Foundations of Data Science № 1

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Problem 1

Vary V_1 from 0 to 100 in steps of 2 (i.e. V_1=0,2,4, . . . , 100) and calculate I_1, I_2 and I_3 as a function of increasing V_1 by solving the system with the standard backslash (MATLAB) command (e.g., $\mathrm{x} = \mathrm{A} \backslash \mathrm{y}$ gives $x = A^{-1}y$). Save your results in a matrix of 3 columns and 51 rows where the first, second and third column are I_1, I_2 and I_3respectively.

```
R1 = 20;
R2 = 15;
R3 = 25;
R4 = 20;
R5 = 30;
R6 = 40;
V2 = 0;
V3 = 200;
% Varying V1 from 0 to 100 in steps of 2
V1_values = 0:2:100;
num_values = length(V1_values);
% Initialize a matrix to store results
currents = zeros(num_values, 3); % 3 columns for I1, I2, I3

for i = 1:num_values
    V1 = V1_values(i);
    % Define the coefficient matrix
    A = [R6+R1+R2, -R1, -R2;
         -R1, R3+R4+R1, -R4;
         -R2, -R4, R5+R4+R2];
    % Define the constant matrix
    B = [V1; V2; V3];
    % Solve the system of equations using the backslash operator
    currents(i, :) = A \ B;
    %  save the results to a file
    file_name = 'Direct_Solution.csv';
    % Write the currents matrix to a CSV file
    csvwrite(file_name, currents);
end
```

1 Problem 2

Repeat part (a), but now solve it with two additional methods: Jacobi Iterations and Gauss-Seidel Iterations. For the iteration methods, begin with the guess $(I_1, I_2, I_3) = (0, 0, 0)$. This will give you two additional matrices of size 3 columns by 51 rows for the Jacobi and Gauss-Seidel respectively.

Timely update iteration results, Jacobi's iteration form(1)

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left[b_i - \sum_{j<i} a_{ij} x_j^{(k)} - \sum_{j>i} a_{ij} x_j^{(k)} \right], \quad i = 1, 2, \ldots, n \tag{1}$$

Can be changed to Gauss-Seidel form(2):

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left[b_i - \sum_{j<i} a_{ij} x_j^{(k+1)} - \sum_{j>i} a_{ij} x_j^{(k)} \right], \quad i = 1, 2, \ldots, n \tag{2}$$

It can be seen that the Gauss-Seidel method uses the results $\{x_1, x_2, \ldots, x_{i-1}\}$ that have been calculated in the current step, that is, formula (2) The part of $\sum_{j<i} a_{ij} x_j^{(k+1)}$.The form of Jacobi iteration and Gauss-Seidel iteration are almost the same. However, Gauss-Seidel takes advantage of the latest iteration results in a timely manner and is intuitively more efficient than Jacobi.

```
% Perform Jacobi iteration
for iter = 1:max_iterations
    x_new = zeros(3, 1);

    for j = 1:3
        sum_term = 0;
        for k = 1:3
            if k ~= j
                sum_term = sum_term + A(j, k) * x(k);
            end
        end
        x_new(j) = (B(j) - sum_term) / A(j, j);
    end

    % Check for convergence
    if norm(x_new - x) < tolerance
        break;
    end

    x = x_new;
end
```

```
% Perform Gauss iteration
for iter = 1:max_iterations
    x_new = zeros(3, 1);

    for j = 1:3
        sum_term = 0;
        for k = 1:3
            if k ~= j
                if k < j
                    sum_term = sum_term + A(j, k) * x_new(k);
                elseif k > j
                    sum_term = sum_term + A(j, k) * x(k);
                end
            end
        end
        x_new(j) = (B(j) - sum_term) / A(j, j);
    end

    % Check for convergence
    if norm(x_new - x) < tolerance
        break;
    end

    x = x_new;
end
```

