CS 228 : Logic in Computer Science

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Recap : Languages, Machines and Logic

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A language $L \subseteq \Sigma^*$ is called FO-definable iff there exists an FO formula φ such that $L = L(\varphi)$.

What we plan to show: L is FO-definable $\Rightarrow$ L is regular. Note that the converse is not true.

Is it Regular? Is it FO-definable?

$\Sigma = \{a, b\}$. Consider the following languages $L \subseteq \Sigma^*$:

- Begins with *a*, ends with *b*, and has a pair of consecutive *a*'s
- Contains a *b* and ends with *aa*
- Contains *abb*
- There are two occurrences of *b* between which only *a*'s occur
- Right before the last position is an *a*
- Even length words

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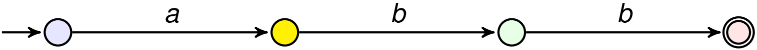

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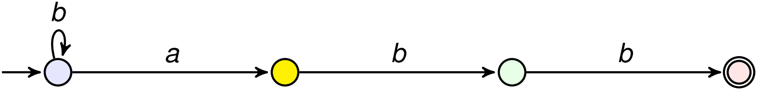

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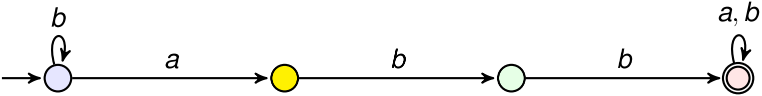

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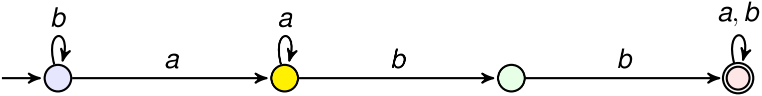

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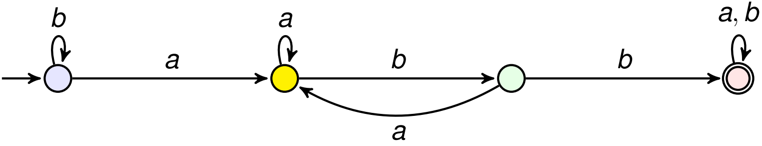

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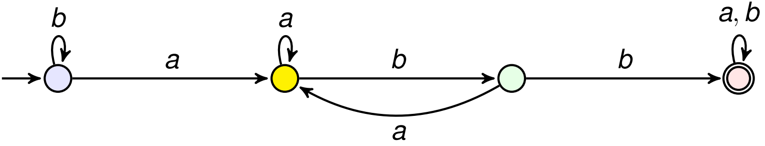

$$\exists x \exists y \exists z (Q_a(x) \wedge Q_b(y) \wedge Q_b(z) \wedge S(x, y) \wedge S(y, z))$$

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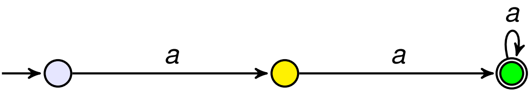

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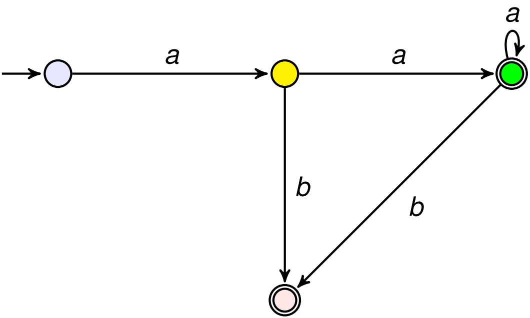

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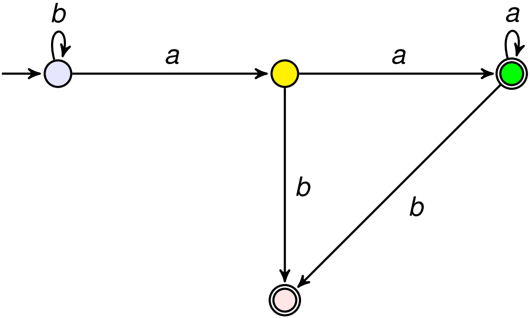

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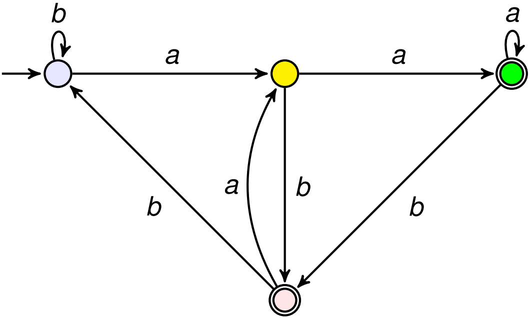

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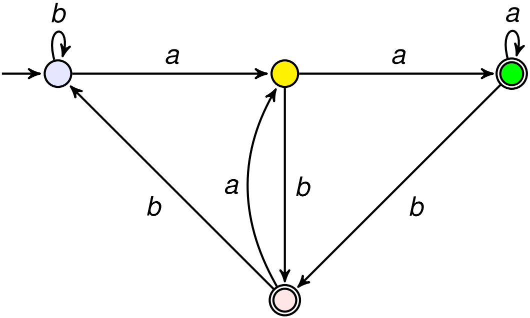

$$\exists x[Q_a(x) \wedge \exists y(S(x, y) \wedge \forall z(z \leqslant y))]$$