CS 228 : Logic in Computer Science

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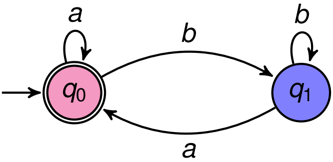
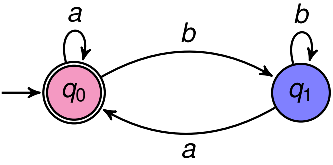
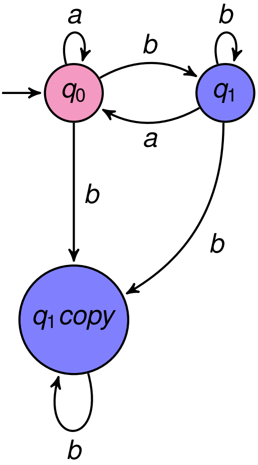
Complementation of DBA

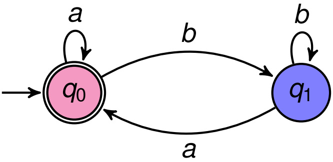


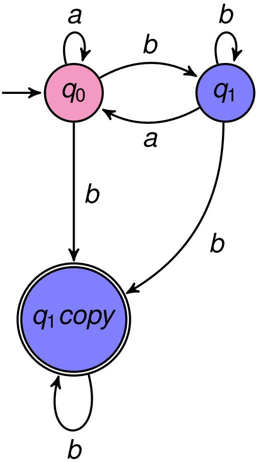
Complementation of DBA





Complementation of DBA





Complementation of DBA

- Given $\mathcal{A}$ is a DBA, and $w \notin L(\mathcal{A})$, then after some finite prefix, the unique run of w settles in bad states.
- Idea for complement: “copy” states of $Q - G$, once you enter this block, you stay there.
- View this as the set of good states, any word w that was rejected by $\mathcal{A}$ has two possible runs in this automaton: the original run, and one another, that will settle in the $Q - G$ copy, and will be accepted.
- What we get now is an NBA for $\overline{L(\mathcal{A})}$, not a DBA.

Complementing NBA non-trivial, can be done.

GNBA

- Generalized NBA, a variant of NBA
- Only difference is in acceptance condition
- Acceptance condition in GNBA is a set $\mathcal{F} = \{F_1, \ldots, F_k\}$, each $F_i \subseteq Q$
- An infinite run ρ is accepting in a GNBA iff

$$\forall F_i \in \mathcal{F}, \mathit{Inf}(\rho) \cap F_i \neq \emptyset$$

- Note that when $\mathcal{F} = \emptyset$, all infinite runs are accepting
- GNBA and NBA are equivalent in expressive power.

Word View

$w = \{a\}\{a, b\}\{\}\ldots, \varphi = a\,\mathrm{U}(\neg a \wedge b)$

- The subformulae of φ are $\{\varphi, a, \neg a \wedge b, \neg a, b\}$
- At each position i of w, some (sub)formulae of φ or their negation are true. Consider maximally consistent such sets wrt φ, call them B_i.
- $B_0 = \{\neg\varphi, \neg b, a, \neg(\neg a \wedge b)\}$,
- $B_1 = \{a, b, \neg(\neg a \wedge b), \neg\varphi\}$,
- $B_2 = \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$.
- $\psi \in B_i$ iff $A_i A_{i+1} A_{i+2} \ldots \models \psi$.

Consistent Sets

- B_i is consistent wrt propositional logic subformulae:
  - $\varphi_1 \wedge \varphi_2 \in B_i \Leftrightarrow \varphi_1 \in B_i \wedge \varphi_2 \in B_i$
  - $\psi \in B_i \Leftrightarrow \neg\psi \notin B_i$
- B_i is consistent wrt until subformulae:
  - $\varphi_2 \in B_i \Rightarrow \varphi_1 \mathsf{U} \varphi_2 \in B_i$
  - $\varphi_1 \mathsf{U} \varphi_2 \in B_i, \varphi_2 \notin B_i \Rightarrow \varphi_1 \in B_i$
- B_i is maximal : for any subformula ψ, $\psi \in B_i \Leftrightarrow \neg\psi \notin B_i$

LTL φ to GNBA G_φ

- States of G_φ are sets B_i
- For a word $w = A_0 A_1 A_2 \ldots$ the sequence of states $\sigma = B_0 B_1 B_2 \ldots$ will be a run for w
- σ will be accepting iff $w \models \varphi$ iff $\varphi \in B_0$
- In general, a run $B_i B_{i+1} \ldots$ for $A_i A_{i+1} \ldots$ is accepting iff $A_i A_{i+1} \ldots \models \psi$ for all $\psi \in B_i$.