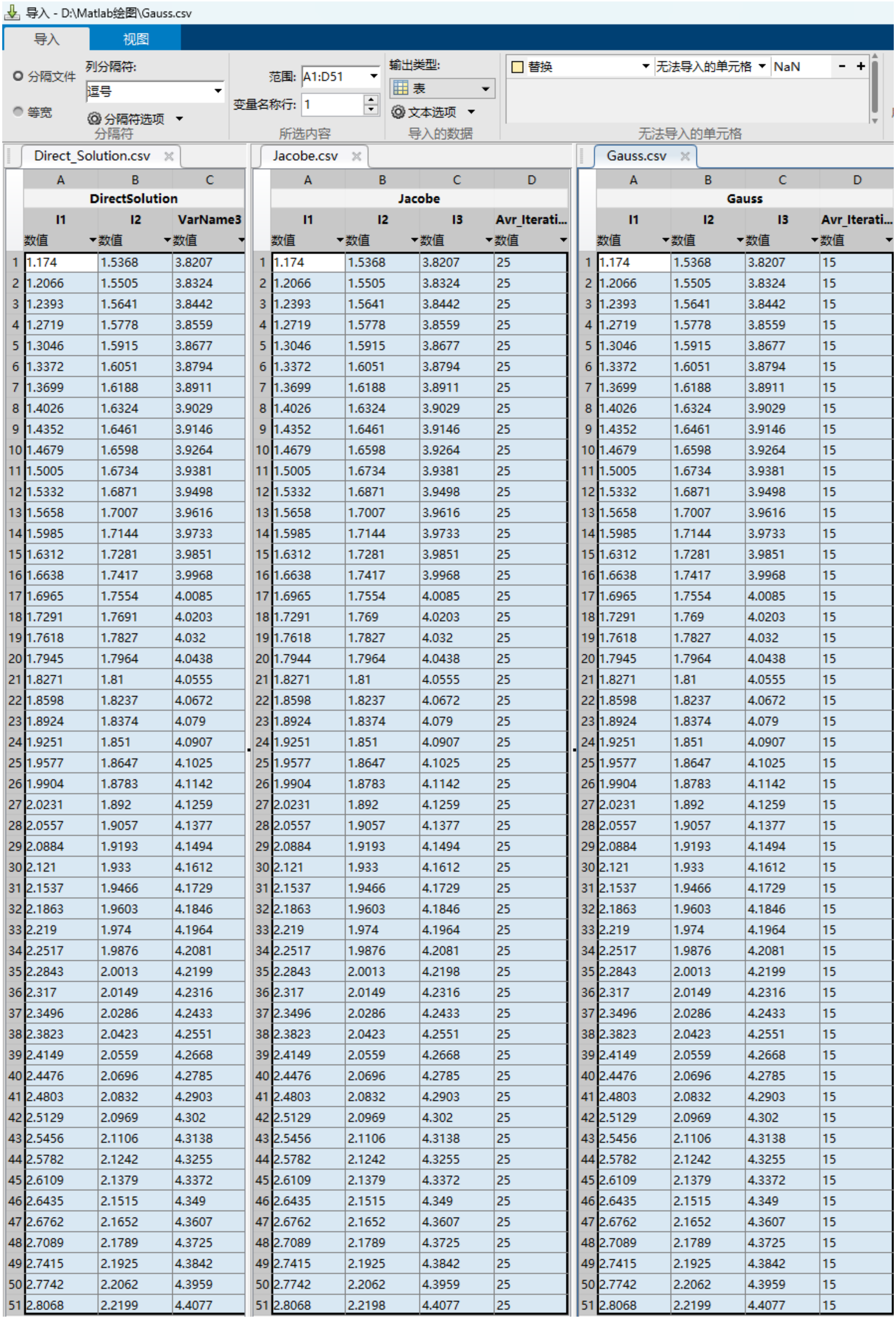
The Results are as follows:

2 Problem 3

For the two iteration methods, what is the average number of iterations required to solve the given equation with accuracy 10^{-6}. The accuracy constraint should be based upon looking at the norm of the difference between successive iterations, $i.e.\ \|x_{n+1} - x_n\|_\infty < 10^{-6}$. Save the two answers (for Jacobi first and Gauss-Seidel second) as a row vector with two components.

