Notations

The book makes use of large number of notations; we have striven to stick to accepted notation and to be consistent throughout the book. The coordinates of a vector are always denoted by a subscript index, $x = (x_i)_{i=1}^n$, while the indices of the elements of sequences are always denoted by a superscript index, $x^1, x^2, \ldots$ The index of a player in a set of players is always denoted by a subscript index, while a time index (in repeated games) is always denoted by a superscript index. The end of the proof of a theorem is indicated by □, the end of an example is indicated by ◀, and the end of a remark is indicated by ⧫.

For convenience we provide a list of the mathematical notation used throughout the book, accompanied by a short explanation and the pages on which they are formally defined. The notations that appear below are those that are used more than once.

0	chance move in an extensive-form game	50
$\vec{0}$	origin of a Euclidean space	570
$\emptyset$	strategy used by a player who has no decision vertices in an extensive-form game	5
$\mathbf{1}_A$	function that is equal to 1 on event A and to 0 otherwise	595
2^Y	collection of all subsets of Y	325
$\|X\|$	number of elements in finite set X	603
$\|\|x\|\|_\infty$	L_∞ norm, $\|\|x\|\|_\infty := \max_{i=1,2,\ldots,n} \|x_i\|$	531
$\|\|x\|\|$	norm of a vector, $\|\|x\|\| := \sqrt{\sum_{l=1}^d (x_l)^2}$	570
$A \vee B$	maximum matching (for men) in a matching problem	895
$A \wedge B$	maximum matching (for women) in a matching problem	896
$A \subseteq B$	set A contains set B or is equal to it	
$A \subset B$	set A strictly contains set B	
$\langle x, y \rangle$	inner product	570
$\langle\!\langle x^0, \ldots, x^k \rangle\!\rangle$	k-dimensional simplex	920
$\succsim_i$	preference relation of player i	14
$\succ_i$	strict preference relation of player i	10
$\approx_i$	indifference relation of player i	10, 897
$\succsim_P$	preference relation of an individual	857
$\succ_Q$	strict preference relation of society	857
$\approx_Q$	indifference relation of society	857
$x \geq y$	$x_k \geq y_k$ for each coordinate k, where x, y are vectors in a Euclidean space	625
$x > y$	$x \geq y$ and $x \neq y$	625

$x \gg y$	$x_k > y_k$ for each coordinate k, where x, y are vectors in a Euclidean space	625
$x + y$	sum of vectors in a Euclidean space, $(x + y)_k := x_k + y_k$	625
xy	coordinatewise product of vectors in a Euclidean space, $(xy)_k := x_k y_k$	625
$x + S$	$x + S := \{x + s : s \in S\}$, where $x \in \mathbb{R}^d$ and $S \subseteq \mathbb{R}^d$	625
xS	$xS := \{xs : s \in S\}$, where $x \in \mathbb{R}^d$ and $S \subseteq \mathbb{R}^d$	625
cx	product of real number c and vector x	625
cS	$cS := \{cs : s \in S\}$, where c is a real number and $S \subseteq \mathbb{R}^d$	625
$S + T$	sum of sets; $S + T := \{x + y : x \in S, y \in T\}$	625
$\lceil c \rceil$	smallest integer greater than or equal to c	534
$\lfloor c \rfloor$	largest integer less than or equal to c	534
$x^\top$	transpose of a vector, column vector that corresponds to row vector x	571
$\operatorname{argmax}_{x \in X} f(x)$	set of all x where function f attains its maximum in the set X	125, 625
$a(i)$	producer i's initial endowment in a market	703
A	set of actions in a decision problem with experts	601
A	set of alternatives	856
A_i	player i's action set in an extensive-form game, $A_i := \cup_{j=1}^{k_i} A(U_i^j)$	221
A_k	possible outcome of a game	13
$A(x)$	set of available actions at vertex x in an extensive-form game	44
$A(U_i)$	set of available actions at information set U_i of player i in an extensive-form game	54
b_i	buyer i's bid in an auction	91, 466
$b(S)$	$b(S) = \sum_{i \in S} b_i$ where $b \in \mathbb{R}^N$	669
$\mathrm{br_I}(y)$	Player I's set of best replies to strategy y	125
$\mathrm{br_{II}}(x)$	Player II's set of best replies to strategy x	125
B_i	player i's belief operator	392
B_i^p	set of states of the world in which the probability that player i ascribes to event E is at least p, $B_i^p(E) := \{\omega \in Y : \pi_i(E \mid \omega) \geq p\}$	426
$\mathrm{BZ}_i(N; v)$	Banzhaf value of a coalitional game	780
$\mathcal{B}$	coalitional structure	673
$\mathcal{B}_i^T$	set of behavior strategies of player i in a T-repeated game	525
$\mathcal{B}_i^\infty$	set of behavior strategies of player i in an infinitely repeated game	538
c	coalitional function of a cost game	661
c_+	maximum of c and 0	840
c_i	$c_i(v_i) := v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}$	501
C	function that dictates the amount that each buyer pays given the vector of bids in an auction	466