LTL to GNBA

- Let $\varphi = \bigcirc a$.
- Subformulae of φ : $\{a, \bigcirc a\}$. Let $A = \{a, \bigcirc a, \neg a, \neg \bigcirc a\}$.
- Possibilities at each state : a maximally consistent subset of A holds
  - $\{a, \bigcirc a\}$
  - $\{\neg a, \bigcirc a\}$
  - $\{a, \neg \bigcirc a\}$
  - $\{\neg a, \neg \bigcirc a\}$
- Our initial state(s) must guarantee truth of $\bigcirc a$. Thus, initial states: $\{a, \bigcirc a\}$ and $\{\neg a, \bigcirc a\}$

LTL to GNBA

$\{a, \bigcirc a\}$

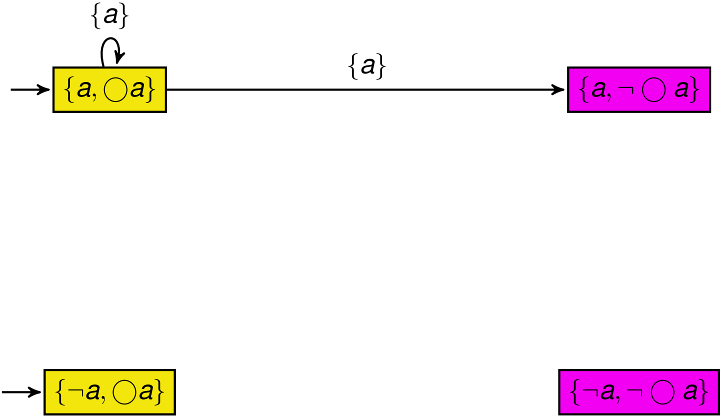
$\{a, \neg \bigcirc a\}$

$\{\neg a, \bigcirc a\}$

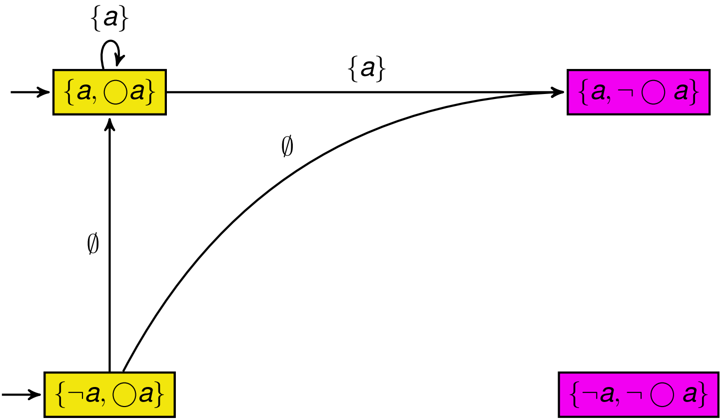
$\{\neg a, \neg \bigcirc a\}$

LTL to GNBA

LTL to GNBA

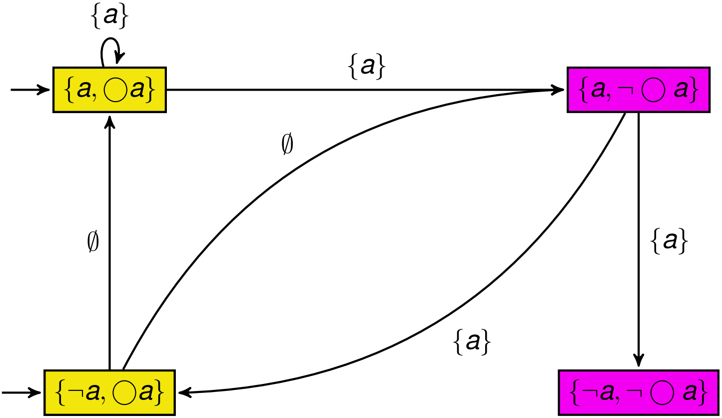


LTL to GNBA



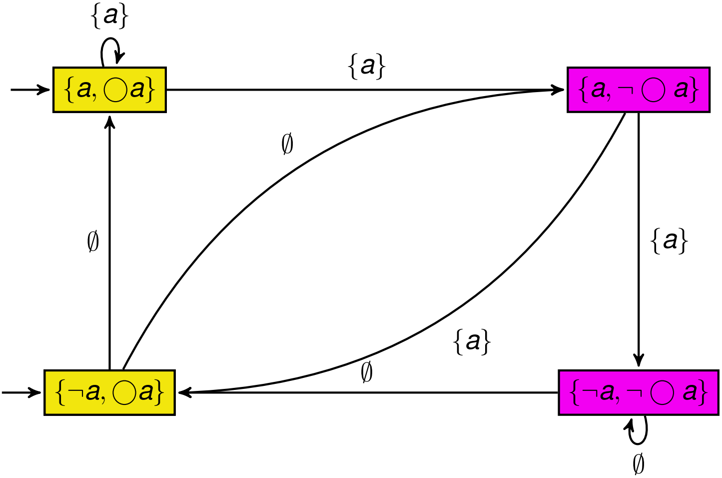
$\{a\}$
$\{a\}$
$\{a, \bigcirc a\}$
$\{a, \neg \bigcirc a\}$
$\emptyset$
$\emptyset$
$\{\neg a, \bigcirc a\}$
$\{\neg a, \neg \bigcirc a\}$

LTL to GNBA



$\{a\}$
$\{a, \bigcirc a\}$
$\{a\}$
$\{a, \neg \bigcirc a\}$
$\emptyset$
$\emptyset$
$\{a\}$
$\{a\}$
$\{\neg a, \bigcirc a\}$
$\{\neg a, \neg \bigcirc a\}$

LTL to GNBA



$\{a\}$
$\{a, \bigcirc a\}$
$\{a\}$
$\{a, \neg \bigcirc a\}$
$\emptyset$
$\emptyset$
$\{a\}$
$\{a\}$
$\emptyset$
$\{\neg a, \bigcirc a\}$
$\{\neg a, \neg \bigcirc a\}$
$\emptyset$

LTL to GNBA

- Claim : Runs from a state labelled set B indeed satisfy B
- No good states. All words having a run from a start state are accepted.
- Automaton for $\neg \bigcirc a$ same, except for the start states.