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IIT Bombay
CS 6001: GT&AMD
Endsem, 2022-23-I
Date: November 18, 2022

CS6001: Game Theory and Algorithmic Mechanism Design

Total: **10 × 4 = 40 marks,** *Duration:* **2 hours 30 minutes, ATTEMPT ALL QUESTIONS**

Instructions:

1. This question paper-cum-answersheet contains a total of 5 sheets of paper (10 pages, page 2 is blank). Please verify.
2. Write your name, roll number, department, section on **every side of every sheet** (except the blank sheet) of this booklet. Use only **black/blue ball-point pen**. The first 5 minutes of additional time is given exclusively for this activity.
3. Write final answers neatly with a pen **only in the given boxes**.
4. Use the rough sheets for scratch works / attempts to solution. **Write only the final solution (which may be a sequence of logical arguments) in a precise and succinct manner in the boxes provided**. Do not provide unnecessarily elaborate steps. The space within the boxes are sufficient for the correct and precise answers.
5. Submit your answerscripts to the teaching staff when you leave the exam hall or the time runs out (whichever is earlier). **Your exam will not be graded if you fail to return the paper.**
6. **This is a closed book, notes, internet exam. No communication device, e.g., cellphones, iPad, etc., is allowed**. Keep it switched off in your bag and keep the bag away from you. If anyone is found in possession of such devices during the exam, that answerscript may be disqualified for evaluation and DADAC may be invoked.

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Problem 1 (1 + 2 + 1 + 1 + 1 + 2 + 2 points). Consider a two agent model with three alternatives $\{a, b, c\}$. Table 1 shows two preference profiles, $P \equiv (P_1, P_2)$ and $P' \equiv (P'_1, P'_2)$, of the agents. Suppose f is an **onto** and **strategyproof** SCF with $f(P_1, P_2) = a$.

P_1	P_2	P'_1	P'_2
a	c	b	a
b	b	a	b
c	a	c	c

Table 1: Two Preference Profiles

(a) Suppose the domain of preferences is of **unrestricted strict preferences**. Then $f(P')$ will be

$f(P') = b$

(b) Explain the answer above, i.e., why $f(P')$ takes that value. You may use any standard result proved in the class.

ONTO + SP ⇒ PE → pareto efficient

Since for P', c is dominated by both a, b.

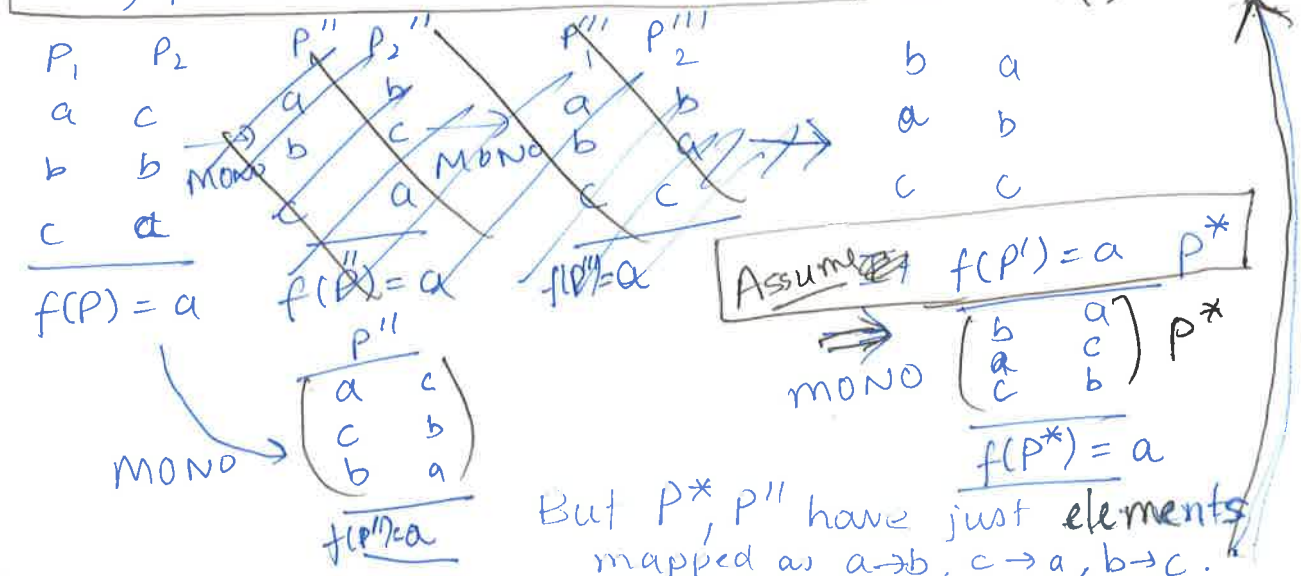
f(P') ≠ c.

