

Homework 1: Analyzing Time Complexity (Big O)

Examine the code and count how many times the operations are executed. Then, detail step by step how to determine the Big O notation.

1.

```
def sum_of_squares(n):
    total = 0
    i = 1
    while i <= n:
        total += i * i
        i += 1
    return total
```

$$\text{Count } (4n+1) + 1 + 1 + 1$$

$$4n+4$$

Operation count:

 $= 1 \text{ operations}$
 $= 1 \text{ operations}$

while

 $\leq, +, *, += : 4 \text{ operations}$

return : 1 operations

$$\text{Count: } 1 + 1 + (4n+1) + 1 = 4n+4$$

Step-by-Step to Find Big O

$$f(n) = 4n+4$$

$$f(n) = 4n + 4n$$

$$f(n) = 8n$$

$$f(n) = O(n), C(8), n_0 = ?$$

$$n \geq n_0$$

$$4n + 4n = 8n$$

$$4n = 4n$$

$$n = 1$$

$$\text{Ans: } f(n) = O(n), C(8), n_0 = 1$$

2.

```
def example1(n):
    i = 0
    while i < n: n+1
        j = 0
        while j <= i: 2
            j += 1
        i += 1
```

$$\text{Count } 1 + (n+1) + 5n$$

$$6n+2$$

Operation count:

$$\begin{array}{l} 1 \\ n+1 \\ 1n \\ 2n \\ 1n \\ 1n \end{array}$$

$$6n+2$$

Step-by-Step to Find Big O

$$f(n) = 6n+2$$

$$f(n) = 6n + 2n$$

$$f(n) = 8n$$

$$f(n) = O(n), C(8), n_0 = ?$$

$$n \geq n_0$$

$$6n + 2n = 8n$$

$$2n = 2n$$

$$n = 1$$

$$\text{Ans: } f(n) = O(n), C(8), n_0 = 1$$

3.

```
def example1(n):
    i = 0 1
    while i < n: n+1
        j = 0 n
        while j < n: n+1
            print(i, j) X
            j += 1 n
        i += 1 n
```

Operation count:

$$\begin{array}{l} 1 \\ n+1 \\ n \\ n+1 \\ n \\ n \end{array} \quad 2n^2 + 4n + 2$$

Count: $1 + (n+1) + n + (2n^2 + n) + n$
 $= 2n^2 + 4n + 2$

Step-by-Step to Find Big O

$$f(n) = 2n^2 + 4n + 2$$

$$f(n) = 2n^2 + 4n^2 + 2n^2$$

$$f(n) = 8n^2$$

$$f(n) = O(n^2), C(8), n_0 = 1$$

$$\begin{aligned} & 1, n, 2n^2 + 4n^2 + 2n^2 = 8n^2 \\ & 2n^2 = 2n^2 \end{aligned}$$

$$n = 1$$

Ans $f(n) = O(n^2), C(8), n_0 = 1$

4.

```
def nested_loop_linear(n):
    i = 0 1
    while i < 3: 4
        j = 0 3
        while j < n: 3(n+1) = 3(4+1) = 15
            print("i =", i, "j =", j)
            j += 1 3(n) = 3x4 = 12
        i += 1 3
```

Operation count:

$$\begin{array}{l} 1 \\ 4 \\ 3 \\ 15 \\ 12 \\ 3 \end{array} \quad 38$$

Count: $1 + 4 + 3 + 15 + 12 + 3$
 $= 38$

Step-by-Step to Find Big O

$$f(n) = O(1), C(38), n_0 = 1$$

5.

```

def nested_loop_linear():
    i = 1
    total = 0
    while i < 3:
        j = 1
        while j < 10:
            total += j * i
            j += 1
        i += 1
    return total

```

Count $4 + 74 + 3 = 84$

Operation count:

1
1
3
2
10
 $10 \times 9 = 90$
9
2
1

Step-by-Step to Find Big O

$$f(n) = O(1), C(84), n_0 = 1$$

6.

```

def cubic_function(n):
    total = 0
    i = 0
    while i < n:
        j = 0
        while j < n:
            k = 0
            while k < n:
                total += 1
            k += 1
        j += 1
    i += 1
    return total

```

Count = $1 + 1 + (n+1) + n + n + (n^2 + n) + n^2 + n^2 +$
 $+ (n^3 + n^2) + n^3 + n^3 + 1$
 $= 3n^3 + 4n^2 + 4n + 4$

Operation count:

1
1
 $(n+1)$
 $n^2 + n$
 n^2
 $(n^2 + n^2)$
 n^3
 n^3
 n
1

Step-by-Step to Find Big O

$$\begin{aligned} f(n) &= 3n^3 + 4n^2 + 4n + 4 \\ f(n) &= 3n^3 + 4n^3 + 4n^3 + 4n^3 \\ f(n) &= 15n^3 \\ f(n) &= O(n^3), C(15), n_0 = ? \\ n_0 &= ? \end{aligned}$$

$$\begin{aligned} 3n^3 + 4n^3 + 4n^3 + 4n^3 &= 16n^3 \\ 16n^3 &= 12n^3 \end{aligned}$$

Ans. $O(n^3), C(15), n_0 = 1$

```

def extra_challenge(n):
    total = 0
    i = 1
    while i <= n: n+1
        j = 1 n
        while j <= i: 3n+1xn
            total += 1
            j += 1 3n2+n
        i += 1 n
    return total 1

```

$$\begin{aligned}
\text{Count} &= 1 + 1 + (n+1) + n + (3n^2+n) + n+1 \\
&= 3n^2 + 4n + 4
\end{aligned}$$

Operation count:

1
 1
 n+1
 n
 n+1 } $\times n$ loop
 n
 n
 $3n^2 + 4n + 4$

Step-by-Step to Find Big O

$$\begin{aligned}
f(n) &= 3n^2 + 4n + 4 \\
f(n) &= 3n^2 + 4n^2 + 4n^2 \\
f(n) &= 11n^2 \\
f(n) &= O(n^2), C(s), n_0 = 9
\end{aligned}$$

$$\begin{aligned}
n_0 &= 3n^2 + 4n^2 + 4n^2 = 11n^2 \\
8n^2 &= 8n^2 \\
n &= 1 \\
\text{Total } O(n^2), C(s), n_0 = 1
\end{aligned}$$