$C(x)$	set of children of vertex x in an extensive-form game	5
$\mathcal{C}(N, v)$	core of a coalitional game	687
$\mathcal{C}(N, v; \mathcal{B})$	core for a coalitional structure	732
$\text{conv}\{x_1, \ldots, x_K\}$	smallest convex set that contains the vectors $\{x_1, \ldots, x_K\}$ Also called the convex hull of $\{x_1, \ldots, x_K\}$	530, 625, 917
d	disagreement point of a bargaining game	625
d_i	debt to creditor i in a bankruptcy problem	833
d^t	distance between average payoff and target set	581
$d(x, y)$	Euclidean distance between two vectors in Euclidean space	571
$d(x, S)$	Euclidean distance between point and set	571
$\mathcal{D}(\alpha, x)$	collection of coalitions whose excess is at least α, $\mathcal{D}(\alpha, x) := \{S \subseteq N, S \neq \emptyset: \quad e(S, x) \geq \alpha\}$	818
$e(S, x)$	excess of coalition S, $e(S, x) := v(S) - x(S)$	802
E	set of vertices of a graph	41, 43
E	estate of bankrupt entity in a bankruptcy problem	833
E	set of experts in a decision problem with experts	601
F	set of feasible payoffs in a repeated game	530, 578
F	social welfare function	857
F_i	cumulative distribution function of buyer i's private values in an auction	466
$F_i(\omega)$	atom of the partition $\mathcal{F}_i$ that contains ω	324
F^N	cumulative distribution function of joint distribution of vector of private values in an auction	466
$\mathcal{F}$	collection of all subgames in the game of chess	5
$\mathcal{F}$	family of bargaining games	625
$\mathcal{F}^N$	family of bargaining games with set of players N	650
$\mathcal{F}_d$	family of bargaining games in $\mathcal{F}$ where the set of alternatives is comprehensive and all alternatives are at least as good as the disagreement point, which is $(0, 0)$	644
$\mathcal{F}_i$	player i's information in an Aumann model of incomplete information	323
g^T	average payoff up to stage T (including) in a repeated game	572
G	graph	41
G	social choice function	865
h	history of a repeated game	525
h_t	history at stage t of a repeated game	602
$H(t)$	set of t-stage histories of a repeated game	525, 601
$H(\infty)$	set of plays in an infinitely repeated game	538
$H(\alpha, \beta)$	hyperplane, $H(\alpha, \beta) := \{x \in \mathbb{R}^d : \langle \alpha, x \rangle = \beta\}$	577, 943
$H^+(\alpha, \beta)$	half-space, $H^+(\alpha, \beta) := \{x \in \mathbb{R}^d : \langle \alpha, x \rangle \geq \beta\}$	577, 943
$H^-(\alpha, \beta)$	half-space, $H^-(\alpha, \beta) := \{x \in \mathbb{R}^d : \langle \alpha, x \rangle \leq \beta\}$	577, 943
i	player	
$-i$	set of all players except of player i	

