1. $B = \left\{ \begin{bmatrix} 2 \\ -5 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix} \right\}$, $[\mathbf{x}]_B = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$

$$-1 \begin{bmatrix} 2 \\ -5 \end{bmatrix} + 5 \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 5 \end{bmatrix} + \begin{bmatrix} -15 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} -17 \\ 10 \end{bmatrix}$$

2. $B = \left\{ \begin{bmatrix} -2 \\ 3 \\ -6 \end{bmatrix}, \begin{bmatrix} 4 \\ -8 \\ -4 \end{bmatrix}, \begin{bmatrix} -8 \\ -7 \\ 5 \end{bmatrix} \right\}$, $[\mathbf{x}]_B = \begin{bmatrix} -6 \\ -1 \\ -2 \end{bmatrix}$

$$-6 \begin{bmatrix} -2 \\ 3 \\ -6 \end{bmatrix} - 1 \begin{bmatrix} 4 \\ -8 \\ -4 \end{bmatrix} - 2 \begin{bmatrix} -8 \\ -7 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 12 \\ -18 \\ 36 \end{bmatrix} + \begin{bmatrix} -4 \\ 8 \\ 4 \end{bmatrix} + \begin{bmatrix} 16 \\ 14 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} 24 \\ 4 \\ 30 \end{bmatrix}$$

3. Find the coordinate vector $[x_\mathcal{B}]$ of x, where

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -5 \end{bmatrix} \right\}, \quad x = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 2 & -1 & 1 \\ 3 & 0 & 1 & 2 \\ -6 & 8 & -5 & -1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -6 & 4 & -1 \\ 0 & 20 & -11 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & -2/3 & 1/6 \\ 0 & 20 & -11 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1/3 & 2/3 \\ 0 & 1 & -2/3 & 1/6 \\ 0 & 0 & 1 & 5/7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 3/7 \\ 0 & 1 & 0 & 9/14 \\ 0 & 0 & 1 & 5/7 \end{bmatrix}$$

$$\begin{bmatrix} 3/7 \\ 9/14 \\ 5/7 \end{bmatrix}$$

4. Given the following basis $\mathcal{B}$, find the change-of-coordinates matrix from $\mathbb{R}^3$ to $\mathcal{B}$.

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -5 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -5 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 & -3 \\ 3 & 0 & 1 \\ -6 & 8 & -5 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 9+12+3 \\ 9+0-1 \\ -18+16+5 \end{bmatrix} = \begin{bmatrix} 24 \\ 8 \\ 3 \end{bmatrix}$$

5. Give a specific example to show that a plane in $\mathbb{R}^3$ NOT going through the origin is NOT isomorphic to $\mathbb{R}^2$ (hint: think of the viewing plane of a camera model).

6. Determine the dimension of the following set and find a basis for it.

$$\left\{ \begin{bmatrix} 2b+3c \\ a+b-2c \\ 4a+b \\ 3a-b-c \end{bmatrix} : a,b,c \in \mathbb{R} \right\}$$

$$\begin{bmatrix} 0 & 2 & 3 \\ 1 & 1 & -2 \\ 4 & 1 & 6 \\ 3 & -1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -2 \\ 0 & 2 & 3 \\ 4 & 1 & 0 \\ 3 & -1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & -2 \\ 0 & 2 & 3 \\ 0 & -3 & 8 \\ 0 & -4 & 5 \end{bmatrix}$$

Dimension **3** $\mathbb{R}^3$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 1 & -2 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 1 & -2 \\ 0 & 2 & 3 \\ 0 & 0 & 25/2 \\ 0 & 0 & 11 \end{bmatrix}$

7. Determine the dimension of the following set of vectors.

$$\begin{bmatrix} 1 \\ 0 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ -4 \\ 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -8 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -6 \\ 9 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 0 & -3 & 1 & 2 \\ 0 & 1 & -4 & -3 & 1 \\ -3 & 2 & 1 & -8 & -6 \\ 2 & -3 & 6 & 7 & 9 \end{pmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -3 & 1 & 2 \\ 0 & 1 & -4 & -3 & 1 \\ 0 & 2 & -8 & -5 & 0 \\ 0 & -3 & 12 & 5 & 5 \end{bmatrix}$$

$$\downarrow$$

(Dimension 3)

$$\begin{bmatrix} 1 & 0 & -3 & 1 & 2 \\ 0 & 1 & -4 & -3 & 1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 6 & 2 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 0 & -3 & 1 & 2 \\ 0 & 1 & -4 & -3 & 1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -4 & 8 \end{bmatrix}$$

8. not attempted

9. Not attempted

10. Zipped