Assignment 4

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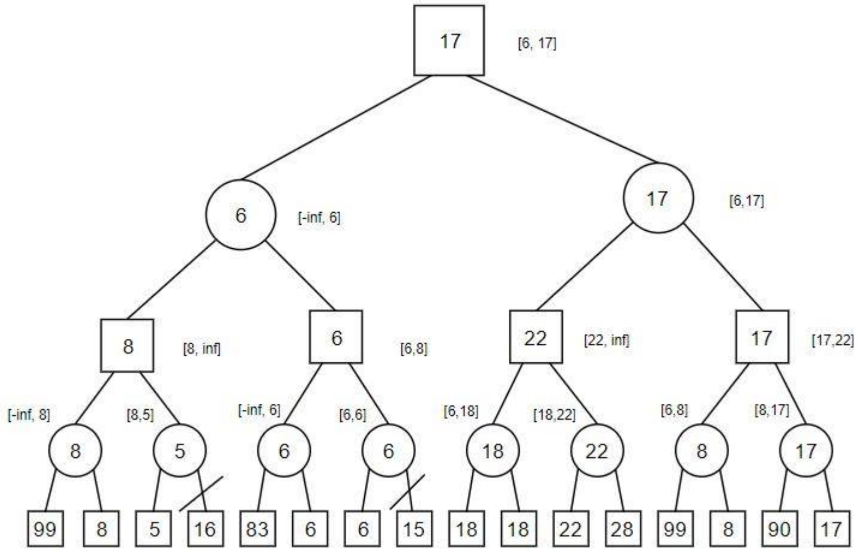
Problem 1

a + b)



c)

Exhaustive minimax and minimax with AB pruning will both choose the right move at the root node because it has the higher score given that both players play optimally every turn. Alpha-beta pruning does not affect the results of the adversarial search. AB pruning simply stops the search on nodes where it is known to not contain a solution. If the nodes are in a good order, the time complexity could be cut in half. Nodes are not always ordered in the best way which is why heuristic are used to reorder the nodes such that you can prune away as much as possible while still keeping a consistent solution.

Problem 2

a.

The variables are $n_{i,j}$ with each variable being part of a domain where $0 < i < 8$. The constraints are that each set of $n_{i,[0:8]}$ and $n_{[0:8],j}$ must be the set [1:9]. Furthermore the grid is divided into 9 subsections of 3x3 boxes. Each of these squares must contain the set [1:9]

b.

Start State: The state where M grid cells are filled with numbers that satisfy the contraint. The start state grid would be the 9x9 grid with 81-M blank cells and M cells filled.

Successor Function: The successor function is placing one number on the grid in place of a blank cell.

Goal Test: The goal test would be to check every cell in the grid, so that no violations are found, and every cell contains a number
Cost Function: The cost function can be defined as the number of possible valid configurations in each blank cell minus the number of possible valid configurations after a cell is filled with a particular number.

The minimum remaining values heuristic would be better for solving this problem. The MRV heuristic is designed to be a "fail-first" heuristic and we are more likely to see a mistake earlier in the process because MRV chooses the node with the fewest possible values, compared to the degree heuristic which chooses the node with the most contraints.

The branching factor at each node is 9, because at each cell we have a total of 9 possible values that can be assigned. The depth to each solution would be the amount of tiles needed to solve the puzzle, so 81-M. The maximum depth of a search space would be 81 because the grid is 81 cells. Since the branching factor is 9, and the maximum depth of the tree is 81 the size of the state space will be 9^{81}

c.

An easy and hard puzzle would be the difference in the amounf of backtracking you need to do in order to solve the problem. A big factor in the amount of backtracking you do is depenent on the heuristic. Because the heuristic determines the cost it can change the order in which tiles are placed, changing the amount of backtracking that occurs. Consider the scenario where we have an optimal heuristic. The difference between a hard and easy puzzle will be determined by the amount of pre assigned numbers M. When M is higher we have more clues to solve the puzzle compared to when M is low and there is a lot of guessing that needs to be done. Therefor an easy puzzle is influenced by a higher M, and a hard puzzle is influenced by lower M.

d.

