Random Variables Are Variables…That Are Random

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Check out the variable **`result`** in the code below.

```
import random

def flip_coin():
    # returns 0 or 1 with prob. 0.5
    return random.choice([0,1])

result = flip_coin()
```

def constant():
return 42

result = constant()

not random

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- Do we know the value of **`result`** before we run the code?

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- Is the value of **`result`** the same every time we run the code?

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- Do we know the value of **`result`** before we run the code? Nope!
- Is the value of **`result`** the same every time we run the code? Nope!

Like **`result`**, a random variable is a variable whose value is uncertain.

Random Variables Are Variables…That Are Random

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"Let $\boldsymbol{X}$ be the result of flipping a coin."

$$P(X = 0) = 0.5$$
$$P(X = 1) = 0.5$$

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"Let X be the result of flipping a coin."

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- Random variables store the outcome of an experiment
- Random variables can be described by their possible outcomes + probabilities
  - Note: random variables can only be numbers (not "heads" or "tails")

Random variables are an abstraction on top of events

Random variables are *not* events

Random Variables vs. Events

X

Let X be a
random variable

Random Variables vs. Events

It is an event when X takes on a value

$$X \qquad\qquad X = 2$$

Let X be a random variable

$X \in \{2, 4, 6\}$

Random Variables vs. Events

It is an event when
X takes on a value

$$X \qquad X = 2 \qquad \underline{P(X = 2)}$$

Let X be a
random variable

So we can still work with
probabilities of events

Examples of Random Variables

"Let X be the result of rolling a dice."

- $P(X = 1) = 1/6$
- $P(X = 2) = 1/6$
- $P(X = 3) = 1/6$
- $P(X = 4) = 1/6$
- $P(X = 5) = 1/6$
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...or, $P(X = x) = 1/6$ for $1 \leq x \leq 6$

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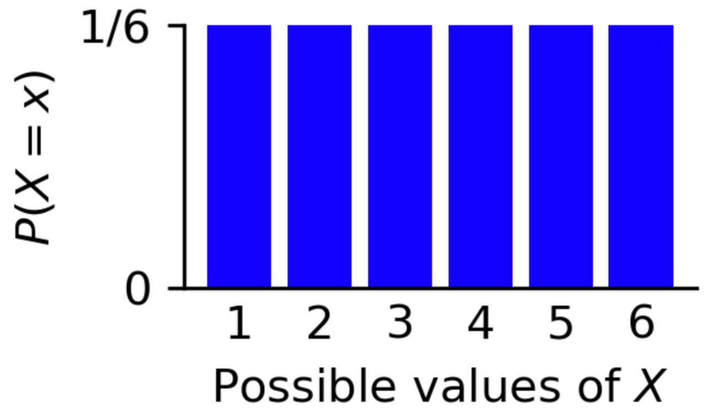

Examples of Random Variables

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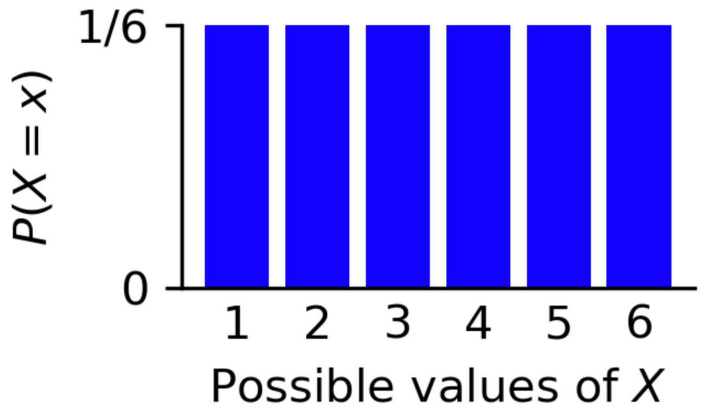

"Let Y be the number of heads seen in 2 coin flips."

- $P(Y=0)=1/4$ (T, T)
- $P(Y=1)=1/2$ (H, T), (T, H)
- $P(Y=2)=1/4$ (H, H)

Examples of Random Variables

"Let X be the result of rolling a dice."

