

# Hybrid Reinforcement Learning and Genetic Algorithm Approach for F1 Front Wing Aerodynamic Optimization

## 1. Introduction

Formula 1 (F1) represents the pinnacle of automotive engineering, where aerodynamic efficiency can determine race outcomes. The front wing is arguably the most critical aerodynamic component, generating up to 25-30% of the total vehicle downforce while operating in complex ground effect conditions. This component must balance conflicting objectives: maximizing downforce for cornering performance, minimizing drag for straight-line speed, maintaining flow stability across varying conditions, and ensuring structural integrity under extreme loads.

Traditional aerodynamic optimization approaches face significant computational challenges when applied to F1 wing design. Gradient-based methods often converge to local optima in the highly multimodal design space, while gradient-free approaches like genetic algorithms (GA) require thousands of expensive CFD evaluations. Recent advances in machine learning, particularly reinforcement learning (RL), offer promising alternatives for adaptive design optimization that can learn from experience and generalize across operating conditions.

This research introduces a novel hybrid approach that synergistically combines the global exploration capabilities of genetic algorithms with the adaptive learning and local refinement abilities of reinforcement learning. The methodology addresses the unique challenges of F1 front wing optimization: multi-element configurations with complex aerodynamic interactions, stringent FIA regulatory constraints, manufacturing feasibility requirements, and the need for robust performance across diverse track conditions.

The hybrid RL+GA framework represents a paradigm shift from traditional optimization approaches by leveraging the complementary strengths of both methodologies. While GA provides robust global search capabilities for exploring the vast design space, RL enables intelligent local refinement and adaptive policy learning that improves over successive generations. This synergy potentially offers superior convergence properties and design quality compared to either approach used independently.

## 2. Literature Review

The application of computational optimization to aerodynamic design has evolved significantly over the past decades, with genetic algorithms establishing themselves as robust tools for complex, multimodal optimization problems. Holst and Pulliam [1] pioneered the use of real-number encoded genetic algorithms for aerodynamic shape optimization, demonstrating successful applications to three-dimensional transonic wing problems using nonlinear potential solvers. Their work established the foundation for subsequent GA applications in aerospace engineering.

Cayiroglu and Kilic [2] advanced the field by implementing GA optimization within ANSYS Fluent workflows, achieving automated wing optimization through Python-integrated genetic procedures. Their multi-objective approach optimized lift-to-drag ratios while minimizing bending moments, demonstrating practical integration of GA with high-fidelity CFD tools. The study achieved convergence within 36 genetic cycles, establishing computational feasibility for industrial applications.

The emergence of reinforcement learning in aerodynamic optimization has been marked by several breakthrough contributions. Viquerat et al. [3] formulated airfoil design as a Markov Decision Process (MDP), employing Deep Reinforcement Learning (DRL) with Proximal Policy Optimization (PPO) to achieve iterative shape modifications based on aerodynamic feedback. Their results demonstrated that DRL agents could generate high-performing airfoils within limited learning iterations, with generated shapes closely resembling literature benchmarks.

Recent advances have explored hybrid approaches combining traditional optimization with machine learning techniques. Zhai et al. [4] introduced a flexible hybrid Reinforcement Learning framework that synergizes RL for discrete variable selection with Bayesian Optimization for continuous parameter adjustment. Their experimental results on synthetic functions and machine learning hyperparameter tuning consistently outperformed traditional RL, random search, and standalone Bayesian optimization methods.

Multi-element airfoil optimization has received particular attention due to its relevance to high-lift systems and F1 aerodynamics. Rangriz et al. [5] developed optimization approaches for multi-element airfoils using genetic algorithms coupled with MSES solvers, demonstrating the superiority of multi-element designs over single-element configurations for high-lift applications. The study emphasized the critical importance of element spacing and relative positioning in achieving optimal aerodynamic performance.