I	function that dictates the winner of an auction given the vector of bids	466
J	number of lotteries that compose a compound lottery	14
$J(x)$	player who chooses a move at vertex x of an extensive-form game	44
$-k$	player who is not k in a two-player game	571
k_i	number of information sets of player i in an extensive-form game	54
K	number of outcomes of a game	16
K_i	player i's knowledge operator	325
$\mathcal{KS}$, $\mathcal{KS}(S)$	Kalai–Smorodinsky solution to bargaining games	648
L	lottery: $L = [p_1(A_1), p_2(A_2), \ldots, p_K(A_K)]$	13
L	number of commodities in a market	703
$\widehat{L}$	compound lottery: $\widehat{L} = [q_1(L_1), \ldots, q_J(L_J)]$	14
$\mathcal{L}$	set of lotteries	13
$\widehat{\mathcal{L}}$	set of compound lotteries	15
$m(\epsilon)$	minimal coordinate of vector ε	264, 268
m_i	number of pure strategies of player i	147
$m_i(S)$	highest possible payoff to player i in a bargaining game	643
M	maximal absolute value of a payoff in a game	521
$M_{m,l}$	space of matrices of dimension $m \times l$	204
$M(\epsilon)$	maximal coordinate of vector ε	264, 268
$\mathcal{M}(N; v; \mathcal{B})$	bargaining set for coalitional structure $\mathcal{B}$	786
n	number of players	77
n	number of buyers in an auction	466
n_x	number of vertices in subgame $\Gamma(x)$	4
N	set of players	43, 833, 660
N	set of buyers in an auction	466
N	set of individuals	856
N	set of producers in a market	703
$\mathbb{N}$	set of natural numbers, $\mathbb{N} := \{1, 2, 3, \ldots\}$	
$\mathcal{N}$	$\mathcal{N}(S, d)$, Nash's solution to bargaining games	630
$\mathcal{N}(N; v)$	nucleolus of a coalitional game	805
$\mathcal{N}(N; v; \mathcal{B})$	nucleolus of a coalitional game for coalitional structure $\mathcal{B}$	805
$\mathcal{N}(N; v; K)$	nucleolus relative to set K	804
O	set of outcomes	13, 43
p	common prior in a Harsanyi game with incomplete information	347
p_k	probability that the outcome of lottery L is A_k	13
p_x	probability distribution over actions at chance move x	50
P	binary relation	857

P	set of all weakly balancing weights for collection $\mathcal{D}^*$ of all coalitions	701
$\mathbf{P}$	common prior in an Aumann model of incomplete information	334
$\mathbf{P}_\sigma(x)$	probability that the play reaches vertex x when the players implement strategy vector σ in an extensive-form game	254
$\mathbf{P}_\sigma(U)$	probability that the play reaches a vertex in information set U when the players implement strategy vector σ in an extensive-form game	273
P^N	vector of preference relations	857
$PO(S)$	set of efficient (Pareto optimal) points in S	627
$PO^W(S)$	set of weakly efficient points in S	627
$\mathcal{P}(A)$	set of all strict preference relations over a set of alternatives A	857
$\mathcal{P}(N)$	collection of nonempty subsets of N, $\mathcal{P}(N) := \{S \subseteq N, S \neq \emptyset\}$	670, 701
$\mathcal{P}^*(A)$	set of all preference relations over a set of alternatives A	857
$\mathcal{PN}(N; v)$	prenucleolus of a coalitional game	805
$\mathcal{PN}(N; v; \mathcal{B})$	prenucleolus of a coalitional game for coalitional structure $\mathcal{B}$	805
q	quota in a weighted majority game	664
$q(w)$	minimal weight of a winning coalition in a weighted majority game, $q(w) := \min_{S \in \mathcal{W}^m} w(S)$	828
$\mathbb{Q}_{++}$	set of positive rational numbers	
r_k	total probability that the result of a compound lottery is A_k	18
$R_1(p)$	set of possible payoffs when Player 1 plays mixed action p, $R_1(p) := \{puq^\top : q \in \Delta(\mathcal{J})\}$	576
$R_2(p)$	set of possible payoffs when Player 2 plays mixed action q, $R_2(p) := \{puq^\top : q \in \Delta(\mathcal{I})\}$	576
$\mathbb{R}$	real line	
$\mathbb{R}_+$	set of nonnegative numbers	
$\mathbb{R}_{++}$	set of positive numbers	
$\mathbb{R}^n$	n-dimensional Euclidean space	
$\mathbb{R}^n_+$	nonnegative orthant in an n-dimensional Euclidean space, $\mathbb{R}^n_+ := \{x \in \mathbb{R}^n : x_i \geq 0, \quad \forall i = 1, 2, \ldots, n\}$	
$\mathbb{R}^S$	$\lvert S\rvert$-dimensional Euclidean space, where each coordinate corresponds to a player in S	669
range(G)	range of a social choice function	870
s	strategy vector	45
$\mathfrak{s}$	function that assigns a state of nature to each state of the world	323
s^t	action vector played at stage t of a repeated game	525
s_i	strategy of player i	45, 56