Algorithm 1: WalkSAT Sudoku Search

```
Data: max-tries, grid with M filled cells
Result: A state where the puzzle is solved
for i := 0 to max-tries do
    if grid is satisfied then
        return grid;
    Choose random cell and change it to random state which maximizes
     heuristic;
    if heuristic is better then
        Change to new grid;
    else
        Continue;
```

Question 3

Proof with resolution inference rule and contradiction

D = Superman is defeated
A = Superman is facing an opponent alone
K = Supermans' opponent is carrying kryptonite
C = Batman coordinates with Lex Luthor
W = Wonder Woman fights with Superman

R1: $D => (A\hat{}K)$
R2: $K => C$
R3: $C => W$
R4:W =>¬A

KB (3-CNF): (¬D ∨ A)ˆ(¬D ∨ K)ˆ(¬K ∨ C)ˆ(¬C ∨ W)ˆ(¬W ∨ ¬A)

KB $\models$ ¬D

(¬D ∨ A)ˆ(¬D ∨ K)ˆ(¬K ∨ C)ˆ(¬C ∨ W)ˆ(¬W ∨ ¬A)ˆ(D)

(A)ˆ(K)ˆ(¬K ∨ C)ˆ(¬C ∨ W)ˆ(¬W ∨ ¬A)ˆ(D)

(A)ˆ(K)ˆ(C)ˆ(W)ˆ(¬W ∨ ¬A)ˆ(D)

(A)ˆ(K)ˆ(C)ˆ(W)ˆ(¬A)ˆ(D)

(A)ˆ(¬A) is a contradiction, therefore KB $\models$ ¬D

Question 4

A)

$\neg P_1 \vee ... \vee \neg P_m \vee Q$

Grouping
$(\neg P_1 \vee ... \vee \neg P_m) \vee Q$

DeMorgan's Law
$\neg(P_1 \wedge ... \wedge P_m) \vee Q$

Implication Law
$(P_1 \wedge ... \wedge P_m) \rightarrow Q$

B)

$\neg P_1 \vee ... \vee \neg P_m \vee Q_1 \vee ... \vee Q_n$

Grouping
$(\neg P_1 \vee ... \vee \neg P_m) \vee (Q_1 \vee ... \vee Q_n)$

DeMorgan's Law
$\neg(P_1 \wedge ... \wedge P_m) \vee (Q_1 \vee ... \vee Q_n)$

Implication Law
$(P_1 \wedge ... \wedge P_m) \rightarrow (Q_1 \vee ... \vee Q_n)$

C)

$(l_1 \vee ... \vee l_{i-1} \vee l_{i+1} \vee ... \vee l_k \vee m_1 \vee ... \vee m_{j-1} \vee m_{j+1} \vee ... \vee m_n)$
$(l_1 \vee ... \vee l_k) \wedge (m_1 \vee ... \vee m_n) \wedge (l_i \leftrightarrow \neg m)$

Question 5

a)

$$P(A,B,C,D,E) = P(A) * P(B) * P(C) * P(D \| A,B) * P(E \| B,C) \tag{5.1}$$
$$= (0.2) * (0.5) * (0.8) * (0.1) * (0.3) = 0.0024 \tag{5.2}$$

b)

$$P(\neg A, \neg B, \neg C, \neg D, \neg E) \tag{5.3}$$

$$= P(\neg A) * P(\neg B) * P(\neg C) * P(\neg D \| \neg A, \neg B) * P(\neg E \| \neg B, \neg C) \tag{5.4}$$

$$= (1 - 0.2) * (1 - 0.5) * (1 - 0.8) * (1 - 0.9) * (1 - 0.2) = 0.0064 \tag{5.5}$$

c)

$$P(\neg A \| B, C, D, E) = \frac{P(\neg A, B, C, D, E)}{P(\neg A, B, C, D, E) + P(A, B, C, D, E)} \tag{5.6}$$

$$= \frac{P(\neg A, B, C, D, E)}{P(B, C, D, E)} \tag{5.7}$$

$$= \frac{P(\neg A) * P(B) * P(C) * P(D \| A, B) * P(E \| B, C)}{\sum_{a}^{A, \neg A} P(a, B, C, D, E)} \tag{5.8}$$

$$= \frac{(1 - 0.2) * (0.5) * (0.8) * (0.6) * (0.3)}{((0.2) * (0.5) * (0.8) * (0.1) * (0.3)) + ((1 - 0.2) * (0.5) * (0.8) * (0.06) * (0.3))} \tag{5.9}$$

$$= \frac{0.0576}{0.6} = 0.96 \tag{5.10}$$

Question 6

a)