Result: It is observed that the average number of Jacobian iterations is **25** and the average number of Gaussian iterations is **15**.

And I feel weird that the number of iterations is the same for each iteration.



Direct_Solution.csv

	A	B	C
	DirectSolution		
	I1	I2	VarName3
	数值	数值	数值
1	1.174	1.5368	3.8207
2	1.2066	1.5505	3.8324
3	1.2393	1.5641	3.8442
4	1.2719	1.5778	3.8559
5	1.3046	1.5915	3.8677
6	1.3372	1.6051	3.8794
7	1.3699	1.6188	3.8911
8	1.4026	1.6324	3.9029
9	1.4352	1.6461	3.9146
10	1.4679	1.6598	3.9264
11	1.5005	1.6734	3.9381
12	1.5332	1.6871	3.9498
13	1.5658	1.7007	3.9616
14	1.5985	1.7144	3.9733
15	1.6312	1.7281	3.9851
16	1.6638	1.7417	3.9968
17	1.6965	1.7554	4.0085
18	1.7291	1.7691	4.0203
19	1.7618	1.7827	4.032
20	1.7945	1.7964	4.0438
21	1.8271	1.81	4.0555
22	1.8598	1.8237	4.0672
23	1.8924	1.8374	4.079
24	1.9251	1.851	4.0907
25	1.9577	1.8647	4.1025
26	1.9904	1.8783	4.1142
27	2.0231	1.892	4.1259
28	2.0557	1.9057	4.1377
29	2.0884	1.9193	4.1494
30	2.121	1.933	4.1612
31	2.1537	1.9466	4.1729
32	2.1863	1.9603	4.1846
33	2.219	1.974	4.1964
34	2.2517	1.9876	4.2081
35	2.2843	2.0013	4.2199
36	2.317	2.0149	4.2316
37	2.3496	2.0286	4.2433
38	2.3823	2.0423	4.2551
39	2.4149	2.0559	4.2668
40	2.4476	2.0696	4.2785
41	2.4803	2.0832	4.2903
42	2.5129	2.0969	4.302
43	2.5456	2.1106	4.3138
44	2.5782	2.1242	4.3255
45	2.6109	2.1379	4.3372
46	2.6435	2.1515	4.349
47	2.6762	2.1652	4.3607
48	2.7089	2.1789	4.3725
49	2.7415	2.1925	4.3842
50	2.7742	2.2062	4.3959
51	2.8068	2.2199	4.4077