- $P(X=1)=1/6$
- $P(X=2)=1/6$
- $P(X=3)=1/6$
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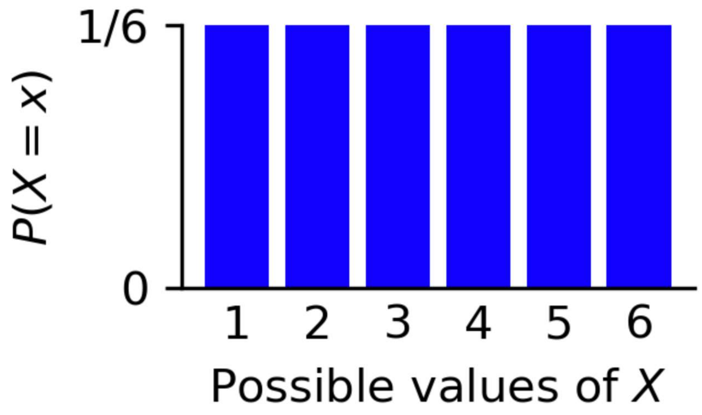

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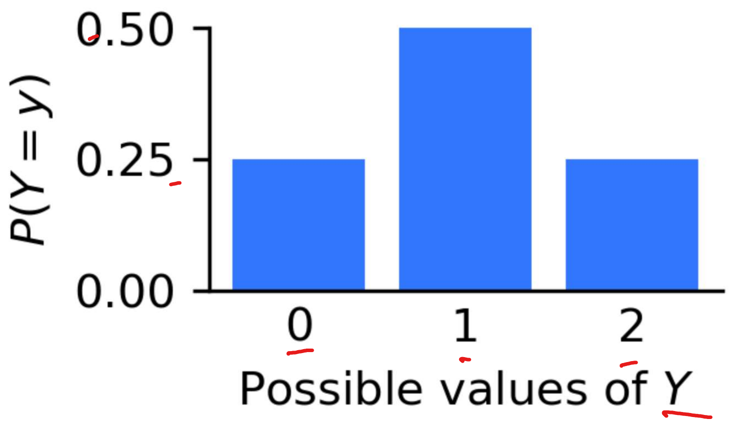
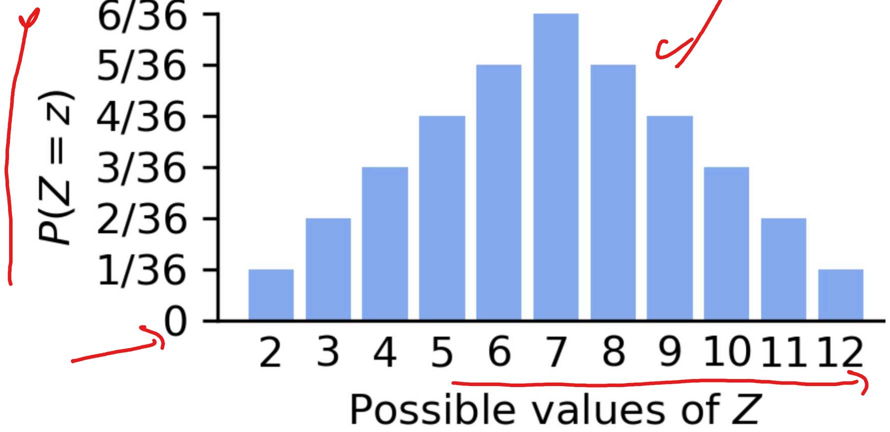


Examples of Random Variables

"Let Z be the sum of rolling two dice."

- $P(Z = 2) = 1/36$
- $P(Z = 3) = 2/36$
- $P(Z = 4) = 3/36$
- $P(Z = 5) = 4/36$
- $P(Z = 6) = 5/36$
- $P(Z = 7) = 6/36$
- $P(Z = 8) = 5/36$
- $P(Z = 9) = 4/36$
- $P(Z = 10) = 3/36$
- $P(Z = 11) = 2/36$
- $P(Z = 12) = 1/36$



$$P(Z = z) = \begin{cases} \frac{z-1}{36} & z \in \mathbb{Z}, 1 \leq z \leq 6 \\ \frac{13-z}{36} & z \in \mathbb{Z}, 7 \leq z \leq 12 \\ 0 & \text{else} \end{cases}$$