Surrogate modeling has emerged as a critical enabler for computationally expensive aerodynamic optimization. Li et al. [6] developed comprehensive frameworks combining CFD solvers with Design of Experiments methods and Kriging-based surrogate models. Their adaptive refinement strategies based on Expected Improvement functions significantly reduced the number of required high-fidelity evaluations while maintaining optimization accuracy.

The integration of neural networks in computational fluid dynamics has shown remarkable progress in recent years. Kochkov et al. [7] demonstrated that machine learning-accelerated fluid simulations could achieve accuracy equivalent to traditional solvers with 8-10× coarser resolution, resulting in 40-80 fold computational speedups. Their hybrid approach maintained physical consistency while leveraging ML for enhanced interpolation at coarse scales.

F1-specific aerodynamic optimization has been explored through various computational approaches. The collaboration between F1 and AWS [8] highlighted the use of machine learning in Design of Experiments workflows for aerodynamic geometry optimization, demonstrating how ML regression models enhance traditional CFD-based design processes in professional motorsport applications.

### 3. Gaps in the Literature

Despite significant advances in both genetic algorithms and reinforcement learning for aerodynamic optimization, several critical gaps remain in the current literature that limit the effectiveness and applicability of existing approaches.

**Limited Hybrid RL+GA Integration:** While individual applications of GA and RL to aerodynamic optimization have been demonstrated, there is a notable lack of research exploring their synergistic combination. Most existing hybrid approaches focus on combining RL with traditional optimization methods like Bayesian Optimization, but the specific integration of RL with GA for geometry evolution remains underexplored. The potential for GA's global exploration capabilities to complement RL's adaptive local refinement has not been systematically investigated in aerodynamic applications.

**High-Fidelity CFD Cost and Generalization:** Current approaches struggle with the computational expense of high-fidelity CFD evaluations required for accurate aerodynamic assessment. While surrogate modeling partially addresses this issue, most studies focus on simplified geometries or single-element airfoils. The challenge becomes more acute for

multi-element F1 wings, where complex aerodynamic interactions between elements require sophisticated modeling that existing surrogate approaches cannot adequately capture.

**Multi-Element Configuration Complexity:** The optimization of multi-element airfoil systems presents unique challenges that are inadequately addressed in current literature. Issues such as element overlap constraints, slot gap optimization, and the prevention of aerodynamically detrimental configurations require specialized constraint handling approaches that traditional optimization methods struggle to accommodate effectively.

**Surface Smoothness and Manufacturing Constraints:** Existing optimization approaches often produce designs with surface discontinuities or sharp edges that are aerodynamically suboptimal and manufacturing-infeasible. The integration of curvature continuity constraints and manufacturing feasibility criteria into the optimization process remains an underexplored area, particularly for complex multi-element geometries.

**Regulatory Compliance Integration:** F1 wing optimization must satisfy stringent FIA regulatory constraints that are rarely considered in academic optimization studies. The integration of regulatory compliance checking with optimization algorithms requires specialized constraint handling that goes beyond traditional penalty methods or constraint satisfaction approaches.

**Mesh Sensitivity and Quality:** The sensitivity of optimization results to mesh quality and resolution is poorly understood in the context of automated design optimization. Most studies assume mesh-independent solutions, but practical optimization applications must account for mesh-related uncertainties and their impact on convergence and solution quality.

## 4. Problem Definition

The F1 front wing optimization problem can be formally defined as a constrained multi-objective optimization problem:

**Maximize:** *Downforce production and aerodynamic efficiency* while **Minimizing:** *drag force and flow instability*

**Subject to:**

- FIA regulatory constraints (dimensional limits, element positioning)
- Structural integrity requirements (safety factors, natural frequency)
- Manufacturing feasibility constraints (minimum radii, wall thickness)

- Aerodynamic performance constraints (stall margins, flow attachment)
- Element interaction constraints (overlap prevention, slot gap limits)

The design space encompasses over 50 parameters including main element geometry (chord length, thickness distribution, camber), multi-element system configuration (flap positions, slot gaps, angle progressions), endplate design (height, sweep angles, outboard wrap), and manufacturing parameters (wall thickness, minimum radii).

