CS 228 : Logic in Computer Science

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GNBA

- Generalized NBA, a variant of NBA
- Only difference is in acceptance condition
- Acceptance condition in GNBA is a set $\mathcal{F} = \{F_1, \ldots, F_k\}$, each $F_i \subseteq Q$
- An infinite run ρ is accepting in a GNBA iff

$$\forall F_i \in \mathcal{F}, \mathit{Inf}(\rho) \cap F_i \neq \emptyset$$

- Note that when $\mathcal{F} = \emptyset$, all infinite runs are accepting
- GNBA and NBA are equivalent in expressive power.

LTL to GNBA

- Let $\varphi = a\,\mathsf{U}(\neg a\,\mathsf{U} c)$. Let $\psi = \neg a\,\mathsf{U} c$
- Subformulae of φ : $\{a, \neg a, c, \psi, \varphi\}$. Let $B = \{a, \neg a, c, \neg c, \psi, \neg\psi, \varphi, \neg\varphi\}$.
- Possibilities at each state : some consistent subset of B holds
  - $\{a, c, \psi, \varphi\}$
  - $\{\neg a, c, \psi, \varphi\}$
  - $\{a, \neg c, \neg\psi, \varphi\}$
  - $\{a, \neg c, \neg\psi, \neg\varphi\}$
  - $\{\neg a, \neg c, \psi, \varphi\}$
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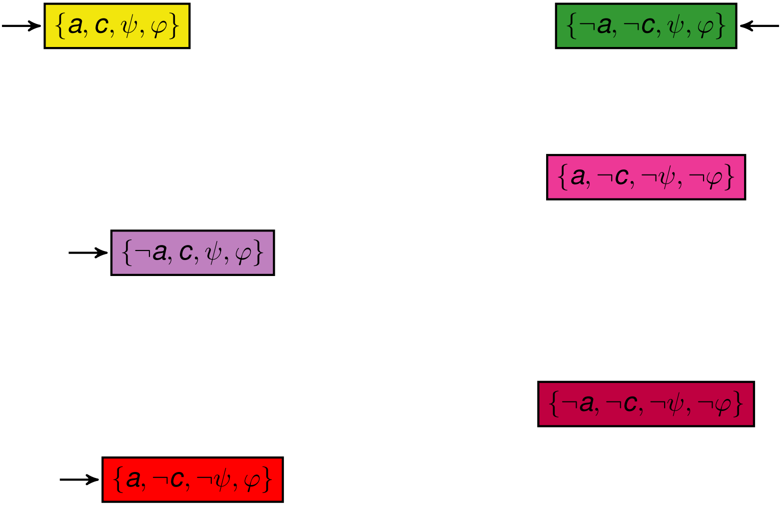
LTL to GNBA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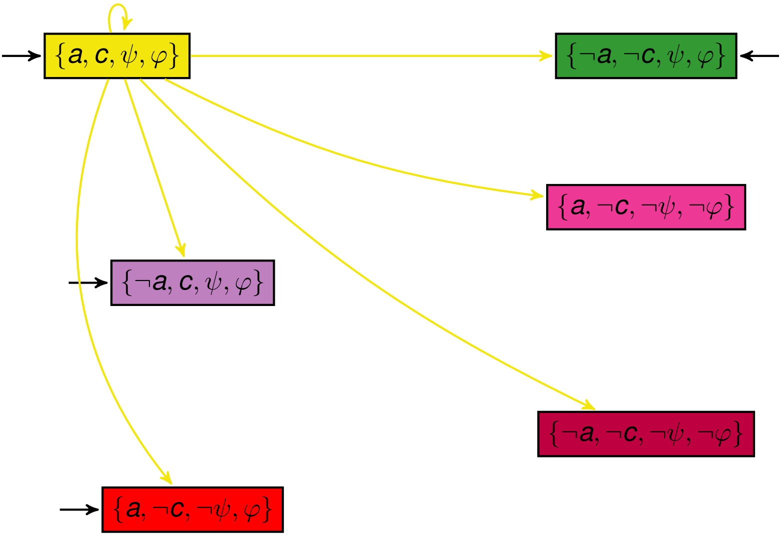
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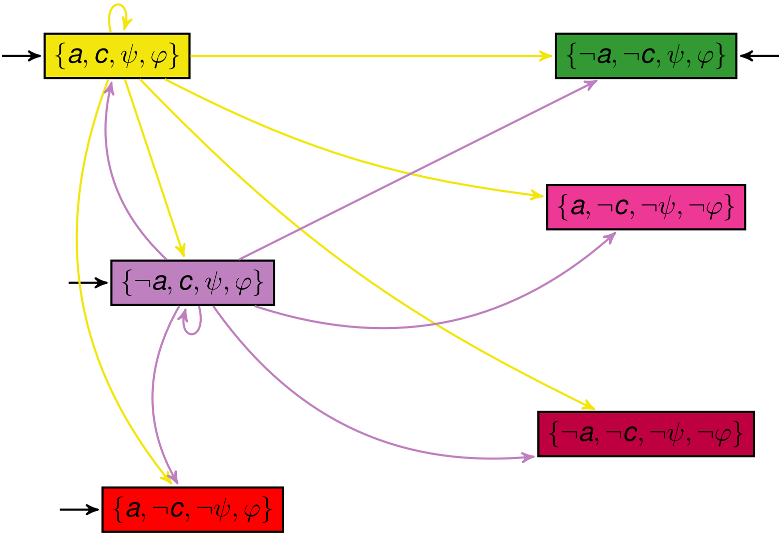
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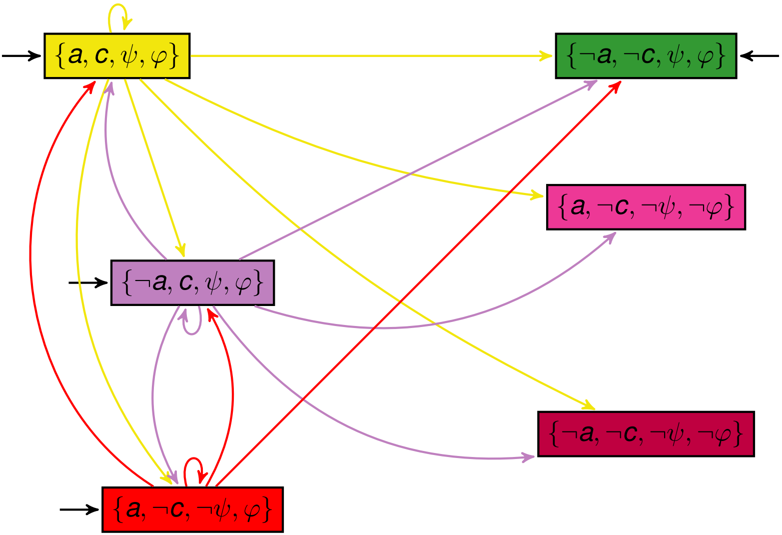