s_t	state of nature that corresponds to type vector t in a Harsanyi game with incomplete information	347
$\mathfrak{s}^{-1}(C)$	set of states of the world that correspond to a state of nature in C, $\mathfrak{s}^{-1}(C) := \{\omega \in Y : s(\omega) \in C\}$	330
S	set of all vectors of pure strategies	77
S	set of states of nature in models of incomplete information	323
S	set of states of nature in a decision problem with experts	601
S	set of alternatives in a bargaining game	625
S_i	set of player i's pure strategies	77
Sh	Shapley value	754
supp	support of a probability distribution	206
supp	support of a vector in $\mathbb{R}^n$	925
t_i	player i's type in models of incomplete information	452
T	set of vectors of types in a Harsanyi model of incomplete information	347
T	number of stages in a finitely repeated game	528
T_i	player i's type set in a Harsanyi model of incomplete information	347
u	payoff function in a strategic-form game	43, 601
u_i	player i's utility function	14
u_i	player i's payoff function	77
u_i	producer i's production function in a market	703
u_t^i	payoff of player i at stage t in a repeated game	527
u^t	vector of payoffs at stage t in a repeated game	527
$u(s)$	outcome of a game under strategy vector s	45
U_i^j	information set of player i in an extensive-form game	54
U_i	mixed extension of player i's payoff function	147
$U(C)$	uniform distribution over set C	
$U[\alpha]$	scalar payoff function generated by projecting the payoffs in direction α in a game with payoff vectors	588
v	value of a two-player zero-sum game	114
v	coalitional function of a coalitional game	660
$\underline{v}$	maxmin value of a two-player non-zero-sum game	113
$\overline{v}$	minmax value of a two-player non-zero-sum game	113
$\overline{v}$	maximal private value of buyers in an auction	471
v_0	root of a game tree	42, 43
v_i	buyer i's private value in an auction	91
v^*	superadditive closure of a coalitional game	732
$\underline{v}_i$	player i's maxmin value in a strategic-form game	103, 104, 176
$\overline{v}_i$	player i's minmax value in a strategic-form game	177, 529
val(A)	value of a two-player zero-sum game whose payoff function is given by matrix A	588
V	set of edges in a graph	41, 43
V	set of individually rational payoffs in a repeated game	530

