INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

Department of Mathematics

SI 427 (Probability Theory)

Tutorial Sheet-III

1. (a conditioning principle) Let $A_1, A_2, \cdots, A_n \in \mathcal{F}$ be such that $P(A_1A_2\cdots A_n) > 0$. Show that
$$\begin{aligned} P(A_1A_2\cdots A_n) &= P(A_1|A_2\cdots A_n) \times P(A_2|A_3\cdots A_n) \times \cdots \\ &\quad \times P(A_{n-1}|A_n) \times P(A_n). \end{aligned}$$

2. Let $\pi = (\pi_1, \cdots \pi_n)$ be a permutation of 1 to n 'picked at random from the set of all permutations of 1 to n. If you are told that $\pi_k > \pi_1, \cdots \pi_k > \pi_{k-1}$, what is the probability that $\pi_k = n$.

3. In an election between two candidates A and B, A wins by a margin of 10 votes and total casted votes are $2n + 10$. Find the probability that A was leading throughout the counting.

4. Let $A, B, C \in \mathcal{F}$ be such that $P(ABC) = P(AB)P(C) = P(A)P(B)P(C)$ and $P(AB) > 0$. Show that
$$P\Big(C|AB\Big) = P(C)\,.$$

5. Let A, B, C be in $\mathcal{F}$ such that
$$P(ABC) > 0,\ P(C|AB) = PC|B)\,.$$
Show that
$$P(A|BC) = P(A|B)\,.$$

6. There are two coins, one is unbiased and other one is biased with probability of geting head as $\frac{1}{3}$. A coin is selected at random and tossed resulting in a head. What is the probability that selected coin was the unbiased one.

7. Let $A_n, B \in \mathcal{F}$ for all n and A_n is an increasing sequence of events such that $A = \cup_{n=1}^{\infty} A_n$. It is given that $P(B) > 0$ and $P(A_n|B) = P(A_n)$ for all n and $P(A^c) = \frac{1}{3}$. Find $P(A|B)$.

8. (Polya urn scheme) An urn has n red balls and m blue balls. A ball is drawn at random from the urn and color is noted and put back with additional 10 balls of the same color. This procedure is repeated for a total of 10 times. What is the probability that 10th draw resulted in a red ball.

9. (Laplace law of succession) Consider $N+1$ urns numbered 0 to N. Each urn has N ball with kth urn containing k red balls and $N-k$ blue balls. An urn is picked at random, and balls are drawn from the urn with replacement in succession. If first m drawings result in red balls, what is the probability that $m+1^{\text{th}}$ draw result in a red ball?

10. Let $A_1 \subseteq A_2 \subseteq \cdots \subseteq A_n$, $A_k \in \mathcal{F}, k = 1, \cdots n$ and $P(A_1) > 0$. Show that

$$P(A_n) = P(A_n|A_{n-1}) \times P(A_{n-1}|A_{n-2}) \times \cdots \times P(A_2|A_1) \times P(A_1).$$