CS 228 : Logic in Computer Science

Krishna. S

Recap : ω-automata

An ω-automaton is a tuple $\mathcal{A} = (Q, \Sigma, \delta, q_0, Acc)$ where

- Q is a finite set of states
- Σ is a finite alphabet
- $\delta : Q \times \Sigma \rightarrow 2^Q$ is a state transition function (if non-deterministic, otherwise, $\delta : Q \times \Sigma \rightarrow Q$)
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A run ρ of $\mathcal{A}$ on an ω-word $\alpha = a_1 a_2 \cdots \in \Sigma^\omega$ is an infinite state sequence $\rho(0)\rho(1)\rho(2)\ldots$ such that

- $\rho(0) = q_0$,
- $\rho(i) = \delta(\rho(i-1), a_i)$ if $\mathcal{A}$ is deterministic,
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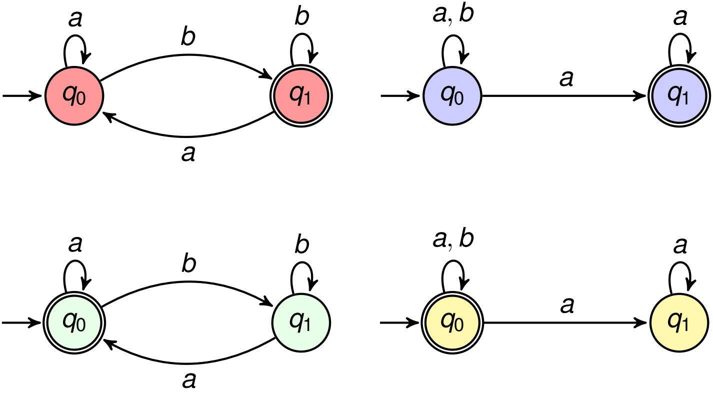
Büchi Acceptance

For Büchi Acceptance, *Acc* is specified as a set of states, $G \subseteq Q$. The ω-word α is accepted if there is a run ρ of α such that $\mathit{Inf}(\rho) \cap G \neq \emptyset$.

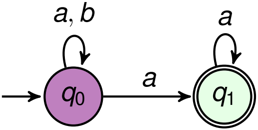
ω-Automata with Büchi Acceptance



- Left (T-B): Inf many b's, Inf many a's
- Right (T-B): Finitely many b's, $(a + b)^\omega$

NBA and DBA

- Is every DBA as expressible as a NBA, like in the case of DFA and NFA?
- Can we do subset construction on NBA and obtain DBA?



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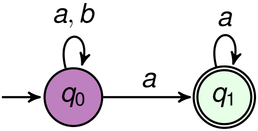

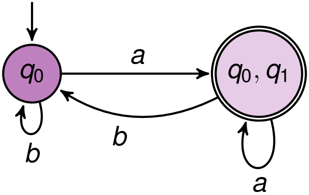

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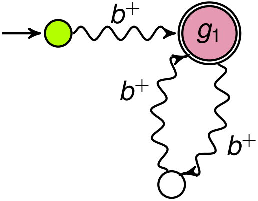

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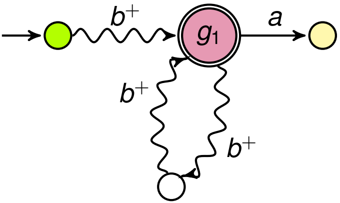

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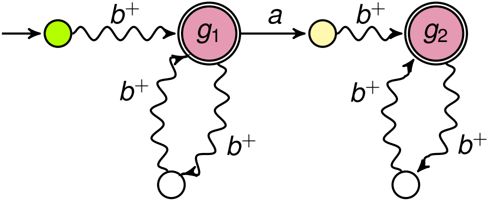

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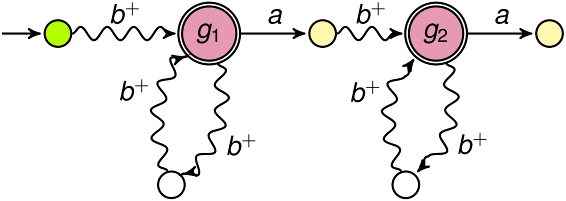

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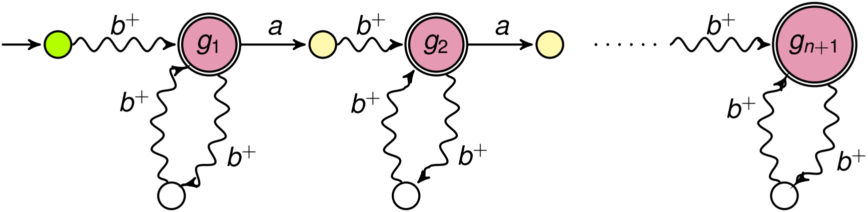

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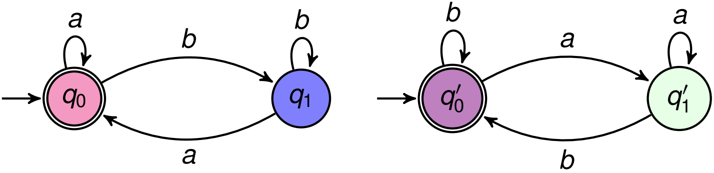

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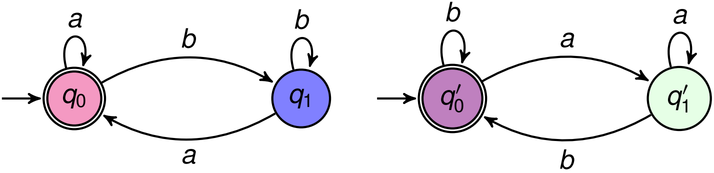

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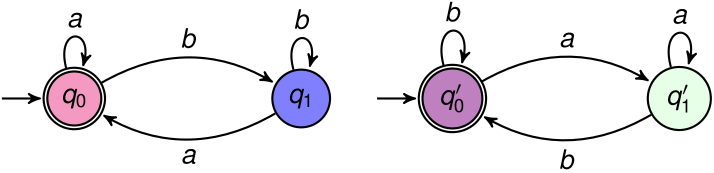

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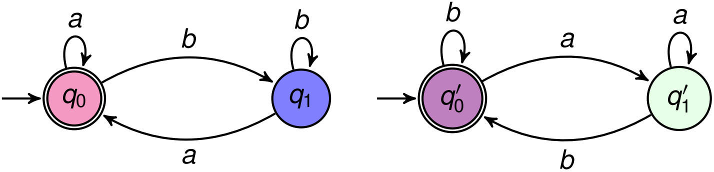

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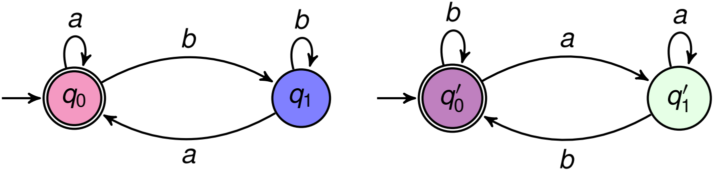

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- Good states=$Q_1 \times G_2 \times \{2\}$ or $G_1 \times Q_2 \times \{1\}$

Emptiness

Given an NBA/DBA $\mathcal{A}$, how do you check if $L(\mathcal{A}) = \emptyset$?

- Enumerate SCCs
- Check if there is an SCC containing a good state