Also, SP ⇒ MONO

It must be dictatorial according to P_1.

Thus, f(P') = b

$f(P^*) = a \Rightarrow f(P'') = b$ (which contradicts Thus f(P') = b)



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(c) Now, suppose that these preferences are generated from a **single-peaked preference** domain with the intrinsic ordering of the alternatives being $a < b < c$. Are the preference profiles in Table 1 valid single-peaked profiles under this setting? (Yes/No)

Yes

(d) Does the conclusion of Part (a) hold in this case? (Yes/No)

No

(e) What will be the value(s) of $f(P')$ in this modified case?

$f(P') = b$ or a

(f) Explain clearly why or why not the earlier proof (of *unrestricted strict preferences*) go through in this case (*single-peaked* domain).

Preference order $a \succ_9 c \succ_9 b$ and $c \succ_9 a \succ_9 b$ are not valid for given single peaked domain.

Since, domain is restricted, proof that f is dictatorial is not possible

Construction of P^* and P'' is not possible since again preference order a, c, b is not possible.

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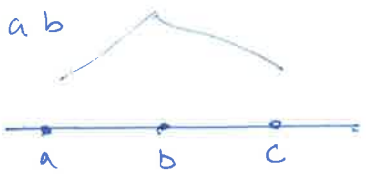
(g) If the conclusion of Part (d) is false, provide a **non-constant** (**onto** and **strategyproof**) SCF f that has $f(P_1', P_2') = a$ in the single-peaked domain.

P_1	P_2	P_1'	P_2'
a	c	b	a
b	b	a	b
c	a	c	c



b b c
a a a
a c b

$f(P)$ = ~~alternative which occurs most in~~ based on most preffered alternative count. breaking ties as a > b > c

i.e. if a, c are most ~~prefred alternative~~ in P_1'', P_2''. $f(P'') = a$

~~Clearly, ONTO~~

a c b
b b a
c a c

~~a c~~ ~~b b~~ ~~c~~ If we know that it is ONTO and SP.

Use median voter SCF.

Let peaks of P_1, P_2 be at the most preferred choice. and the ~~two~~ phantom peaks can be ~~placed anywhere~~. placed at $p < a < b < c$.

No, clearly a will be choosed by SCF. For P, $F(P) = \text{median}(p, a, c) = a$

For P', $F(P) = \text{median}(p, b, a) = a$

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Problem 2 (1 + 2 + 1 + 2 + 1 + 3 points). Consider the following Arrovian social welfare function (ASWF) setup. A committee of 11 members is asked to rank three colors: red, blue, and green, from most preferred to least preferred (consider only strict preferences). The committee members simultaneously announce their strict preference relations over the three colors. If red is the most preferred color of *at least* **five** members of the committee, red is determined to be the prettiest color. Otherwise, if blue is the most preferred color of at least five members of the committee, blue is determined to be the prettiest color. Otherwise, green is determined to be the prettiest color. The prettiest color gets the top position in the consolidated ranking of the ASWF. Once the prettiest color is determined, the remaining two colors are then ranked by the simple majority rule, i.e., if the majority prefers color a over color b, then a will get the second position and b the last. This is how the social welfare function is constructed.