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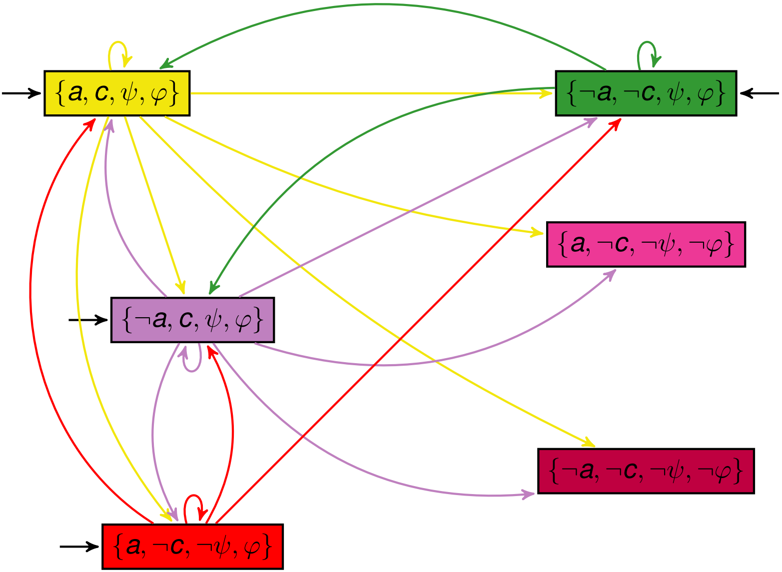
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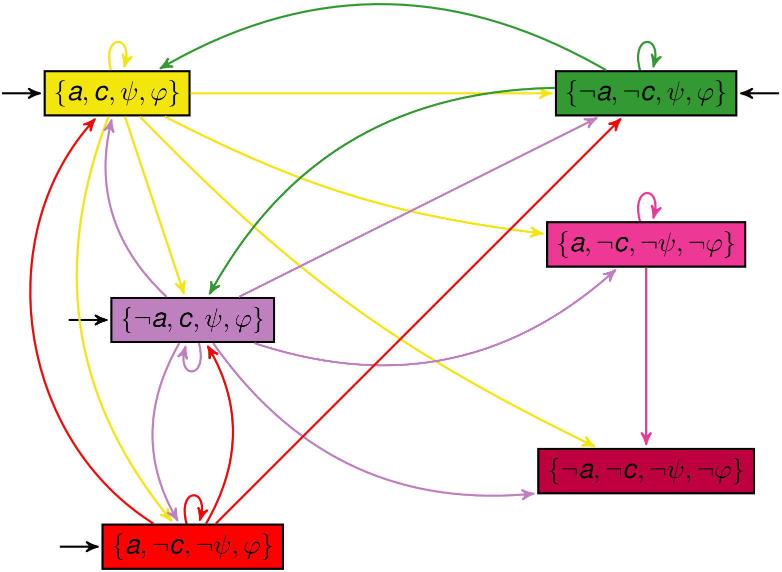
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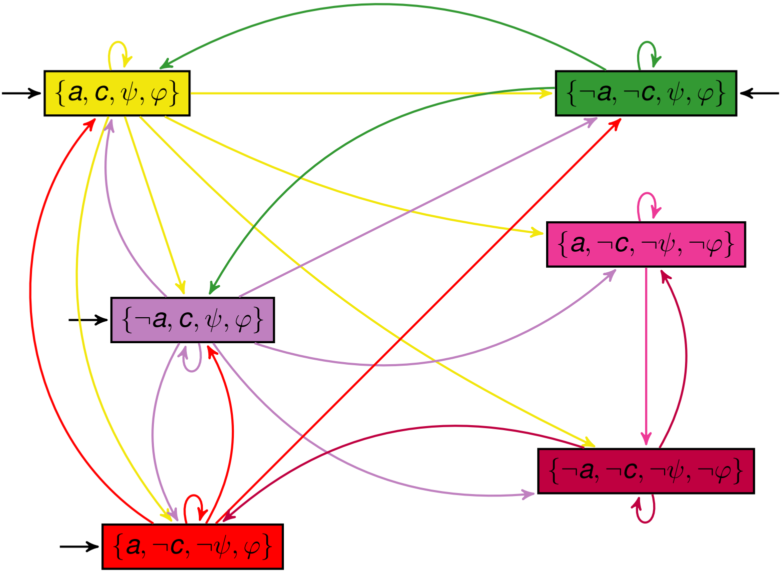
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GNBA Acceptance Condition