V_0	set of vertices in an extensive-form game where a chance move takes place	43
V_i	set of player i's decision points in an extensive-form game	43
V_i	random variable representing buyer i's private value in an auction	467
$\mathbb{V}$	buyer's set of possible private values in a symmetric auction	471
$\mathbb{V}_i$	buyer i's set of possible private values	466
$\mathbb{V}^N$	set of vectors of possible private values: $\mathbb{V}^N := \mathbb{V}_1 \times \mathbb{V}_2 \times \cdots \times \mathbb{V}_n$	466
w_i	player i's weight in a weighted majority game	664
$\mathcal{W}^m$	collection of minimal winning coalitions in a simple monotonic game	826
x_{-i}	$x_{-i} := (x_j)_{j \neq i}$	85
$x(S)$	$x(S) := \sum_{i \in S} x_i$, where $x \in \mathbb{R}^N$	669
X	$X := \times_{i \in N} X_i$	2
X_k	space of belief hierarchies of order k	442
X_{-i}	$X_{-i} := \times_{j \neq i} X_j$	85
$X(n)$	standard $(n-1)$-dimensional simplex, $X(n) := \{x \in \mathbb{R}^n : \sum_{i=1}^n x_i = 1, \quad x_i \geq 0 \quad \forall i\}$	935
$X(N; v)$	set of imputations in a coalitional game, $X(N; v) := \{x \in \mathbb{R}^n : x(N) = v(N), x_i \geq v(i) \quad \forall i \in N\}$	674, 802
$X^0(N; v)$	set of preimputations, $X^0(N; v) := \{x \in \mathbb{R}^N : x(N) = v(N)\}$	805
$X(\mathcal{B}; v)$	set of imputations for coalitional structure $\mathcal{B}$, $X(\mathcal{B}; v) := \{x \in \mathbb{R}^N : x(S) = v(S) \quad \forall S \in \mathcal{B}, x_i \geq v_i \quad \forall i\}$	674
$X^0(\mathcal{B}; v)$	set of preimputations for coalitional structure $\mathcal{B}$, $X^0(\mathcal{B}; v) := \{x \in \mathbb{R}^N : x(S) = v(S) \quad \forall S \in \mathcal{B}\}$	805
Y	set of states of the world	323, 334
$\widetilde{Y}(\omega)$	minimal belief subspace in state of the world ω	401
$\widetilde{Y}_i(\omega)$	minimal belief subspace of player i in state of the world ω	403
Z_k	space of coherent belief hierarchies of order k	445
$Z(P, Q; R)$	preference relation in which alternatives in R are preferred to alternatives not in R, the preference over alternatives in R is determined by P, and the preference over alternatives not in R is determined by Q	866
$Z(P^N, Q^N; R)$	preference profile in which the preference of individual i is $Z(P_i, Q_i; R)$	867
β_i	buyer i's strategy in an auction	467
β_i	buyer i's strategy in a selling mechanism	495
β_i^*	buyer i's strategy in a direct selling mechanism in which he reports his private value	495
Γ	extensive-form game	43, 50, 54
Γ	extension of a strategic-form game to mixed strategies	147

Γ_T	T-stage repeated game	528
Γ_λ	discounted game with discount factor λ	544
Γ_∞	infinitely repeated game	539
$\Gamma(x)$	subgame of an extensive-form game that starts at vertex x	4, 45, 55
$\Gamma^*(p)$	extended game that includes a chance move that selects a vector of recommendations according to the probability distribution p in the definition of a correlated equilibrium	305
$\Delta(S)$	set of probability distributions over S	146
ε	vector of constraints in the definition of perfect equilibrium	264
ε_i	vector of constraints of player i in the definition of perfect equilibrium	264
$\varepsilon_i(s_i)$	minimal probability in which player i selects pure strategy s_i in the definition of perfect equilibrium	264
$\theta(x)$	vector of excesses in decreasing order	802
θ_i^k	$A_k \approx [\theta_i^k(A_K), (1-\theta_i^k)(A_0)]$	20
λ	discount factor in a repeated game	543
λ_α	egalitarian solution with angle α of bargaining games	640
μ^k	belief hierarchy of order k	442
χ^S	incidence vector of a coalition	693
Π	belief space: $\Pi = (Y, \mathcal{F}, s, (\pi_i)_{i\in N})$	466
π_i	player i's belief in a belief space	387
σ	strategy in a decision problem with experts	601
σ_i	mixed strategy of player i	146
σ_{-k}	strategy of the player who is not player k in a two-player game	571
Σ_i	set of mixed strategies of player i	147
τ_i	strategy in a game with an outside observer $\Gamma^*(p)$	305
τ_i	player i's strategy in a repeated game	525, 538
τ_i^*	strategy in a game with an outside observer in which player i follows the observer's recommendation	306
$\varphi, \varphi(S, d)$	solution concept for bargaining games	626
φ	solution concept for coalitional games	673
φ	solution concept for bankruptcy problems	833
Ω	universal belief space	453