Jacobe.csv

	A	B	C	D
	Jacobe			
	I1	I2	I3	Avr_Iterati...
	数值	数值	数值	数值
1	1.174	1.5368	3.8207	25
2	1.2066	1.5505	3.8324	25
3	1.2393	1.5641	3.8442	25
4	1.2719	1.5778	3.8559	25
5	1.3046	1.5915	3.8677	25
6	1.3372	1.6051	3.8794	25
7	1.3699	1.6188	3.8911	25
8	1.4026	1.6324	3.9029	25
9	1.4352	1.6461	3.9146	25
10	1.4679	1.6598	3.9264	25
11	1.5005	1.6734	3.9381	25
12	1.5332	1.6871	3.9498	25
13	1.5658	1.7007	3.9616	25
14	1.5985	1.7144	3.9733	25
15	1.6312	1.7281	3.9851	25
16	1.6638	1.7417	3.9968	25
17	1.6965	1.7554	4.0085	25
18	1.7291	1.769	4.0203	25
19	1.7618	1.7827	4.032	25
20	1.7944	1.7964	4.0438	25
21	1.8271	1.81	4.0555	25
22	1.8598	1.8237	4.0672	25
23	1.8924	1.8374	4.079	25
24	1.9251	1.851	4.0907	25
25	1.9577	1.8647	4.1025	25
26	1.9904	1.8783	4.1142	25
27	2.0231	1.892	4.1259	25
28	2.0557	1.9057	4.1377	25
29	2.0884	1.9193	4.1494	25
30	2.121	1.933	4.1612	25
31	2.1537	1.9466	4.1729	25
32	2.1863	1.9603	4.1846	25
33	2.219	1.974	4.1964	25
34	2.2517	1.9876	4.2081	25
35	2.2843	2.0013	4.2198	25
36	2.317	2.0149	4.2316	25
37	2.3496	2.0286	4.2433	25
38	2.3823	2.0423	4.2551	25
39	2.4149	2.0559	4.2668	25
40	2.4476	2.0696	4.2785	25
41	2.4803	2.0832	4.2903	25
42	2.5129	2.0969	4.302	25
43	2.5456	2.1106	4.3138	25
44	2.5782	2.1242	4.3255	25
45	2.6109	2.1379	4.3372	25
46	2.6435	2.1515	4.349	25
47	2.6762	2.1652	4.3607	25
48	2.7089	2.1789	4.3725	25
49	2.7415	2.1925	4.3842	25
50	2.7742	2.2062	4.3959	25
51	2.8068	2.2198	4.4077	25

Gauss.csv

	A	B	C	D
	Gauss			
	I1	I2	I3	Avr_Iterati...
	数值	数值	数值	数值
1	1.174	1.5368	3.8207	15
2	1.2066	1.5505	3.8324	15
3	1.2393	1.5641	3.8442	15
4	1.2719	1.5778	3.8559	15
5	1.3046	1.5915	3.8677	15
6	1.3372	1.6051	3.8794	15
7	1.3699	1.6188	3.8911	15
8	1.4026	1.6324	3.9029	15
9	1.4352	1.6461	3.9146	15
10	1.4679	1.6598	3.9264	15
11	1.5005	1.6734	3.9381	15
12	1.5332	1.6871	3.9498	15
13	1.5658	1.7007	3.9616	15
14	1.5985	1.7144	3.9733	15
15	1.6312	1.7281	3.9851	15
16	1.6638	1.7417	3.9968	15
17	1.6965	1.7554	4.0085	15
18	1.7291	1.769	4.0203	15
19	1.7618	1.7827	4.032	15
20	1.7945	1.7964	4.0438	15
21	1.8271	1.81	4.0555	15
22	1.8598	1.8237	4.0672	15
23	1.8924	1.8374	4.079	15
24	1.9251	1.851	4.0907	15
25	1.9577	1.8647	4.1025	15
26	1.9904	1.8783	4.1142	15
27	2.0231	1.892	4.1259	15
28	2.0557	1.9057	4.1377	15
29	2.0884	1.9193	4.1494	15
30	2.121	1.933	4.1612	15
31	2.1537	1.9466	4.1729	15
32	2.1863	1.9603	4.1846	15
33	2.219	1.974	4.1964	15
34	2.2517	1.9876	4.2081	15
35	2.2843	2.0013	4.2199	15
36	2.317	2.0149	4.2316	15
37	2.3496	2.0286	4.2433	15
38	2.3823	2.0423	4.2551	15
39	2.4149	2.0559	4.2668	15
40	2.4476	2.0696	4.2785	15
41	2.4803	2.0832	4.2903	15
42	2.5129	2.0969	4.302	15
43	2.5456	2.1106	4.3138	15
44	2.5782	2.1242	4.3255	15
45	2.6109	2.1379	4.3372	15
46	2.6435	2.1515	4.349	15
47	2.6762	2.1652	4.3607	15
48	2.7089	2.1789	4.3725	15
49	2.7415	2.1925	4.3842	15
50	2.7742	2.2062	4.3959	15
51	2.8068	2.2199	4.4077	15

Figure 1: Results