Homework 3 Due: before/at the last lecture

Problem 1

Consider the following curve fitting problem:

- data: $\{x_k, y_k\}_{k=1}^n$ are given in the `data1.csv`, `data2.csv` files;
- model: $f(x) = ax + b\exp(cx) + d$ with parameters a, b, c, d;
- cost function: mean-square error $E_2 = \frac{1}{2}\sum_{k=1}^n [f(x_k) - y_k]^2$.

Now solve the following problems:

- Suppose that a, c have been given as $a = 0, c = 2$ (i.e. remove term ax and treat $\exp(cx)$ as $\exp(2x)$). Provided with `data1.csv`, use the least-square method to solve this curve fitting problem. You need to first derive the least-square solution in mathematics, and then computing the solution from data using MATLAB/Python.
  (Hints: introduce new data variable $z = \exp(cx)$.)
- Consider the curve fitting problem with unknown a, c, d and known $b = 1$ (i.e. the second term is $\exp(cx)$), using dataset `data2.csv`. Write **your own gradient descent method** to solve the optimization problem $\min_{a,c,d} E_2$, using backtracking line search and initial guess $a = 0.1, c = 2, d = 2.5$, to obtain the optimal values of a, c, d.

Note: your homework solutions include the codes, results, and any essential mathematical paperwork (like computing the least-square solution, the gradient).