

**Homework 1 Due: 11:59pm, 5 December, 2023**

**Problem 1**

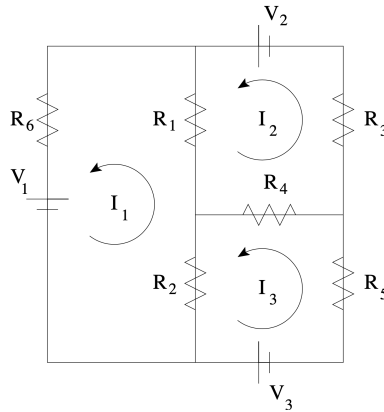
Consider the circuit depicted in the figure. By using the two following facts: (i) the voltage drop across a resistor is  $V = IR$  and (ii) the sum of all voltage drops in a closed loop sum to zero. The currents  $I_1$ ,  $I_2$  and  $I_3$  are determined from the  $3 \times 3$  system:

$$R_6 I_1 + R_1(I_1 - I_2) + R_2(I_1 - I_3) = V_1$$

$$R_3 I_2 + R_4(I_2 - I_3) + R_1(I_2 - I_1) = V_2$$

$$R_5 I_3 + R_4(I_3 - I_2) + R_2(I_3 - I_1) = V_3$$

where  $R_1 = 20$ ,  $R_2 = 15$ ,  $R_3 = 25$ ,  $R_4 = 20$ ,  $R_5 = 30$ ,  $R_6 = 40$ ,  $V_2 = 0$ ,  $V_3 = 200$ , and  $V_1$  will be variable. In this form, the associated matrix is strictly diagonal dominant.



- (a) Vary  $V_1$  from 0 to 100 in steps of 2 (i.e.  $V_1 = 0, 2, 4, \dots, 100$ ) and calculate  $I_1$ ,  $I_2$  and  $I_3$  as a function of increasing  $V_1$  by solving the system with the standard backslash (MATLAB) command (e.g.,  $x = A \setminus y$  gives  $x = A^{-1}y$ ). Save your results in a matrix of 3 columns and 51 rows where the first, second and third column are  $I_1$ ,  $I_2$  and  $I_3$  respectively.

- (b) Repeat part (a), but now solve it with two additional methods: Jacobi Iterations and Gauss-Seidel Iterations. For the iteration methods, begin with the guess  $(I_1, I_2, I_3) = (0, 0, 0)$ . This will give you two additional matrices of size 3 columns by 51 rows for the Jacobi and Gauss-Seidel respectively.
- (c) For the two iteration methods, what is the **average** number of iterations required to solve the given equation with accuracy  $10^{-6}$ . The accuracy constraint should be based upon looking at the norm of the difference between successive iterations, i.e.  $\|x_{n+1} - x_n\|_\infty < 10^{-6}$ . Save the two answers (for Jacobi first and Gauss-Seidel second) as a row vector with two components.