- $\psi = \neg a \, \mathsf{U} c$
- $\varphi = a \, \mathsf{U} \psi$
- $F_1 = \{B \mid \psi \in B \rightarrow c \in B\}$
- $F_2 = \{B \mid \varphi \in B \rightarrow \psi \in B\}$
- $\mathcal{F} = \{F_1, F_2\}$

Final States

$\{a, c, \psi, \varphi\} \in F_1, F_2$

$\{\neg a, \neg c, \psi, \varphi\} \in F_2$

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  - $\psi \in B \rightarrow \neg\psi \notin B$ and $\psi \notin B \rightarrow \neg\psi \in B$
  - Whenever $\psi_1 \, \mathsf{U} \psi_2 \in Cl(\varphi)$,
    - $\psi_2 \in B \rightarrow \psi_1 \, \mathsf{U} \psi_2 \in B$
    - $\psi_1 \, \mathsf{U} \psi_2 \in B$ and $\psi_2 \notin B \rightarrow \psi_1 \in B$

Putting Together

Given φ over AP, construct $A_\varphi = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$,

- $Q = \{B \mid B \subseteq Cl(\varphi)$ is consistent $\}$
- $Q_0 = \{B \mid \varphi \in B\}$
- $\delta : Q \times 2^{AP} \to 2^Q$ is such that

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- $\delta : Q \times 2^{AP} \to 2^Q$ is such that
  - For $C = B \cap AP$, $\delta(B, C)$ is enabled and is defined as :
  - If $\bigcirc\psi \in Cl(\varphi)$, $\bigcirc\psi \in B$ iff $\psi \in \delta(B, C)$
  - If $\varphi_1 \, \mathsf{U} \varphi_2 \in Cl(\varphi)$,
    $\varphi_1 \, \mathsf{U} \varphi_2 \in B$ iff $(\varphi_2 \in B \vee (\varphi_1 \in B \wedge \varphi_1 \, \mathsf{U} \varphi_2 \in \delta(B, C)))$

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    $\varphi_1 \mathsf{U} \varphi_2 \in B$ iff $(\varphi_2 \in B \vee (\varphi_1 \in B \wedge \varphi_1 \mathsf{U} \varphi_2 \in \delta(B, C)))$
- $\mathcal{F} = \{F_{\varphi_1 \mathsf{U} \varphi_2} \mid \varphi_1 \mathsf{U} \varphi_2 \in Cl(\varphi)\}$, with
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- Prove that $L(\varphi) = L(A_\varphi)$

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- LTL $\varphi \rightsquigarrow$ NBA A_φ : Number of states in $A_\varphi \leqslant |\varphi|.2^{|\varphi|}$