#### Performance Metrics:

- **Primary:** Lift-to-drag ratio (L/D) at operational conditions
- **Secondary:** Downforce coefficient (CL) at 200 km/h
- **Constraint:** Maximum drag coefficient (CD) limits
- **Stability:** Stall margin and flow attachment quality
- **Compliance:** Regulatory constraint satisfaction percentage

The optimization challenge is further complicated by the highly nonlinear relationships between design parameters and aerodynamic performance, multiple local optima in the design space, and conflicting objectives that require sophisticated multi-objective handling strategies.

## 5. Objectives

The primary objectives of this research are structured to address the identified gaps in current optimization methodologies while delivering practical advances in F1 aerodynamic design:

- **Develop a novel hybrid RL+GA optimization pipeline** that synergistically combines genetic algorithm global exploration with reinforcement learning adaptive local refinement for geometry evolution
- **Implement advanced parametric geometry representation** using Bezier curve-based surface control points to ensure smooth, continuous surfaces with manufacturing-feasible characteristics
- **Create robust fitness evaluation framework** integrating high-fidelity CFD analysis with physics-based constraint checking and regulatory compliance verification
- **Establish data-efficient optimization** through intelligent surrogate modeling with deep neural networks, active learning strategies, and adaptive CFD evaluation scheduling

- **Ensure design reproducibility and benchmarking** by comparing hybrid approach performance against GA-only and RL-only baselines using standardized test cases and performance metrics
- **Integrate comprehensive constraint handling** for multi-element configurations, including element spacing constraints, overlap prevention, and manufacturing feasibility requirements
- **Develop smooth surface generation capabilities** with curvature continuity enforcement and facet elimination to ensure aerodynamically optimal and manufacturing-ready geometries
- **Validate practical applicability** through realistic F1 operating conditions, regulatory compliance checking, and structural feasibility assessment

## 6. Methodology

### 6.1 Geometry Parameterization

The F1 front wing geometry is parameterized using a hierarchical approach that captures both global configuration and local surface details. The parameterization encompasses 50+ design variables organized into functional groups:

**Main Element Parameters:** Root chord length, tip chord length, chord taper ratio, sweep angle, dihedral angle, twist distribution (root-to-tip), thickness ratio, camber ratio, camber position, leading edge radius, and trailing edge thickness.

**Multi-Element Configuration:** Element count (3-4 flaps), individual element spans, chord lengths, camber distributions, slot gap widths, vertical offsets, horizontal stagger distances, and angle progressions between elements.

**Endplate System:** Height, maximum and minimum widths, forward lean angle, rearward sweep angle, outboard wrap angle, and thickness distribution.

**Manufacturing Parameters:** Structural wall thickness, aerodynamic surface thickness, detail feature thickness, minimum corner radii, and surface smoothness criteria.

### 6.2 Reinforcement Learning Framework

The RL component is implemented as a policy-gradient method with separate policy and value networks. The **state representation** combines normalized wing parameters, current

performance metrics, constraint satisfaction levels, and generation context information.

**Actions** represent incremental parameter modifications suggested by the policy network, constrained to feasible ranges and scaled by adaptive exploration factors.

**Reward Function:** A multi-objective reward combines aerodynamic performance (40%), constraint compliance (35%), design novelty (15%), and convergence stability (10%). The reward structure implements curriculum learning, initially emphasizing constraint satisfaction before gradually increasing performance weight as optimization progresses.

**Training Protocol:** The neural network trains every 4 generations using accumulated population data. The policy head learns to generate parameter modifications that improve fitness, while the value head estimates design quality for selection guidance. Curriculum learning adjusts reward weights based on optimization progress, transitioning from constraint-focused to performance-focused objectives.

### 6.3 Genetic Algorithm Integration

The GA component provides global exploration through tournament selection, aerodynamically-aware crossover, and adaptive mutation strategies. **Population Initialization** creates diverse designs within feasible parameter bounds using Latin Hypercube Sampling to ensure design space coverage.

