

Problem Set 8

1. Let $AP = \{p, q, r\}$. Formulate the following as LT properties:
 - (a) Eventually false
 - (b) p occurs exactly twice, and q never occurs between two occurrences of p
 - (c) If r occurs only finitely often, then p continuously occurs from some point
 - (d) r is true continuously upto somepoint, and at the next point, both p, q hold, and then q and r alternate infinitely often
 - (e) Infinitely often there are two consecutive occurrences of p
 - (f) Between every consecutive occurrences of p , there is a q , and there is a prefix of r 's of even length
2. Let TS and TS' be two transition systems without terminal states on the same set of atomic propositions AP . Then show that $Traces(TS) = Traces(TS')$ iff TS and TS' satisfy the same set of LT properties.
3. Consider a set of atomic propositions AP . Consider the following logic \mathcal{X} defined as follows:

$$\varphi ::= (a \in AP) \mid \varphi \wedge \varphi \mid \neg \varphi \mid \varphi \Delta \varphi$$

with semantics as follows:

Given a word $w = A_0 A_1 \dots$ over 2^{AP} and a position $i \in \mathbb{N}$, we define

- (a) $w, i \models a$ iff $a \in A_i$ for $a \in AP$
- (b) $w, i \models \varphi_1 \wedge \varphi_2$ iff $w, i \models \varphi_1$ and $w, i \models \varphi_2$
- (c) $w, i \models \neg \varphi$ iff $w, i \not\models \varphi$
- (d) $w, i \models \varphi \Delta \psi$ iff $\exists j > i, w, j \models \psi$ and $\forall i < k < j, w, k \models \varphi$.

Comment on the equivalence of LTL and \mathcal{X} .

4. Exercises 5.1, 5.2, 5.5, 5.6 and 5.7 from Baier-Katoen.