$P(Burglary \| JohnCalls = True, MaryCalls = True)$
B = Burglary
E = Earthquake
A = Alarm
J = JohnCalls
M = MaryCalls

$$P(B \| J, M) = \alpha P(B) * \sum_{E} P(E) * \sum_{A} P(A|B, E) * P(J \| A) * P(M \| A) \tag{1}$$

$$f_0(B) = P(B) = [\frac{P(B)}{P(\neg B)}] = [\frac{0.001}{0.999}] \tag{2}$$

$$f_1(E) = P(E) = [\frac{P(E)}{P(\neg E)}] = [\frac{0.002}{0.998}] \tag{3}$$

$$f_2(A, B, E) = P(A \| B, E) = [\frac{P(B)}{P(\neg B}] = [\frac{0.001}{0.999}] \tag{4}$$

$$= [\frac{P(A| \ B, E) P(A \| \neg B, E)}{P(A| \ B, E) P(A \| \neg B, \neg E)}] = [\frac{0.95 \ 0.29}{0.94 \ 0.001}] \tag{5}$$

and

$$[\frac{P(\neg A|\ B,E)P(\neg A\|\neg B,E)}{P(\neg A|\ B,E)P(\neg A\|\neg B,\neg E)}] = [\frac{0.05\ 0.71}{0.06\ 0.999}] \tag{6}$$

$$f_3(A) = P(J\|A) = [\frac{P(J\|A)}{P(J\|\neg A}] = [\frac{0.9}{0.05}] \tag{7}$$

$$f_4(A) = P(M\|A) = [\frac{P(J\|A)}{P(J\|\neg A}] = [\frac{0.7}{0.01}] \tag{8}$$

Combine the factors into a new query

$$P(B\|J,M) = \alpha f_0(B) * \sum_E f_1(E) * \sum_A f_2(A,B,E) * f_3(A) * f_4(A) \tag{9}$$

Perform variable elimination wth $f_2, f_3, and\, f_4$

$$f_5(B,E) = \sum_A f_2(A,B,E) * f_3(A) * f_4(A) \tag{10}$$

$$= f_2(B,E,A) * f_3(A) * f_4(A) + f_2(B,E,\neg A) * f_3(\neg A) * f_4(\neg A) \tag{11}$$

$$= [\frac{0.95\ 0.29}{0.94\ 0.001}] * 0.9 * 0.7 + [\frac{0.05\ 0.71}{0.06\ 0.999}] * 0.5 * 0.1 = [\frac{0.598\ 0.183}{0.592\ 0.001}] \tag{12}$$

Query without $f_2, f_3, and\, f_4$

$$P(B\|J,M) = \alpha f_0(B) * \sum_E f_1(E) * f_5(B,E) \tag{13}$$

Perform variable elimination removing earthquake from $f_1 and\, f_5$

$$f_6(B) = \alpha f_0(B) * \sum_E f_1(E) * f_5(B,E) = f_1(E) * f_5(B,E) + f_1(\neg E) * f_5(B,\neg E) \tag{14}$$

$$= 0.002 * [\frac{0.598}{0.183}] + 0.998 * [\frac{0.592}{0.001}] = [\frac{0.592}{0.001}] \tag{15}$$

Query without earthquake

$$P(B\|J,M) = \alpha f_0(B) * f_6(B) \tag{16}$$

$$= \alpha * [\frac{0.001}{0.999}] * [\frac{0.592}{0.001}] = \alpha * [\frac{0.000592}{0.001}] = (0.284, 0.716) \tag{17}$$

$$P(B\|J,M) = 0.284\, , P(\neg B\|J,M) = 0.716 \tag{18}$$

b)