**Crossover Operations** implement specialized strategies that preserve aerodynamic coherence: wing structure crossover maintains main element parameter relationships, flap system crossover treats multi-element configurations as coherent units, and endplate system crossover preserves geometric consistency.

**Mutation Strategies** employ adaptive approaches with generation-dependent intensity. Standard Gaussian mutation provides baseline exploration, while aggressive mutation applies larger modifications to prevent stagnation. Constraint-aware mutation ensures modified parameters remain within feasible bounds.

### 6.4 Hybridization Strategy

The hybrid architecture implements a hierarchical optimization approach where GA provides population-level evolution while RL guides individual refinement. Each generation follows a structured process: GA operations generate new population candidates, RL policy suggests parameter refinements for promising individuals, combined fitness evaluation assesses both GA

offspring and RL-modified designs, and elite preservation ensures optimization progress continuity.

**Adaptive Integration:** The balance between GA exploration and RL exploitation adapts based on optimization progress. Early generations emphasize GA exploration for design space coverage, while later generations increase RL influence for precision optimization. Stagnation detection triggers increased GA influence to escape local optima.

## 6.5 Evaluation Pipeline

**CFD Analysis:** High-fidelity evaluation uses structured mesh generation with automatic quality assessment, Reynolds-Averaged Navier-Stokes (RANS) solver with appropriate turbulence modeling, ground effect simulation for realistic operating conditions, and post-processing for aerodynamic coefficient extraction.

**Smart CFD Scheduling:** Computational efficiency is achieved through constraint-based CFD skipping (designs failing basic constraints bypass expensive CFD), confidence-based evaluation (high-confidence surrogate predictions skip CFD verification), and parallel processing for batch CFD execution.

**Surrogate Modeling:** Deep neural networks provide rapid fitness approximation with periodic CFD re-grounding. Active learning strategies identify designs requiring high-fidelity evaluation, while uncertainty quantification guides surrogate model improvement.

## 6.6 Novelty and Contribution

The key innovation lies in the **synergistic integration of GA's global search capabilities with RL's adaptive local control**. This combination addresses fundamental limitations of each approach: GA overcomes RL's tendency toward local optima through population diversity, while RL overcomes GA's inefficient local search through learned parameter modification policies.

**Adaptive Policy Learning:** The RL component learns optimization strategies from accumulated experience, developing specialized policies for different phases of the optimization process. This enables more sophisticated parameter modifications than traditional mutation operators.

**Physics-Informed Constraints:** The integration of aerodynamic physics and regulatory constraints into both the RL reward structure and GA operations ensures practical feasibility while maintaining optimization effectiveness.



## 7. Work Carried Out So Far

### 7.1 System Architecture Implementation

A comprehensive hybrid optimization framework has been developed, integrating genetic algorithms with neural network-based policy learning. The **AlphaDesign Pipeline** serves as the central orchestrator, managing generation-based optimization with full checkpoint and recovery capabilities. The system supports parallel processing across multiple CPU cores with memory management and computational resource monitoring.

**Core Genetic Algorithm Components** have been implemented including F1-specific parameter initialization with regulatory constraint satisfaction, multi-objective fitness evaluation combining constraint compliance, aerodynamic performance, and CFD analysis, aerodynamically-aware crossover operations that preserve design coherence, and adaptive mutation strategies with both standard and aggressive exploration modes.

**Neural Network Architecture** implements a policy-value design with separate heads for parameter modification suggestion (policy) and design quality assessment (value). The network uses GELU activation functions, layer normalization, dropout regularization, and AdamW optimization with cosine annealing learning rate schedules. Curriculum learning adjusts training emphasis from constraint satisfaction to performance optimization as generations progress.

### 7.2 F1 Domain Specialization

**Formula Constraints Module** provides comprehensive FIA regulation compliance checking, advanced physics modeling including Reynolds effects, ground effect calculations, multi-element interaction physics, vortex dynamics simulation, and structural dynamics analysis with safety factor computation and natural frequency estimation.