(a) Is the social welfare function described here **dictatorial**? (Yes/No)

No

(b) Justify your answer above, i.e., if yes, explain why, if no, provide a counterexample.

The given social welfare function is rather anonymous. If we interchange preferences of the players, the output will remain same. on contrary

Assume a player P_1 is dictatorial and its preference is R > B > G and all other 10 players have preferrence B > G > R.

Clearly, Blue will selected

Hence, it cannot be dictatorial

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(c) Does the social welfare function described here satisfy the **unanimity** property?

Yes

Every order =

(d) Justify your answer above, i.e., if yes, explain why, if no, provide a counterexample.

Unanimity

if $a P_i b \ \forall i, \forall b \neq a \Rightarrow a F(\hat{R}) b \ \forall \ b \neq a$

Clearly, if either of the color is most preferred by all members, then the color will get top position by the Arrovian Function.

The second preference is given according to majority rule. If all players have same preference, clearly the second preferred color will be in majority over third preference.

Hence, it is unanimous.

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(e) Does the social welfare function described here satisfy the **independence of irrelevant alternatives** property?

No

(f) Justify your answer above, i.e., if yes, explain why, if no, provide a counterexample.

Consider the following two cases P, P'

5× P_1	P_2 × 6	P
B	G	first five have preference P_1 & last 6 have preference P_2
R	R	
G	B	

5× P_1'	P_2' × 6	P'
B	R	first five has preference P_1' and last 6 have preference P_2'.
R	G	
G	B	

$P|_{B,R} = P'|_{B,R}$

but, [B F(P) R] while [R F(P') B]

blue wins since it is preferred most by 5 members. and red is not top preference of any member

red wins since it is preffered most by atleast 5 members.

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Problem 3 ((2 + 3) + (2 + 3) points). One object is being sold using Myerson's optimal auction. There are three buyers with types t_1, t_2, and t_3 respectively and their virtual valuations are given as follows.

$$w_1(t_1) = 2t_1 - 2,\ w_2(t_2) = 2t_2 - 3,\ w_3(t_3) = 2t_3 - 4.$$

(a) If the reported types are $t_1 = 1, t_2 = 2, t_3 = 3$, which player wins the auction?

$w_1(t_1) = 0 \quad w_2(t_2) = 1 \quad w_3(t_3) = 2$

Player 3 wins

(b) How much do these players pay respectively? (express the answers rounded to one decimal point)

Player 1: 0

Player 2: 0

Player 3: 2.5

(c) If the reported types were $t_1 = 5, t_2 = 10, t_3 = 7$, which player wins the auction?

$w_1(t_1) = 8 \quad w_2(t_2) = 17 \quad w_3(t_3) = 10$

Player 2 wins

(d) How much do these players pay respectively? (express the answers rounded to one decimal point)

Player 1: 0

Player 2: 6.5

Player 3: 0

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Problem 4 ((0.5 × 2 + 2 × 2) + (0.5 × 2 + 2 × 2) points). Consider two players, 1 and 2, are being allocated two objects A and B for which they have the valuations as shown in the table below (rows denote the players and the columns the different bundles of the objects – $\emptyset$ denotes the empty bundle).

	$\emptyset$	A	B	$\{A, B\}$
1	0	7	0	10
2	0	0	5	9

(a) If the objects are allocated using the VCG mechanism, which object goes to whom? (Provide only the player number)

A goes to player: 1

B goes to player: 2

(b) What will be the payments of players 1 and 2 respectively?

Player 1: 4

Player 2: 5

(c) In the same setup as before, consider the following change. Objects A and B are *no longer sold separately*, rather they are **sold as a bundle**.

If the objects are allocated using the VCG mechanism, which object goes to whom? (Provide only the player number)

A goes to player: 1

B goes to player: 1

(d) What will be the payments of players 1 and 2 respectively?

Player 1: 9

Player 2: 0

END OF QUESTION PAPER. GOOD LUCK!