When using the variable elimination method we have 16 multiplication operations, 2 divison operations, and 7 addition operations for a total of 25 operations. Using the enumeration algorithm we have 18 multiplication operations, 2 divison operations, and 7 addition operations for a total of 27 operations.

c)

Using enumeration to compute $P(X_1 \| X_n = true)$ has $O(2^n)$ time complexity because the depth of the binary tree is n-2, and we need to evaluate the complete binary tree. Using variable elimination requires a polytree, and we can express the network to show that the equation depends only on n-1 variables making the time complexity O(n).

$$P(X_1 \| X_n = true) \tag{1}$$

$$= \alpha P(X) ... \sum_{X_{n-2}} P(X_{n-2} \| X_{n-3}) * \sum_{X_{n-1}} P(X_{n-2} \| X_{n-3}) * P(X_n = true \| X_{n-1}) \tag{2}$$

Question 7

a)

$$P(X_i \| MB(X_i)) = P(X_i \| parents(X_i), Y, Z_{i1}, ..., Z_{nj}) \tag{1}$$

$$= \alpha P(X_i \| parents(X_i), Y, Z_{i1}, ..., Z_{nj}) \times P(Y \| parents(X_i), X_i, Z_{i1}, ..., Z_{nj}) \tag{2}$$

$$= \alpha P(X_i \| parents(X_i)) \times P(Y \| parents(Y_j), X_i, Z_{i1}, ..., Z_{nj}) \tag{3}$$

$$= \alpha P(X_i \| parents(X_i)) \prod_{Y_j \in children(X_i)} P(Y_j \| parents(Y_j)) \tag{4}$$

$$= \alpha P(X \| U_1, ..., U_m) \prod_{Y_i \in children(X_i)} P(Y_i \| Z_{i1} ...) \tag{5}$$

b)

Because we are given Sprinkler = true, and Wet Grass = true, we are left with two boolean variables Cloudy and Rain. This makes for 4 possible states.

c)

$$\begin{array}{cc} & \text{Current State} \\ & \begin{array}{cccc} (c,r) & (c,\neg r) & (\neg c,r) & (\neg c,\neg r) \end{array} \\ \text{Next State} \begin{array}{c} (c,r) \\ (c,\neg r) \\ (\neg c,r) \\ (\neg c,\neg r) \end{array} & \begin{bmatrix} .62 & .21 & .28 & 0 \\ .62 & .12 & 0 & .48 \\ .22 & 0 & .39 & .39 \\ 0 & .02 & .11 & .87 \end{bmatrix} \end{array}$$

Question 8

a)

Expected Net Gain, given no test:
$0.7(\$4000) + 0.3(\$2600) - \$3000 = \$580$

b)

$P(q^+) = 0.7$
$P(q^-) = 0.3$
$P(pass|q^+) = 0.8$
$P(pass|q^-) = 0.35$

$P(pass) = P(pass|q^+)P(q^+) + P(pass|q^-)P(q^-) = 0.8 \cdot 0.7 + 0.35 \cdot 0.3 = 0.665$

$P(q^+|pass) = \frac{P(q^+,pass)}{P(pass)} = \frac{0.8 \cdot 0.7}{0.665} = 0.8421$

$P(q^-|pass) = \frac{P(q^-,pass)}{P(pass)} = \frac{0.35 \cdot 0.3}{0.665} = 0.1579$

$P(q^+|fail) = \frac{P(q^+,fail)}{P(fail)} = \frac{0.2 \cdot 0.7}{0.335} = 0.4179$

$P(q^-|fail) = \frac{P(q^-,fail)}{P(fail)} = \frac{0.65 \cdot 0.3}{0.335} = 0.5820$

c)

Given the car passed inspection, the expected net gain is:
$0.8421(\$4000) + 0.1579(\$2600) - \$3000 - \$100 = \$678.94$

Given the car failed inspection, the expected net gain is:
$0.4179(\$4000) + 0.582(\$2600) - \$3000 - \$100 = \$84.8$

d)

Overall net gain given test:
$0.7 \cdot \$678.94 + 0.3 \cdot \$84.8 = \$500.7$
Therefore, the expected monetary gain is greater when you don't bring the car to the mechanic.