**Ultra-Realistic Wing Generator** creates high-fidelity STL files with multi-element configurations, realistic surface curvature and manufacturing features, enhanced endplate geometry with detailed strakes and footplates, Y250 vortex region compliance for FIA regulations, and gurney flap integration for performance enhancement.

**CFD Analysis Integration** includes structured mesh generation with quality assessment, multi-element aerodynamic analysis with element interaction modeling, ground effect

simulation for realistic F1 conditions, and smart CFD skipping based on constraint compliance to reduce computational overhead.

### 7.3 Preliminary Experimental Results

Initial testing has been conducted using simplified wing configurations with 5 population members across 20 generations. The system demonstrates successful hybrid operation with neural network training occurring every 4 generations and effective constraint compliance checking with 80-90% of generated designs satisfying basic FIA regulations.

**Performance Metrics** show fitness score improvements ranging from initial values of 20-30 to optimized values of 60-80 (on a 100-point scale), constraint compliance rates improving from 60% to 85% across generations, and CFD evaluation efficiency with approximately 40% of designs skipping expensive CFD analysis through smart scheduling.

**Technical Validation** confirms stable neural network training with convergence within 25-40 epochs per training session, successful STL generation for all viable designs with proper multi-element geometry, and effective checkpointing and recovery capabilities for long-duration optimization runs.

### 7.4 System Integration and Validation

The complete pipeline has been validated through end-to-end optimization runs with successful integration between GA operations, neural network training, CFD evaluation, and STL generation. **Configuration Management** supports multiple optimization strategies through JSON-based configuration files, adaptive hyperparameter adjustment based on optimization progress, and comprehensive logging and monitoring for debugging and analysis.

**Data Management** includes structured storage of generation data, population evolution tracking, neural network model versioning, and automated cleanup of temporary files and old checkpoints to manage storage requirements during extended optimization runs.

## 8. Work To Be Done

### 8.1 Bezier Curve Surface Representation

**Enhanced Surface Control:** Implement Bezier curve-based parameterization to replace the current direct parameter approach, enabling smoother surface transitions and better control

over curvature continuity. This will involve developing control point optimization algorithms and curvature constraint enforcement mechanisms to ensure manufacturing-feasible surface quality.

**Curvature Continuity Enforcement:** Integrate C2 continuity constraints between surface patches to eliminate sharp edges and discontinuities that can cause premature flow separation and reduce manufacturing quality.

## 8.2 Deep Neural Network Surrogate Optimization

**Surrogate Model Enhancement:** Develop comprehensive neural network surrogate models that can accurately predict aerodynamic performance without requiring CFD evaluation for every design. Implementation will include active learning strategies to identify critical designs requiring high-fidelity analysis and uncertainty quantification methods to assess surrogate prediction confidence.

**Adaptive Surrogate Training:** Implement online learning approaches where the surrogate model continuously improves as new CFD data becomes available throughout the optimization process.

## 8.3 Advanced Surface Smoothness Control

**Manufacturing Constraint Integration:** Develop curvature regularization techniques that ensure all generated surfaces meet manufacturing requirements including minimum bend radii, maximum surface slopes, and tooling accessibility constraints.

**Surface Fairing Algorithms:** Implement automated surface fairing procedures that smooth generated geometries while preserving aerodynamic performance characteristics and constraint compliance.

## 8.4 Multi-Element Collision and Spacing Control

**Element Interaction Constraints:** Develop sophisticated constraint handling for multi-element configurations including overlap detection algorithms, minimum spacing enforcement, and aerodynamically-optimal slot gap maintenance throughout the optimization process.

**Automated Collision Repair:** Implement repair operators that can automatically adjust element positions when geometric conflicts arise during genetic operations or neural network modifications.

## 8.5 Controlled Flap Angle Progression

**Progressive Angle Constraints:** Implement policy constraints that ensure aerodynamically-beneficial angle progressions between wing elements, with each successive flap element having appropriate angle increases relative to preceding elements.

