborate steps. The space within the boxes are sufficient for the correct and precise answers.
5. Submit your answerscripts to the teaching staff when you leave the exam hall or the time runs out (whichever is earlier). **Your exam will not be graded if you fail to return the paper**.
6. **This is a closed book, notes, internet exam. No communication device, e.g., cellphones, iPad, etc., is allowed**. Keep it switched off in your bag and keep the bag away from you. If anyone is found in possession of such devices during the exam, that answerscript may be disqualified for evaluation and DADAC may be invoked.

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Problem 1 (1 + 2 + 1 + 1 + 1 + 2 + 2 points). Consider a two agent model with three alternatives $\{a, b, c\}$. Table 1 shows two preference profiles, $P \equiv (P_1, P_2)$ and $P' \equiv (P'_1, P'_2)$, of the agents. Suppose f is an **onto** and **strategyproof** SCF with $f(P_1, P_2) = a$.

P_1	P_2	P'_1	P'_2
a	c	b	a
b	b	a	b
c	a	c	c

Table 1: Two Preference Profiles

(a) Suppose the domain of preferences is of **unrestricted strict preferences**. Then $f(P')$ will be

b

(b) Explain the answer above, i.e., why $f(P')$ takes that value. You may use any standard result proved in the class.

This can be proved in multiple ways. The easiest is by using Gibbard-Satterthwaite theorem. Since this is the unrestricted domain and the SCF is onto and strategyproof, then by G-S theorem, it must be dictatorial. Since the outcome at P is the most preferred alternative of agent 1, the outcome at P′ should also be the most preferred alternative of agent 1. Hence, f(P′) = b.

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(c) Now, suppose that these preferences are generated from a **single-peaked preference** domain with the intrinsic ordering of the alternatives being $a < b < c$. Are the preference profiles in Table 1 valid single-peaked profiles under this setting? (Yes/No)

Yes, the disallowed preferences in this domain with the above intrinsic ordering are $a \succ c \succ b$ *and* $c \succ a \succ b$ *which are not present here.*

(d) Does the conclusion of Part (a) hold in this case? (Yes/No)

No.

(e) What will be the value(s) of $f(P')$ in this modified case?

Can be either a or b.

(f) Explain clearly why or why not the earlier proof (of *unrestricted strict preferences*) go through in this case (*single-peaked* domain).

In the unrestricted domain, the proof is constructive, and it can pick a preference of any kind, in particular, those preferences that are disallowed in the single-peaked domain. Those preference orders are not possible to construct in the single-peaked domain. Therefore, the same proof does not go through.

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(g) If the conclusion of Part (d) is false, provide a **non-constant** (**onto** and **strategyproof**) SCF f that has $f(P_1', P_2') = a$ in the single-peaked domain.

"Pick the leftmost peak" (where left is w.r.t. the intrinsic ordering of the alternatives) is a non-constant, onto, and strategyproof SCF that gives f(P') = a.

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Problem 2 (1 + 2 + 1 + 2 + 1 + 3 points). Consider the following Arrovian social welfare function (ASWF) setup. A committee of 11 members is asked to rank three colors: red, blue, and green, from most preferred to least preferred (consider only strict preferences). The committee members simultaneously announce their strict preference relations over the three colors. If red is the most preferred color of *at least* **five** members of the committee, red is determined to be the prettiest color. Otherwise, if blue is the most preferred color of at least five members of the committee, blue is determined to be the prettiest color. Otherwise, green is determined to be the prettiest color. The prettiest color gets the top position in the consolidated ranking of the ASWF. Once the prettiest color is determined, the remaining two colors are then ranked by the simple majority rule, i.e., if the majority prefers color a over color b, then a will get the second position and b the last. This is how the social welfare function is constructed.

(a) Is the social welfare function described here **dictatorial**? (Yes/No)

No.

(b) Justify your answer above, i.e., if yes, explain why, if no, provide a counterexample.

Suppose, it is a dictatorial ASWF. Then there exists an agent whose preference order is always chosen as the outcome of the ASWF. Suppose, this agent has green as the most favorite color, and all other agents have red as the most preferred choice, then red will be the most preferred color in the final outcome. This violates dictatorship.

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(c) Does the social welfare function described here satisfy the **unanimity** property?

Yes.

(d) Justify your answer above, i.e., if yes, explain why, if no, provide a counterexample.

If all the agents have the same order of preferences over the colors, then it is trivial to check that the same preference order will emerge as the final outcome of the ASWF. Hence, this ASWF is unanimous.

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(e) Does the social welfare function described here satisfy the **independence of irrelevant alternatives** property?

No.

(f) Justify your answer above, i.e., if yes, explain why, if no, provide a counterexample.

Consider the following preference profile.

		Outcomes	
4 agents:	$R \succ G \succ B$		*why? because, B is the most preferred color by the description of the ASWF, then between G and R, G wins.*
6 agents:	$B \succ G \succ R$	$B \succ G \succ R$	
1 agent:	$B \succ R \succ G$		

and then consider the following preference profile (which is a slight modification of the above)

4 agents:	$R \succ G \succ B$		*why? because, now R is the most preferred color by the description of the ASWF, then between G and B, B wins.*
6 agents:	$B \succ G \succ R$	$R \succ B \succ G$	
1 agent:	$R \succ B \succ G$		

The relative position of R and G did not change in this two preference profiles, but the outcome has the positions of R and G flipped. This violates IIA.

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Problem 3 ((2 + 3) + (2 + 3) points). One object is being sold using Myerson's optimal auction. There are three buyers with types t_1, t_2, and t_3 respectively and their virtual valuations are given as follows.

$$w_1(t_1) = 2t_1 - 2, \; w_2(t_2) = 2t_2 - 3, \; w_3(t_3) = 2t_3 - 4.$$

(a) If the reported types are $t_1 = 1, t_2 = 2, t_3 = 3$, which player wins the auction?

3 wins the auction.

(b) How much do these players pay respectively? (express the answers rounded to one decimal point)

Player 1: *0*

Player 2: *0*

Player 3: *2.5*

(c) If the reported types were $t_1 = 5, t_2 = 10, t_3 = 7$, which player wins the auction?

2 wins the auction.

(d) How much do these players pay respectively? (express the answers rounded to one decimal point)

Player 1: *0*

Player 2: *6.5*

Player 3: *0*

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Problem 4 ((0.5 × 2 + 2 × 2) + (0.5 × 2 + 2 × 2) points). Consider two players, 1 and 2, are being allocated two objects A and B for which they have the valuations as shown in the table below (rows denote the players and the columns the different bundles of the objects – $\emptyset$ denotes the empty bundle).

	$\emptyset$	A	B	$\{A, B\}$
1	0	7	0	10
2	0	0	5	9

(a) If the objects are allocated using the VCG mechanism, which object goes to whom? (Provide only the player number)

A goes to player: 1

B goes to player: 2

(b) What will be the payments of players 1 and 2 respectively?

Player 1: 9-5 = 4

Player 2: 10-7 = 3

(c) In the same setup as before, consider the following change. Objects A and B are *no longer sold separately*, rather they are **sold as a bundle**.

If the objects are allocated using the VCG mechanism, which object goes to whom? (Provide only the player number)

A goes to player: 1

B goes to player: 1

(d) What will be the payments of players 1 and 2 respectively?

Player 1: 9

Player 2: 0

END OF QUESTION PAPER. GOOD LUCK!