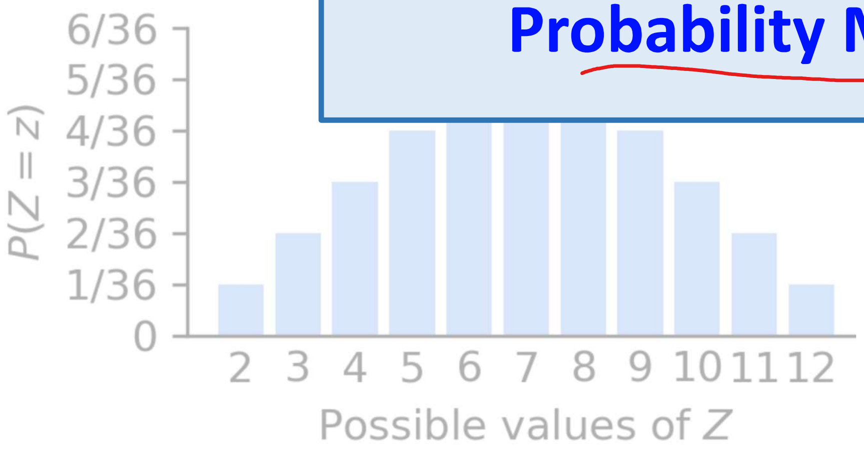
Examples of Random Variables

"Let Z be the sum of rolling two dice."

- $P(Z = 2) = 1/36$
- $P(Z = 3) = 2/36$
- $P(Z =$ 4
- $P(Z =$ 5
- $P(Z = 6) = 5/36$
- $P(Z = 7) = 6/36$
- $P(Z = 10) = 3/36$
- $P(Z = 11) = 2/36$
- /36

There's a name for what we're describing, when we list out all possible outcomes + their probabilities:

Probability Mass Function (PMF)



$\mathbb{Z}, 1 \leq z \leq 6$

$$P(Z = z) = \begin{cases} \frac{13-z}{36} & z \in \mathbb{Z}, 7 \leq z \leq 12 \\ 0 & \text{else} \end{cases}$$

Probability Mass Functions

Random Variables & Functions

"Let Y be the number of heads seen in 2 coin flips."

If this is a number

$$P(Y = 2)$$

Then this is a number (between 0 and 1)

Random Variables & Functions

"Let Y be the number of heads seen in 2 coin flips."

If this is a variable

$$\boxed{P(Y = k)}$$

Then this is a function

$f_Y(k) : \to [0,1]$ $\cancel{\{0,1\}}$

Random Variables & Functions

"Let Y be the number of heads seen in 2 coin flips."

...and get out their probabilities!

$$0.5$$

$$P(Y = k)$$

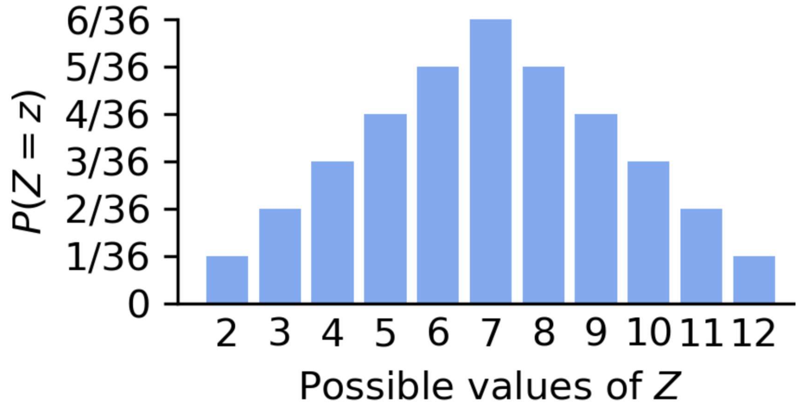
$$k = 1$$

We can put in different inputs...

The relationship between values a random variable can take on, and the corresponding probability, is a ***function***!

Probability Mass Function: Representations

$$P(Z = z) = \begin{cases} \frac{z-1}{36} & z \in \mathbb{Z}, 1 \leq z \leq 6 \\ \frac{13-z}{36} & z \in \mathbb{Z}, 7 \leq z \leq 12 \\ 0 & \text{else} \end{cases}$$




```
def event_probability(z):
    # probability mass function of Z
    if not z.is_integer() or z > 12 or z < 1:
        return 0

    if z < 7:
        return (z - 1) / 36
    else:
        return (13 - z) / 36
```

All of these are different ways we can represent probability mass functions!