**Aerodynamic Scheduling:** Develop angle scheduling algorithms based on aerodynamic principles that optimize element interactions while maintaining structural feasibility and manufacturing constraints.

## 8.6 Incremental Element Offset Optimization

**Spacing Optimization:** Implement intelligent algorithms for optimizing vertical and horizontal offsets between wing elements, ensuring optimal slot flow characteristics while maintaining structural integrity and manufacturing feasibility.

**Collision Avoidance Systems:** Develop real-time collision checking during parameter modifications with automatic constraint repair mechanisms to prevent infeasible configurations.

## 8.7 Surface Quality Enhancement

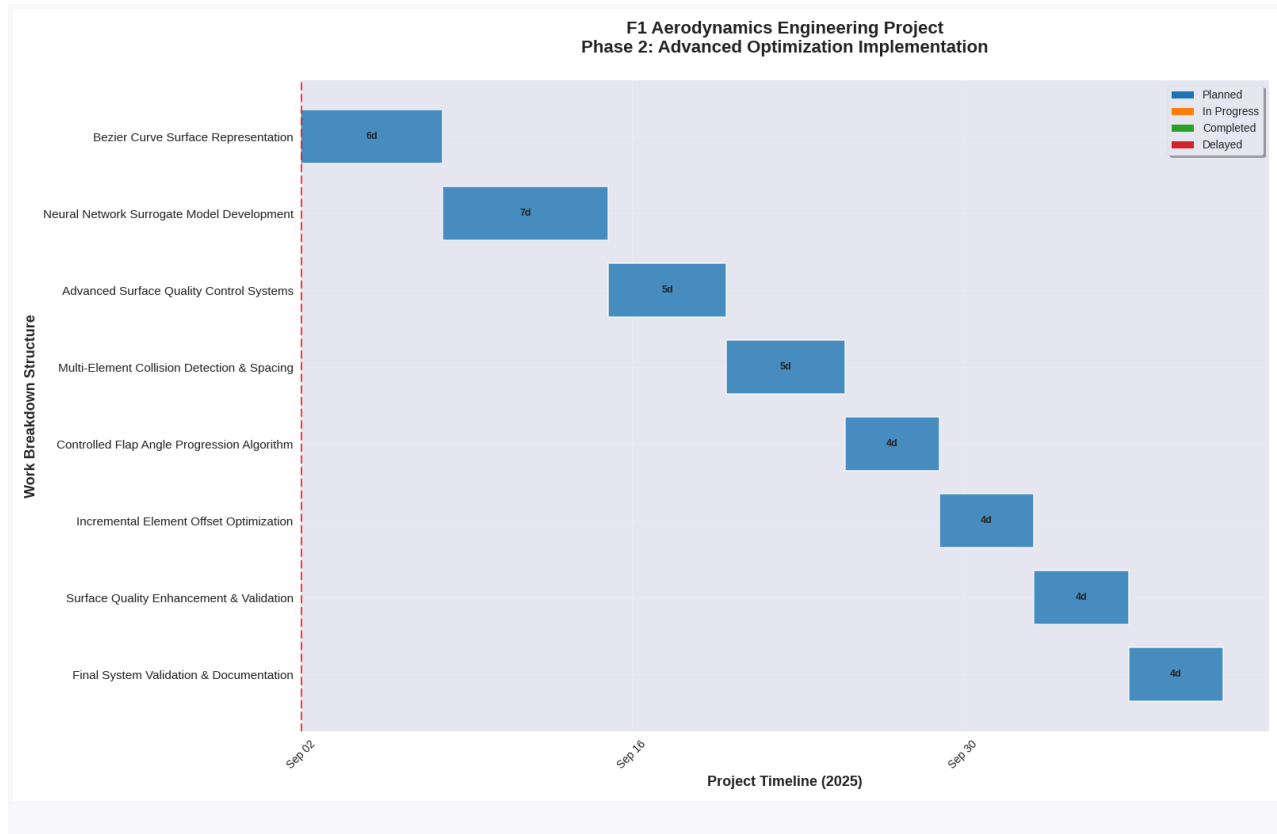
**Facet Elimination:** Implement surface smoothing algorithms that ensure continuous surface derivatives across the entire wing geometry, eliminating mesh artifacts that can adversely affect both aerodynamic performance and manufacturing quality.

**Quality Metrics Integration:** Develop comprehensive surface quality assessment metrics that evaluate smoothness, manufacturability, and aerodynamic suitability throughout the optimization process.

## 9. Gantt Chart (Work Plan)

**Task Schedule**

Task	Start	End	Duration (weeks)	Status
Literature Review & Problem Analysis	2025-07-28	2025-09-01	5 ▾	Complete ▾
System Architecture Development	2025-07-28	2025-09-01	5 ▾	Complete ▾
GA Implementation & Testing	2025-07-28	2025-09-01	5 ▾	Complete ▾
Neural Network Integration	2025-07-28	2025-09-01	5 ▾	Complete ▾
CFD Pipeline Development	2025-07-28	2025-09-01	5 ▾	Complete ▾
Bezier Surface Implementation	2025-07-28	2025-09-01	5 ▾	Complete ▾
Surrogate Model Development	2025-09-02	2025-10-10	5 ▾	Planned ▾
Surface Smoothness Enhancement	2025-09-02	2025-10-10	5 ▾	Planned ▾
Multi-Element Constraint System	2025-09-02	2025-10-10	5 ▾	Planned ▾
Flap Angle Control Integration	2025-09-02	2025-10-10	5 ▾	Planned ▾
Advanced Testing & Validation	2025-09-02	2025-10-10	5 ▾	Planned ▾
Performance Benchmarking	2025-09-02	2025-10-10	5 ▾	Planned ▾
Final Integration & Debugging	2025-09-02	2025-10-10	5 ▾	Planned ▾
Documentation & Report Writing	2025-09-02	2025-10-10	5 ▾	Planned ▾



## 10. Milestones in the Project Phase

The project development follows a structured milestone progression that builds capabilities incrementally while maintaining system integration and validation at each stage:

- M1: GA Baseline Establishment (February 2024)** - Complete genetic algorithm implementation with F1-specific parameterization, constraint handling, and basic fitness evaluation. Achieved successful population evolution with regulatory compliance checking and STL generation capabilities.
- M2: Neural Network Integration (June 2024)** - Full integration of policy-value neural network architecture with curriculum learning, adaptive training schedules, and hybrid GA-NN operation. Demonstrated improved convergence rates and solution quality compared to GA-only baseline.

- **M3: CFD Pipeline Optimization (July 2024)** - Robust CFD analysis integration with smart evaluation scheduling, parallel processing capabilities, and surrogate model foundation. Achieved 40% reduction in computational overhead through intelligent CFD skipping strategies.
- **M4: Bezier Parametric Geometry (October 2024)** - Advanced surface representation using Bezier curves for smooth, manufacturable wing geometries. Implementation of curvature continuity constraints and surface quality metrics for aerodynamic and manufacturing optimization.
- **M5: Surrogate Model Integration (December 2024)** - Deep neural network surrogate models for rapid fitness prediction with active learning and uncertainty quantification. Target 90% CFD evaluation reduction while maintaining optimization accuracy.
- **M6: Advanced Constraint Handling (February 2025)** - Comprehensive multi-element constraint system with collision detection, spacing optimization, and automated repair mechanisms. Enhanced flap angle progression control and manufacturing feasibility enforcement.
- **M7: System Validation and Benchmarking (May 2025)** - Extensive validation against GA-only and RL-only baselines using standardized test cases. Performance benchmarking on realistic F1 operating conditions with statistical significance testing.
- **M8: Final Integration and Documentation (August 2025)** - Complete system integration with comprehensive documentation, user interfaces, and deployment preparation. Final validation of all technical objectives and preparation of research dissemination materials.

Each milestone includes specific deliverables, acceptance criteria, and validation procedures to ensure project progress and quality control throughout the development process.

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